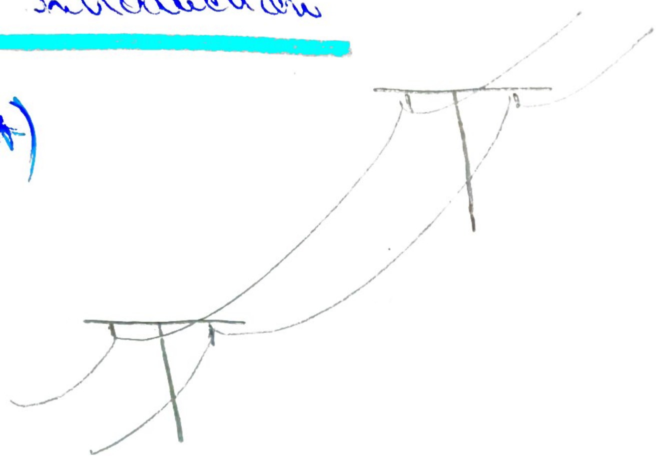
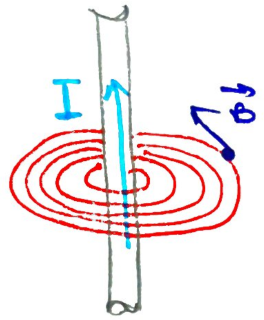
VIII The magnetic field

1. Introduction

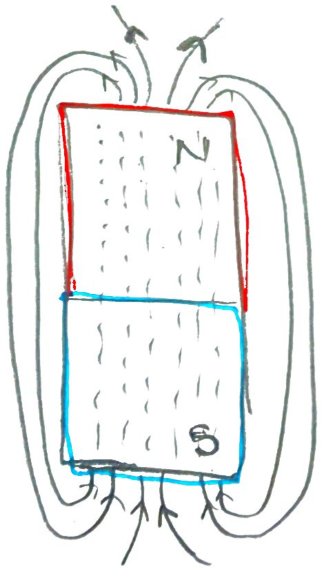
A)



B)

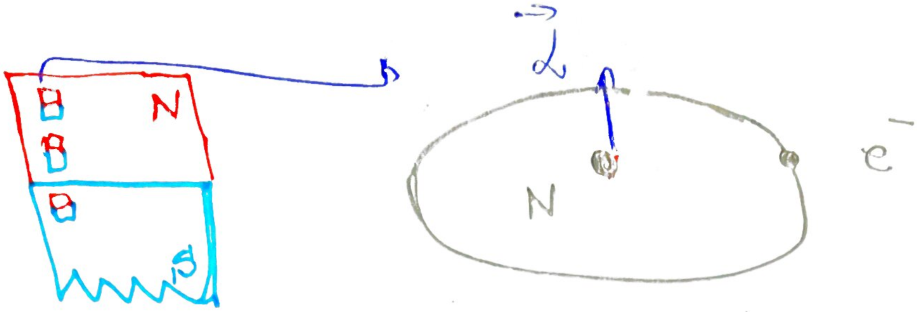


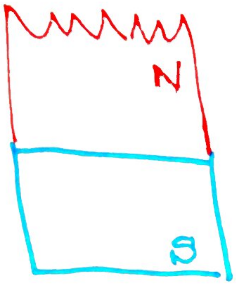
c)



$\boxed{\text{Magnetic field lines are closed lines.}}$

- 2 magnetic poles;





