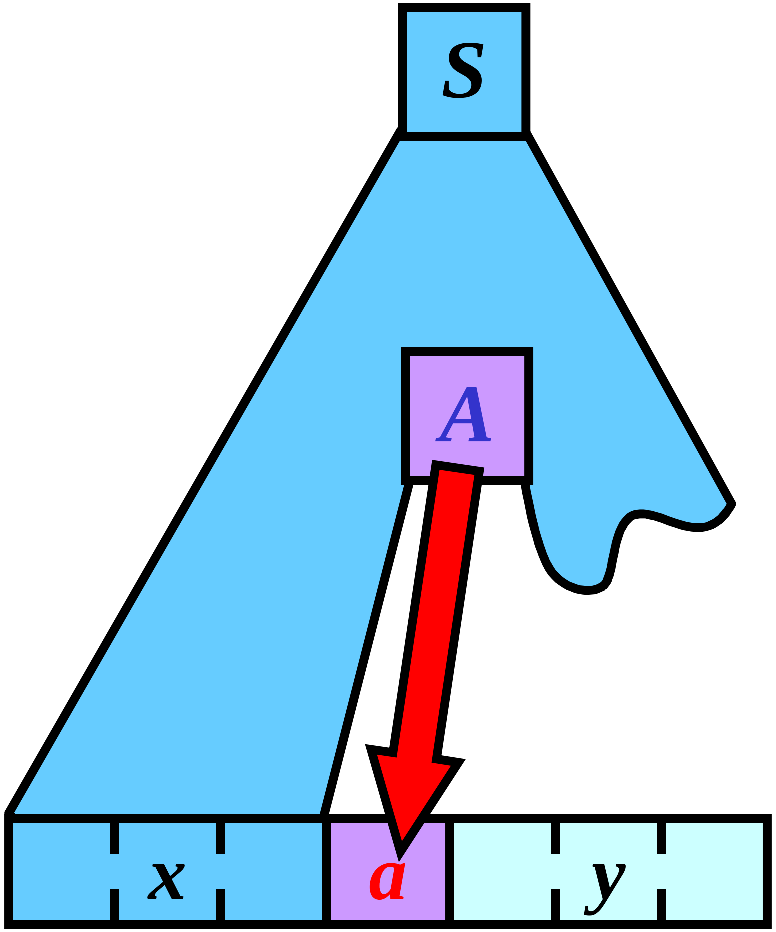
Part VII.
Top-Down Parsing

Top-Down Parsing: Introduction

Problem:



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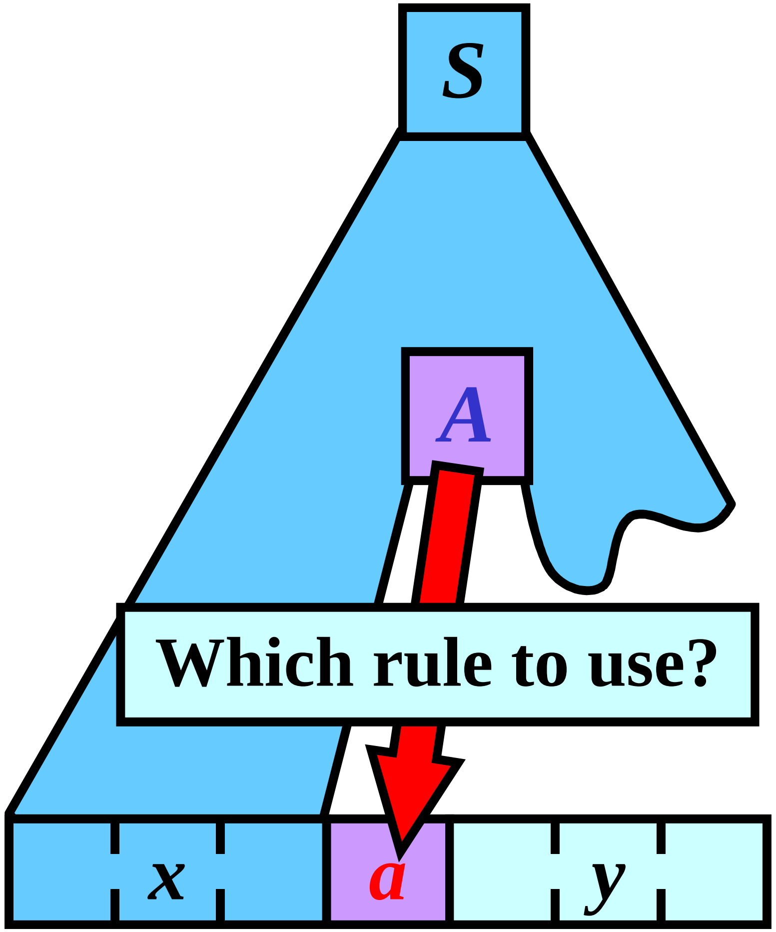

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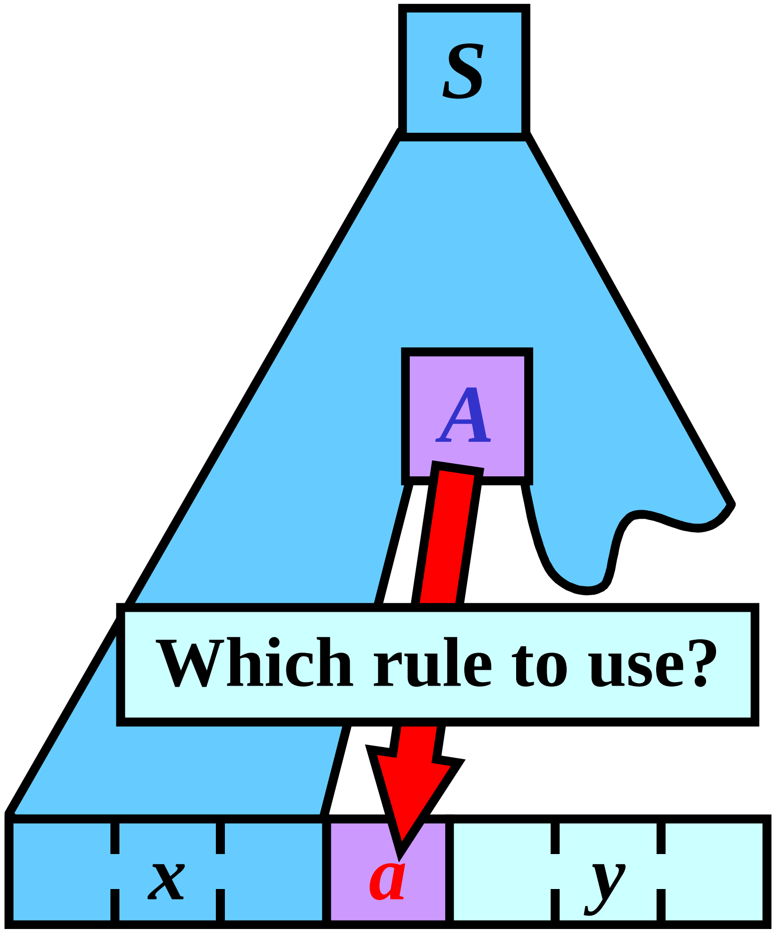

Basic idea:

Table:

α	...	*a*	...
...			
A		α(*A*, *a*)	
...			

Use rule ***r***: ***A*** $\rightarrow$ ***x***

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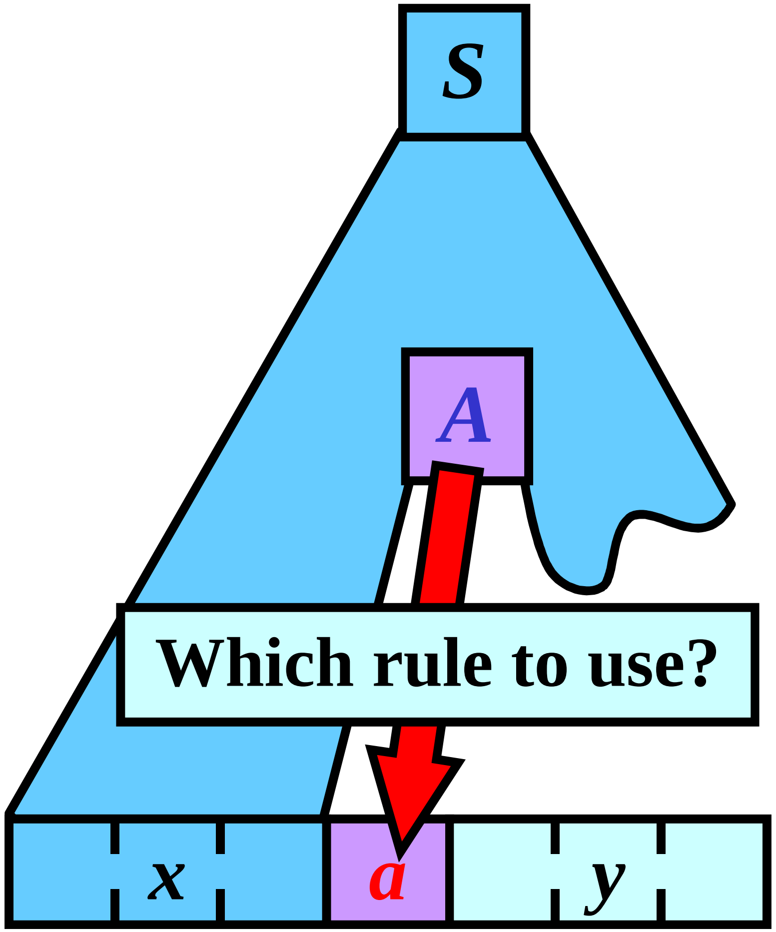

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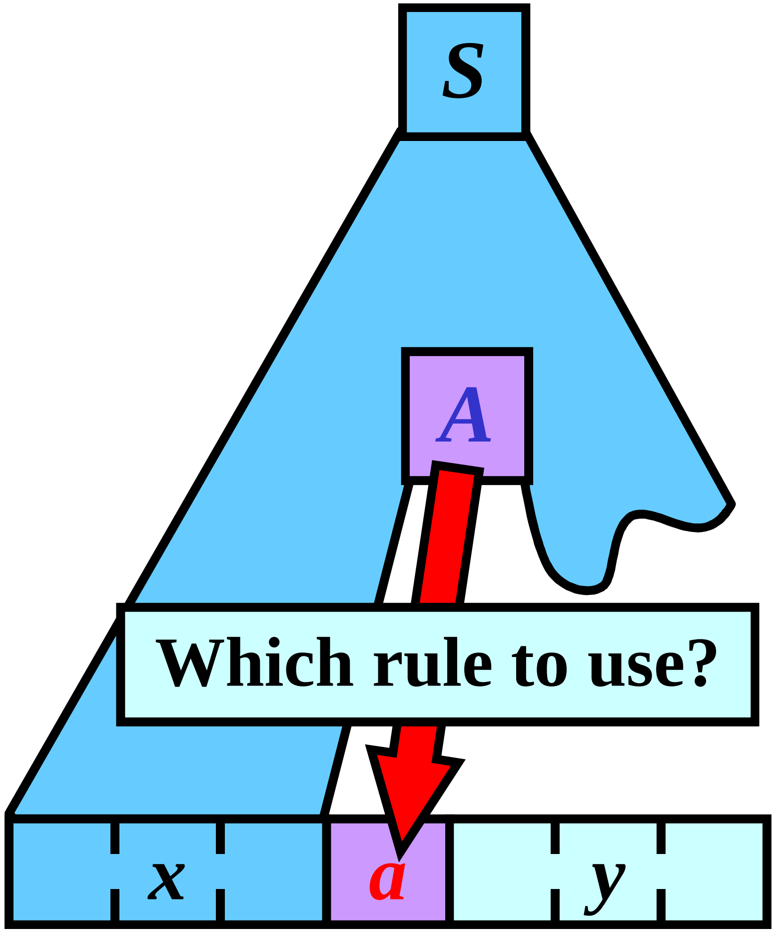

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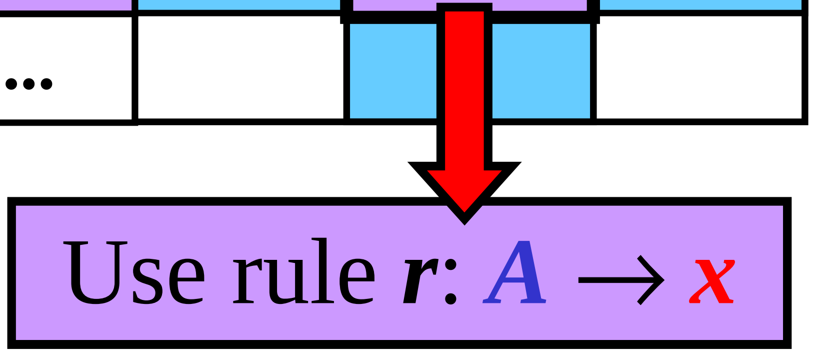
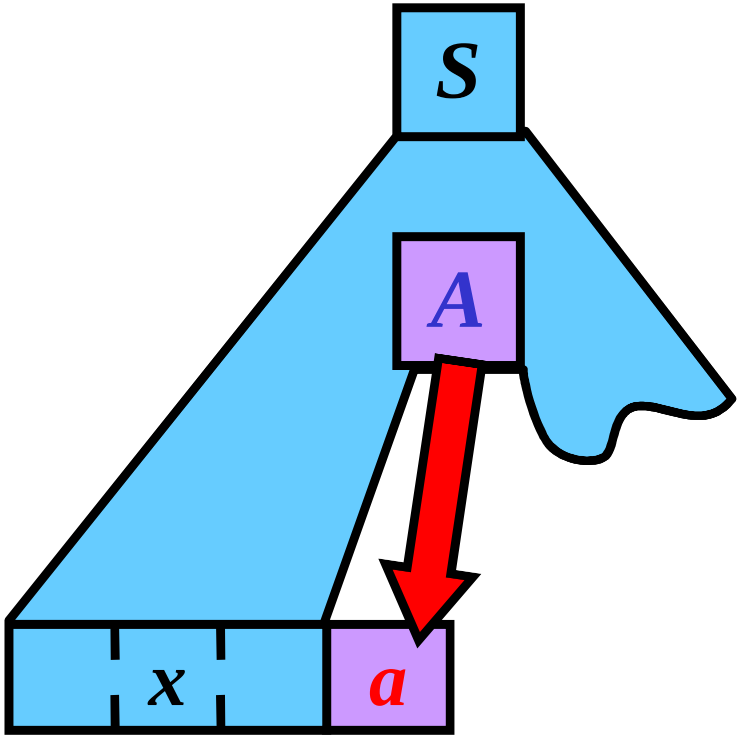


Question: Could you construct this table for **any** CFG?

Answer: NO

A Table-Based Selection of a Rule



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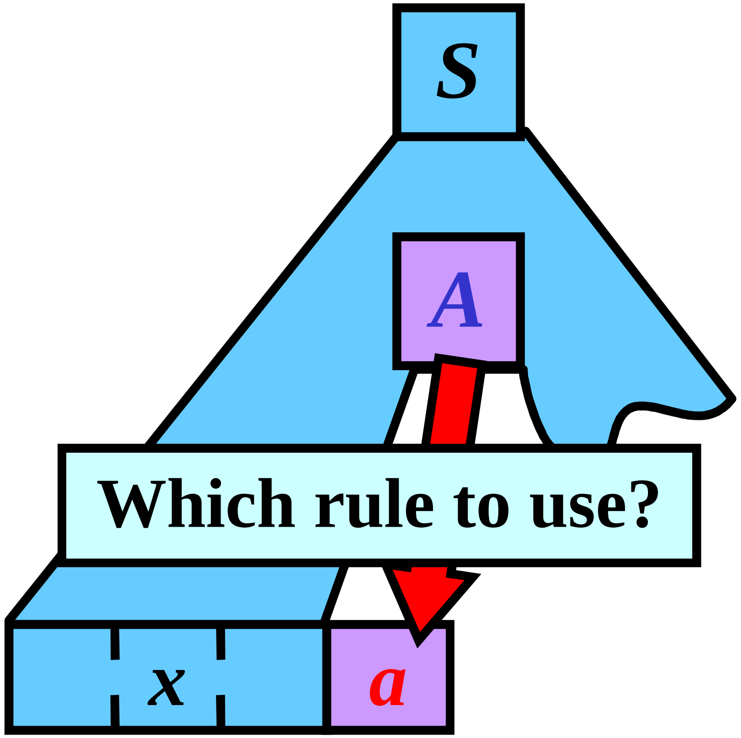

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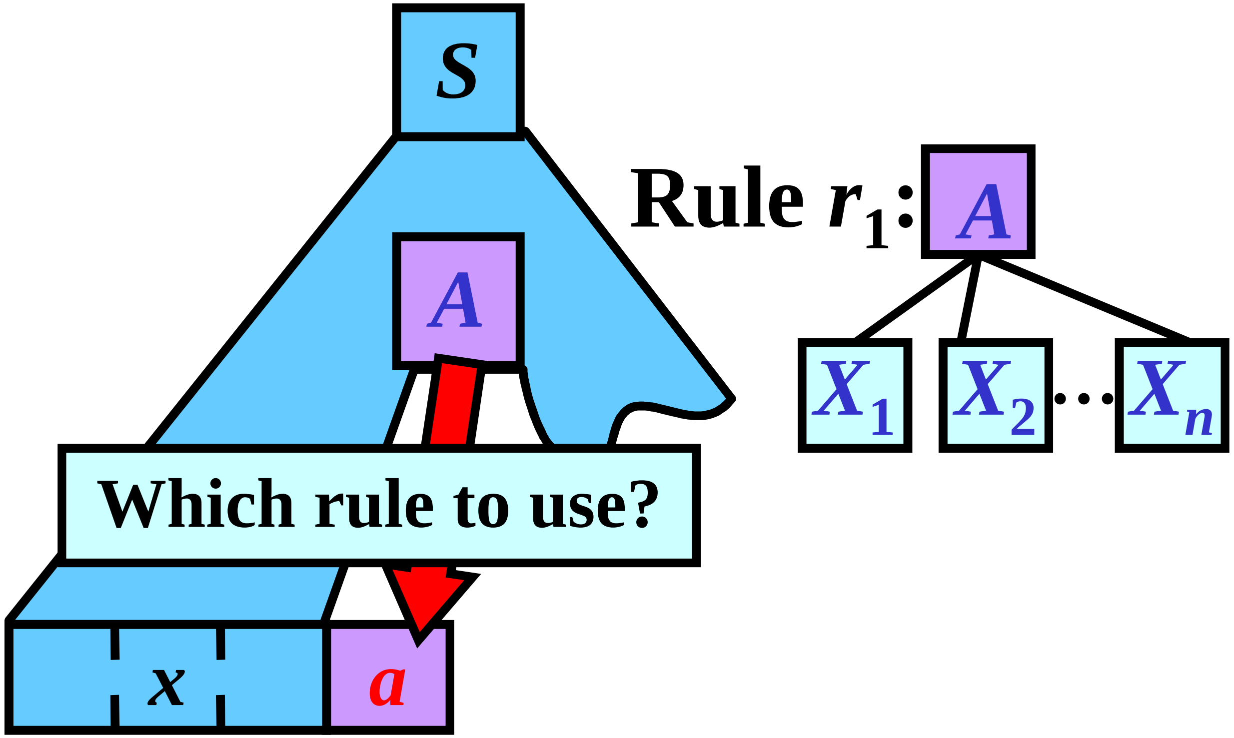

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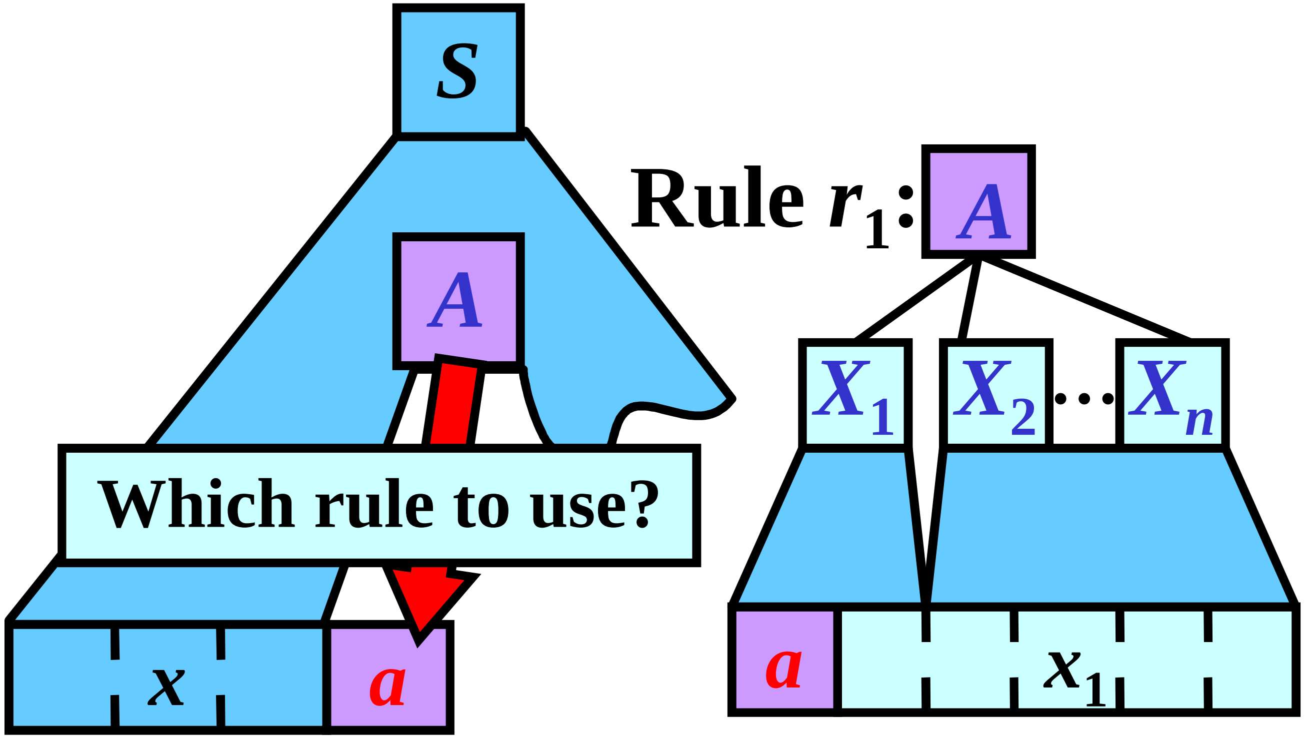

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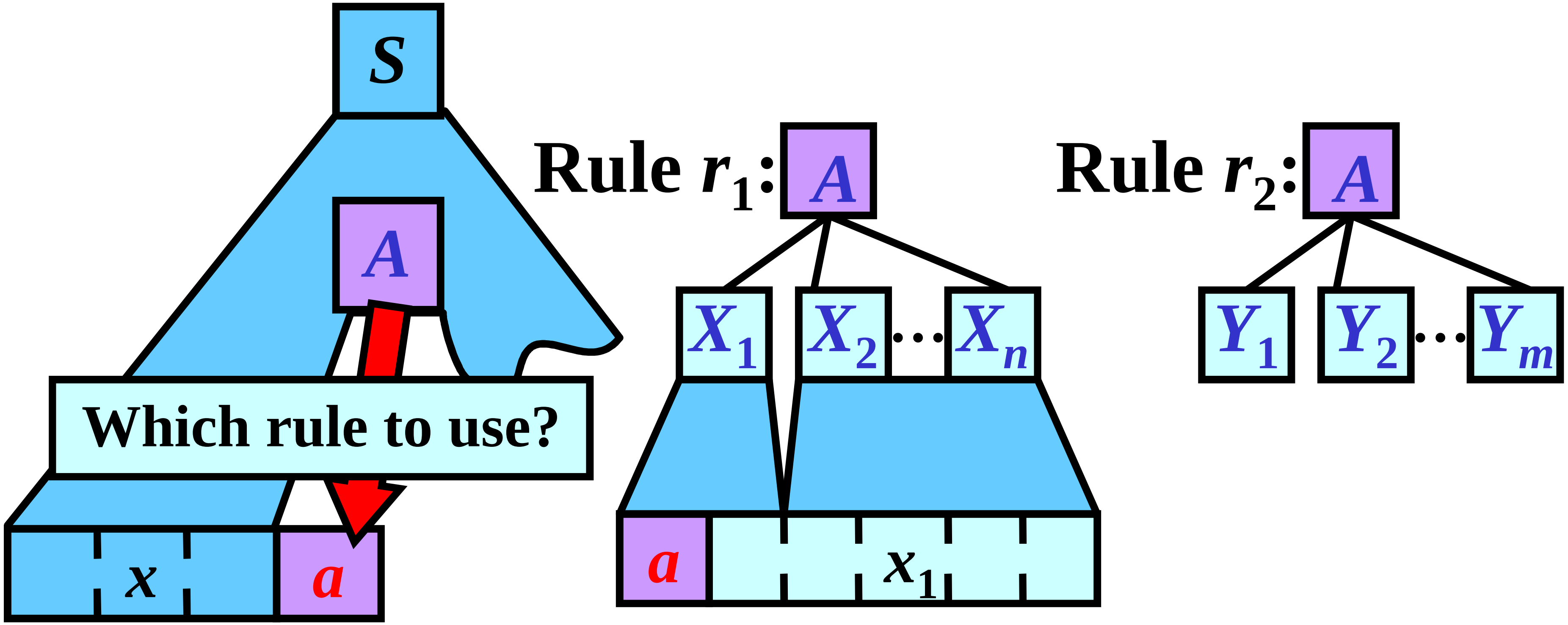

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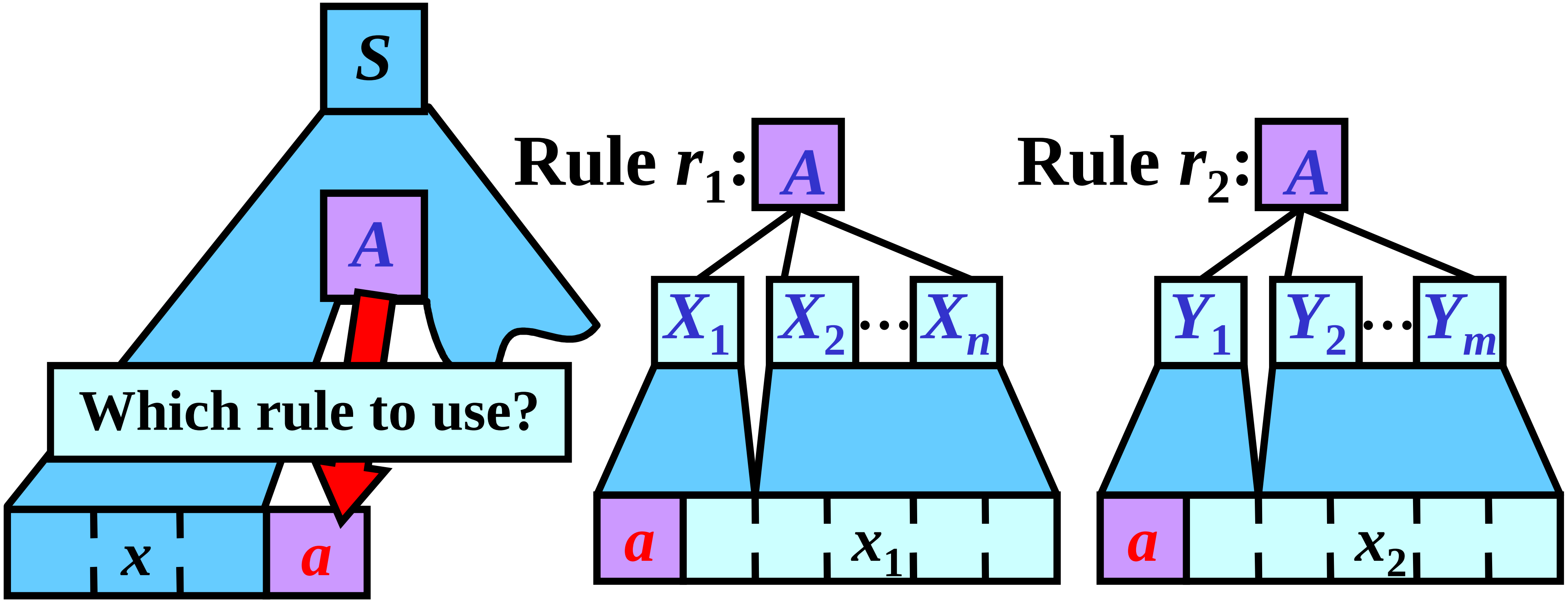

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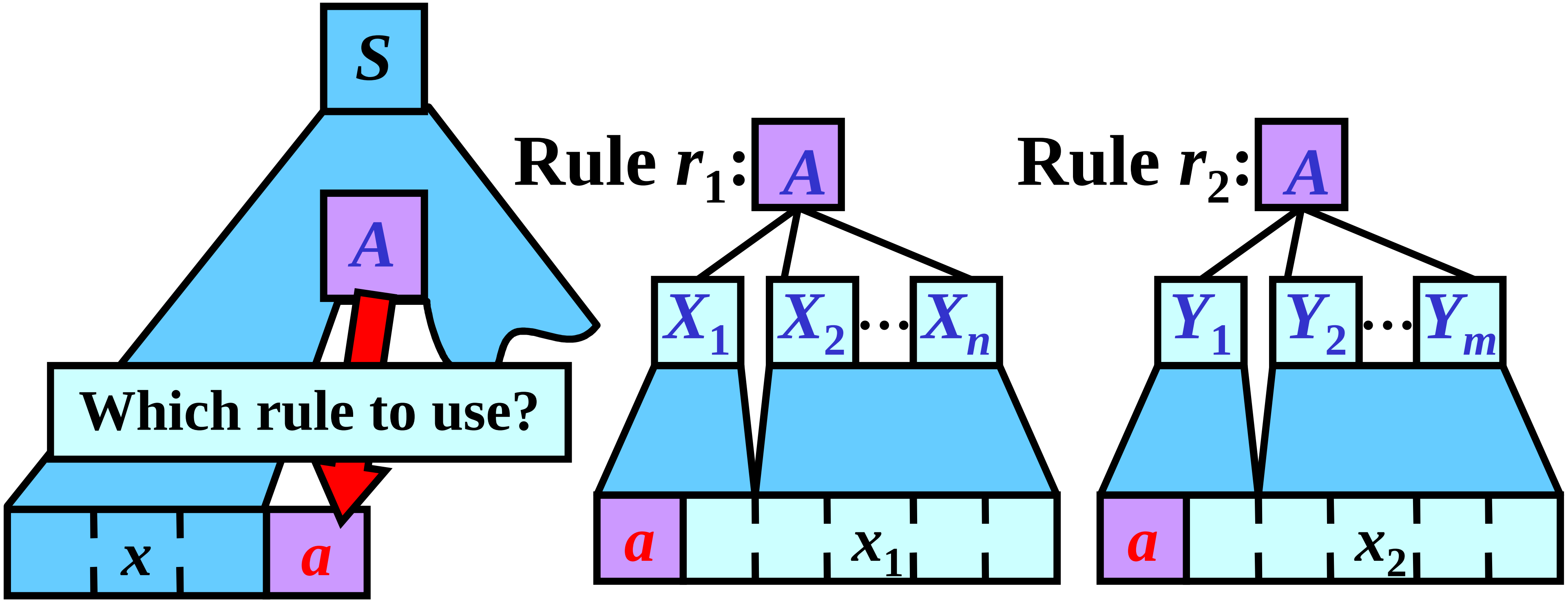

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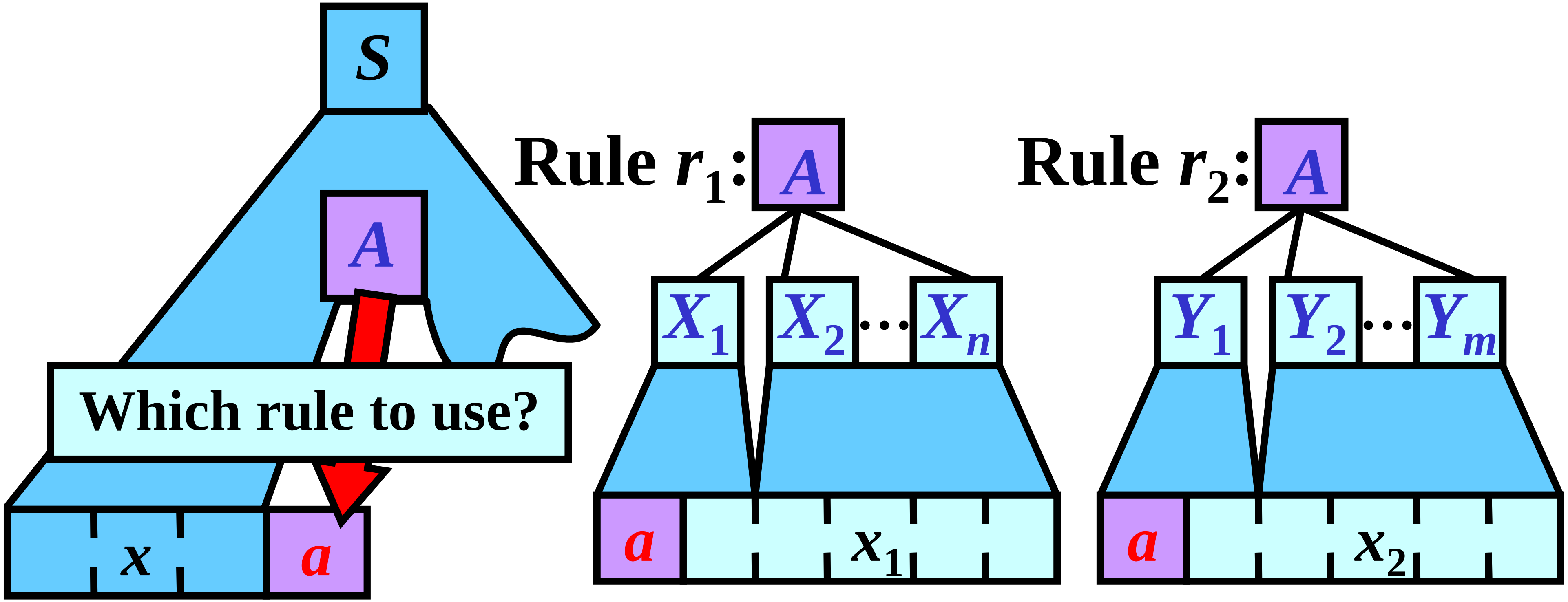

Table:

α	...	a	...
...			
A		α(A, a)	
...			

?

Use rule r_1: $A \to X_1X_2\ldots X_n$

Use rule r_2: $A \to Y_1Y_2\ldots Y_m$

Set *First*

Gist: *First*(*x*) is the set of all terminals that can begin a string derivable from *x*.

Definition: Let $G = (N, T, P, S)$ be a CFG. For every $x \in (N \cup T)^*$,we define the set $First(x)$ as $First(x) = \{a: a \in T, x \Rightarrow^* ay; y \in (N \cup T)^*\}$.

Illustration: $x = \boxed{X_1}\,\boxed{X_2}\cdots\boxed{X_n}$

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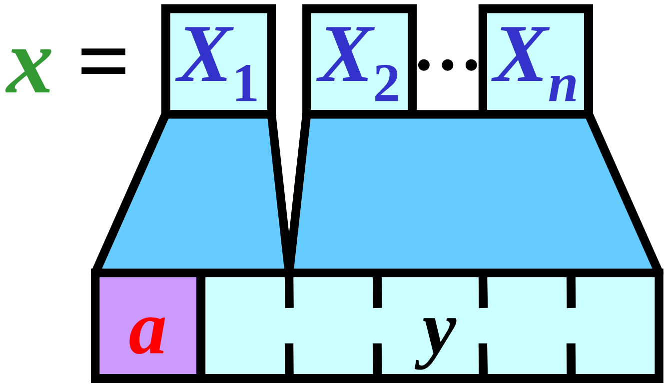

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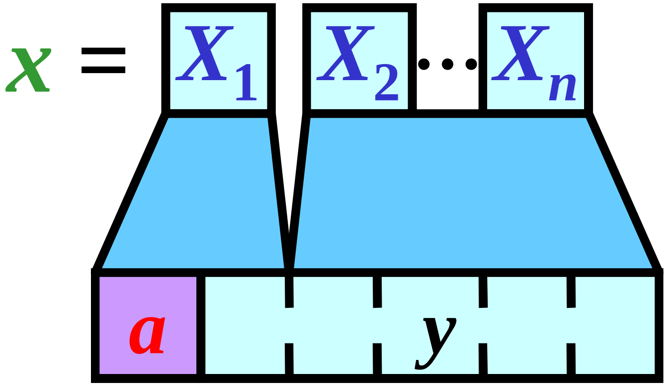

$x = X_1X_2\dots X_n \Rightarrow^* ay$

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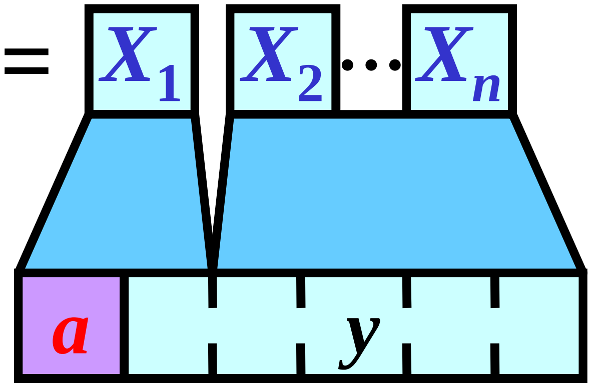
Illustration: $x =$



$$x = X_1X_2\dots X_n \Rightarrow^* ay$$

$$a \in First(x)$$

LL Grammars without ε-rules

Definition: Let $G = (N, T, P, S)$ be a CFG <u>without ε-rules</u>. G is an *LL grammar* if for every $a \in T$ and every $A \in N$ there is **no more than one** rule $A \rightarrow X_1X_2...X_n \in P$ such that $a \in First(X_1X_2...X_n)$

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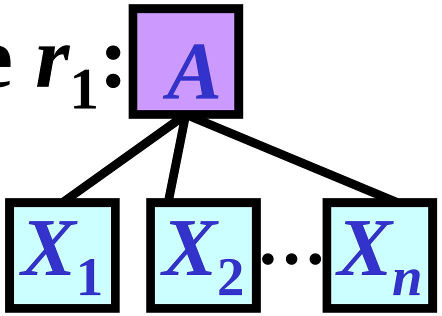
Rule r_1:



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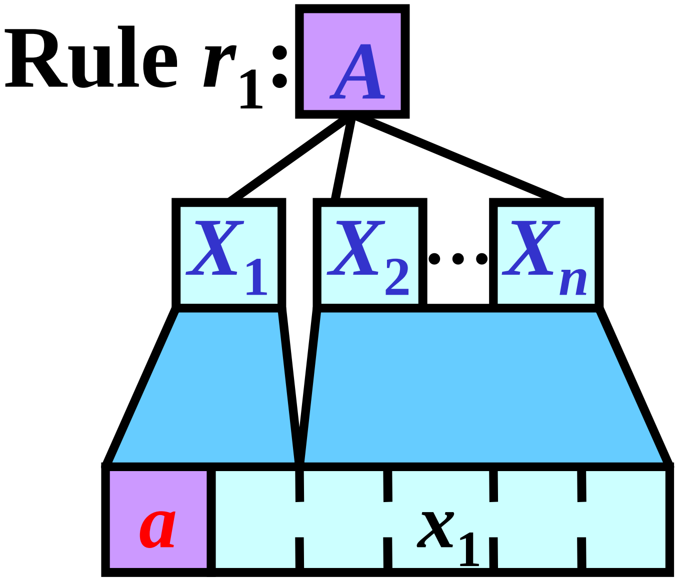

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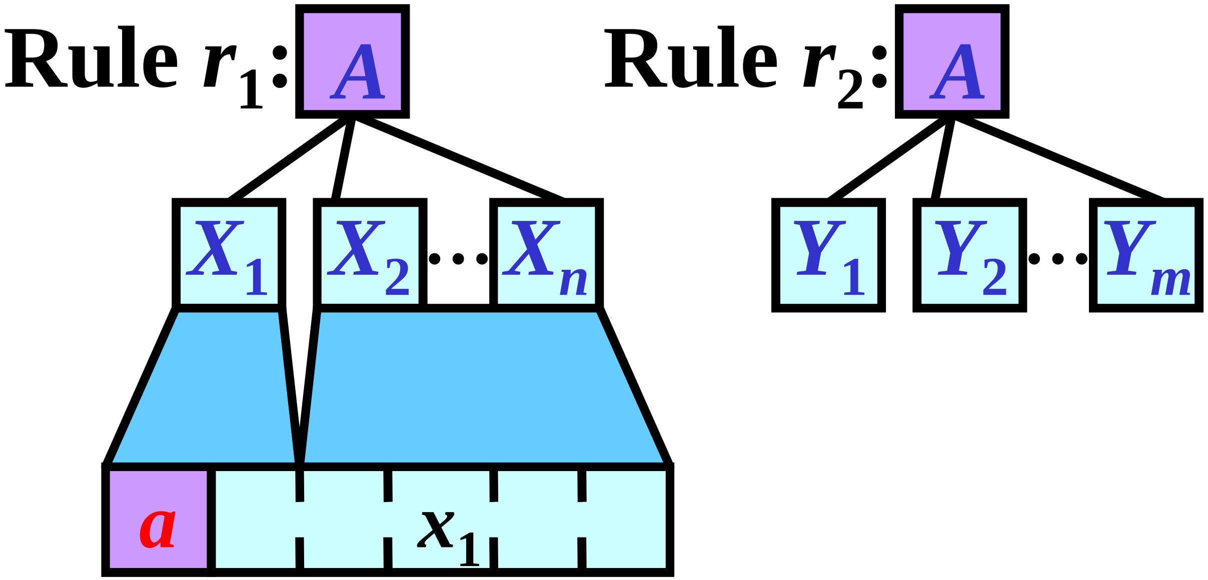

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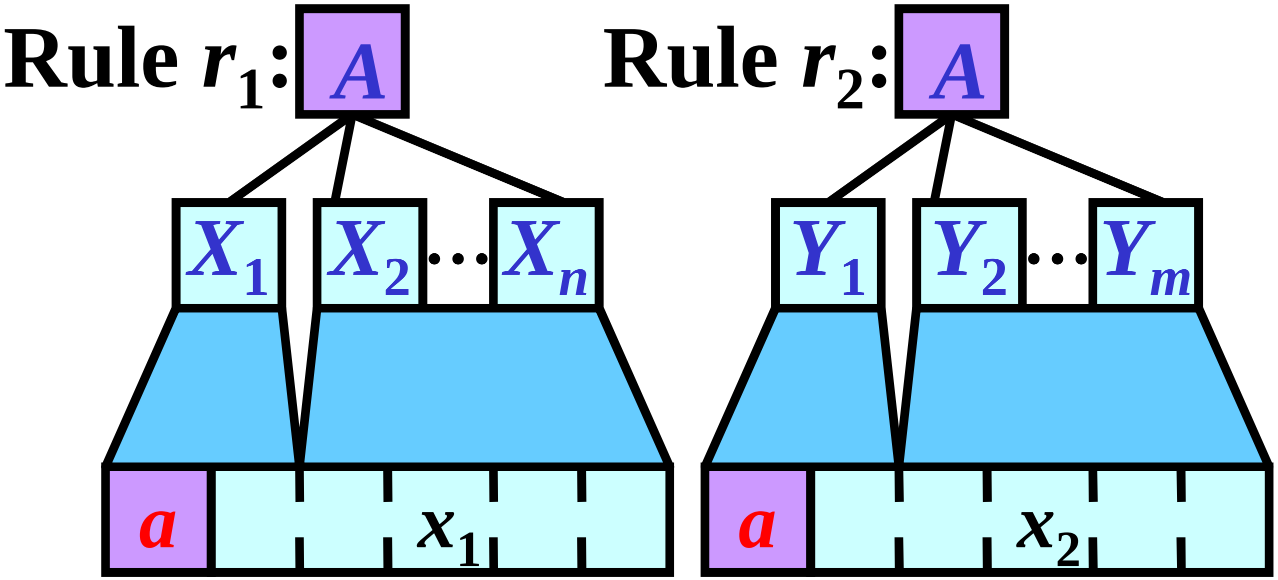

$a \in First(X_1X_2...X_n)$ $a \in First(Y_1Y_2...Y_m)$

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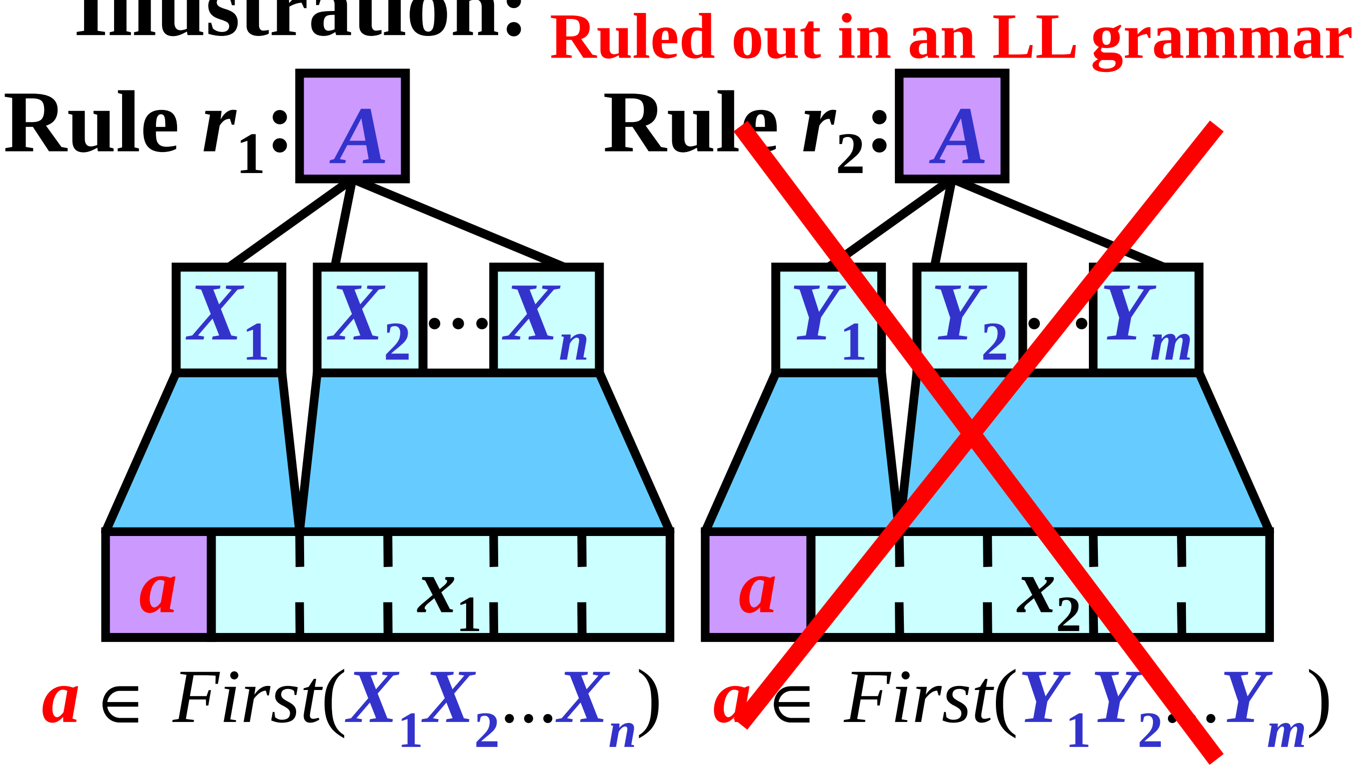

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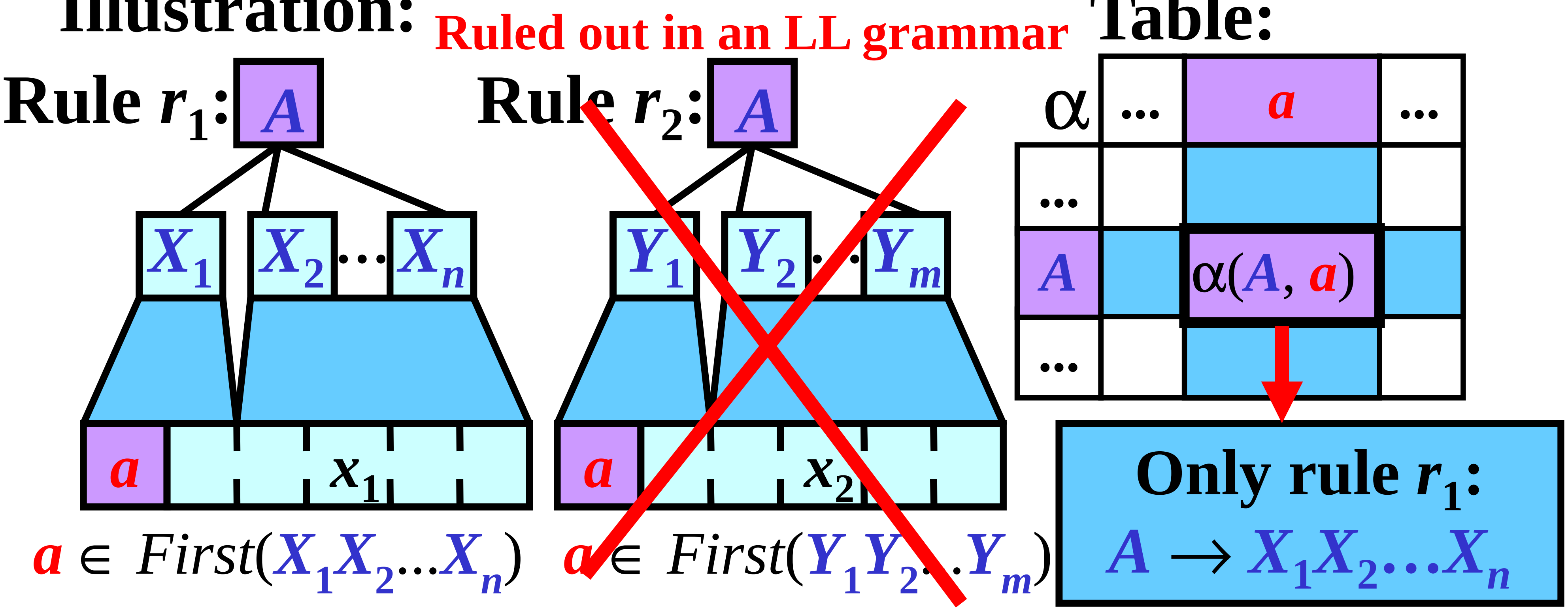

Simple *P*rogramming *L*anguage (SPL)

1: <prog> → **begin** <st-list>
2: <st-list> → <stat> **;** <st-list>
3: <st-list> → **end**
4: <stat> → **read** **id**
5: <stat> → **write** <item>
6: <stat> → **id** **:=** **add** **(** <item> <it-list>
7: <it-list> → **,** <item> <it-list>
8: <it-list> → **)**
9: <item> → **int**
10: <item> → **id**

Note: G_{SPL} **is LL grammar**

Example:

```
begin
  read i;
  j := add(i, 1);
  write j;
end
```

$\in$ SPL

Algorithm: *First*(X)

- **Input:** $G = (N, T, P, S)$ **without ε-rules**
- **Output:** $First(X)$ for every $X \in N \cup T$

- **Method:**
- for each $a \in T$: $First(a) := \{a\}$
- **Apply the following rule until no *First* set can be changed:**
- if $A \rightarrow X_1X_2\ldots X_n \in P$, then add $First(X_1)$ to $First(A)$

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Illustration:

1) for each $a \in T$:

$First(a) := \{a\}$

because $a \Rightarrow^0 a$

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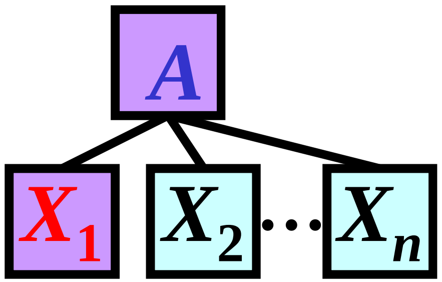

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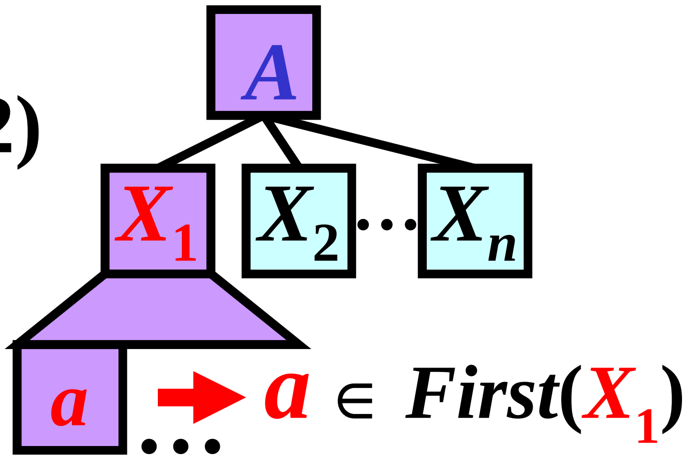

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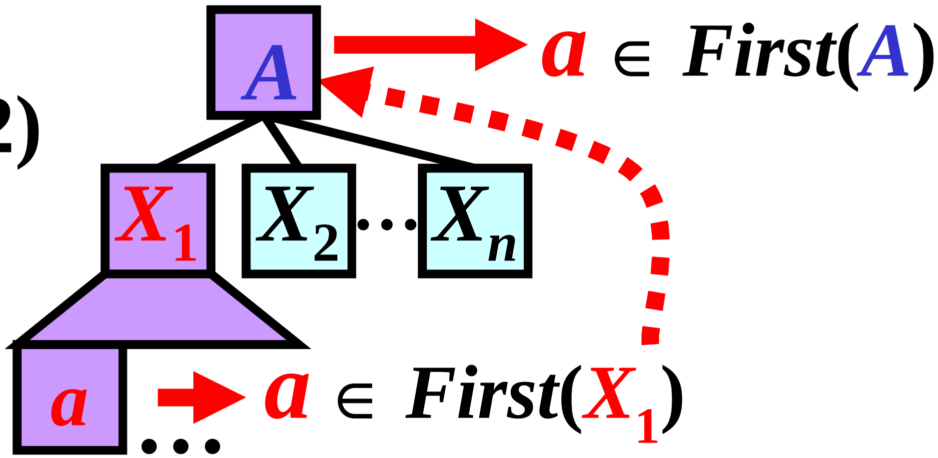

First(*X*) for SPL: Example

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<item> $\rightarrow$ **id** $\in P$**:** **add** *First*(**id**) **to** *First*(**<item>**)

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Summary: *First*(**<item>**) **= {id, int}**

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<st-list> → **<stat>** … ∈ *P*:	**add** *First*(**<stat>**)	**to** *First*(**<st-list>**)
Summary: *First*(**<st-list>**)	= {**id**, **write**, **read**, **end**}	
<prog> → **begin** … ∈ *P*:	**add** *First*(**begin**)	**to** *First*(**<prog>**)
Summary: *First*(**<prog>**)	= {**begin**}	

Construction of LL Table

α	…	*a*	…
…			
A		α(*A*, *a*)	
…			

Construction of LL Table

α	…	***a***	…
…			
A		α(***A***, ***a***)	
…			

$\alpha(A, a) = A \rightarrow X_1X_2\ldots X_n \in P$ if $a \in First(X_1)$; otherwise, $\alpha(A, a)$ is blank $\Rightarrow$ **ERROR**

Construction of LL Table

α	…	a	…
…			
A		$\alpha(A, a)$	
…			

$\alpha(A, a) = A \to X_1X_2\ldots X_n \in P$ if $a \in First(X_1)$; otherwise, $\alpha(A, a)$ is blank $\Rightarrow$ **ERROR**

Task: LL table for SPL

	id	int	:=	…
<prog>				
<st-list>				
<stat>				
<it-list>				
<item>				

Rule r: $A \to X_1X_2\ldots X_n$	$First(X_1)$
1: <prog> → begin …	{begin}
2: <st-list> → <stat>…	{id, write, read}
3: <st-list> → end	{end}
4: <stat> → read …	{read}
5: <stat> → write …	{write}
6: <stat> → id …	{id}
7: <it-list> → , …	{,}
8: <it-list> →)	{)}
9: <item> → int	{int}
10: <item> → id	{id}

Construction of LL Table

α	…	a	…
…			
A		$\alpha(A, a)$	
…			

$\alpha(A, a) = A \rightarrow X_1X_2\ldots X_n \in P$
if $a \in First(X_1)$; otherwise,
$\alpha(A, a)$ is blank $\Rightarrow$ **ERROR**

Task: LL table for SPL

	id	int	:=	…
<prog>				
<st-list>	2			
<stat>				
<it-list>				
<item>				

id $\in$ *First*(<stat>)

Rule r: $A \rightarrow X_1X_2\ldots X_n$	$First(X_1)$
1: <prog> → begin …	{begin}
2: <st-list> → <stat>…	{id, write, read}
3: <st-list> → end	{end}
4: <stat> → read …	{read}
5: <stat> → write …	{write}
6: <stat> → id …	{id}
7: <it-list> → , …	{,}
8: <it-list> →)	{)}
9: <item> → int	{int}
10: <item> → id	{id}

Construction of LL Table

α	…	a	…
…			
A		α(A, a)	
…			

α(A, a) = $A \to X_1X_2…X_n \in P$
if $a \in First(X_1)$; otherwise,
α(A, a) is blank ⇒ **ERROR**

Task: LL table for SPL

	id	int	:=	…
<prog>				
<st-list>	2			
<stat>	6			
<it-list>				
<item>				

id ∈ *First*(<stat>)

id ∈ *First*(id)

Rule r: $A \to X_1X_2…X_n$	***First*(X_1)**
1: <prog> → begin …	{begin}
2: <st-list> → <stat>…	{id, write, read}
3: <st-list> → end	{end}
4: <stat> → read …	{read}
5: <stat> → write …	{write}
6: <stat> → id …	{id}
7: <it-list> → , …	{,}
8: <it-list> →)	{)}
9: <item> → int	{int}
10: <item> → id	{id}

Construction of LL Table

α	...	a	...
...			
A		$\alpha(A, a)$	
...			

