

A Fast Solver For Differential Equations



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Abstract

plenty of waffle, plenty of waffle, plenty of waffle, plenty of waffle, plenty
of waffle, plenty of waffle, plenty of waffle, plenty of waffle.

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Einstein's paper: [?]

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Sample Title

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Appendix A

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Appendix B

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