# A Fast Solver For Differential Equations

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# Abstract

Several methods are presented to solve a class of differential equations which are discretized to a linear system and kept in memory using a sparse structure. I have implemented, tested and compared both classic and more recent methods for solving the resulting linear equation. The main results are about the convergence rates of the Multigrid method and of the Preconditioned Conjugate Gradients method. An empasize is put on the setting of the parameters and of how they affect the solver overall. Some interesting results are presented towards the end regarding the comparison of different parameters selections, including an adaptive method.

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Einstein's paper: [?]

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# Sample Title

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# Appendix B

# Sample Title

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