$\alpha(A, a) = A \to X_1X_2\ldots X_n \in P$ if $a \in First(X_1)$; otherwise, $\alpha(A, a)$ is blank $\Rightarrow$ **ERROR**

Task: LL table for SPL

	id	int	:=	...
<prog>				
<st-list>	2			
<stat>	6			
<it-list>				
<item>	10			

2 ← id ∈ *First*(<stat>)

6 ← id ∈ *First*(id)

10 ← id ∈ *First*(id)

Rule r: $A \to X_1X_2\ldots X_n$	$First(X_1)$
1: <prog> → begin …	{begin}
2: <st-list> → <stat>…	{id, write, read}
3: <st-list> → end	{end}
4: <stat> → read …	{read}
5: <stat> → write …	{write}
6: <stat> → id …	{id}
7: <it-list> → , …	{,}
8: <it-list> →)	{)}
9: <item> → int	{int}
10: <item> → id	{id}

Construction of LL Table

α	…	*a*	…
…			
A		α(*A*, *a*)	
…			

$\alpha(A, a) = A \rightarrow X_1X_2\ldots X_n \in P$
if $a \in First(X_1)$; otherwise,
$\alpha(A, a)$ is blank ⇒ **ERROR**

Task: LL table for SPL

	id	int	:=	…
<prog>				
<st-list>	2			
<stat>	6			
<it-list>				
<item>	10			

id ∈ *First*(<stat>)

id ∈ *First*(id)

id ∈ *First*(id)

Construct the rest analogically.

Rule *r*: $A \rightarrow X_1X_2\ldots X_n$		*First*(X_1)
1: <prog>	→ begin …	{begin}
2: <st-list>	→ <stat>…	{id, write, read}
3: <st-list>	→ end	{end}
4: <stat>	→ read …	{read}
5: <stat>	→ write …	{write}
6: <stat>	→ id …	{id}
7: <it-list>	→ , …	{,}
8: <it-list>	→)	{)}
9: <item>	→ int	{int}
10: <item>	→ id	{id}

Parsing Based on LL Table: Example

1: <prog> → **begin** <st-list>
2: <st-list> → <stat> **;** <st-list>
3: <st-list> → **end**
4: <stat> → **read** **id**
5: <stat> → **write** <item>
6: <stat> → **id** **:=** **add** **(** …
7: <it-list> → **,** <item> <it-list>
8: <it-list> → **)**
9: <item> → **int**
10: <item> → **id**

	beg	end	rd	wr	id	int	,	(	)	;	:=	add
<prog>	1											
<st-list>		3	2	2	2							
<stat>			4	5	6							
<it-list>							7		8			
<item>					10	9						

<prog>

Source program:

```
begin write 25; end
```

Lexical Analyzer

Parsing Based on LL Table: Example

1: <prog> → **begin** <st-list>
2: <st-list> → <stat> **;** <st-list>
3: <st-list> → **end**
4: <stat> → **read** **id**
5: <stat> → **write** <item>
6: <stat> → **id** **:=** **add** **(** ...
7: <it-list> → **,** <item> <it-list>
8: <it-list> → **)**
9: <item> → **int**
10: <item> → **id**

	beg	end	rd	wr	id	int	,	(	)	;	:=	add
<prog>	1											
<st-list>		3	2	2	2							
<stat>			4	5	6							
<it-list>							7		8			
<item>					10	9						

<prog>

Source program:

`begin write 25; end`

Lexical Analyzer

begin

Parsing Based on LL Table: Example

1: <prog> → **begin** <st-list>
2: <st-list> → <stat> **;** <st-list>
3: <st-list> → **end**
4: <stat> → **read** **id**
5: <stat> → **write** <item>
6: <stat> → **id** **:=** **add** **(** …
7: <it-list> → **,** <item> <it-list>
8: <it-list> → **)**
9: <item> → **int**
10: <item> → **id**

	beg	end	rd	wr	id	int	,	(	)	;	:=	add
<prog>	1											
<st-list>		3	2	2	2							
<stat>			4	5	6							
<it-list>							7		8			
<item>					10	9						

<prog>

Source program:

```
begin write 25; end
```

Lexical Analyzer

begin

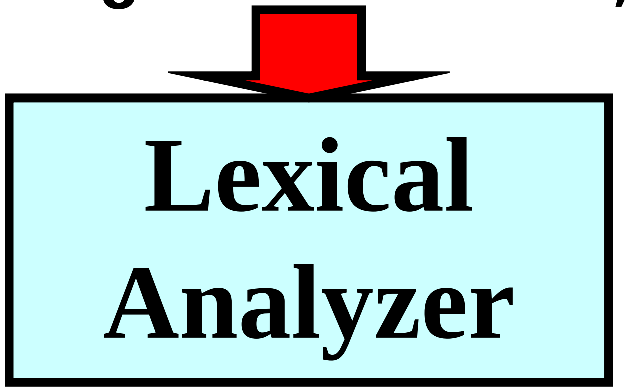
Parsing Based on LL Table: Example

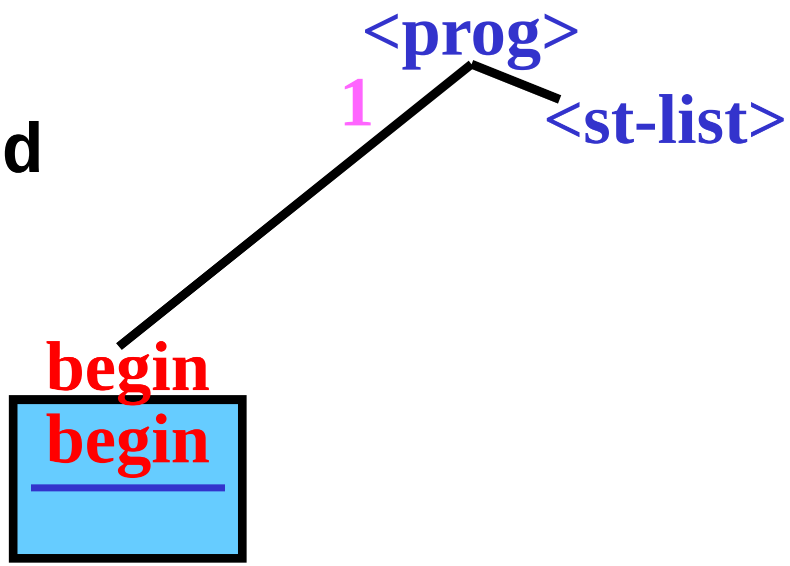
1: <prog> → **begin** <st-list>
2: <st-list> → <stat> **;** <st-list>
3: <st-list> → **end**
4: <stat> → **read** **id**
5: <stat> → **write** <item>
6: <stat> → **id** **:=** **add** **(** …
7: <it-list> → **,** <item> <it-list>
8: <it-list> → **)**
9: <item> → **int**
10: <item> → **id**

	beg	end	rd	wr	id	int	,	(	)	;	:=	add
<prog>	1											
<st-list>		3	2	2	2							
<stat>			4	5	6							
<it-list>							7		8			
<item>					10	9						

Source program:

```
begin write 25; end
```





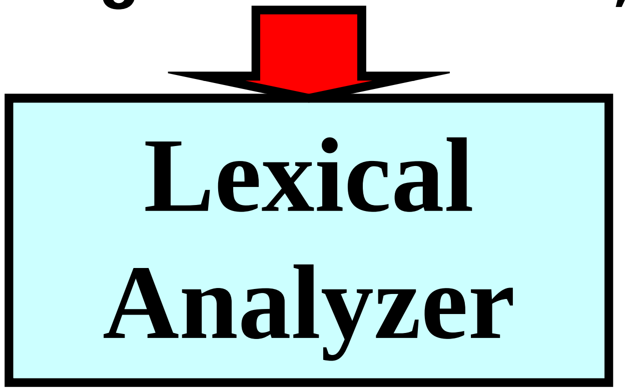
Parsing Based on LL Table: Example

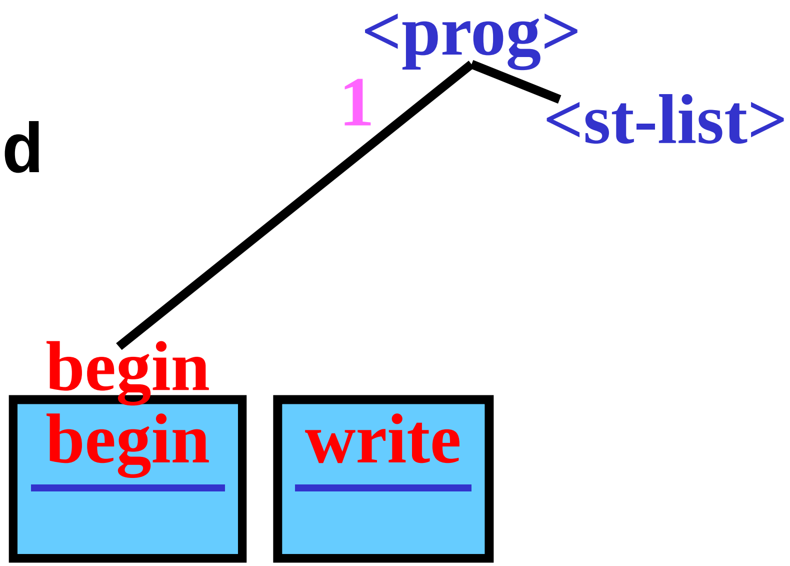
1: <prog> → **begin** <st-list>
2: <st-list> → <stat> **;** <st-list>
3: <st-list> → **end**
4: <stat> → **read** **id**
5: <stat> → **write** <item>
6: <stat> → **id** **:=** **add** **(** …
7: <it-list> → **,** <item> <it-list>
8: <it-list> → **)**
9: <item> → **int**
10: <item> → **id**

	beg	end	rd	wr	id	int	,	(	)	;	:=	add
<prog>	1											
<st-list>		3	2	2	2							
<stat>			4	5	6							
<it-list>							7		8			
<item>					10	9						

Source program:

`begin write 25; end`





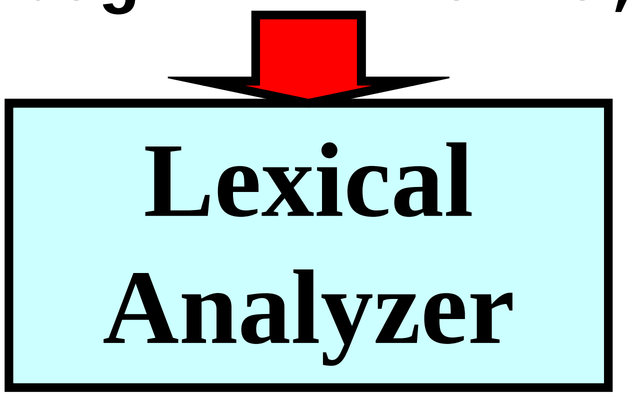
Parsing Based on LL Table: Example

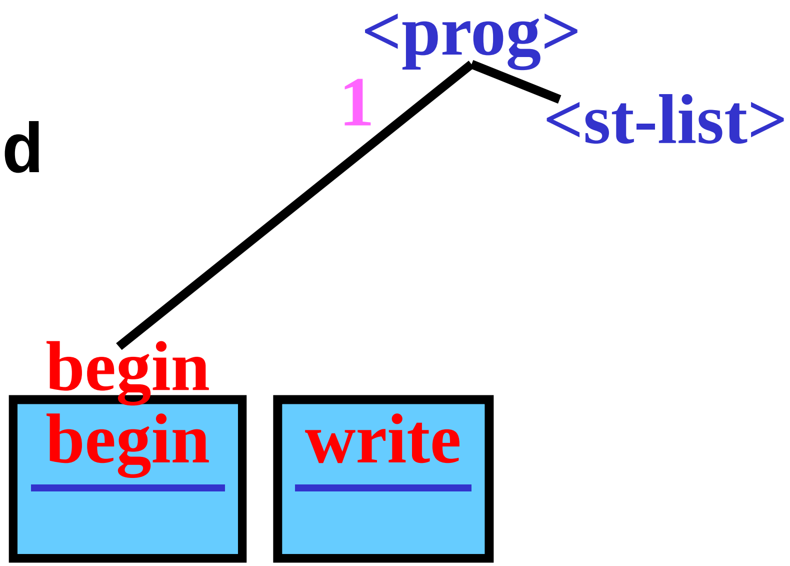
1: <prog> → **begin** <st-list>
2: <st-list> → <stat> **;** <st-list>
3: <st-list> → **end**
4: <stat> → **read** **id**
5: <stat> → **write** <item>
6: <stat> → **id** **:=** **add** **(** …
7: <it-list> → **,** <item> <it-list>
8: <it-list> → **)**
9: <item> → **int**
10: <item> → **id**

	beg	end	rd	wr	id	int	,	(	)	;	:=	add
<prog>	1											
<st-list>		3	2	2	2							
<stat>			4	5	6							
<it-list>							7		8			
<item>					10	9						

Source program:

`begin write 25; end`





Parsing Based on LL Table: Example

1: <prog> → begin <st-list>
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3: <st-list> → end
4: <stat> → read id
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6: <stat> → id := add (…
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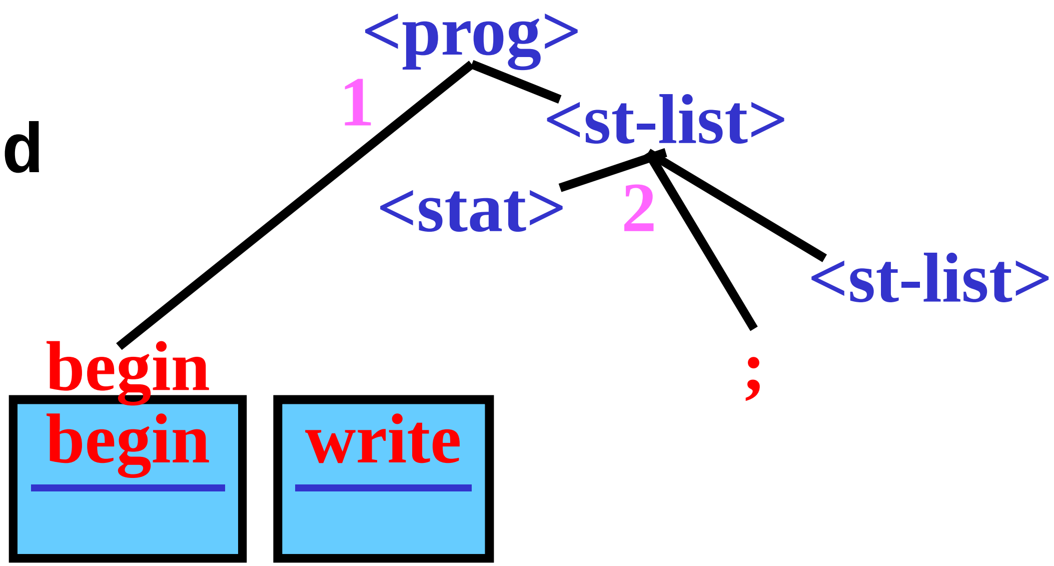
8: <it-list> →)
9: <item> → int
10: <item> → id

	beg	end	rd	wr	id	int	,	(	)	;	:=	add
<prog>	1											
<st-list>		3	2	2	2							
<stat>			4	5	6							
<it-list>							7		8			
<item>					10	9						

Source program:

```
begin write 25; end
```

Lexical Analyzer



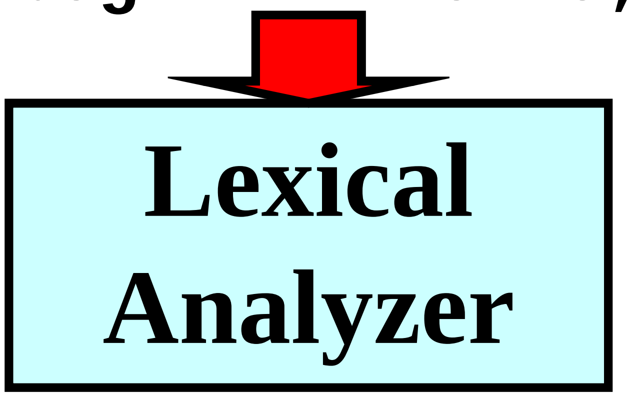
Parsing Based on LL Table: Example

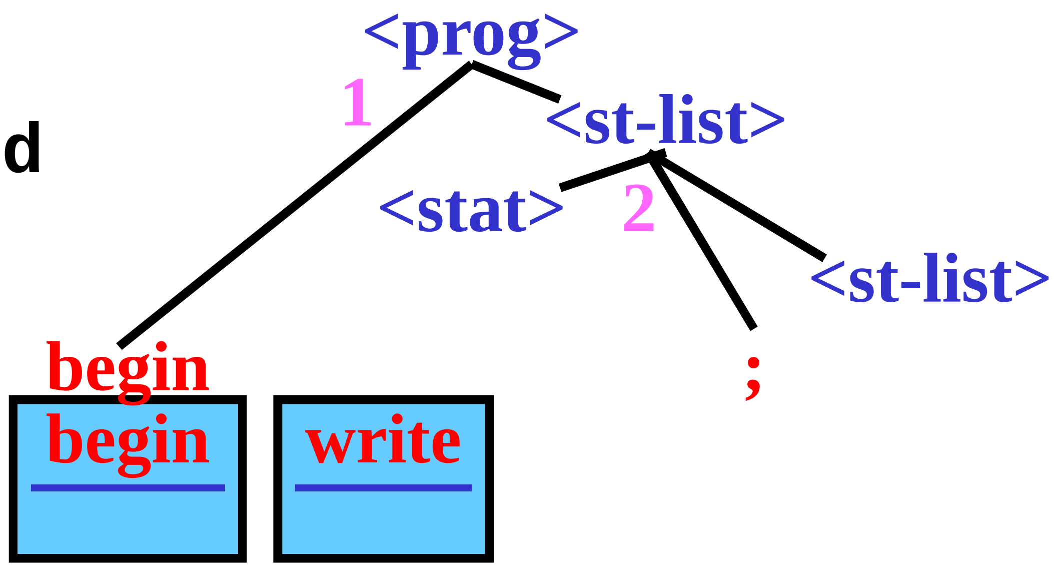
1: <prog> → begin <st-list>
2: <st-list> → <stat> ; <st-list>
3: <st-list> → end
4: <stat> → read id
5: <stat> → write <item>
6: <stat> → id := add (…
7: <it-list> → , <item> <it-list>
8: <it-list> →)
9: <item> → int
10: <item> → id

	beg	end	rd	wr	id	int	,	(	)	;	:=	add
<prog>	1											
<st-list>		3	2	2	2							
<stat>			4	$\boxed{5}$	6							
<it-list>							7		8			
<item>					10	9						

Source program:

```
begin write 25; end
```





Parsing Based on LL Table: Example

1: <prog> → **begin** <st-list>
2: <st-list> → <stat> **;** <st-list>
3: <st-list> → **end**
4: <stat> → **read** **id**
5: <stat> → **write** <item>
6: <stat> → **id** **:=** **add** **(** …
7: <it-list> → **,** <item> <it-list>
8: <it-list> → **)**
9: <item> → **int**
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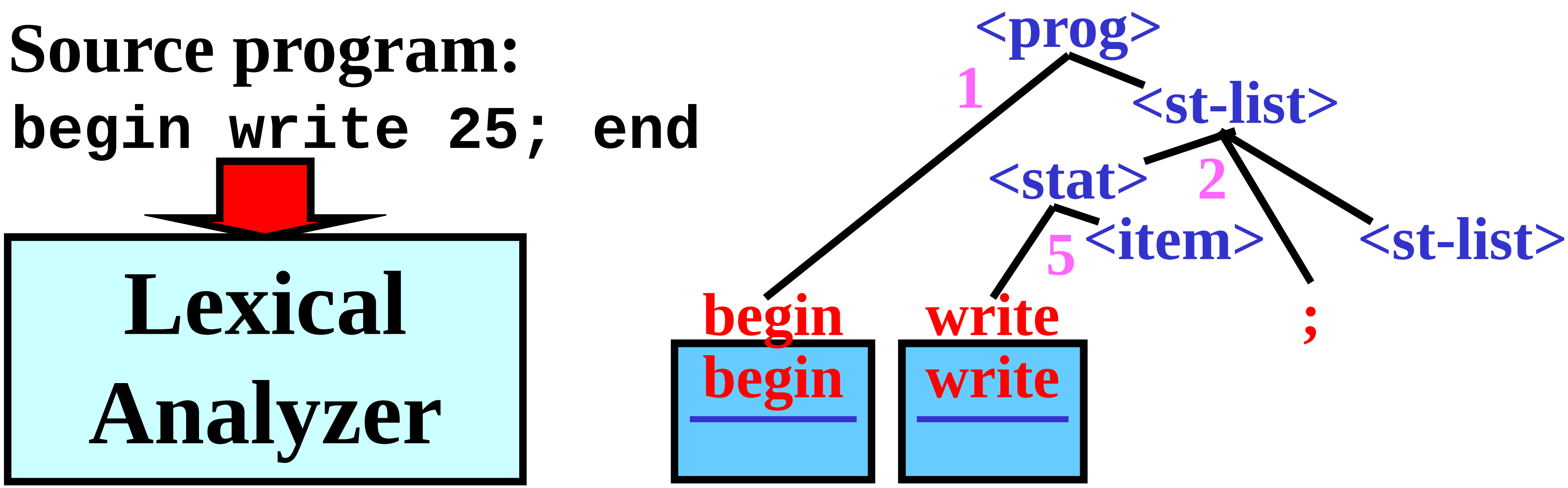
	beg	end	rd	wr	id	int	,	(	)	;	:=	add
<prog>	1											
<st-list>		3	2	2	2							
<stat>			4	5	6							
<it-list>							7		8			
<item>					10	9						

Source program:

```
begin write 25; end
```



Parsing Based on LL Table: Example

1: <prog> → **begin** <st-list>
2: <st-list> → <stat> **;** <st-list>
3: <st-list> → **end**
4: <stat> → **read id**
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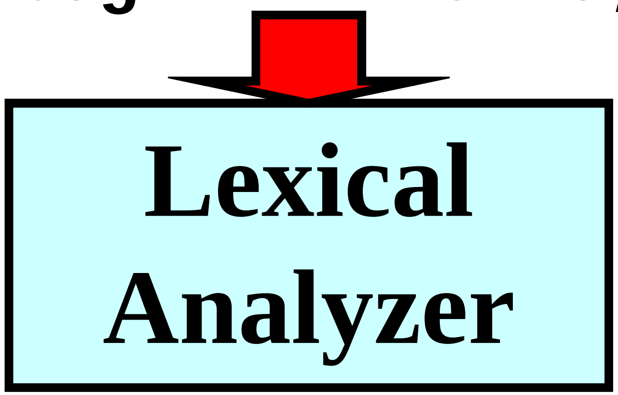
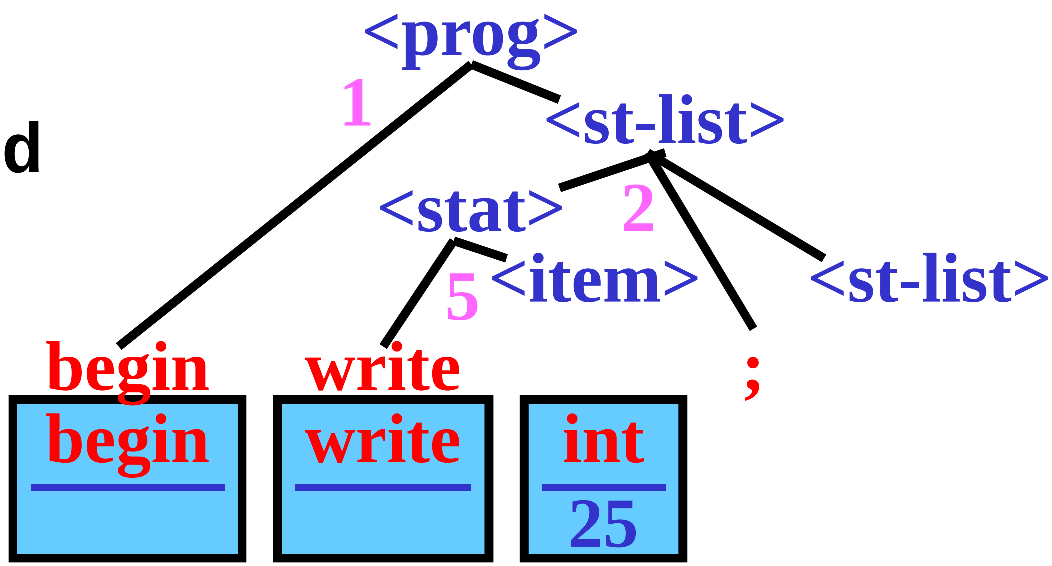
6: <stat> → **id := add (** …
7: <it-list> → **,** <item> <it-list>
8: <it-list> → **)**
9: <item> → **int**
10: <item> → **id**

	beg	end	rd	wr	id	int	,	(	)	;	:=	add
<prog>	1											
<st-list>		3	2	2	2							
<stat>			4	5	6							
<it-list>							7		8			
<item>					10	9						

Source program:

```
begin write 25; end
```





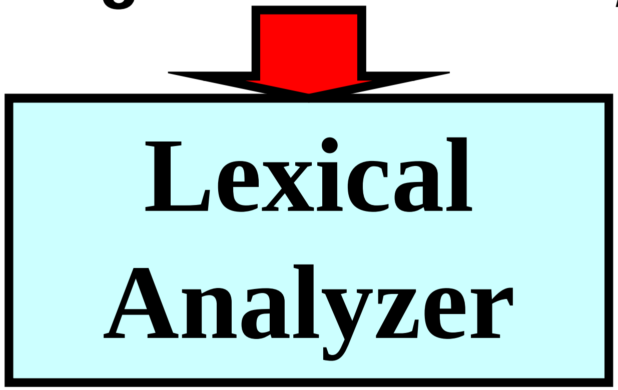
Parsing Based on LL Table: Example

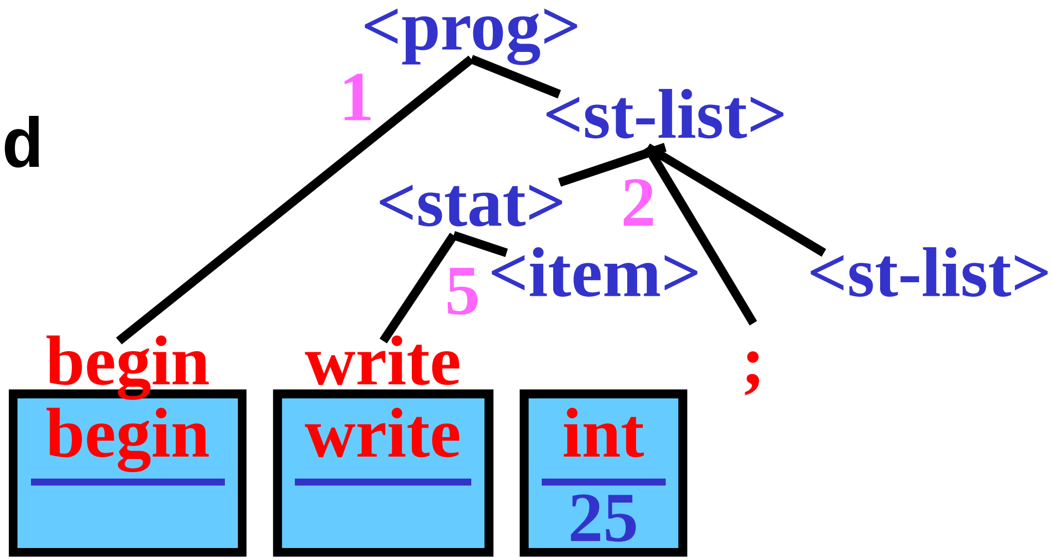
1: <prog> → **begin** <st-list>
2: <st-list> → <stat> **;** <st-list>
3: <st-list> → **end**
4: <stat> → **read id**
5: <stat> → **write** <item>
6: <stat> → **id := add (** …
7: <it-list> → **,** <item> <it-list>
8: <it-list> → **)**
9: <item> → **int**
10: <item> → **id**

	beg	end	rd	wr	id	int	,	(	)	;	:=	add
<prog>	1											
<st-list>		3	2	2	2							
<stat>			4	5	6							
<it-list>							7		8			
<item>					10	$\boxed{9}$						

Source program:

```
begin write 25; end
```





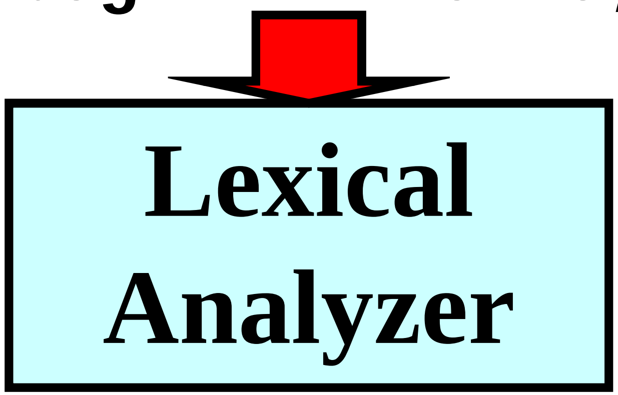
Parsing Based on LL Table: Example

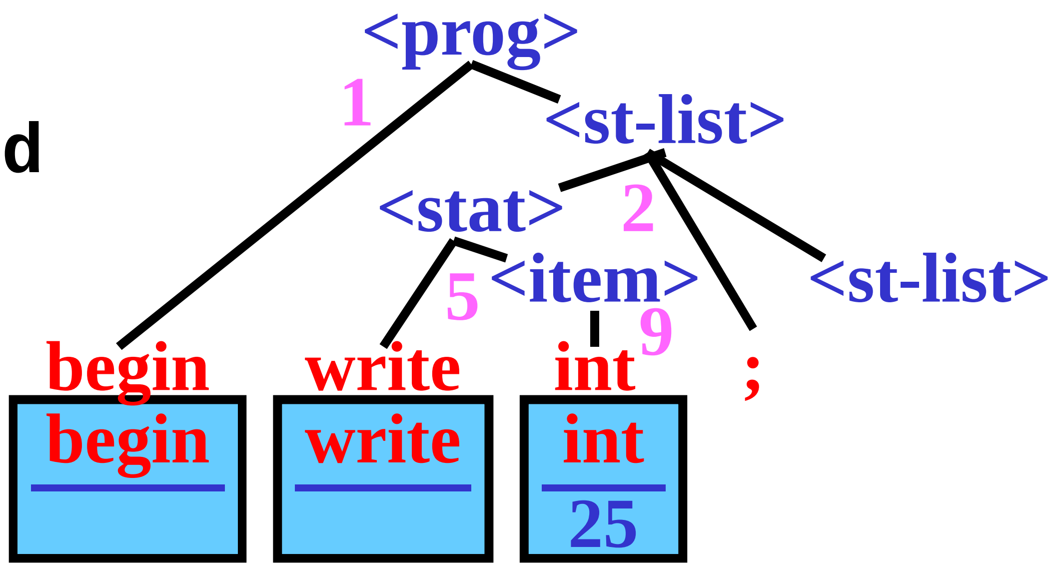
1: <prog> → begin <st-list>
2: <st-list> → <stat> ; <st-list>
3: <st-list> → end
4: <stat> → read id
5: <stat> → write <item>
6: <stat> → id := add (…
7: <it-list> → , <item> <it-list>
8: <it-list> →)
9: <item> → int
10: <item> → id

	beg	end	rd	wr	id	int	,	(	)	;	:=	add
<prog>	1											
<st-list>		3	2	2	2							
<stat>			4	5	6							
<it-list>							7		8			
<item>					10	9						

Source program:

`begin write 25; end`





Parsing Based on LL Table: Example

1: <prog> → **begin** <st-list>
2: <st-list> → <stat> **;** <st-list>
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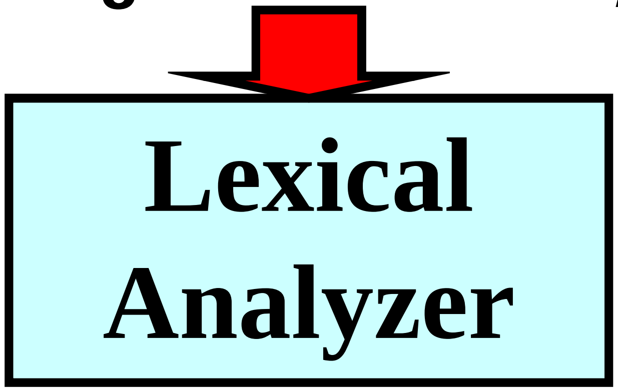
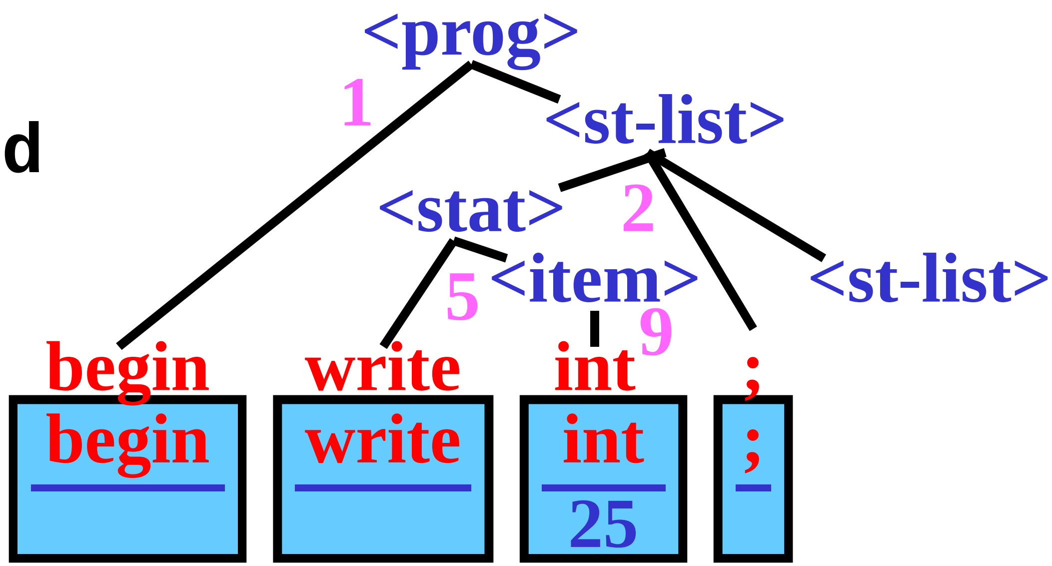
6: <stat> → **id** **:=** **add** **(** …
7: <it-list> → **,** <item> <it-list>
8: <it-list> → **)**
9: <item> → **int**
10: <item> → **id**

	beg	end	rd	wr	id	int	,	(	)	;	:=	add
<prog>	1											
<st-list>		3	2	2	2							
<stat>			4	5	6							
<it-list>							7		8			
<item>					10	9						

Source program:

```
begin write 25; end
```





Parsing Based on LL Table: Example

1: <prog> → begin <st-list>
2: <st-list> → <stat> ; <st-list>
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4: <stat> → read id
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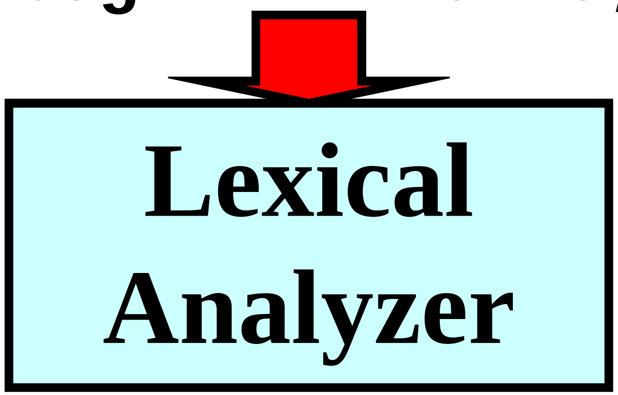
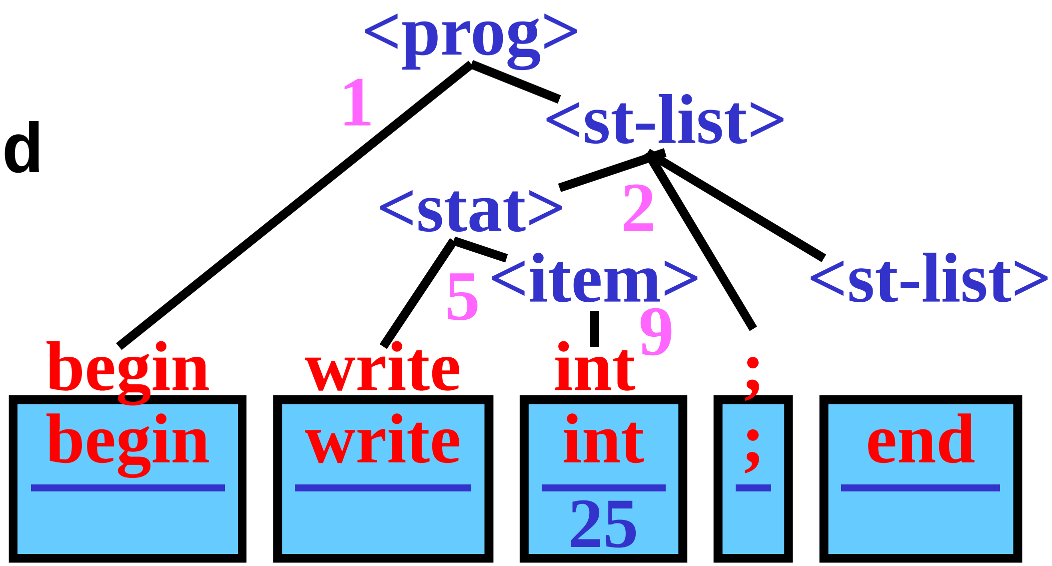
6: <stat> → id := add (…
7: <it-list> → , <item> <it-list>
8: <it-list> →)
9: <item> → int
10: <item> → id

	beg	end	rd	wr	id	int	,	(	)	;	:=	add
<prog>	1											
<st-list>		3	2	2	2							
<stat>			4	5	6							
<it-list>							7		8			
<item>					10	9						

Source program:

```
begin write 25; end
```





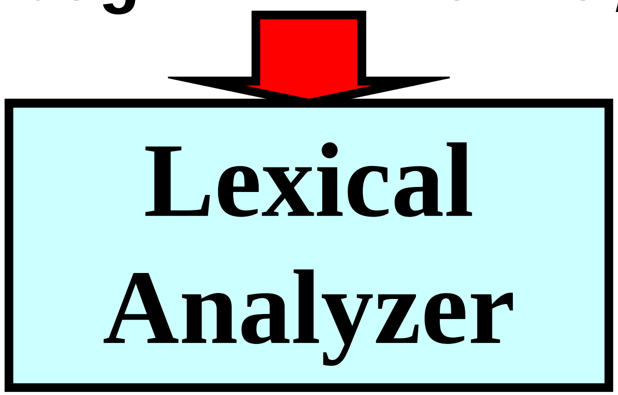
Parsing Based on LL Table: Example

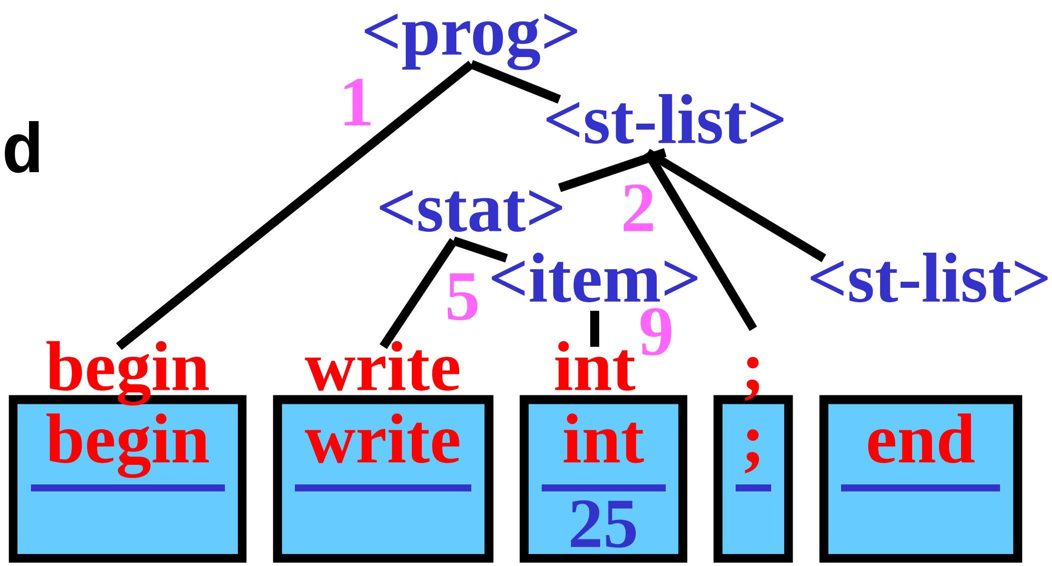
1: <prog> → **begin** <st-list>
2: <st-list> → <stat> **;** <st-list>
3: <st-list> → **end**
4: <stat> → **read** **id**
5: <stat> → **write** <item>
6: <stat> → **id** **:=** **add** **(** …
7: <it-list> → **,** <item> <it-list>
8: <it-list> → **)**
9: <item> → **int**
10: <item> → **id**

	beg	end	rd	wr	id	int	,	(	)	;	:=	add
<prog>	1											
<st-list>		$\boxed{3}$	2	2	2							
<stat>			4	5	6							
<it-list>							7		8			
<item>					10	9						

Source program:

```
begin write 25; end
```





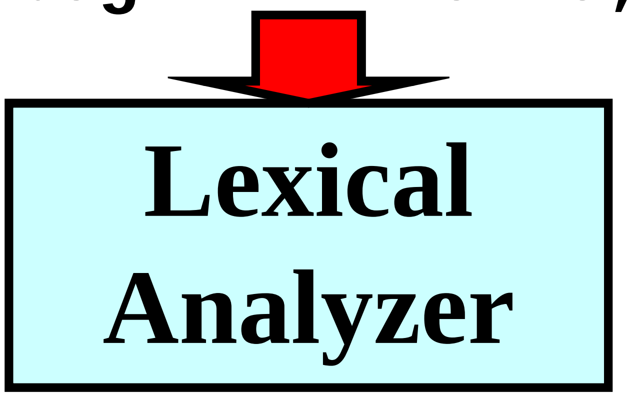
Parsing Based on LL Table: Example

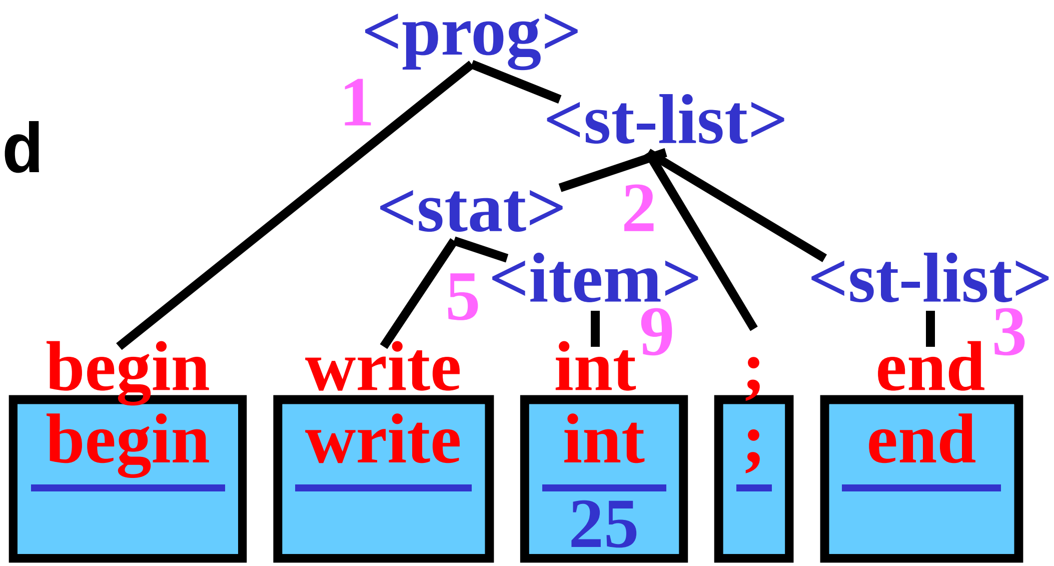
1: <prog> → **begin** <st-list>
2: <st-list> → <stat> **;** <st-list>
3: <st-list> → **end**
4: <stat> → **read** **id**
5: <stat> → **write** <item>
6: <stat> → **id** **:=** **add** **(** …
7: <it-list> → **,** <item> <it-list>
8: <it-list> → **)**
9: <item> → **int**
10: <item> → **id**

	beg	end	rd	wr	id	int	,	(	)	;	:=	add
<prog>	1											
<st-list>		3	2	2	2							
<stat>			4	5	6							
<it-list>							7		8			
<item>					10	9						

Source program:

`begin write 25; end`

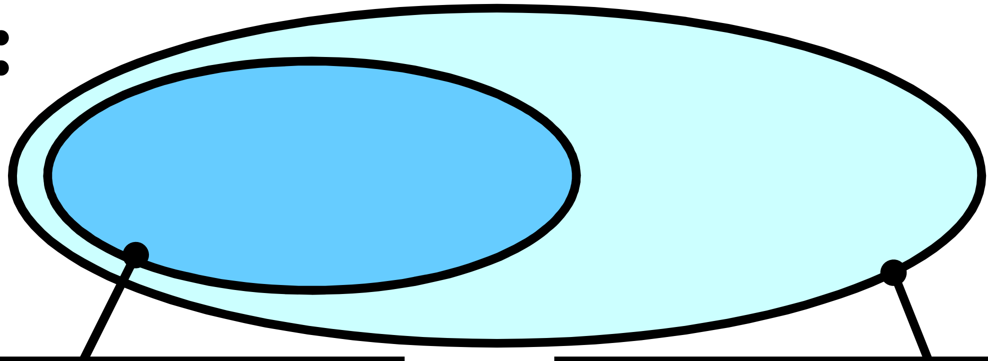




LL Grammars: Useful Transformations

Generaly: CFG are stronger that LL grammars

Illustration:



The family of languages generated by LL grammars $\subset$ **The family of languages generated by CFGs**

- **Some** CFGs can be converted to equivalent LL grammars

Basic conversions:

1) Factorization

2) Left recursion replacement

Note: A rule of the form $A \rightarrow Ax$, where $A \in N$, $x \in (N \cup T)^*$ is called a *left recursive rule*.

