Learning Conditional Random Field parameters from training data

A review

Przemyslaw Polewski

Photogrammetry and Remote Sensing
Technische Universität München

Department of Geoinformatics
Hochschule München

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Outline

1. Conditional Random Fields
2. Direct methods
   - Probabilistic modeling
   - Grid search
3. Maximum likelihood variations
   - Pseudo-likelihood
   - Marginal approximations
4. Max-margin learning
   - Structured SVM
5. Software libraries

Conditional Random Fields

Definition

- Consider set of objects $O = \{o_1, \ldots, o_n\}$ with features $X = \{x_1, \ldots, x_n\}$ and labels $Y = \{y_1, \ldots, y_n\}$.

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Equivalent energy formulation:

$$E(y) = -\log P(y|x) = \sum_{c \in \mathscr{C}(G)} \psi_c(y_c|x) - \log Z(x) \tag{2}$$

Conditional Random Fields

Notation

$$E(y;\overbrace{\Theta}^{\text{param.}}) = \sum_{c \in \underbrace{\mathscr{C}(G)}_{\text{cliques of G}}} \overbrace{\psi_c(y_c|x;\Theta)}^{\text{clique potentials}} - \log \underbrace{Z(x;\Theta)}_{\text{partition function}}$$

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- Θ: vector of parameters $\theta_1, \theta_2, \ldots, \theta_m$
- Z: function of data and parameters, but not of labels

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- Inference: minimize E w.r.t. **labels** y given parameters

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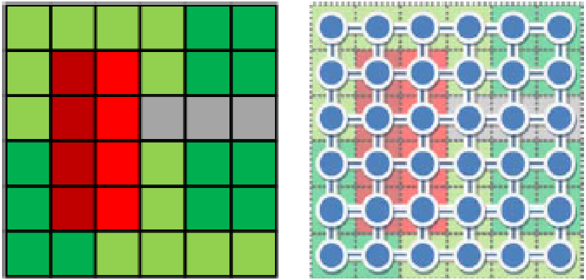
Two complementary tasks:

- Inference: minimize E w.r.t. **labels** y given parameters
- Learning: minimize E w.r.t. **parameters** Θ given training labels

Conditional Random Fields

Neighborhood examples

Raster image



ⓒWegner, 2011

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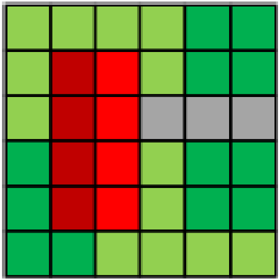

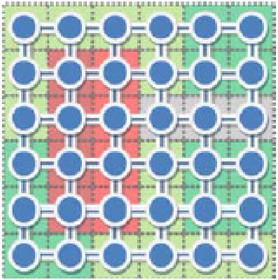

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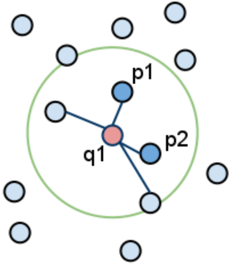
Point cloud



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- Drawback: need a lot of training examples for rare class combinations
- E.g. for unary and binary cliques:

$$E(y) = -\sum_i \log P(y_i|x) - \sum_i \sum_{j \in N(i)} \log P(\underbrace{y_i, y_j}_{\text{for all class pairs}}|x) \tag{3}$$

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Classification of urban ALS point clouds [Niemeyer et al., 2014]

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> "A comparison of three different versions of a CRF-based classifier has shown that Random Forests are well suited for the computation of unary and pairwise potentials needed for CRFs(. . .)"
>
> — Niemeyer et al.

©Niemeyer et al. 2014

Grid search / empirical approach

- Applicable when number of parameters is small and their domain is known a priori better than $(-\infty;\infty)$ (e.g. [0;1])
- Grid search on parameter combinations $\Theta^k \in \mathbf{\Theta}$ with repeated inference:

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for $k = 1$ to $|\boldsymbol{\Theta}|$ **do**
    $y^k = \arg\min_y E(y|x; \Theta^k)$
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Will (almost) never find the true optimal weights

- Discrete grid $\neq$ continuous weights

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Example

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$$\psi(y_i, y_j; x) = [y_i = y_j] \cdot \boldsymbol{w_1} \cdot \frac{N_a}{N_{k_i}} \cdot [\boldsymbol{w_2} + (1 - \boldsymbol{w_2}) \cdot e^{-\frac{d_{ij}(x)^2}{2\sigma^2}}]$$

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"The weight parameters w_1 and w_2 could be set based on (...). Here, they are set to values that were found empirically."

— Weinmann et al.

