IV WAVES

1. Mechanical waves are mechanical oscillations transmitted into an elastic media.

Mechanical waves are characterized by energy transport and not by mass transport.

2. Sources of oscillations

If a particle from an elastic media is forced to oscillate then the particle becomes a source of oscillation for all particles.

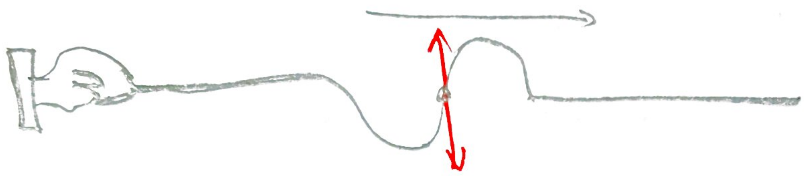
If a particle from an elastic media starts to oscillate, all the other particles from that elastic media will oscillate after a time period and will reproduce the motion of the first particle.

The propagation [transmition] of an oscillation into an elastic media is achieved from near to near due to the interaction forces that exists between the neighbour particles.

The transmition takes some time (is not instantaneous) and is produced with a finite velocity.

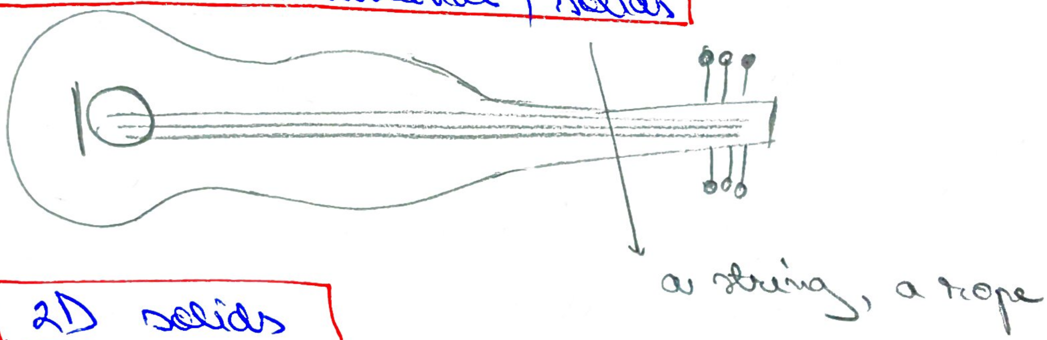
3. Types of waves

3.1. The transverse waves

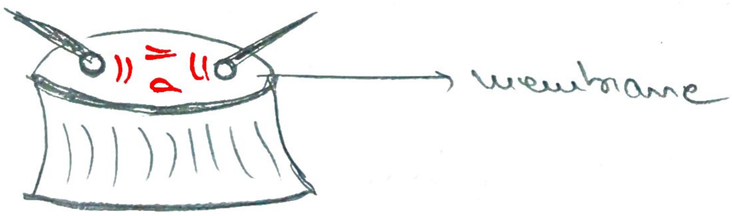


If the direction of oscillation of particles from an elastic media is perpendicular to the direction of propagation then the waves are <u>transverse waves</u>.

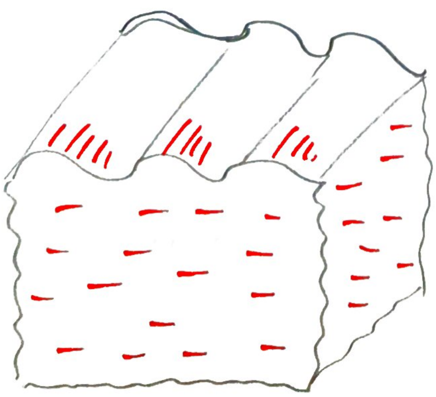
A) 1D (one dimensional) solids



B) 2D solids

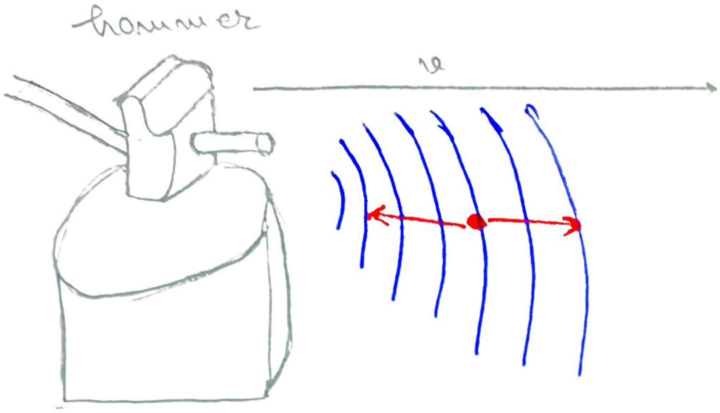


C) surface of liquids



3.2. Longitudinal waves.

(A)