Factorization

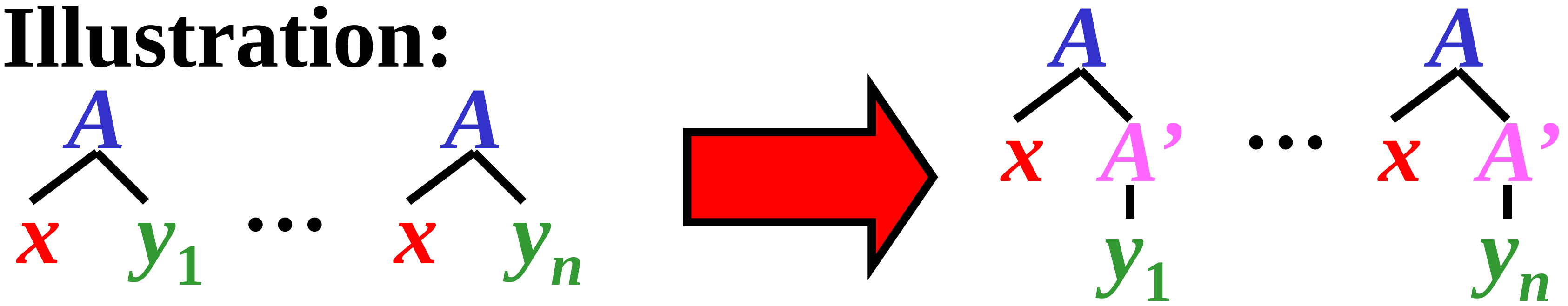
Idea: Replace rules of the form

$$A \rightarrow xy_1,\ A \rightarrow xy_2,\ \ldots,\ A \rightarrow xy_n \text{ with}$$

$$A \rightarrow xA',\ A' \rightarrow y_1,\ A' \rightarrow y_2,\ \ldots,\ A' \rightarrow y_n,$$

where A' is a new nonterminal

Illustration:



Factorization

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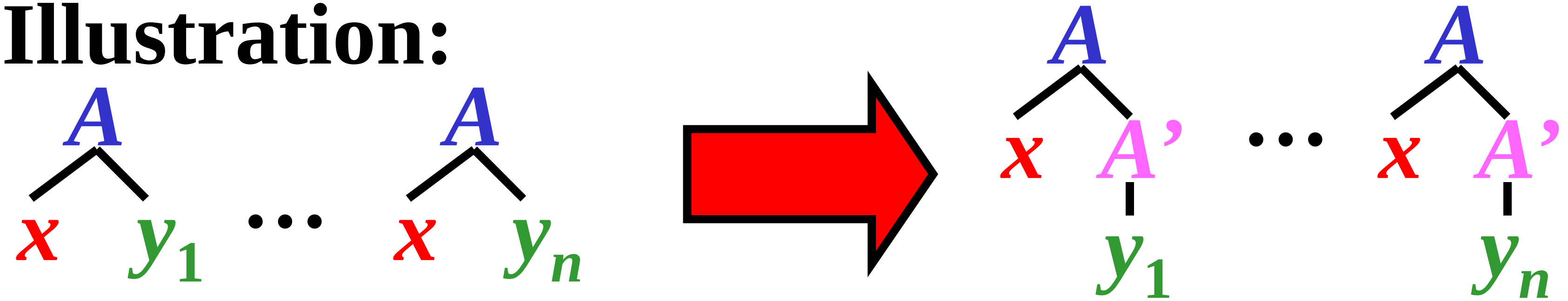
Illustration:



Example:

<stat> → <u>**write**</u> <u>**id**</u>

<stat> → <u>**write**</u> <u>**int**</u>

Factorization

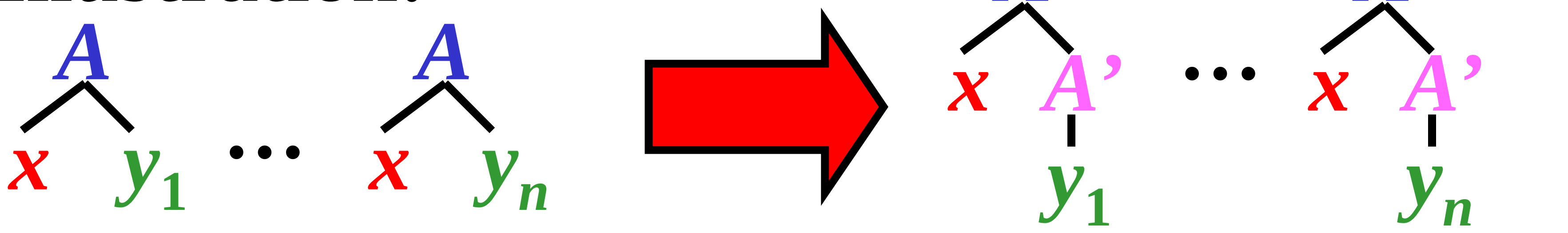
Idea: Replace rules of the form

$$A \rightarrow xy_1,\ A \rightarrow xy_2,\ \ldots,\ A \rightarrow xy_n \text{ with}$$

$$A \rightarrow xA',\ A' \rightarrow y_1,\ A' \rightarrow y_2,\ \ldots,\ A' \rightarrow y_n,$$

where A' is a new nonterminal

Illustration:



Example:

<stat> → write id
<stat> → write int

<stat> → write <item>
<item> → id
<item> → int

Left Recursion Replacement

Idea: Replace rules of the form $A \to Ax$, $A \to y$ with $A \to yA'$, $A' \to xA'$, $A' \to \varepsilon$, where A' is a new nonterminal.

Illustration:

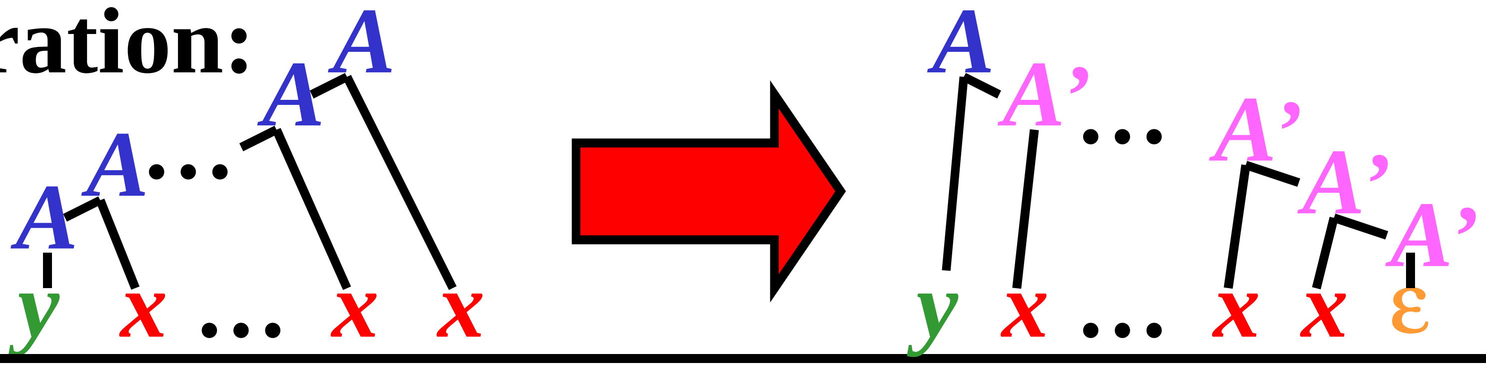


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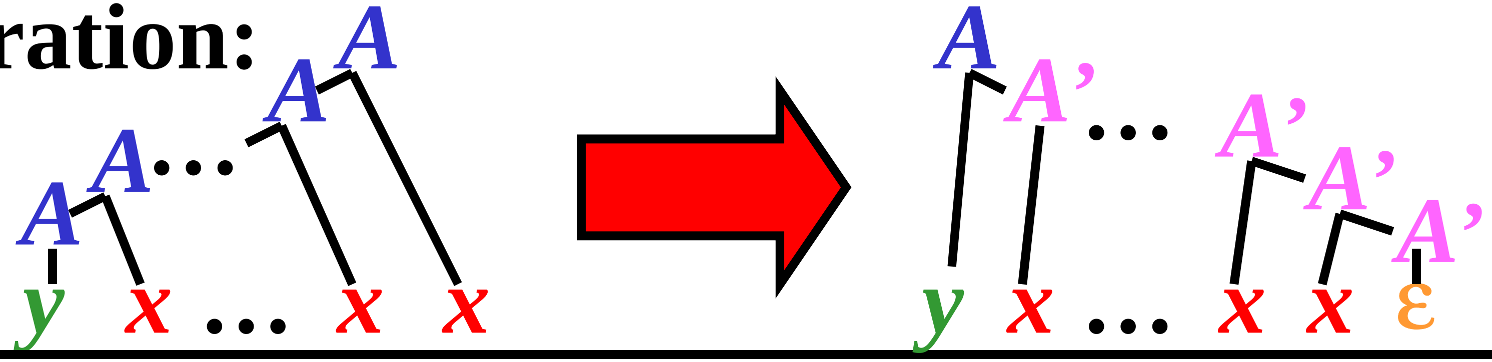
Illustration:



Example:

$E \rightarrow E+T$

$E \rightarrow T$

$T \rightarrow T*F$

$T \rightarrow F$

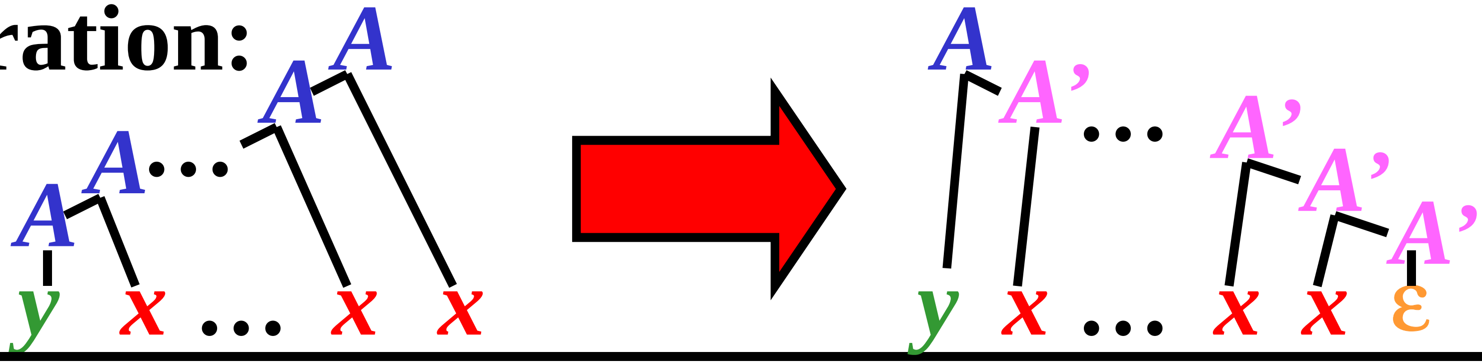
$F \rightarrow (E)$

$F \rightarrow i$

Left Recursion Replacement

Idea: Replace rules of the form $A \rightarrow Ax$, $A \rightarrow y$ with $A \rightarrow yA'$, $A' \rightarrow xA'$, $A' \rightarrow \varepsilon$, where A' is a new nonterminal.

Illustration:



Example:

$$\left.\begin{array}{l} E \rightarrow E+T \\ E \rightarrow T \end{array}\right\} \Rightarrow E \rightarrow TE',\ E' \rightarrow +TE',\ E' \rightarrow \varepsilon$$

$$\left.\begin{array}{l} T \rightarrow T*F \\ T \rightarrow F \end{array}\right\} \Rightarrow T \rightarrow FT',\ T' \rightarrow *FT',\ T' \rightarrow \varepsilon$$

$F \rightarrow (E)$ $\qquad$ $F \rightarrow (E)$

$F \rightarrow i$ $\qquad$ $F \rightarrow i$

LL Grammars with ε-rules: Introduction

Why ε-rules?

- elimination of the left recursion introduces ε-rule
- ε-rules often make the language specification clearer

Simplification of this part:

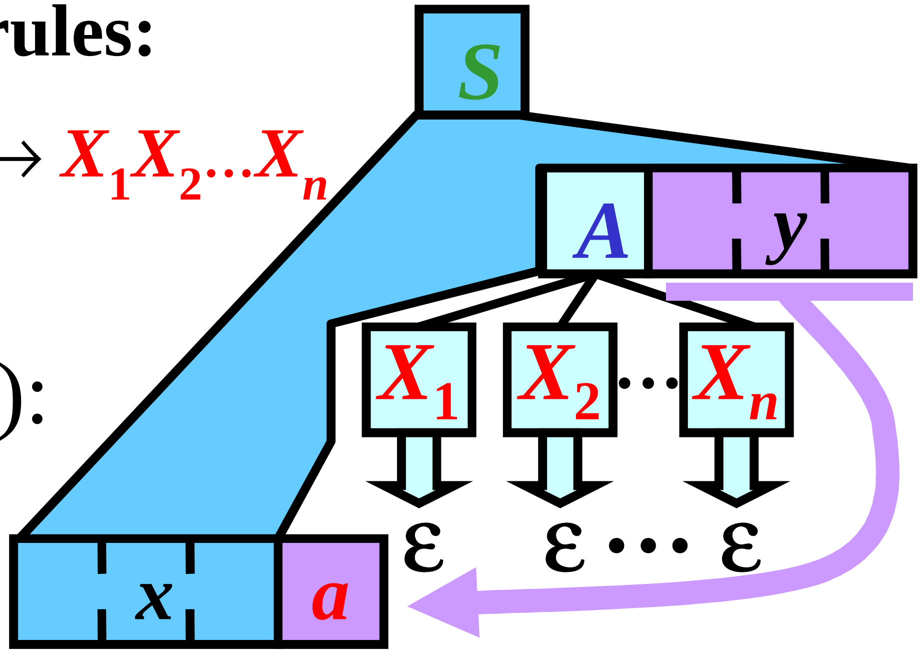
Assume that every input string of tokens ends with $.

Note: **$** acts as an *end marker*.

Main problem with ε-rules:

Rule ***r*****:** $A \rightarrow X_1X_2\ldots X_n$

Maybe: $a \notin First(A)$:



Note: We must define other sets: *Empty*, *Follow* and *Predict*.

Grammar for Arithmetical Expressions

- $G_{expr3} = (N, T, P, \boldsymbol{E})$, where
- $N = \{\boldsymbol{E}, \boldsymbol{E'}, \boldsymbol{T}, \boldsymbol{T'}, \boldsymbol{F}\}$,
- $T = \{\boldsymbol{i}, \mathbf{+}, \mathbf{*}, \mathbf{(}, \mathbf{)}\}$,
- $P = \{$ 1: $\boldsymbol{E} \rightarrow \boldsymbol{TE'}$, 2: $\boldsymbol{E'} \rightarrow \mathbf{+}\boldsymbol{TE'}$,
  3: $\boldsymbol{E'} \rightarrow \varepsilon$, 4: $\boldsymbol{T} \rightarrow \boldsymbol{FT'}$,
  5: $\boldsymbol{T'} \rightarrow \mathbf{*}\boldsymbol{FT'}$, 6: $\boldsymbol{T'} \rightarrow \varepsilon$,
  7: $\boldsymbol{F} \rightarrow \mathbf{(}\boldsymbol{E}\mathbf{)}$, 8: $\boldsymbol{F} \rightarrow \boldsymbol{i}$ $\}$

Example:

$$\boldsymbol{(i + i)*(i + i)} \in \boldsymbol{L(G_{expr3})}$$

Set *Empty*

Gist: *Empty*(*x*) is the set that includes ε if *x* derives the empty string; otherwise, *Empty*(*x*) is empty

Definition: Let $G = (N, T, P, S)$ be a CFG.
$Empty(x) = \{\varepsilon\}$ if $x \Rightarrow^* \varepsilon$; otherwise,
$Empty(x) = \emptyset$, where $x \in (N \cup T)^*$.

Illustration: $x = \boxed{X_1}\ \boxed{X_2} \cdots \boxed{X_n}$

Set *Empty*

Gist: ***Empty*(*x*) is the set that includes ε if *x* derives the empty string; otherwise, *Empty*(*x*) is empty**

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$Empty(x) = \{\varepsilon\}$ if $x \Rightarrow^* \varepsilon$; otherwise,
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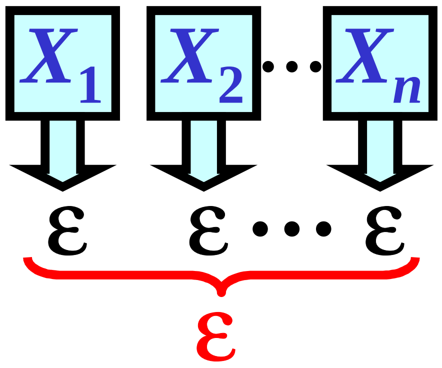
Illustration: x =



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$Empty(x) = \{\varepsilon\}$ if $x \Rightarrow^* \varepsilon$; otherwise,
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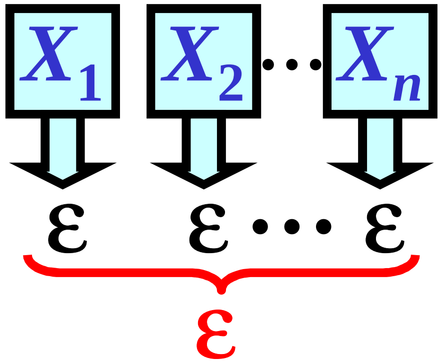
Illustration: $x =$



$x = X_1X_2\dots X_n \Rightarrow^* \varepsilon$

$Empty(x) = \{\varepsilon\}$

Algorithm: *Empty*(X)

- **Input:** $G = (N, T, P, S)$
- **Output:** $Empty(X)$ for every $X \in N \cup T$

- **Method:**
- **for each** $a \in T$: $Empty(a) := \emptyset$
- **for each** $A \in N$:

  if $A \to \varepsilon \in P$ **then** $Empty(A) := \{\varepsilon\}$

  else $Empty(A) := \emptyset$
- **Apply the following rule until no *Empty* set can be changed:**
  - **if** $A \to X_1X_2\ldots X_n \in P$ **and** $Empty(X_i) = \{\varepsilon\}$ for all $i = 1, \ldots, n$ **then** $Empty(A) := \{\varepsilon\}$

Previous Algorithm: Illustration

1) for each $a \in T$: *Empty*(a) := $\emptyset$ because $a \not\Rightarrow^* \varepsilon$

2) for each r**:** $A \rightarrow \varepsilon \in P$: *Empty*($A$) := {ε} because $A \Rightarrow^1 \varepsilon$ [r]

---

3) Apply the following rules until no *Empty* set can be changed:

Previous Algorithm: Illustration

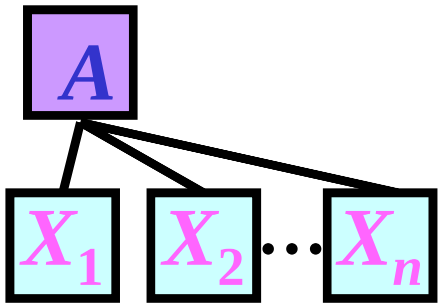
1) for each $\boldsymbol{a} \in T$: $Empty(\boldsymbol{a}) := \emptyset$ because $\boldsymbol{a} \not\Rightarrow^* \varepsilon$

2) for each $\boldsymbol{r}$**:** $\boldsymbol{A} \rightarrow \varepsilon \in \boldsymbol{P}$: $Empty(\boldsymbol{A}) := \{\varepsilon\}$ because $\boldsymbol{A} \Rightarrow^1 \varepsilon$ **[*r*]**

---

3) Apply the following rules until no *Empty* set can be changed:

- **if** $\boldsymbol{A} \rightarrow \boldsymbol{X_1 X_2 \ldots X_n} \in P$ **and** $Empty(\boldsymbol{X_i}) = \{\varepsilon\}$ for all $i = 1, \ldots, n$ **then** $Empty(\boldsymbol{A}) := \{\varepsilon\}$



Previous Algorithm: Illustration

1) for each $a \in T$: $Empty(a) := \emptyset$ because $a \not\Rightarrow^* \varepsilon$
2) for each r: $A \rightarrow \varepsilon \in P$: $Empty(A) := \{\varepsilon\}$ because $A \Rightarrow^1 \varepsilon$ [r]

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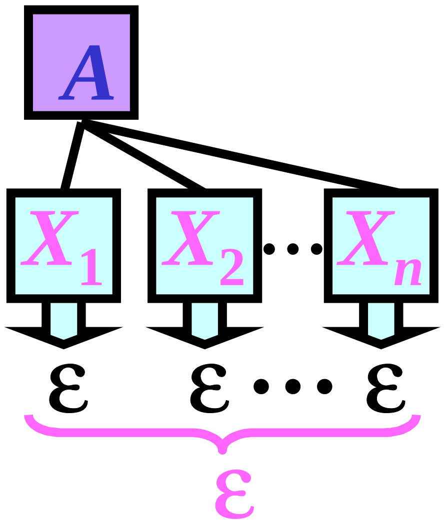
- **if** $A \rightarrow X_1X_2\ldots X_n \in P$ **and** $Empty(X_i) = \{\varepsilon\}$ for all $i = 1, \ldots, n$ **then** $Empty(A) := \{\varepsilon\}$



Previous Algorithm: Illustration

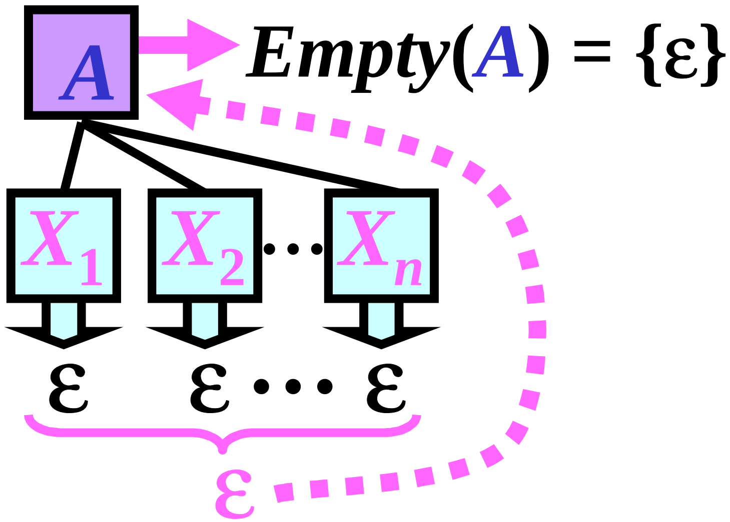
1) for each $\boldsymbol{a} \in T$: $Empty(\boldsymbol{a}) := \varnothing$ because $\boldsymbol{a} \not\Rightarrow^* \varepsilon$

2) for each $\boldsymbol{r}$**:** $\boldsymbol{A} \rightarrow \varepsilon \in \boldsymbol{P}$: $Empty(\boldsymbol{A}) := \{\varepsilon\}$ because $\boldsymbol{A} \Rightarrow^1 \varepsilon$ **[$\boldsymbol{r}$]**

---

3) Apply the following rules until no *Empty* set can be changed:

- **if** $\boldsymbol{A} \rightarrow \boldsymbol{X_1X_2}\ldots\boldsymbol{X_n} \in P$ **and** $Empty(\boldsymbol{X_i}) = \{\varepsilon\}$ for all $i = 1, \ldots, n$ **then** $Empty(\boldsymbol{A}) := \{\varepsilon\}$



Empty(*X*) for G_{expr3}: Example

$G_{expr3} = (N, T, P, E)$, where: $N = \{E, E', T, T', F\}$, $T = \{i, +, *, (,)\}$,

$P = \{$ 1: $E \rightarrow TE'$, 2: $E' \rightarrow +TE'$, 3: $E' \rightarrow \varepsilon$, 4: $T \rightarrow FT'$
5: $T' \rightarrow *FT'$, 6: $T' \rightarrow \varepsilon$, 7: $F \rightarrow (E)$, 8: $F \rightarrow i$ $\}$

Initialization:

Empty(i) := $\emptyset$	*Empty*(E) := $\emptyset$
Empty(+) := $\emptyset$	*Empty*(E') := $\{\varepsilon\}$
Empty(*) := $\emptyset$	*Empty*(T) := $\emptyset$
Empty(( ) := $\emptyset$	*Empty*(T') := $\{\varepsilon\}$
Empty( )) := $\emptyset$	*Empty*(F) := $\emptyset$

- **No *Empty* set can be changed.**

Algorithm: *First*(*X*)

- **Input:** $G = (N, T, P, S)$
- **Output:** $First(X)$ for every $X \in N \cup T$

- **Method:**
- **for each** $a \in T$: $First(a) := \{a\}$
- **for each** $A \in N$: $First(A) := \emptyset$
- **Apply the following rule until no *First* set can be changed:**
- **if** $A \to X_1X_2\ldots X_{k-1}X_k\ldots X_n \in P$ **then**
  - add all symbols from $First(X_1)$ to $First(A)$
  - **if** $Empty(X_i) = \{\varepsilon\}$ for all $i = 1,\ldots, k$-1, where $k \leq n$ **then** add all symbols from $First(X_k)$ to $First(A)$

Previous Algorithm: Illustration

1) for each $\boldsymbol{a} \in T$: $First(\boldsymbol{a}) := \{\boldsymbol{a}\}$ because $\boldsymbol{a} \Rightarrow^{\mathbf{0}} \boldsymbol{a}$

2) for each $\boldsymbol{A} \in N$: $First(\boldsymbol{A}) := \emptyset$ (inicialization)

---

3) Apply the following rules until no *First* set or *Empty* set can be changed:

- **if** $\boldsymbol{A} \rightarrow \boldsymbol{X_1 X_2 \ldots X_{k-1} X_k \ldots X_n} \in P$ **then**

Previous Algorithm: Illustration

1) for each $a \in T$: *First*(a) := {a} because $a \Rightarrow^0 a$

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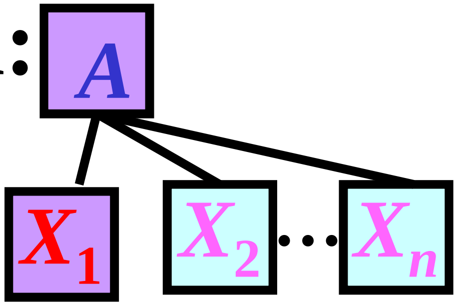
---

3) Apply the following rules until no *First* set or *Empty* set can be changed:

- **if** $A \rightarrow X_1X_2\ldots X_{k-1}X_k\ldots X_n \in P$ **then**

  3a) add all symbols from *First*(X_1) to *First*(A)

3a:



Previous Algorithm: Illustration

1) for each $a \in T$: *First*(a) := {a} because $a \Rightarrow^0 a$

2) for each $A \in N$: *First*(A) := ∅ (inicialization)

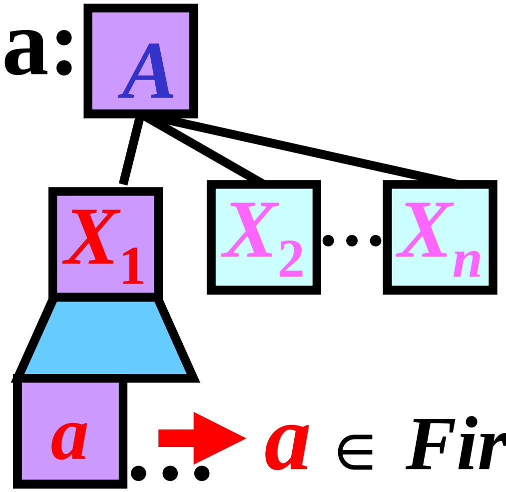
---

3) Apply the following rules until no *First* set or *Empty* set can be changed:

• **if** $A \rightarrow X_1X_2\ldots X_{k-1}X_k\ldots X_n \in P$ **then**

3a) add all symbols from *First*(X_1) to *First*(A)

3a:



$a \in$ ***First***(X_1)

Previous Algorithm: Illustration

1) for each $a \in T$: *First*(a) := {a} because $a \Rightarrow^0 a$

2) for each $A \in N$: *First*(A) := ∅ (inicialization)

---

3) Apply the following rules until no *First* set or *Empty* set can be changed:

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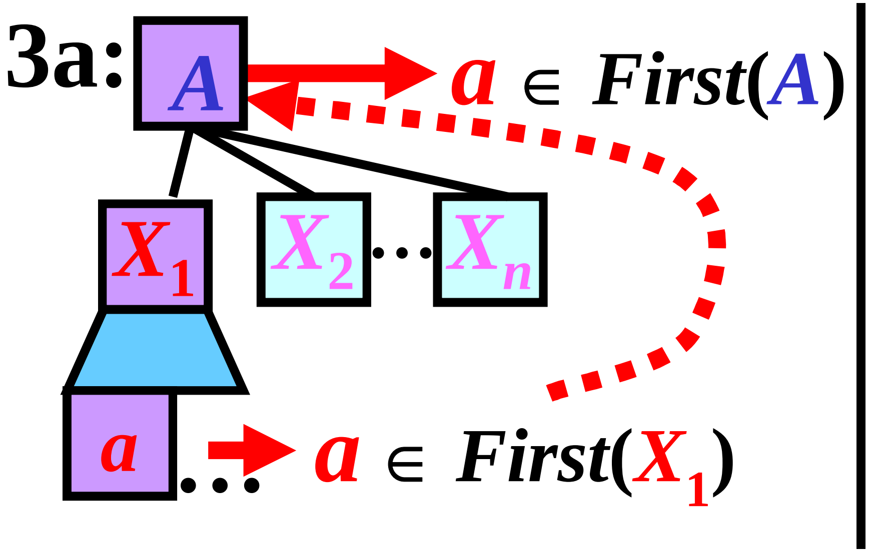
  3a) add all symbols from *First*(X_1) to *First*(A)



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1) for each $a \in T$: $First(a) := \{a\}$ because $a \Rightarrow^{0} a$

2) for each $A \in N$: $First(A) := \emptyset$ (inicialization)

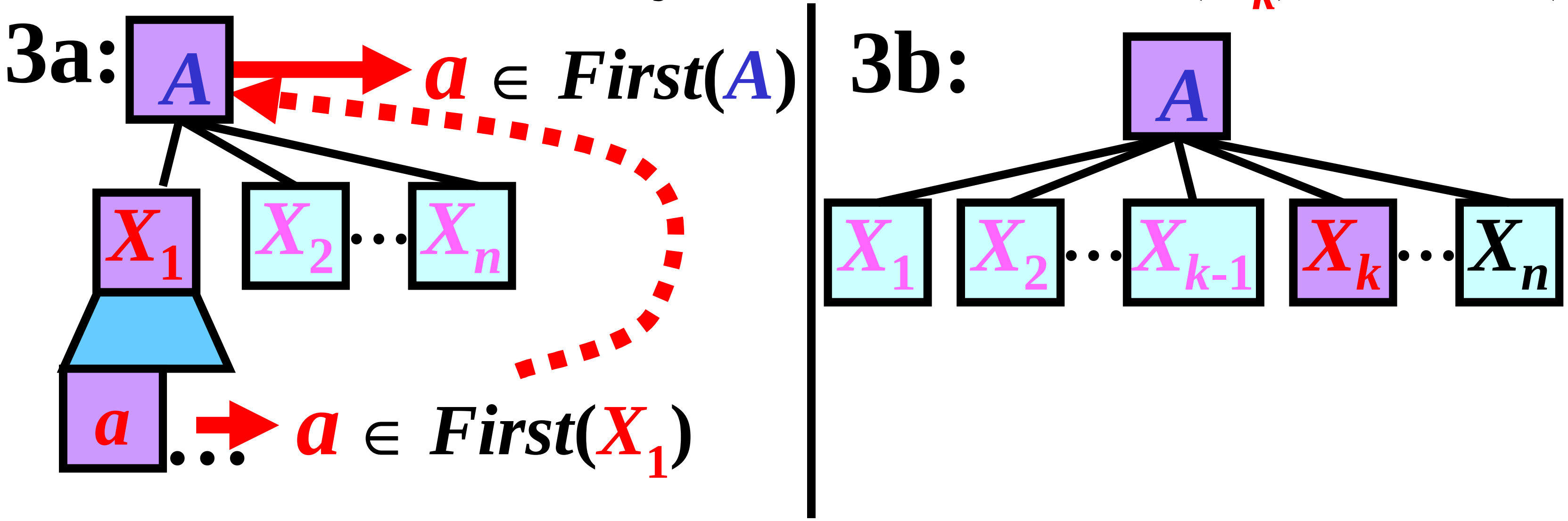
---

3) Apply the following rules until no *First* set or *Empty* set can be changed:

- **if** $A \rightarrow X_1X_2\ldots X_{k-1}X_k\ldots X_n \in P$ **then**

  3a) add all symbols from $First(X_1)$ to $First(A)$

  3b) if $Empty(X_i) = \{\varepsilon\}$ for all $i = 1, \ldots, k\text{-}1$, where $k \leq n$
  then add all symbols from $First(X_k)$ to $First(A)$:



Previous Algorithm: Illustration

1) for each $a \in T$: $First(a) := \{a\}$ because $a \Rightarrow^0 a$

2) for each $A \in N$: $First(A) := \varnothing$ (inicialization)

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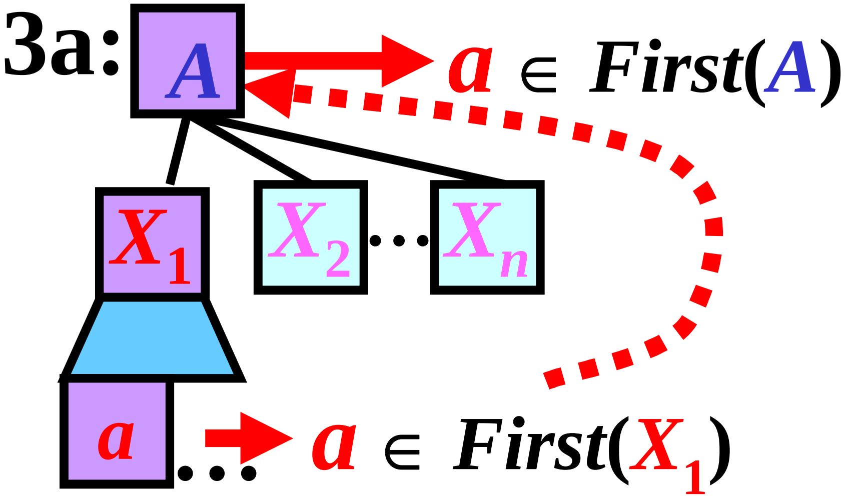
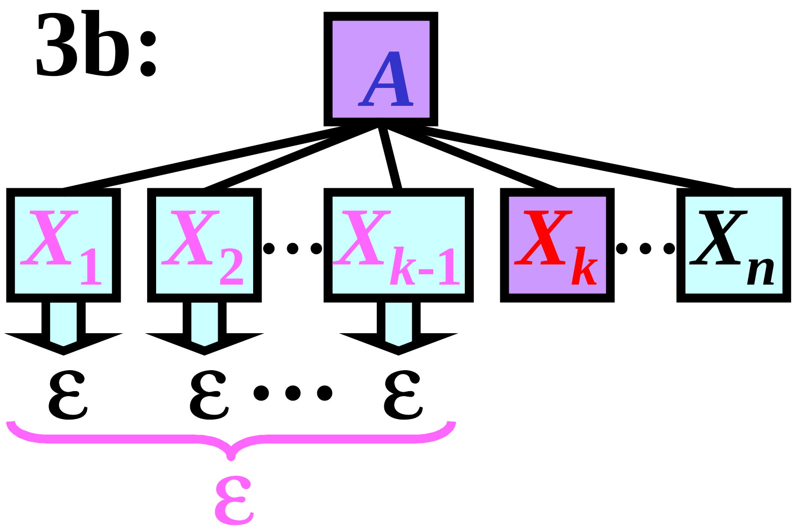
3) Apply the following rules until no *First* set or *Empty* set can be changed:

- **if** $A \rightarrow X_1X_2\ldots X_{k-1}X_k\ldots X_n \in P$ **then**
  - **3a)** add all symbols from $First(X_1)$ to $First(A)$
  - **3b) if** $Empty(X_i) = \{\varepsilon\}$ for all $i = 1, \ldots, k\text{-}1$, where $k \leq n$ **then** add all symbols from $First(X_k)$ to $First(A)$:





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1) for each $a \in T$: $First(a) := \{a\}$ because $a \Rightarrow^0 a$

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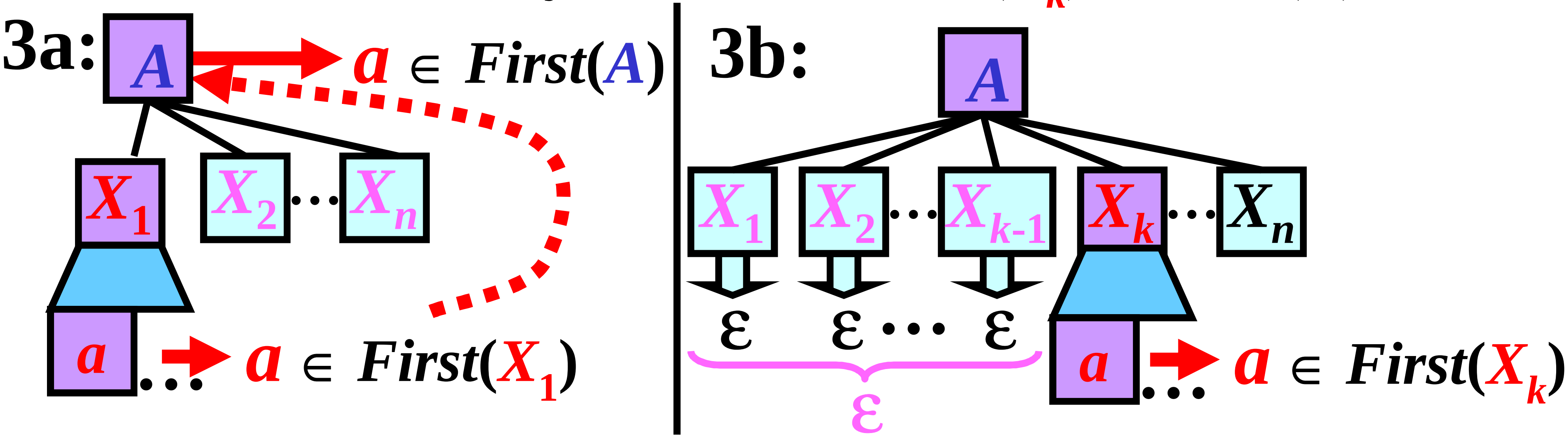
---

3) Apply the following rules until no *First* set or *Empty* set can be changed:

- **if** $A \to X_1X_2\ldots X_{k-1}X_k\ldots X_n \in P$ **then**

  3a) add all symbols from $First(X_1)$ to $First(A)$

  3b) if $Empty(X_i) = \{\varepsilon\}$ for all $i = 1, \ldots, k$-1, where $k \leq n$ **then** add all symbols from $First(X_k)$ to $First(A)$:



Previous Algorithm: Illustration

1) for each $a \in T$: $First(a) := \{a\}$ because $a \Rightarrow^0 a$

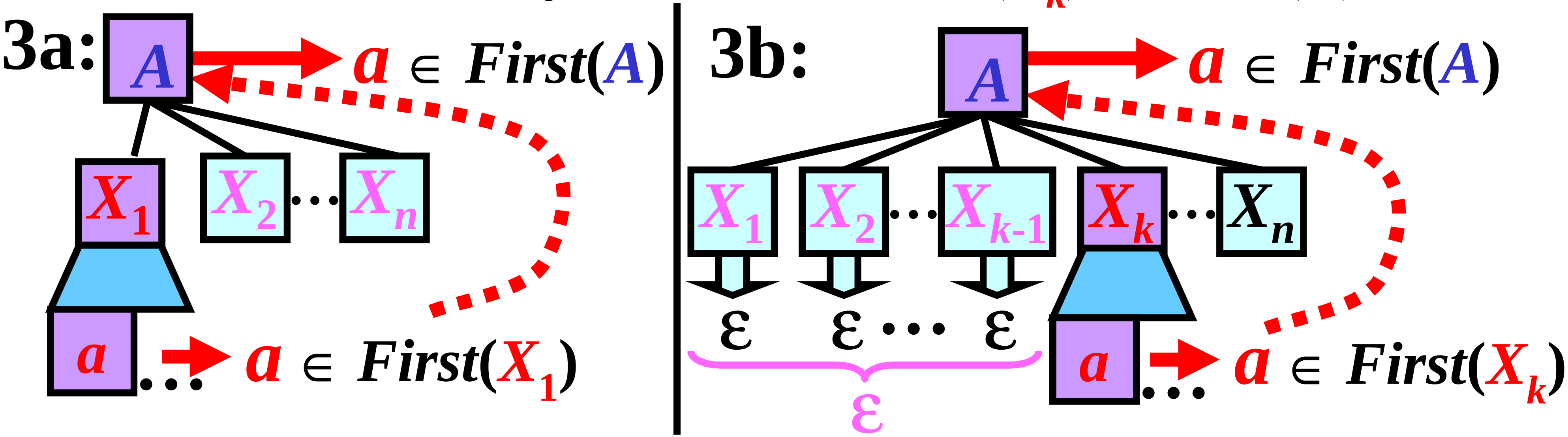
2) for each $A \in N$: $First(A) := \emptyset$ (inicialization)

3) Apply the following rules until no *First* set or *Empty* set can be changed:

- **if** $A \rightarrow X_1X_2\ldots X_{k-1}X_k\ldots X_n \in P$ **then**

3a) add all symbols from $First(X_1)$ to $First(A)$

3b) if $Empty(X_i) = \{\varepsilon\}$ for all $i = 1, \ldots, k$-1, where $k \leq n$ **then** add all symbols from $First(X_k)$ to $First(A)$:



First(*X*) for G_{expr3}: Example

Initialization:

First(***i***)	:= {***i***}	*First*(***E***)	:= ∅
First(+)	:= {+}	*First*(***E'***)	:= ∅
First(*)	:= {*}	*First*(***T***)	:= ∅
First(**(**)	:= { **(** }	*First*(***T'***)	:= ∅
First(**)**)	:= { **)** }	*First*(***F***)	:= ∅

$First(X)$ for G_{expr3}: Example

Initialization:

$First(\ i\ )$	$:= \{i\}$	$First(E)$	$:= \emptyset$
$First(\ +\ )$	$:= \{+\}$	$First(E')$	$:= \emptyset$
$First(\ *\ )$	$:= \{*\}$	$First(T)$	$:= \emptyset$
$First(\ (\ )$	$:= \{\ (\ \}$	$First(T')$	$:= \emptyset$
$First(\ )\ )$	$:= \{\ )\ \}$	$First(F)$	$:= \emptyset$

$F \to i \in P$: **add** $First(i) = \{i\}$ **to** $First(F)$

$F \to (E) \in P$: **add** $First(() = \{(\}$ **to** $First(F)$

Summary: $First(F) = \{i, (\}$

First(*X*) for G_{expr3}: Example

Initialization:

First(**i**)	:= {**i**}	*First*(***E***)	:= ∅
First(+)	:= {+}	*First*(***E'***)	:= ∅
First(*)	:= {*}	*First*(***T***)	:= ∅
First(**(**)	:= { **(** }	*First*(***T'***)	:= ∅
First(**)**)	:= { **)** }	*First*(***F***)	:= ∅

F → **i** ∈ *P*: **add** *First*(**i**) = {**i**} **to** *First*(***F***)
F → **(*E*)** ∈ *P*: **add** *First*(**(**) = {**(**} **to** *First*(***F***)
Summary: *First*(*F*) = {i, (}

T' → ***FT'*** ∈ *P*: **add** *First* (*) = {*} **to** *First*(***T'***)
Summary: *First*(*T'*) = {*}

First(*X*) for G_{expr3}: Example

Initialization:

First(**i**)	:= {**i**}	*First*(***E***)	:= ∅
First(+)	:= {+}	*First*(***E'***)	:= ∅
First(*)	:= {*}	*First*(***T***)	:= ∅
First(**(**)	:= { **(** }	*First*(***T'***)	:= ∅
First(**)**)	:= { **)** }	*First*(***F***)	:= ∅

F → **i** ∈ *P*: **add** *First*(**i**) = {**i**} **to** *First*(***F***)
F → **(*E*)** ∈ *P*: **add** *First*(**(**) = {**(**} **to** *First*(***F***)
Summary: *First*(*F*) = {i, (}

T' → ***FT'*** ∈ *P*: **add** *First* (*****) = {*****} **to** *First*(***T'***)
Summary: *First*(*T'*) = {*}

T → ***FT'*** ∈ *P*: **add** *First* (***F***) = {**i**, **(**} **to** *First*(***T***)
Summary: *First*(*T*) = {i, (}

$First(X)$ for G_{expr3}: Example

Initialization:

$First(\ i\ )$	$:= \{i\}$	$First(E)$	$:= \emptyset$
$First(\ +\ )$	$:= \{+\}$	$First(E')$	$:= \emptyset$
$First(\ *\ )$	$:= \{*\}$	$First(T)$	$:= \emptyset$
$First(\ (\ )$	$:= \{\ (\ \}$	$First(T')$	$:= \emptyset$
$First(\ )\ )$	$:= \{\ )\ \}$	$First(F)$	$:= \emptyset$

$F \rightarrow i \in P$: **add** $First(i) = \{i\}$ **to** $First(F)$

$F \rightarrow (E) \in P$: **add** $First(() = \{(\}$ **to** $First(F)$

Summary: $First(F) = \{i, (\}$

$T' \rightarrow *FT' \in P$: **add** $First\ (*) = \{*\}$ **to** $First(T')$

Summary: $First(T') = \{*\}$

$T \rightarrow FT' \in P$: **add** $First\ (F) = \{i, (\}$ **to** $First(T)$

Summary: $First(T) = \{i, (\}$

$E' \rightarrow +TE' \in P$: **add** $First\ (+) = \{+\}$ **to** $First(E')$

Summary: $First(E') = \{+\}$

First(*X*) for G_{expr3}: Example

Initialization:

First(**i**)	:= {**i**}	*First*(**E**)	:= ∅
First(+)	:= {+}	*First*(**E'**)	:= ∅
First(*****)	:= {*****}	*First*(**T**)	:= ∅
First(**(**)	:= { **(** }	*First*(**T'**)	:= ∅
First(**)**)	:= { **)** }	*First*(**F**)	:= ∅

F → ***i*** ∈ *P*: **add** *First*(***i***) = {***i***} **to** *First*(***F***)
F → **(*E*)** ∈ *P*: **add** *First*(**(**) = {**(**} **to** *First*(***F***)
Summary: *First*(*F*) = {*i*, (}

T' → ****FT'*** ∈ *P*: **add** *First* (*****) = {*****} **to** *First*(***T'***)
Summary: *First*(*T'*) = {*}

T → ***FT'*** ∈ *P*: **add** *First* (***F***) = {***i*, (**} **to** *First*(***T***)
Summary: *First*(*T*) = {*i*, (}

E' → **+*TE'*** ∈ *P*: **add** *First* (+) = {+} **to** *First*(***E'***)
Summary: *First*(*E'*) = {+}

E → ***TE'*** ∈ *P*: **add** *First* (***T***) = {***i*, (**} **to** *First*(***E***)
Summary: *First*(*E*) = {*i*, (}

First(*X*) for G_{expr3}: Example

Initialization:			
First(**i**)	:= {**i**}	*First*(***E***)	:= ∅
First(+)	:= {+}	*First*(***E'***)	:= ∅
First(*)	:= {*}	*First*(***T***)	:= ∅
First(**(**)	:= { **(** }	*First*(***T'***)	:= ∅
First(**)**)	:= { **)** }	*First*(***F***)	:= ∅

F → **i** ∈ *P*: **add** *First*(**i**) = {**i**} **to** *First*(***F***)
F → **(*E*)** ∈ *P*: **add** *First*(**(**) = {**(**} **to** *First*(***F***)
Summary: *First*(*F*) = {i, (}

T' → ***FT'*** ∈ *P*: **add** *First* (*) = {*} **to** *First*(***T'***)
Summary: *First*(*T'*) = {*}

T → ***FT'*** ∈ *P*: **add** *First* (***F***) = {**i**, **(**} **to** *First*(***T***)
Summary: *First*(*T*) = {i, (}

E' → **+*TE'*** ∈ *P*: **add** *First* (+) = {+} **to** *First*(***E'***)
Summary: *First*(*E'*) = {+}

E → ***TE'*** ∈ *P*: **add** *First* (***T***) = {**i**, **(**} **to** *First*(***E***)
Summary: *First*(*E*) = {i, (}

- **No *First* set can be changed.**

$First(X)$ & $Empty(X)$ for G_{expr3}: Summary

$G_{expr3} = (N, T, P, E)$, where: $N = \{E, E', T, T', F\}$, $T = \{i, +, *, (,)\}$,
$P = \{$ 1: $E \to TE'$, 2: $E' \to +TE'$, 3: $E' \to \varepsilon$, 4: $T \to FT'$
5: $T' \to *FT'$, 6: $T' \to \varepsilon$, 7: $F \to (E)$, 8: $F \to i$ $\}$

Set *Empty* for all $X \in N \cup T$:

$Empty(\,i\,) := \emptyset$	$Empty(E) := \emptyset$
$Empty(\,+\,) := \emptyset$	$Empty(E') := \{\varepsilon\}$
$Empty(\,*\,) := \emptyset$	$Empty(T) := \emptyset$
$Empty(\,(\,) := \emptyset$	$Empty(T') := \{\varepsilon\}$
$Empty(\,)\,) := \emptyset$	$Empty(F) := \emptyset$

Set *First* for all $X \in N \cup T$:

$First(\,i\,) := \{i\}$	$First(E) := \{i, (\}$
$First(\,+\,) := \{+\}$	$First(E') := \{+\}$
$First(\,*\,) := \{*\}$	$First(T) := \{i, (\}$
$First(\,(\,) := \{\,(\,\}$	$First(T') := \{*\}$
$First(\,)\,) := \{\,)\,\}$	$First(F) := \{i, (\}$

Note: for each $a \in T$: $Empty(a) = \emptyset$, $First(a) = \{a\}$

Algorithm: $First(X_1X_2\ldots X_n)$

- **Input:** $G = (N, T, P, S)$; $First(X)$ & $Empty(X)$ for every $X \in N \cup T$; $x = X_1X_2\ldots X_n$, where $x \in (N \cup T)^+$
- **Output:** $First(X_1X_2\ldots X_n)$

- **Method:**
- $First(X_1X_2\ldots X_n) := First(X_1)$
- **Apply the following rule until nothing can be added to $First(X_1X_2 \ldots X_{k-1} X_k \ldots X_n)$:**
  - **if** $Empty(X_i) = \{\varepsilon\}$ for all $i = 1,\ldots,k\text{-}1$, where $k \leq n$ **then** add all symbols from $First(X_k)$ to $First(X_1X_2\ldots X_n)$

! Note: $First(\varepsilon) = \emptyset$

Illustration:

$X_1\, X_2 \ldots X_{k-1}\, X_k \ldots X_n$

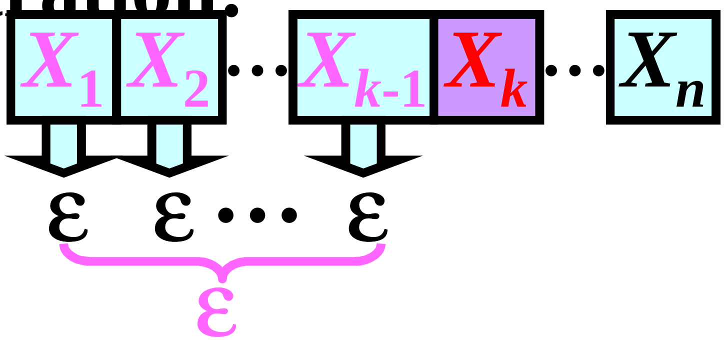
Algorithm: $First(X_1X_2\ldots X_n)$

- **Input:** $G = (N, T, P, S)$; $First(X)$ & $Empty(X)$ for every $X \in N \cup T$; $x = X_1X_2\ldots X_n$, where $x \in (N \cup T)^+$
- **Output:** $First(X_1X_2\ldots X_n)$

- **Method:**
- $First(X_1X_2\ldots X_n) := First(X_1)$
- **Apply the following rule until nothing can be added to $\boldsymbol{First(X_1X_2\ldots X_{k-1}X_k\ldots X_n)}$:**
  - **if** $Empty(\boldsymbol{X_i}) = \{\varepsilon\}$ for all $i = 1,\ldots,k$-1, where $k \leq n$ **then** add all symbols from $First(\boldsymbol{X_k})$ to $First(X_1X_2\ldots X_n)$

! Note: $First(\varepsilon) = \emptyset$

Illustration:



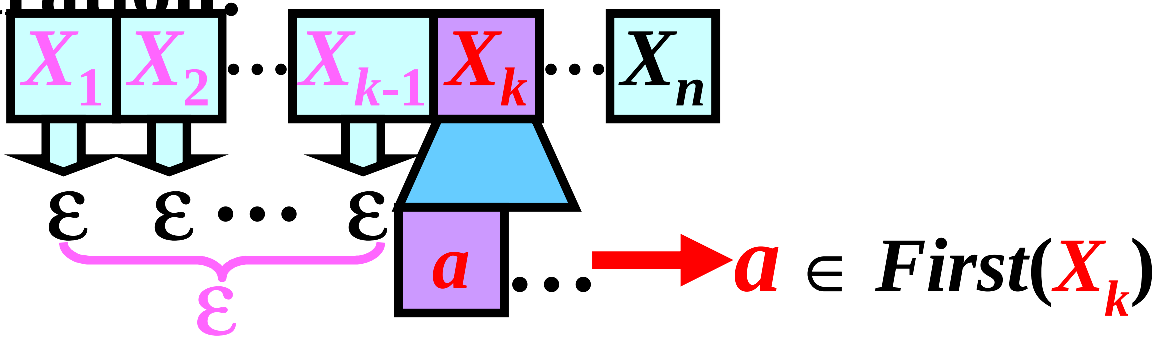
Algorithm: $First(X_1X_2\ldots X_n)$

- **Input:** $G = (N, T, P, S)$; $First(X)$ & $Empty(X)$ for every $X \in N \cup T$; $x = X_1X_2\ldots X_n$, where $x \in (N \cup T)^+$
- **Output:** $First(X_1X_2\ldots X_n)$

- **Method:**
- $First(X_1X_2\ldots X_n) := First(X_1)$
- **Apply the following rule until nothing can be added to** $\boldsymbol{First(X_1X_2 \ldots X_{k-1} X_k \ldots X_n)}$**:**
  - **if** $Empty(\boldsymbol{X_i}) = \{\varepsilon\}$ for all $i = 1,\ldots,k\text{-}1$, where $k \leq n$ **then** add all symbols from $First(\boldsymbol{X_k})$ to $First(X_1X_2\ldots X_n)$

! Note: $First(\varepsilon) = \emptyset$

Illustration:



