Superconductivity: Electron-phonon Coupling and Unconventional Pairing with Repulsive Interaction

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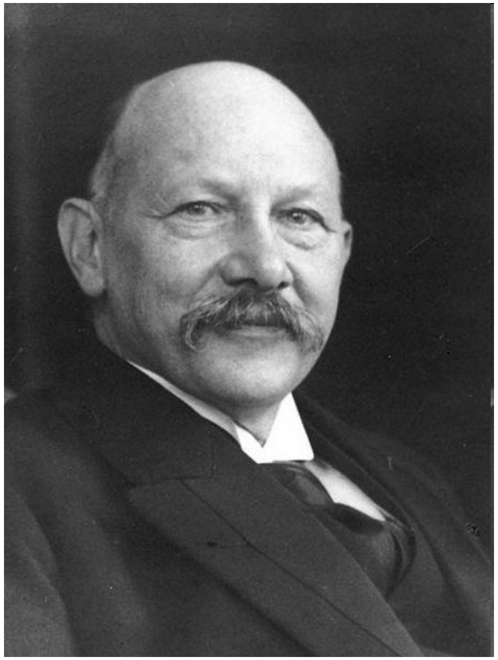
National Research Nuclear University "MEPhI“

MEPhI was founded in 1942 as the Moscow Mechanical Institute of Munitions

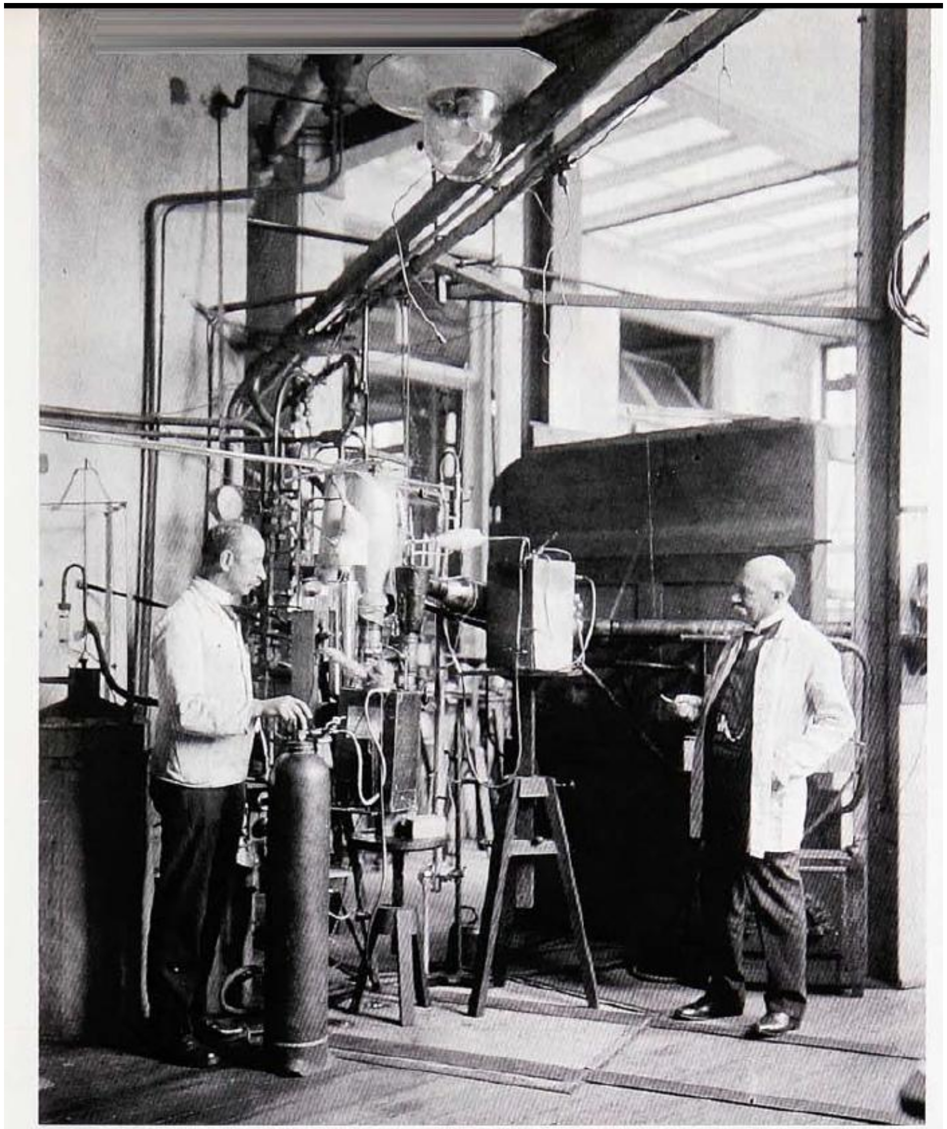
1 Historical overview
2 Some theoretical concepts and e-ph coupling.
3 Superconductivity with repulsion.



Heike Kamerlingh Onnes (21 September 1853 – 21 February 1926) was a Dutch physisist. His scientific career was spent exploring extremely cold refrigeration techniques and the associated phenomena.