Learning parameters with maximum likelihood

The problem

Likelihood function:

$$\mathscr{L}(\Theta) = P(y|x;\Theta) = \frac{1}{Z(x,\Theta)} \exp[-\sum_{c \in \mathscr{C}(G)} \psi_c(y_c|x;\Theta)]$$

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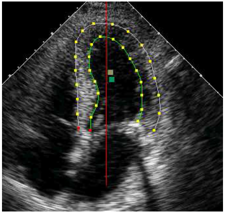
. . . **need an approximation !**

Taming the partition function

Method I

Case study: Heart Motion Abnormality Detection [Schmidt et al., 2008]

- Binary classification, up to pairwise cliques



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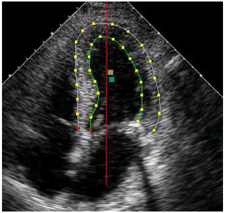

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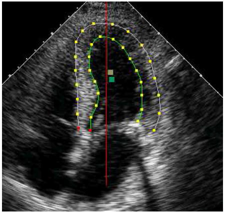

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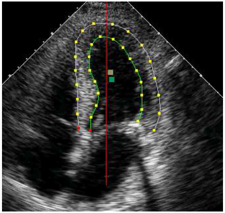

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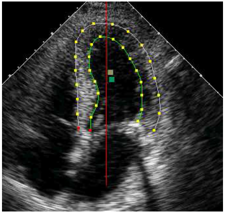
$$\nabla \ell(\Theta) = F(x,y) - \underbrace{\sum_{y'} P(y'|x,\Theta) F(x,y')}_{\text{computationally infeasible}}$$



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$$Z_i = \underbrace{\sum_{y_i'}}_{\text{only over current variable}} \exp(\Theta^T F_i(x, y_i'))$$

Taming the partition function

Method II

Instead of approximating entire likelihood, try to estimate the problematic sum [Kumar et al., 2005]. Assume linear potentials ($\Theta = (w, v)$):

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Under this model, the (log) likelihood gradient is:

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Need to estimate the expectation:

$$< y_i >_{\Theta;x} = \sum_{y'} P(y'|x;\Theta) \cdot y'_i$$

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  Obtain estimated marginal probabilities per variable:
  $P_i(y_i|x;\Theta) = \sum_{y_1} \ldots \sum_{y_{i-1}} \sum_{y_{i+1}} \ldots \sum_{y_n} P(y_1, \ldots, y_n|x;\Theta)$
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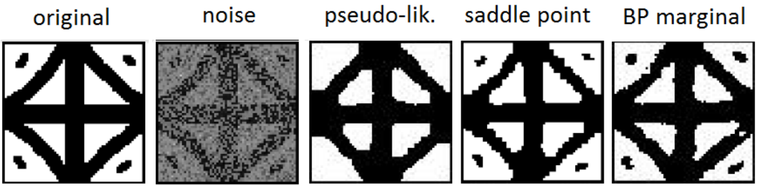
  Then, $< y_i >_{\Theta;x} \approx \hat{y}_i$.

Taming the partition function

Learning/inference results



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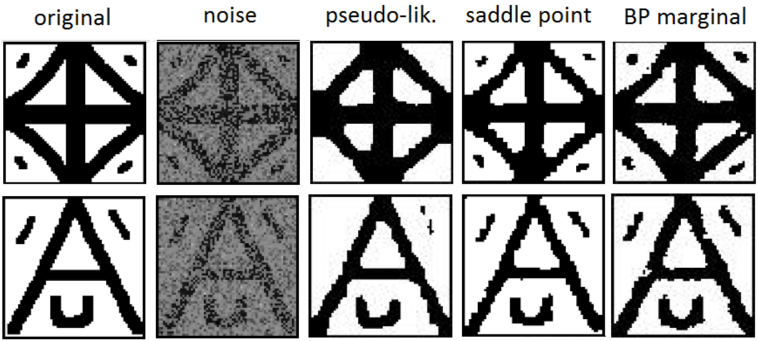

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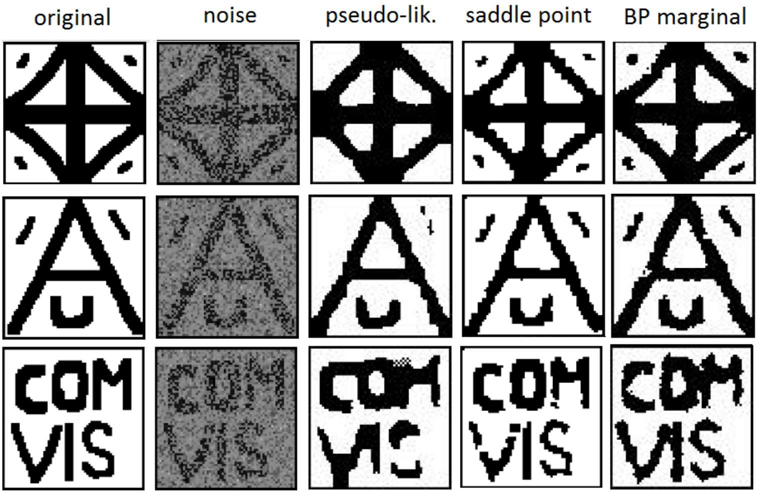