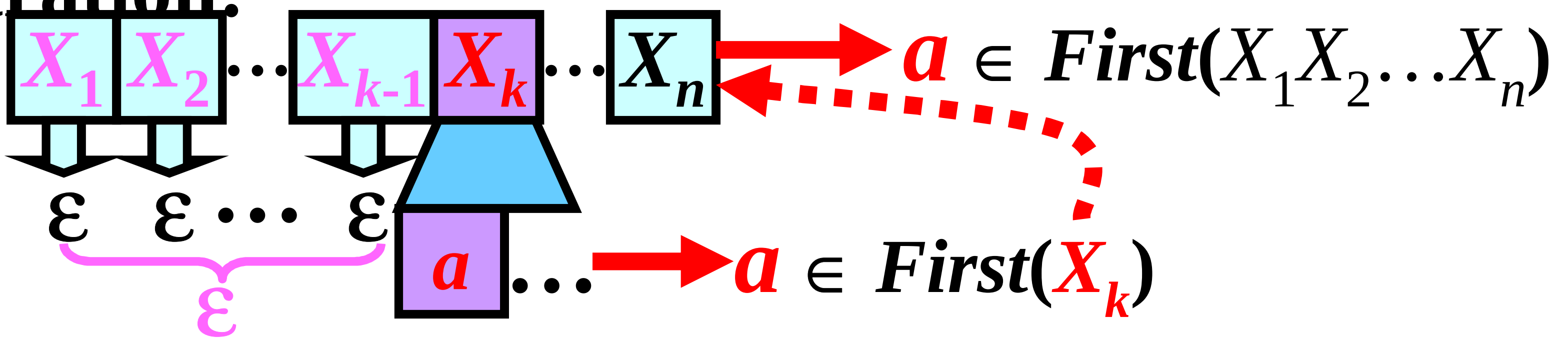
Algorithm: $First(X_1X_2\ldots X_n)$

- **Input:** $G = (N, T, P, S)$; $First(X)$ & $Empty(X)$ for every $X \in N \cup T$; $x = X_1X_2\ldots X_n$, where $x \in (N \cup T)^+$
- **Output:** $First(X_1X_2\ldots X_n)$

- **Method:**
- $First(X_1X_2\ldots X_n) := First(X_1)$
- **Apply the following rule until nothing can be added to** $\boldsymbol{First(X_1X_2\ldots X_{k-1}X_k\ldots X_n)}$**:**
  - **if** $Empty(\boldsymbol{X_i}) = \{\varepsilon\}$ for all $i = 1,\ldots,k$-1, where $k \le n$ **then** add all symbols from $First(\boldsymbol{X_k})$ to $First(X_1X_2\ldots X_n)$

! Note: $First(\varepsilon) = \emptyset$

Illustration:



$First(X_1X_2\ldots X_n)$: Example

$G_{expr3} = (N, T, P, E)$, where: $N = \{E, E', T, T', F\}$, $T = \{i, +, *, (,)\}$,
$P = \{$ 1: $E \to TE'$, 2: $E' \to +TE'$, 3: $E' \to \varepsilon$, 4: $T \to FT'$
5: $T' \to *FT'$, 6: $T' \to \varepsilon$, 7: $F \to (E)$, 8: $F \to i$ $\}$

Set *Empty* & *First* for all $X \in N$:				
	$Empty(E)$	$:= \emptyset$	$First(E)$	$:= \{i, (\}$
	$Empty(E')$	$:= \{\varepsilon\}$	$First(E')$	$:= \{+\}$
	$Empty(T)$	$:= \emptyset$	$First(T)$	$:= \{i, (\}$
	$Empty(T')$	$:= \{\varepsilon\}$	$First(T')$	$:= \{*\}$
	$Empty(F)$	$:= \emptyset$	$First(F)$	$:= \{i, (\}$

Task: $First(E'T'FET)$

1) $First(\underline{E'}T'FET) := First(E') = \{+\}$

2) $First(E'\underline{T'}FET)$: add $First(T') = \{*\}$ to $First(E'T'FET)$

$Empty(E') = \{\varepsilon\}$

3) $First(E'T'\underline{F}ET)$: add $First(F) = \{i, (\}$ to $First(E'T'FET)$

$Empty(E') = Empty(T') = \{\varepsilon\}$

Summary: $First(E'T'FET) = \{+, *, i, (\}$

Algorithm: $Empty(X_1X_2\ldots X_n)$

- **Input:** $G = (N, T, P, S)$; $Empty(X)$ for every $X \in N \cup T$;
  $x = X_1X_2\ldots X_n$, where $x \in (N \cup T)^+$
- **Output:** $Empty(X_1X_2\ldots X_n)$

- **Method:**
- **if** $Empty(X_i) = \{\varepsilon\}$ for all $i = 1,\ldots,n$ **then**
  $Empty(X_1X_2\ldots X_n) := \{\varepsilon\}$

  else

  $Empty(X_1X_2\ldots X_n) := \emptyset$

! Note: $Empty(\varepsilon) = \{\varepsilon\}$

Illustration: $\boxed{X_1}\boxed{X_2}\ldots\boxed{X_n}$

Algorithm: $Empty(X_1X_2\ldots X_n)$

- **Input:** $G = (N, T, P, S)$; $Empty(X)$ for every $X \in N \cup T$;
  $x = X_1X_2\ldots X_n$, where $x \in (N \cup T)^+$
- **Output:** $Empty(X_1X_2\ldots X_n)$

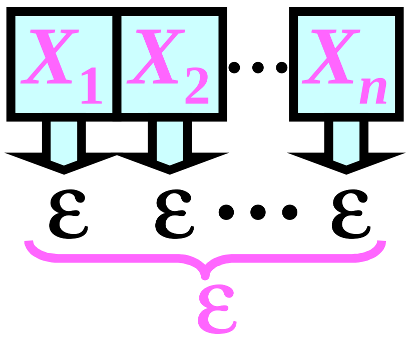
- **Method:**
- **if** $Empty(X_i) = \{\varepsilon\}$ for all $i = 1,\ldots,n$ **then**
  $Empty(X_1X_2\ldots X_n) := \{\varepsilon\}$

  else

  $Empty(X_1X_2\ldots X_n) := \emptyset$

! Note: $Empty(\varepsilon) = \{\varepsilon\}$

Illustration:



Algorithm: $Empty(X_1X_2\ldots X_n)$

- **Input:** $G = (N, T, P, S)$; $Empty(X)$ for every $X \in N \cup T$;
  $x = X_1X_2\ldots X_n$, where $x \in (N \cup T)^+$
- **Output:** $Empty(X_1X_2\ldots X_n)$

- **Method:**
- **if** $Empty(X_i) = \{\varepsilon\}$ for all $i = 1,\ldots,n$ **then**

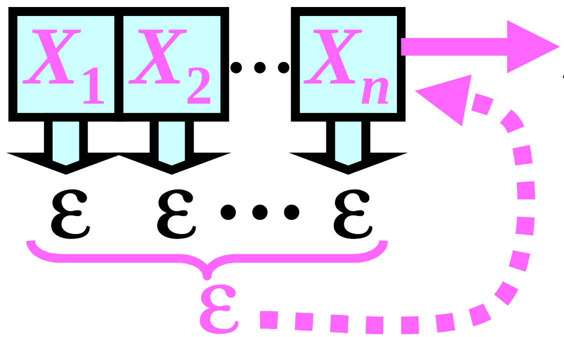
  $Empty(X_1X_2\ldots X_n) := \{\varepsilon\}$

  else

  $Empty(X_1X_2\ldots X_n) := \varnothing$

! Note: $Empty(\varepsilon) = \{\varepsilon\}$

Illustration:



$\boldsymbol{Empty}(X_1X_2\ldots X_n) = \{\varepsilon\}$

$Empty(X_1X_2\ldots X_n)$: Example

$G_{expr3} = (N, T, P, E)$, where: $N = \{E, E', T, T', F\}$, $T = \{i, +, *, (,)\}$,

$P = \{$ 1: $E \rightarrow TE'$, 2: $E' \rightarrow +TE'$, 3: $E' \rightarrow \varepsilon$, 4: $T \rightarrow FT'$
5: $T' \rightarrow *FT'$, 6: $T' \rightarrow \varepsilon$, 7: $F \rightarrow (E)$, 8: $F \rightarrow i$ $\}$

Set *Empty* for all $X \in N$:

$Empty(E) := \emptyset$
$Empty(E') := \{\varepsilon\}$
$Empty(T) := \emptyset$
$Empty(T') := \{\varepsilon\}$
$Empty(F) := \emptyset$

Task: $Empty(E'T')$

$Empty(E') = Empty(T') = \{\varepsilon\}$, **so** $Empty(E'T') = \{\varepsilon\}$

Set *Follow*

Gist: ***Follow*(*A*) is the set of all terminals that can come right after *A* in a sentential form of *G***

Definition: Let $G = (N, T, P, S)$ be a CFG. For every $A \in N$, we define the set $Follow(A)$ as
$$Follow(A) = \{a: a \in T, S \Rightarrow^* xAay, x, y \in (N \cup T)^*\}$$
$$\cup \{\$: S \Rightarrow^* xA, x \in (N \cup T)^*\}$$

Illustration:

Set *Follow*

Gist: ***Follow*(*A*) is the set of all terminals that can come right after *A* in a sentential form of *G***

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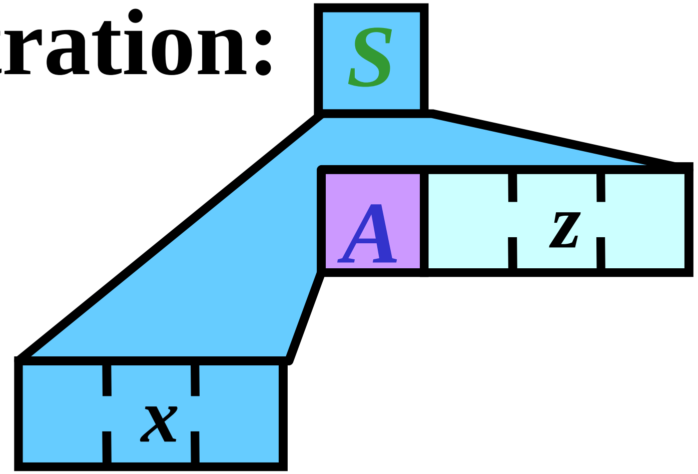
Illustration:



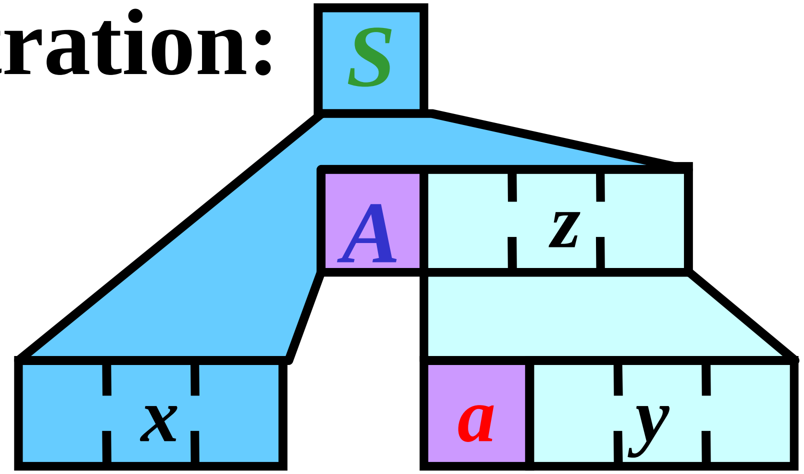
$S \Rightarrow^* xAz$

Set *Follow*

Gist: *Follow*(*A*) is the set of all terminals that can come right after *A* in a sentential form of *G*

Definition: Let $G = (N, T, P, S)$ be a CFG. For every $A \in N$, we define the set $Follow(A)$ as
$Follow(A) = \{a: a \in T, S \Rightarrow^* xAay, x, y \in (N \cup T)^*\}$
$\cup \{\$: S \Rightarrow^* xA, x \in (N \cup T)^*\}$

Illustration:



$S \Rightarrow^* xAz \Rightarrow^* xAay$

$a \in Follow(A)$

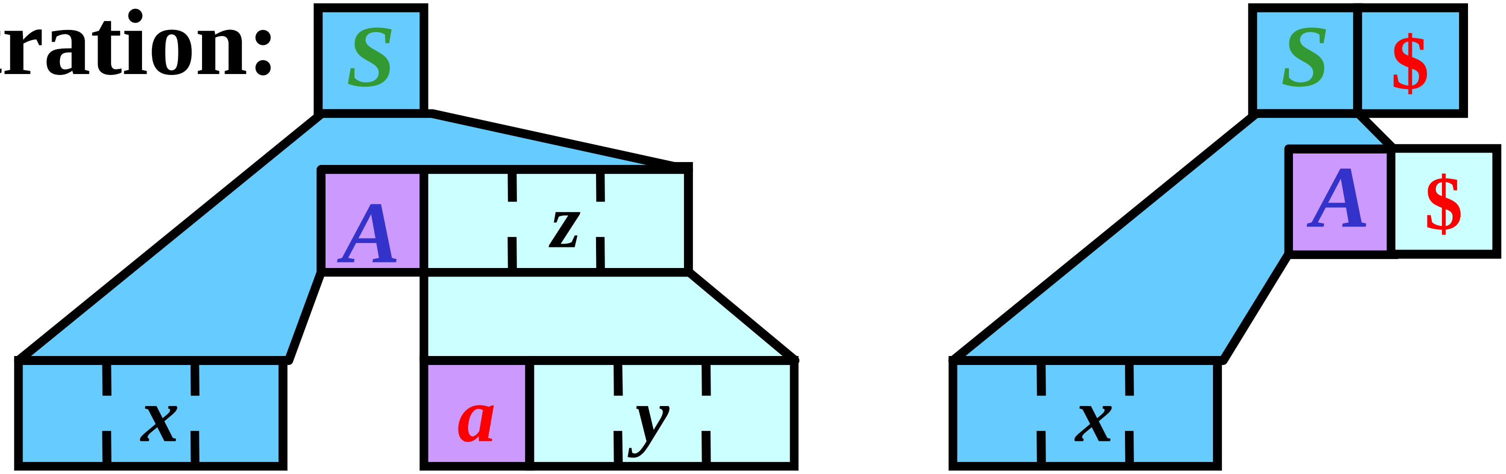
Set *Follow*

Gist: ***Follow*(*A*) is the set of all terminals that can come right after *A* in a sentential form of *G***

Definition: Let $G = (N, T, P, S)$ be a CFG. For every $A \in N$, we define the set $Follow(A)$ as

$$Follow(A) = \{a: a \in T, S \Rightarrow^* xAay, x, y \in (N \cup T)^*\} \cup \{\$: S \Rightarrow^* xA, x \in (N \cup T)^*\}$$

Illustration:



$S \Rightarrow^* xAz \Rightarrow^* xAay$

$a \in Follow(A)$

$S \Rightarrow^* xA$

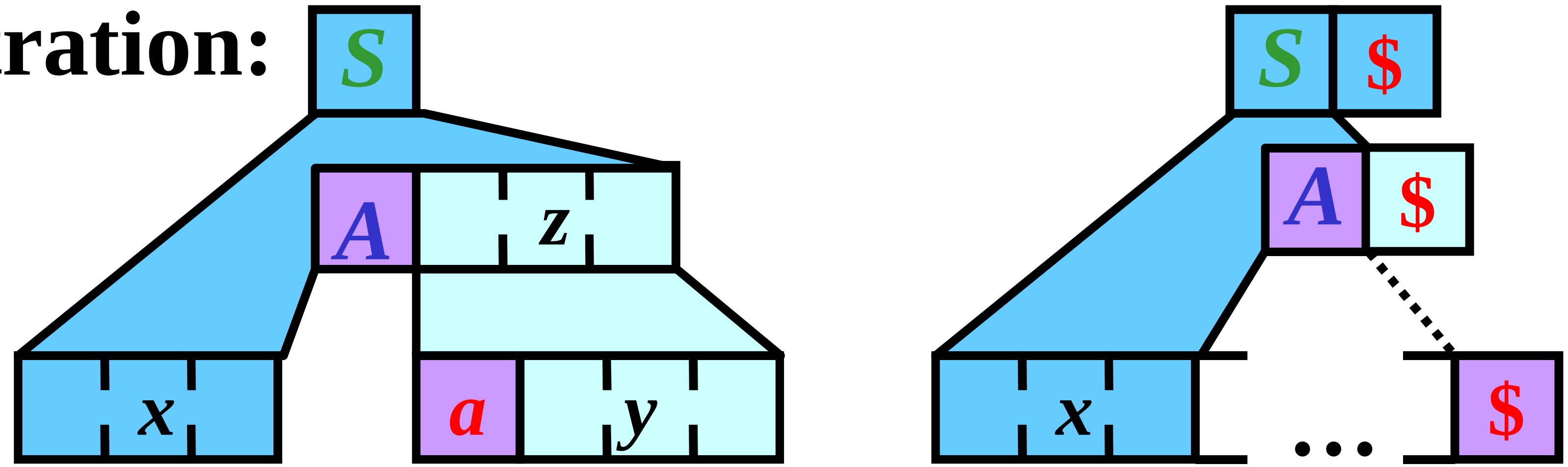
Set *Follow*

Gist: *Follow*(A) is the set of all terminals that can come right after A in a sentential form of G

Definition: Let $G = (N, T, P, S)$ be a CFG. For every $A \in N$, we define the set *Follow*(A) as

$$Follow(A) = \{a: a \in T, S \Rightarrow^* xAay, x, y \in (N \cup T)^*\}$$
$$\cup \{\$: S \Rightarrow^* xA, x \in (N \cup T)^*\}$$

Illustration:



$S \Rightarrow^* xAz \Rightarrow^* xAay$

$a \in Follow(A)$

$S \Rightarrow^* xA$

$\$ \in Follow(A)$

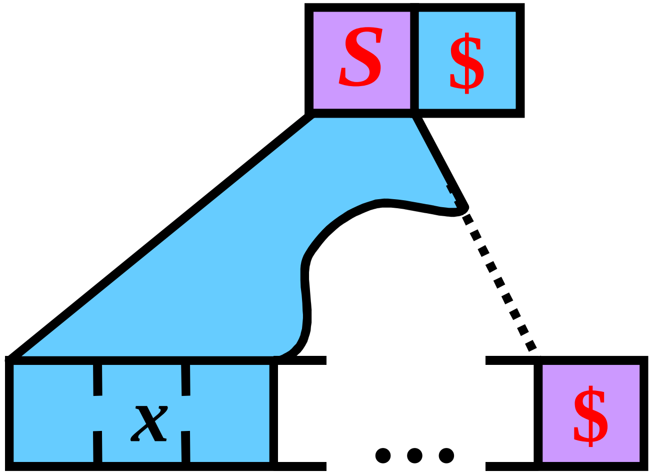
Algorithm: *Follow*(*A*)

- **Input:** $G = (N, T, P, S)$;
- **Output:** *Follow*(A) for every $A \in N$

- **Method:**
- *Follow*(S) := {**$**};
- **Apply the following rules until no *Follow* set can be changed:**
- **if** $A \rightarrow xBy \in P$ **then**
  - **if** $y \neq \varepsilon$ **then**
    add all symbols from *First*(y) to *Follow*(B);
  - **if** *Empty*(y) = {ε} **then**
    add all symbols from *Follow*(A) to *Follow*(B);

Previous Algorithm: Illustration

1) *Follow*(***S***) := {**$**}

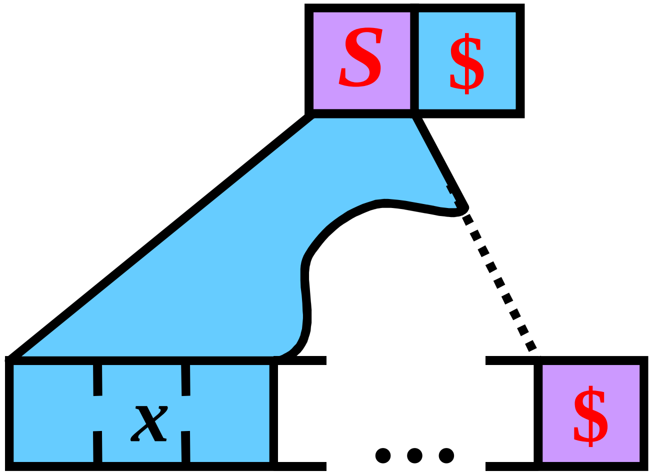


2) Apply the following rules until no *Follow* set can be changed:

• **if** $\boldsymbol{A} \rightarrow \boldsymbol{xBy} \in P$ **then**

Previous Algorithm: Illustration

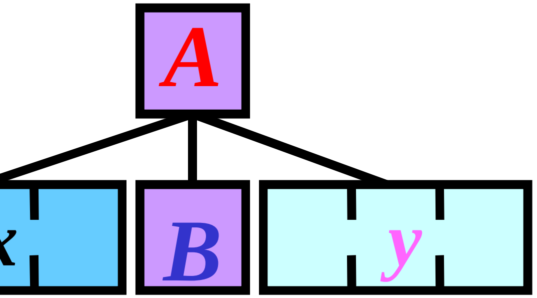
1) *Follow*(***S***) := {**$**}



2) Apply the following rules until no *Follow* set can be changed:

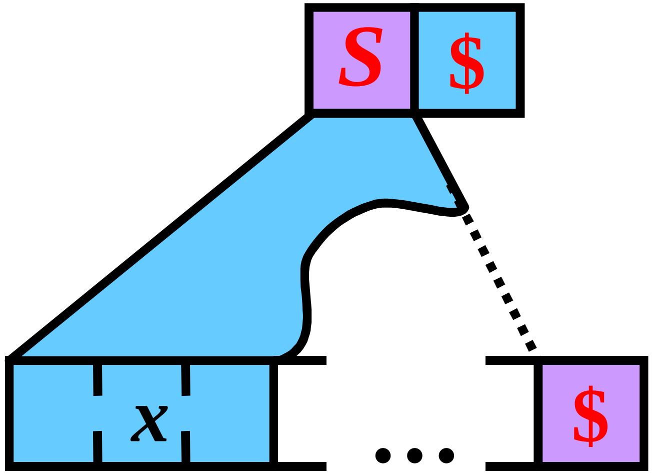
- **if** ***A*** $\to$ ***xBy*** $\in$ *P* **then**
  2a) if ***y*** $\neq$ ε **then** add all symbols from *First*(***y***) to *Follow*(***B***)

2a:



Previous Algorithm: Illustration

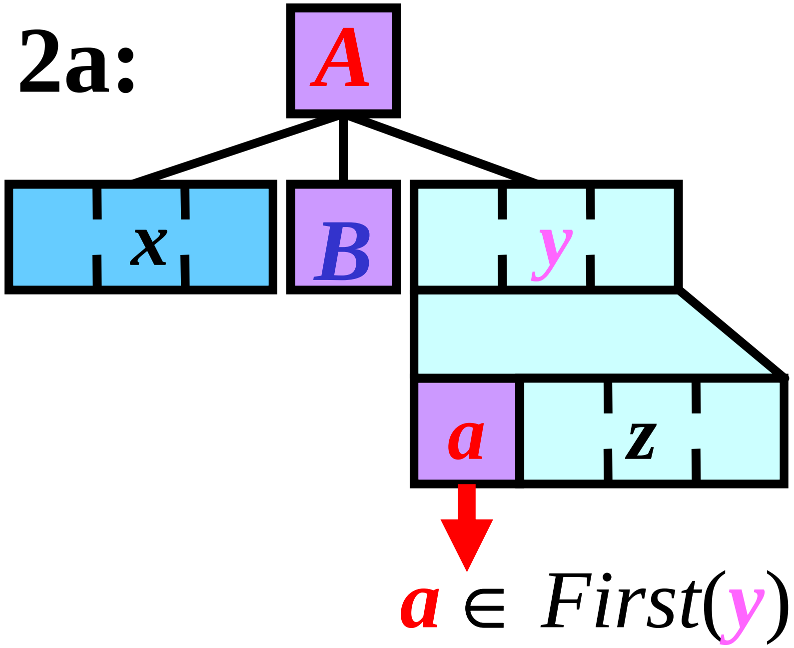
1) *Follow*(***S***) := {**$**}



2) Apply the following rules until no *Follow* set can be changed:

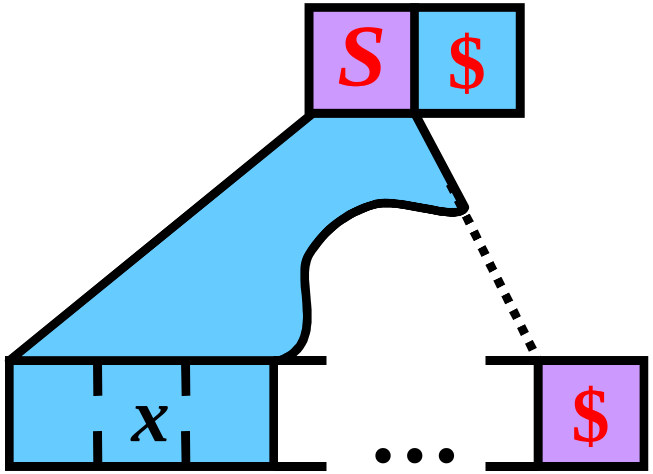
- **if** $A \rightarrow xBy \in P$ **then**
  2a) if $y \neq \varepsilon$ **then** add all symbols from *First*(*y*) to *Follow*(*B*)

2a:



Previous Algorithm: Illustration

1) *Follow*(***S***) := {**\$**}

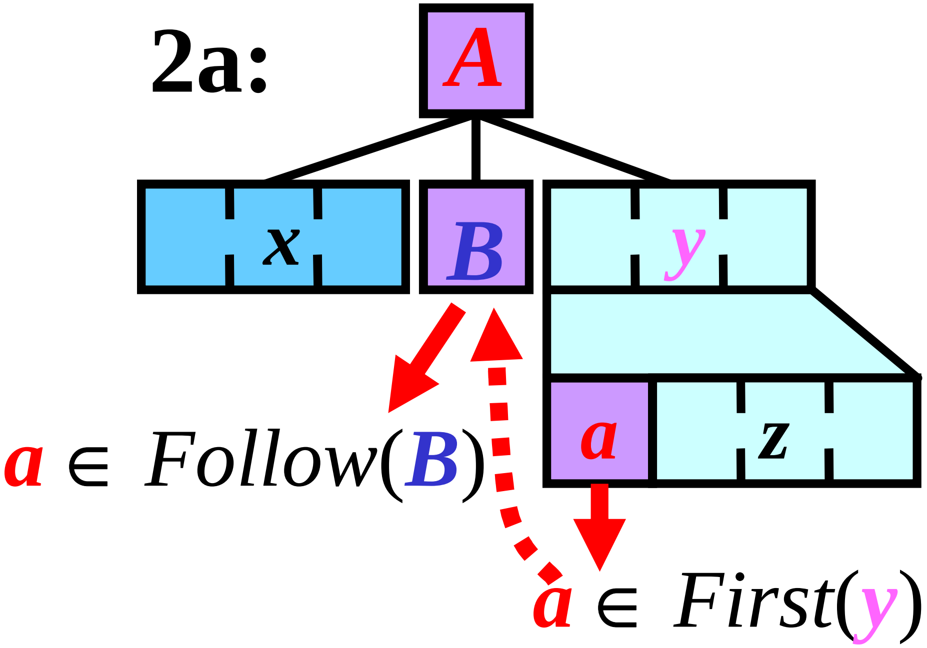


2) Apply the following rules until no *Follow* set can be changed:

- **if** $A \rightarrow xBy \in P$ **then**

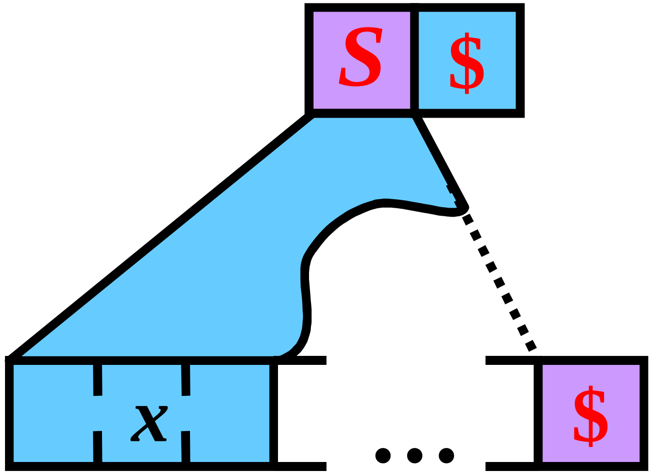
  2a) if $y \neq \varepsilon$ **then** add all symbols from *First*(*y*) to *Follow*(***B***)

2a:



Previous Algorithm: Illustration

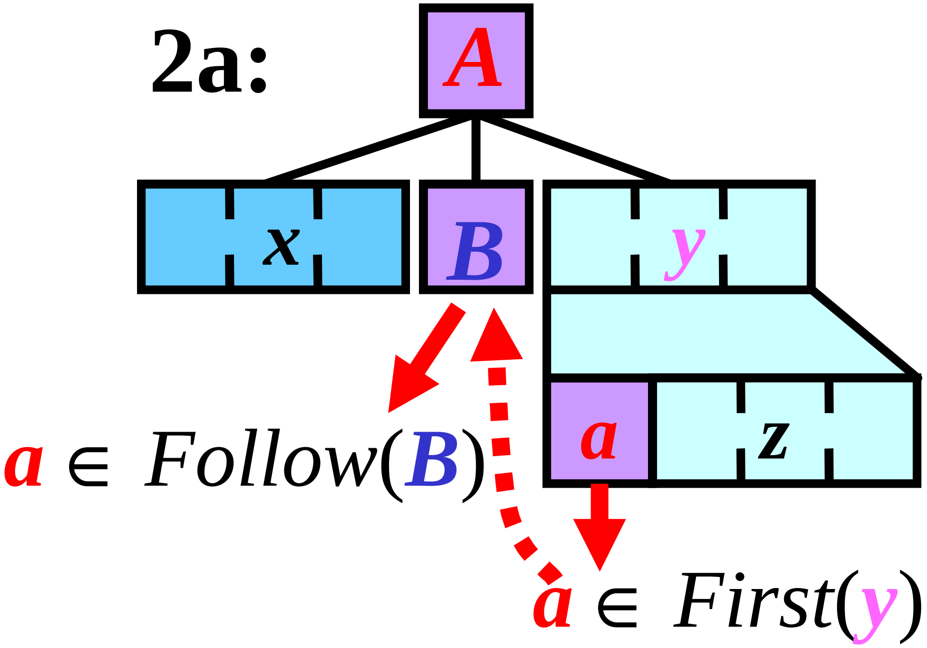
1) *Follow*(***S***) := {**$**}



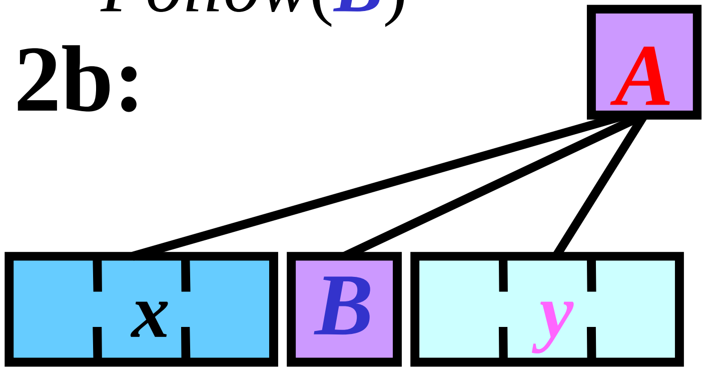
2) Apply the following rules until no *Follow* set can be changed:

- **if *A*** $\rightarrow$ ***xBy*** $\in$ *P* **then**
  - **2a) if *y*** $\neq \varepsilon$ **then** add all symbols from *First*(*y*) to *Follow*(***B***)
  - **2b) if *Empty*(*y*) = {ε} then** add all symbols from *Follow*(***A***) to *Follow*(***B***)

2a:

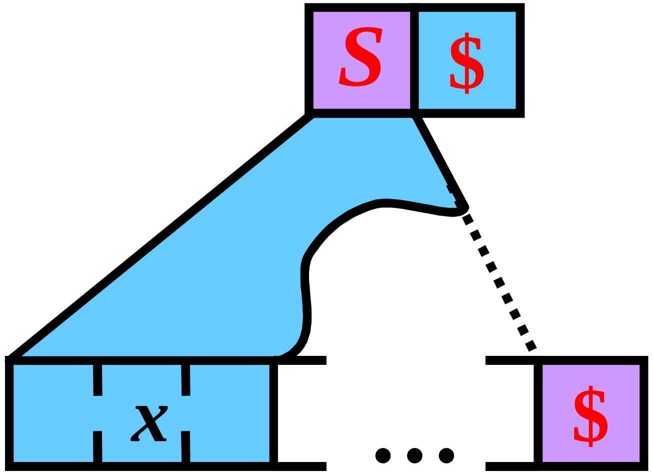


2b:



Previous Algorithm: Illustration

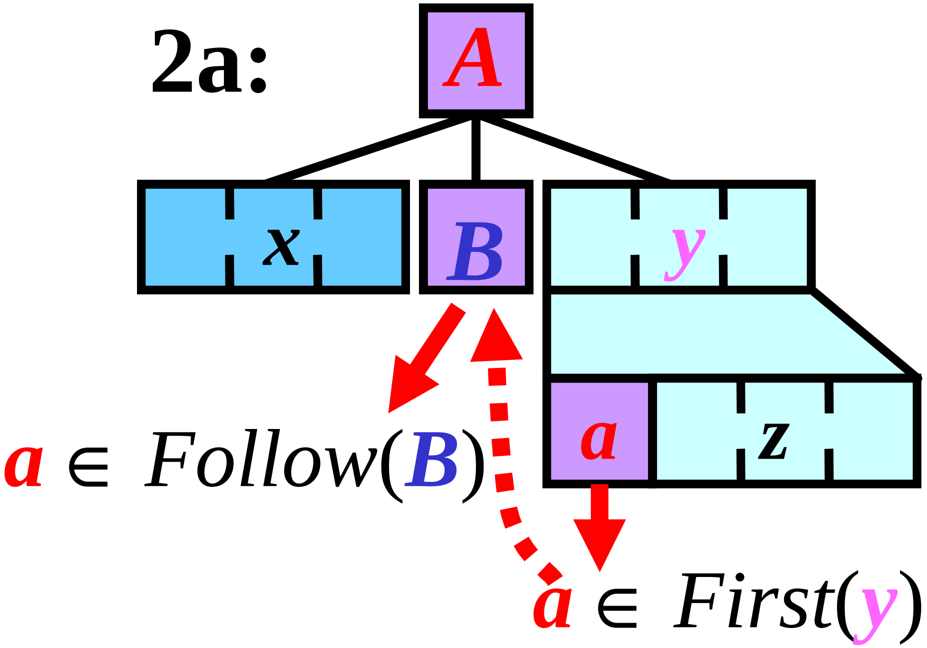
1) *Follow*(*S*) := {$}



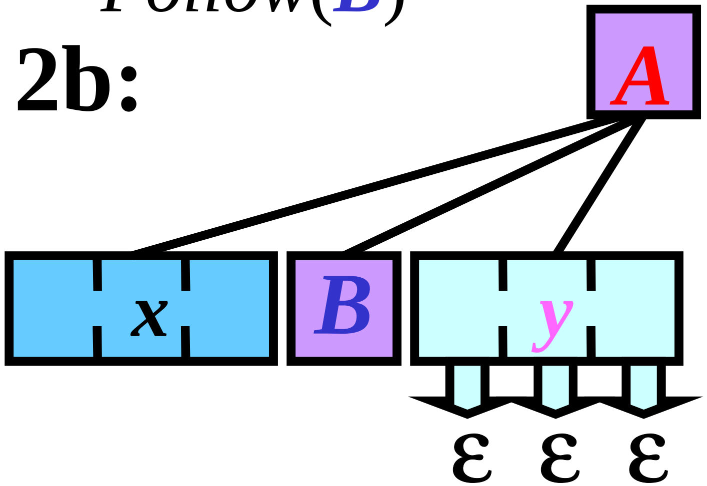
2) Apply the following rules until no *Follow* set can be changed:

- **if** $A \to xBy \in P$ **then**
  - **2a) if** $y \neq \varepsilon$ **then** add all symbols from *First*(y) to *Follow*(B)
  - **2b) if** *Empty*(y) = {ε} **then** add all symbols from *Follow*(A) to *Follow*(B)

2a:

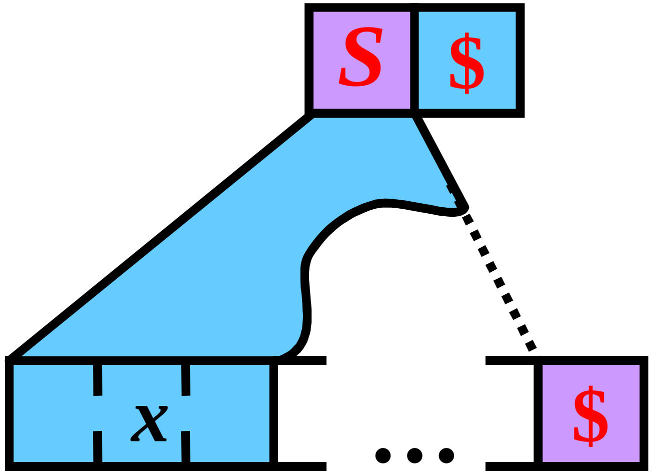


2b:



Previous Algorithm: Illustration

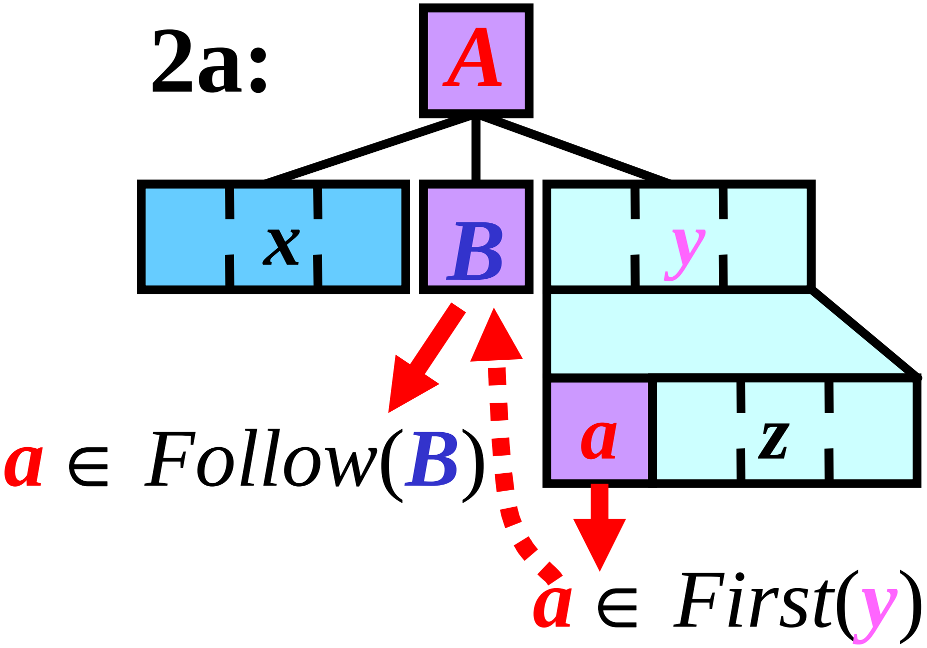
1) *Follow*(***S***) := {**$**}



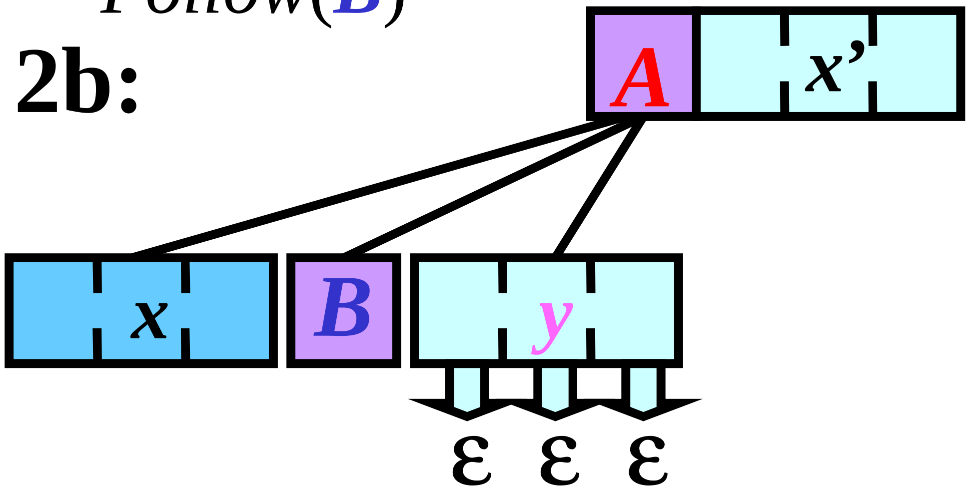
2) Apply the following rules until no *Follow* set can be changed:

- **if *A* → x*By* ∈ *P* then**
  - **2a) if *y* ≠ ε then** add all symbols from *First*(*y*) to *Follow*(***B***)
  - **2b) if *Empty*(*y*) = {ε} then** add all symbols from *Follow*(***A***) to *Follow*(***B***)

2a:

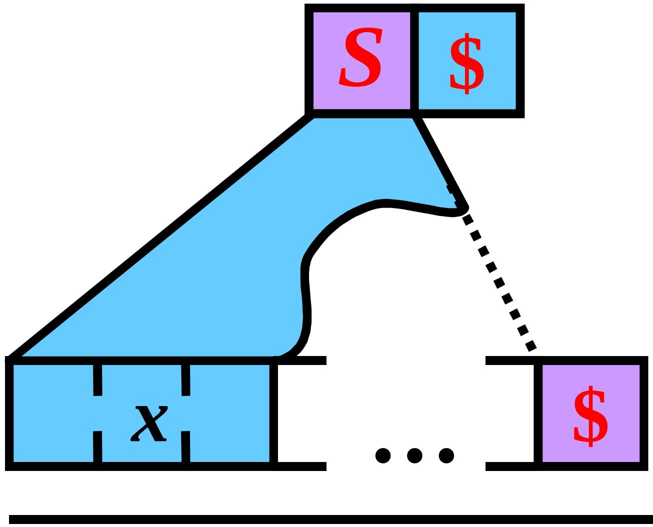


2b:



Previous Algorithm: Illustration

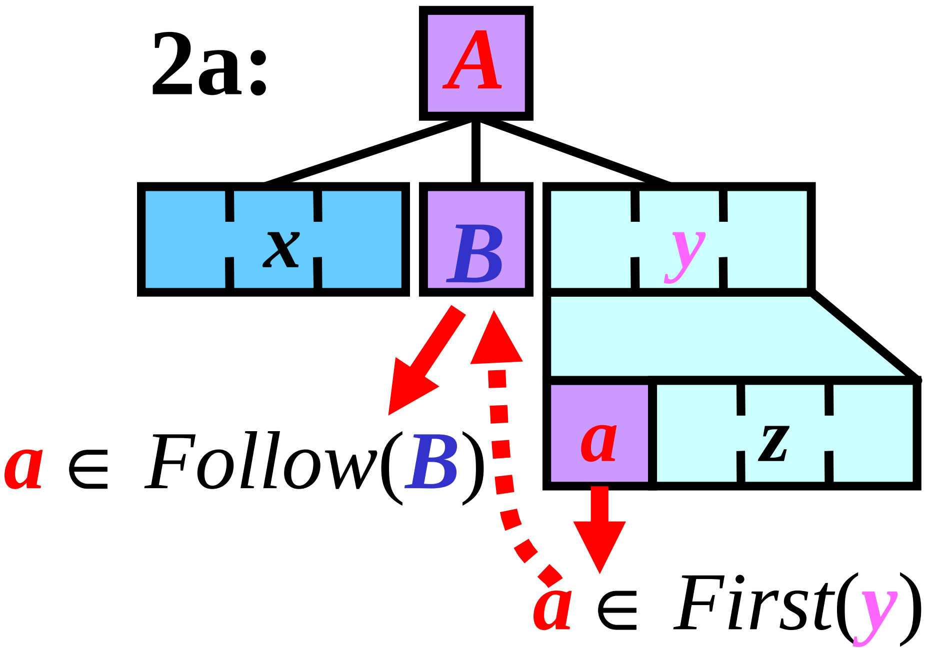
1) *Follow*(***S***) := {**$**}



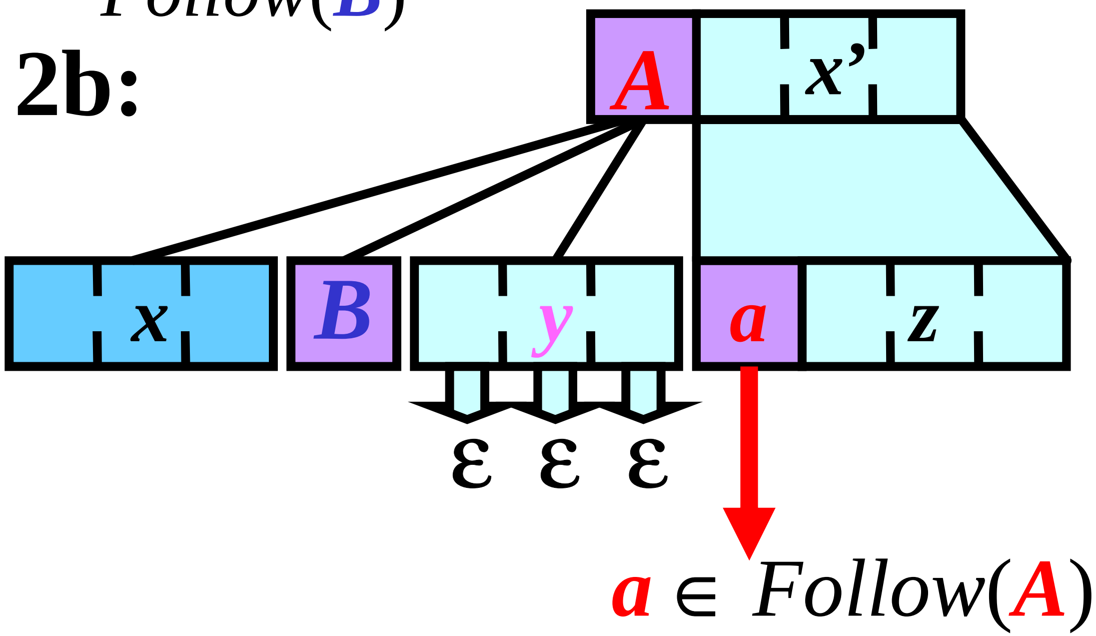
2) Apply the following rules until no *Follow* set can be changed:

- **if *A*** → ***xBy*** ∈ *P* **then**
  - **2a) if *y*** ≠ ε **then** add all symbols from *First*(*y*) to *Follow*(***B***)
  - **2b) if *Empty*(*y*) = {ε} then** add all symbols from *Follow*(***A***) to *Follow*(***B***)

2a:

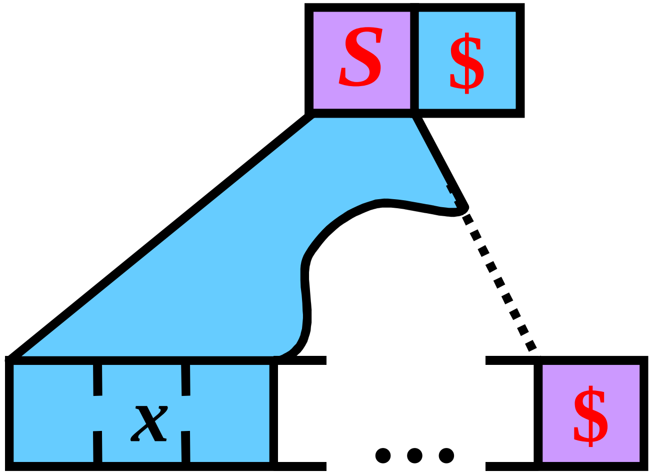


2b:



Previous Algorithm: Illustration

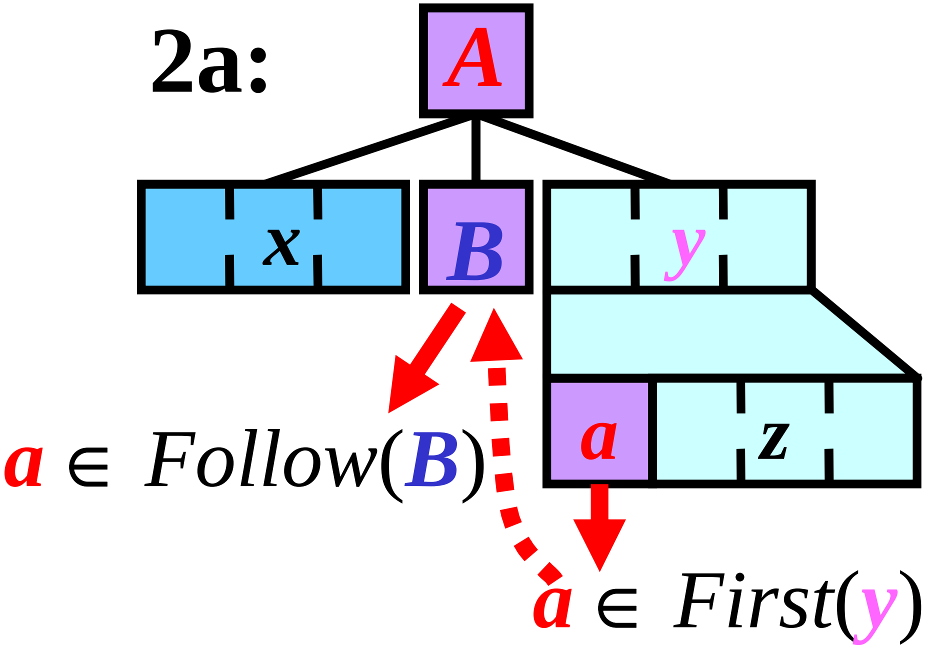
1) *Follow*(***S***) := {**$**}



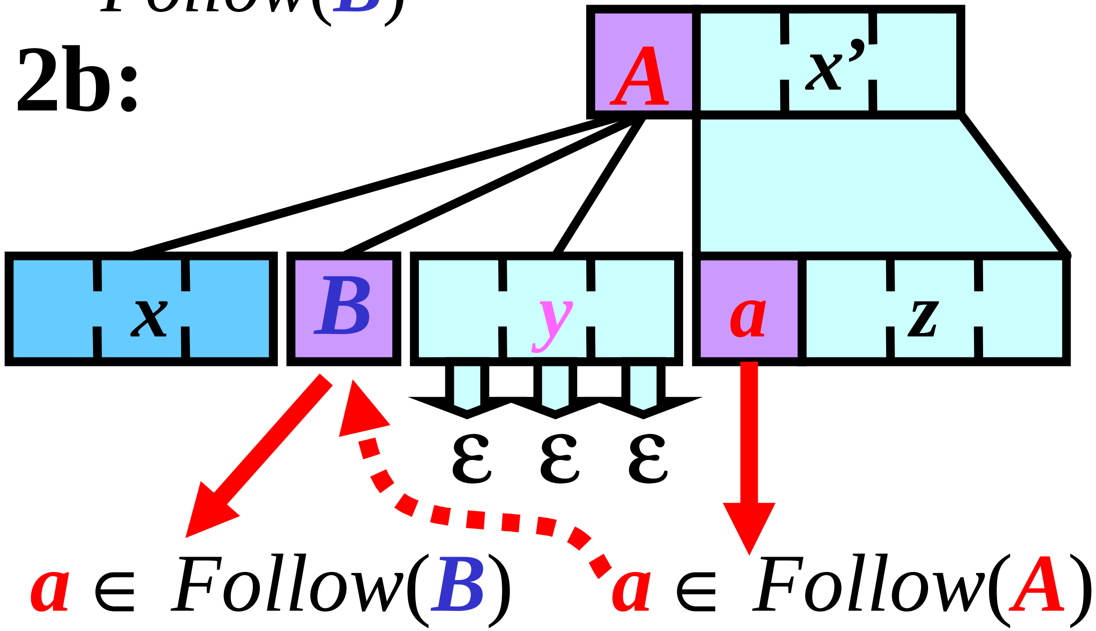
2) Apply the following rules until no *Follow* set can be changed:

- **if** ***A*** $\rightarrow$ ***xBy*** $\in$ *P* **then**
  - **2a) if** ***y*** $\neq \varepsilon$ **then** add all symbols from *First*(***y***) to *Follow*(***B***)
  - **2b) if** *Empty*(***y***) **=** {ε} **then** add all symbols from *Follow*(***A***) to *Follow*(***B***)

2a:



2b:



Follow(*X*) for G_{expr3}: Example 1/3

First(***E***)	:= {***i***, **(**}	*Empty*(***E***)	:= $\emptyset$	*Follow*(***E***)	:= $\emptyset$
First(***E'***)	:= {+}	*Empty*(***E'***)	:= {ε}	*Follow*(***E'***)	:= $\emptyset$
First(***T***)	:= {***i***, **(**}	*Empty*(***T***)	:= $\emptyset$	*Follow*(***T***)	:= $\emptyset$
First(***T'***)	:= {*****}	*Empty*(***T'***)	:= {ε}	*Follow*(***T'***)	:= $\emptyset$
First(***F***)	:= {***i***, **(**}	*Empty*(***F***)	:= $\emptyset$	*Follow*(***F***)	:= $\emptyset$

$Follow(X)$ for G_{expr3}: Example 1/3

$First(E)$	:= {**i**, **(**}	$Empty(E)$	:= $\emptyset$	$Follow(E)$	:= $\emptyset$
$First(E')$	:= {+}	$Empty(E')$	:= {ε}	$Follow(E')$	:= $\emptyset$
$First(T)$	:= {**i**, **(**}	$Empty(T)$	:= $\emptyset$	$Follow(T)$	:= $\emptyset$
$First(T')$	:= {*****}	$Empty(T')$	:= {ε}	$Follow(T')$	:= $\emptyset$
$First(F)$	:= {**i**, **(**}	$Empty(F)$	:= $\emptyset$	$Follow(F)$	:= $\emptyset$

