X The electromagnetic wave

1. The Maxwell's equations without sources

$\rho_i = 0$, $j_i = 0$

$$\boxed{\nabla \vec{E} = 0}$$

$$\boxed{\nabla \vec{B} = 0}$$

$$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

$$\boxed{\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}}$$

Method 1:

$$\nabla \times (\nabla \times \vec{E}) = \nabla \times \left(-\frac{\partial \vec{B}}{\partial t}\right) = -\frac{\partial (\nabla \times \vec{B})}{\partial t} =$$

$$= -\frac{\partial \left(\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)}{\partial t} =$$

$$= -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

/* $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{a}(\vec{b} \cdot \vec{c}) - (\vec{a} \cdot \vec{b}) \cdot \vec{c}$ */

Method 2:

$$\nabla \times (\nabla \times \vec{E}) = \nabla(\underbrace{\nabla \vec{E}}_{=0}) - (\nabla \nabla) \vec{E}$$

$$= -\nabla^2 \vec{E} .$$

$$\nabla = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$$

$$V = 3xy + 7x^2z - 9yz$$

$$\nabla V = (3y + 14xz)\vec{i} + (3x + 9z)\vec{j} + (7x^2 - 9y)\vec{k}$$

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\boxed{\nabla^2 \vec{E} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0}$$

$$\boxed{\nabla^2 \vec{B} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} = 0}$$

2. The electromagnetic field equations

⇒ electromagnetic wave

$$\boxed{\vec{E}(x,t) = \vec{E}_m \sin(\omega t - kx)}$$

$$\boxed{\vec{B}(x,t) = \vec{B}_m \sin(\omega t - kx)}$$

$$\frac{\partial \vec{B}}{\partial t} = \omega \vec{B}_m \cos(\omega t - kx)$$

$$\boxed{\frac{\partial^2 \vec{B}}{\partial t} = -\omega^2 \vec{B}_m \sin(\omega t - kx)}$$

$$\nabla \vec{B} = \frac{d\vec{B}}{dx} = -k\vec{B}_m \cos(\omega t - kx)$$

$$\nabla^2 \vec{B} = \frac{d^2 \vec{B}}{dx^2} = -k^2 \vec{B}_m \sin(\omega t - kx)$$

$$\vec{B}(x,t) = \boxed{-\frac{1}{\omega^2} \frac{\partial^2 \vec{B}}{\partial t^2} = -\frac{1}{k^2} \nabla^2 \vec{B}}$$

$$\nabla^2 \vec{B} - \frac{k^2}{\omega^2} \cdot \frac{\partial^2 \vec{B}}{\partial t^2} = 0$$

$$\left.\begin{aligned} k &= \frac{2\pi}{\lambda} \\ \omega &= \frac{2\pi}{T} \end{aligned}\right\} \Rightarrow \frac{k}{\omega} = \frac{\frac{2\pi}{\lambda}}{\frac{2\pi}{T}} = \frac{T}{\lambda} \Rightarrow \frac{k}{\omega} = \frac{1}{c}$$

$$\nabla^2 \vec{B} - \frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} = 0$$

$$\boxed{\nabla^2 \vec{B} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} = 0}$$

$$\frac{1}{c^2} = \mu_0 \varepsilon_0 \Rightarrow \boxed{c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}}$$

the velocity of electromagnetic WAVE

5th Maxwell equation

$$\frac{1}{4\pi\varepsilon_0} = 9\cdot 10^9 \frac{Nm^2}{C^2}$$

$$\mu_0 = 4\pi\cdot 10^{-7}\frac{V}{A}$$

$$\boxed{c = 2{,}99792246\cdot 10^8\ m/s}$$

↳ velocity of light

- $\nabla \times \vec{E} = ?$

$$\vec{E}(x,t) = \underbrace{E(x,t)}_{\text{magnitude}}\cdot \underbrace{\vec{j}}_{\text{direction}}$$

$$\nabla \times \vec{E} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E(x,t) & 0 \end{vmatrix} =$$

$$= \vec{i}\left(-\underbrace{\frac{\partial E(x,t)}{\partial z}}_{=0}\right) - \vec{j}(0) + \vec{k}\left(\frac{\partial E(x,t)}{\partial x} - 0\right)$$

versor

$$= \underline{\vec{k}\, E_m \cos(\omega t - kx)(-k)} \qquad k = \frac{2\pi}{\lambda}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = \underline{-\omega B_m \cos(\omega t - kx)\vec{k}}$$