$$\boxed{\vec{\mu} = \frac{e}{2 m_e}\vec{L}}$$

e : elementary charge

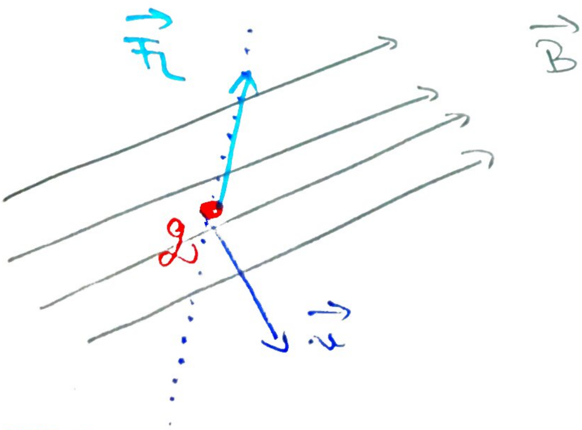
m_e : mass of the electron

μ: magnetic moment

can produce a local change

$$M = \frac{\sum_i \vec{\mu}_i}{V}$$ — magnetization

2. The Lorentz force



$$\boxed{\vec{F}_L = q\vec{v} \times \vec{B}}$$

3. The magnetic flux

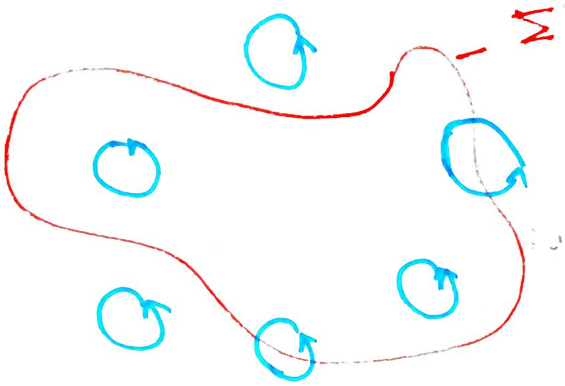
$$\Phi_m = \vec{B} \cdot \vec{S}$$

$$\boxed{\Phi_m = \int_S \vec{B}\, d\vec{s}}$$

Exam

4. Gauss' law for the magnetic field

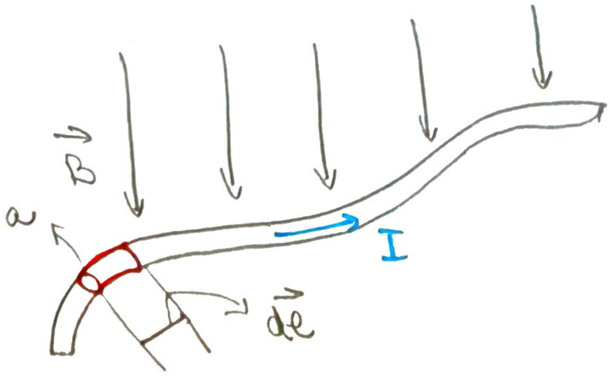
$$\Phi_m = \oint_\Sigma \vec{B}\, d\vec{s} = ?$$



$$\boxed{\oint_\Sigma \vec{B}\, d\vec{s} = 0}$$

Exam

5. The magnetic force of an element of current



$$\vec{F}_L = q\vec{v} \times \vec{B}$$

$$\Downarrow$$

$$d\vec{F} = dQ\,\vec{v} \times \vec{B}$$

$$dQ = dN e = n \cdot e \cdot a \cdot dl$$

Note: $n = \dfrac{dN}{dV} \Rightarrow dN = n \cdot a \cdot dl$

$$d\vec{F} = n \cdot e \cdot a \cdot dl\,\vec{v} \times \vec{B}$$

$$= \underbrace{n \cdot e \cdot v}_{j} \cdot a \cdot d\vec{l} \times B$$

$$\boxed{\vec{j} = n \cdot e \cdot \vec{v}}$$

the electric current density

$$I = j \cdot a = n \cdot e \cdot v \cdot a$$

$$\boxed{d\vec{F} = \left(I\,d\vec{l}\right) \times \vec{B}}$$

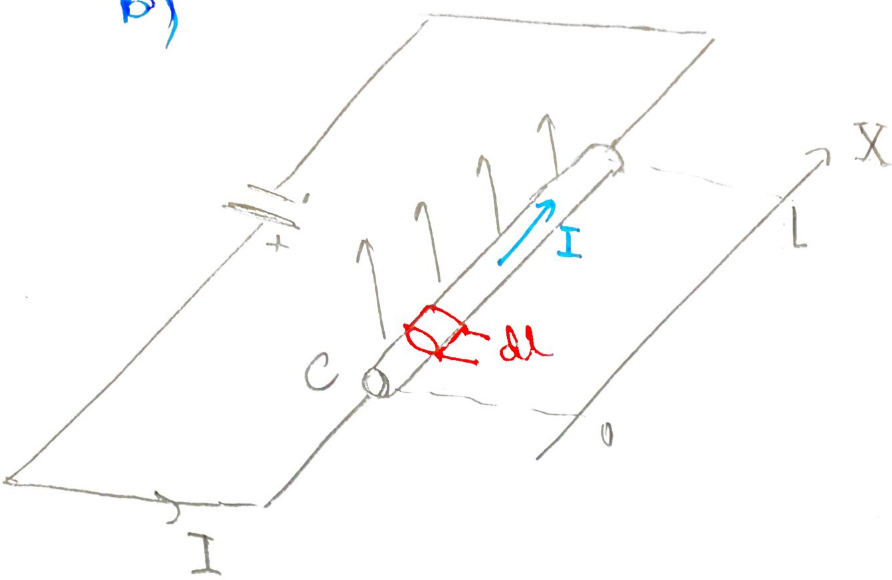
An element of current

6. LAPLACE force

A) $\int d\vec{F} = \int \left(I\,d\vec{l}\right) \times \vec{B}$

$$\boxed{\vec{F} = \int_C I\,d\vec{l} \times \vec{B}}$$

B)