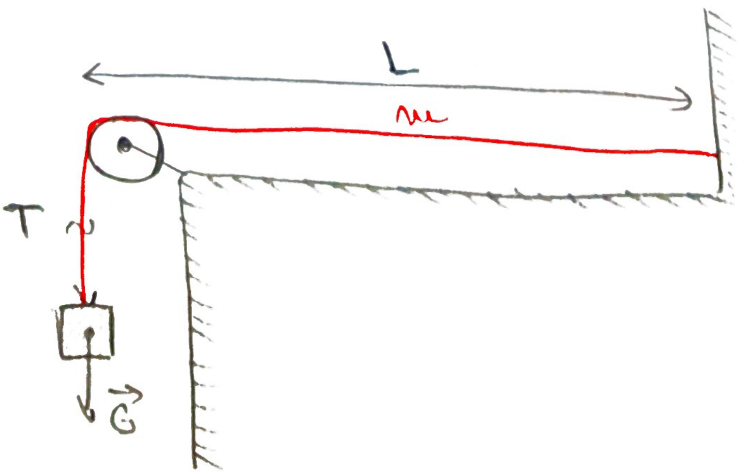
If the direction of oscillation of a particle of an elastic media is parallel with the direction of propagation of the wave than that wave is called longitudinal wave.

(B)
- 3D solids
- bulk liquids
- in gases

4. Velocity of waves

4.1. Velocity of transverse waves in solids

v_s^* : velocity of transverse waves in solids



$$\boxed{v_s^t = \sqrt{\frac{T}{\mu}}}$$

$\mu = \frac{m}{L}$ reduced mass (linear density)

4.2. Velocity of longitudinal waves in solids

v_s^l : velocity in longitudinal waves in solids;

$$\boxed{v_s^l = \sqrt{\frac{E}{\rho}}}$$

$\rho = \frac{m}{V}$ - the density

E = the modulus of elasticity / Young's modulus of elasticity

4.3. The velocity of longitudinal waves in liquids

v_l^l = the velocity of longitudinal waves in liquids;

$$\boxed{v_l^l = \sqrt{\frac{\chi}{\rho}}}$$

ρ: density

χ (chi) : the modulus of compresibility

4.4. The velocity of longitudinal waves in gases

v_g^l : the velocity of longitudinal waves in gases;

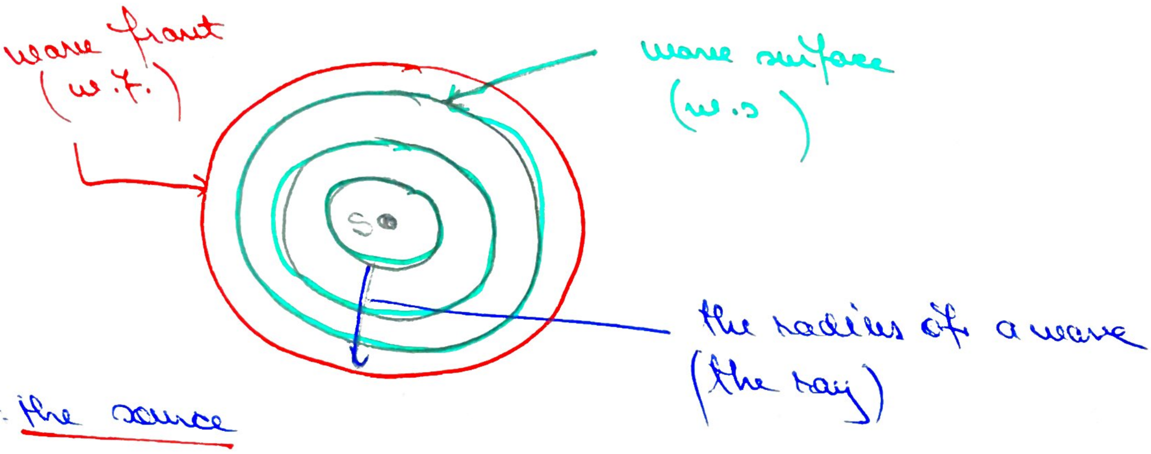
$$\boxed{v_g^l = \sqrt{\gamma \frac{P}{\rho}}}$$

P : pressure

$\gamma = \frac{C_p}{C_v}$: the adiabatic coefficient

C_p, C_v - the molar heats at constant pressure and volume;

5. Characteristics of waves



S: the source

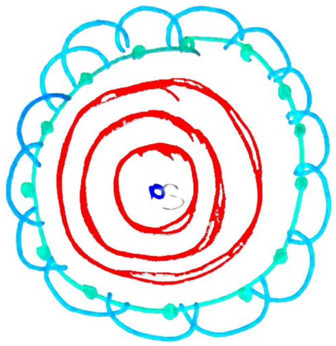
wave surface: the geometrical space of all adjacent points that oscillate in phase;

wave front: the most distant wave surface is called a wave front;

The wave front separates the points that are already in oscillation from the points that will oscillate later.

ray (radius) - a line perpendicular to the wave surface;