0) $Follow(E)$:= {$}

Follow(*X*) for G_{expr3}: Example 1/3

First(***E***)	:= {***i***, **(**}	*Empty*(***E***)	:= ∅	*Follow*(***E***)	:= ∅
First(***E'***)	:= {**+**}	*Empty*(***E'***)	:= {ε}	*Follow*(***E'***)	:= ∅
First(***T***)	:= {***i***, **(**}	*Empty*(***T***)	:= ∅	*Follow*(***T***)	:= ∅
First(***T'***)	:= {*****}	*Empty*(***T'***)	:= {ε}	*Follow*(***T'***)	:= ∅
First(***F***)	:= {***i***, **(**}	*Empty*(***F***)	:= ∅	*Follow*(***F***)	:= ∅

0) *Follow*(***E***) := {**$**}

1) $\boldsymbol{F} \rightarrow \textbf{(}\ \boldsymbol{E}\ \underbrace{\textbf{)}}_{\neq \varepsilon} \in P$**:**

Follow(*X*) for G_{expr3}: Example 1/3

First(***E***)	:= {***i***, **(**}	*Empty*(***E***)	:= ∅	*Follow*(***E***)	:= ∅
First(***E'***)	:= {**+**}	*Empty*(***E'***)	:= {ε}	*Follow*(***E'***)	:= ∅
First(***T***)	:= {***i***, **(**}	*Empty*(***T***)	:= ∅	*Follow*(***T***)	:= ∅
First(***T'***)	:= {*****}	*Empty*(***T'***)	:= {ε}	*Follow*(***T'***)	:= ∅
First(***F***)	:= {***i***, **(**}	*Empty*(***F***)	:= ∅	*Follow*(***F***)	:= ∅

0) *Follow*(***E***) := {**$**}

1) $\boldsymbol{F} \rightarrow \textbf{(}\ \boldsymbol{E}\ \underbrace{\textbf{)}}_{\neq \varepsilon} \in P$**:** **add** *First*(**)**) = {**)**} **to** *Follow*(***E***)

$Follow(X)$ for G_{expr3}: Example 1/3

$First(E)$	$:= \{i, (\}$	$Empty(E)$	$:= \emptyset$	$Follow(E)$	$:= \emptyset$
$First(E')$	$:= \{+\}$	$Empty(E')$	$:= \{\varepsilon\}$	$Follow(E')$	$:= \emptyset$
$First(T)$	$:= \{i, (\}$	$Empty(T)$	$:= \emptyset$	$Follow(T)$	$:= \emptyset$
$First(T')$	$:= \{*\}$	$Empty(T')$	$:= \{\varepsilon\}$	$Follow(T')$	$:= \emptyset$
$First(F)$	$:= \{i, (\}$	$Empty(F)$	$:= \emptyset$	$Follow(F)$	$:= \emptyset$

0) $Follow(E) := \{\$\}$

1) $F \rightarrow (\ E\ \underbrace{)}_{\neq \varepsilon} \in P$: **add** $First()) = \{)\}$ **to** $Follow(E)$

Summary: $Follow(E) = \{\$,)\}$

Follow(X) for G_{expr3}: Example 1/3

First(***E***)	:= {***i***, **(**}	*Empty*(***E***)	:= ∅	*Follow*(***E***)	:= ∅
First(***E'***)	:= {**+**}	*Empty*(***E'***)	:= {ε}	*Follow*(***E'***)	:= ∅
First(***T***)	:= {***i***, **(**}	*Empty*(***T***)	:= ∅	*Follow*(***T***)	:= ∅
First(***T'***)	:= {*****}	*Empty*(***T'***)	:= {ε}	*Follow*(***T'***)	:= ∅
First(***F***)	:= {***i***, **(**}	*Empty*(***F***)	:= ∅	*Follow*(***F***)	:= ∅

0) *Follow*(***E***) := {**$**}

---

1) ***F*** → **(** ***E*** $\underbrace{\textbf{)}}_{\neq \varepsilon}$ ∈ *P*: **add** *First*(**)**) = {**)**} **to** *Follow*(***E***)

Summary: *Follow*(***E***) = {**$**, **)**}

---

2) ***E*** → ***TE'*** $\underbrace{\ }_{\varepsilon}$ ∈ *P*:

ε: *Empty*(ε) = {ε}

$Follow(X)$ for G_{expr3}: Example 1/3

$First(E)$	$:= \{i, (\}$	$Empty(E)$	$:= \emptyset$	$Follow(E)$	$:= \emptyset$
$First(E')$	$:= \{+\}$	$Empty(E')$	$:= \{\varepsilon\}$	$Follow(E')$	$:= \emptyset$
$First(T)$	$:= \{i, (\}$	$Empty(T)$	$:= \emptyset$	$Follow(T)$	$:= \emptyset$
$First(T')$	$:= \{*\}$	$Empty(T')$	$:= \{\varepsilon\}$	$Follow(T')$	$:= \emptyset$
$First(F)$	$:= \{i, (\}$	$Empty(F)$	$:= \emptyset$	$Follow(F)$	$:= \emptyset$

0) $Follow(E) := \{\$\}$

---

1) $F \rightarrow (E \underbrace{)}_{\neq \varepsilon} \in P$**:** **add** $First()) = \{)\}$ **to** $Follow(E)$

Summary: $Follow(E) = \{\$,)\}$

---

2) $E \rightarrow TE' \underbrace{}_{\varepsilon:\ Empty(\varepsilon) = \{\varepsilon\}} \in P$**:** **add** $Follow(E) = \{\$,)\}$ **to** $Follow(E')$

Follow(*X*) for G_{expr3}: Example 1/3

First(***E***) := {***i***, **(**}	*Empty*(***E***) := ∅	*Follow*(***E***) := ∅
First(***E'***) := {**+**}	*Empty*(***E'***) := {ε}	*Follow*(***E'***) := ∅
First(***T***) := {***i***, **(**}	*Empty*(***T***) := ∅	*Follow*(***T***) := ∅
First(***T'***) := {*****}	*Empty*(***T'***) := {ε}	*Follow*(***T'***) := ∅
First(***F***) := {***i***, **(**}	*Empty*(***F***) := ∅	*Follow*(***F***) := ∅

0) *Follow*(***E***) := {**$**}

1) ***F*** → **(** ***E*** $\underbrace{\textbf{)}}_{\neq \varepsilon}$ ∈ *P*: **add** *First*(**)**) = {**)**} **to** *Follow*(***E***)

Summary: *Follow*(***E***) = {**$**, **)**}

2) ***E*** → ***T*** ***E'*** $\underbrace{}_{\varepsilon:\ Empty(\varepsilon) = \{\varepsilon\}}$ ∈ *P*: **add** *Follow*(***E***) = {**$**, **)**} **to** *Follow*(***E'***)

E → ***T*** $\underbrace{\textit{\textbf{E'}}}_{\neq \varepsilon}$ ∈ *P*:

Follow(*X*) for G_{expr3}: Example 1/3

First(***E***)	:=	{**i**, **(**}	*Empty*(***E***)	:=	∅	*Follow*(***E***)	:=	∅
First(***E'***)	:=	{**+**}	*Empty*(***E'***)	:=	{ε}	*Follow*(***E'***)	:=	∅
First(***T***)	:=	{**i**, **(**}	*Empty*(***T***)	:=	∅	*Follow*(***T***)	:=	∅
First(***T'***)	:=	{*****}	*Empty*(***T'***)	:=	{ε}	*Follow*(***T'***)	:=	∅
First(***F***)	:=	{**i**, **(**}	*Empty*(***F***)	:=	∅	*Follow*(***F***)	:=	∅

0) *Follow*(***E***) := {**$**}

1) $F \to (E\underbrace{)}_{\neq \varepsilon} \in P$: **add** *First*(**)**) = {**)**} **to** *Follow*(***E***)

Summary: *Follow*(***E***) = {**$**, **)**}

2) $E \to TE'\underbrace{\ }_{\varepsilon:\ Empty(\varepsilon) = \{\varepsilon\}} \in P$: **add** *Follow*(***E***) = {**$**, **)**} **to** *Follow*(***E'***)

$E \to T\underbrace{E'}_{\neq \varepsilon} \in P$: **add** *First*(***E'***) = {**+**} **to** *Follow*(***T***)

Follow(*X*) for G_{expr3}: Example 1/3

First(***E***)	:= {**i**, **(**}	*Empty*(***E***)	:= ∅	*Follow*(***E***)	:= ∅
First(***E'***)	:= {**+**}	*Empty*(***E'***)	:= {ε}	*Follow*(***E'***)	:= ∅
First(***T***)	:= {**i**, **(**}	*Empty*(***T***)	:= ∅	*Follow*(***T***)	:= ∅
First(***T'***)	:= {*****}	*Empty*(***T'***)	:= {ε}	*Follow*(***T'***)	:= ∅
First(***F***)	:= {**i**, **(**}	*Empty*(***F***)	:= ∅	*Follow*(***F***)	:= ∅

0) *Follow*(***E***) := {**$**}

1) $F \rightarrow (E \underbrace{)}_{\neq \varepsilon} \in P$: **add** *First*()) = {)} **to** *Follow*(***E***)

Summary: *Follow*(***E***) = {**$**, **)**}

2) $E \rightarrow TE' \underbrace{\ }_{\varepsilon:\ Empty(\varepsilon) = \{\varepsilon\}} \in P$: **add** *Follow*(***E***) = {**$**, **)**} **to** *Follow*(***E'***)

$E \rightarrow T\underbrace{E'}_{\neq \varepsilon} \in P$: **add** *First*(***E'***) = {**+**} **to** *Follow*(***T***)

$E \rightarrow T\underbrace{E'} \in P$:

Empty(***E'***) = {ε}

Follow(*X*) for G_{expr3}: Example 1/3

First(*E*) := {i, (}	*Empty*(*E*) := ∅	*Follow*(*E*) := ∅
First(*E'*) := {+}	*Empty*(*E'*) := {ε}	*Follow*(*E'*) := ∅
First(*T*) := {i, (}	*Empty*(*T*) := ∅	*Follow*(*T*) := ∅
First(*T'*) := {*}	*Empty*(*T'*) := {ε}	*Follow*(*T'*) := ∅
First(*F*) := {i, (}	*Empty*(*F*) := ∅	*Follow*(*F*) := ∅

0) *Follow*(*E*) := {$}

---

1) $F \rightarrow (E \underbrace{)}_{\neq \varepsilon} \in P$: **add** *First*()) = {)} **to** *Follow*(*E*)

Summary: *Follow*(*E*) = {$,)}

---

2) $E \rightarrow TE' \underbrace{}_{\varepsilon:\ Empty(\varepsilon) = \{\varepsilon\}} \in P$: **add** *Follow*(*E*) = {$,)} **to** *Follow*(*E'*)

$E \rightarrow T\underbrace{E'}_{\neq \varepsilon} \in P$: **add** *First*(*E'*) = {+} **to** *Follow*(*T*)

$E \rightarrow T\underbrace{E'}_{Empty(E') = \{\varepsilon\}} \in P$: **add** *Follow*(*E*) = {$,)} **to** *Follow*(*T*)

Follow(*X*) for G_{expr3}: Example 1/3

First(*E*) := {i, (}	*Empty*(*E*) := ∅	*Follow*(*E*) := ∅
First(*E'*) := {+}	*Empty*(*E'*) := {ε}	*Follow*(*E'*) := ∅
First(*T*) := {i, (}	*Empty*(*T*) := ∅	*Follow*(*T*) := ∅
First(*T'*) := {*}	*Empty*(*T'*) := {ε}	*Follow*(*T'*) := ∅
First(*F*) := {i, (}	*Empty*(*F*) := ∅	*Follow*(*F*) := ∅

0) *Follow*(*E*) := {$}

---

1) $F \rightarrow (E \underbrace{)}_{\neq \varepsilon} \in P$: **add** *First*()) = {)} **to** *Follow*(*E*)

Summary: *Follow*(*E*) = {$,)}

---

2) $E \rightarrow TE' \underbrace{\;}_{\varepsilon:\ Empty(\varepsilon) = \{\varepsilon\}} \in P$: **add** *Follow*(*E*) = {$,)} **to** *Follow*(*E'*)

$E \rightarrow T\underbrace{E'}_{\neq \varepsilon} \in P$: **add** *First*(*E'*) = {+} **to** *Follow*(*T*)

$E \rightarrow T\underbrace{E'} \in P$: **add** *Follow*(*E*) = {$,)} **to** *Follow*(*T*)

Empty(*E'*) = {ε}

Summary: *Follow*(*E'*) = {$,)}, *Follow*(*T*) = {+, $,)}

Follow(*X*) for G_{expr3}: Example 2/3

First(***E***)	:= {***i***, **(**}	*Empty*(***E***)	:= $\emptyset$	*Follow*(***E***)	:= {**$**, **)**}
First(***E'***)	:= {+}	*Empty*(***E'***)	:= {ε}	*Follow*(***E'***)	:= {**$**, **)**}
First(***T***)	:= {***i***, **(**}	*Empty*(***T***)	:= $\emptyset$	*Follow*(***T***)	:= {+, **$**, **)**}
First(***T'***)	:= {*}	*Empty*(***T'***)	:= {ε}	*Follow*(***T'***)	:= $\emptyset$
First(***F***)	:= {***i***, **(**}	*Empty*(***F***)	:= $\emptyset$	*Follow*(***F***)	:= $\emptyset$

Follow(*X*) for G_{expr3}: Example 2/3

First(*E*)	:= {**i**, **(**}	*Empty*(*E*)	:= ∅	*Follow*(*E*)	:= {**$**, **)**}
First(*E'*)	:= {**+**}	*Empty*(*E'*)	:= {ε}	*Follow*(*E'*)	:= {**$**, **)**}
First(*T*)	:= {**i**, **(**}	*Empty*(*T*)	:= ∅	*Follow*(*T*)	:= {**+**, **$**, **)**}
First(*T'*)	:= {*****}	*Empty*(*T'*)	:= {ε}	*Follow*(*T'*)	:= ∅
First(*F*)	:= {**i**, **(**}	*Empty*(*F*)	:= ∅	*Follow*(*F*)	:= ∅

3) $E' \to +TE' \underbrace{}_{\varepsilon} \in P$: **add** *Follow*(*E'*) = {**$**, **)**} **to** *Follow*(*E'*)

ε: *Empty*(ε) = {ε}

$E' \to +T\underbrace{E'}_{\neq \varepsilon} \in P$: **add** *First*(*E'*) = {**+**} **to** *Follow*(*T*)

$E' \to +T\underbrace{E'} \in P$: **add** *Follow*(*E'*) = {**$**, **)**} **to** *Follow*(*T*)

Empty(*E'*) = {ε}

Summary: Nothing is changed

Follow(*X*) for G_{expr3}: Example 2/3

First(*E*) := {**i**, **(**}	*Empty*(*E*) := ∅	*Follow*(*E*) := {$,)}
First(*E'*) := {+}	*Empty*(*E'*) := {ε}	*Follow*(*E'*) := {$,)}
First(*T*) := {**i**, **(**}	*Empty*(*T*) := ∅	*Follow*(*T*) := {+, $,)}
First(*T'*) := {*}	*Empty*(*T'*) := {ε}	*Follow*(*T'*) := ∅
First(*F*) := {**i**, **(**}	*Empty*(*F*) := ∅	*Follow*(*F*) := ∅

3) $E' \to +TE'\underbrace{}_{\varepsilon} \in P$: **add** *Follow*(*E'*) = {$,)} **to** *Follow*(*E'*)

ε: *Empty*(ε) = {ε}

$E' \to +T\underbrace{E'}_{\neq \varepsilon} \in P$: **add** *First*(*E'*) = {+} **to** *Follow*(*T*)

$E' \to +T\underbrace{E'} \in P$: **add** *Follow*(*E'*) = {$,)} **to** *Follow*(*T*)

Empty(*E'*) = {ε}

Summary: Nothing is changed

4) $T \to FT'\underbrace{}_{\varepsilon} \in P$: **add** *Follow*(*T*) = {+, $,)} **to** *Follow*(*T'*)

ε: *Empty*(ε) = {ε}

$T \to F\underbrace{T'}_{\neq \varepsilon} \in P$: **add** *First*(*T'*) = {*} **to** *Follow*(*F*)

$T \to F\underbrace{T'} \in P$: **add** *Follow*(*T*) = {+, $,)} **to** *Follow*(*F*)

Empty(*T'*) = {ε}

Summary: *Follow*(*T'*) = {+, $,)}, *Follow*(*F*) = {*, +, $,)}

Follow(*X*) for G_{expr3}: Example 3/3

First(***E***)	:= {***i***, **(**}	*Empty*(***E***)	:= ∅	*Follow*(***E***)	:= {**$**, **)**}
First(***E'***)	:= {+}	*Empty*(***E'***)	:= {ε}	*Follow*(***E'***)	:= {**$**, **)**}
First(***T***)	:= {***i***, **(**}	*Empty*(***T***)	:= ∅	*Follow*(***T***)	:= {+, **$**, **)**}
First(***T'***)	:= {*}	*Empty*(***T'***)	:= {ε}	*Follow*(***T'***)	:= {+, **$**, **)**}
First(***F***)	:= {***i***, **(**}	*Empty*(***F***)	:= ∅	*Follow*(***F***)	:= {*, +, **$**, **)**}

Follow(*X*) for G_{expr3}: Example 3/3

First(*E*)	:= {i, (}	*Empty*(*E*)	:= ∅	*Follow*(*E*)	:= {$,)}
First(*E'*)	:= {+}	*Empty*(*E'*)	:= {ε}	*Follow*(*E'*)	:= {$,)}
First(*T*)	:= {i, (}	*Empty*(*T*)	:= ∅	*Follow*(*T*)	:= {+, $,)}
First(*T'*)	:= {*}	*Empty*(*T'*)	:= {ε}	*Follow*(*T'*)	:= {+, $,)}
First(*F*)	:= {i, (}	*Empty*(*F*)	:= ∅	*Follow*(*F*)	:= {*, +, $,)}

5) $T' \rightarrow *FT'\underbrace{\,}_{\varepsilon} \in P$: **add** *Follow*(*T'*) = {+, $,)} **to** *Follow*(*T'*)

ε: *Empty*(ε) = {ε}

$T' \rightarrow *F\underbrace{T'}_{\neq \varepsilon} \in P$: **add** *First*(*T'*) = {*} **to** *Follow*(*F*)

$T' \rightarrow *F\underbrace{T'}_{} \in P$: **add** *Follow*(*T'*) = {+, $,)} **to** *Follow*(*F*)

Empty(*T'*) = {ε}

End: Nothing is changed.

Follow(*X*) for G_{expr3}: Example 3/3

First(***E***) := {***i***, **(**}	*Empty*(***E***) := ∅	*Follow*(***E***) := {**$**, **)**}
First(***E'***) := {+}	*Empty*(***E'***) := {ε}	*Follow*(***E'***) := {**$**, **)**}
First(***T***) := {***i***, **(**}	*Empty*(***T***) := ∅	*Follow*(***T***) := {+, **$**, **)**}
First(***T'***) := {*}	*Empty*(***T'***) := {ε}	*Follow*(***T'***) := {+, **$**, **)**}
First(***F***) := {***i***, **(**}	*Empty*(***F***) := ∅	*Follow*(***F***) := {*, +, **$**, **)**}

5) $T' \rightarrow *FT'\underbrace{}_{\varepsilon} \in P$: **add** *Follow*(***T'***) = {+, **$**, **)**} **to** *Follow*(***T'***)

ε: *Empty*(ε) = {ε}

$T' \rightarrow *F\underbrace{T'}_{\neq \varepsilon} \in P$: **add** *First*(***T'***) = {*} **to** *Follow*(***F***)

$T' \rightarrow *F\underbrace{T'} \in P$: **add** *Follow*(***T'***) = {+, **$**, **)**} **to** *Follow*(***F***)

Empty(***T'***) = {ε}

End: Nothing is changed.

Summary:

Follow(***E***) := {**$**, **)**}
Follow(***E'***) := {**$**, **)**}
Follow(***T***) := {+, **$**, **)**}
Follow(***T'***) := {+, **$**, **)**}
Follow(***F***) := {*, +, **$**, **)**}

Set *Predict*

Gist: ***Predict*($A \rightarrow x$) is the set of all terminals that can begin a string obtained by a derivation started by using $A \rightarrow x$.**

Definition: Let $G = (N, T, P, S)$ be a CFG. For every $\mathbf{A} \rightarrow \mathbf{x} \in P$, we define *Predict*($\mathbf{A} \rightarrow \mathbf{x}$) so that

- if *Empty*($\mathbf{x}$) = {ε} then

  Predict($\mathbf{A} \rightarrow \mathbf{x}$) = *First*($\mathbf{x}$) ∪ *Follow*($\mathbf{A}$)

- if *Empty*($\mathbf{x}$) = ∅ then

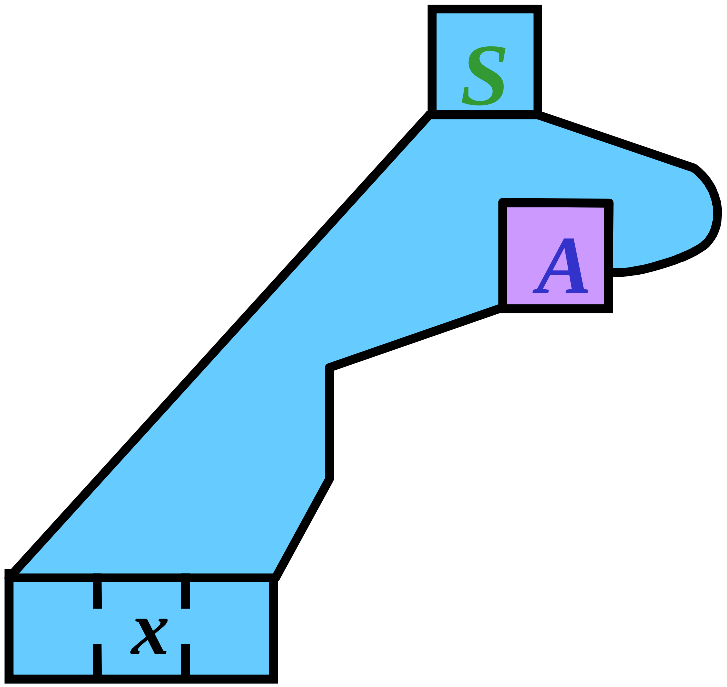
  Predict($\mathbf{A} \rightarrow \mathbf{x}$) = *First*($\mathbf{x}$)

Set $Predict(A \to X_1X_2...X_n)$: Illustration

$Empty(\boldsymbol{X_1X_2...X_n}) = \varnothing$ vs. $Empty(\boldsymbol{X_1X_2...X_n}) = \{\varepsilon\}$

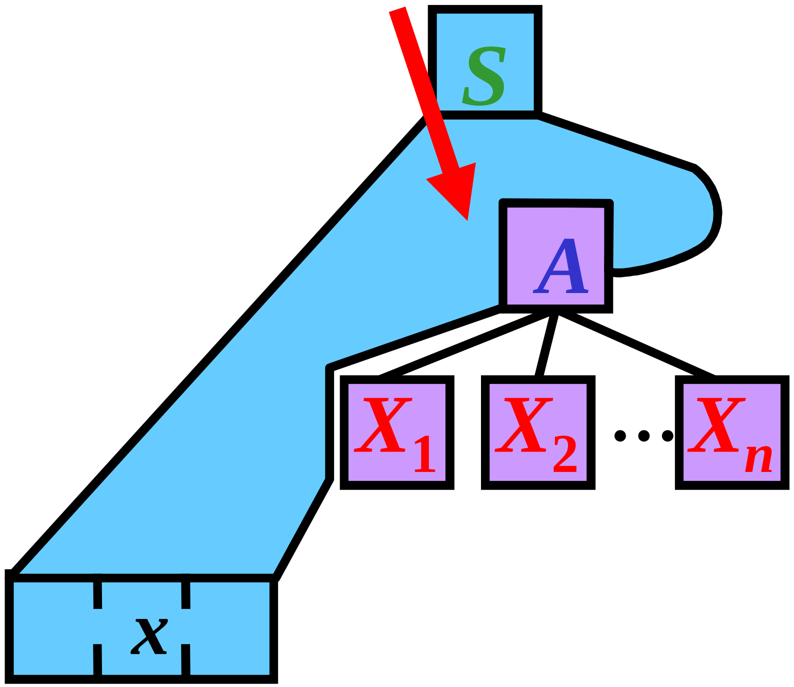
Set *Predict*($A \to X_1X_2...X_n$): Illustration

Empty($X_1X_2...X_n$) = ∅ vs. *Empty*($X_1X_2...X_n$) = {ε}



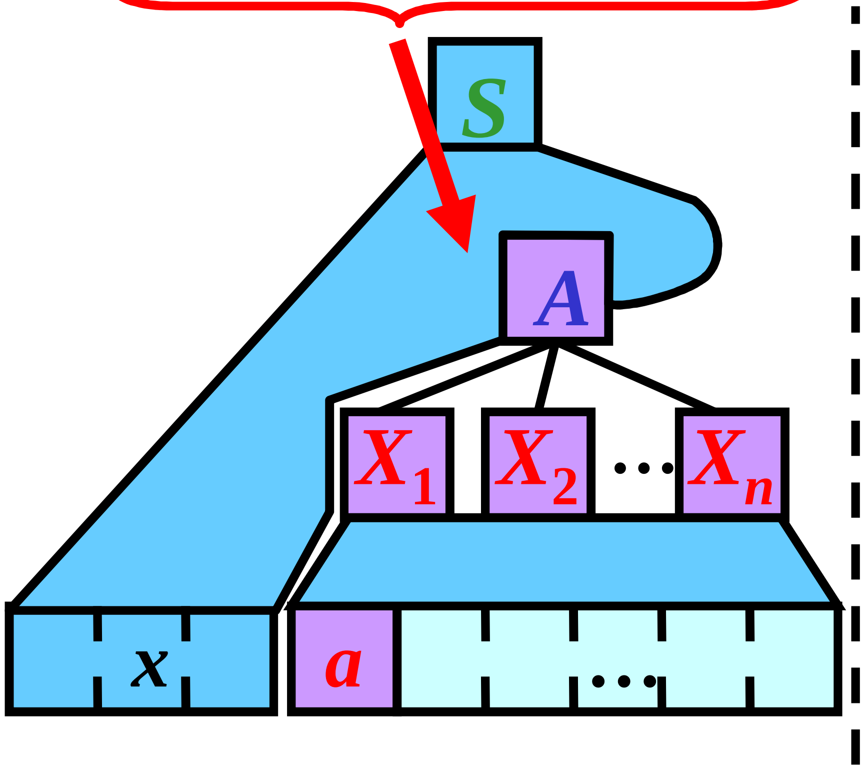
Set $Predict(A \rightarrow X_1X_2...X_n)$: Illustration

$\underbrace{Empty(X_1X_2...X_n) = \emptyset}$ vs. $Empty(X_1X_2...X_n) = \{\varepsilon\}$



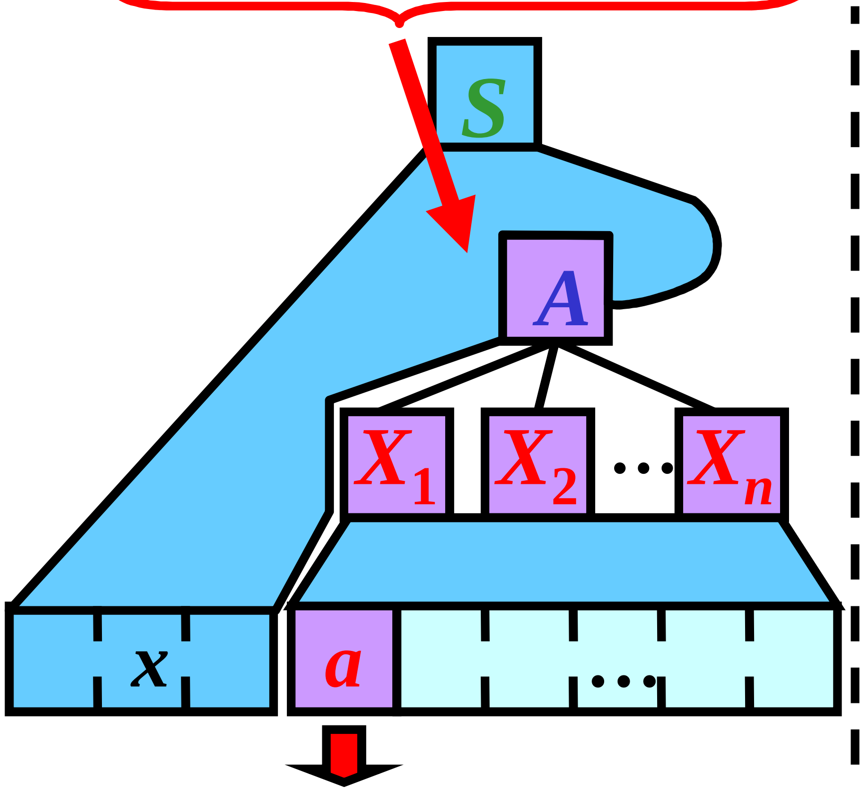
Set *Predict*($A \to X_1X_2...X_n$): Illustration

$\underbrace{Empty(X_1X_2...X_n) = \emptyset}$ vs. $Empty(X_1X_2...X_n) = \{\varepsilon\}$



Set $Predict(A \to X_1X_2\ldots X_n)$: Illustration

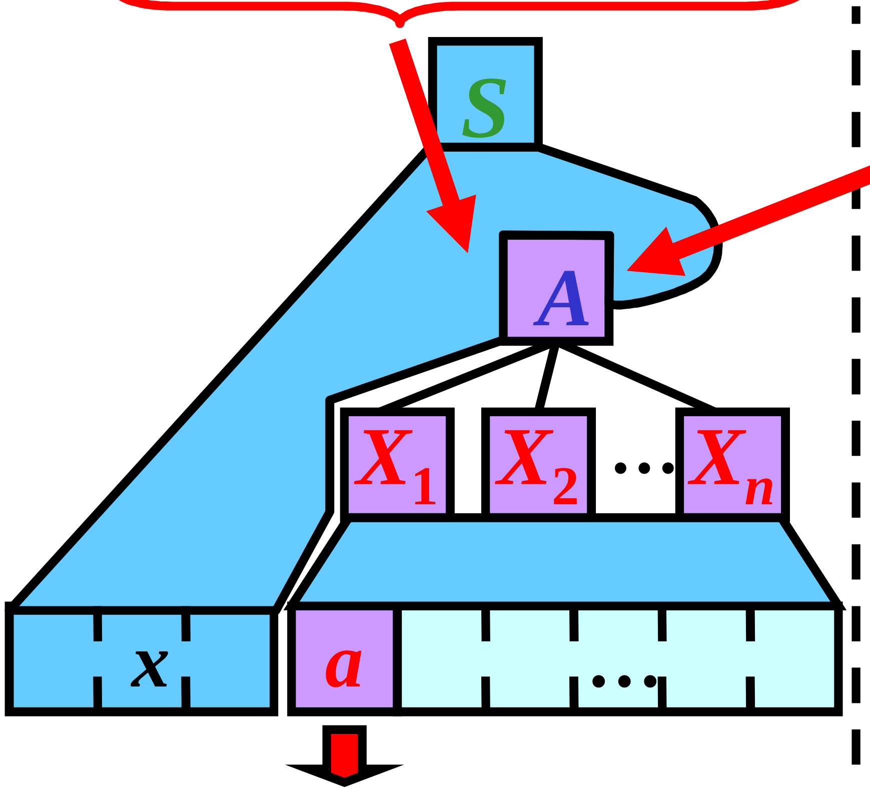
$\underbrace{Empty(X_1X_2\ldots X_n) = \emptyset}$ vs. $Empty(X_1X_2\ldots X_n) = \{\varepsilon\}$



$a \in First(X_1X_2\ldots X_n)$

Set $Predict(A \to X_1X_2\ldots X_n)$: Illustration

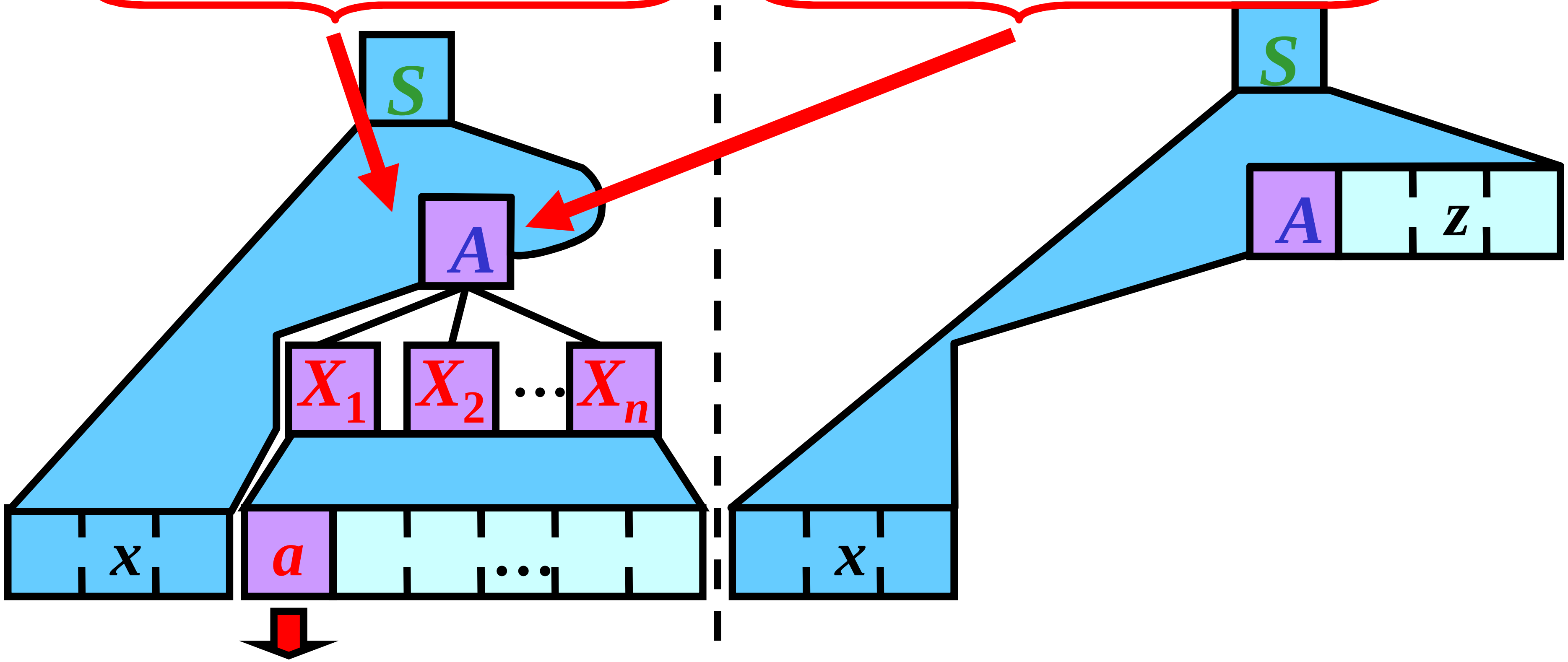
$\underbrace{Empty(X_1X_2\ldots X_n) = \emptyset}$ vs. $\underbrace{Empty(X_1X_2\ldots X_n) = \{\varepsilon\}}$



$a \in First(X_1X_2\ldots X_n)$

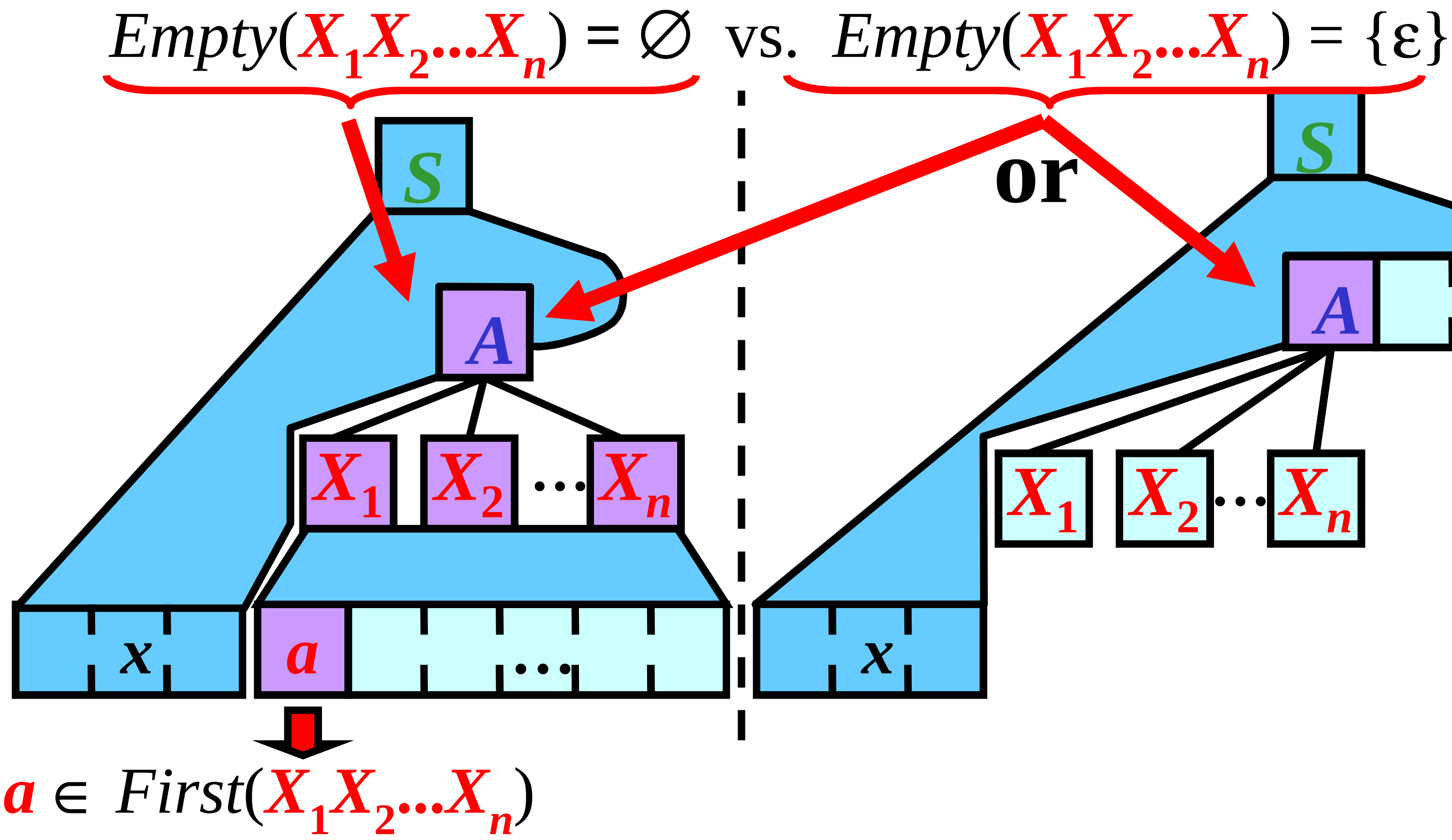
Set *Predict*($A \to X_1X_2 \dots X_n$): Illustration

Empty($X_1X_2 \dots X_n$) = $\emptyset$ vs. *Empty*($X_1X_2 \dots X_n$) = $\{\varepsilon\}$

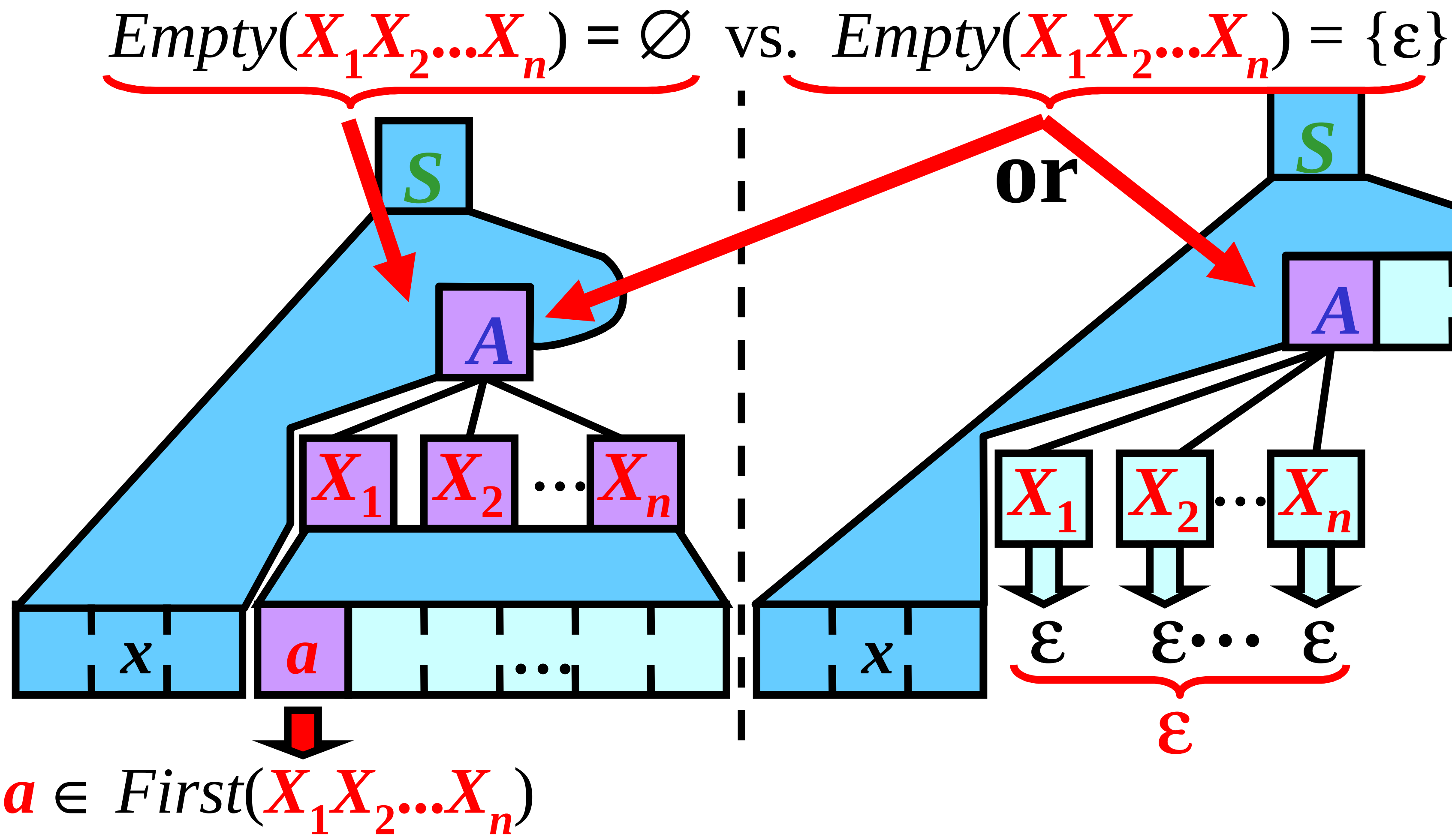


$a \in$ *First*($X_1X_2 \dots X_n$)

Set $Predict(A \rightarrow X_1X_2...X_n)$: Illustration

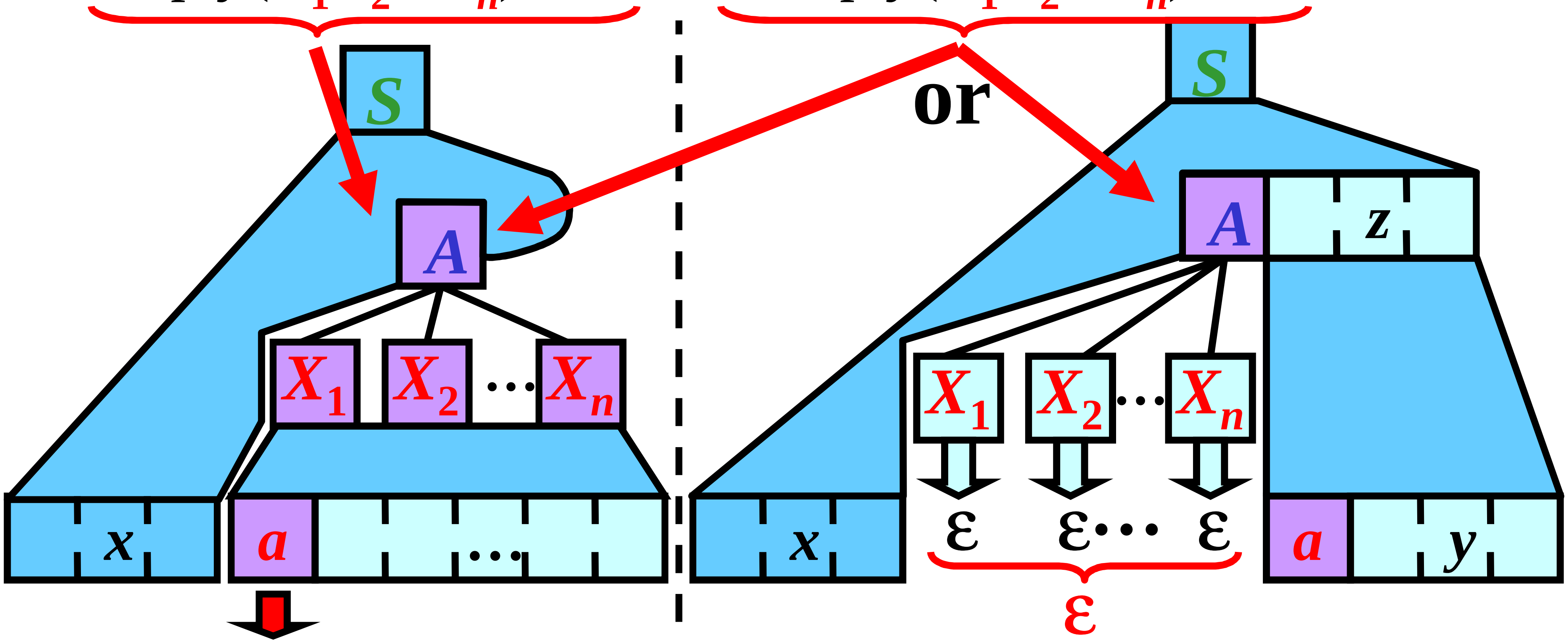


Set *Predict*($A \rightarrow X_1X_2...X_n$): Illustration



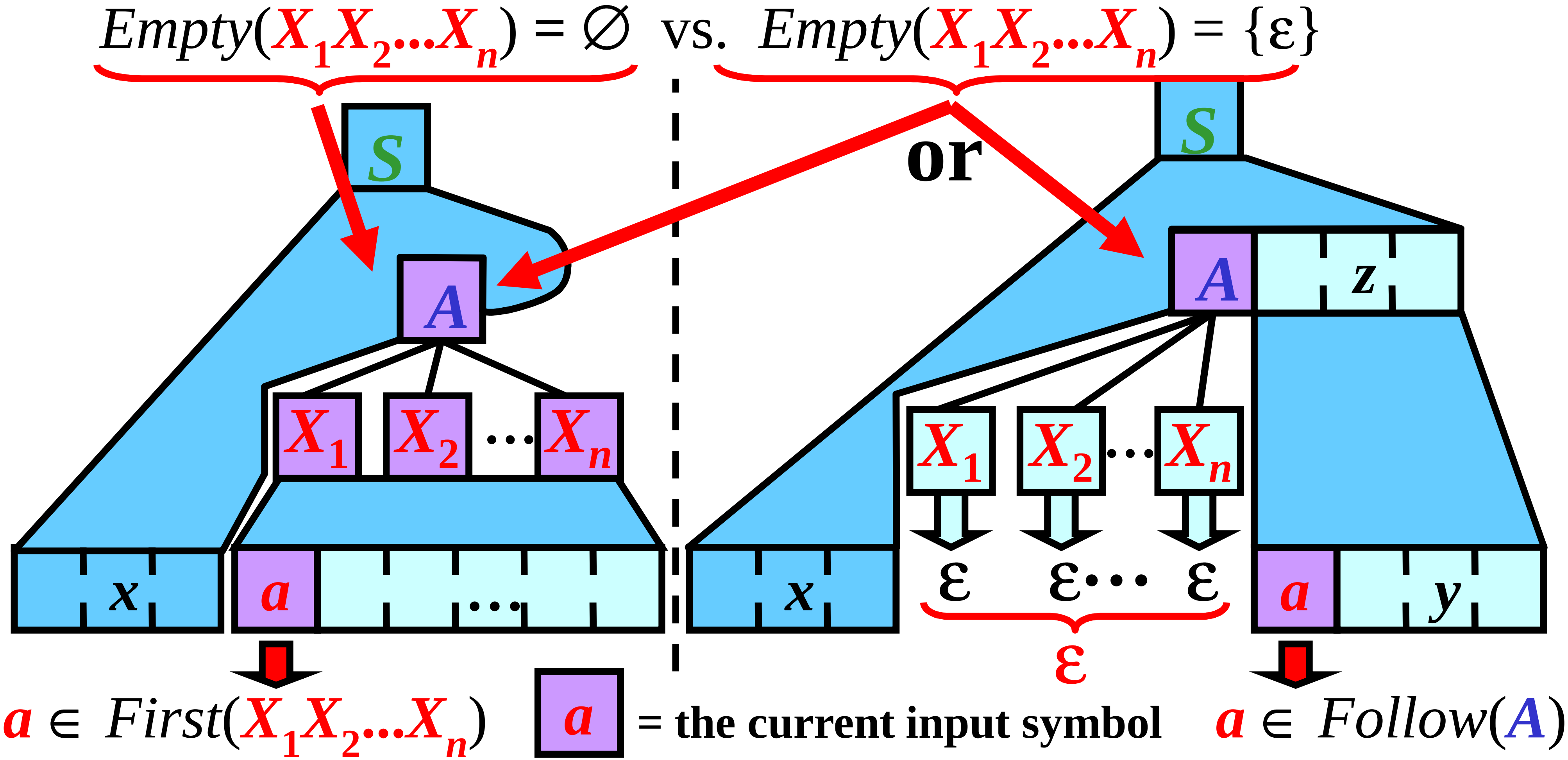
Set *Predict*($A \rightarrow X_1X_2...X_n$): Illustration

$Empty(X_1X_2...X_n) = \emptyset$ vs. $Empty(X_1X_2...X_n) = \{\varepsilon\}$

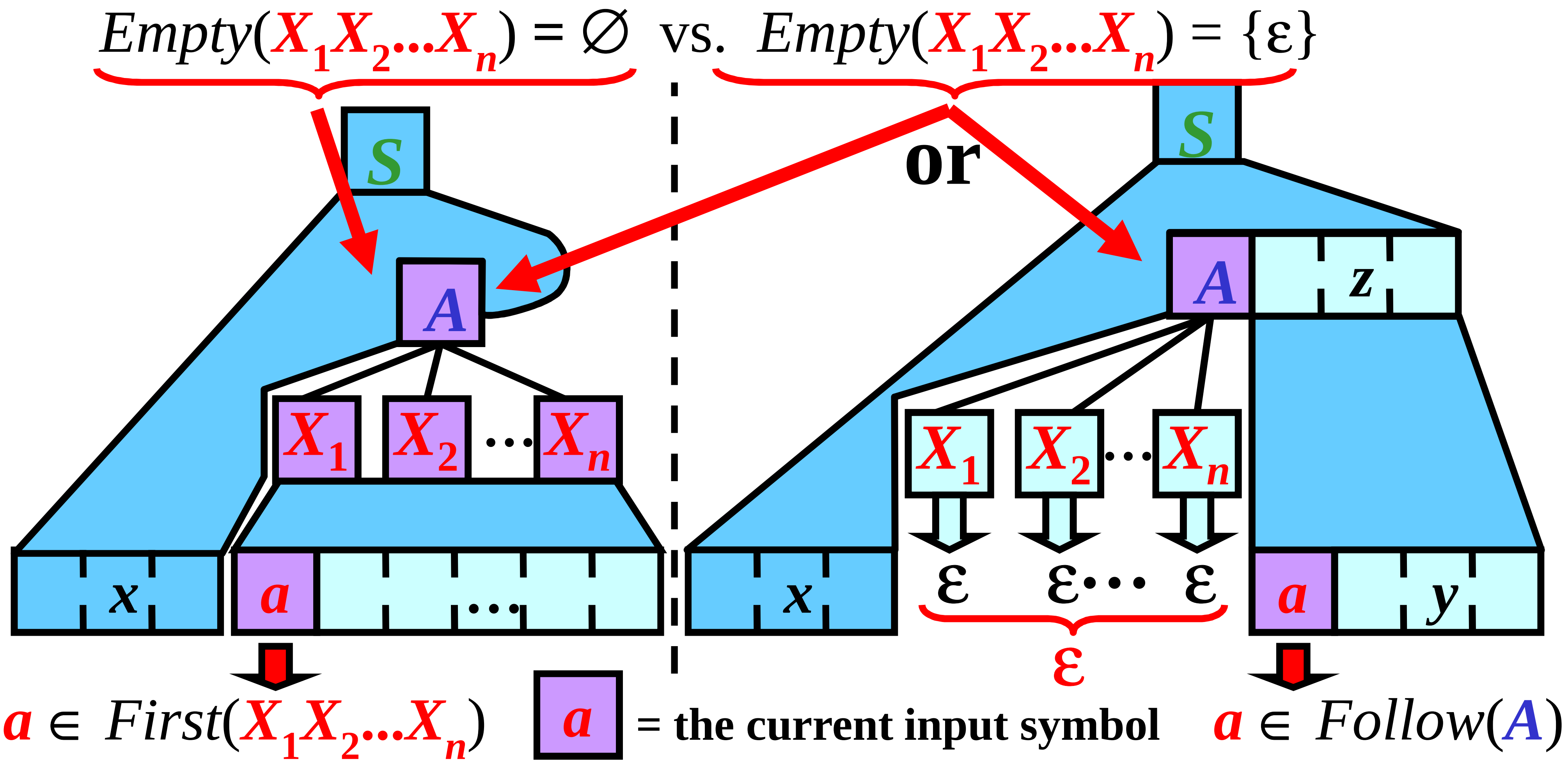


$a \in First(X_1X_2...X_n)$

Set $Predict(A \to X_1X_2...X_n)$: Illustration



Set *Predict*($A \rightarrow X_1X_2...X_n$): Illustration



Summary: if *Empty*($X_1X_2...X_n$) = {ε} then

Predict($A \rightarrow X_1X_2...X_n$) = *First*($X_1X_2...X_n$) ∪ *Follow*($A$);

otherwise, *Predict*($A \rightarrow X_1X_2...X_n$) = *First*($X_1X_2...X_n$)