• 1908 he was the first physicist to liquify helium. Reducing the pressure of helium allows to reach 0.9K. Birthday of low temperature physics.
• 1911 The resistivity of mercury drops unexpectedly to zero at 4.2K. Birthday of superconductivity.
• 1913 Nobel prize in physics.
• In the field of superconductivity and superfluidity it was awarded 7 Nobel prices, 6 of them in the last 40 years (the last in 2003).

Few historical facts



Kamerlingh Onnes and his assistant
Gerrit Jahn Flim in Cryogenic Lab.

1 Problem of obtaining the required amount of helium gas. They manage to get 360 liters of helium gas from the sand from North Carolina.

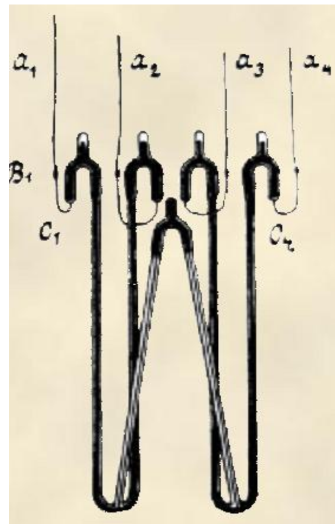
2 Measurements of resistivity of pure metals at low T. First platinum and gold. Then mercury was chosen because of easy purification.

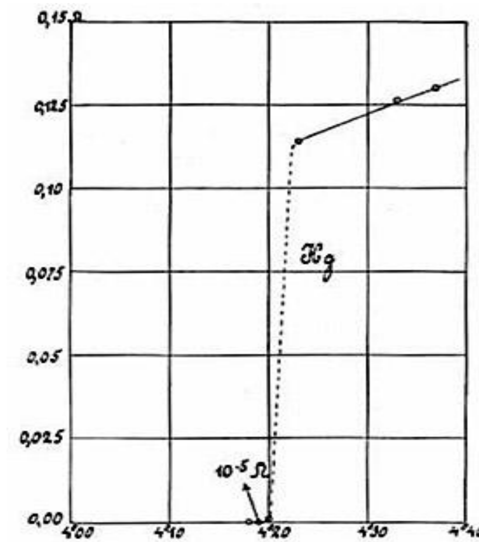
3 Zero resistance has been found. But it was attributed to a short circuit somewhere in the cryostat.

Because of deficit of helium the pressure in the cryostat was low, to ensure that helium does not escape from the cryostat.

Boiling point of He is 4.2 K. Critical temperature of mercury is 4.15K

The finite resistivity was found, because student was not able to keep the pressure low!





Gilles Holst, who made the first measurements of superconductivity.

Very soon it was discovered superconductivity in tin and lead. Then it was discovered, that external magnetic field as well as superconducting current suppresses superconductivity. It was attempts to measure isotope effect and Meissner -Ochsenfeld effect(1933).

“There is no doubt that very pure gold and platinum are superconducting at low temperatures.”

London Equations

$$\boxed{\Lambda \nabla \times \mathbf{j}_s = -\mathbf{H}/c, \qquad \Lambda = \frac{m}{e^2 n_s}}$$

The meaning of London theory:

$$\frac{\partial \mathbf{v}_s}{\partial t} = -(\mathbf{v}_s \nabla)\mathbf{v}_s + \frac{e}{m}\mathbf{E} + \frac{e}{mc}\mathbf{v}_s \times \mathbf{H} \equiv$$

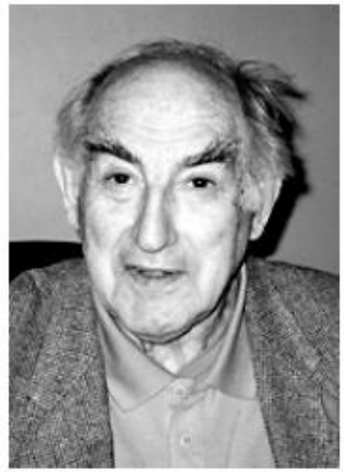
$$\frac{e}{m}\mathbf{E} - \nabla \frac{\mathbf{v}_s^2}{2} + \mathbf{v}_s \times (\nabla \times \mathbf{v}_s + \frac{e}{mc}\mathbf{H})$$

$$\boxed{\lambda^2 = \frac{mc^2}{4\pi e^2 n_s}}$$

London penetration depth

The Nobel Prize in Physics 2003

"for pioneering contributions to the theory of superconductors and superfluids"



Alexei A. Abrikosov

1/3 of the prize

USA and Russia

Argonne National Laboratory
Argonne, IL, USA

Vitaly L. Ginzburg

1/3 of the prize

Russia

P.N. Lebedev Physical Institute
Moscow, Russia

Anthony J. Leggett

1/3 of the prize

United Kingdom and USA

University of Illinois
Urbana, IL, USA

Ginzburg-Landau Theory

(Nobel prize 2003).

