VI The ELECTRIC FIELD

1. The electric charge

notation: q, Q

$[Q]_{IS} = C$

- there are 2 types of electric charge.
  - positive ⊕
  - negative ⊖

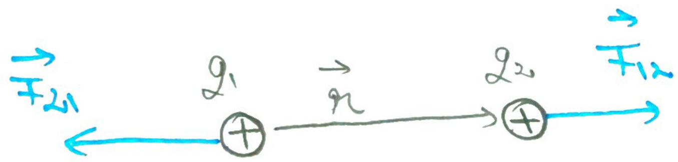
- ∃ of an elementary charge e

$$\boxed{e = 1{,}6 \cdot 10^{-19}\ C}$$

EXAM

! $\boxed{Q = N \cdot e}$; $N = 0, 1, 2, \ldots$

2. Coulomb's law



$$\boxed{\vec{F}_{12} = k \cdot \frac{q_1 \cdot q_2}{r^2} \cdot \frac{\vec{r}}{|\vec{r}|}}$$

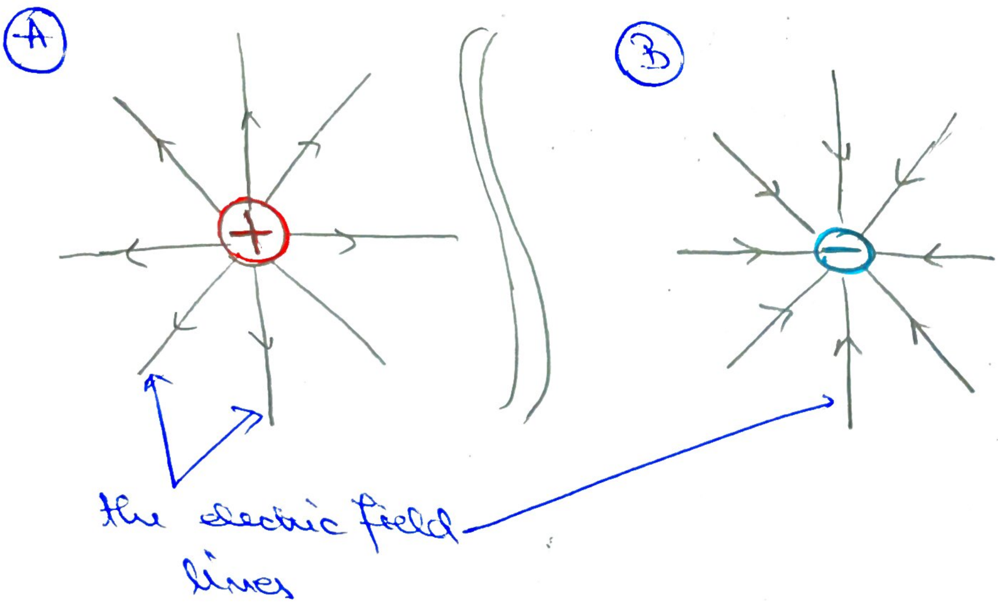
$$\boxed{k = \frac{1}{4\pi \varepsilon_0} = 9 \cdot 10^9 \ \frac{N \cdot m^2}{C^2}}$$

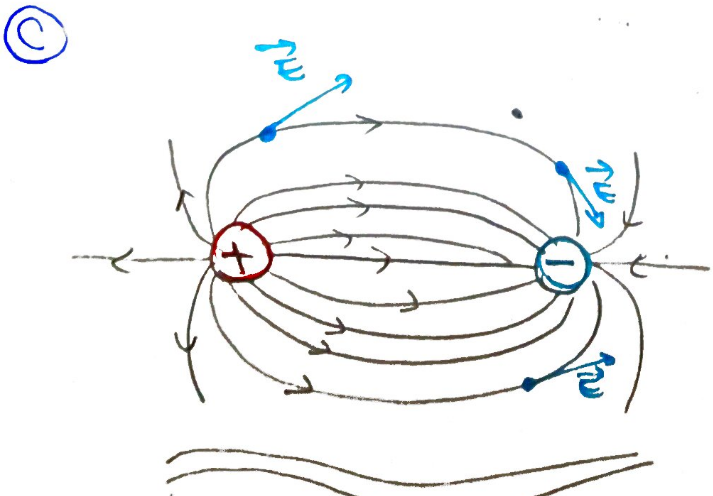
ε_0: the electric permittivity of void

→ Between any two electrical charges exists a force of interaction directly proportional with the product of their electrical charge and inversely proportional with the square of the distance between

- **force** – of attraction between electrical charges with opposite signs;
  – of rejection between electrical charges of the same sign;

