

Samodejno odvajanje

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1 Samodejno odvajanje

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2 Levi in desni odvodi

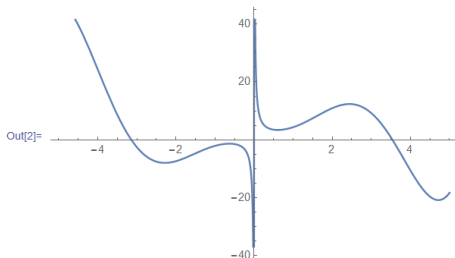
1 Samodejno odvajanje

2 Levi in desni odvodi

3 Lipschitzeve konstante

```
In[1]:= f[x_] := Exp[x] / x + 2 x^2 Sin[x];
```

```
In[2]:= Plot[f[x], {x, -5, 5}]
```



```
In[3]:= Table[N[D[f[x], {x, k}] /. x -> 1, 17], {k, 0, 14}]
```

```
Out[3]= {4.4012237980748382, 4.4464885509678655, 8.7236422450199560, -10.131192415931451,
-2.6926621035325515, -123.30646837902937, 782.11725700134034, -5018.9509084565035,
40210.422879416369, -362926.18094394889, 3.6289716219290460 × 106, -3.9916719028555494 × 107,
4.7900135379519123 × 108, -6.2270209235552409 × 109, 8.7178291535039949 × 1010}
```

```
(*
```

```
Haskell]:
```

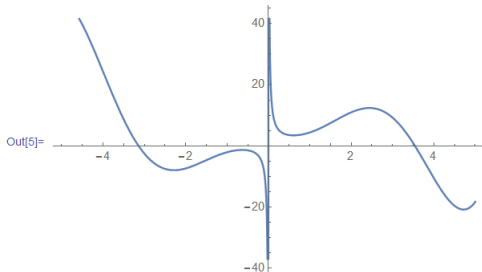
```
f (idD 1);
```

```
4.401223798074838,4.4464885509678655,8.723642245019956,-10.131192415931453,
-2.692662103532559,-123.30646837902931,782.1172570013406,-5018.950908456501,
40210.42287941638,-362926.1809439487,3628971.6219290467,-3.991671902855549e7,
4.7900135379519105e8,-6.227020923555244e9,8.71782915350399e10, ...
```

```
*)
```

```
In[4]:= f[x_] := Exp[x] / x + 2 x^2 Sin[x];
```

```
In[5]:= Plot[f[x], {x, -5, 5}]
```



```
In[7]:= Table[N[D[f[x], {x, k}] /. x -> 0.01, 17], {k, 0, 14}]
```

```
Out[7]= {101.005, -9999.5, 2. × 106, -6. × 108, 2.4 × 1011, -1.2 × 1014, 7.2 × 1016, -5.04 × 1019, 4.032 × 1022,  
-3.6288 × 1025, 3.6288 × 1028, -3.99168 × 1031, 4.79002 × 1034, -6.22702 × 1037, 8.71783 × 1040}
```

```
(*
```

```
Haskell:
```

```
f (idD 0.01);
```

```
101.00501870838346,-9999.496054149928,2000000.4558366945,-5.999999877499917e8,  
2.39999999998017e11,-1.200000000003981e14,7.2e16,-5.03999999999999e19,  
4.032000000000001e22,-3.628799999999995e25,3.6288e28,-3.991679999999992e31,  
4.790015999999998e34,-6.227020799999999e37,8.71782912e40, ...
```

```
*)
```



```
g(x) = x^2  
g (idD 4)  
16.0, 8.0, 2.0, 0
```