$\Psi(\mathbf{r})$ Order parameter – scalar complex function

$$F_s = F_0 + \frac{H^2}{8\pi} + \frac{1}{4m}\left| -i\hbar\nabla\Psi - \frac{2e}{c}\mathbf{A}\Psi \right|^2 + \alpha|\Psi|^2 + \frac{\beta}{2}|\Psi|^4$$

$$\frac{1}{4m}\left(-i\hbar\nabla - \frac{2e}{c}\mathbf{A}\right)^2\Psi + \alpha\Psi + \frac{\beta}{2}|\Psi|^2\Psi = 0$$

$$\mathbf{j}_s = -\frac{ie\hbar}{2m}\left(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*\right) - \frac{2e}{mc}|\Psi|^2\mathbf{A}$$

! Interesting story about charge 2e in GL theory!

$$\xi^2 = \frac{\hbar^2}{4m|\alpha|}$$

-Coherence length

$$\kappa = \frac{\lambda}{\xi} < \frac{1}{\sqrt{2}}$$

- Type I superconductors

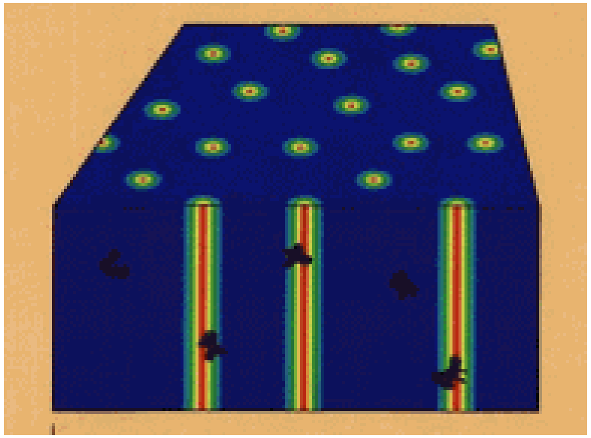
$$\lambda^2 = \frac{mc\beta}{8\pi e^2|\alpha|}$$

-Penetration depth

$$\kappa > \frac{1}{\sqrt{2}}$$

- Type II superconductors

Vortices in in type II superconductors Abrikosov phase



Magnetic field

Magnetic field penetrates to the superconductor in the form of tubes. Outside of this tubes field is absent

Microscopic theory of superconductivity (1956)

The Nobel Prize in Physics 1972

"for their jointly developed theory of superconductivity, usually called the BCS-theory"

John Bardeen

1/3 of the prize

USA

University of Illinois
Urbana, IL, USA

Leon Neil Cooper

1/3 of the prize

USA

Brown University
Providence, RI, USA

John Robert Schrieffer

1/3 of the prize

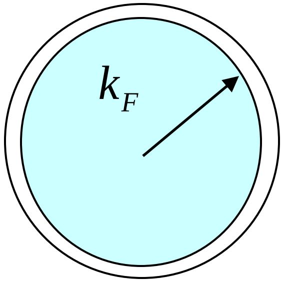
USA

University of Pennsylvania
Philadelphia, PA, USA

Microscopic theory of superconductivity (1956)

Superconducting transition is the phase transition of the second order which is described by the complex order parameter:

$$\Delta_{s_1 s_2}(\vec{r}_1, \vec{r}_2) \propto \left\langle \psi_{s_1}(\vec{r}_1) \psi_{s_2}(\vec{r}_2) \right\rangle \qquad \Delta_{s_1 s_2}(\vec{R}, \vec{r}) = -\Delta_{s_2 s_1}(\vec{R}, -\vec{r})$$



$$\varepsilon(k) = \sqrt{\xi_k^2 + |\Delta_k|^2} \qquad \frac{2\Delta}{k_B T_c} = 3.5 - 8$$

Elementary excitations have gapped spectrum.

The ground state is described by the order parameter which have the meaning of the quantum mechanical wave function.

The properties of the system at large energies are not changed with respect to that of the normal state.