6. The Huygens' principle

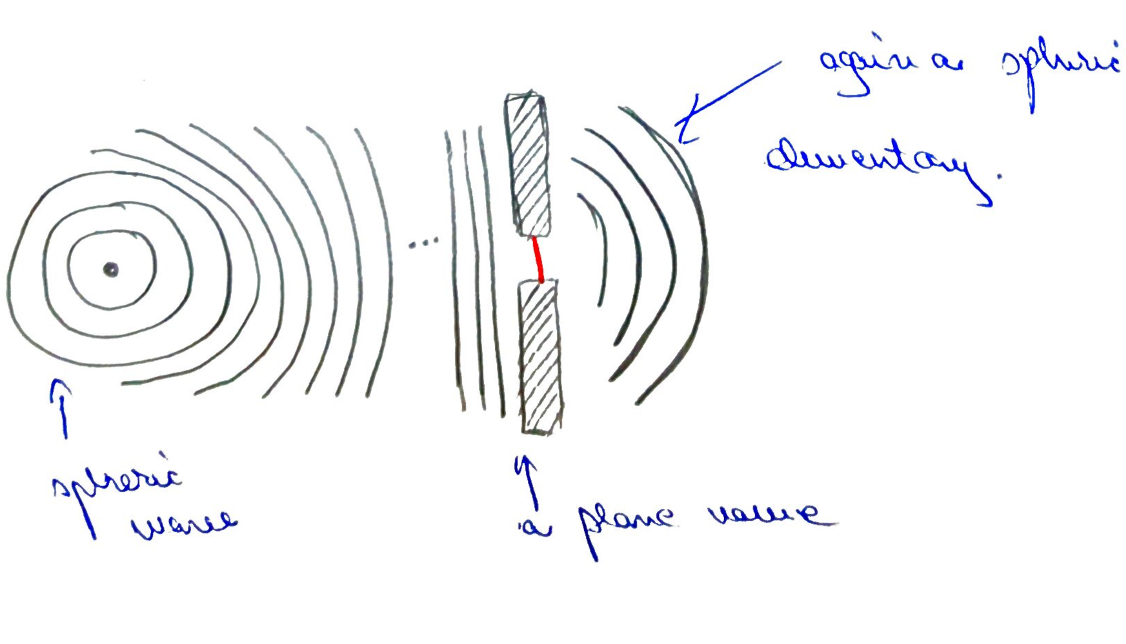


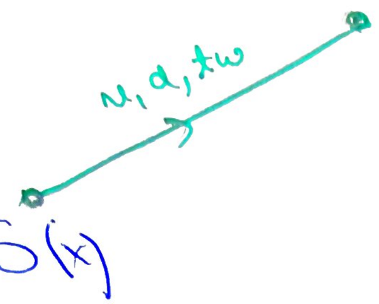
Each point from a front wave is considered as the secondary source of a new elementary wave with the same velocity, frequency and wavelength as the initial wave.

EXAM : applications of Huygens' principle

The new front wave can be found as an envelope of all elementary waves.

7. Spheric and plane waves





$$y_P(t_P) = A\sin(\omega \cdot t_P)$$

$$y_S(t_S) = A\sin(\omega t_S)$$

$$t_S = t_P + t$$

$$v = \frac{d}{t_w} \Rightarrow t_w = \frac{v}{d}$$

$$t_P = t_S - \frac{d}{v}$$

$$y_P(t) = A\sin\left[\omega\left(t_S - \frac{d}{v}\right)\right]$$

the elongation

$$\psi(d,t) = A\sin\left[\omega\left(t - \frac{d}{v}\right)\right]$$

wave function

$$\omega = \frac{2\pi}{T}$$

$$\psi(d,t) = A\sin\left[2\pi\left(\frac{t}{T} - \frac{d}{T \cdot v}\right)\right]$$

$$\lambda = T \cdot v$$

the wavelength

$$\boxed{\psi(d,t) = A\sin\left[2\pi\left(\frac{t}{T} - \frac{d}{\lambda}\right)\right]}$$

