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A modified common weight model for maximum discrimination in technology selection

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This paper provides a modification of a recently developed common weight model for technology selection. The discrimination power of the existing model depends on a discriminating parameter. Although larger values of this parameter can give more discrimination, it should also be small enough to guarantee the existence of at least one efficient DMU. Hence, different values of this parameter should be examined to determine the proper value. In the model proposed here, the largest possible value of the parameter is utilised to maximise the discrimination power, and the existence of an efficient DMU is guaranteed by adding a few constraints. In addition, the paper presents comments on existing models, and the properties and the advantages of the model are explained. The contents of the paper are illustrated by several numerical examples.

Keywords: data envelopment analysis; multi-criteria decision making; common weights; technology selection

1. Introduction

Conventional data envelopment analysis (DEA) (Charnes *et al.* 1978) uses different weights to evaluate decision-making units (DMUs). However, in some applications of DEA, a common set of weights is preferred (Cook *et al.* 1990, Roll *et al.* 1991). In two recent papers by Karsak and Ahiska (2005, 2008), they utilised a common weight DEA model for technology selection. With the assumption that there are n DMUs with one input and s outputs, they proposed a model for the exact input and output data as follows:

$$\text{Min } M - k \sum_{j \in EF} d_j, \tag{1}$$

subject to:

$$\begin{aligned} & M \geq d_j, \quad j = 1, 2, \ldots, n, \\ & \frac{\sum_{r=1}^{s} u_r y_{rj}}{x_j} + d_j = 1, \quad j = 1, 2, \ldots, n, \\ & u_r \geq \varepsilon, \quad r = 1, 2, \ldots, s, \\ & d_j \geq 0, \quad j = 1, 2, \ldots, n. \end{aligned}$$

Here, x_j and y_{rj} are the input and output r of the jth DMU ($r = 1, 2, \ldots, s,\ j = 1, 2, \ldots, n$), $k \in [0, 1]$ is a discriminating parameter, EF is the set of mini-max efficient DMUs which is determined by solving model (1) with $k = 0$, and ε is a small positive scalar. The model has also been extended to be used when both exact and ordinal outputs need to be considered.

One issue with model (1) is determining a proper value for the parameter k. Indeed, on one hand, increasing the value of k in the model can decrease the number of efficient DMUs and provide more discrimination among DMUs, and, on the other, increasing the value of k excessively can make all DMUs inefficient, which is not desirable.

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To determine a proper value for k, Karsak and Ahiska (2005) proposed augmenting the value of k from zero with a pre-determined step size, which can cause some difficulties. To overcome the drawbacks of this approach, Amin *et al.* (2006) proposed a mixed-integer model. By referring to certain drawbacks with the model proposed by Amin *et al.* (2006), Karsak and Ahiska (2008) used a bisection search algorithm to determine the parameter value in model (1), which improves their earlier work. Amin (2008) proposed a nonlinear mixed-integer model to determine a single efficient DMU.

In this paper, we comment on existing approaches and propose a new modification to model (1). Instead of searching for a proper value for the parameter, the maximum value of k, $k = 1$, is used, which maximises the discrimination power, and the existence of at least one efficient DMU is guaranteed by some additional constraints. The modified model is a mixed-integer linear programming model. To decrease the (worst case) computational complexity and provide a more (theoretically) efficient approach, we propose an alternative approach that solves a set of linear models instead of solving this mixed-integer model.

The rest of the paper is organised as follows. Section 2 provides a review of the existing methods and their difficulties. The modified approach is developed in Section 3. Section 4 illustrates the contents of the paper with some numerical examples. The conclusion is presented in Section 5.

2. Review of the methods and some comments

As explained above, the model proposed by Karsak and Ahiska contains the parameter k. Amin *et al.* (2006) proposed the following mixed-integer linear model, which does not need this parameter:

$$\text{Min } M, \tag{2}$$

subject to

$$\begin{aligned}
&M - d_j + \alpha_j \geq 0, \quad j = 1, 2, \ldots, n,\\
&\frac{\sum_{r=1}^{s} u_r y_{rj}}{x_j} + d_j - \alpha_j = 1, \quad j = 1, 2, \ldots, n,\\
&\sum_{j=1}^{n} d_j = n - 1,\\
&u_r \geq \varepsilon, \quad r = 1, 2, \ldots, s,\\
&0 \leq \alpha_j \leq 1, \quad j = 1, 2, \ldots, n,\\
&d_j \in \{0, 1\}, \quad j = 1, 2, \ldots, n.
\end{aligned}$$