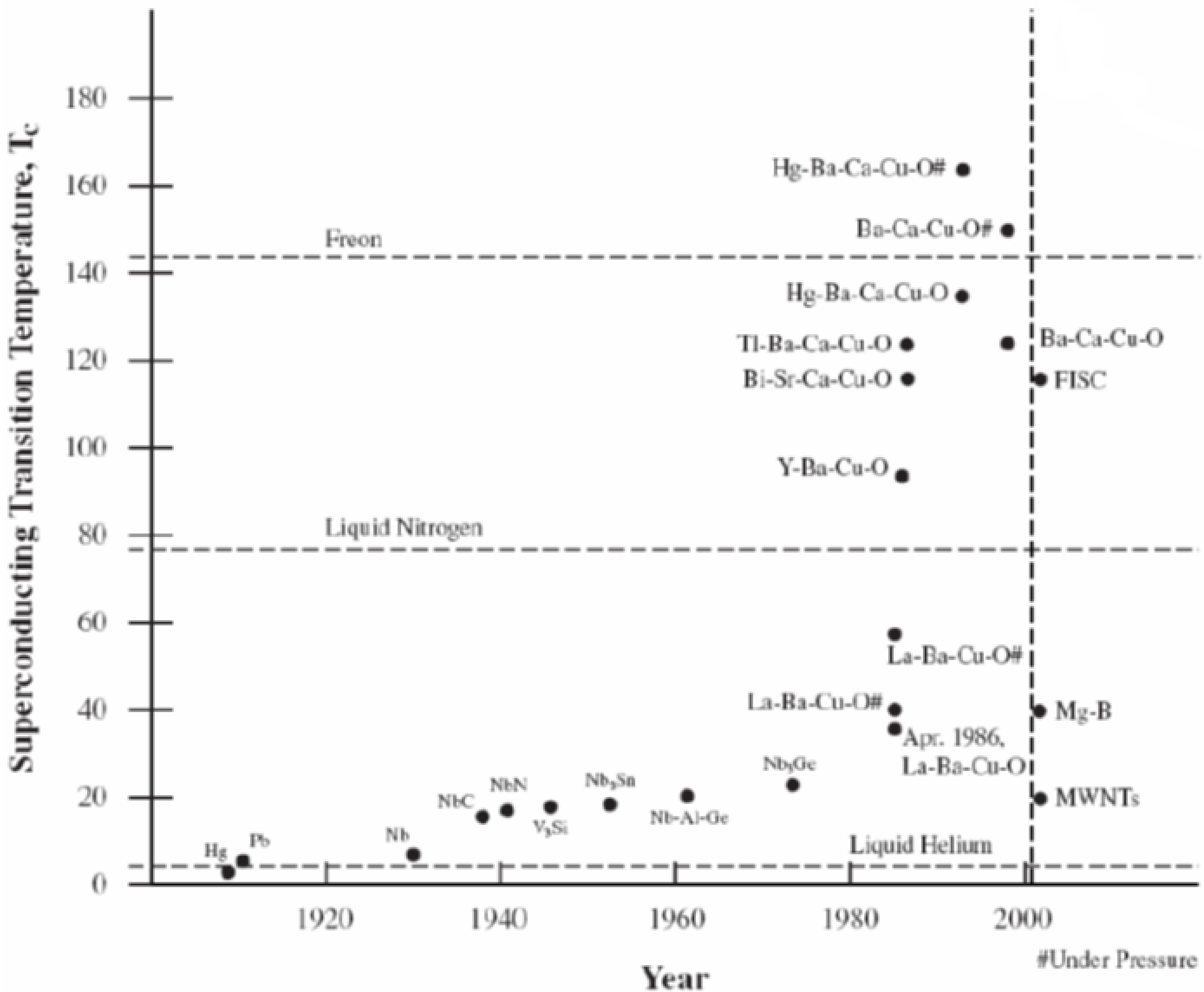
Critical temperature is determined by the interactions in the normal metallic state, i.e. by Coulomb pseudopotential and coupling to phonons.

Discovery of high temperature superconductivity (1986)

The Nobel Prize in Physics 1987

"for their important break-through in the discovery of superconductivity in ceramic materials"

J. Georg Bednorz	K. Alexander Müller
1/2 of the prize	1/2 of the prize
Federal Republic of Germany	Switzerland
IBM Zurich Research Laboratory Rüschlikon, Switzerland	IBM Zurich Research Laboratory Rüschlikon, Switzerland



Superconducting Transition Temperature, T_c
180
160
140
120
100
80
60
40
20
0
Freon
Liquid Nitrogen
Liquid Helium
Hg-Ba-Ca-Cu-O#
Ba-Ca-Cu-O#
Hg-Ba-Ca-Cu-O
Tl-Ba-Ca-Cu-O
Ba-Ca-Cu-O
Bi-Sr-Ca-Cu-O
FISC
Y-Ba-Cu-O
La-Ba-Cu-O#
La-Ba-Cu-O#
Mg-B
Apr. 1986,
La-Ba-Cu-O
MWNTs
Nb_3Ge
Nb_3Sn
NbN
NbC
V_3Si
Nb-Al-Ge
Nb
Hg
Pb
1920
1940
1960
1980
2000
Year
#Under Pressure

Eliashberg theory of superconductivity

Coulomb pseudopotential: $\mu_c = V(q \approx 0)N(0)$

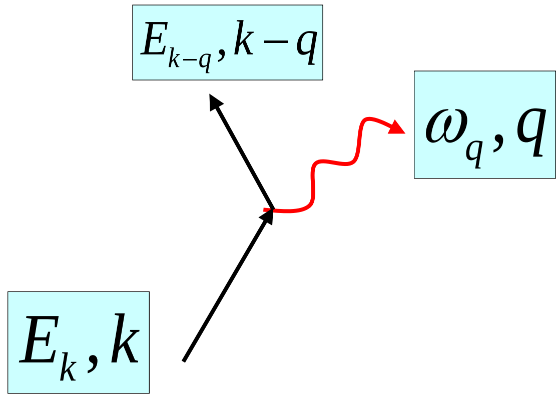
This parameter is dimensionless and tells us if the correlations are weak or strong in the metal.

$$\mu_c^* = \frac{\mu_c}{[1+\mu_c \ln(E_F/\omega_{ph})]}$$

Coulomb pseudopotential is renormalized due to retardation effect

Electron-phonon interaction in metals

$$H_{e-ph} = \sum_{k,q,\alpha} g(k,q) u_{q,\alpha} c^{\dagger}_{k+q} c_k$$



Isotropic case, nothing is momentum dependent:

$$Q(\omega,\xi,\xi') = \frac{1}{\hbar N(0)} \sum_{k,q} g(k,q)^2 \delta(\omega-\omega_q)\delta(E_k - E_F - \xi)\delta(E_{k+q} - E_F - \xi')$$