the equation of a plane wave

$$\psi(d,t) = A\sin\left(\frac{2\pi}{T}t - \frac{2\pi d}{\lambda}\right)$$

$$\omega = \frac{2\pi}{T} = 2\pi\nu$$

Note : $|\vec{k}| = k = \frac{2\pi}{\lambda}$ – the wave vector

$$\boxed{\psi(d,t) = A\sin(\omega t - kd)}$$

equation of a plan wave

$$\boxed{\psi(\vec{r},t) = A\sin(\omega t - \vec{k}\vec{r})}$$

equation of a spherical wave

$$\vec{k} = k_x\vec{i} + 0\vec{j} + 0\vec{k}$$

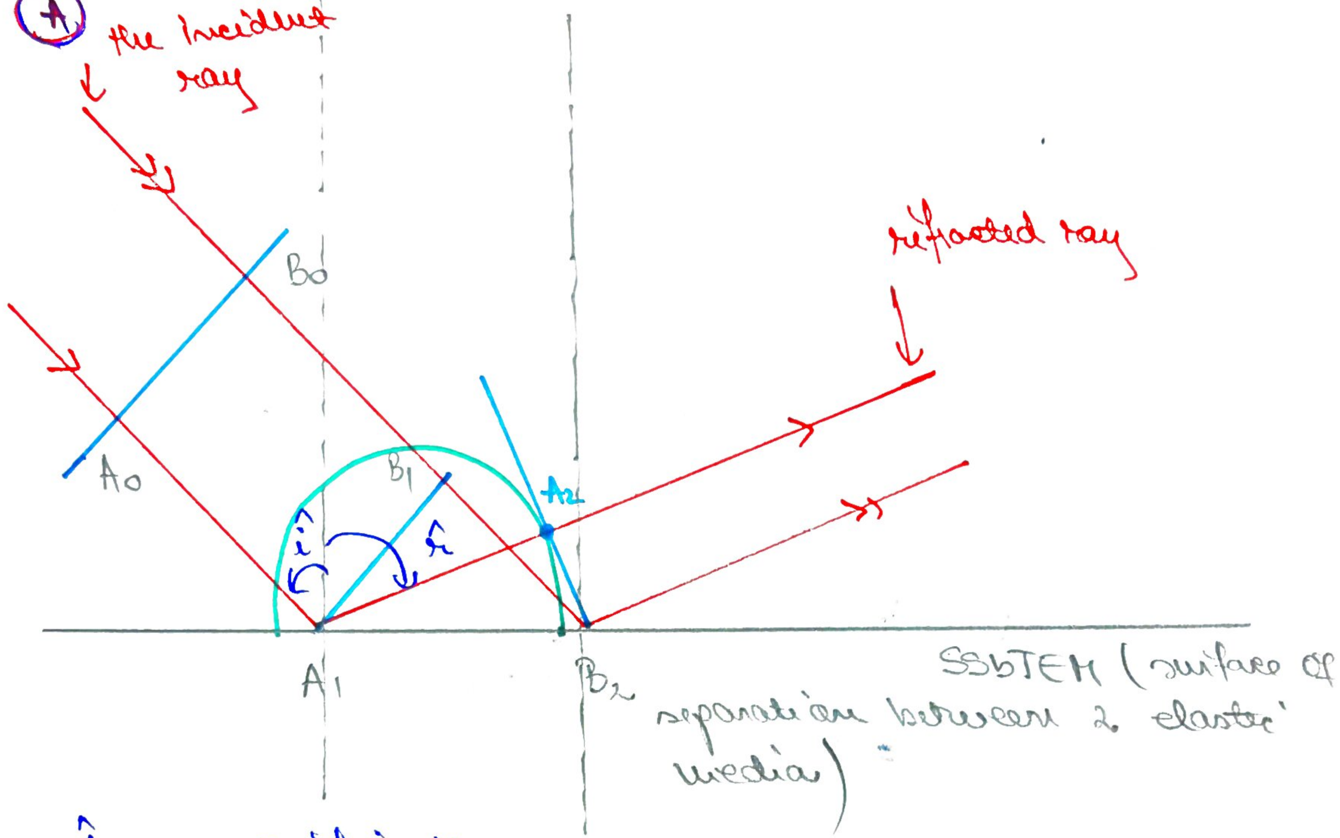
$$\vec{r} = x\cdot\vec{i} + y\cdot\vec{j} + z\cdot\vec{k}$$

$$\vec{k}\cdot\vec{r} = k_x\cdot x = \frac{2\pi}{\lambda}\cdot x$$

$$\psi(x,t) = A\sin(\omega t - kx)$$

9. The wave reflection

(A)



$\hat{i}$: angle of incidence
$\hat{r}$: angle of reflection

(B) Laws of reflection

$\boxed{\text{Law 1}}$: The incident ray, the normal and the reflected ray are in the same plane.

