Samodejno odvajanje

Andrej Borštnik, Barbara Bajcer

Fakulteta za matematiko in fiziko

8. januar 2015

Kratek pregled

1 Samodejno odvajanje

Kratek pregled

1 Samodejno odvajanje

2 Levi in desni odvodi

Kratek pregled

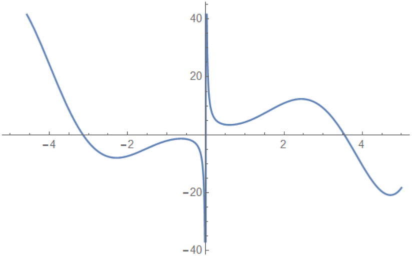
1 Samodejno odvajanje

2 Levi in desni odvodi

3 Lipschitzeve konstante

```
In[1]:= f[x_] := Exp[x] / x + 2 x^2 Sin[x];

In[2]:= Plot[f[x], {x, -5, 5}]
```

Out[2]=

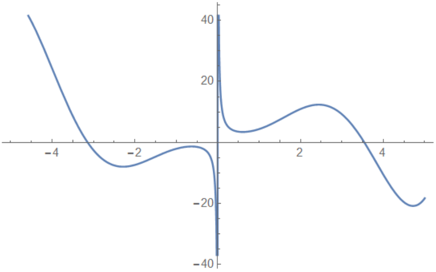



```
In[3]:= Table[N[D[f[x], {x, k}] /. x → 1, 17], {k, 0, 14}]

Out[3]= {4.4012237980748382, 4.4464885509678655, 8.7236422450199560, -10.131192415931451,
 -2.6926621035325515, -123.30646837902937, 782.11725700134034, -5018.9509084565035,
 40 210.422879416369, -362 926.18094394889, 3.6289716219290460 × 10^6, -3.9916719028555494 × 10^7,
 4.7900135379519123 × 10^8, -6.2270209235552409 × 10^9, 8.7178291535039949 × 10^10}

(*
Haskell:
 f (idD 1);
4.401223798074838,4.446488550967865 5,8.723642245019956,-10.131192415931453,
-2.69266210353255 9,-123.30646837902931,782.1172570013406,-5018.950908456501,
40210.42287941638,-362926.1809439487,3628971.6219290467,-3.991671902855549e7,
4.7900135379519105e8,-6.227020923555244e9,8.71782915350399e10, ...
*)
```

In[4]:=
```
f[x_] := Exp[x] / x + 2 x^2 Sin[x];
```

In[5]:=
```
Plot[f[x], {x, -5, 5}]
```

Out[5]=



In[7]:=
```
Table[N[D[f[x], {x, k}] /. x → 0.01, 17], {k, 0, 14}]
```

Out[7]= $\{101.005, -9999.5, 2.\times10^{6}, -6.\times10^{8}, 2.4\times10^{11}, -1.2\times10^{14}, 7.2\times10^{16}, -5.04\times10^{19}, 4.032\times10^{22}, -3.6288\times10^{25}, 3.6288\times10^{28}, -3.99168\times10^{31}, 4.79002\times10^{34}, -6.22702\times10^{37}, 8.71783\times10^{40}\}$

```
(*
Haskell:
 f (idD 0.01);
101.00501870838346,-9999.496054149928,2000000.4558366945,-5.999999877499917e8,
2.399999999998017e11,-1.2000000000003981e14,7.2e16,-5.0399999999999999e19,
4.032000000000001e22,-3.6287999999999995e25,3.6288e28,-3.9916799999999992e31,
4.790015999999998e34,-6.227020799999999e37,8.71782912e40, ...
*)
```

```
g(x) = x^2
g (idD 4)
16.0, 8.0, 2.0, 0
```

```
g(x) = x^2
g (idD 4)
16.0, 8.0, 2.0, 0

h(x) = x^6
h (idD 1)
1.0, 6.0, 30.0, 120.0, 360.0, 720.0, 720.0, 0
```

```
g(x) = x^2
g (idD 4)
16.0, 8.0, 2.0, 0

h(x) = x^6
h (idD 1)
1.0, 6.0, 30.0, 120.0, 360.0, 720.0, 720.0, 0

k(x) = sin(x^2)
k (idD 1)
0.8414709848078965, 1.0806046117362795, -2.2852793274953065,
-14.420070264639875, -22.568626742439122, 87.08875465286715,
746.5137961465103, 2028.9913034612805, -4355.06268259366,
-69627.83387598622, -308672.6345339131, Interrupted.
```

```
g(x) = x^2
g (idD 4)
16.0, 8.0, 2.0, 0

h(x) = x^6
h (idD 1)
1.0, 6.0, 30.0, 120.0, 360.0, 720.0, 720.0, 0

k(x) = sin(x^2)
k (idD 1)
0.8414709848078965, 1.0806046117362795, -2.2852793274953065,
-14.420070264639875, -22.568626742439122, 87.08875465286715,
746.5137961465103, 2028.9913034612805, -4355.06268259366,
-69627.83387598622, -308672.6345339131, Interrupted.
```

Kjer funkcija ni odvedljiva, dobimo čudne stvari:

```
g(x) = x^2
g (idD 4)
16.0, 8.0, 2.0, 0

h(x) = x^6
h (idD 1)
1.0, 6.0, 30.0, 120.0, 360.0, 720.0, 720.0, 0

k(x) = sin(x^2)
k (idD 1)
0.8414709848078965, 1.0806046117362795, -2.2852793274953065,
-14.420070264639875, -22.568626742439122, 87.08875465286715,
746.5137961465103, 2028.9913034612805, -4355.06268259366,
-69627.83387598622, -308672.6345339131, Interrupted.
```