$$\vec{F} = \int_C I \, \vec{dl} \times \vec{B}$$

$$\vec{F} = I \int_C \vec{dl} \times \vec{B}$$

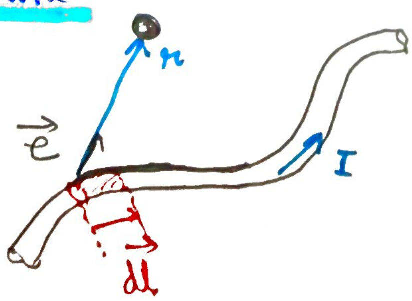
$$F = I \left(\int_C dl \right) B$$

$$F = ILB$$

$$\boxed{F_{Lp} = BIL}$$

7. The Biot-Savart law

7.1. The magnetic field produced by an element of current



universal constant

μ_0 = the magnetic permeability of void

$$\boxed{\vec{dB} = \frac{\mu_0}{4\pi} \frac{(I \vec{dl}) \times \vec{e}}{r^2}}$$

B-S law differential form.

7.2 $$\int d\vec{B} = \int_C \frac{\mu_0}{4\pi} \frac{I\, \vec{dl} \times \vec{e}}{r^2}$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{4\pi} \int_C \frac{\vec{dl} \times \vec{e}}{r^2}}$$

B-S law
the integral form

7.3 Examples

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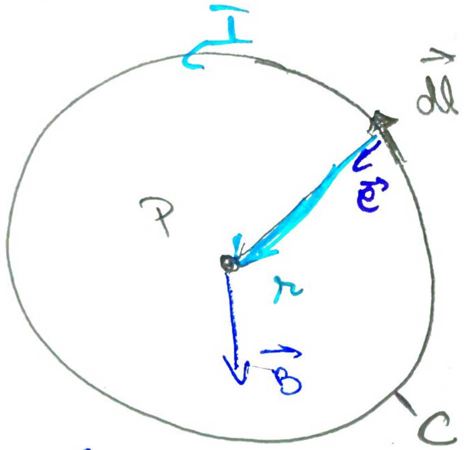


$$|\vec{e}| = 1$$

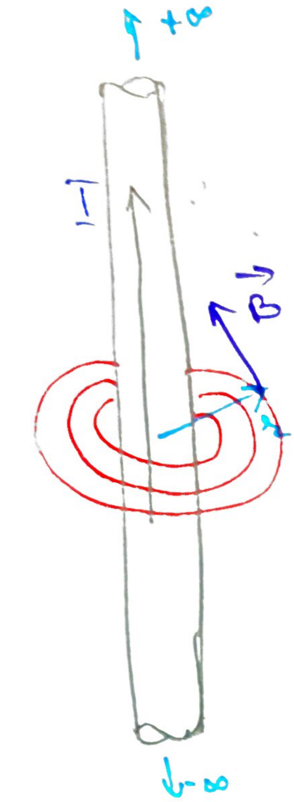
$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_C \frac{\vec{dl} \times \vec{e}}{r^2}$$

$$B = \frac{\mu_0 I}{4\pi r^2} \int dl$$

$$B = \frac{\mu_0 I}{4\pi r^2} 2\pi r$$

$$\boxed{B = \frac{\mu_0 I}{2r}}$$

(B)

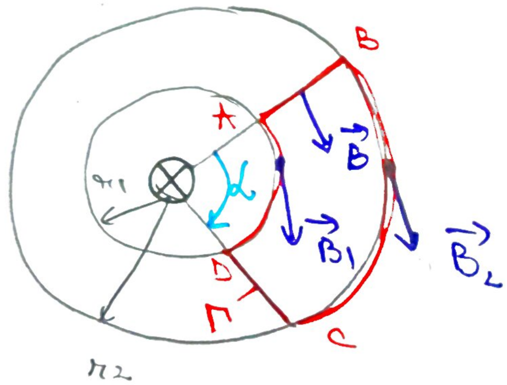


$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

8. The Ampere's law

$$\oint_{\Gamma} \vec{B}\,\vec{dl} = ?$$

(A)



(the current goes in)

$$\oint \vec{B}\,\vec{dl} = \int_A^B \vec{B}\,\vec{dl} + \int_B^C \vec{B}\,\vec{dl} + \int_C^D \vec{B}\,\vec{dl} + \int_D^A \vec{B}\,\vec{dl} =$$