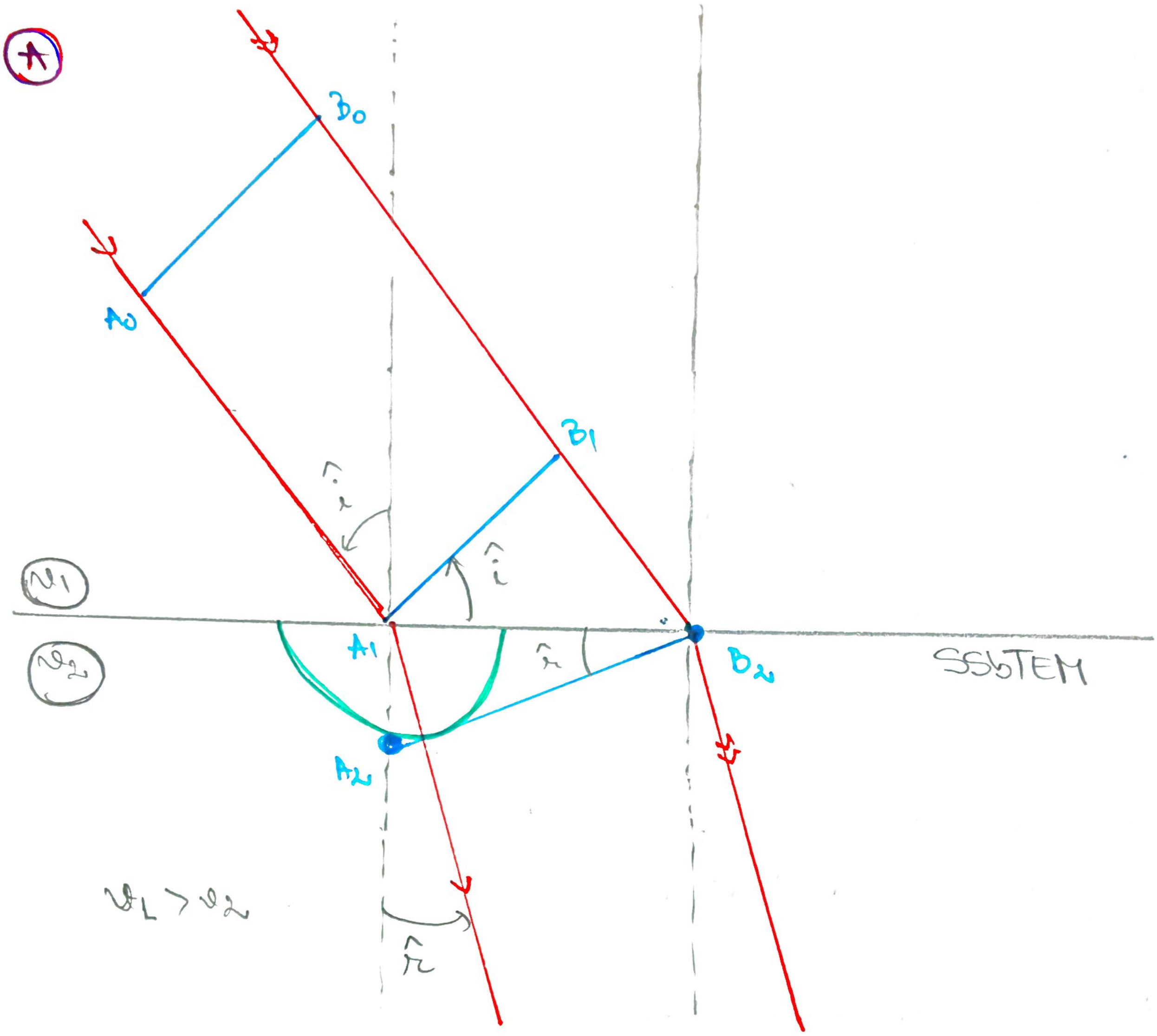
$\boxed{\text{Law 2}}$: The angle of incidence is equal to the angle of reflection.

The phenomena in which a propagating wave is drawn back into the original media is called wave reflection.

$\boxed{\text{EXAM}}$: draw the picture + apply the Huygen's principle

10. The wave refraction

Ⓐ



Ⓑ **Laws of refraction**

Law 1: The incident ray, the normal and the refracted ray are in the same plane

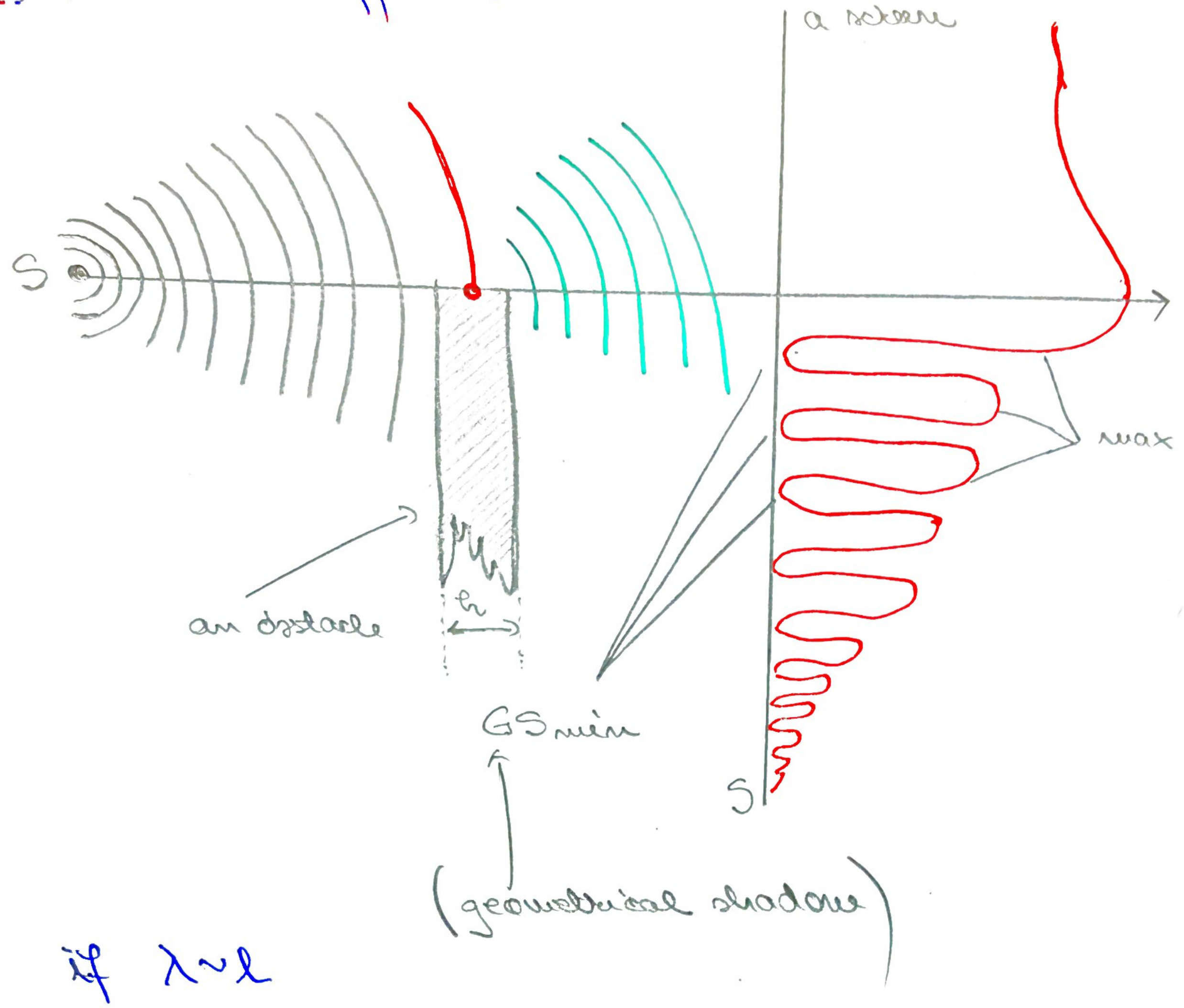
Law 2: $\boxed{\dfrac{\sin \hat{i}}{\sin \hat{r}} = \dfrac{v_1}{v_2} = n_{21}}$