$Predict(A \rightarrow x)$ for G_{expr3}: Example 1/2

$First(E)$	:= {**i**, **(**}	$Empty(E)$	:= $\emptyset$	$Follow(E)$	:= {$,)}
$First(E')$	:= {+}	$Empty(E')$	:= {ε}	$Follow(E')$	:= {$,)}
$First(T)$	:= {**i**, **(**}	$Empty(T)$	:= $\emptyset$	$Follow(T)$	:= {+, $,)}
$First(T')$	:= {*}	$Empty(T')$	:= {ε}	$Follow(T')$	:= {+, $,)}
$First(F)$	:= {**i**, **(**}	$Empty(F)$	:= $\emptyset$	$Follow(F)$	:= {*, +, $,)}

$Predict(A \to x)$ for G_{expr3}: Example 1/2

$First(E)$:= {**i**, **(**}	$Empty(E)$:= $\varnothing$	$Follow(E)$:= {$,)}
$First(E')$:= {+}	$Empty(E')$:= {ε}	$Follow(E')$:= {$,)}
$First(T)$:= {**i**, **(**}	$Empty(T)$:= $\varnothing$	$Follow(T)$:= {+, $,)}
$First(T')$:= {*}	$Empty(T')$:= {ε}	$Follow(T')$:= {+, $,)}
$First(F)$:= {**i**, **(**}	$Empty(F)$:= $\varnothing$	$Follow(F)$:= {*, +, $,)}

1: $E \to TE'$

$Empty(TE') = \varnothing$ because $Empty(T) = \varnothing$

$Predict(1) := First(TE') = First(T) =$ {**i**, **(**}

$Predict(A \to x)$ for G_{expr3}: Example 1/2

$First(E) := \{i, (\}$	$Empty(E) := \varnothing$	$Follow(E) := \{\$,)\}$
$First(E') := \{+\}$	$Empty(E') := \{\varepsilon\}$	$Follow(E') := \{\$,)\}$
$First(T) := \{i, (\}$	$Empty(T) := \varnothing$	$Follow(T) := \{+, \$,)\}$
$First(T') := \{*\}$	$Empty(T') := \{\varepsilon\}$	$Follow(T') := \{+, \$,)\}$
$First(F) := \{i, (\}$	$Empty(F) := \varnothing$	$Follow(F) := \{*, +, \$,)\}$

1: $E \to TE'$

$Empty(TE') = \varnothing$ because $Empty(T) = \varnothing$

$Predict(1) := First(TE') = First(T) = \{i, (\}$

2: $E' \to +TE'$

$Empty(+TE') = \varnothing$ because $Empty(+) = \varnothing$

$Predict(2) := First(+TE') = First(+) = \{+\}$

$Predict(A \to x)$ for G_{expr3}: Example 1/2

$First(E) := \{i, (\}$	$Empty(E) := \varnothing$	$Follow(E) := \{\$,)\}$
$First(E') := \{+\}$	$Empty(E') := \{\varepsilon\}$	$Follow(E') := \{\$,)\}$
$First(T) := \{i, (\}$	$Empty(T) := \varnothing$	$Follow(T) := \{+, \$,)\}$
$First(T') := \{*\}$	$Empty(T') := \{\varepsilon\}$	$Follow(T') := \{+, \$,)\}$
$First(F) := \{i, (\}$	$Empty(F) := \varnothing$	$Follow(F) := \{*, +, \$,)\}$

1: $E \to TE'$

$Empty(TE') = \varnothing$ because $Empty(T) = \varnothing$

$Predict(1) := First(TE') = First(T) = \{i, (\}$

2: $E' \to +TE'$

$Empty(+TE') = \varnothing$ because $Empty(+) = \varnothing$

$Predict(2) := First(+TE') = First(+) = \{+\}$

3: $E' \to \varepsilon$

$Empty(\varepsilon) = \{\varepsilon\}$

$Predict(3) := First(\varepsilon) \cup Follow(E') = \varnothing \cup \{\$,)\} = \{\$,)\}$

$Predict(A \rightarrow x)$ for G_{expr3}: Example 1/2

$First(E)$	$:= \{i, (\}$	$Empty(E)$	$:= \emptyset$	$Follow(E)$	$:= \{\$,)\}$
$First(E')$	$:= \{+\}$	$Empty(E')$	$:= \{\varepsilon\}$	$Follow(E')$	$:= \{\$,)\}$
$First(T)$	$:= \{i, (\}$	$Empty(T)$	$:= \emptyset$	$Follow(T)$	$:= \{+, \$,)\}$
$First(T')$	$:= \{*\}$	$Empty(T')$	$:= \{\varepsilon\}$	$Follow(T')$	$:= \{+, \$,)\}$
$First(F)$	$:= \{i, (\}$	$Empty(F)$	$:= \emptyset$	$Follow(F)$	$:= \{*, +, \$,)\}$

1: $E \rightarrow TE'$

$Empty(TE') = \emptyset$ because $Empty(T) = \emptyset$

$Predict(1) := First(TE') = First(T) = \{i, (\}$

2: $E' \rightarrow +TE'$

$Empty(+TE') = \emptyset$ because $Empty(+) = \emptyset$

$Predict(2) := First(+TE') = First(+) = \{+\}$

3: $E' \rightarrow \varepsilon$

$Empty(\varepsilon) = \{\varepsilon\}$

$Predict(3) := First(\varepsilon) \cup Follow(E') = \emptyset \cup \{\$,)\} = \{\$,)\}$

4: $T \rightarrow FT'$

$Empty(FT') = \emptyset$ because $Empty(F) = \emptyset$

$Predict(4) := First(FT') = First(F) = \{i, (\}$

Predict($A \to x$) for G_{expr3}: Example 2/2

First(***E***)	:= {***i***, **(**}	*Empty*(***E***)	:= ∅	*Follow*(***E***)	:= {**$**, **)**}
First(***E'***)	:= {+}	*Empty*(***E'***)	:= {ε}	*Follow*(***E'***)	:= {**$**, **)**}
First(***T***)	:= {***i***, **(**}	*Empty*(***T***)	:= ∅	*Follow*(***T***)	:= {+, **$**, **)**}
First(***T'***)	:= {*****}	*Empty*(***T'***)	:= {ε}	*Follow*(***T'***)	:= {+, **$**, **)**}
First(***F***)	:= {***i***, **(**}	*Empty*(***F***)	:= ∅	*Follow*(***F***)	:= {*****, +, **$**, **)**}

Predict($A \rightarrow x$) for G_{expr3}: Example 2/2

$First(E) := \{i, (\}$	$Empty(E) := \emptyset$	$Follow(E) := \{\$,)\}$
$First(E') := \{+\}$	$Empty(E') := \{\varepsilon\}$	$Follow(E') := \{\$,)\}$
$First(T) := \{i, (\}$	$Empty(T) := \emptyset$	$Follow(T) := \{+, \$,)\}$
$First(T') := \{*\}$	$Empty(T') := \{\varepsilon\}$	$Follow(T') := \{+, \$,)\}$
$First(F) := \{i, (\}$	$Empty(F) := \emptyset$	$Follow(F) := \{*, +, \$,)\}$

5: $T' \rightarrow *FT'$

$Empty(*FT') = \emptyset$ because $Empty(*) = \emptyset$

$Predict(5) := First(*FT') = First(*) = \{*\}$

Predict($A \to x$) for G_{expr3}: Example 2/2

First(***E***)	:= {***i***, **(**}	*Empty*(***E***)	:= ∅	*Follow*(***E***)	:= {$, **)**}
First(***E'***)	:= {+}	*Empty*(***E'***)	:= {ε}	*Follow*(***E'***)	:= {$, **)**}
First(***T***)	:= {***i***, **(**}	*Empty*(***T***)	:= ∅	*Follow*(***T***)	:= {+, $, **)**}
First(***T'***)	:= {*****}	*Empty*(***T'***)	:= {ε}	*Follow*(***T'***)	:= {+, $, **)**}
First(***F***)	:= {***i***, **(**}	*Empty*(***F***)	:= ∅	*Follow*(***F***)	:= {*, +, $, **)**}

5: ***T'*** → *****FT'***

Empty(*****FT'***) = ∅ because *Empty*(*****) = ∅

Predict(5) := *First*(*****FT'***) = *First*(*****) = {*****}

6: ***T'*** → ε

Empty(ε) = {ε}

Predict(6) := *First*(ε) ∪ *Follow*(***T'***) = ∅ ∪ {+, $, **)**} = {+, $, **)**}

$Predict(A \to x)$ for G_{expr3}: Example 2/2

$First(E) := \{i, (\}$	$Empty(E) := \varnothing$	$Follow(E) := \{\$,)\}$
$First(E') := \{+\}$	$Empty(E') := \{\varepsilon\}$	$Follow(E') := \{\$,)\}$
$First(T) := \{i, (\}$	$Empty(T) := \varnothing$	$Follow(T) := \{+, \$,)\}$
$First(T') := \{*\}$	$Empty(T') := \{\varepsilon\}$	$Follow(T') := \{+, \$,)\}$
$First(F) := \{i, (\}$	$Empty(F) := \varnothing$	$Follow(F) := \{*, +, \$,)\}$

5: $T' \to *FT'$

$Empty(*FT') = \varnothing$ because $Empty(*) = \varnothing$

$Predict(5) := First(*FT') = First(*) = \{*\}$

6: $T' \to \varepsilon$

$Empty(\varepsilon) = \{\varepsilon\}$

$Predict(6) := First(\varepsilon) \cup Follow(T') = \varnothing \cup \{+, \$,)\} = \{+, \$,)\}$

7: $F \to (E)$

$Empty((E)) = \varnothing$ because $Empty(() = \varnothing$

$Predict(7) := First((E)) = First(() = \{(\}$

$Predict(A \rightarrow x)$ for G_{expr3}: Example 2/2

$First(E) := \{i, (\}$	$Empty(E) := \emptyset$	$Follow(E) := \{\$,)\}$
$First(E') := \{+\}$	$Empty(E') := \{\varepsilon\}$	$Follow(E') := \{\$,)\}$
$First(T) := \{i, (\}$	$Empty(T) := \emptyset$	$Follow(T) := \{+, \$,)\}$
$First(T') := \{*\}$	$Empty(T') := \{\varepsilon\}$	$Follow(T') := \{+, \$,)\}$
$First(F) := \{i, (\}$	$Empty(F) := \emptyset$	$Follow(F) := \{*, +, \$,)\}$

5: $T' \rightarrow *FT'$

$Empty(*FT') = \emptyset$ because $Empty(*) = \emptyset$

$Predict(5) := First(*FT') = First(*) = \{*\}$

6: $T' \rightarrow \varepsilon$

$Empty(\varepsilon) = \{\varepsilon\}$

$Predict(6) := First(\varepsilon) \cup Follow(T') = \emptyset \cup \{+, \$,)\} = \{+, \$,)\}$

7: $F \rightarrow (E)$

$Empty((E)) = \emptyset$ because $Empty(() = \emptyset$

$Predict(7) := First((E)) = First(() = \{(\}$

8: $F \rightarrow i$

$Empty(i) = \emptyset$

$Predict(8) := First(i) = \{i\}$

Construction of LL Table

α	...	a	...
...			
A		$\alpha(A, a)$	
...			

Construction of LL Table

α	…	a	…
…			
A		$\alpha(A, a)$	
…			

$\alpha(A, a) = A \to X_1X_2\ldots X_n \in P$ if $a \in Predict(A \to X_1X_2\ldots X_n)$; otherwise, $\alpha(A, a)$ is blank.

Construction of LL Table

α	...	*a*	...
...			
A		α(*A*, *a*)	
...			

$\alpha(A, a) = A \rightarrow X_1X_2\ldots X_n \in P$ if $a \in Predict(A \rightarrow X_1X_2\ldots X_n)$;
otherwise, $\alpha(A, a)$ is blank.

Task: LL table for G_{expr3}

	i	+	*	(	)	$
E						
E'						
T						
T'						
F						

Rule *r*	***Predict*(*r*)**
1: $E \rightarrow TE'$	{*i*, (}
2: $E' \rightarrow +TE'$	{+}
3: $E' \rightarrow \varepsilon$	{$,)}
4: $T \rightarrow FT'$	{*i*, (}
5: $T' \rightarrow *FT'$	{*}
6: $T' \rightarrow \varepsilon$	{+, $,)}
7: $F \rightarrow (E)$	{(}
8: $F \rightarrow i$	{*i*}

Construction of LL Table

α	…	*a*	…
…			
A		α(*A*, *a*)	
…			

$\alpha(A, a) = A \rightarrow X_1X_2\ldots X_n \in P$ if
$a \in Predict(A \rightarrow X_1X_2\ldots X_n)$;
otherwise, $\alpha(A, a)$ is blank.

Task: LL table for G_{expr3}

	i	+	*	(	)	$
E	1					
E'						
T						
T'						
F						

$i \in$ ***Predict***(1)

Rule *r*	***Predict*(*r*)**
1: $E \rightarrow TE'$	{*i*, (}
2: $E' \rightarrow +TE'$	{+}
3: $E' \rightarrow \varepsilon$	{$,)}
4: $T \rightarrow FT'$	{*i*, (}
5: $T' \rightarrow *FT'$	{*}
6: $T' \rightarrow \varepsilon$	{+, $,)}
7: $F \rightarrow (E)$	{(}
8: $F \rightarrow i$	{*i*}

Construction of LL Table

α	...	*a*	...
...			
A		α(*A*, *a*)	
...			

α(*A*, *a*) = $A \to X_1X_2\ldots X_n \in P$ if $a \in Predict(A \to X_1X_2\ldots X_n)$; otherwise, α(*A*, *a*) is blank.

Task: LL table for G_{expr3}

	i	+	*	(	)	$
E	1					
E'						
T	4					
T'						
F						

i ∈ ***Predict***(1)

i ∈ ***Predict***(4)

Rule *r*	***Predict***(*r*)
1: $E \to TE'$	{*i*, (}
2: $E' \to +TE'$	{+}
3: $E' \to \varepsilon$	{$,)}
4: $T \to FT'$	{*i*, (}
5: $T' \to *FT'$	{*}
6: $T' \to \varepsilon$	{+, $,)}
7: $F \to (E)$	{(}
8: $F \to i$	{*i*}

Construction of LL Table

α	…	a	…
…			
A		$\alpha(A, a)$	
…			

$\alpha(A, a) = A \to X_1X_2\ldots X_n \in P$ if $a \in Predict(A \to X_1X_2\ldots X_n)$; otherwise, $\alpha(A, a)$ is blank.

Task: LL table for G_{expr3}

	i	+	*	(	)	$
E	1					
E'						
T	4					
T'						
F	8					

$i \in Predict(1)$

$i \in Predict(4)$

$i \in Predict(8)$

Rule r	$Predict(r)$
1: $E \to TE'$	{i, (}
2: $E' \to +TE'$	{+}
3: $E' \to \varepsilon$	{$,)}
4: $T \to FT'$	{i, (}
5: $T' \to *FT'$	{*}
6: $T' \to \varepsilon$	{+, $,)}
7: $F \to (E)$	{(}
8: $F \to i$	{i}

Construction of LL Table

α	...	*a*	...
...			
A		α(*A*, *a*)	
...			

$\alpha(A, a) = A \to X_1X_2\ldots X_n \in P$ if $a \in Predict(A \to X_1X_2\ldots X_n)$; otherwise, $\alpha(A, a)$ is blank.

Task: LL table for G_{expr3}

	i	+	*	(	)	$
E	1					
E'						
T	4					
T'						
F	8					

$i \in Predict(1)$

$i \in Predict(4)$

$i \in Predict(8)$

Rule *r*	*Predict*(*r*)
1: $E \to TE'$	{*i*, (}
2: $E' \to +TE'$	{+}
3: $E' \to \varepsilon$	{$,)}
4: $T \to FT'$	{*i*, (}
5: $T' \to *FT'$	{*}
6: $T' \to \varepsilon$	{+, $,)}
7: $F \to (E)$	{(}
8: $F \to i$	{*i*}

Construct the rest analogically.

Parsing Based on LL Table: Example

	i	+	*	(	)	$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

1: $E \rightarrow TE'$
2: $E' \rightarrow +TE'$
3: $E' \rightarrow \varepsilon$
4: $T \rightarrow FT'$
5: $T' \rightarrow *FT'$
6: $T' \rightarrow \varepsilon$
7: $F \rightarrow (E)$
8: $F \rightarrow i$

Question: $i * i \in L(G_{expr3})$**?**

E

$i \quad * \quad i \quad \boxed{\$}$

Parsing Based on LL Table: Example

	i	+	*	(	)	$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

1: $E \to TE'$
2: $E' \to +TE'$
3: $E' \to \varepsilon$
4: $T \to FT'$
5: $T' \to *FT'$
6: $T' \to \varepsilon$
7: $F \to (E)$
8: $F \to i$

Question: ***i*** ***** ***i*** $\in L(G_{expr3})$**?**

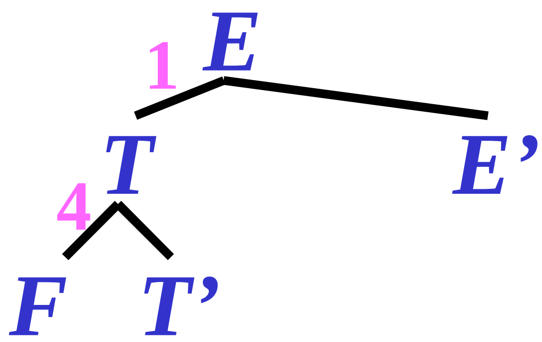
i * *i* $

Parsing Based on LL Table: Example

	i	+	*	(	)	$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

1: $E \rightarrow TE'$
2: $E' \rightarrow +TE'$
3: $E' \rightarrow \varepsilon$
4: $T \rightarrow FT'$
5: $T' \rightarrow *FT'$
6: $T' \rightarrow \varepsilon$
7: $F \rightarrow (E)$
8: $F \rightarrow i$

Question: $i * i \in L(G_{expr3})$?



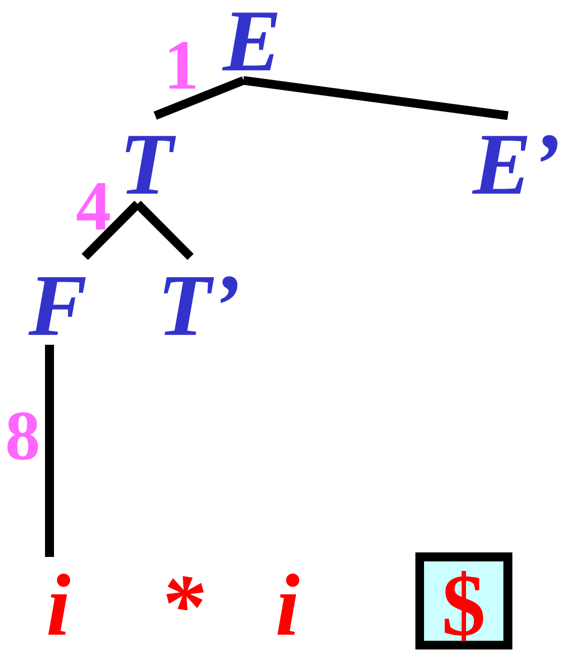
i * *i* $

Parsing Based on LL Table: Example

	i	+	*	(	)	$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

1: $E \to TE'$
2: $E' \to +TE'$
3: $E' \to \varepsilon$
4: $T \to FT'$
5: $T' \to *FT'$
6: $T' \to \varepsilon$
7: $F \to (E)$
8: $F \to i$

Question: $i * i \in L(G_{expr3})$?

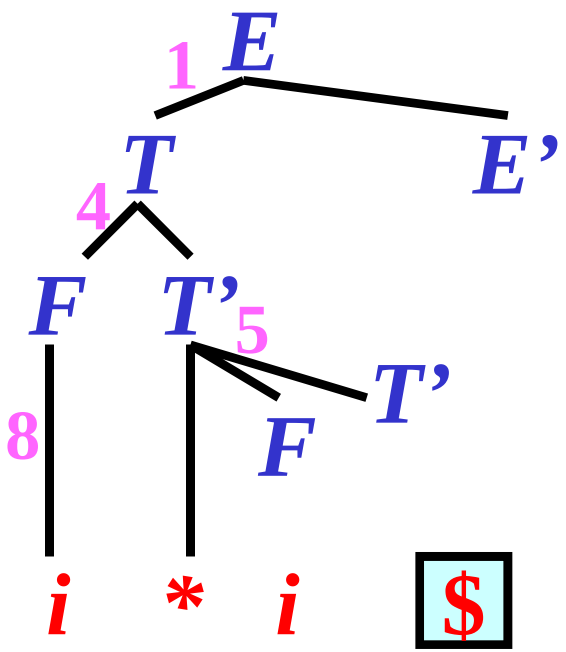


Parsing Based on LL Table: Example

	i	+	*	(	)	$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

1: $E \rightarrow TE'$
2: $E' \rightarrow +TE'$
3: $E' \rightarrow \varepsilon$
4: $T \rightarrow FT'$
5: $T' \rightarrow *FT'$
6: $T' \rightarrow \varepsilon$
7: $F \rightarrow (E)$
8: $F \rightarrow i$

Question: $i * i \in L(G_{expr3})$?



Parsing Based on LL Table: Example

	i	+	*	(	)	$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

1: $E \to TE'$
2: $E' \to +TE'$
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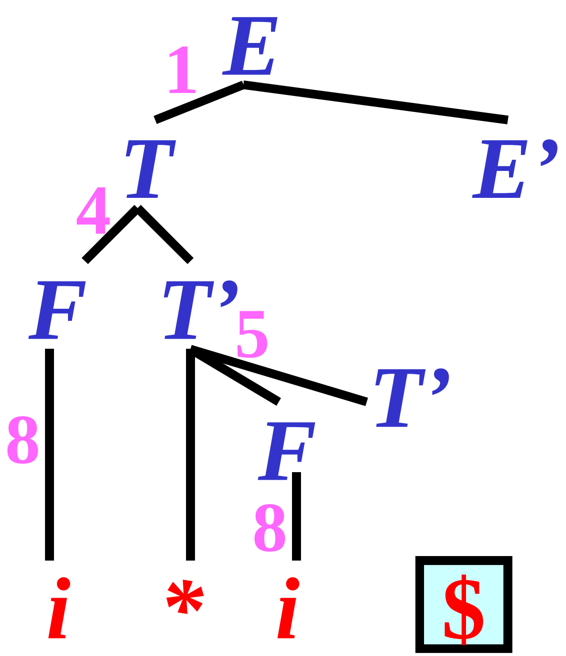

Parsing Based on LL Table: Example

	i	+	*	(	)	\$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

1: $E \rightarrow TE'$
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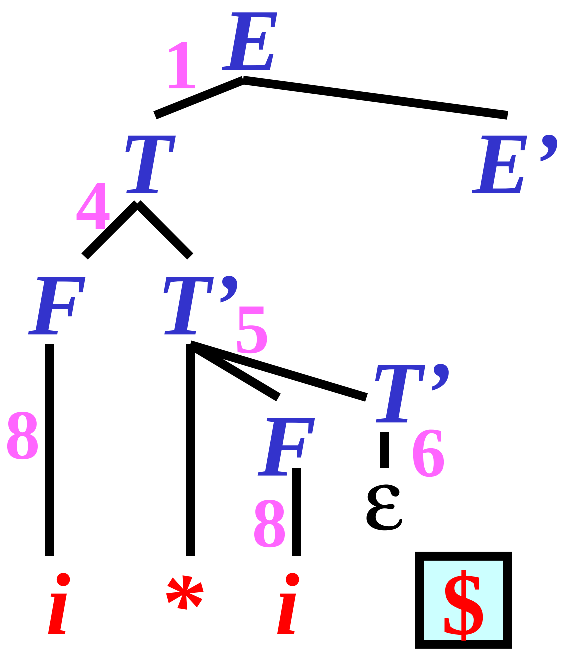

Parsing Based on LL Table: Example

	i	+	*	(	)	$
E	1			1		
E'		2			3	3
T	4			4		
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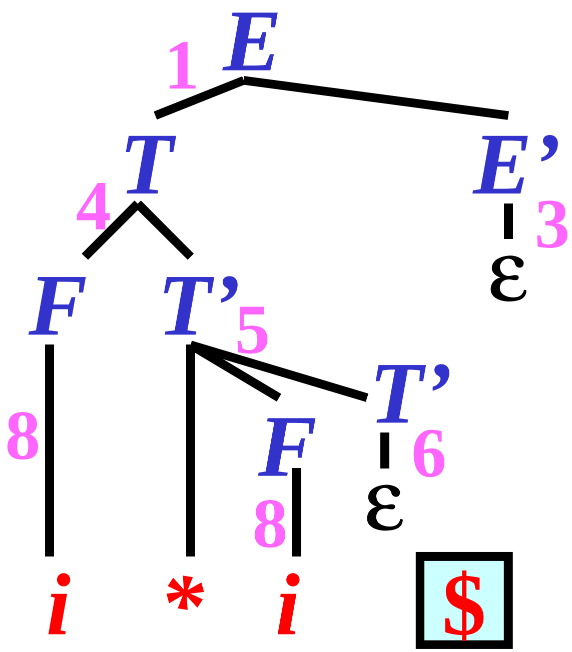

LL Grammars with ε-rules: Definition

Definition: Let $G = (N, T, P, S)$ be a CFG. G is an *LL grammar* if for every $a \in T$ and every $A \in N$ there is **no more than one** A-rule $A \rightarrow X_1X_2...X_n \in P$ such that $a \in Predict(A \rightarrow X_1X_2...X_n)$

Illustration:

LL Grammars with ε-rules: Definition

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Illustration:

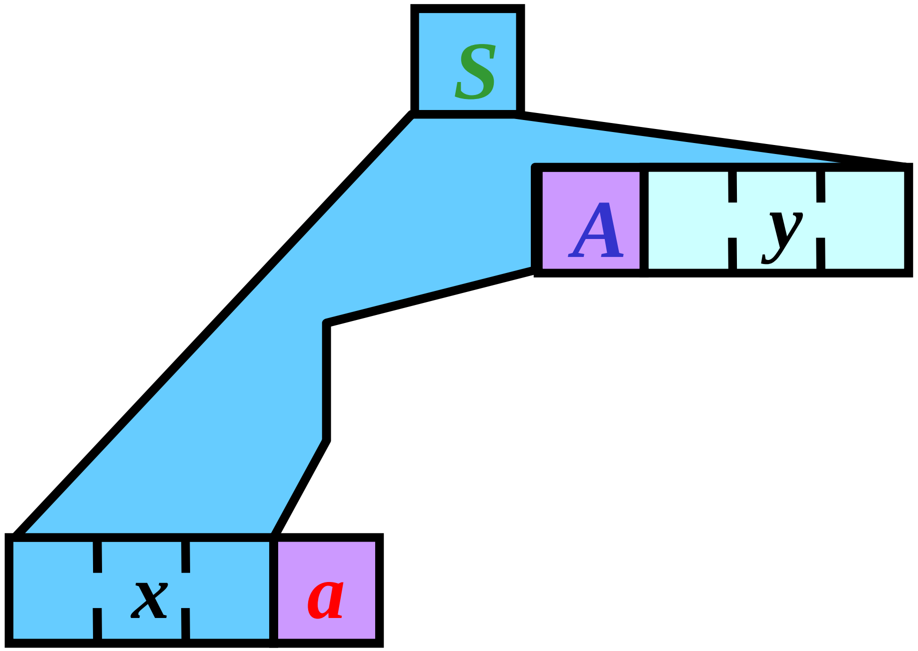

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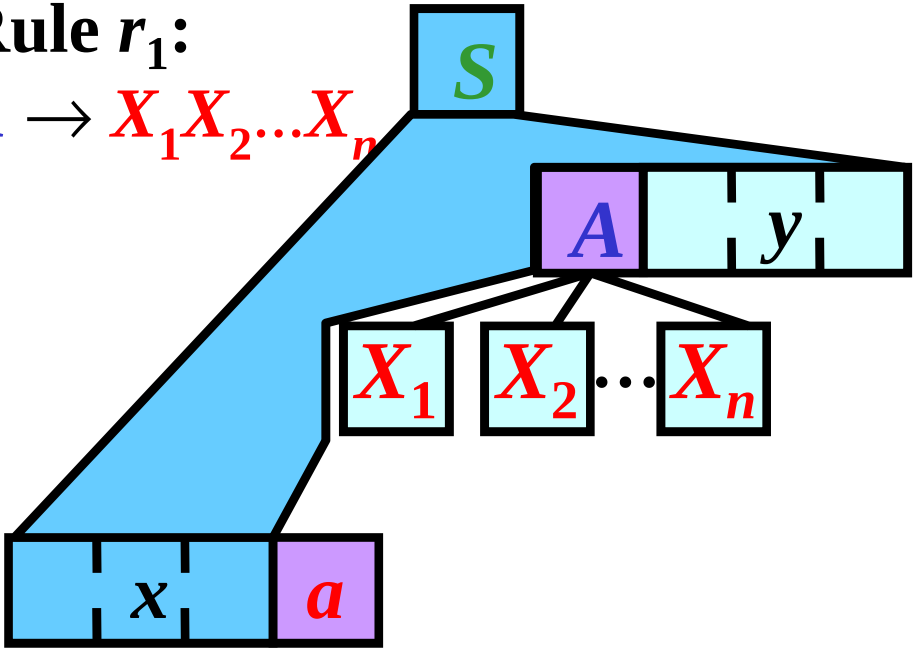
Illustration:

Rule r_1:

$A \rightarrow X_1X_2\ldots X_n$



LL Grammars with ε-rules: Definition

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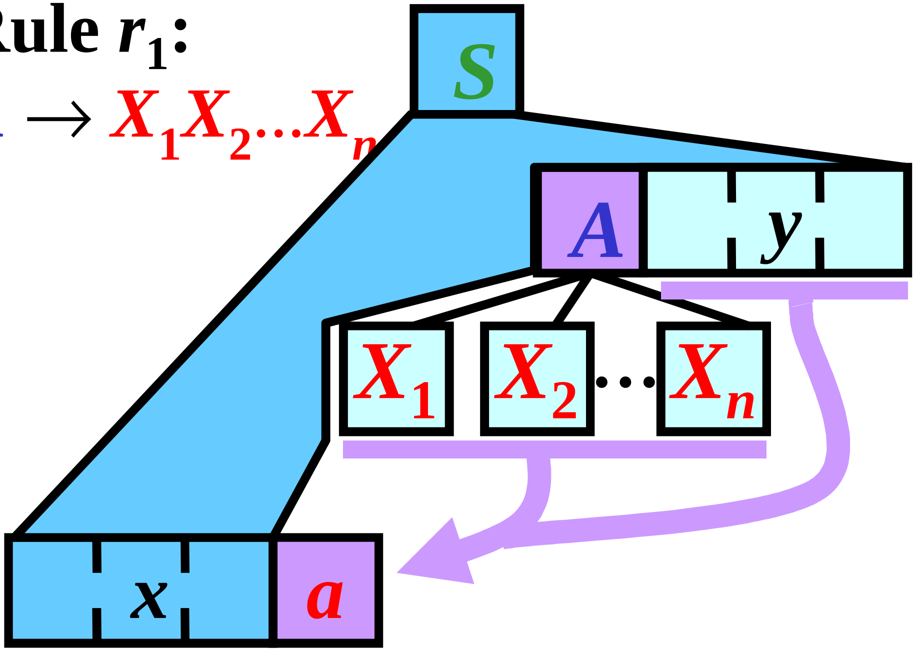
Illustration:

Rule r_1:

$A \rightarrow X_1X_2...X_n$



$a \in Predict(A \rightarrow X_1X_2...X_n)$

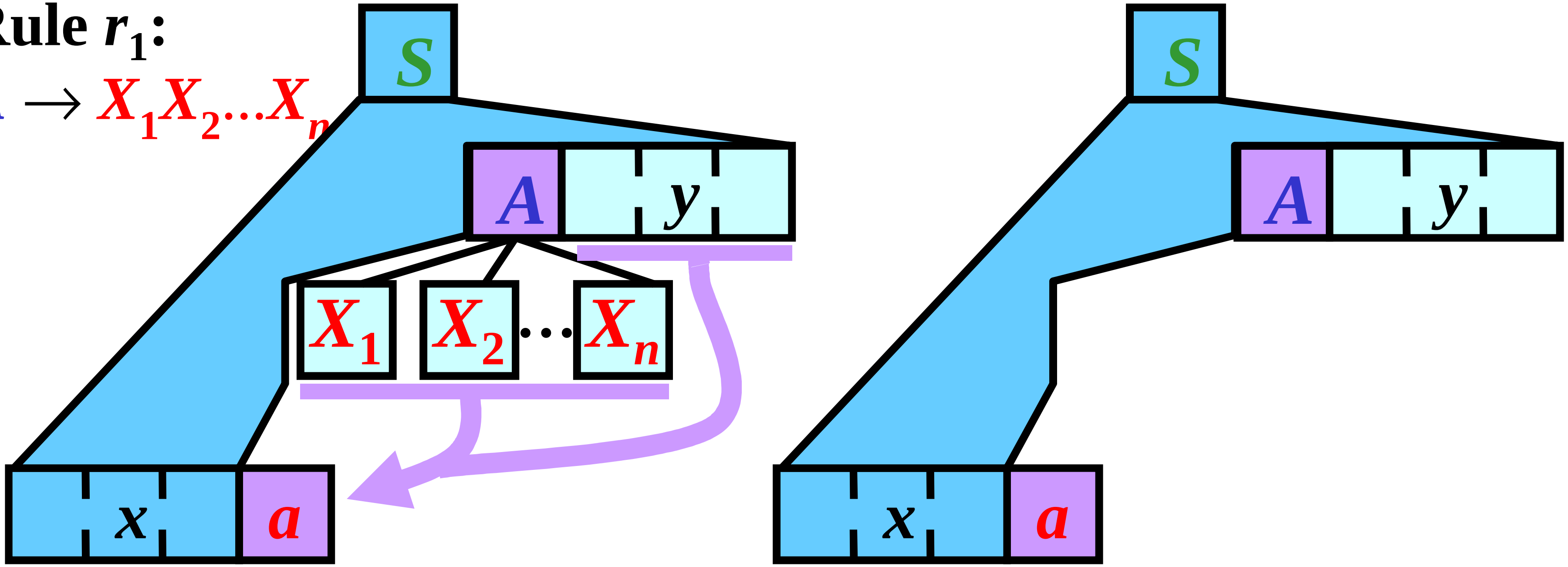
LL Grammars with ε-rules: Definition

Definition: Let $G = (N, T, P, S)$ be a CFG. G is an *LL grammar* if for every $a \in T$ and every $A \in N$ there is **no more than one** A-rule $A \rightarrow X_1X_2\ldots X_n \in P$ such that $a \in Predict(A \rightarrow X_1X_2\ldots X_n)$

Illustration:

Rule r_1:

$A \rightarrow X_1X_2\ldots X_n$



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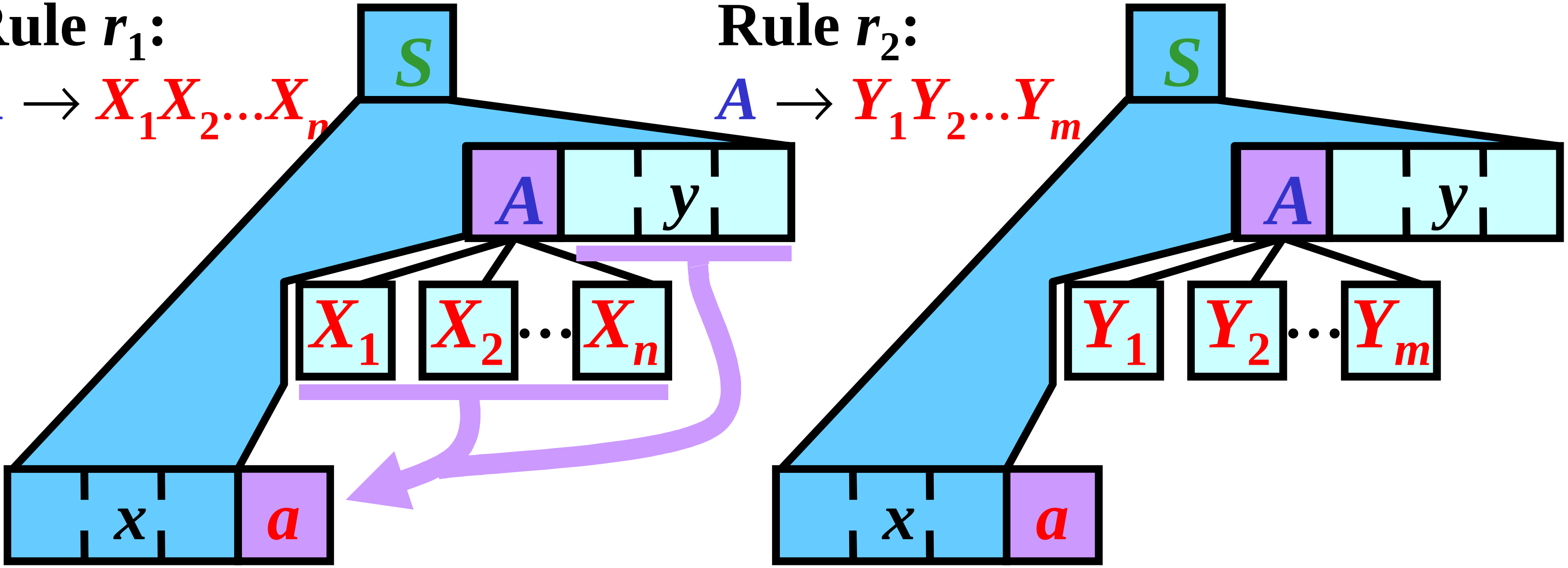
LL Grammars with ε-rules: Definition

Definition: Let $G = (N, T, P, S)$ be a CFG. G is an *LL grammar* if for every $a \in T$ and every $A \in N$ there is **no more than one** A-rule $A \rightarrow X_1X_2...X_n \in P$ such that $a \in Predict(A \rightarrow X_1X_2...X_n)$

Illustration:

Rule r_1: $A \rightarrow X_1X_2...X_n$

Rule r_2: $A \rightarrow Y_1Y_2...Y_m$

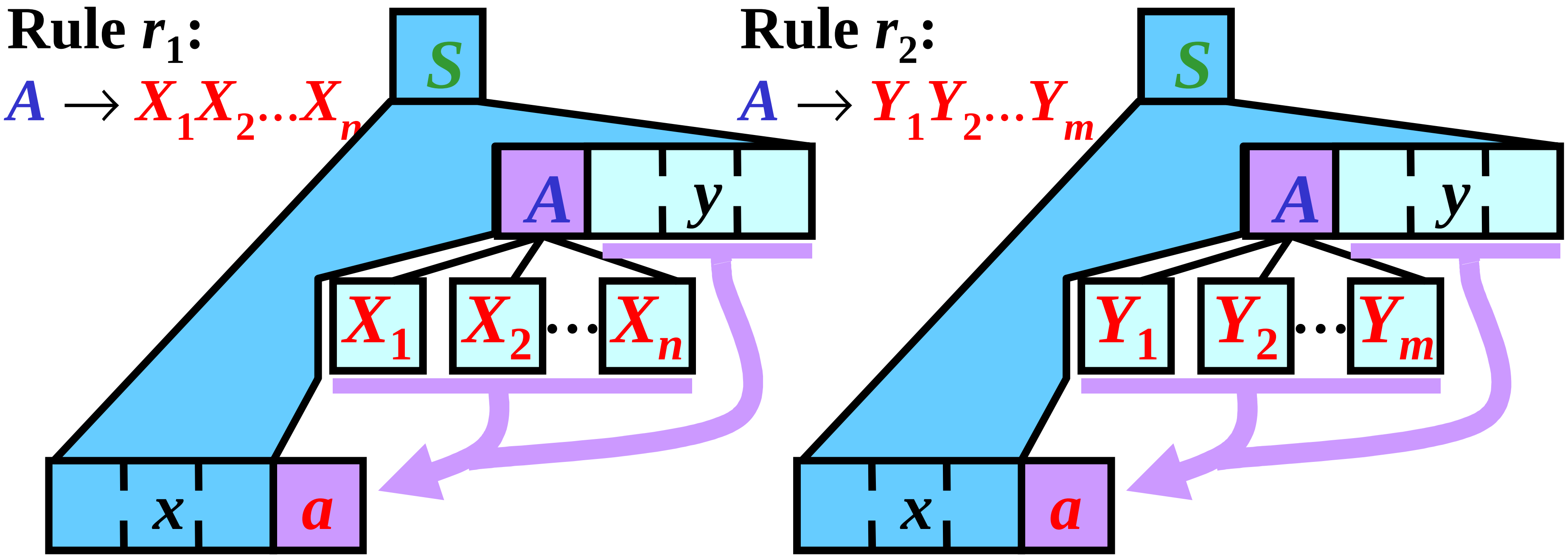


$a \in Predict(A \rightarrow X_1X_2...X_n)$

LL Grammars with ε-rules: Definition

Definition: Let $G = (N, T, P, S)$ be a CFG. G is an *LL grammar* if for every $a \in T$ and every $A \in N$ there is **no more than one** A-rule $A \to X_1X_2\ldots X_n \in P$ such that $a \in Predict(A \to X_1X_2\ldots X_n)$

Illustration:



$a \in Predict(A \to X_1X_2\ldots X_n)$ $a \in Predict(A \to Y_1Y_2\ldots Y_m)$

LL Grammars with ε-rules: Definition

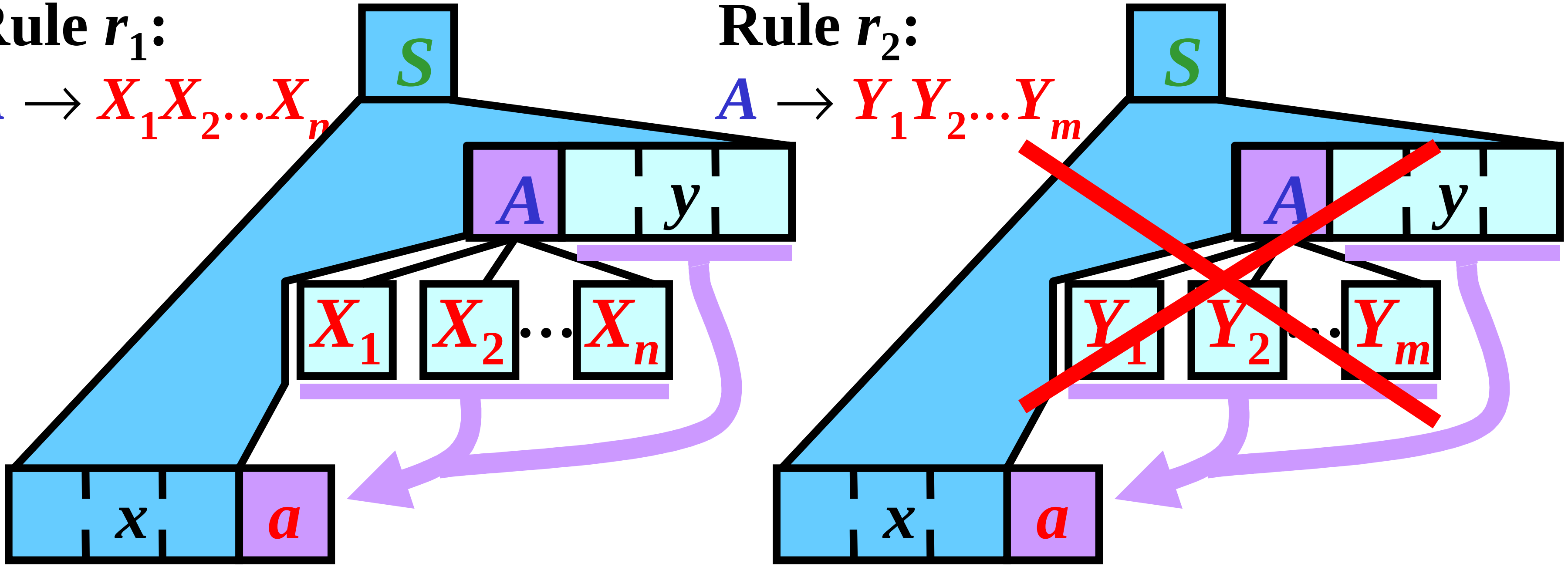
Definition: Let $G = (N, T, P, S)$ be a CFG. G is an *LL grammar* if for every $a \in T$ and every $A \in N$ there is **no more than one** A-rule $A \rightarrow X_1X_2...X_n \in P$ such that $a \in Predict(A \rightarrow X_1X_2...X_n)$

Illustration:

Ruled out in an LL grammar

Rule r_1: $A \rightarrow X_1X_2...X_n$

Rule r_2: $A \rightarrow Y_1Y_2...Y_m$



$a \in Predict(A \rightarrow X_1X_2...X_n)$ $a \in Predict(A \rightarrow Y_1Y_2...Y_m)$

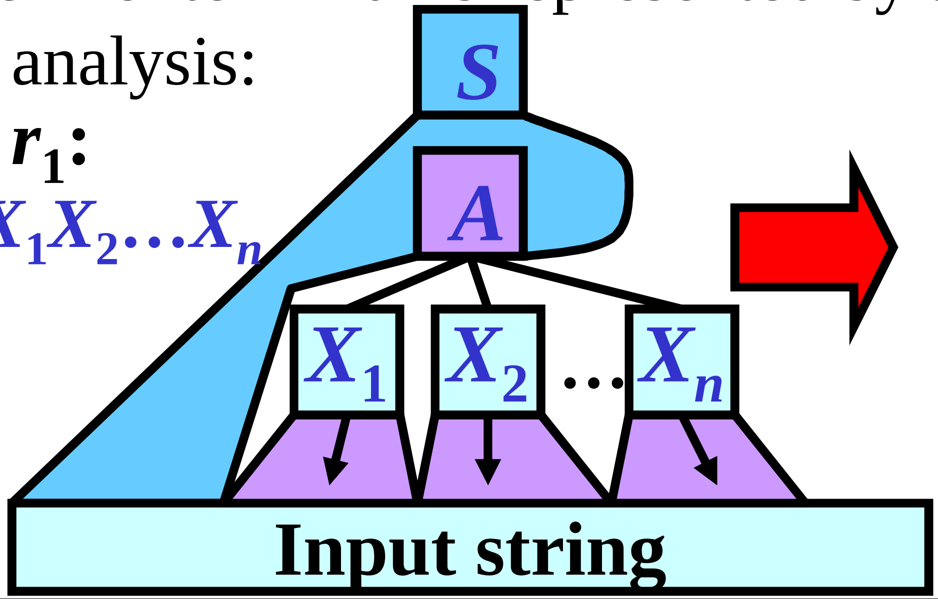
LL Analyzer Implementation

1) Recursive-Descent Parsing

- Each nonterminal is represented by a procedure, which perform its analysis:

Rule r_1:

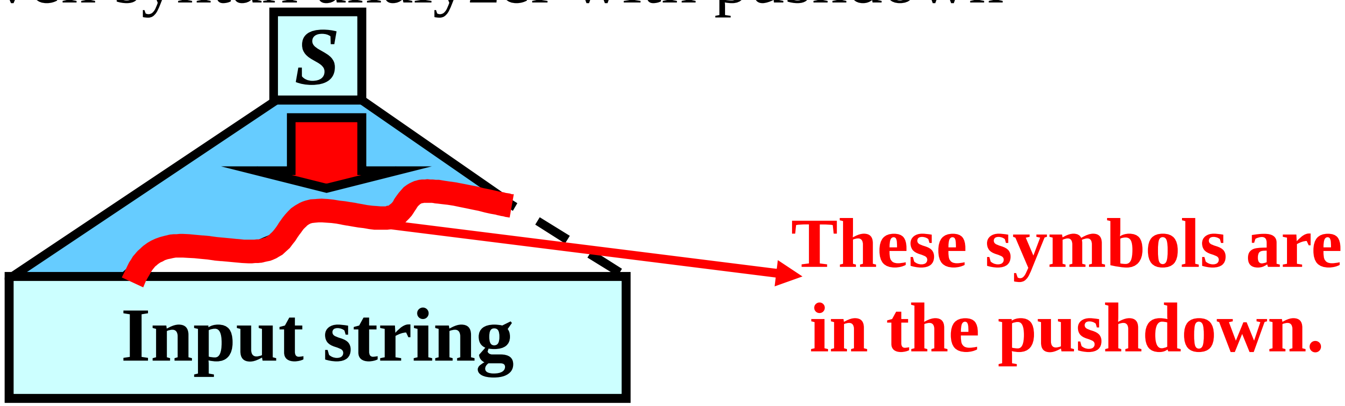
$A \rightarrow X_1X_2\ldots X_n$



function *A*: **boolean**;
begin
{X_1 analysis}
{X_2 analysis}
…
{X_n analysis}
end

2) Predictive Parsing

- Table-driven syntax analyzer with pushdown



Recursive Descent: Example 1/4

```
Procedure GetNextToken;
begin
{ this procedure get the next token to global variable "token"}
end
```

• For $E \in N$: Rule 1: $E \rightarrow TE'$

```
function E: boolean;
begin
   E := false;
   if token in ['i', '('] then
      { simulation of rule 1: E → TE' }
       E := T and E1;
end;
```

	i	+	*	(	)	$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

• For $T \in N$: Rule 4: $T \rightarrow FT'$

```
function T: boolean;
begin
   T := false;
   if token in ['i', '('] then
      { simulation of rule 4: T → FT' }
      T := F and T1;
end;
```

	i	+	*	(	)	$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

Recursive Descent: Example 2/4

- For $E' \in N$: Rules 2: $E' \rightarrow +TE'$, 3: $E' \rightarrow \varepsilon$

	i	+	*	(	)	$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

```
function E1: boolean;
begin
   E1 := false;
   if token = '+' then begin
      { simulation of rule 2: E' → +TE' }
      GetNextToken;
      E1 := T and E1;
   end
   else
   if token in [')', '$'] then
      { simulation of rule 3: E' → ε}
      E1 := true;
end;
```

Recursive Descent: Example 3/4

- For $T' \in N$: Rules 5: $T' \to *FT'$, 6: $T' \to \varepsilon$

```
function T1: boolean;
begin
  T1 := false;
  if token = '*' then begin
     { simulation of rule 5: T' → *FT' }
     GetNextToken;
     T1 := F and T1;
  end
  else
  if token in ['+', ')', '$'] then
     { simulation of rule 6: T' → ε}
     T1 := true;
end;
```

	i	+	*	(	)	$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

Recursive Descent: Example 4/4

- For $F \in N$: Rules 7: $F \rightarrow (E)$, 8: $F \rightarrow i$

```
function F: boolean;
begin
   F := false;
   if token = '(' then begin
      { simulation of rule 7: F → (E) }
      GetNextToken;
      if E then begin
         F := (token = ')');
         GetNextToken;
      end;
   end
   else
   if token = 'i' then begin
      { simulation of rule 8: F → i }
      F := true;
      GetNextToken;
   end;
end;
```

	i	+	*	(	)	$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

Main body:

```
begin
   GetNextToken;
   if E then
      write('OK')
   else
      write('ERROR')
end.
```

Recursive Descent: Illustration for $i*i\$$

Start:

Input string:

i * i $

Recursive Descent: Illustration for $i{*}i\$$

Start: **GetNextToken;**
Call E;

Input string:

$\boxed{i} * \; i \; \$$

Recursive Descent: Illustration for *i***i*$

Start: **GetNextToken;**
Call E;

E:

For token = *i*:
Call T, Call E1

Input string:

$\boxed{i}$ * *i* $

Recursive Descent: Illustration for *i***i*$

Start: **GetNextToken;**
Call E;

Input string:

$\boxed{i}$ * *i* $

***E*:**

For token = *i*:
Call T, Call E1

***T*:**

For token = *i*:
Call F, Call T1

Recursive Descent: Illustration for *i***i*$

Start: **GetNextToken;**
Call E;

Input string:

***i* * *i* $**

***E*:**

For token = *i*:
Call T, Call E1

***T*:**

For token = *i*:
Call F, Call T1

***F*:**

For token = *i*:
GetNextToken;
***Return* TRUE;**

Recursive Descent: Illustration for *i*i*$

Start: **GetNextToken;**
***Call E*;**

Input string:

i	*******

***i* $**

***E*:**

For token = *i*:
Call T*, *Call E1

***T*:**

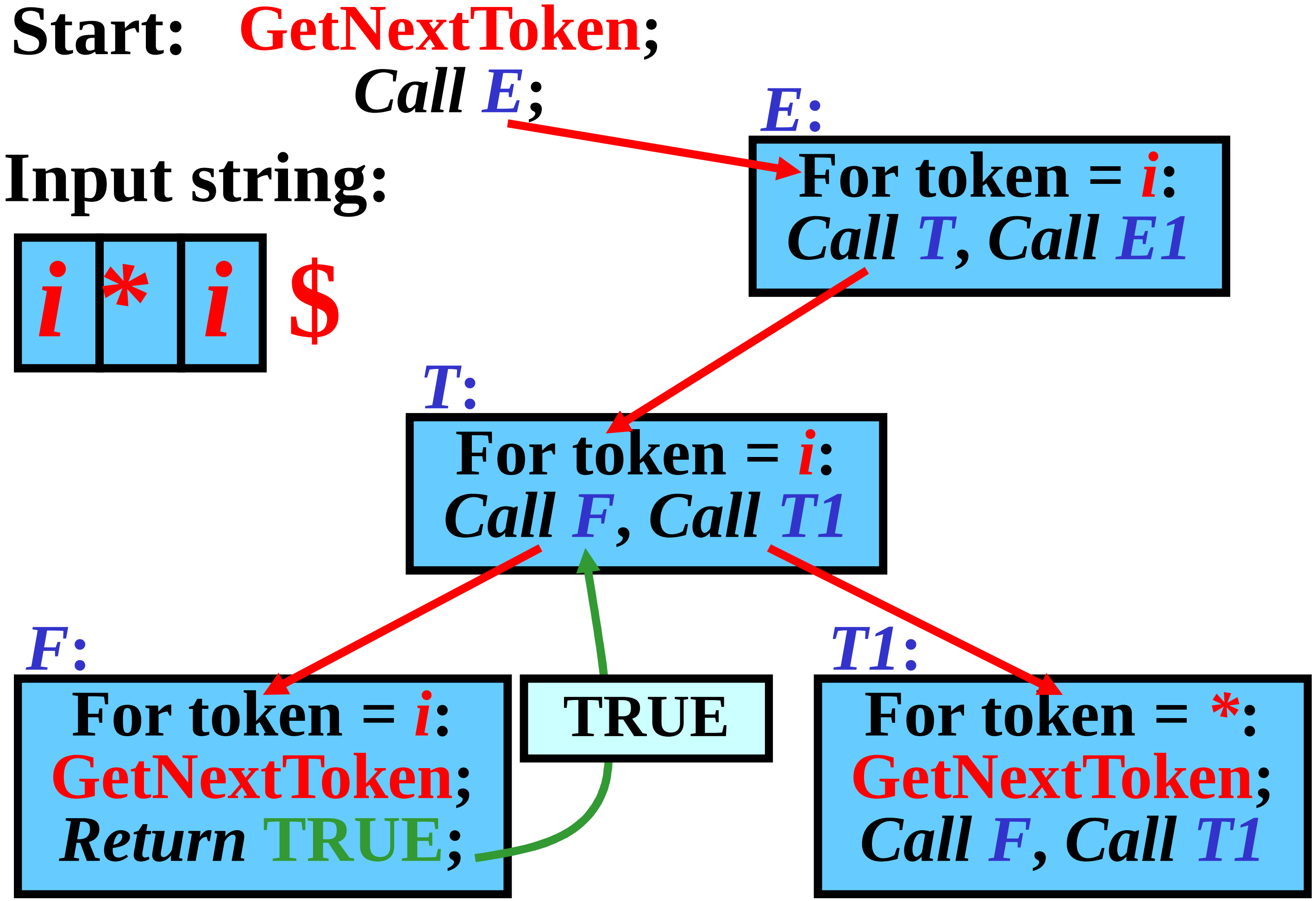
For token = *i*:
Call F*, *Call T1

***F*:**

For token = *i*:
GetNextToken;
***Return* TRUE;**

TRUE

Recursive Descent: Illustration for *i***i*$



Recursive Descent: Illustration for *i*i$*

Start: **GetNextToken**;
Call ***E***;

Input string:

i	*****	***i***	**$**

***E*:**
For token = *i*:
Call T*, *Call E1

***T*:**
For token = *i*:
Call F*, *Call T1

***F*:**
For token = *i*:
GetNextToken;
***Return* TRUE;**

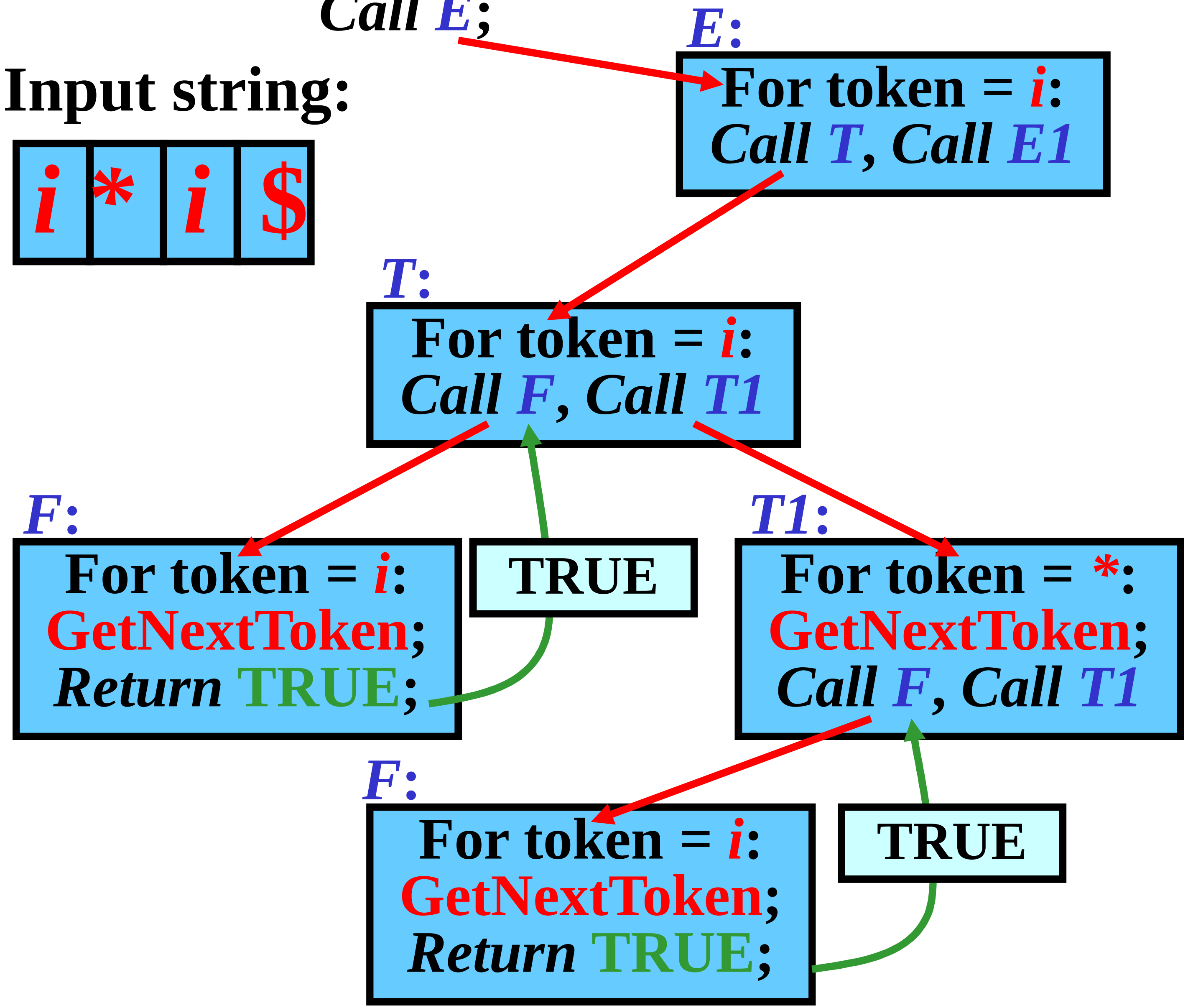
TRUE

***T1*:**
For token = *:
GetNextToken;
Call F*, *Call T1

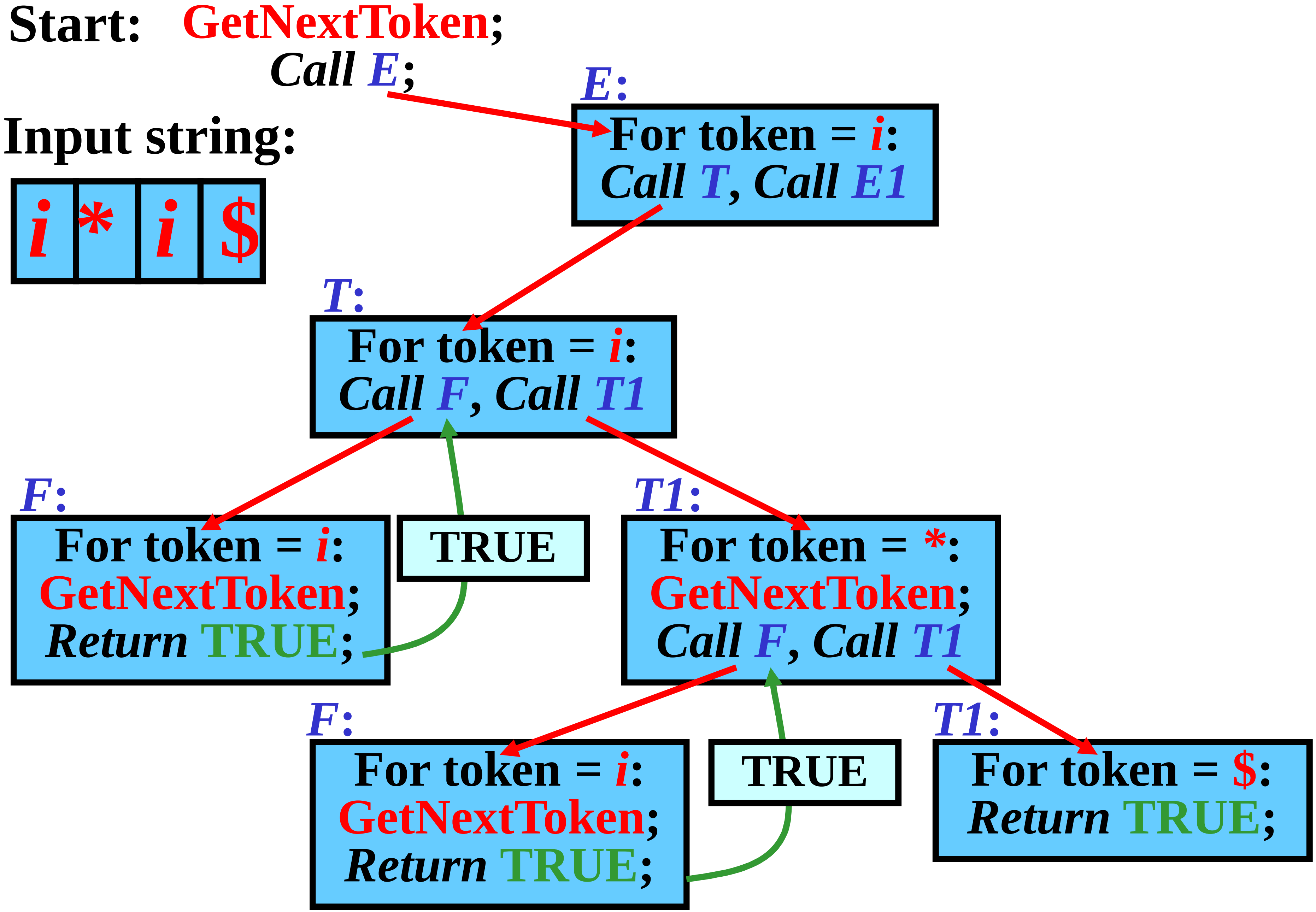
***F*:**
For token = *i*:
GetNextToken;
***Return* TRUE;**

Recursive Descent: Illustration for *i*i$*

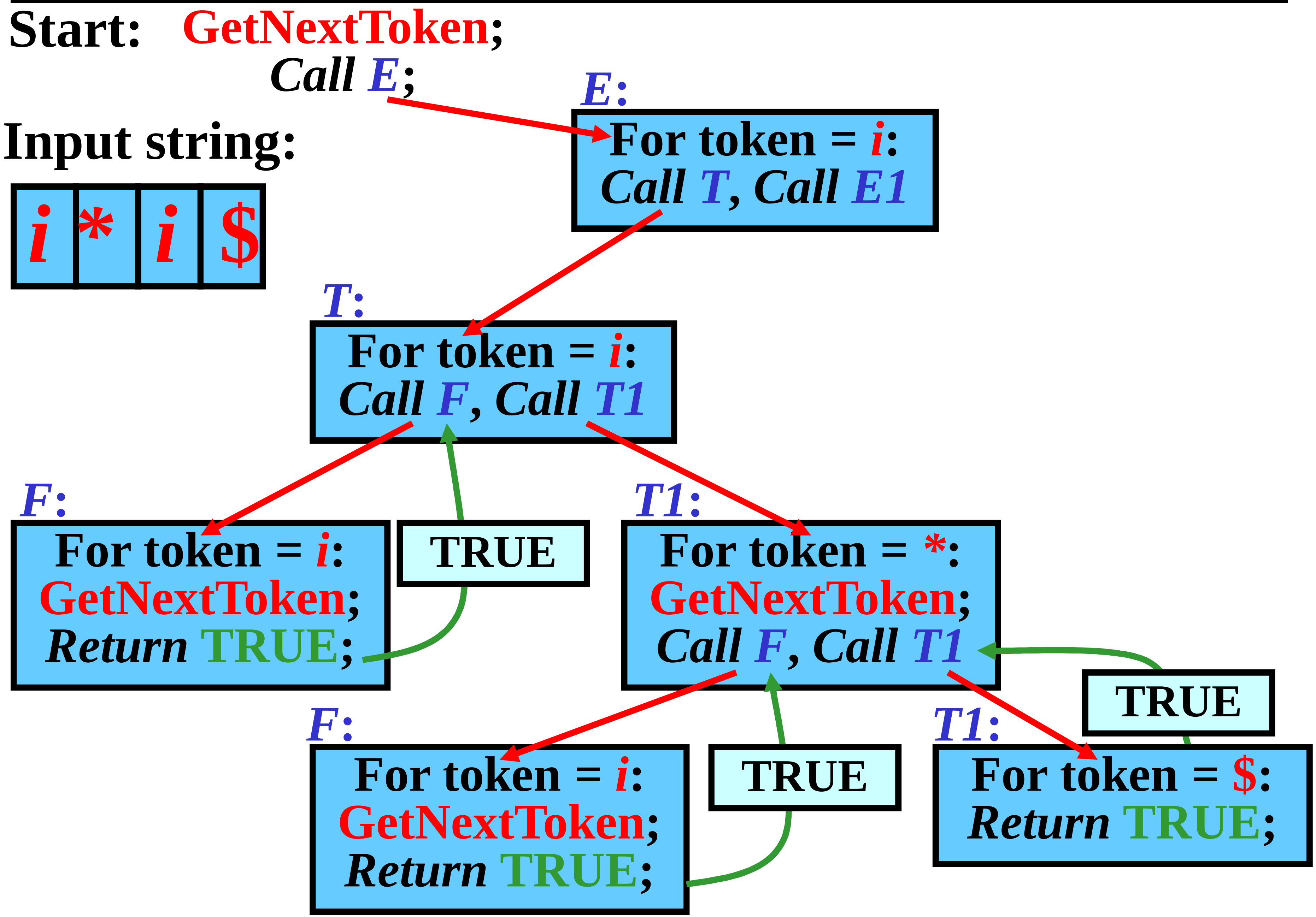
Start: **GetNextToken;**
Call E;



Recursive Descent: Illustration for *i*i*$

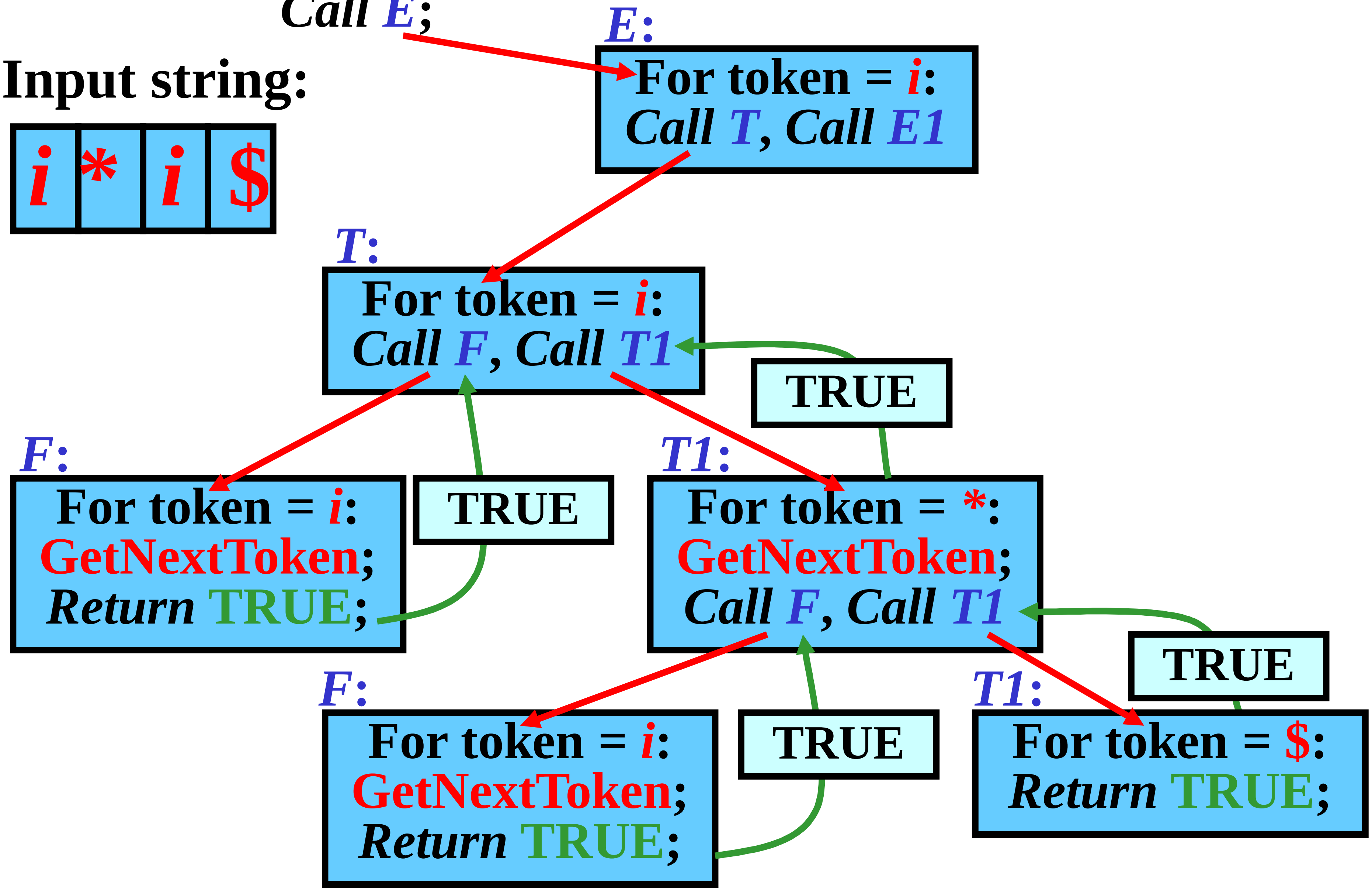


Recursive Descent: Illustration for $i*i\$$

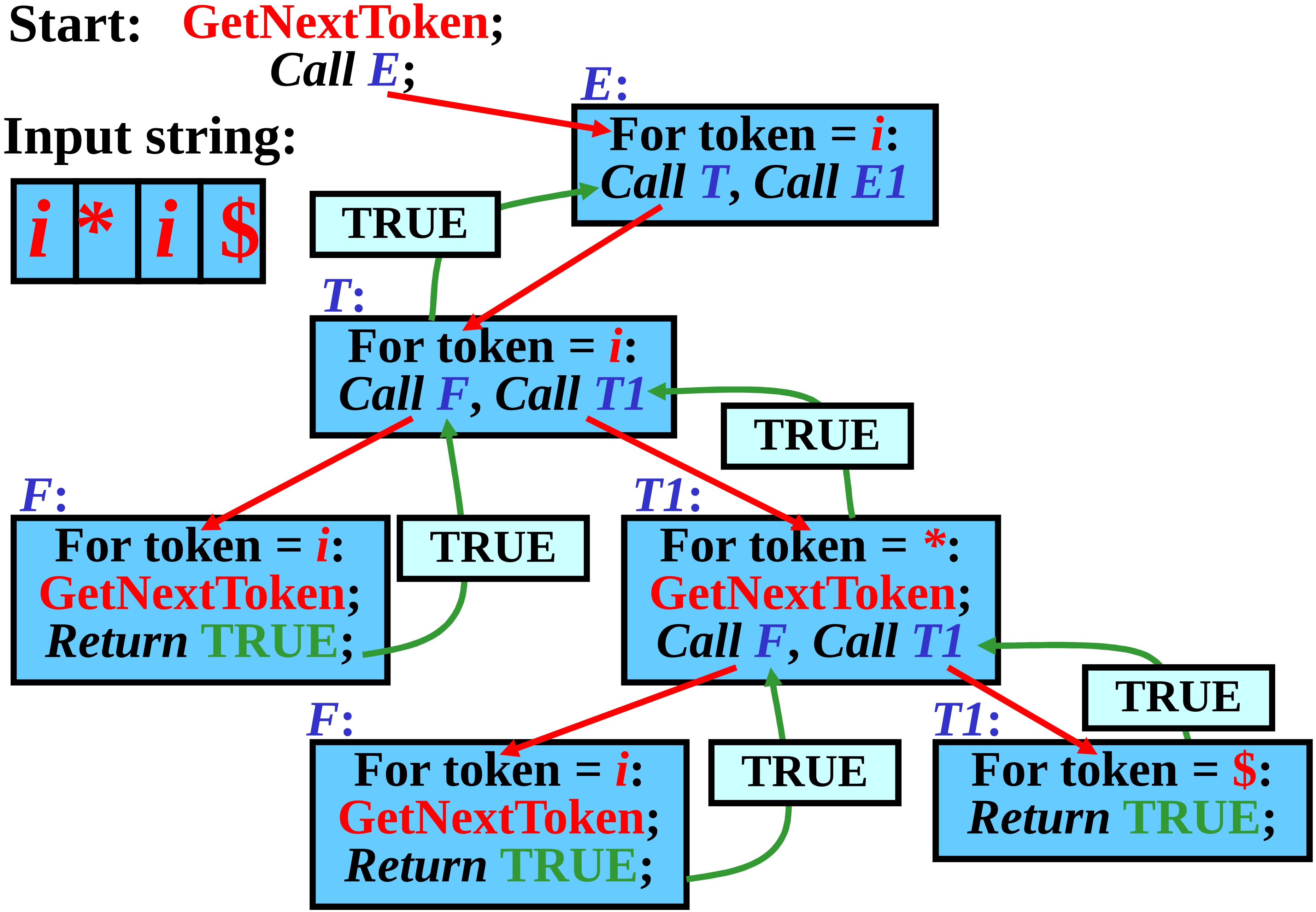


Recursive Descent: Illustration for *i***i*$

Start: **GetNextToken;**
Call E;



Recursive Descent: Illustration for *i***i*$

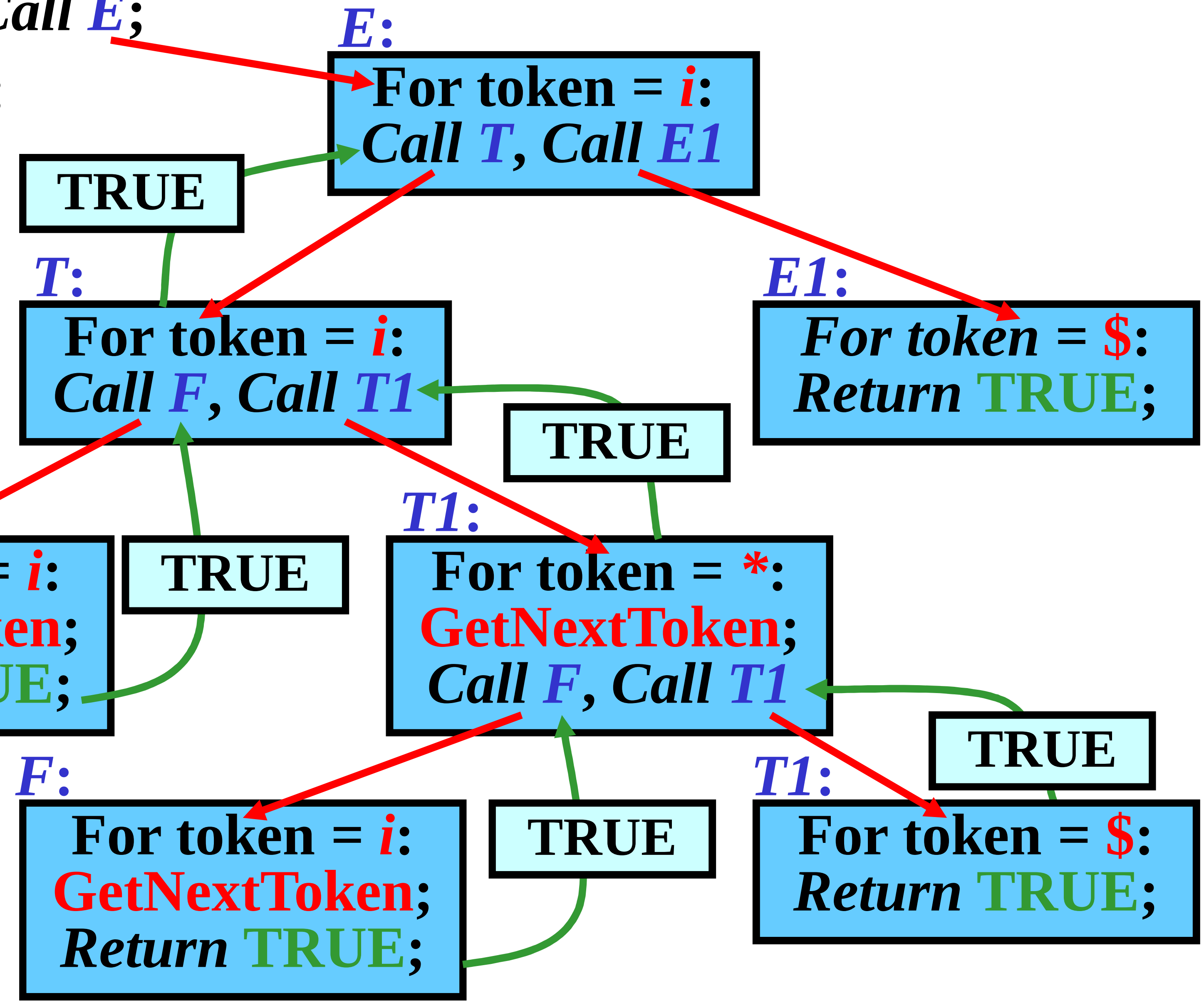


Recursive Descent: Illustration for *i*i$*

Start: **GetNextToken**;
Call E;

Input string:

i	*	i	$

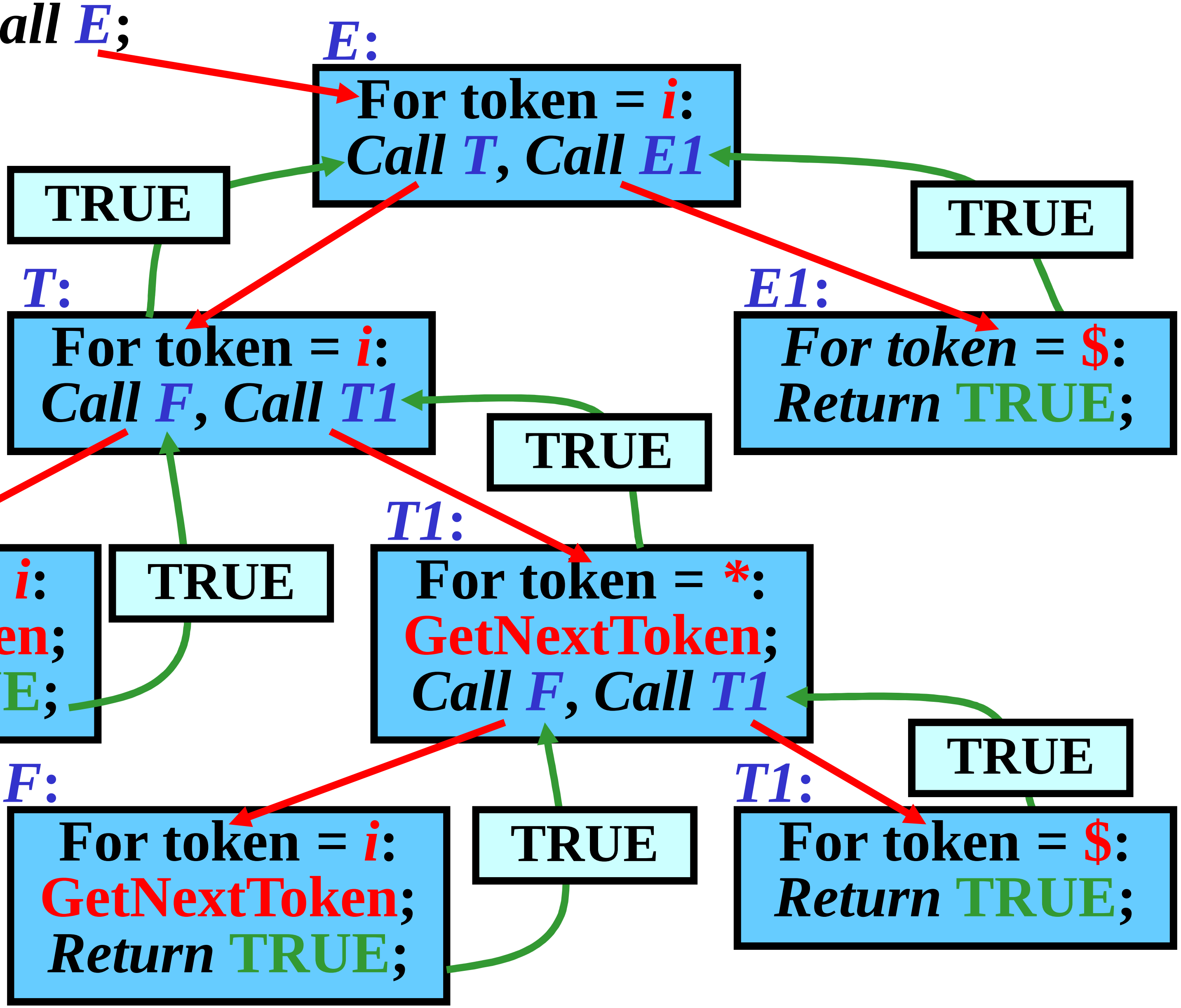


Recursive Descent: Illustration for *i***i*$

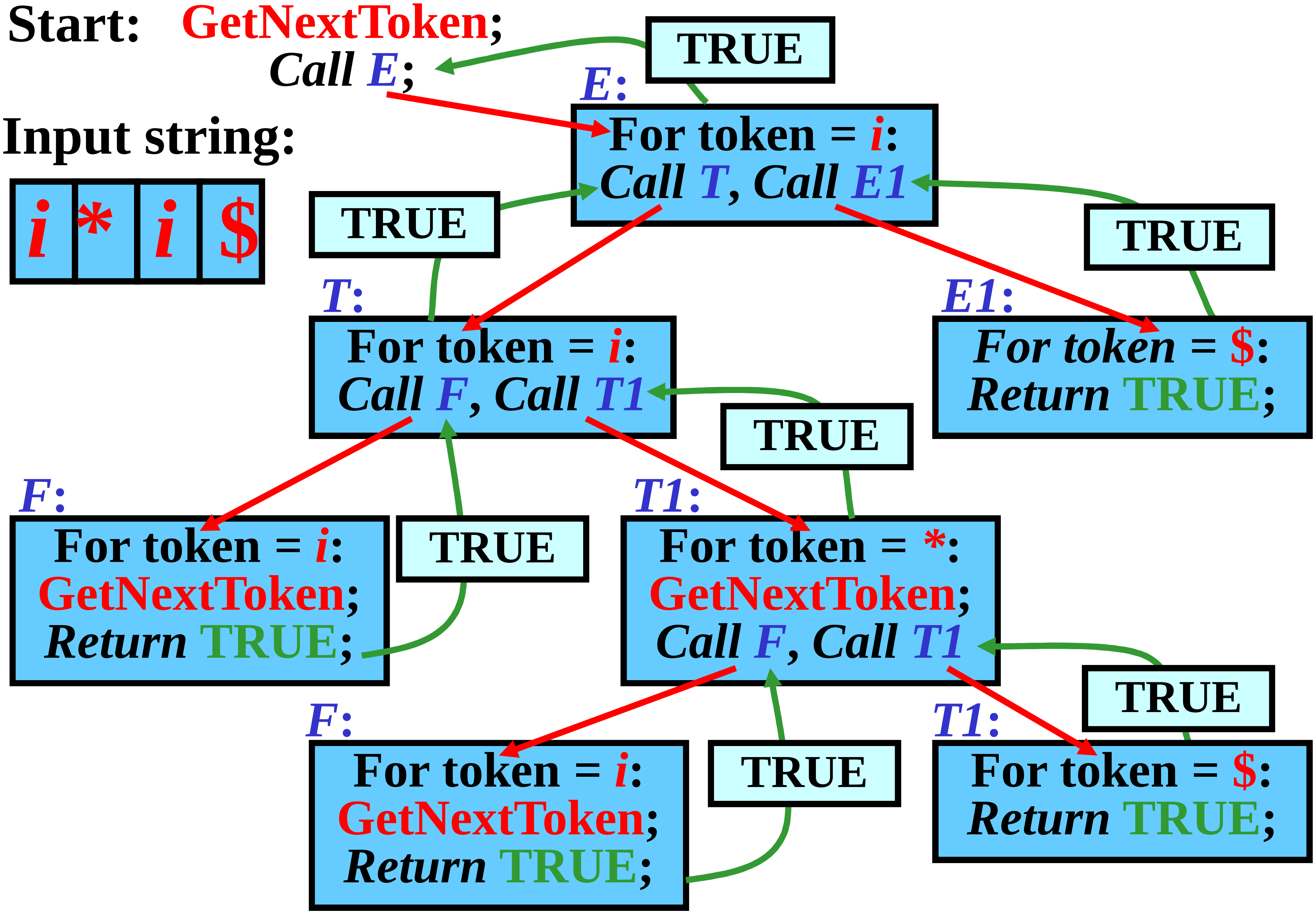
Start: **GetNextToken;**
Call E;

Input string:

i	*	*i*	$

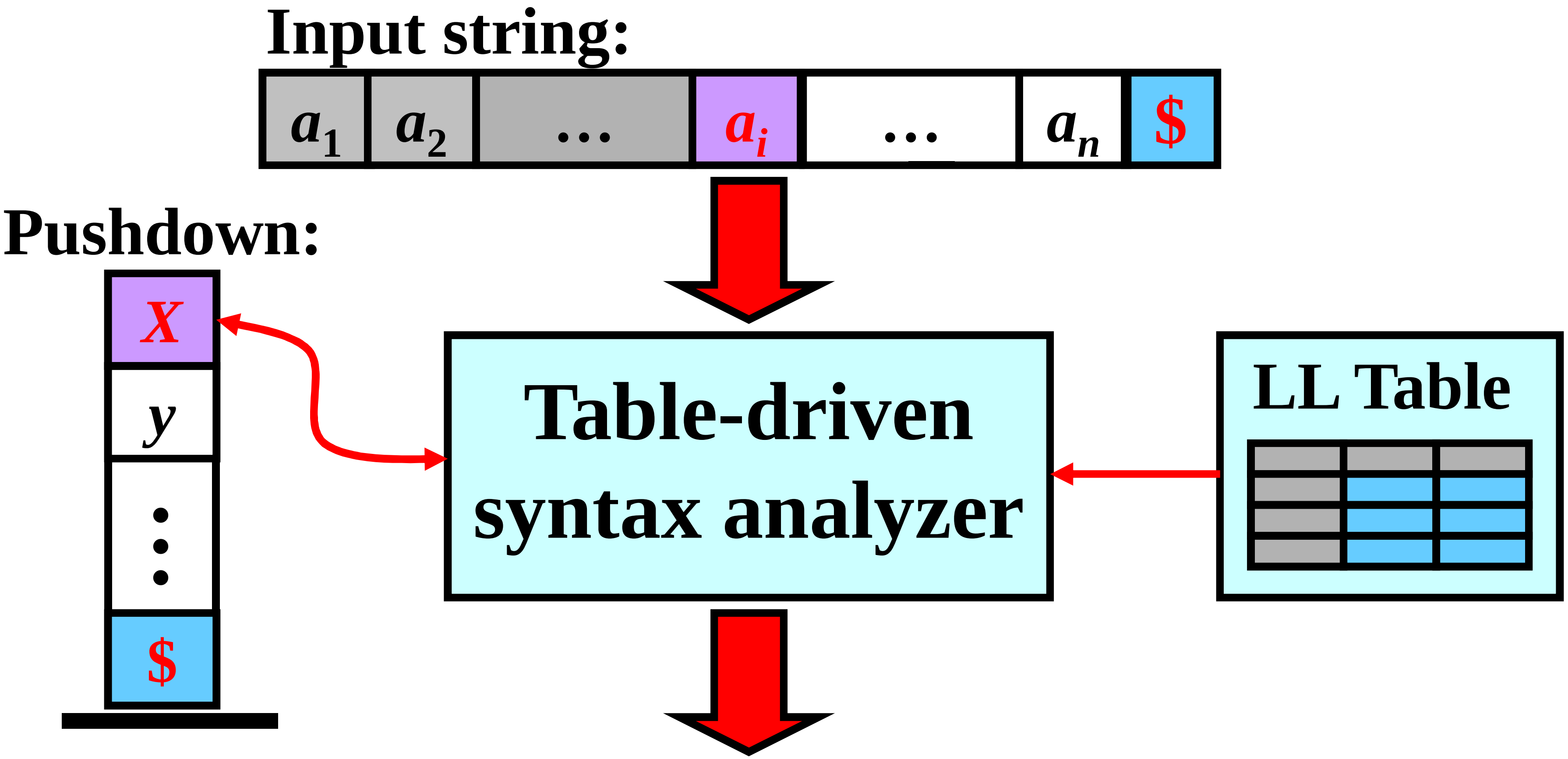


Recursive Descent: Illustration for *i***i*$



Predictive Parsing

- **Model of table-driven syntax analyzer:**



Left parse = sequence of rules used in the leftmost derivation of the input string.

Table-Driven Parsing: Algorithm

- **Input:** LL-table for $G = (N, T, P, S)$; $x \in T^*$
- **Output:** Left parse of x if $x \in L(G)$; otherwise, error

---

- **Method:**
- push(**\$**) & push(S) onto the pushdown;
- **while** the pushdown is not empty **do**
  - let X = the pushdown top and a = the current token
  - **case** X **of:**
    - X = **\$**: **if** a = **\$** **then** **success** **else** **error**;
    - $X \in T$: **if** $X = a$ **then** pop(X) & read next a from input string **else** **error**;
    - $X \in N$: **if** r: $X \to x \in$ LL-table[X, a] **then** replace X with reversal(x) on the pushdown & write r to output **else** **error**;

end

Table-Driven Parsing: Example

	i	+	*	(	)	$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

Rules:

1: $E \rightarrow TE'$

2: $E' \rightarrow +TE'$

3: $E' \rightarrow \varepsilon$

4: $T \rightarrow FT'$

5: $T' \rightarrow *FT'$

6: $T' \rightarrow \varepsilon$

7: $F \rightarrow (E)$

8: $F \rightarrow i$

Input string: *i* * *i* $

Pushdown	Input	Rule	Derivation

Table-Driven Parsing: Example

	i	+	*	(	)	$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

Input string: *i* * *i* $

Pushdown	Input	Rule	Derivation
$*E*	*i***i*$	1: $E \rightarrow TE'$	$\underline{E} \Rightarrow \underline{T}E'$

Rules:

1: $E \rightarrow TE'$

2: $E' \rightarrow +TE'$

3: $E' \rightarrow \varepsilon$

4: $T \rightarrow FT'$

5: $T' \rightarrow *FT'$

6: $T' \rightarrow \varepsilon$

7: $F \rightarrow (E)$

8: $F \rightarrow i$

Table-Driven Parsing: Example

	i	+	*	(	)	$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

Input string: *i* * *i* $

Pushdown	Input	Rule	Derivation
$\$E$	$i*i\$$	1: $E \rightarrow TE'$	$\underline{E} \Rightarrow \underline{T}E'$
$\$E'T$	$i*i\$$	4: $T \rightarrow FT'$	$\Rightarrow \underline{F}T'E'$

Rules:

1: $E \rightarrow TE'$

2: $E' \rightarrow +TE'$

3: $E' \rightarrow \varepsilon$

4: $T \rightarrow FT'$

5: $T' \rightarrow *FT'$

6: $T' \rightarrow \varepsilon$

7: $F \rightarrow (E)$

8: $F \rightarrow i$

Table-Driven Parsing: Example

	i	+	*	(	)	$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

Input string: *i* * *i* $

Pushdown	Input	Rule	Derivation
$*E*	*i**i*$	1: $E \rightarrow TE'$	$\underline{E} \Rightarrow \underline{T}E'$
$*E'T*	*i**i*$	4: $T \rightarrow FT'$	$\Rightarrow \underline{F}T'E'$
$*E'T'F*	*i**i*$	8: $F \rightarrow i$	$\Rightarrow i\underline{T'}E'$

Rules:

1: $E \rightarrow TE'$

2: $E' \rightarrow +TE'$

3: $E' \rightarrow \varepsilon$

4: $T \rightarrow FT'$

5: $T' \rightarrow *FT'$

6: $T' \rightarrow \varepsilon$

7: $F \rightarrow (E)$

8: $F \rightarrow i$

Table-Driven Parsing: Example

	i	+	*	(	)	$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

Input string: ***i* * *i* $**

Pushdown	Input	Rule	Derivation
$*E*	*i***i*$	1: $E \rightarrow TE'$	$\underline{E} \Rightarrow \underline{T}E'$
$*E'T*	*i***i*$	4: $T \rightarrow FT'$	$\Rightarrow \underline{F}T'E'$
$*E'T'F*	*i***i*$	8: $F \rightarrow i$	$\Rightarrow i\underline{T'}E'$
$*E'T'i*	*i***i*$		

Rules:

1: $E \rightarrow TE'$

2: $E' \rightarrow +TE'$

3: $E' \rightarrow \varepsilon$

4: $T \rightarrow FT'$

5: $T' \rightarrow *FT'$

6: $T' \rightarrow \varepsilon$

7: $F \rightarrow (E)$

8: $F \rightarrow i$

Table-Driven Parsing: Example

	i	+	*	(	)	$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
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Rules:

1: $E \to TE'$
2: $E' \to +TE'$
3: $E' \to \varepsilon$
4: $T \to FT'$
5: $T' \to *FT'$
6: $T' \to \varepsilon$
7: $F \to (E)$
8: $F \to i$

Input string: *i* * *i* $

Pushdown	Input	Rule	Derivation
$\$E$	$i*i\$$	1: $E \to TE'$	$\underline{E} \Rightarrow \underline{T}E'$
$\$E'T$	$i*i\$$	4: $T \to FT'$	$\Rightarrow \underline{F}T'E'$
$\$E'T'F$	$i*i\$$	8: $F \to i$	$\Rightarrow i\underline{T'}E'$
$\$E'T'i$	$i*i\$$		
$\$E'T'$	$*i\$$	5: $T' \to *FT'$	$\Rightarrow i*\underline{F}T'E'$

Table-Driven Parsing: Example

	i	+	*	(	)	$
E	1			1		
E’		2			3	3
T	4			4		
T’		6	5		6	6
F	8			7		

Input string: i * i $

Pushdown	Input	Rule	Derivation
$E	i*i$	1: $E \rightarrow TE'$	$E \Rightarrow TE'$
$E’T	i*i$	4: $T \rightarrow FT'$	$\Rightarrow FT'E'$
$E’T’F	i*i$	8: $F \rightarrow i$	$\Rightarrow iT'E'$
$E’T’i	i*i$		
$E’T’	*i$	5: $T' \rightarrow *FT'$	$\Rightarrow i*FT'E'$
$E’T’F*	*i$		

Rules:

1: $E \rightarrow TE'$
2: $E' \rightarrow +TE'$
3: $E' \rightarrow \varepsilon$
4: $T \rightarrow FT'$
5: $T' \rightarrow *FT'$
6: $T' \rightarrow \varepsilon$
7: $F \rightarrow (E)$
8: $F \rightarrow i$

Table-Driven Parsing: Example

	i	+	*	(	)	$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

Input string: *i* * *i* $

Pushdown	Input	Rule	Derivation
$*E*	*i***i*$	1: *E* → *TE'*	*E* ⇒ *TE'*
$*E'T*	*i***i*$	4: *T* → *FT'*	⇒ *FT'E'*
$*E'T'F*	*i***i*$	8: *F* → *i*	⇒ *iT'E'*
$*E'T'i*	*i***i*$		
$*E'T'*	**i*$	5: *T'* → **FT'*	⇒ *i***FT'E'*
$*E'T'F**	**i*$		
$*E'T'F*	*i*$	8: *F* → *i*	⇒ *i***iT'E'*

Rules:

1: *E* → *TE'*
2: *E'* → +*TE'*
3: *E'* → ε
4: *T* → *FT'*
5: *T'* → **FT'*
6: *T'* → ε
7: *F* → (*E*)
8: *F* → *i*

Table-Driven Parsing: Example

	i	+	*	(	)	$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
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Rules:

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Input string: *i* * *i* $

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$\$E$	$i*i\$$	1: $E \rightarrow TE'$	$\underline{E} \Rightarrow \underline{T}E'$
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$\$E'T'i$	$i*i\$$		
$\$E'T'$	$*i\$$	5: $T' \rightarrow *FT'$	$\Rightarrow i*\underline{F}T'E'$
$\$E'T'F*$	$*i\$$		
$\$E'T'F$	$i\$$	8: $F \rightarrow i$	$\Rightarrow i*i\underline{T'}E'$
$\$E'T'i$	$i\$$		

Table-Driven Parsing: Example

	i	+	*	(	)	$
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Input string: *i * i $*

Pushdown	Input	Rule	Derivation
$\$E$	$i*i\$$	1: $E \rightarrow TE'$	$E \Rightarrow TE'$
$\$E'T$	$i*i\$$	4: $T \rightarrow FT'$	$\Rightarrow FT'E'$
$\$E'T'F$	$i*i\$$	8: $F \rightarrow i$	$\Rightarrow iT'E'$
$\$E'T'i$	$i*i\$$		
$\$E'T'$	$*i\$$	5: $T' \rightarrow *FT'$	$\Rightarrow i*FT'E'$
$\$E'T'F*$	$*i\$$		
$\$E'T'F$	$i\$$	8: $F \rightarrow i$	$\Rightarrow i*iT'E'$
$\$E'T'i$	$i\$$		
$\$E'T'$	$\$$	6: $T' \rightarrow \varepsilon$	$\Rightarrow i*iE'$

Rules:

1: $E \rightarrow TE'$

2: $E' \rightarrow +TE'$

3: $E' \rightarrow \varepsilon$

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6: $T' \rightarrow \varepsilon$

7: $F \rightarrow (E)$

8: $F \rightarrow i$

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	i	+	*	(	)	$
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Input string: $i * i \$$

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$\$E$	$i*i\$$	1: $E \rightarrow TE'$	$E \Rightarrow TE'$
$\$E'T$	$i*i\$$	4: $T \rightarrow FT'$	$\Rightarrow FT'E'$
$\$E'T'F$	$i*i\$$	8: $F \rightarrow i$	$\Rightarrow iT'E'$
$\$E'T'i$	$i*i\$$		
$\$E'T'$	$*i\$$	5: $T' \rightarrow *FT'$	$\Rightarrow i*FT'E'$
$\$E'T'F*$	$*i\$$		
$\$E'T'F$	$i\$$	8: $F \rightarrow i$	$\Rightarrow i*iT'E'$
$\$E'T'i$	$i\$$		
$\$E'T'$	$\$$	6: $T' \rightarrow \varepsilon$	$\Rightarrow i*iE'$
$\$E'$	$\$$	3: $E' \rightarrow \varepsilon$	$\Rightarrow i*i$

Rules:

1: $E \rightarrow TE'$
2: $E' \rightarrow +TE'$
3: $E' \rightarrow \varepsilon$
4: $T \rightarrow FT'$
5: $T' \rightarrow *FT'$
6: $T' \rightarrow \varepsilon$
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Input string: *i* * *i* $

Pushdown	Input	Rule	Derivation
$*E*	*i***i*$	1: $E \rightarrow TE'$	$E \Rightarrow TE'$
$*E'T*	*i***i*$	4: $T \rightarrow FT'$	$\Rightarrow FT'E'$
$*E'T'F*	*i***i*$	8: $F \rightarrow i$	$\Rightarrow iT'E'$
$*E'T'i*	*i***i*$		
$*E'T'*	**i*$	5: $T' \rightarrow *FT'$	$\Rightarrow i*FT'E'$
$*E'T'F**	**i*$		
$*E'T'F*	*i*$	8: $F \rightarrow i$	$\Rightarrow i*iT'E'$
$*E'T'i*	*i*$		
$*E'T'*	$	6: $T' \rightarrow \varepsilon$	$\Rightarrow i*iE'$
$*E'*	$	3: $E' \rightarrow \varepsilon$	$\Rightarrow i*i$
$	$		

Rules:

1: $E \rightarrow TE'$
2: $E' \rightarrow +TE'$
3: $E' \rightarrow \varepsilon$
4: $T \rightarrow FT'$
5: $T' \rightarrow *FT'$
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Table-Driven Parsing: Example

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E'		2			3	3
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Input string: *i* * *i* $

Rules:

1: $E \to TE'$
2: $E' \to +TE'$
3: $E' \to \varepsilon$
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5: $T' \to *FT'$
6: $T' \to \varepsilon$
7: $F \to (E)$
8: $F \to i$

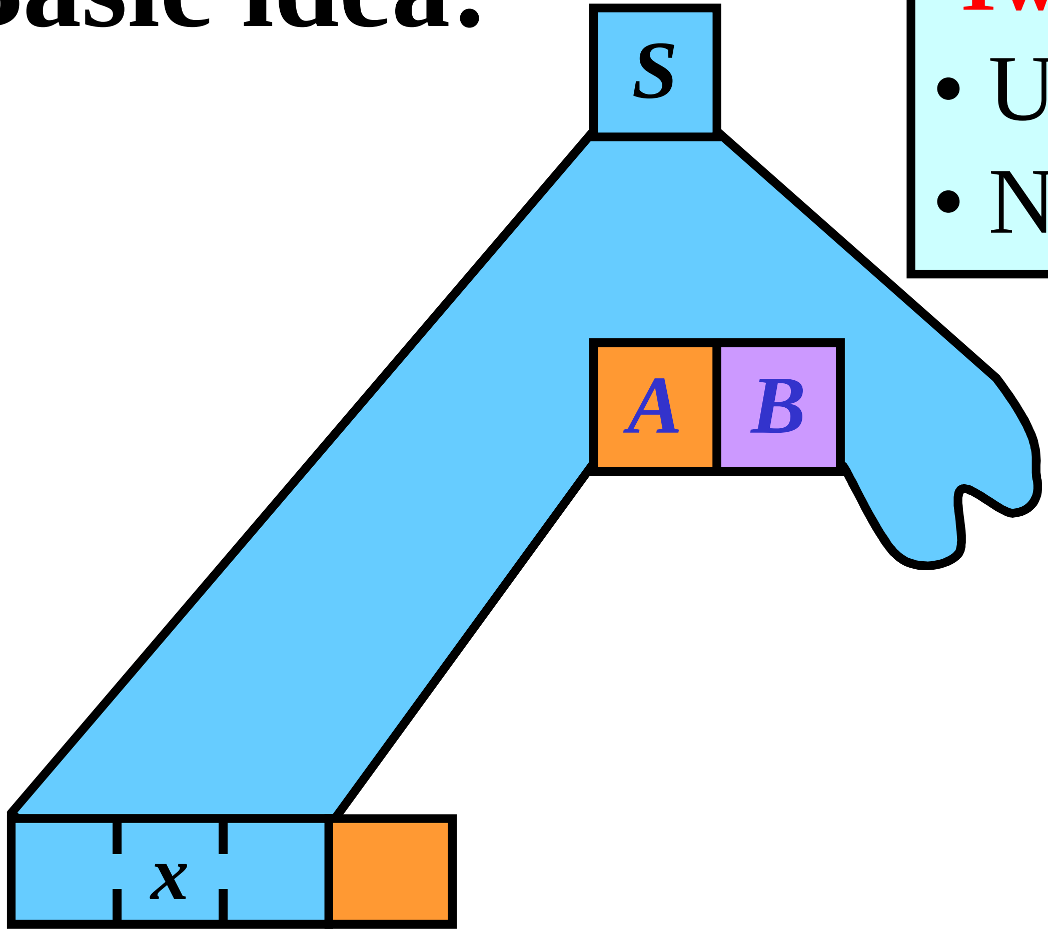
Pushdown	Input	Rule	Derivation
$\$E$	$i*i\$$	1: $E \to TE'$	$E \Rightarrow TE'$
$\$E'T$	$i*i\$$	4: $T \to FT'$	$\Rightarrow FT'E'$
$\$E'T'F$	$i*i\$$	8: $F \to i$	$\Rightarrow iT'E'$
$\$E'T'i$	$i*i\$$		
$\$E'T'$	$*i\$$	5: $T' \to *FT'$	$\Rightarrow i*FT'E'$
$\$E'T'F*$	$*i\$$		
$\$E'T'F$	$i\$$	8: $F \to i$	$\Rightarrow i*iT'E'$
$\$E'T'i$	$i\$$		
$\$E'T'$	$\$$	6: $T' \to \varepsilon$	$\Rightarrow i*iE'$
$\$E'$	$\$$	3: $E' \to \varepsilon$	$\Rightarrow i*i$
$\$$	$\$$		