Electron-phonon interaction in superconductors

$$\boxed{Q(\omega,\xi,\xi')}$$ is complicated function

$$E_k, E_{k+q} \approx E_F >> \omega_q \approx \omega_D \Rightarrow \boxed{Q(\omega,\xi,\xi') \approx Q(\omega,0,0) \equiv \alpha^2 F(\omega)}$$

$\boxed{\alpha^2 F(\omega)}$ is spectral function of electron-phonon interaction or Eliashberg function. This function describes electron-phonon interaction in superconductors.

$$\boxed{\lambda \left\langle \omega^n \right\rangle = 2\int_0^\infty d\omega \frac{\alpha^2 F(\omega)\omega^n}{\omega}}$$ n-th moment of Eliashberg function

$$\boxed{\lambda = 2\int_0^\infty d\omega \frac{\alpha^2 F(\omega)}{\omega}}$$ Dimensionless electron-phonon coupling constant

Superconducting metals

There are two dimensionless parameters to characterize superconducting metals:

$$\boxed{\lambda = 2\int_0^{\infty} d\omega \frac{\alpha^2 F(\omega)}{\omega}}$$

Electron-phonon coupling constant

$$\boxed{\mu_c^* = \frac{\mu_c}{[1+\mu_c \ln(E_F/\omega_{ph})]}}$$

Coulomb pseudopotential, reduced due to retardation effects

$$\Downarrow$$

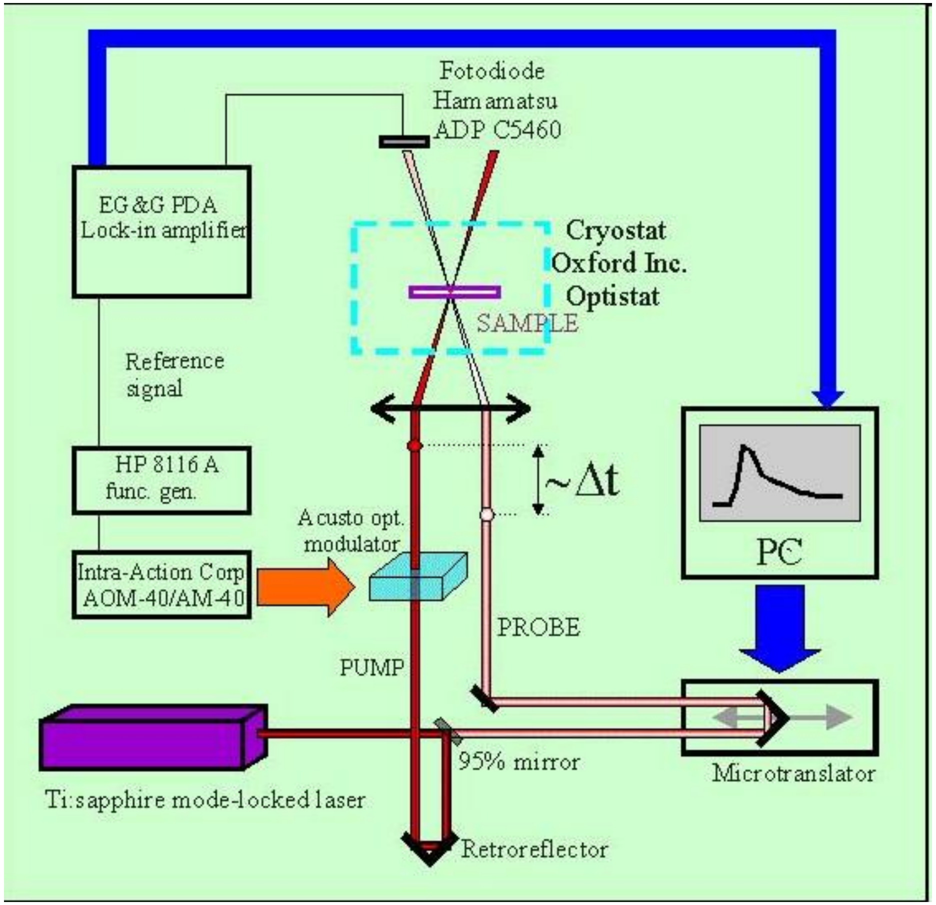
$$\boxed{\Delta \rightarrow \Delta(\omega)}$$

Kinetics of hot electrons in the normal states.
Pump-probe spectroscopy.

$$\frac{\Delta R}{R}(t); \qquad \frac{\Delta T}{T}(t);$$

$$\tau_i(T,P), A_i(T,P)$$

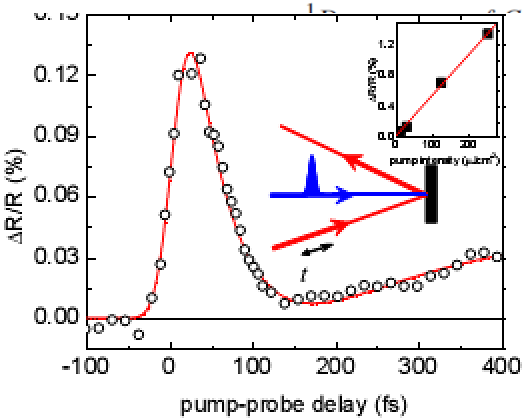
$$\mu_c; \qquad \lambda\langle\omega^n\rangle$$