$\Delta A_1B_1B_2$ $\quad \sin \hat{i} = \dfrac{B_1B_2}{A_1B_2}$

$\Delta A_1A_2B_2$ $\quad \sin \hat{r} = \dfrac{A_1A_2}{A_1B_2}$

$$\frac{\sin \hat{i}}{\sin \hat{r}} = \frac{\frac{B_1B_2}{A_1B_2}}{\frac{A_1A_2}{A_1A_2}} = \frac{v_1 \cdot t}{v_2 \cdot t} = \frac{v_1}{v_2}$$

11. The wave diffraction



if $\lambda \sim l$

12. The wave interference

Ⓐ Def: The superposition of two or more coherent waves resulting into a new wave pattern of minima and maxima is called wave interference.

Def: The attribute of two waves, or part of the same wave, to have the relative phase constant in time is called coherence.

$$\varphi_1(t) = \omega_1 t + \varphi_{01}$$

$$\varphi_2(t) = \omega_2 t + \varphi_{02}$$

$$\varphi_2(t) - \varphi_1(t) = (\omega_2 - \omega_1)t + \varphi_{02} - \varphi_{01}$$

$\omega_1 = \omega_2 = \omega$

$2\pi \nu_1 = 2\pi \nu_2 \Rightarrow \nu_1 = \nu_2 = \nu$

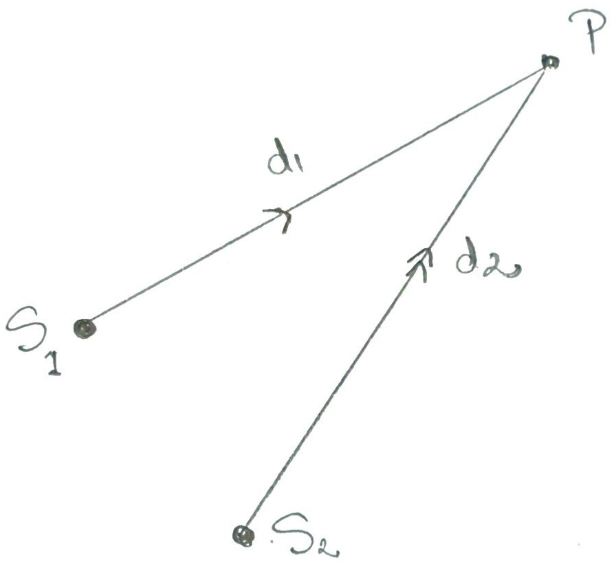
$$\frac{2\pi}{T_1} = \frac{2\pi}{T_2} = \frac{2\pi}{T}$$

$v_1 = v_2 = v$

$\lambda_1 = \lambda_2 = \lambda.$

$$\frac{2\pi}{\lambda_1} = \frac{2\pi}{\lambda_2} = \frac{2\pi}{\lambda} = k$$

B



$\psi_1(d_1,t) = A\sin(\omega t - kd_1)$

$\psi_2(d_2,t) = A\sin(\omega t - kd_2)$

$\psi(d_1,d_2,t) = \psi_1(d_1,t) + \psi(d_2,t)$

/* $\sin a + \sin b = 2\cos\frac{a-b}{2}\sin\frac{a+b}{2}$ */

$$\psi(d_1,d_2,t) = 2A\cos\left(\frac{\cancel{\omega t} - kd_1 - \cancel{\omega t} + kd_2}{2}\right) \cdot \sin\left(\frac{\omega t - kd_1 + \omega t - kd_2}{2}\right)$$