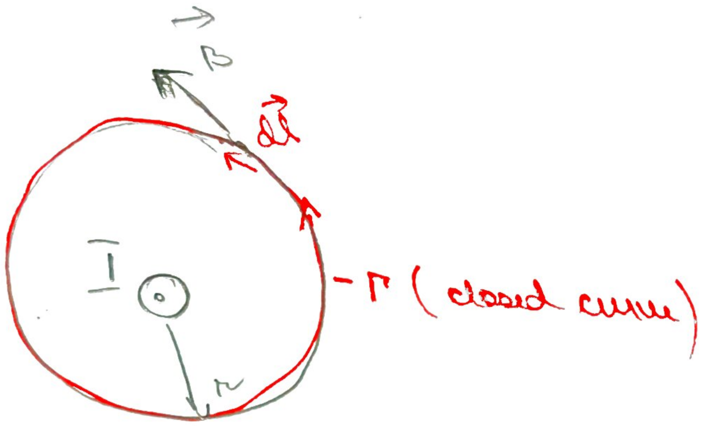
$$= 0 + B_2 \int_B^C dl + 0 - B_1 \int_A^D dl =$$

$$= \frac{\mu_0 I}{2\pi R_2}\,\overset{\frown}{BC} - \frac{\mu_0 I}{2\pi R_1}\,\overset{\frown}{DA} =$$

$$= \frac{\mu_0 I}{4\pi}\left(\frac{\cancel{L\cdot r_2}}{\cancel{r_2}} - \frac{\cancel{L\cdot r_1}}{\cancel{r_1}}\right) \Rightarrow$$

$$\boxed{\oint_\Gamma \vec{B}\,\vec{dl} = 0}$$

(B)



$$\oint_\Gamma \vec{B}\,\vec{dl} = B\oint_\Gamma dl = \frac{\mu_0 I}{2\pi r}\cdot 2\pi r \Rightarrow \boxed{\oint_\Gamma \vec{B}\,\vec{dl} = \mu_0 I}$$

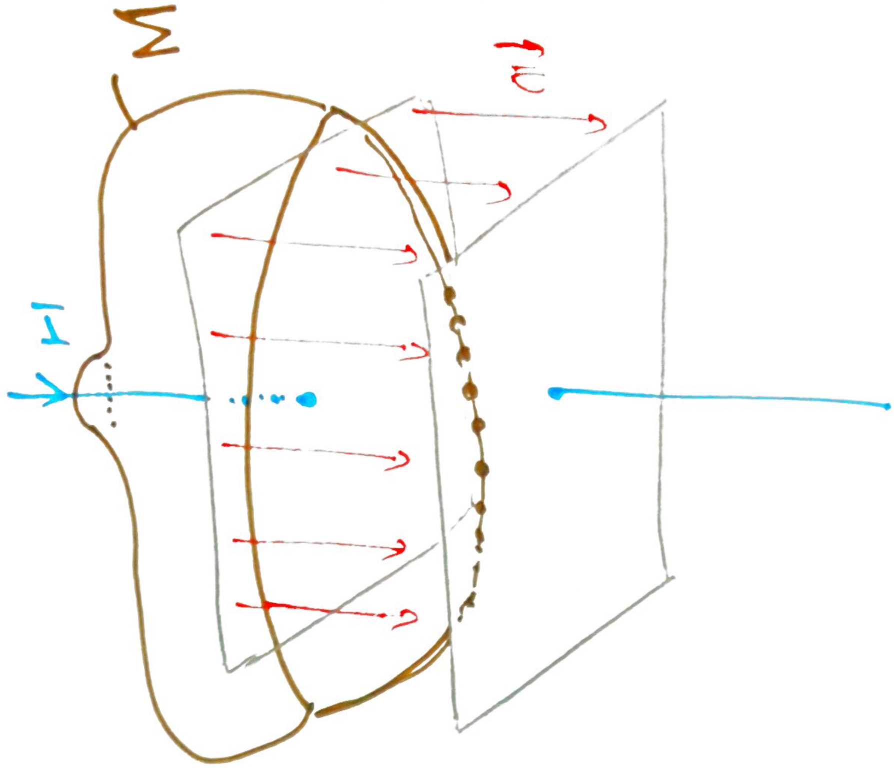
$$\boxed{\oint_\Gamma \vec{B}\,\vec{dl} = \mu_0 I_{inside}}$$ Ampere's law

(C) $j = \frac{dI}{dS} \rightarrow I = \int_S \vec{j}\,\vec{ds}$

$$\boxed{\oint_\Gamma \vec{B}\,\vec{dl} = \int_S \mu_0 \vec{j}_{int}\,\vec{ds}}$$

any surface based on the curve Γ

D) The displacement current



$$\boxed{\vec{j}_d = \varepsilon_0 \frac{d\vec{E}}{dt}}$$

displacement current density

$$\vec{j}_{in} \longrightarrow \vec{j}_t = \vec{j}_{in} + \vec{j}_d$$

$$\boxed{\oint_{\Gamma} \vec{B}\,\vec{dl} = \iint_S \left(\mu_0 \vec{j}_{in} + \mu_0 \varepsilon_0 \frac{d\vec{E}}{dt}\right) d\vec{S}}$$

ε_0 : electric permittivity