3. The electric field

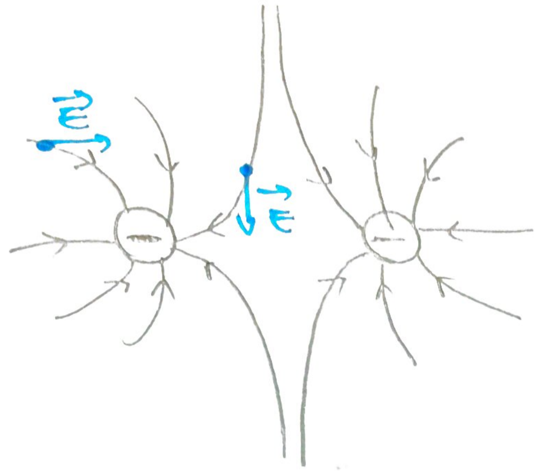




EXAM

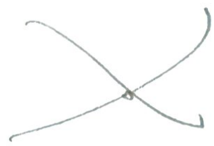
– drawings.

D



$\vec{E}$: the electric field intensity

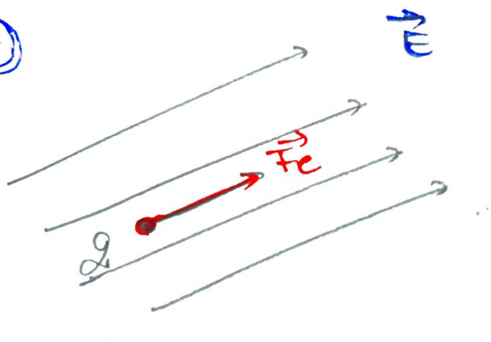
(Suppose there's an intersection point between two of the electric field lines:



We would not know to which line is the vector $\vec{E}$ tangent).

4. The electric force

A



$$\boxed{\vec{F_e} = q\vec{E}}$$

B

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Q \cdot q}{r^2} \cdot \frac{\vec{r}}{|\vec{r}|}$$

$$\vec{F} = q\,\frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} \cdot \frac{\vec{r}}{|\vec{r}|}$$

$$\boxed{\vec{F} = q\vec{E}}$$

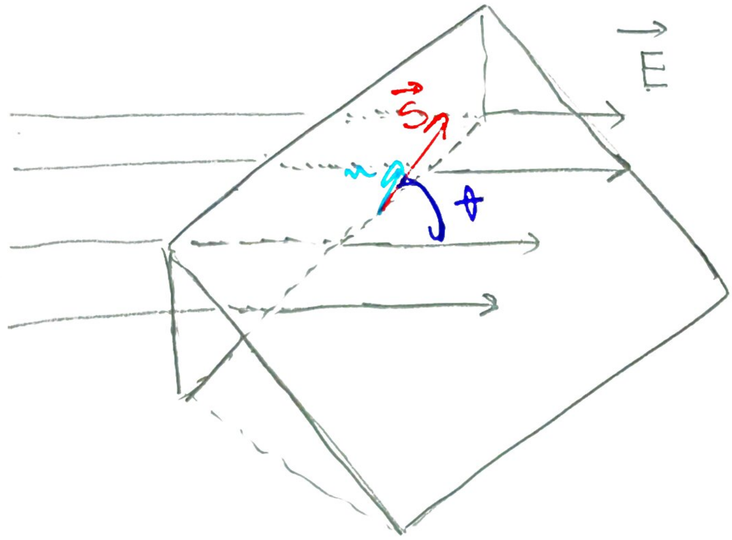
$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} \cdot \frac{\vec{r}}{|\vec{r}|}}$$

5. The electric flux

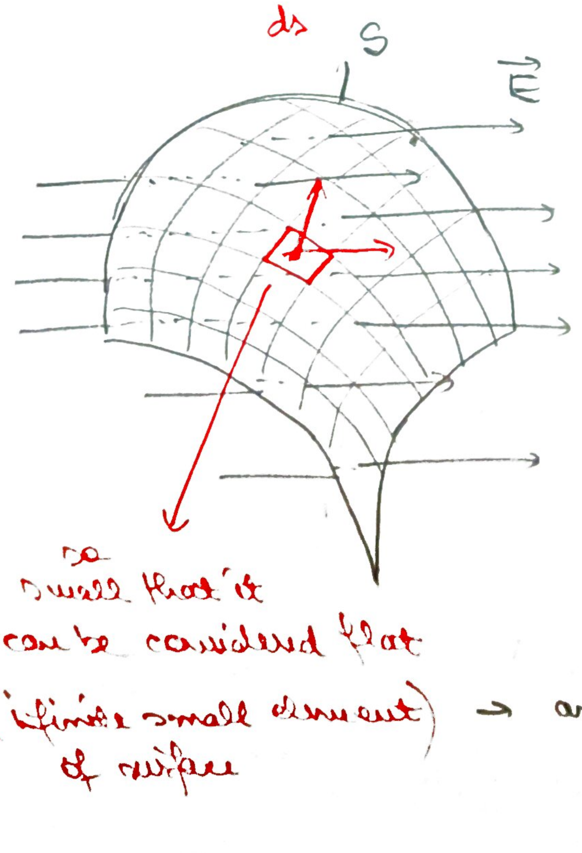
notation: ϕ_e

A

$$\boxed{\phi_e = \vec{E} \cdot \vec{S}} \qquad \boxed{\phi_e = E \cdot S \cdot \cos\theta}$$