Notation: $d_2 - d_1 = \delta$

$$\boxed{\psi(\delta,t) = 2A\cos\left(\frac{k\cdot\delta}{2}\right)\sin\left(\omega t - k\cdot\frac{d_1+d_2}{2}\right)}$$

equation of the wave after interference

C) Minimum

$$k = \frac{2\pi}{\lambda}$$

$$\cos\left(\frac{\pi\delta}{\lambda}\right) = 0 \Rightarrow \frac{\pi\delta}{\lambda} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots$$

$$\frac{\pi\delta_n}{\lambda} = (2n+1)\frac{\pi}{2} \qquad n = 0, 1, 2 \ldots$$

$$\boxed{\delta_n^{min} = (2n+1)\frac{\lambda}{2}}$$

D) Maximum

$$\cos\left(\frac{\pi\delta}{\lambda}\right) = \pm 1$$

$$\frac{\pi\delta}{\lambda} = 0, \pi, 2\pi, 4\pi \ldots$$

$$\frac{\pi\delta_n}{2} = 2n\frac{\pi}{2} \qquad n = 0, 1, 2 \ldots$$

$$\boxed{\delta_n^{max} = 2n\frac{\lambda}{2}}$$

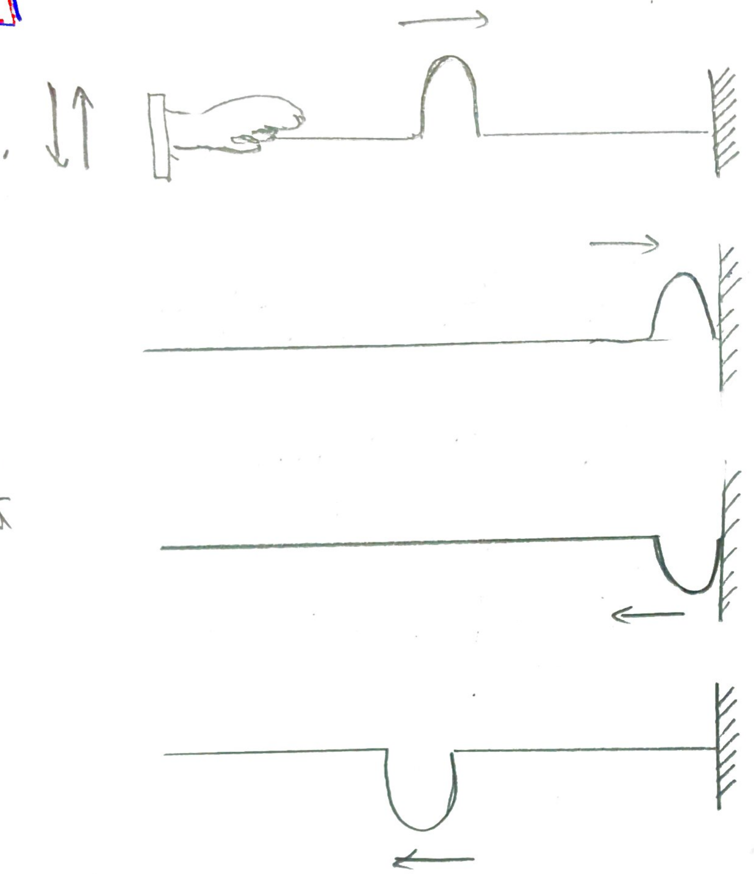
EXAM:
- min, max;
- example of wave;
- wave interference;
- apply the Huygens' principle