Let M^*, u_r^*, $r = 1, 2, \ldots, s$, d_j^*, α_j^*, $j = 1, 2, \ldots, n$, be an optimal solution for model (2). From the constraints, it is seen that, for only one DMU, say DMUp, we have $d_p^* = 0$, and $d_j^* = 1$ for $j \neq p$, $j = 1, 2, \ldots, n$. On the other hand, from the constraints, we have $(\sum_{r=1}^{s} u_r^* y_{rj}/x_j) = 1 - (d_j^* - \alpha_j^*)$, $j = 1, 2, \ldots, n$. Hence, the maximum score is obtained for DMU(s) with a minimum value of $d_j^* - \alpha_j^*$. Since $d_p^* - \alpha_p^* = -\alpha_p^* \leq 0$ and $d_j^* - \alpha_j^* = 1 - \alpha_j^* \geq 0$, $j \neq p$, we have $(\sum_{r=1}^{s} u_r^* y_{rp})/x_p = 1 + \alpha_p^* \geq 1$ and $(\sum_{r=1}^{s} u_r^* y_{rj})/x_j = \alpha_j^* \leq 1$, $j \neq p$. Therefore, DMUp will be efficient, and is one of the best DMUs corresponding to the optimal weights. If we have $\alpha_p^* > 0$, then DMUp is the single best DMU, since we have $(\sum_{r=1}^{s} u_r^* y_{rp})/x_p > 1$ and $(\sum_{r=1}^{s} u_r^* y_{rj})/x_j \leq 1$, $j \neq p$. Note that, in this case, we may have $(\sum_{r=1}^{s} u_r^* y_{rj})/x_j = 1$ (i.e. $d_j^* - \alpha_j^* = 0$), even for an inefficient DMUj, $j \neq p$, and we cannot consider these DMUs as the best DMUs.

Note: Karsak and Ahiska (2008) suggest that DMUs with $d_j^* - \alpha_j^* = 0$ should be considered as the best efficient DMU(s). From the previous discussion, since the value of $d_p^* - \alpha_p^*$ can be negative, this suggestion is not correct unless $\alpha_p^* = 0$. This error was repeated by Amin (2008). He defines $A = \{j : d_j^* = \alpha_j^*\}$ and claims that DMUj is efficient for each $j \in A$, which is correct only if $\alpha_p^* = 0$ (this is illustrated by numerical examples in Section 4).

To provide a single efficient DMU, Amin (2008) proposed the following nonlinear mixed-integer model:

$$\text{Min } M, \tag{3}$$

subject to

$$
\begin{aligned}
& M - d_j \geq 0, \quad j = 1, 2, \ldots, n, \\
& \frac{\sum_{r=1}^{s} u_r y_{rj}}{x_j} + d_j = 1, \quad j = 1, 2, \ldots, n, \\
& \sum_{j=1}^{n} \delta_j = n - 1, \\
& \delta_j - d_j \beta_j = 0, \quad j = 1, 2, \ldots, n, \\
& u_r \geq \varepsilon, \quad r = 1, 2, \ldots, s, \\
& \delta_j \in \{0, 1\}, \quad d_j \geq 0, \beta_j \geq 1, \quad j = 1, 2, \ldots, n.
\end{aligned}
$$

Let this model be feasible and M^*, u_r^*, $r = 1, 2, \ldots, s$, d_j^*, δ_j^*, and $\beta_j^*, j = 1, 2, \ldots, n$, be an optimal solution for model (3). From the constraints, it is seen that, for only one DMU, say DMUp, we have $\delta_p^* = 0$, and $\delta_j^* = 1$ for $j \neq p$, $j = 1, 2, \ldots, n$. In addition, $\delta_j^* = 0$ if and only if $d_j^* = 0$ (since $\beta_j^* \geq 1$). Therefore, we have $d_p^* = 0$ and $d_j^* > 0$ for $j \neq p$, $j = 1, 2, \ldots, n$, that is $(\sum_{r=1}^{s} u_r^* y_{rp})/x_p = 1$ and $(\sum_{r=1}^{s} u_r^* y_{rj})/x_j < 1$, $j \neq p$.

As also mentioned by Karsak and Ahiska (2008), there exist some cases for which more discrimination among efficient DMUs is not possible. In such cases, as will be seen below from numerical examples, model (3) is infeasible. In addition, this model is a nonlinear mixed-integer model, which is very difficult to solve relative to a linear model.

In the following section, a new modified model will be proposed that uses the maximum value of the discrimination parameter k, $k = 1$, and is feasible and has a (finite) optimal solution.

In all models of this paper it is assumed that $\varepsilon \in (0, \varepsilon^*]$, where ε^* is the maximum admissible value of ε (Amin *et al*. 2006, Amin and Emrouznejad 2007). It can be shown that all models of this paper, except model (3), are feasible when $\varepsilon \in (0, \varepsilon^*]$.