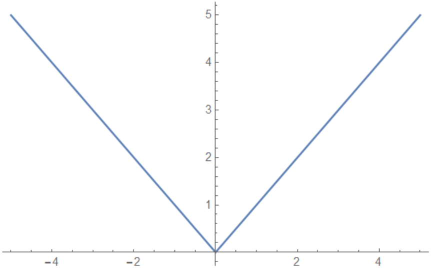
Kjer funkcija ni odvedljiva, dobimo čudne stvari:

```
f (idD 0)
Infinity, NaN, NaN, NaN, NaN, NaN, NaN, NaN, NaN, NaN, NaN, NaN, NaN,
NaN, NaN, NaN, NaN, ...
```

In[8]:=
```
g[x_] := Abs[x];
```

In[9]:=
```
Plot[g[x], {x, -5, 5}]
```

Out[9]=

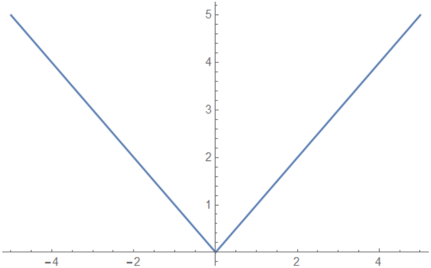


In[10]:=
```
Table[N[D[g[x], {x, k}] /. x → 0, 17], {k, 0, 14}]
```

Out[10]= $\{0, \text{Abs}'[0], \text{Abs}''[0], \text{Abs}^{(3)}[0], \text{Abs}^{(4)}[0], \text{Abs}^{(5)}[0], \text{Abs}^{(6)}[0], \text{Abs}^{(7)}[0], \text{Abs}^{(8)}[0], \text{Abs}^{(9)}[0], \text{Abs}^{(10)}[0], \text{Abs}^{(11)}[0], \text{Abs}^{(12)}[0], \text{Abs}^{(13)}[0], \text{Abs}^{(14)}[0]\}$

```
In[8]:= g[x_] := Abs[x];

In[9]:= Plot[g[x], {x, -5, 5}]
```

Out[9]=




```
In[10]:= Table[N[D[g[x], {x, k}] /. x → 0, 17], {k, 0, 14}]
```

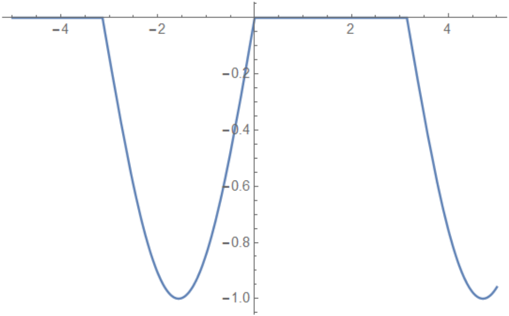
Out[10]= $\{0, \mathrm{Abs}'[0], \mathrm{Abs}''[0], \mathrm{Abs}^{(3)}[0], \mathrm{Abs}^{(4)}[0], \mathrm{Abs}^{(5)}[0], \mathrm{Abs}^{(6)}[0], \mathrm{Abs}^{(7)}[0], \mathrm{Abs}^{(8)}[0], \mathrm{Abs}^{(9)}[0], \mathrm{Abs}^{(10)}[0], \mathrm{Abs}^{(11)}[0], \mathrm{Abs}^{(12)}[0], \mathrm{Abs}^{(13)}[0], \mathrm{Abs}^{(14)}[0]\}$

Haskell:

```
g (idD 0)
D 0.0 -1.0 1.0
```

In[11]:=
```
h[x_] := Min[Sin[x], 0];
Plot[h[x], {x, -5, 5}]
```

Out[12]=

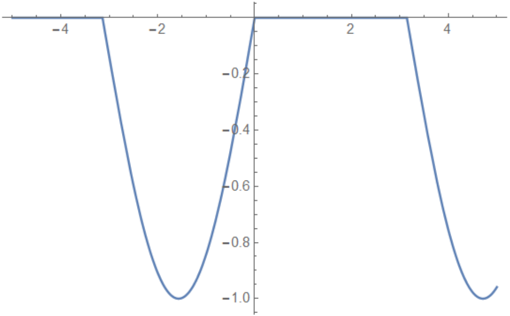


In[13]:=
```
D[h[x], x]
```

Out[13]=
$$\begin{cases} \mathrm{Cos}[x] & \mathrm{Sin}[x] < 0 \\ 0 & \mathrm{True} \end{cases}$$

```
In[11]:= h[x_] := Min[Sin[x], 0];
         Plot[h[x], {x, -5, 5}]
```

Out[12]=




```
In[13]:= D[h[x], x]

Out[13]= { Cos[x]   Sin[x] < 0
         { 0        True
```

Haskell:

```
h (idD 0)
D 0.0 1.0 0.0
```

Lipschitzeva funkcija: $\exists C \geq 0 : |f(x) - f(y)| \leq C|x - y|, \forall x, y$

Lipschitzeva funkcija: $\exists C \geq 0 : |f(x) - f(y)| \leq C|x - y|, \forall x, y$

Lokalno: $C_1|x - y| \leq |f(x) - f(y)| \leq C_2|x - y|, \forall x, y \in [a - eps, a + eps]$