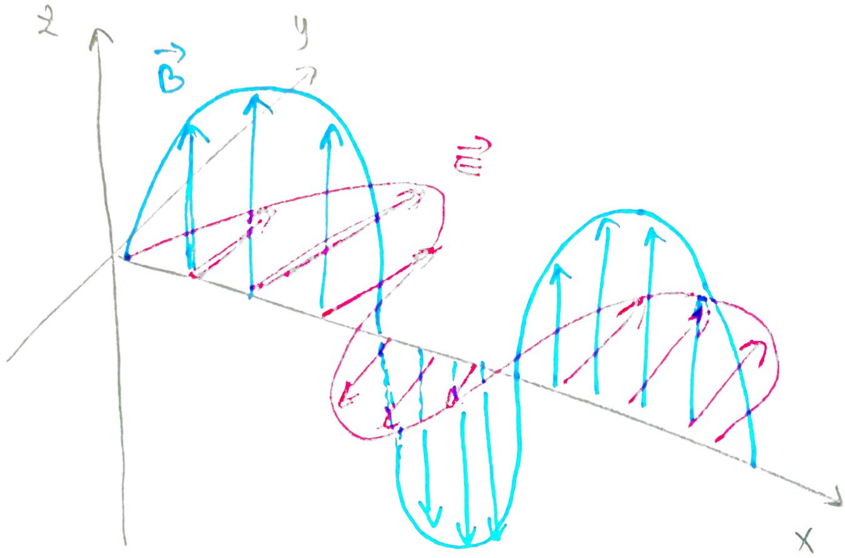
$$\Rightarrow \quad kE_m = \omega B_m$$

$$E_m = \frac{\omega}{k} B_m$$

$$\boxed{E_m = c\cdot B_m}$$

$$\boxed{E = c \cdot B}$$

EXAM

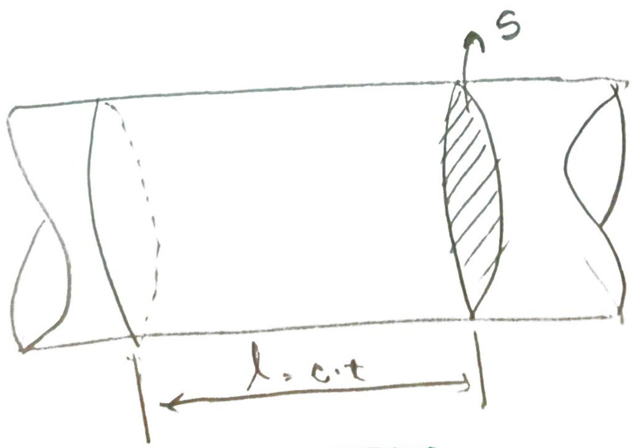


3. The electromagnetic wave intensity

$$\boxed{I \overset{def}{:=} \frac{1}{S} \left\langle \frac{dW}{dt} \right\rangle_t}$$

W : wave energy

$\langle \ \rangle$: the time average



Note : $\boxed{\underline{\underline{w}} \overset{dif}{:=} \left\langle \frac{dW}{dV} \right\rangle_t}$ density of energy

$$\boxed{w = \left\langle \frac{dW}{d(S \cdot c t)} \right\rangle_t}$$

$$w = \frac{1}{c} \cdot \underbrace{\frac{1}{S} \left\langle \frac{dW}{dt} \right\rangle_t}_{I}$$

$$\boxed{I = c \cdot w}$$

$$w_{em} = w_e + w_m$$

$$\boxed{w_e = \frac{\varepsilon_0 E^2}{2}}$$

$$w_m = \frac{B^2}{2\mu_0}$$

$$w_e = \frac{\varepsilon_0 E^2}{2} = \frac{\varepsilon_0 c^2 B^2}{2} = \frac{\cancel{\varepsilon_0} B^2}{2 \cancel{\varepsilon_0} \mu_0} = \frac{B^2}{2\mu_0} = w_m$$