$g(x) = x^2$ $g \text{ (idD 4)}$

16.0, 8.0, 2.0, 0

 $h(x) = x^6$ $h \text{ (idD 1)}$

1.0, 6.0, 30.0, 120.0, 360.0, 720.0, 720.0, 0

```
g(x) = x^2
```

```
g (idD 4)
```

```
16.0, 8.0, 2.0, 0
```

```
h(x) = x^6
```

```
h (idD 1)
```

```
1.0, 6.0, 30.0, 120.0, 360.0, 720.0, 720.0, 0
```

```
k(x) = sin(x^2)
```

```
k (idD 1)
```

```
0.8414709848078965, 1.0806046117362795, -2.2852793274953065,  
-14.420070264639875, -22.568626742439122, 87.08875465286715,  
746.5137961465103, 2028.9913034612805, -4355.06268259366,  
-69627.83387598622, -308672.6345339131, Interrupted.
```

```
g(x) = x^2
```

```
g (idD 4)
```

```
16.0, 8.0, 2.0, 0
```

```
h(x) = x^6
```

```
h (idD 1)
```

```
1.0, 6.0, 30.0, 120.0, 360.0, 720.0, 720.0, 0
```

```
k(x) = sin(x^2)
```

```
k (idD 1)
```

```
0.8414709848078965, 1.0806046117362795, -2.2852793274953065,  
-14.420070264639875, -22.568626742439122, 87.08875465286715,  
746.5137961465103, 2028.9913034612805, -4355.06268259366,  
-69627.83387598622, -308672.6345339131, Interrupted.
```

Kjer funkcija ni odvedljiva, dobimo čudne stvari:

$$g(x) = x^2$$

$$g \text{ (idD 4)}$$

$$16.0, 8.0, 2.0, 0$$

$$h(x) = x^6$$

$$h \text{ (idD 1)}$$

$$1.0, 6.0, 30.0, 120.0, 360.0, 720.0, 720.0, 0$$

$$k(x) = \sin(x^2)$$

$$k \text{ (idD 1)}$$

$$0.8414709848078965, 1.0806046117362795, -2.2852793274953065, \\ -14.420070264639875, -22.568626742439122, 87.08875465286715, \\ 746.5137961465103, 2028.9913034612805, -4355.06268259366, \\ -69627.83387598622, -308672.6345339131, \text{ Interrupted.}$$

Kjer funkcija ni odvedljiva, dobimo čudne stvari:

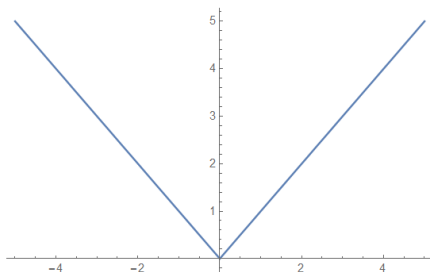
$$f \text{ (idD 0)}$$

$$\text{Infinity, NaN, NaN, NaN, NaN, NaN, NaN, NaN, NaN, NaN, NaN, NaN, NaN, NaN, NaN, NaN, ...}$$

```
In[8]:= g[x_] := Abs[x];
```

```
In[9]:= Plot[g[x], {x, -5, 5}]
```

Out[9]=



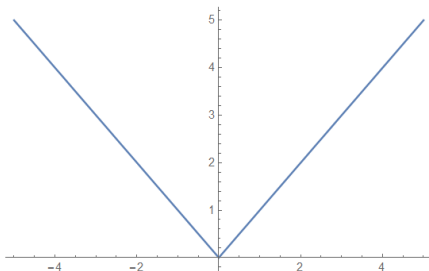
```
In[10]:= Table[N[D[g[x], {x, k}] /. x -> 0, 17], {k, 0, 14}]
```

```
Out[10]= {0, Abs'[0], Abs''[0], Abs(3)[0], Abs(4)[0], Abs(5)[0], Abs(6)[0], Abs(7)[0],  
Abs(8)[0], Abs(9)[0], Abs(10)[0], Abs(11)[0], Abs(12)[0], Abs(13)[0], Abs(14)[0]}
```

```
In[8]:= g[x_] := Abs[x];
```

```
In[9]:= Plot[g[x], {x, -5, 5}]
```

Out[9]=



```
In[10]:= Table[N[D[g[x], {x, k}] /. x -> 0, 17], {k, 0, 14}]
```

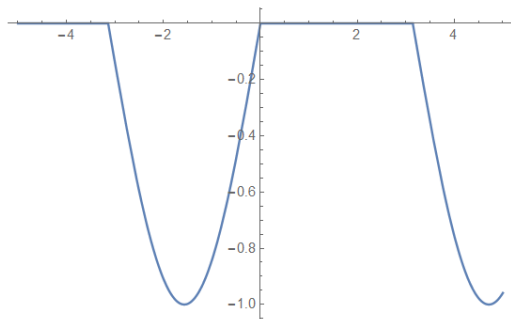
```
Out[10]= {0, Abs'[0], Abs''[0], Abs(3)[0], Abs(4)[0], Abs(5)[0], Abs(6)[0], Abs(7)[0],  
Abs(8)[0], Abs(9)[0], Abs(10)[0], Abs(11)[0], Abs(12)[0], Abs(13)[0], Abs(14)[0]}
```