PRL **105,** 257001 (2010) PHYSICAL REVIEW LETTERS week ending 17 DECEMBER 2010

Electron-Phonon Coupling in High-Temperature Cuprate Superconductors Determined from Electron Relaxation Rates

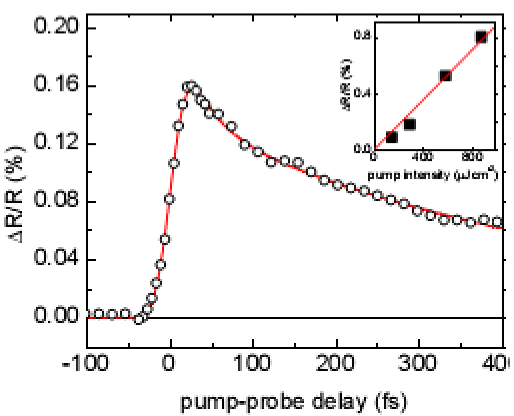
C. Gadermaier,[1,*] A. S. Alexandrov,[2,1] V. V. Kabanov,[1] P. Kusar,[1] T. Mertelj,[1] X. Yao,[3] C. Manzoni,[4] D. Brida,[4] G. Cerullo,[4] and D. Mihailovic[1]



YBaCuO

$$\lambda\langle\omega^2\rangle = 400 \pm 100 meV^2 \rightarrow \lambda = 0.6 \pm 0.2 \quad \text{YBaCuO}$$

$$\lambda\langle\omega^2\rangle = 800 \pm 200 meV^2 \rightarrow \lambda = 1.2 \pm 0.5 \quad \text{LaSrCuO}$$



LaSrCuO

Iron-based pnictides

	$\lambda\langle(\hbar\omega)^2\rangle$ (meV^2)	$\lambda\langle(\hbar\omega)^2\rangle/\lambda$ (meV^2)	λ	$2\Delta_{SDW}/k_B T_{SDW}$
$SrFe_2As_2$	110 ± 10	430	~0.25	7.2 ± 1
SmFeAsO (Ref. 24)	135 ± 10	770	~0.18	~5
$BaFe_2As_2$ [a]	~65 [b]	430	~0.15	4.7 ± 1.6

PRB, 82, 012505 (2010)

Background

VOLUME 15, NUMBER 12 PHYSICAL REVIEW LETTERS 20 SEPTEMBER 1965

NEW MECHANISM FOR SUPERCONDUCTIVITY*

W. Kohn

University of California, San Diego, La Jolla, California

and

J. M. Luttinger

Columbia University, New York, New York

(Received 16 August 1965)

For electrons in metals

$$u(\vec{r}) = +(4\pi a/m)\delta(\vec{r}),$$

$$u(0) = \pi^2/mk_F$$

$$(kT_c/\epsilon_{0,l}) \sim e^{-40} \sim 10^{-17},$$

$$(kT_c/\epsilon_0) \sim \exp[-(2l)^4].$$

Secondly, by taking metals with different parameters and nonspherical Fermi surfaces, it may prove possible to enhance the effect appreciably. In fact, it is known that some

Order parameter

$$\Delta_{s_1 s_2}(\vec{r}_1, \vec{r}_2) = \Delta_{s_1 s_2}(\vec{R}, \vec{r}) = -\Delta_{s_2 s_1}(\vec{R}, -\vec{r})$$

In our simple case we consider uniform case i.e

$$\Delta_{s_1 s_2}(\vec{R}, \vec{r}) = \Delta_{s_1 s_2}(\vec{r}) \Rightarrow \Delta(\vec{q})$$

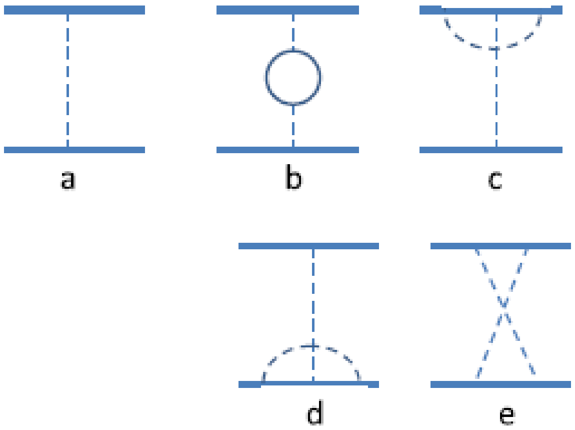
More details

$$\Delta(k) = \int d\xi \frac{\tanh(\xi / 2T_c)}{2\xi} \frac{\Omega}{(2\pi\hbar)^d} \sum_{k'} K(k, k')\Delta(k')$$

Equation for λ

$$\boxed{\lambda\Delta(\mathbf{p}) = \frac{\Omega}{(2\pi\hbar)^d} \oint \frac{dS}{v_F(S)} K(\mathbf{p}, \mathbf{p}')\Delta(\mathbf{p}')}$$

$$\begin{aligned} K(\mathbf{p}, \mathbf{p}') = v(\mathbf{p} - \mathbf{p}') + v(\mathbf{p} - \mathbf{p}') \sum_{\mathbf{k}} [2v(\mathbf{p} - \mathbf{p}') \\ - v(\mathbf{k} + \mathbf{p}') - v(\mathbf{k} - \mathbf{p})] Q(\mathbf{p} - \mathbf{p}', \mathbf{k}) \\ - \sum_{\mathbf{k}} v(\mathbf{p} - \mathbf{k}) v(\mathbf{k} + \mathbf{p}') Q(\mathbf{p} + \mathbf{p}', \mathbf{k}) \end{aligned}$$



$$Q(\mathbf{q}, \mathbf{k}) = (f_{\mathbf{k}} - f_{\mathbf{k}-\mathbf{q}})/(E_{\mathbf{k}} - E_{\mathbf{k}-\mathbf{q}})$$

This diagrams are very difficult to evaluate if repulsion V(q) is q-dependent! Therefore most of the previous and recent studies confined to hard sphere (Hubbard U) repulsion V(q)=const!