$$E = cB$$

$$\Rightarrow \quad w_{em} = 2 w_e$$

$$I = c \cdot w_{em} \Rightarrow \boxed{I = c \cdot \varepsilon_0 E^2} = c \cdot \frac{B^2}{\mu_0}$$

$$\boxed{I = c \varepsilon_0 E B}$$

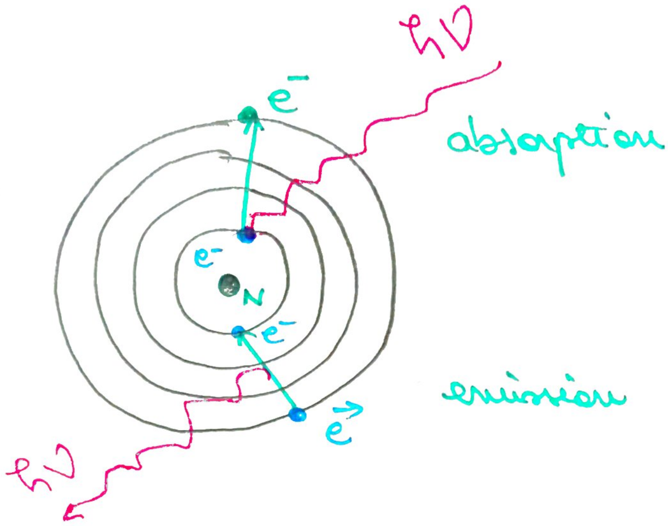
$$\boxed{\vec{P} = c^2 \varepsilon_0 \vec{E} \times \vec{B}}$$ the POINTING vector

(points along the direction of propagation of the electromagnetic wave)

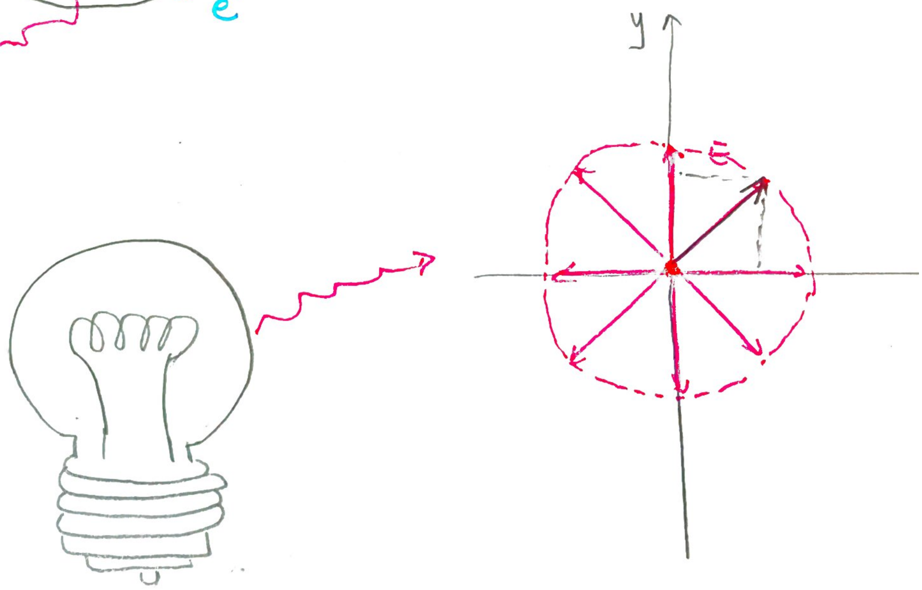
$$|\vec{p}| = I$$

4. The polarization of light

(A) The absorption and emission of light



(B)



$$\begin{cases} E_x = a \sin(\omega t - \varphi_x) \\ E_y = b \sin(\omega t - \varphi_y) \end{cases}$$

$$\frac{E_x}{a} = \sin(\omega t - \varphi_x)$$

$$\frac{E_y}{b} = \sin(\omega t - \varphi_x + \varphi_x - \varphi_y)$$

Note $\Delta\varphi = \varphi_x - \varphi_y$

$$\frac{E_y}{b} = \underbrace{\sin(\omega t - \varphi_x)}_{\frac{E_x}{a}} \cos\Delta\varphi + \underbrace{\cos(\omega t - \varphi_x)}_{\sqrt{1-\sin^2(\omega t - \varphi_x)}} \sin\Delta\varphi$$