Haskell:

```
g (idD 0)
```

```
D 0.0 -1.0 1.0
```

```
In[11]:= h[x_] := Min[Sin[x], 0];
Plot[h[x], {x, -5, 5}]
```

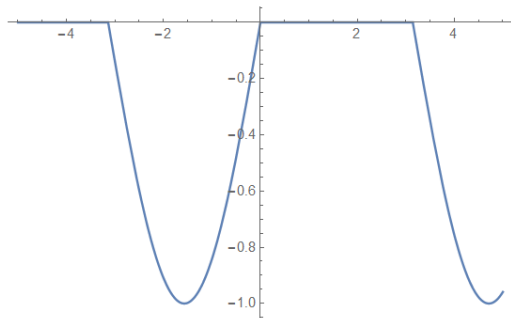


Out[12]=

```
In[13]:= D[h[x], x]
```

```
Out[13]= { Cos[x] Sin[x] < 0
           0 True }
```

```
In[11]:= h[x_] := Min[Sin[x], 0];
Plot[h[x], {x, -5, 5}]
```



Out[12]=

```
In[13]:= D[h[x], x]
```

```
Out[13]= { Cos[x] Sin[x] < 0
           0 True }
```

Haskell:

```
h (idD 0)
D 0.0 1.0 0.0
```


Lipschitzeva funkcija: $\exists C \geq 0 : |f(x) - f(y)| \leq C|x - y|, \forall x, y$

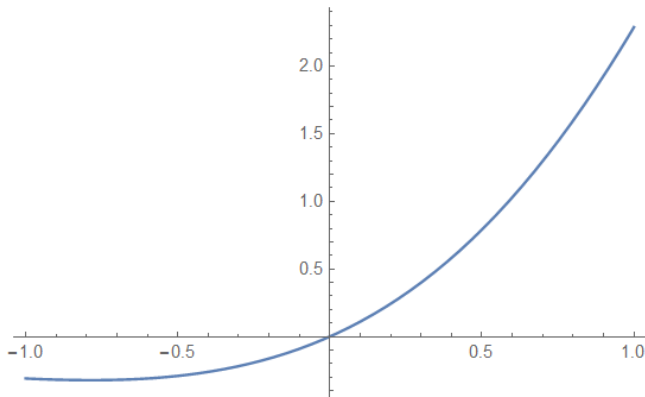
Lipschitzeva funkcija: $\exists C \geq 0 : |f(x) - f(y)| \leq C|x - y|, \forall x, y$

Lokalno: $C_1|x - y| \leq |f(x) - f(y)| \leq C_2|x - y|, \forall x, y \in [a - \textit{eps}, a + \textit{eps}]$

```
In[3]:= f[x_] := Exp[x] * Sin[x];
```

```
In[4]:= Plot[f[x], {x, -1, 1}]
```

```
Out[4]=
```



```
In[11]:= D[f[x], x] /. x -> 0.5
```

```
Out[11]= 2.23733
```

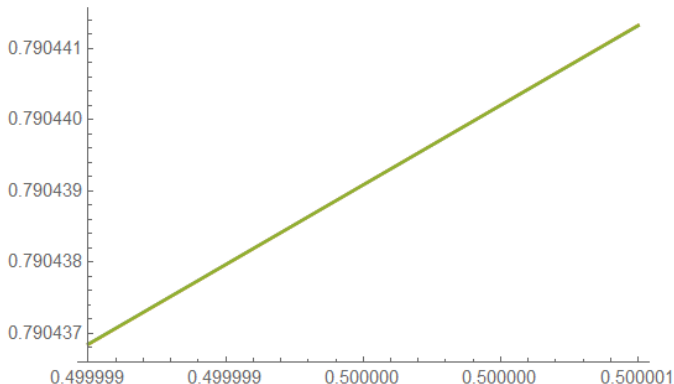
f (constL 0.5 0.001 0.001)

L 0.7904390832136149 2.2409056829475054 2.2366443367212314 1.0e-3

```
f (constL 0.5 0.001 0.001)
```

```
L 0.7904390832136149 2.2409056829475054 2.2366443367212314 1.0e-3
```

```
Plot[{f[x], f[0.5] + (x - 0.5) * 2.2409056829475054,  
      f[0.5] + (x - 0.5) * 2.2366443367212314}, {x, 0.499999, 0.500001}]
```