B



→ an infinite small electric flux

$d\phi_e = \vec{E}\cdot d\vec{S} \quad \Big| \int$

$$\int d\phi_e = \int_S \vec{E}\, d\vec{s}$$

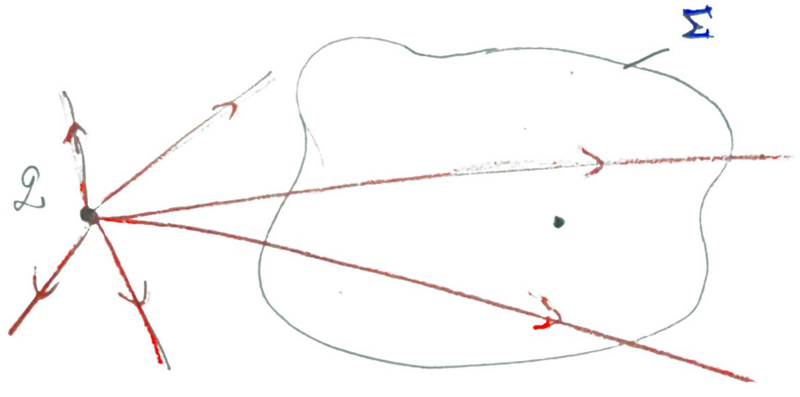
$$\boxed{\phi_e = \int_S \vec{E}\, d\vec{s}.}$$

6. The Gauss' law for the electric field

$$\phi_e = \oint_\Sigma \vec{E}\, d\vec{s} = ?$$

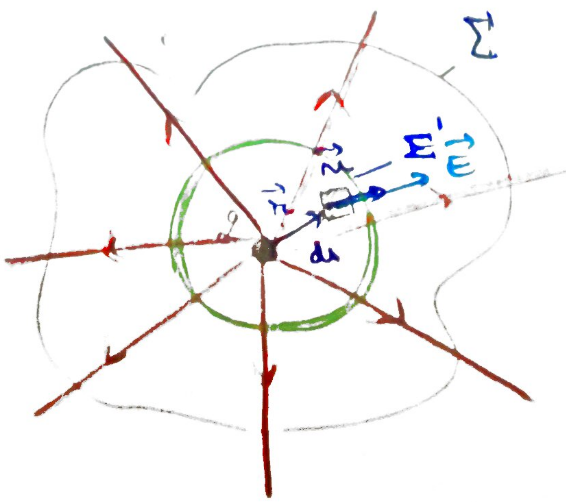
(closed surface)

6.1



$$\oint_\Sigma \vec{E}\, d\vec{s} = 0$$

6.2



$$\oint_\Sigma \vec{E}\, d\vec{s} = \oint_{\Sigma'} \vec{E}\, d\vec{s}'$$

$$= \oint_{\Sigma'} \underbrace{\vec{E}\,\vec{n}}_{\text{constant}}\, ds =$$

$$= E \oint_{\Sigma'} ds$$

$\vec{ds} = \vec{n} \cdot ds$

$\vec{E} \cdot \vec{ds} = \vec{E} \cdot \vec{n} \cdot ds$

$$\oint_{\Sigma} \vec{E} \cdot \vec{ds} = \oint_{\Sigma'} \vec{E} \cdot \vec{ds} = \oint_{\Sigma'} \underbrace{\vec{E} \cdot \vec{n}}_{\text{const}} \cdot ds = E \oint_{\Sigma'} ds = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} A_{\Sigma'} =$$

$$= \frac{q}{4\pi\varepsilon_0 r^2} \cdot 4\pi r^2 \Rightarrow \oint_{\Sigma} \vec{E} \cdot \vec{ds} = \frac{q}{\varepsilon_0}$$