Diagrams b),c),d) cancel each other. Diagram e) is known Lindhard function:

$$\Gamma_a \propto U(0); \qquad \Gamma_e \propto U(0)^2 \chi(q) = U(0)^2 [1 + \frac{k_F^2 - q^2/4}{qk_F} \ln(\frac{k_F + q/2}{k_F - q/2})]/2$$

$$\lambda_l = \int_0^{\pi} d\theta \sin\theta \, P_l(\cos\theta) \, (\Gamma_a + \Gamma_e)$$

Hubbard model: $\Gamma_a = const$ $\Rightarrow$ $\lambda_{la} \propto \delta_{l0}$

$$\lambda_{le} \propto 1/l^4$$

$$\Downarrow$$

Weakly interacting Hubbard model is superconducting in 3D

$$V(q) = \frac{4\pi e^2}{q^2+\kappa^2} \Rightarrow U = V(0) = \frac{\pi^2}{mk_F}$$

$$\Rightarrow$$

$$\lambda_1 = (2\ln 2 - 1)/40 = 0.0096$$

$$\lambda_2 = (8 - 11\ln 2)/420 = 0.00089$$

In 2D the screening is different and the pairing is possible only because of nontrivial Fermi surface.

The exchange diagram e) is given by:

$$\chi(q) = \frac{m}{2\pi}\left[1 - \frac{\mathrm{Re}\sqrt{q^2 - (2k_F)^2}}{q}\right].$$

Pairing is absent because it does not have negative contributions for any l ! To have superconductivity in 2D it is important to take into account nonspherical shape of Fermi surface.

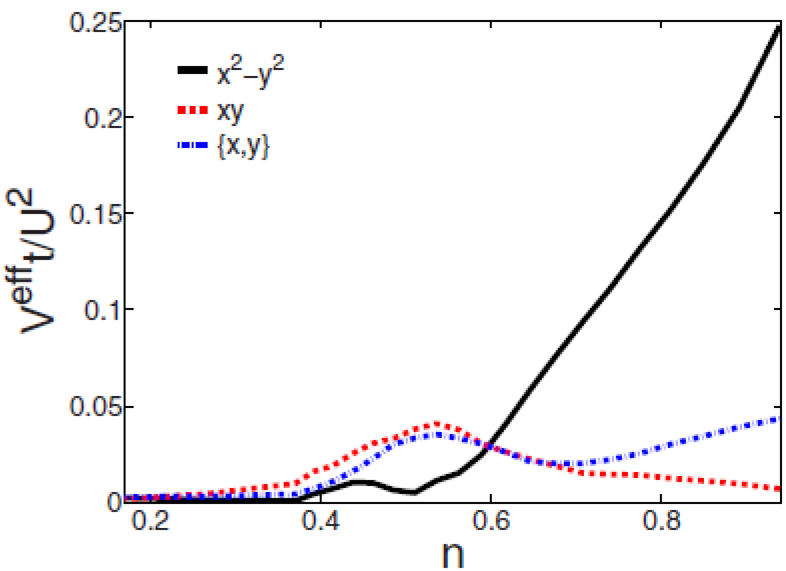
M.A. Baranov, A.V. Chubukov M.Yu. Kagan Int. J. of Mod. Phys. (1992).

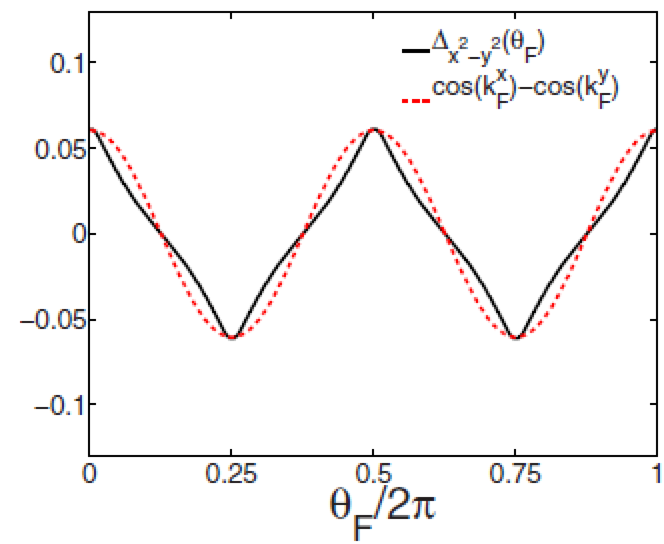
PHYSICAL REVIEW B **81**, 224505 (2010)

Superconductivity in the repulsive Hubbard model: An asymptotically exact weak-coupling solution

S. Raghu,[1] S. A. Kivelson,[1] and D. J. Scalapino[1,2]

Only diagrams a) and e) are not equal to 0!





