OVM Homework Exercises I (WI3405TU)

1 : Submission deadline : 25th October 2024 (electronically via Brightspace)

Please include all your answers clearly in the main body of your report (PDF), and attach the programming code in the appendix for TA's review. The use of LaTeX is highly recommended. Additionally, submit your code as a separate file (preferably as a Jupyter notebook) for implementation checking. Note that if your solutions are unclear or if we are unable to run your code (in Python or Matlab) correctly, you may not receive full credit. Example code for the programming exercises can be found in the textbook and on the author's website.

1 (0.5P) Central Limit Theorem.
Please reproduce the results on slide 13 of Lecture 3.

Hint : First, define a generator for random variable X (Bernoulli or Uniform) using a programming language. Then create a new random variable Z through the summation of i.i.d. sequence $X_1, X_2, \ldots, X_n$, where $n = 1, 2, 3, 30, 100$. Next, draw m samples from Z to estimate the corresponding density function, for example, $m = 10000$.

2 (3P) Stock price model :

a). Load the daily close prices (i.e., Column "Close" in the provided CSV file `DailyData - STOCK_US_XNAS_AAPL.csv`[1]) and plot the price movement over time.

Next compute the daily return $r_i = \frac{S_i - S_{i-1}}{S_{i-1}}$ and daily logarithmic return $\hat{r}_i = \ln\left(\frac{S_i}{S_{i-1}}\right)$, and create plots (similar to the plot on page 6 of Lecture 4) to illustrate the patterns of both returns.

Under what circumstances can these returns be assumed identical ?

b). Statistics : First, plot the histogram of daily logarithmic returns, compute the sample mean $\hat{\mu}$ and variance $\hat{\sigma}^2$, and produce the quantile-quantile plot, i.e., daily logarithmic returns vs. normal distribution $N(\hat{\mu}, \hat{\sigma}^2)$.

Second, repeat the above process for the weekly close prices in the provided CSV file `WeeklyData - STOCK_US_XNAS_AAPL.csv`, and make a comparison between weekly and daily returns.

c). When using a continuous-time stochastic equation (see Equation (6.8) in the book) to model the stock price, please estimate the **annual** drift μ and volatility σ by means of the result in 2.b). Hint : Equation (6.9) is useful for the parameter estimation.

3 (3P). Simulation of stock price paths.

a). We have the discrete price model $S(t_{i+1}) = S(t_i)(1 + \mu\delta t + \sigma\sqrt{\delta t}Z)$, where $Z \sim \mathcal{N}(0, 1)$, $T = 1$, $S_0 = 170$, $\sigma = 0.344$, and $\mu = 0.1$.

First, choose the number of time intervals $L = 5$ and plot the price evolution of S_t for

1. The data of Apple stock prices can be downloaded from the website https ://www.marketwatch.com/investing/stock/aapl/ under the "Historical quotes" tab.

$t = i \cdot \Delta t$ with $i = 0, \ldots, L$ and $\Delta t = \frac{T}{L}$.

Second, repeat the simulation by $M = 5000$ times, and store the simulated S_T to make the histogram.

Third, repeat the simulation for increasing $L = 10, 20, 50, 100$ to see how the histogram of S_T changes.

b). We would like to compare the simulations with the theoretical continuous-time price model, i.e., $S_T = S_0 e^{(\mu - \frac{1}{2}\sigma^2)T + \sigma\sqrt{T}Z}$:

First use the continuous-time model to generate a quantile-quantile plot for S_T simulated using the discrete model (set $L = 100$ and $M = 5000$).

Second, vary the value of M, for example, $M = 50$, $M = 500$, $M = 5000$, $M = 50000$ to compute the sample mean and variance of S_T.

Third, compare these two statistic measures (mean and variance) with their theoretical value as M changes.

c). Use the above discrete model to generate the graphs in Figure 8.1 and study the behavior of the sum-of-square increments for asset prices.

4(2P). Black-Scholes option pricing model :

a). Solve Exercise 8.6.

b). Solve Exercise 8.8.

5(1.5P). Delta hedging :

a). Image you perform portfolio replicating for a European put option, with strike price $E = 170$, stock price $S_0 = 170$, expiry time $T = 1$ year, risk-free interest rate $r = 0.05$, volatility $\sigma = 0.344$, drift $\mu = 0.1$, please generate plots of the delta hedging process for two scenarios, in-the-money and out-of-the-money (refer to Figure 9.1, Figure 9.2 in the book). Suppose the time step size is $\Delta t = 0.01$.

b). Generate figures simiar to Figure 9.4 using two different drift values, $\mu = \pm 0.1$ and analyze your result.