13. Standing waves

(A) Types of reflections

I

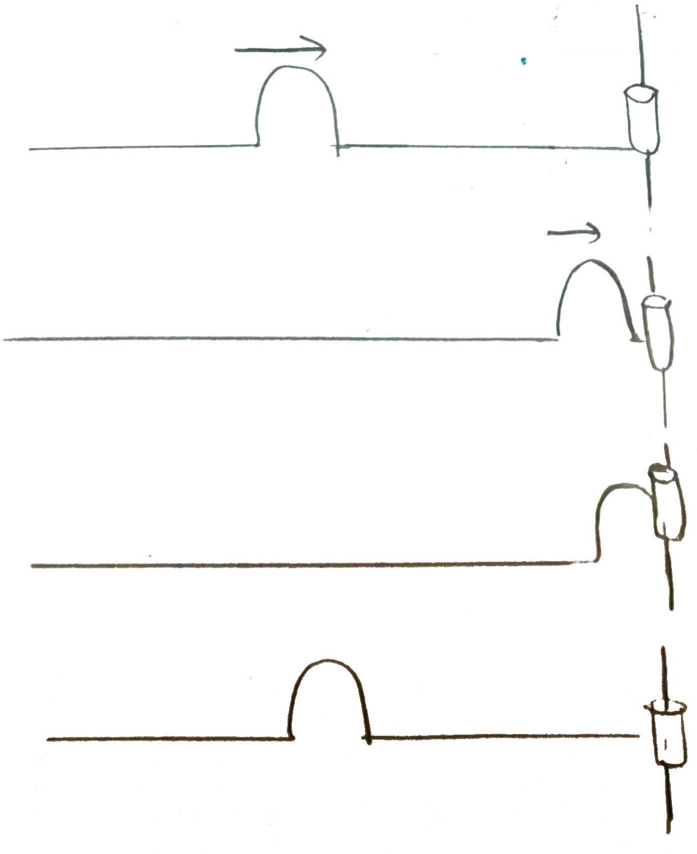
fixed



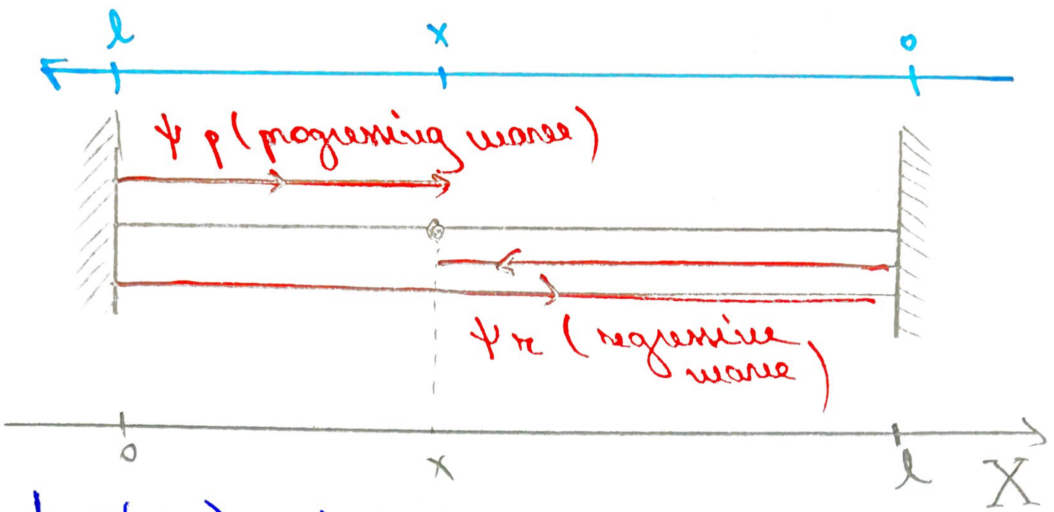
with a lot of

$\frac{1}{2}, \frac{T}{2}, \frac{T}{4}$

II



B



$$\psi p(x,t) = A\sin(\omega t - kx)$$

$$\psi r(x,t) = A\sin[\omega t - k(2l - x) - \pi]$$

$$\psi(x,t) = \psi p(x,t) + \psi r(x,t)$$

$$\psi(x,t) = 2A\cos\frac{\omega t - kx - \omega t + k(2l - x) + \pi}{2} \cdot$$

$$\cdot \sin\left(\frac{\omega t - kx + \omega t - k(2l - x) - \pi}{2}\right)$$

$$\psi(x,t) = 2A\cos\left[k(l - \underset{\overset{\parallel}{x}}{x}) + \frac{\pi}{2}\right]\sin\left(\omega t - kl - \frac{\pi}{2}\right)$$

$$\boxed{\psi(x,t) = 2A\sin(kx)\cos(\omega t - kl)}$$

(a) maximum

$$\sin(kx) = \pm 1 \;; \quad k = \frac{2\pi}{\lambda}$$

$$\frac{2\pi x}{\lambda} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots, (2n+1)\frac{\pi}{2}$$

$$\frac{2\pi x}{\lambda} = (2m+1)\frac{\pi}{2}$$

$$\boxed{X_n^{max} = (2n+1)\frac{\lambda}{4}} \qquad n = 0, 1, 2 \dots$$

b) minimum

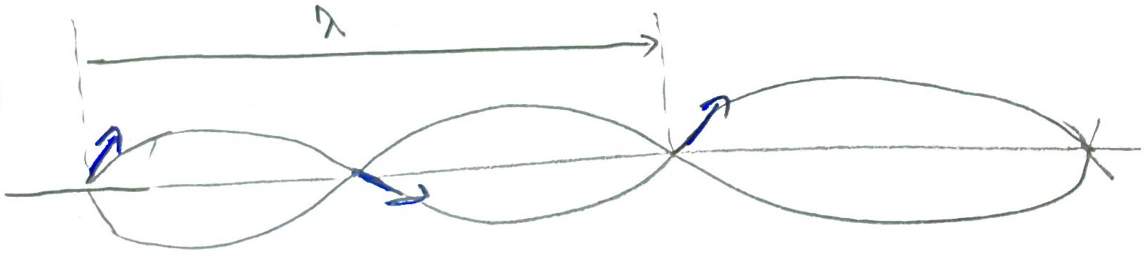
$$\sin(kx) = 0$$

$$\frac{2\pi x}{\lambda} = 0, \pi, 2\pi \dots$$

$$\frac{2\pi x_n}{\lambda} = n\pi \frac{2}{2}$$

$$\boxed{X_n^{min} = 2n\frac{\lambda}{4}} \qquad n = 0, 1, 2, 3 \dots$$

c)



$$i = X_{n+1}^{min} - X_n^{min} =$$

$$= 2(n+1)\frac{\lambda}{4} - 2n\frac{\lambda}{4}$$

$$\boxed{i = \frac{\lambda}{2}}$$: the distance between two minimum is half of the wavelength

$$\boxed{\lambda = 2i}$$