©Kumar et al., 2005

Max-margin learning

Introduction

- Abandons probabilistic perspective
- Aims at maximizing energy *margin* between object labels
- Uses *linear* potentials, can be kernelized

Max-margin learning

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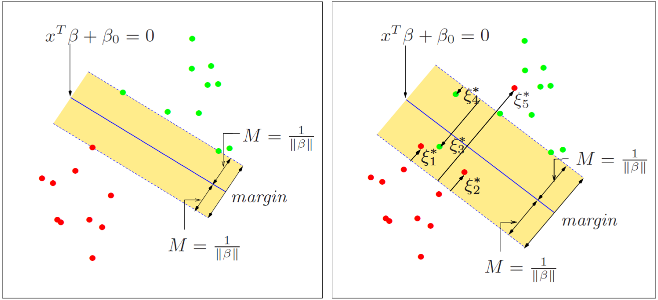
- Aims at maximizing energy *margin* between object labels
- Uses *linear* potentials, can be kernelized
- Generalizes ideas from support vector machine



Max-margin learning

Basic formulation

Let $\psi(y_i) = w^T F(x_i, y_i), \psi(y_i, y_j) = v^T F(x_{i,j}, y_i, y_j)$ be the potentials.

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- $\Theta = (w, v)$: compound weight, $F(x, y)$: compound feature
- $\Delta(y, z) \geq 0$: distance between two labelings y and z
- Energy: $E(y|x; \Theta) = \Theta^T F(x, y)$

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Structured SVM [Taskar et al., 2003, Tsochantaridis et al., 2005]

$$
\begin{aligned}
&\text{maximize } \gamma \\
&\qquad \text{s.t. } ||\Theta|| \leq 1 \\
&\qquad\qquad \forall_{y' \neq y} \underbrace{\Theta^T F(x, y')}_{E(y')} - \underbrace{\Theta^T F(x, y)}_{E(y)} \geq \Delta(y, y') \cdot \gamma
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$$

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$$\text{s.t. } \zeta_i \geq 0$$

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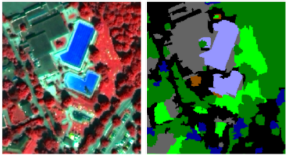
Example: semantic segmentation of urban scenes [Volpi and Ferrari, 2015]

> "In our setting, the potentials employed are linear with respect to the parameter vector.(...)To learn the CRF parameters **w** we adopt the margin rescaling variant of the SSVM(...)"
>
> — Volpi and Ferrari



ⓒVolpi and Ferrari

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Main challenge

Constraints revisited:

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- Exponential in input data length ! Infeasible to add all constraints at once

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Possible solution [Tsochantaridis et al., 2005]:

- Add constraints iteratively
- At every step, find most violated constraint
- Re-solve with augmented constraint set

Max-margin learning

Determining most violated constraint

Find the labeling minimizing the loss-augmented energy:

$$\hat{y} = \arg\max_{y'} E(y'|x, \underbrace{\Theta}_{\text{current est.}}) - \Delta(y', y)$$

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Assume Δ decomposes according to nodes: $\Delta(y, y') = \sum_i \delta(y_i, y_i')$. Define augmented unary potentials:

$$\bar{\psi}(y_i') = \psi(y_i') - \delta(y_i, y_i')$$

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Perform inference in new network with energy $E(y'; \bar{\psi}(\cdot), \psi(\cdot, \cdot))$ using preferred method (QPBO, linear programming relaxation, simulated annealing,...)

Available software libraries

- UGM: Matlab code for undirected graphical models (`http://www.cs.ubc.ca/~schmidtm/Software/UGM.html`)
- SVM$^{\text{struct}}$: Structured SVM API in Python, C++, Matlab (`http://www.cs.cornell.edu/people/tj/svm_light/svm_struct.html`)
- PyStruct - Structured Learning in Python (`http://pystruct.github.io/`)
- OpenGM: a C++ template library for discrete factor graph models (`http://hciweb2.iwr.uni-heidelberg.de/opengm/`)

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- Numerous learning methods with varying degrees of complexity and capabilities
- Two main paradigms: probabilistic perspective, max-margin learning
- Many specializations have been developed (chain graphs, trees, submodular potentials,. . .)
- Direct methods may still be useful

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The end

Thank you for your attention.