$$\frac{E_y}{b} - \frac{E_x}{a}\cos\Delta\varphi = \sqrt{1-\frac{E_x^2}{a^2}}\,\sin\Delta\varphi \;\Big|^2$$

$$\frac{E^2}{b^2} + \frac{E_x^2}{a^2}\cos^2\Delta\varphi - \frac{2E_xE_y}{a\cdot b}\cos\Delta\varphi = \left(1-\frac{E_x^2}{a^2}\right)\sin^2\Delta\varphi$$

$$\frac{E_x^2}{a^2} + \frac{E_y^2}{b^2} - \frac{2E_xE_y}{ab}\cos\Delta\varphi = \sin^2\Delta\varphi$$

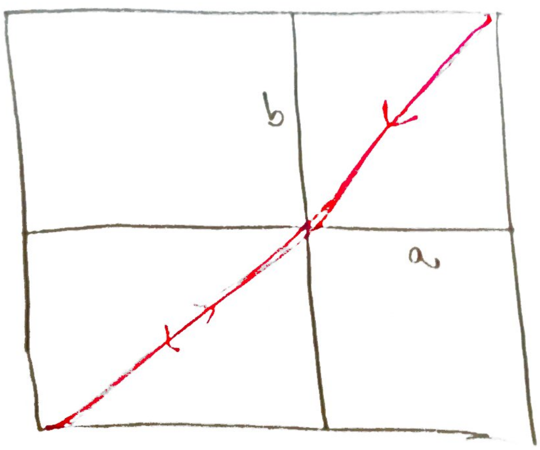
! If $\Delta\varphi \neq f(t)$ time independent $\Rightarrow$

$\Rightarrow$ polarized light

c) Lissajous figures

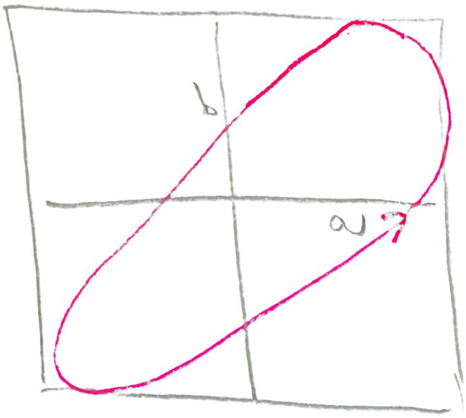
(Types of light polarization)