Računanje integralov

- Mathematica:

```
N[Integrate[f[x], {x, -1, 1}], 15]  
0.663493666631241
```

Računanje integralov

- Mathematica:

```
N[Integrate[f[x], {x, -1, 1}], 15]  
0.663493666631241
```

- Haskell:

```
zgornja meja = 0.6635009523819319
```

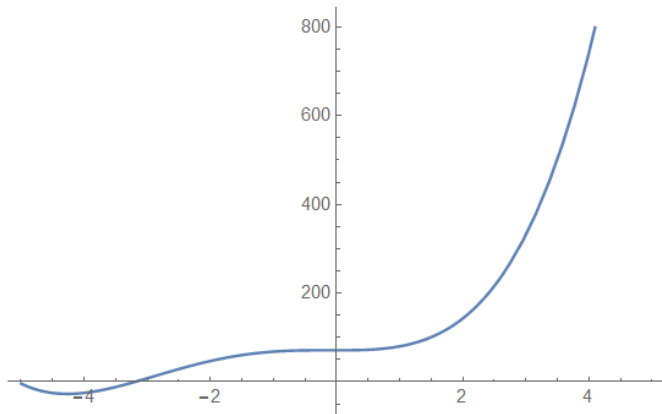
```
spodnja meja = 0.6634863808534411
```

```
integral = 0.6634936666176671
```



```
In[15]:= g[x_] := x^4 + 6 x^3 + 2 x^2 + 71;  
Plot[g[x], {x, -5, 5}]
```

Out[16]=



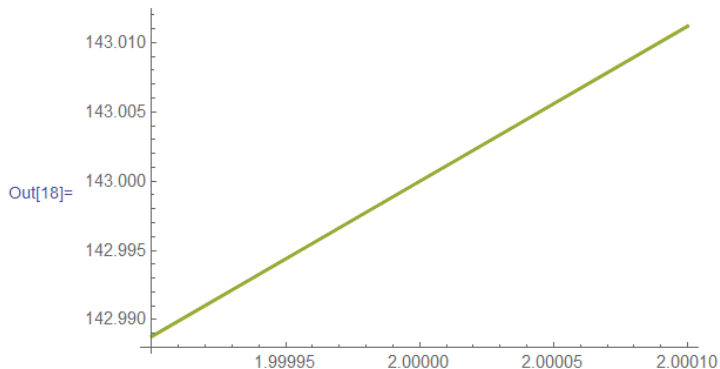
```
In[20]:= D[g[x], x] /. x -> 2
```

Out[20]= 112

```
g (idL 2 0.001 0.001)
```

```
L 143.0 112.30429833519612 111.81973011116794 1.0e-3
```

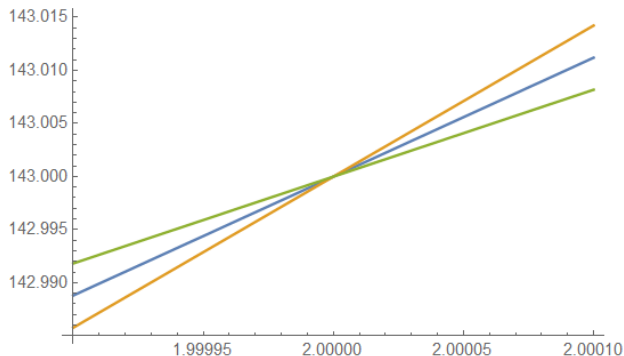
```
In[18]:= Plot[{g[x], g[2] + (x - 2) * 112.30429833519612,  
              g[2] + (x - 2) * 111.81973011116794}, {x, 2 - 0.0001, 2 + 0.0001}]
```



```
g (idL 2 0.001 0.001)
```

```
L 143.0 112.30429833519612 111.81973011116794 1.0e-3
```

```
Plot[{fg[x], fg[2] + (x - 2) * (112.30429833519612 + 30),  
      fg[2] + (x - 2) * (111.81973011116794 - 30)}, {x, 2 - 0.0001, 2 + 0.0001}]
```



Računanje integralov

- Mathematica:

```
N[Integrate[g[x], {x, -5, 5}], 15]  
2126.66666666667
```

Računanje integralov

- Mathematica:

```
N[Integrate[g[x], {x, -5, 5}], 15]  
2126.66666666667
```

- Haskell:

```
zgornja meja = 2128.700864620524
```

```
spodnja meja = 2124.6324687729543
```

```
integral = 2126.6666666970277
```