3. The modified approach

In order to eliminate the search for an appropriate value for the discrimination parameter k in model (1), the following mixed-integer model is proposed:

$$
\text{Min } M - \sum_{j \in EF} d_j, \tag{4}
$$

subject to

$$
\begin{aligned}
& M - d_j \geq 0, \quad j = 1, 2, \ldots, n, \\
& \frac{\sum_{r=1}^{s} u_r y_{rj}}{x_j} + d_j = 1, \quad j = 1, 2, \ldots, n, \\
& d_j - t_j \leq 0, \quad j \in EF, \\
& \sum_{j \in EF} t_j = |EF| - 1, \\
& u_r \geq \varepsilon, \quad r = 1, 2, \ldots, s, \\
& t_j \in \{0, 1\}, \quad d_j \geq 0, \quad j = 1, 2, \ldots, n.
\end{aligned}
$$

In this model, the maximum possible value of k, $k = 1$, is selected to provide maximum discrimination power. $|EF|$ is the number of mini-max efficient DMUs, which is determined by solving model (1) with $k = 0$ and $\varepsilon \in (0, \varepsilon^*]$.

It can readily be seen that this model is feasible and has a (finite) optimal solution. In fact, for a given $\varepsilon \in (0, \varepsilon^*]$, every mini-max efficient DMU is DEA efficient. Hence, for any $k \in EF$ there exist $\bar{u}_r \geq \varepsilon, r = 1, 2, \ldots, s$, such that $(\sum_{r=1}^{s} \bar{u}_r y_{rk}/x_k) = 1$ and $(\sum_{r=1}^{s} \bar{u}_r y_{rj}/x_j) \leq 1$, $j \neq k$. It can now readily be seen that $M = 1$, $u_r = \bar{u}_r$, $\quad r = 1, 2, \ldots, s$, $t_k = 0$, $t_j = 1$, $j \neq k$, $d_j = 1 - (\sum_{r=1}^{s} \bar{u}_r y_{rj}/x_j)$, $\quad j = 1, 2, \ldots, n$, is a feasible solution for model (4). In addition, the objective function clearly has a lower bound (since $M \geq 0$ and $d_j \leq 1$ we have $M - \sum_{j \in EF} d_j \geq -|EF|$). Therefore, the model has a (finite) optimal solution (Bazaraa *et al*. 1990).

Let M^*, u_r^*, $r = 1, 2, \ldots, s$, d_j^*, $j = 1, 2, \ldots, n$, t_j^*, $j \in EF$, be an optimal solution for model (4). From the constraints, it can be seen that, for one (and only one) $p \in EF$, we have $t_p^* = 0$, and since $d_j \geq 0$ and $d_j - t_j \leq 0$, $j \in EF$, we should also have $d_p^* = 0$. Indeed, instead of searching for an appropriate value of k that can provide convenient discrimination and can also determine at least one mini-max efficient DMU with zero value for the deviation from efficiency, some constraints have been added to give zero optimal value to at least one deviation variable, that is d_p, when the maximum possible value of k, $k = 1$, is selected. With the assumption that $d_p^* = 0$, in the optimal solution of this model, DMUp can be considered as the best efficient DMU, if $d_j^* \neq 0$, for all $j \in EF$, $j \neq p$. Otherwise, all the DMUs in EF with zero deviation from efficiency can be considered as the best, and more information is necessary if selecting a single best efficient DMU is desirable.

An advantage of model (4) is that it can be solved using existing software packages for mixed-integer linear programs. In addition, the number of mini-max efficient DMUs (i.e. $|EF|$) is usually small and model (4) can be solved easily, without a major complexity problem. However, at least theoretically, we can decrease the computational complexity by using the following linear models, where $k \in EF$:

$$Z_k = \text{Min } M - \sum_{j \in EF} d_j, \tag{5}$$

subject to

$$\begin{aligned}
&M - d_j \geq 0, \quad j = 1, 2, \ldots, n, \\
&\frac{\sum_{r=1}^{s} u_r y_{rj}}{x_j} + d_j = 1, \quad j = 1, 2, \ldots, n, \\
&d_k = 0, \\
&u_r \geq \varepsilon, \quad r = 1, 2, \ldots, s, \\
&d_j \geq 0, \quad j = 1, 2, \ldots, n.
\end{aligned}$$

The optimal value of (4) is seen to be the minimum value of the optimal values of these models (note that we have considered all possible values of $t_j, j \in EF$, in model (4)). Indeed, if we assume that $Z_p = \text{Min}_{k \in EF} Z_k$, then this model will solve model (4) when $k = p$ is selected. The computational effort of using this approach for solving model (4) is no more than when solving $|EF|$ linear models.

4. Numerical examples

This section illustrates the contents of the paper with some numerical examples.