i) $\Delta\varphi = 0$ $\quad \left(\frac{E_x}{a} - \frac{E_y}{b}\right)^2 = 0$



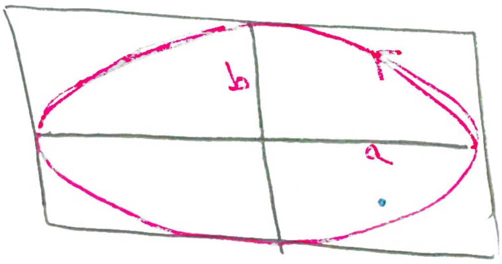
linear polarized light

ii) $0 < \Delta\varphi < \frac{\pi}{2}$



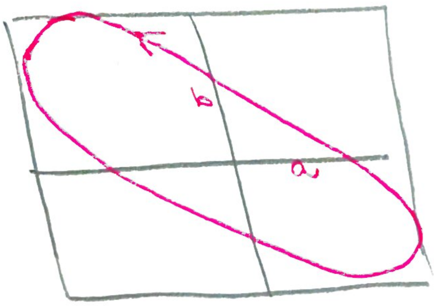
eliptical polarised light

iii) $\Delta\varphi = \frac{\pi}{2}$

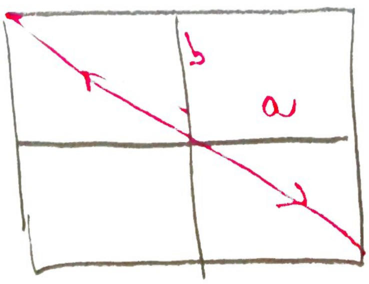


$$\frac{E_x^2}{a^2} + \frac{E_y^2}{b^2} = 1$$

iv) $\frac{\pi}{2} < \Delta\varphi < \pi$

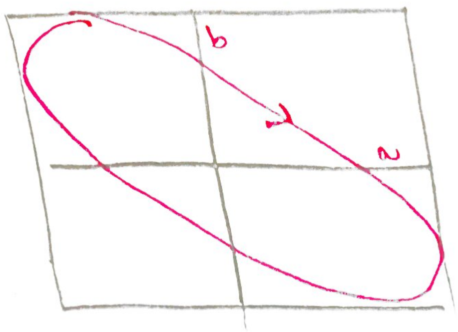


v) $\Delta\varphi = \pi$



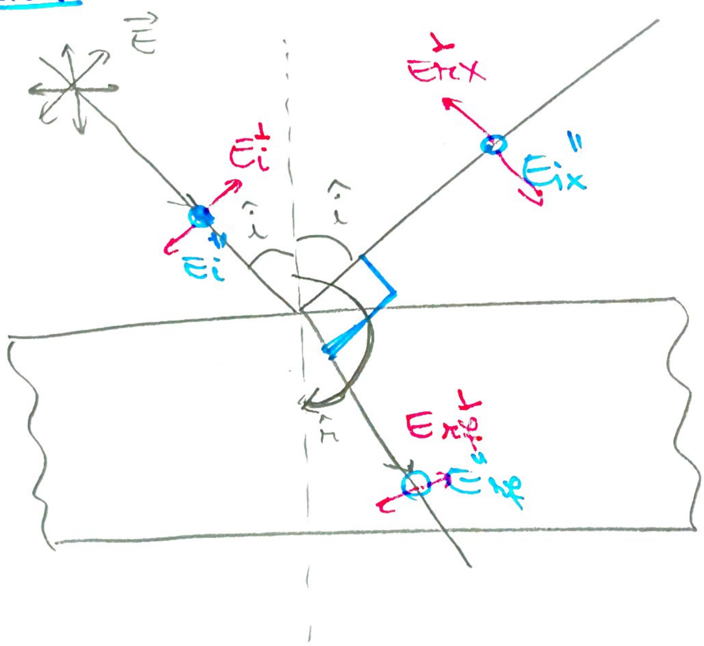
$$\left(\frac{E_x}{a} + \frac{E_y}{b} \right)^2 = 0$$

vi) $\pi < \Delta\varphi < \frac{3\pi}{2}$



D) Methods to produce a polarized

D1. In homogeneous media by reflexion and refraction



- Fresnel's relationships

$$E_{rx}^{\parallel} = -E_i^{\parallel} \cdot \frac{tg(\hat{i}-\hat{r})}{tg(\hat{i}+\hat{r})}$$

$$E_{r} = -E_i^{\perp} \cdot \frac{\sin(\hat{i}-\hat{r})}{\sin(\hat{i}+\hat{r})}$$

$$E_{rf}^{\parallel} = E_i^{\parallel} \frac{2\cos(\hat{i})\sin(\hat{r})}{\sin(\hat{i}+\hat{r})\sin(\hat{i}-\hat{r})}$$

$$E_{rf}^{\perp} = E_i^{\perp} \frac{2\cos(\hat{i})\sin(\hat{r})}{\sin(\hat{i}+\hat{r})}$$

$$\frac{\sin(\hat{i})}{\sin(\hat{r})} = \frac{v_1}{v_2} = n_{12}$$

$$E''_{ref} = 0 \Rightarrow \cos(\hat{i}+\hat{r}) = 0$$

$$\underline{\underline{\hat{i} + \hat{r} = 90^0}}$$

reflected refracted

If the ~~refracted~~ ray is perpendicular on the ~~reflected~~ ray, then the refracted ray is linearly polarized.

and $\hat{i}+\hat{r} = 90$

if $\hat{i}+\hat{r} = 90$ $\quad \underline{i = i_B}$

$$\Rightarrow \hat{r} = 90^0 - i_B$$

$$\frac{\sin(i_B)}{\sin(90^0 - i_B)} = \frac{\sin(i_B)}{\cos(i_B)} = n_{12} \text{ or } \boxed{tg(i_B) = n_{12}}$$

i_B = the BREWSTER angle