PHYSICAL REVIEW B 83, 094518 (2011)

Superconductivity from repulsive interactions in the two-dimensional electron gas

S. Raghu[1,2] and S. A. Kivelson[1]

In the small r_s limit, the Coulomb interactions are sufficiently well screened that it may be reasonable to treat them as weak and short-ranged

$$V(q) = \frac{4\pi e^2}{q^2 + \kappa^2(q)} \quad \text{where}\ \kappa^2(q) = 2k_F^2 r_s \chi(q)/3 << k_F^2!$$

$$r_s = 1.92 e^2 / \hbar v_F < 1$$

Therefore screening radius is large in comparison with the distance between particles. q-dependence is important in metals with small $r_s << 1$

Note that the line from my paper with Raghu, (''in the small rs limit, the Coulomb interactions are sufficiently well screened that it may be reasonable to treat them as weak and short-ranged'' contains the carefully inserted modifier, "may," as this is not something we could establish.

PRL 106, 136403 (2011) PHYSICAL REVIEW LETTERS week ending 1 APRIL 2011

Unconventional High-Temperature Superconductivity from Repulsive Interactions: Theoretical Constraints

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(Received 27 January 2011; published 31 March 2011)

Where superconductivity is possible?

$$V(q)=\frac{4\pi e^2}{q^2+\kappa^2}\ ,\kappa >> \mathrm{k_F} \Rightarrow V(q)=\frac{4\pi e^2}{\kappa^2}[1-\frac{q^2}{\kappa^2}+\frac{q^4}{\kappa^4}+....]$$

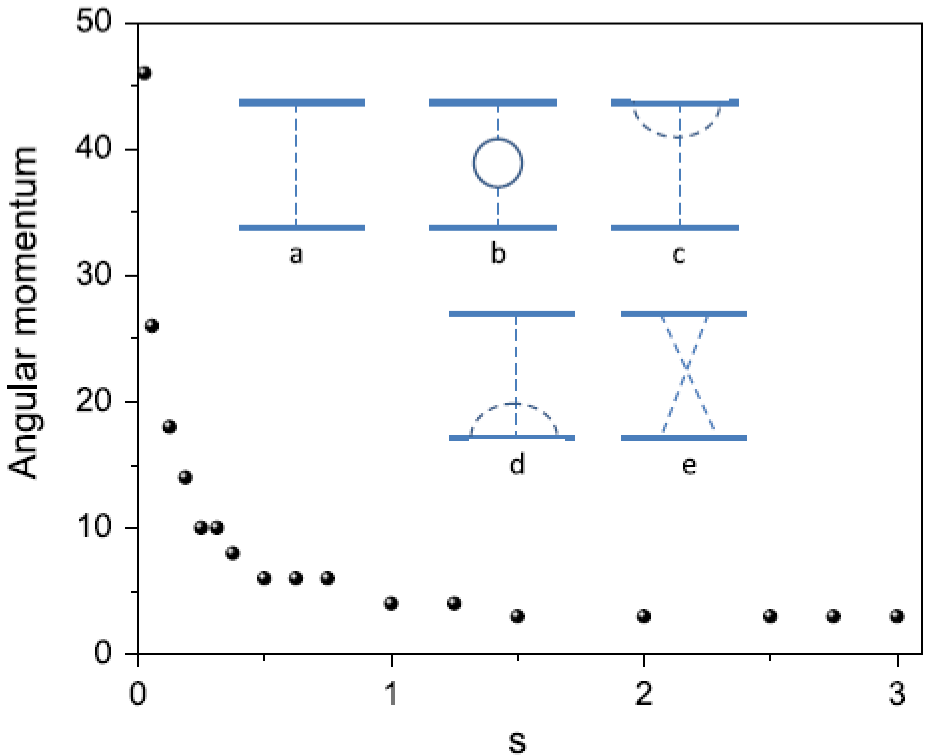
$$\lambda_1/s=\left(\frac{k_F}{\kappa}\right)^4\left(3/4-2s(2\ln 2-1)/5\right)\Rightarrow r_s>36!$$

$$\lambda_2/s=\left(\frac{k_F}{\kappa}\right)^4\left(16k_F^2/\kappa^2 15-4s(8-11\ln 2)/105\right)\Rightarrow\left(\frac{\kappa}{k_F}\right)^2>450/r_s!$$

P-wave superconductivity is outside of applicability of the theory, d-wave superconductivity requires screening radius which is at leas 1 order of magnitude shorter than lattice constant!

Self-consistent calculations

$$\left(\frac{\kappa}{k_F}\right)^2 \longrightarrow s(q) = 4s\left[\frac{1}{2} + \frac{k_F^2 - q^2/4}{2qk_F}\ln\frac{k_F + q/2}{k_F - q/2}\right].$$



$(\lambda_3 \approx 0.0011 \text{ for } s = 3)$

$T_c \approx (E_F/k_B)\exp(-1/\lambda)$

$s = r_s / 6$

Conclusions

Thank you for the attention