Example 1: In this example, it is shown that model (3) may be infeasible. Consider the data in Table 1. For these data, model (3) is infeasible corresponding to all admissible values of ε. In fact, from the constraints of model (3) for the data of Table 1, we have $d_1 = 1 - u_1 - 2u_2 = d_2$, $d_3 = 1 - u_1 - u_2$, and, since only one d_j, $j = 1, 2, 3$, can be zero in each feasible solution of model (3), we should have $d_3 = 1 - u_1 - u_2 = 0$. But, since the weights are assumed to be positive ($u_j \geq \varepsilon > 0, j = 1, 2$), this gives $d_1 = d_2 = 1 - u_1 - 2u_2 < 1 - u_1 - u_2 = 0$, which is not feasible.

In this example, applying the proposed approach of this paper gives both DMUs 1 and 2 as the most efficient, as expected. Note that the ratios of output to input are the same for these two efficient DMUs, so further discrimination among these DMUs is not possible. The same result is obtained if we apply the proposed models of Karsak and Ahiska (2005, 2008).

Example 2: In this example, the proposed approach of this paper is applied to the robot selection problem investigated by Braglia and Petroni (1999) and Karsak and Ahiska (2005, 2008). The problem involves the evaluation of 12 robots with respect to four attributes as outputs and one attribute, the cost, as the single input. The data are shown in Table 2. To compare the results, the models are solved by considering $\varepsilon = 0.00001$. By solving model (1) with $k = 0$, DMU5 and DMU12 are determined as mini-max efficient DMUs. Now, solving the modified model (4) (or using linear programming models (5)) with $EF = \{5, 12\}$ gives DMU12 as the best DMU, which is the

Table 1. Data for three DMUs.

DMU	Input	Output 1	Output 2
1	1	1	2
2	2	2	4
3	1	1	1

Table 2. Inputs and outputs of 12 industrial robots.

Robot	Cost	Handling coefficient	Load capacity (kg)	1/Repeatability (mm^2)	Velocity (m/s)
1	100,000	0.995	85	1.70	3.00
2	75,000	0.933	45	2.50	3.60
3	56,250	0.875	18	5.00	2.20
4	28,125	0.409	16	1.70	1.50
5	46,875	0.818	20	5.00	1.10
6	78,125	0.664	60	2.50	1.35
7	87,500	0.880	90	2.00	1.40
8	56,250	0.633	10	8.00	2.50
9	56,250	0.653	25	4.00	2.50
10	87,500	0.747	100	2.00	2.50
11	68,750	0.880	100	4.00	1.50
12	43,750	0.633	70	5.00	3.00

same result obtained by Karsak and Ahiska. Amin *et al*. (2006) applied their model (2) to these data and obtained the following optimal solution:

$$d_1^* = d_2^* = d_3^* = d_4^* = d_5^* = d_6^* = d_7^* = d_8^* = d_9^* = d_{10}^* = d_{11}^* = 1, \quad d_{12}^* = 0, \quad \alpha_1^* = 0.7621, \quad \alpha_2^* = 0.8887,$$
$$\alpha_3^* = 0.9428, \quad \alpha_4^* = \alpha_5^* = 1, \quad \alpha_6^* = 0.6306, \quad \alpha_7^* = 0.7566, \quad \alpha_8^* = 0.7276, \quad \alpha_9^* = 0.8028,$$
$$\alpha_{10}^* = 0.7546, \quad \alpha_{11}^* = 1, \quad \alpha_{12}^* = 0.2766.$$

It can be seen that $d_j^* - \alpha_j^* = 0$ ($j = 4, 5, 11$), however DMUs 4 and 11 are not even DEA efficient (see Table 2 of Karsak and Ahiska (2005)). This shows that DMUs with $d_j^* - \alpha_j^* = 0$ (in model (2)) are not necessarily efficient, which confirms the previously mentioned existing error in the papers of Karsak and Ahiska (2008) and Amin (2008) (see the note in Section 2).

5. Conclusion

In two recent papers by Karsak and Ahiska (2005, 2008), they proposed a model that uses a discriminating parameter. They proposed two methods to search for a proper value for this parameter to attain the maximum possible discrimination. Since larger values of the parameter can provide more discrimination among DMUs, it is proposed in this paper to use the largest possible value for k, which is 1. But, with this value of the parameter, all DMUs can be found to be inefficient. To overcome this problem, constraints are added to the problem that can guarantee the existence of at least one efficient DMU. The obtained model is a mixed-integer linear programming model. A more (theoretically) efficient approach also has been proposed that solves a few linear models instead of solving this mixed-integer model. Advantages of the proposed approach are that it does not contain any parameter (in comparison with the Karsak and Ahiska (2005, 2008) models) and it is linear and has a feasible solution corresponding to all the admissible values of ε (in comparison with the proposed model of Amin (2008)). The paper also comments on existing papers by referring to some of their drawbacks.

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