$$\boxed{\oint_{\Sigma} \vec{E} \, \vec{ds} = \frac{q_i}{\varepsilon_0}}$$

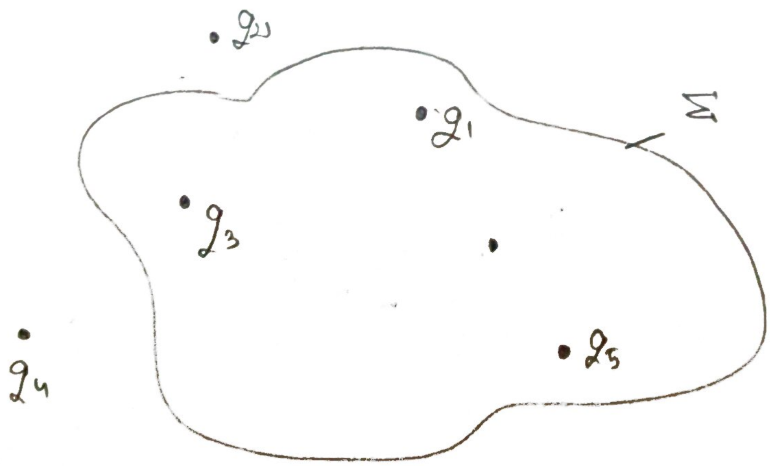
The Gauss law

q_i : inside

6.3 Example:

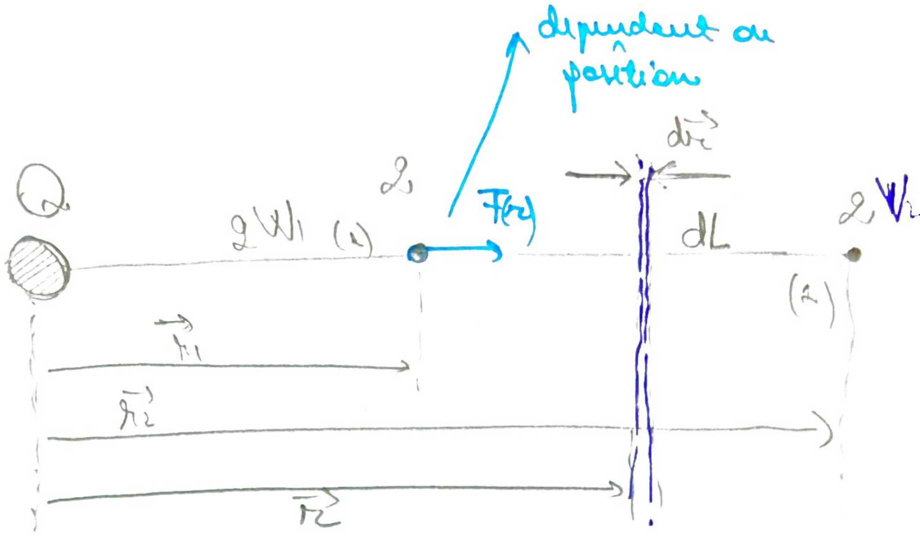
$$\oint_{\Sigma} \vec{E} \, \vec{ds} = \frac{q_1 + q_3 + q_5}{\varepsilon_0}$$

EXAM



7. The mechanic work L

$L = \vec{F} \cdot \vec{s}$



$dL = \vec{F} \cdot d\vec{r} \quad \Big| \int$

$\int dL = \int_{\vec{r_1}}^{\vec{r_2}} \vec{F}\, d\vec{r}$

$L = \int_{\vec{r_1}}^{\vec{r_2}} q \vec{E} d\vec{r} \Rightarrow L = q \int_{\vec{r_1}}^{\vec{r_2}} E dr$

(every angle in the point A)

$L = W_1 \cdot W_2$

Notation: $V = \dfrac{W}{q}$ electric potential

$\Delta V = V_2 - V_1 = \dfrac{W_2 - W_1}{q_2 \cdot q_1}$

$\Delta V = \dfrac{-L}{q} = \dfrac{-q \int_{s_1}^{s_2} E d\vec{r}}{q} \Rightarrow$

↓

works with arrows $\Rightarrow$ $\boxed{\Delta V = - \int_{\vec{r_1}}^{\vec{r_2}} \vec{E} \cdot d\vec{r}}$

$$\boxed{\vec{E} = \Delta V}$$

$$E = -\frac{dV}{dr} \Rightarrow dV = -\vec{E} \cdot \vec{dr}$$

$$\nabla = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} \cdot \frac{\partial}{\partial k}\vec{k} \Rightarrow \nabla = \frac{d}{dr}$$

(nabla)

$$V(x, y, z)$$

For example: $V(x,y,z) = x^2 y + 2xz^2 + 5e^{-2z} + 2$

$$E = -\frac{dV}{dr} \Rightarrow dV = -\vec{E} \cdot \vec{dr}$$

$$\int_{V_1}^{V_2} dV = -\int_{\vec{r}_1}^{\vec{r}_2} \vec{E}\,\vec{dr}$$

$$\Delta V = V_2 - V_1 = -\int_{r_1}^{r_2} \vec{E}\,\vec{dr}$$