Success

Left parse: 1485863

Handling Errors: Introduction

Basic idea:

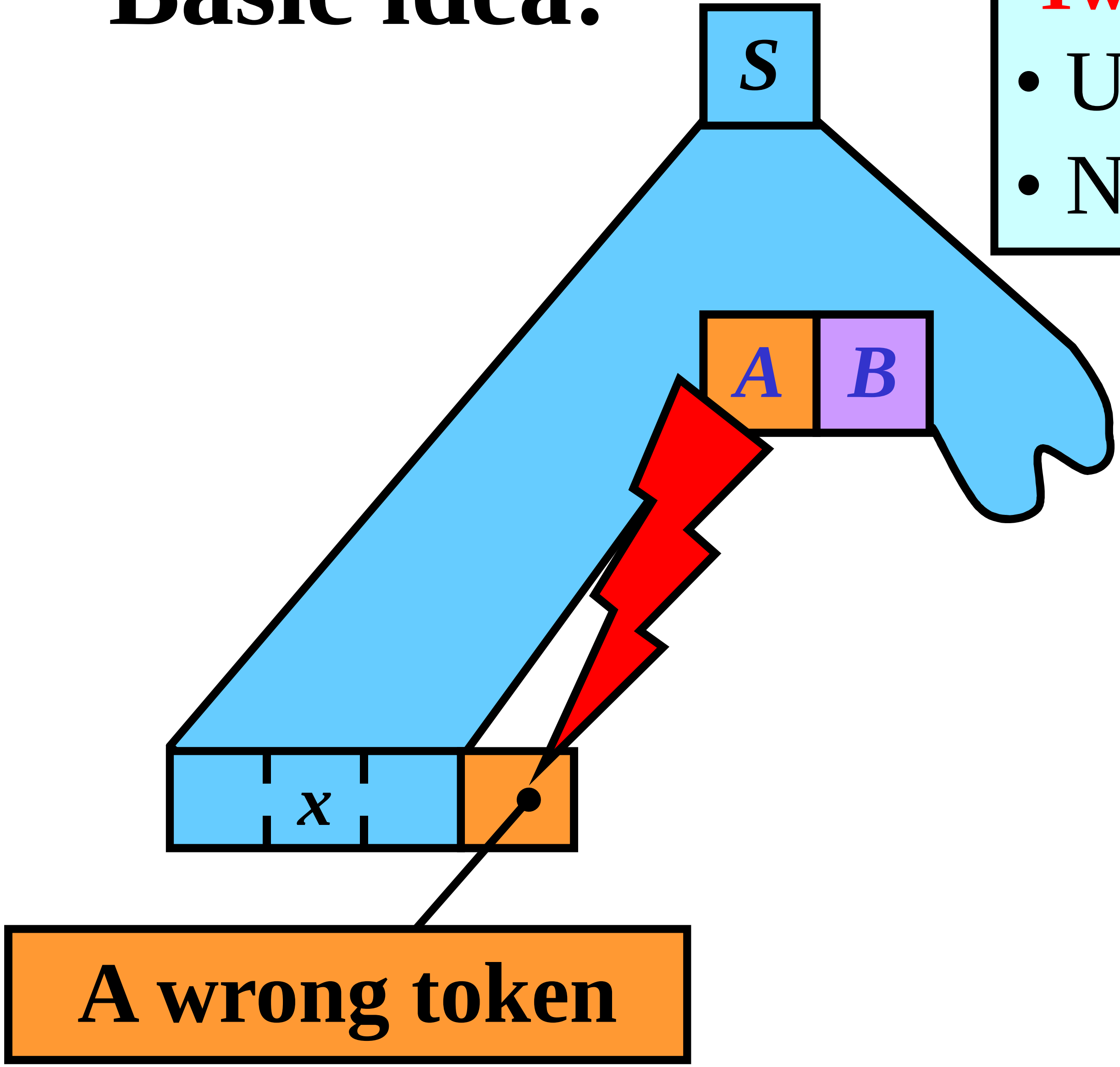


Two kinds of errors:

- Unexpected token
- No rule applicable

Handling Errors: Introduction

Basic idea:



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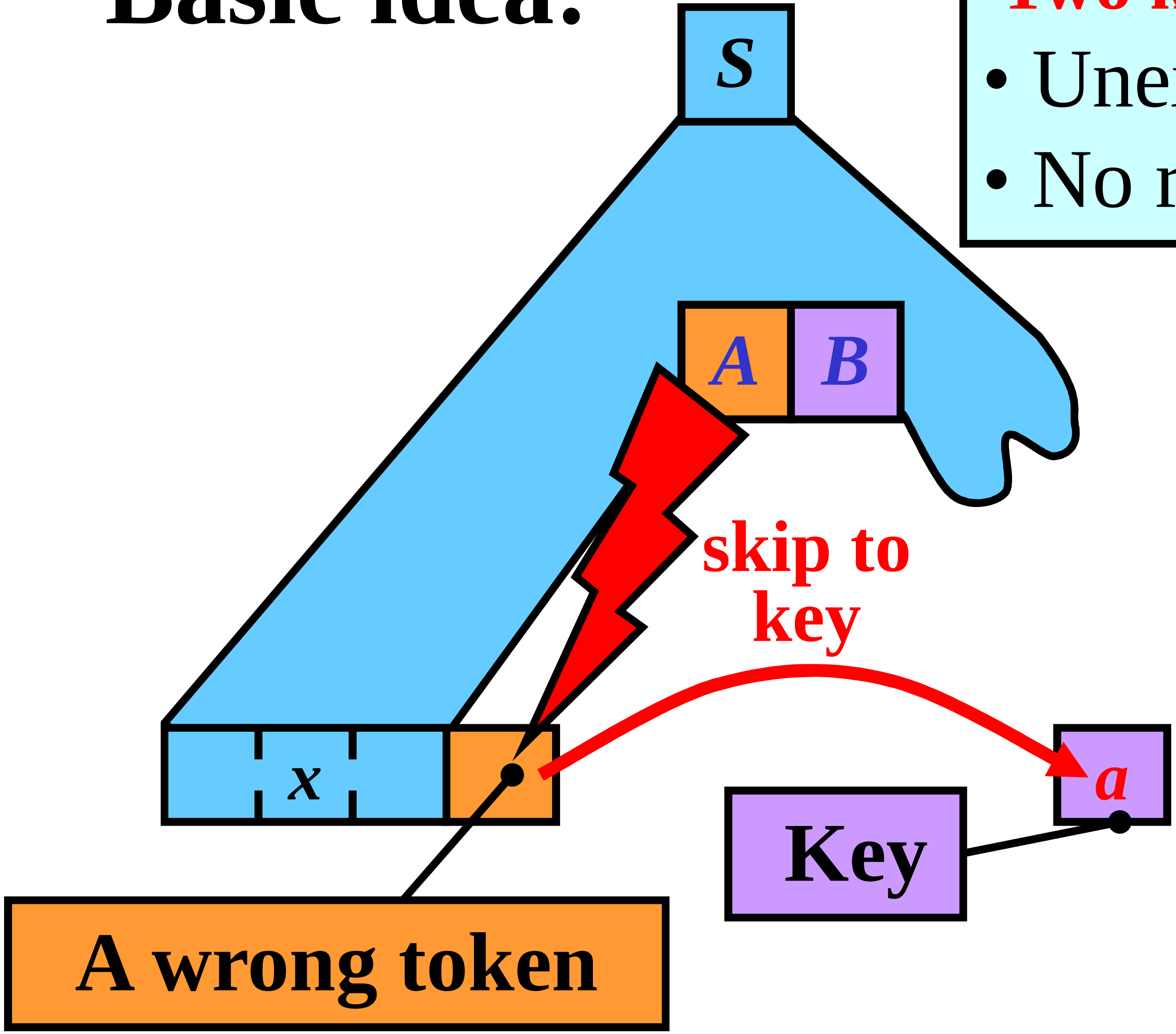
Handling Errors: Introduction

Basic idea:



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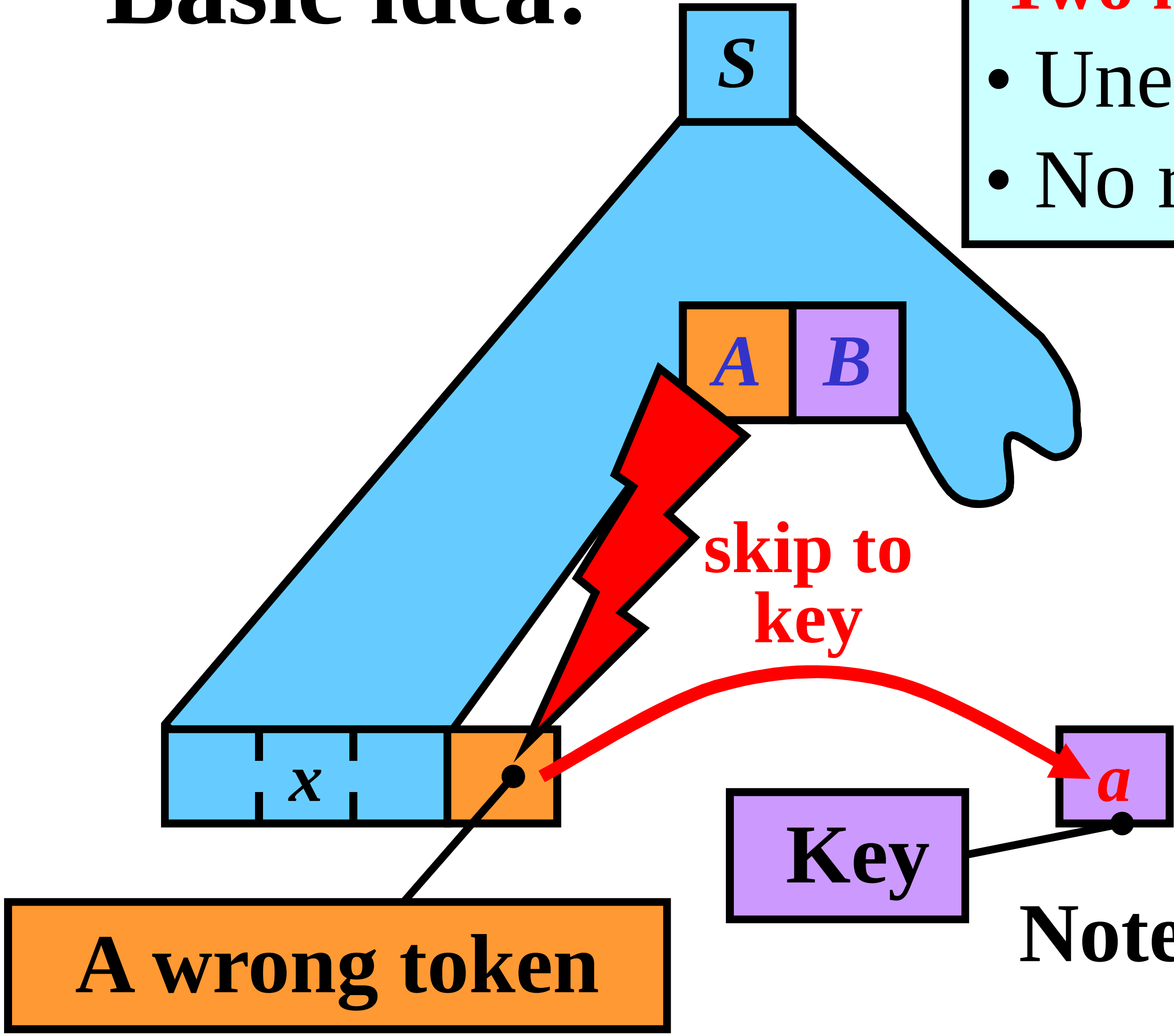
Handling Errors: Introduction

Basic idea:



Two kinds of errors:

- Unexpected token
- No rule applicable

Note: $a \in Follow(A)$

Handling Errors: Introduction

Basic idea:

Two kinds of errors:

- Unexpected token
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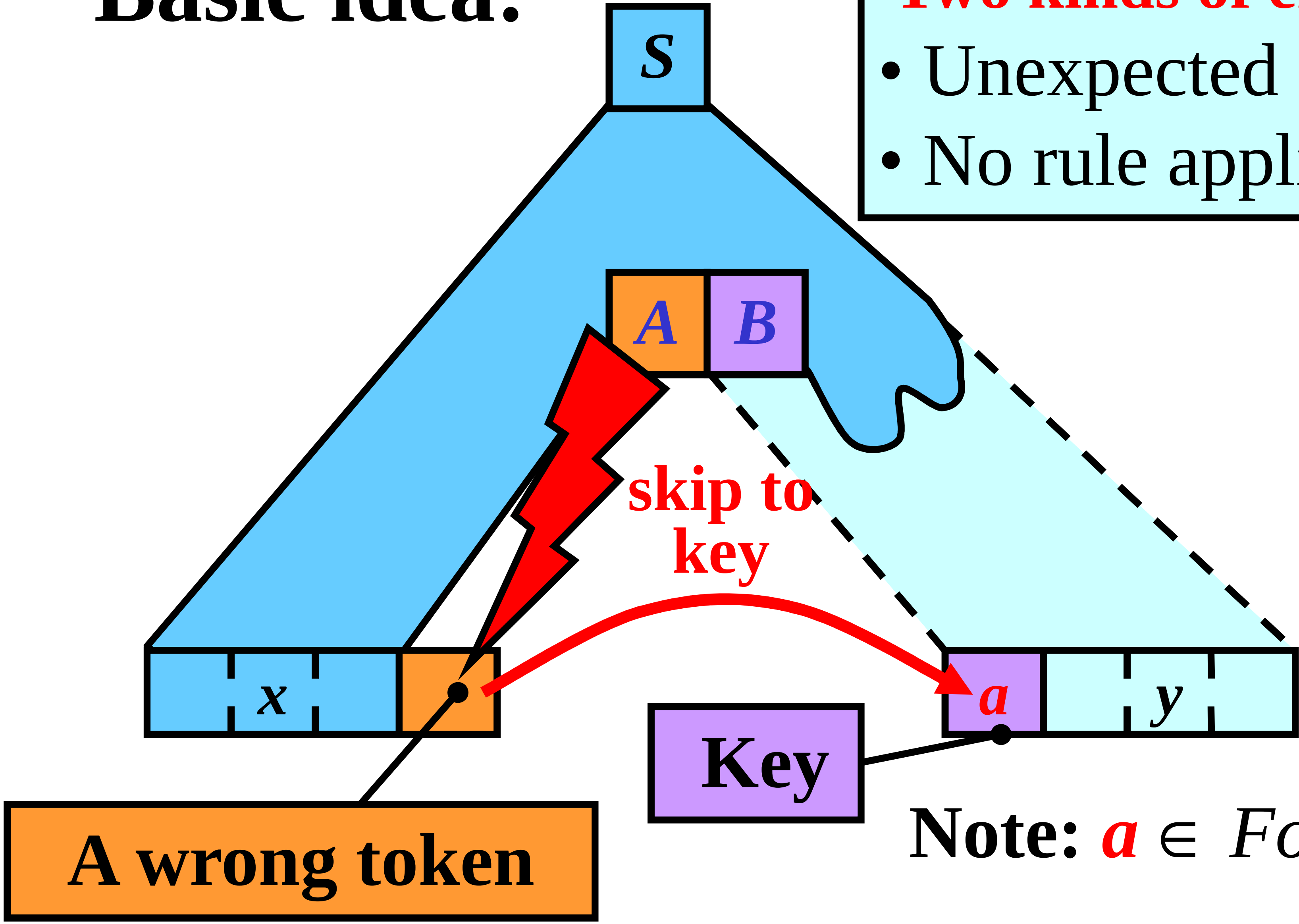
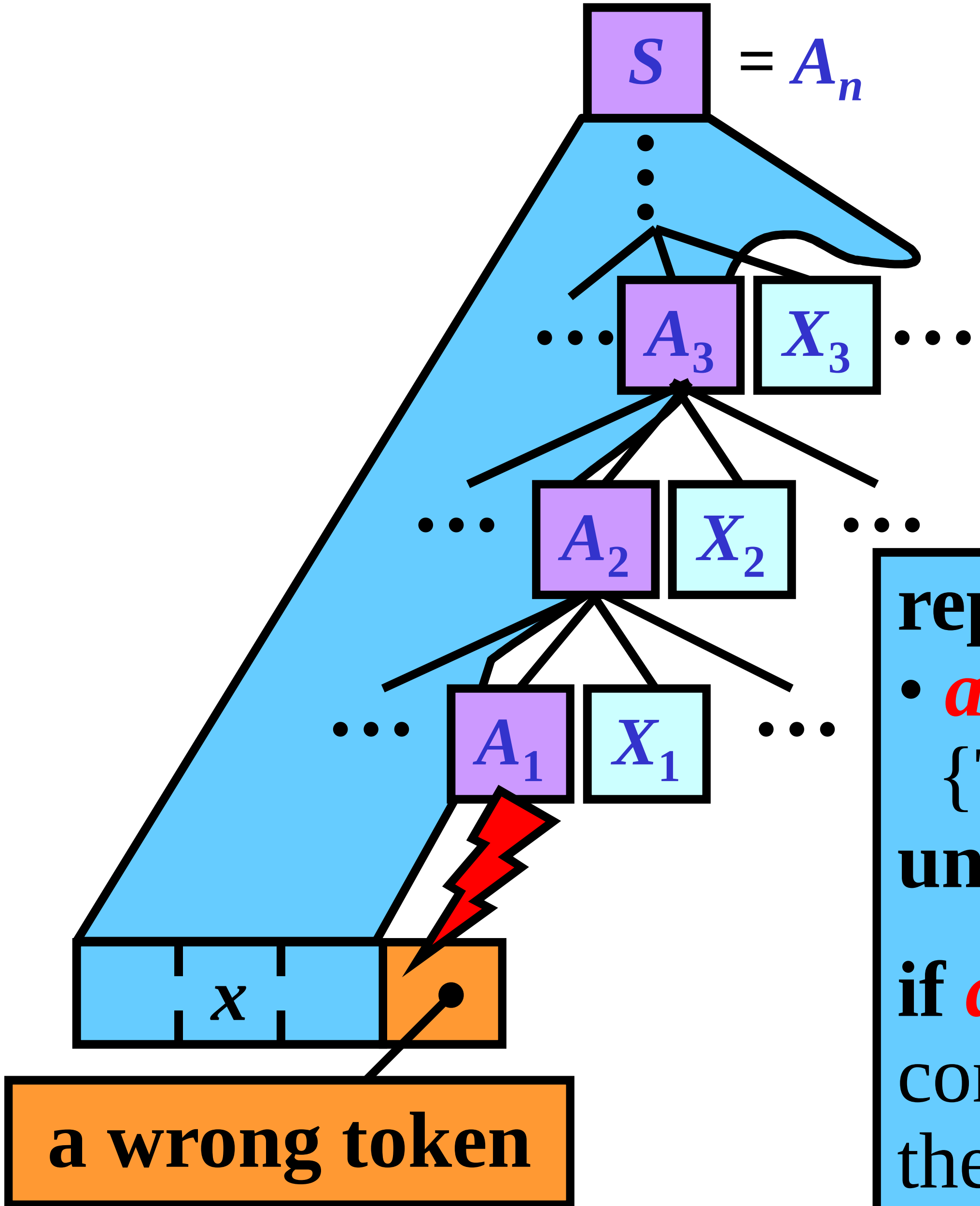


Note: $a \in Follow(A)$

Panic-Mode (Hartmann) Error Recovery



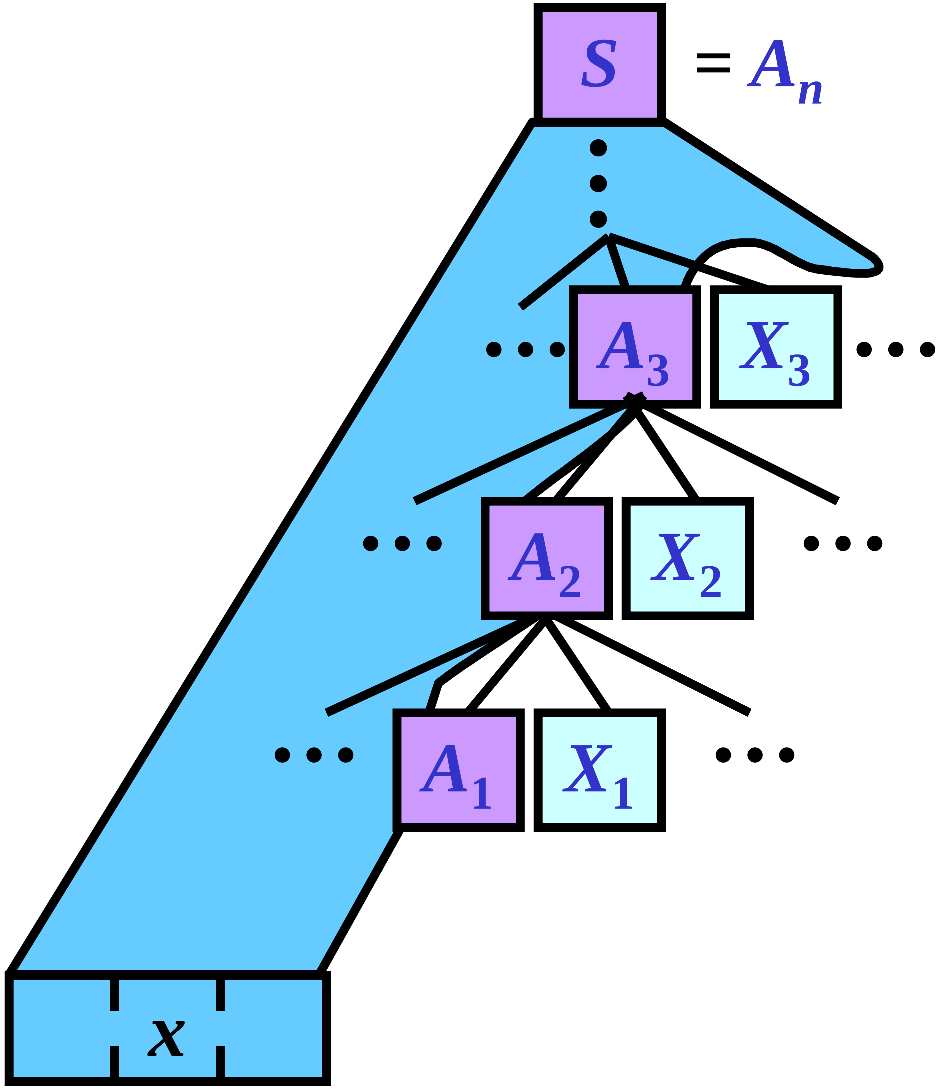
- Let ***Context***(A_1) =
  $\textbf{\textit{Follow}}(A_1) \cup$
  $\textbf{\textit{Follow}}(A_2) \cup$
  $\ldots$
  $\textbf{\textit{Follow}}(A_n)$

repeat
- a := GetNextToken;
  {These tokens are skipped}

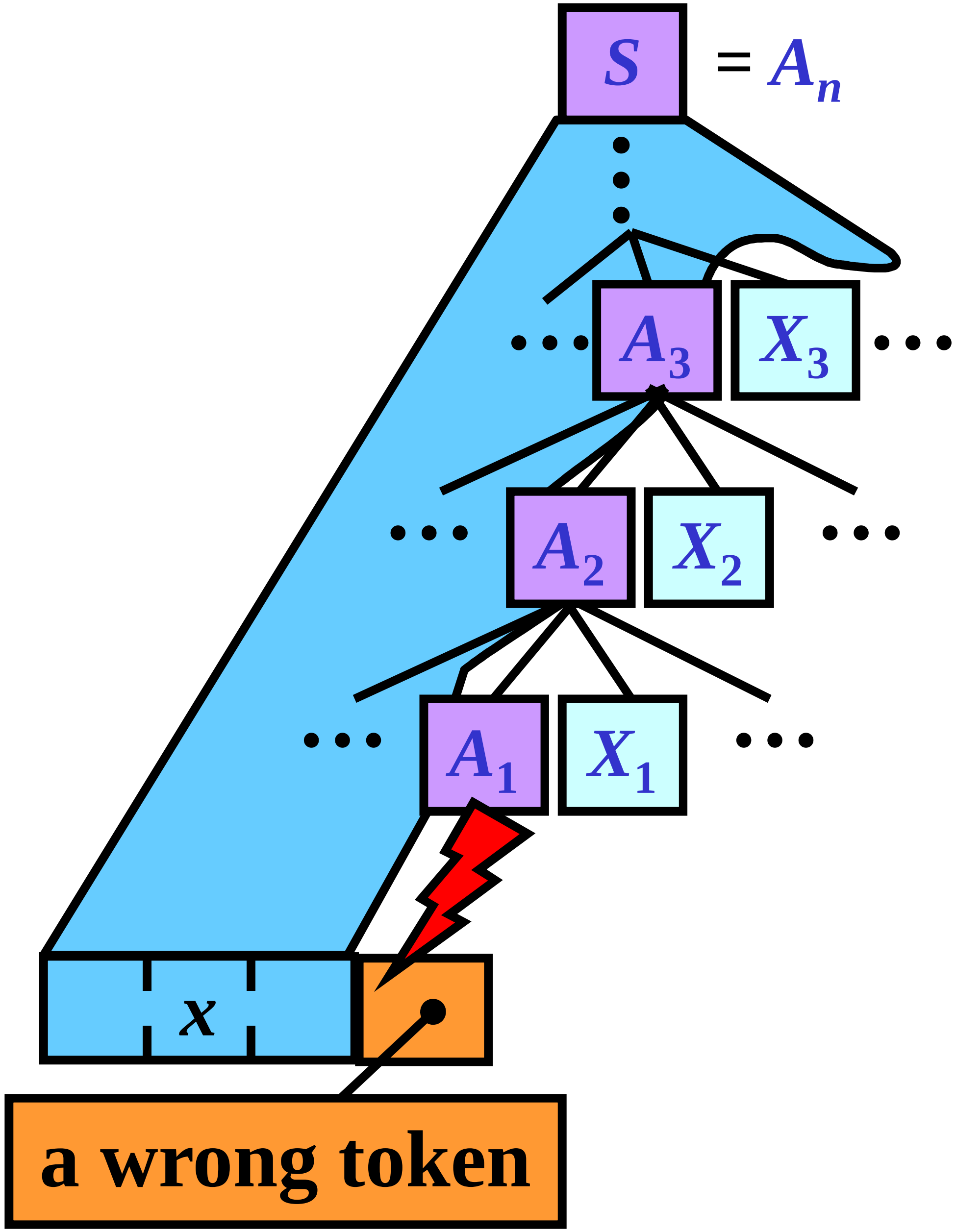
until a in ***Context***(A_1)

if a in ***Follow***(A_i) **then** continue with parsing from the symbol X_i.

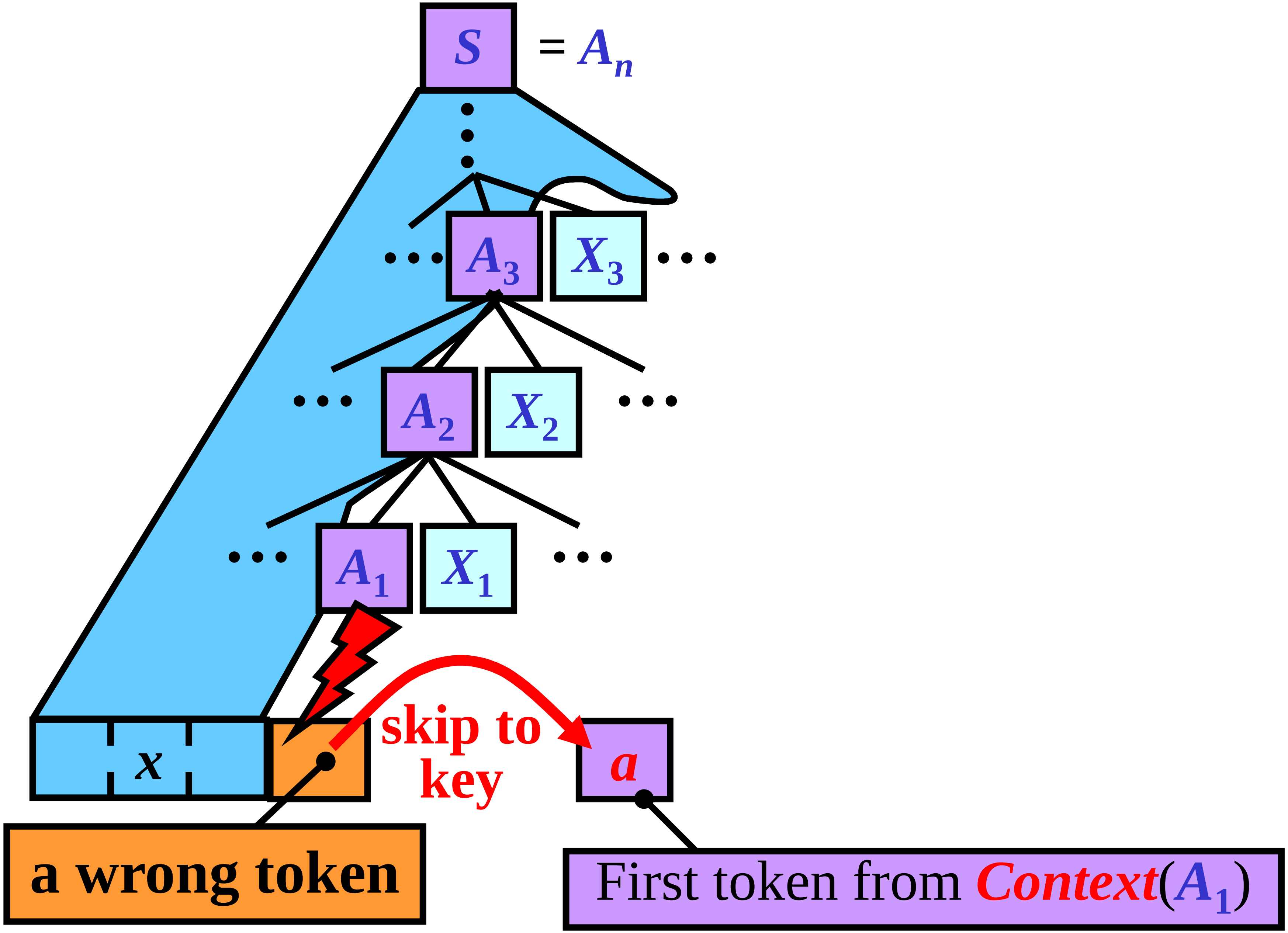
Panic-Mode Recovery: Illustration 1/2



Panic-Mode Recovery: Illustration 1/2

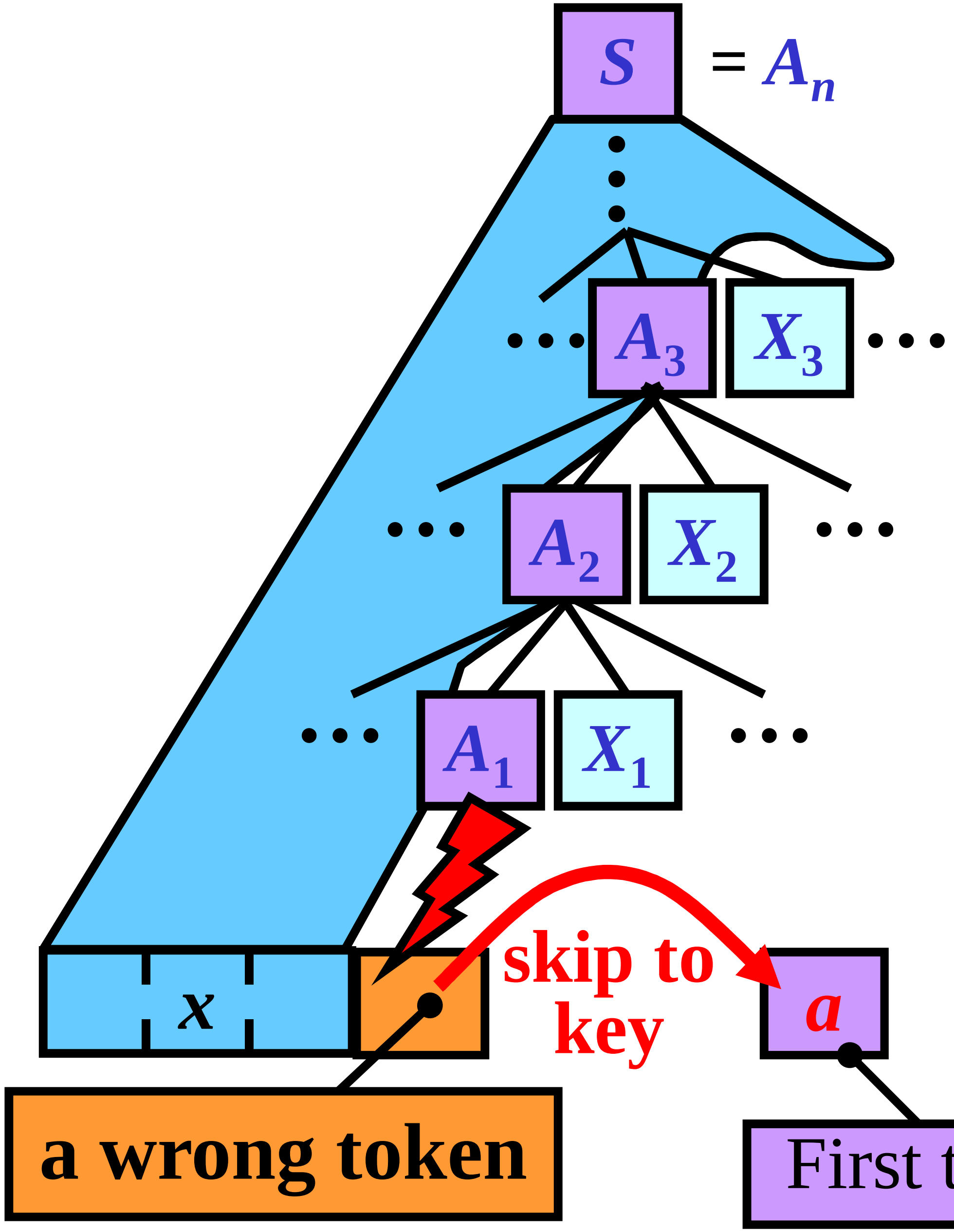


Panic-Mode Recovery: Illustration 1/2

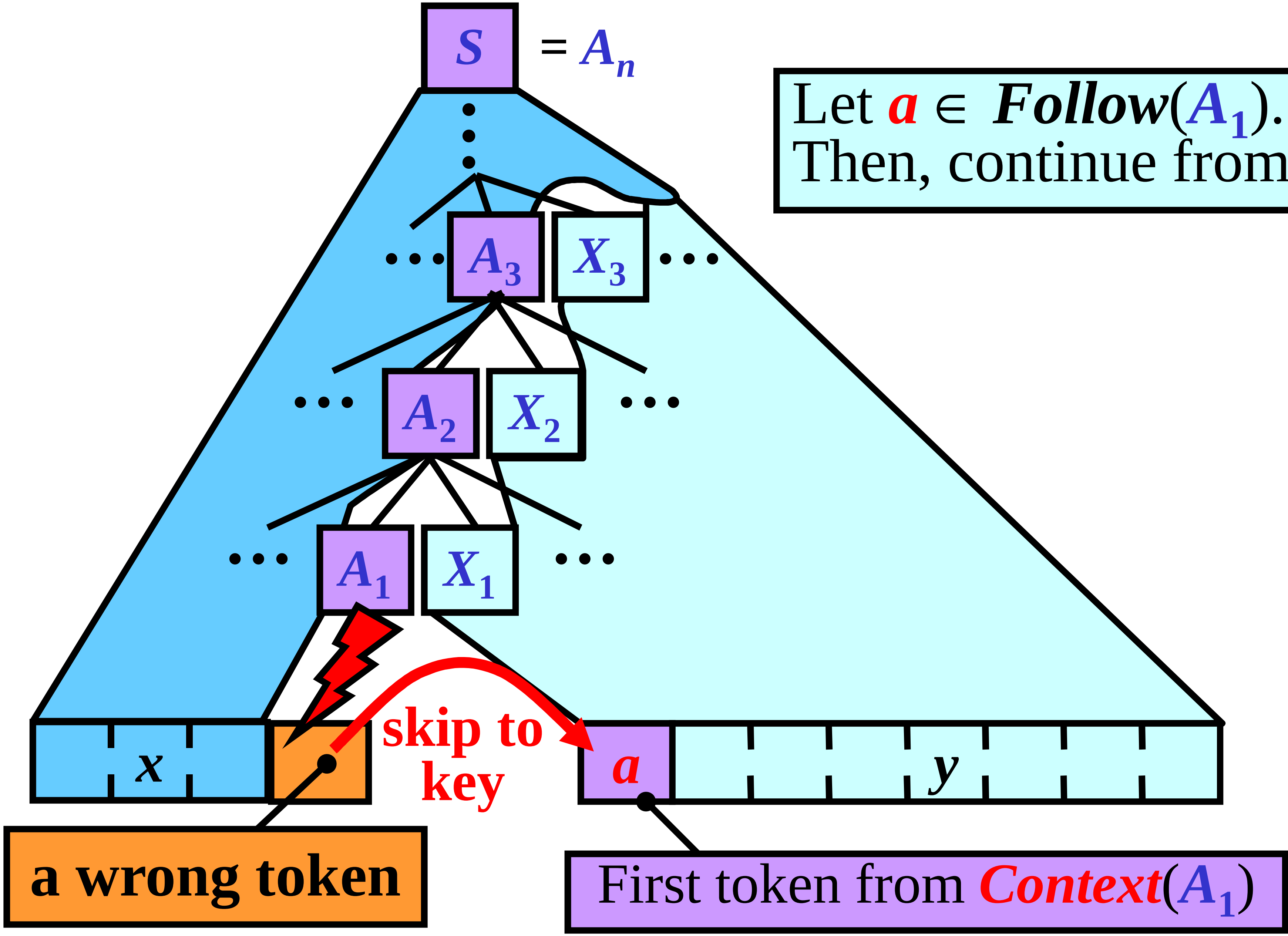


Panic-Mode Recovery: Illustration 1/2

Let $a \in$ ***Follow***(A_1).
Then, continue from X_1

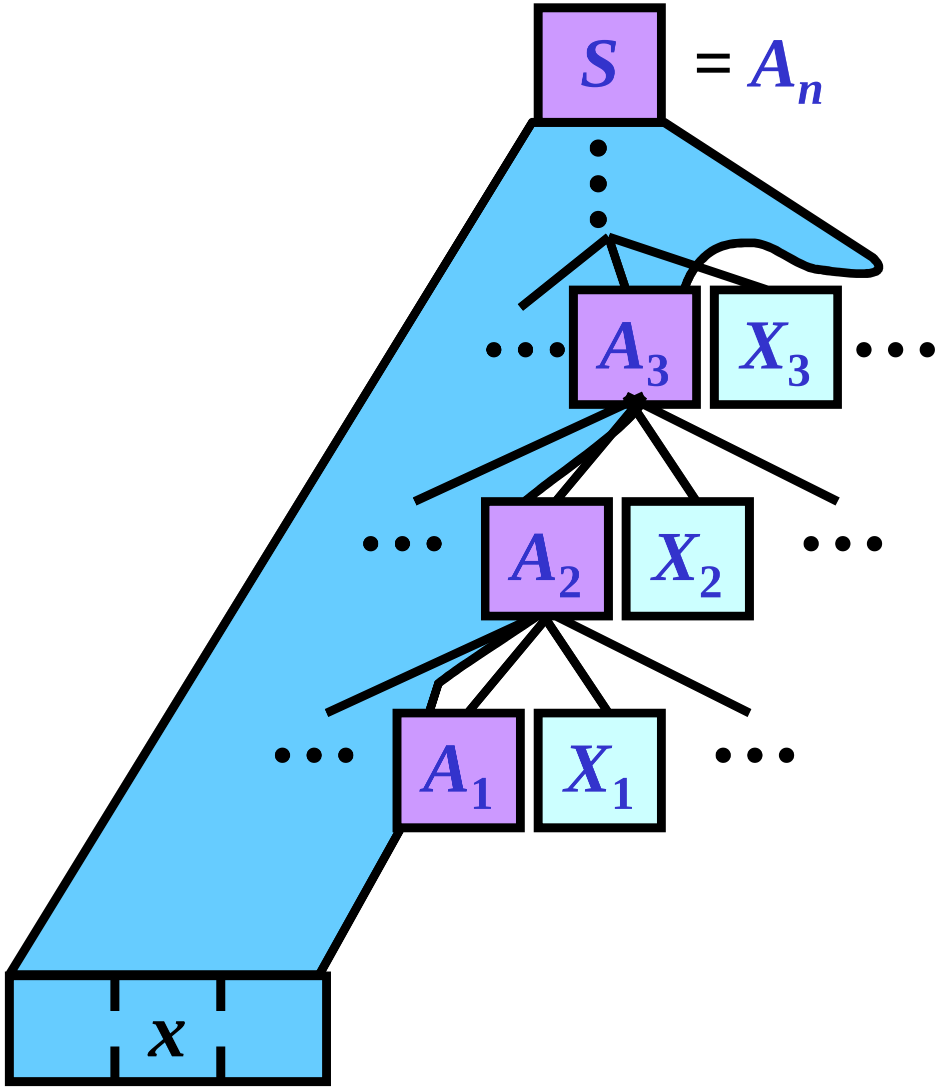


Panic-Mode Recovery: Illustration 1/2

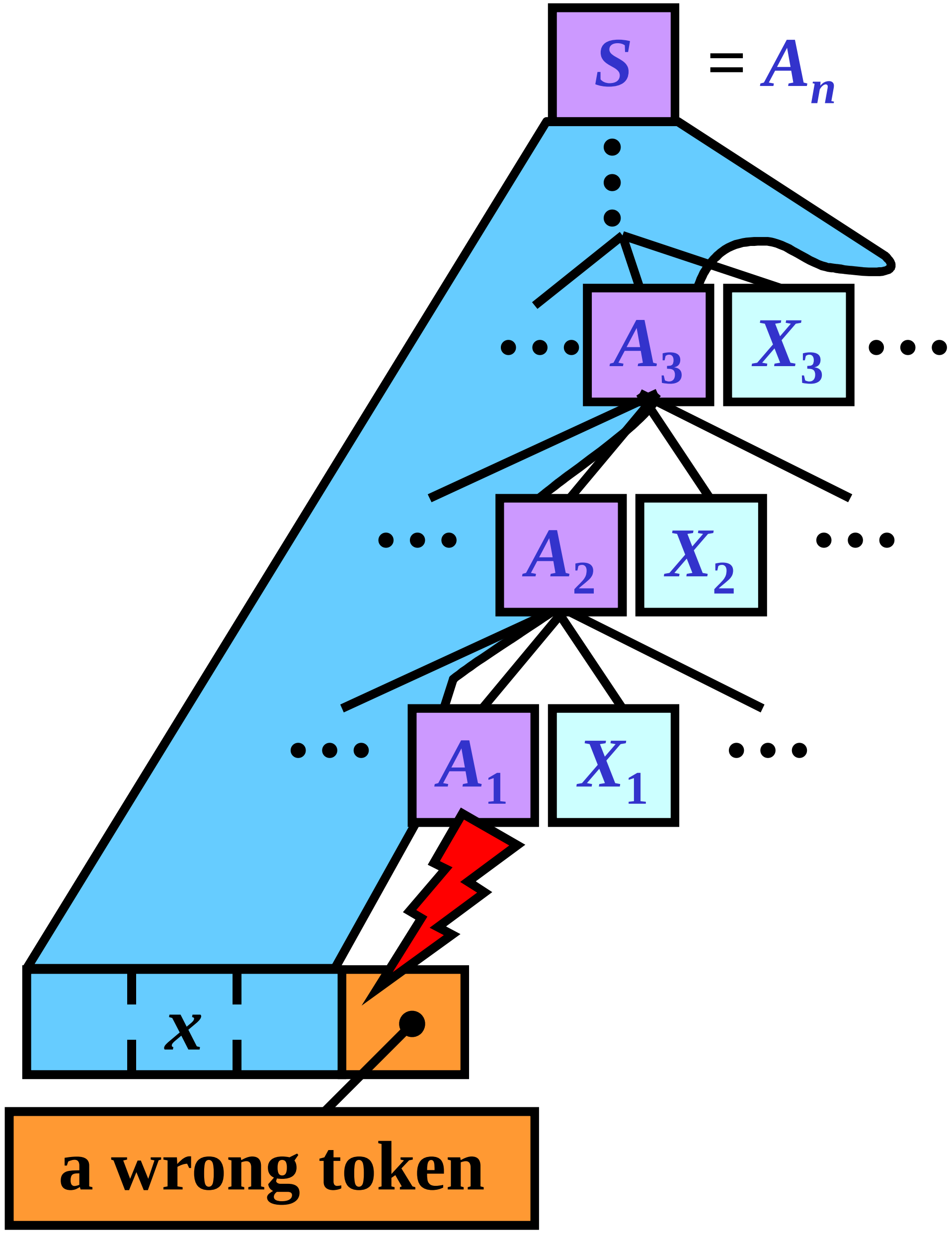


Let $a \in$ ***Follow***(A_1).
Then, continue from X_1

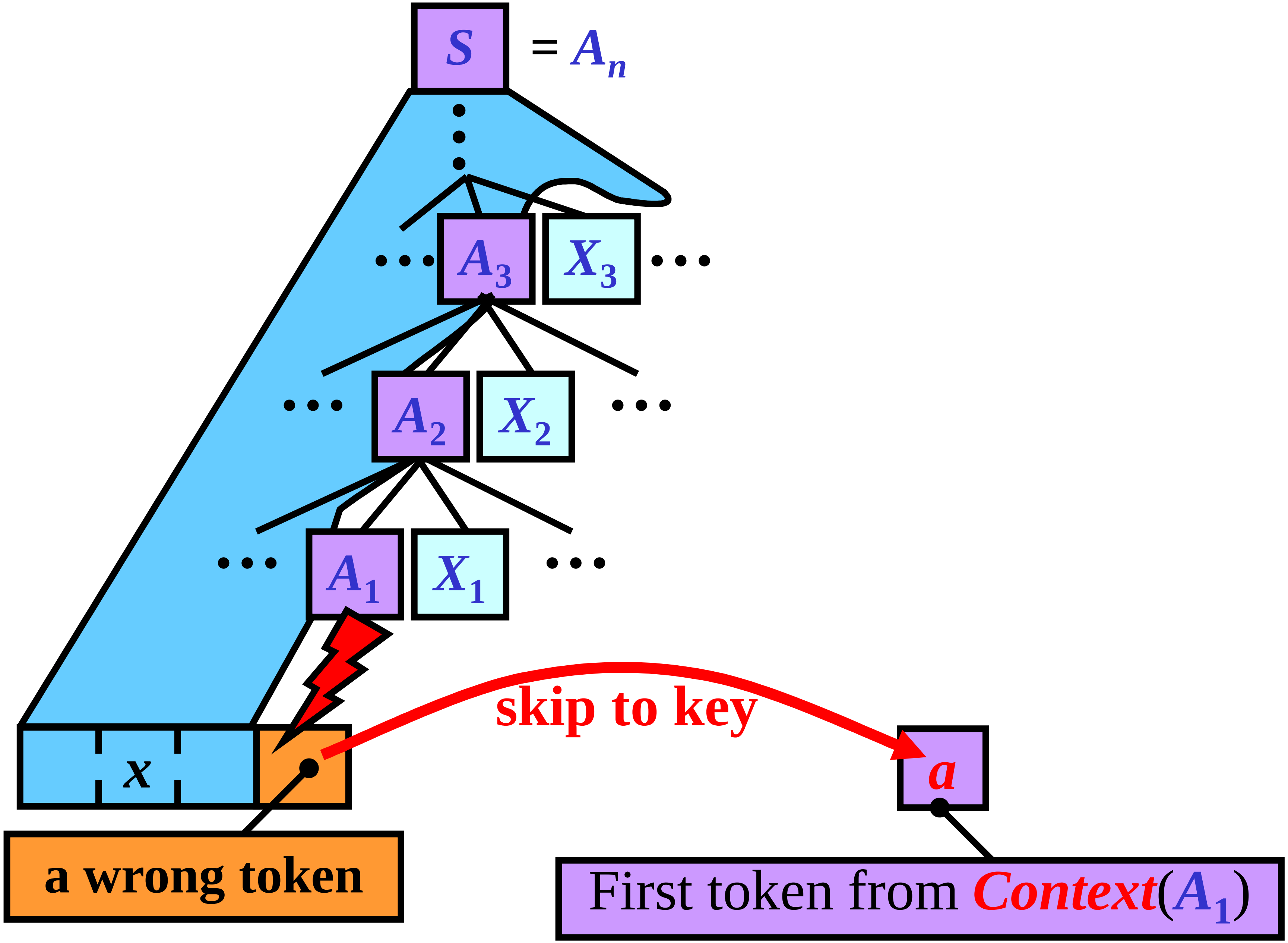
Panic-Mode Recovery: Illustration 2/2



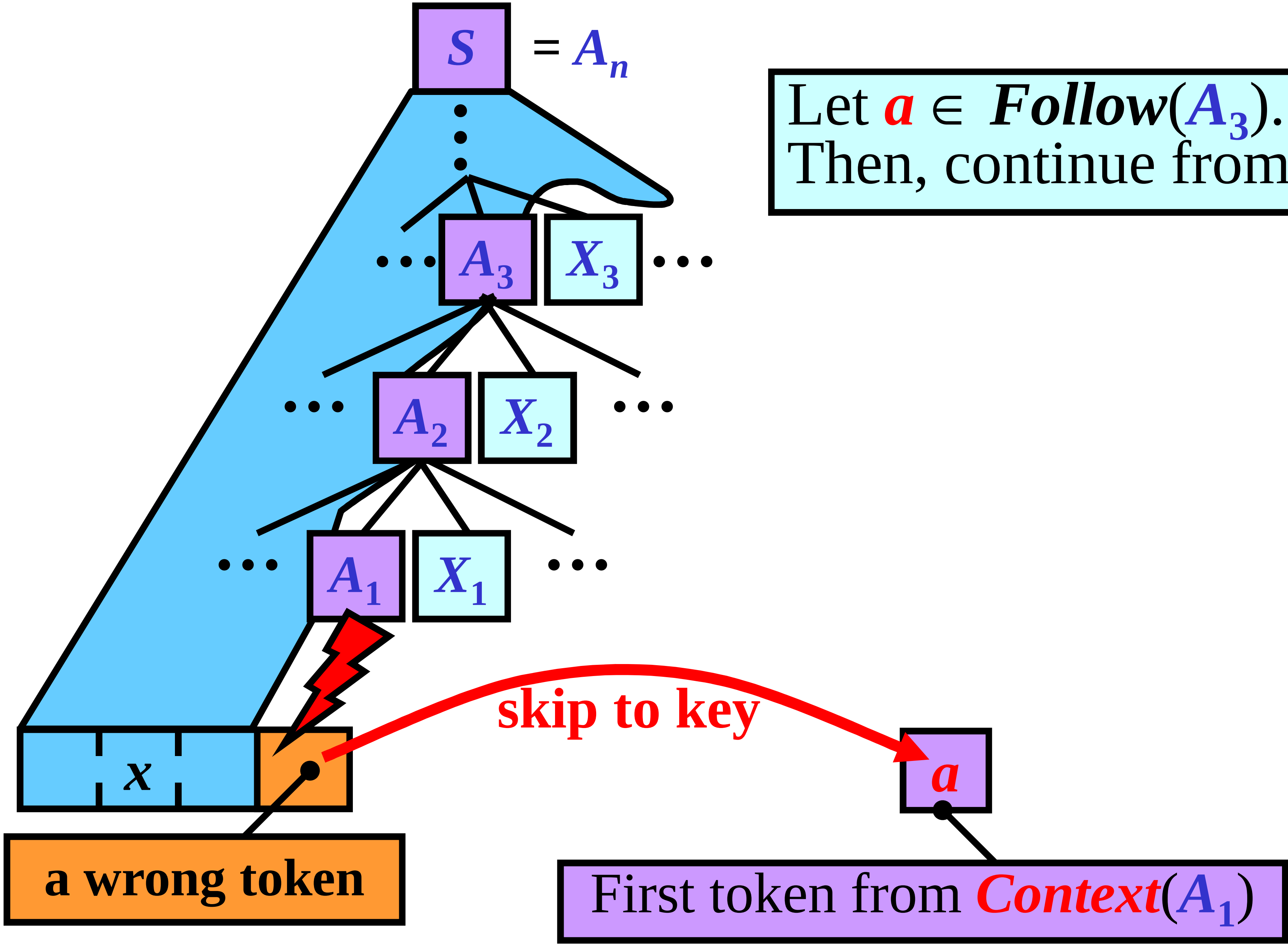
Panic-Mode Recovery: Illustration 2/2



Panic-Mode Recovery: Illustration 2/2

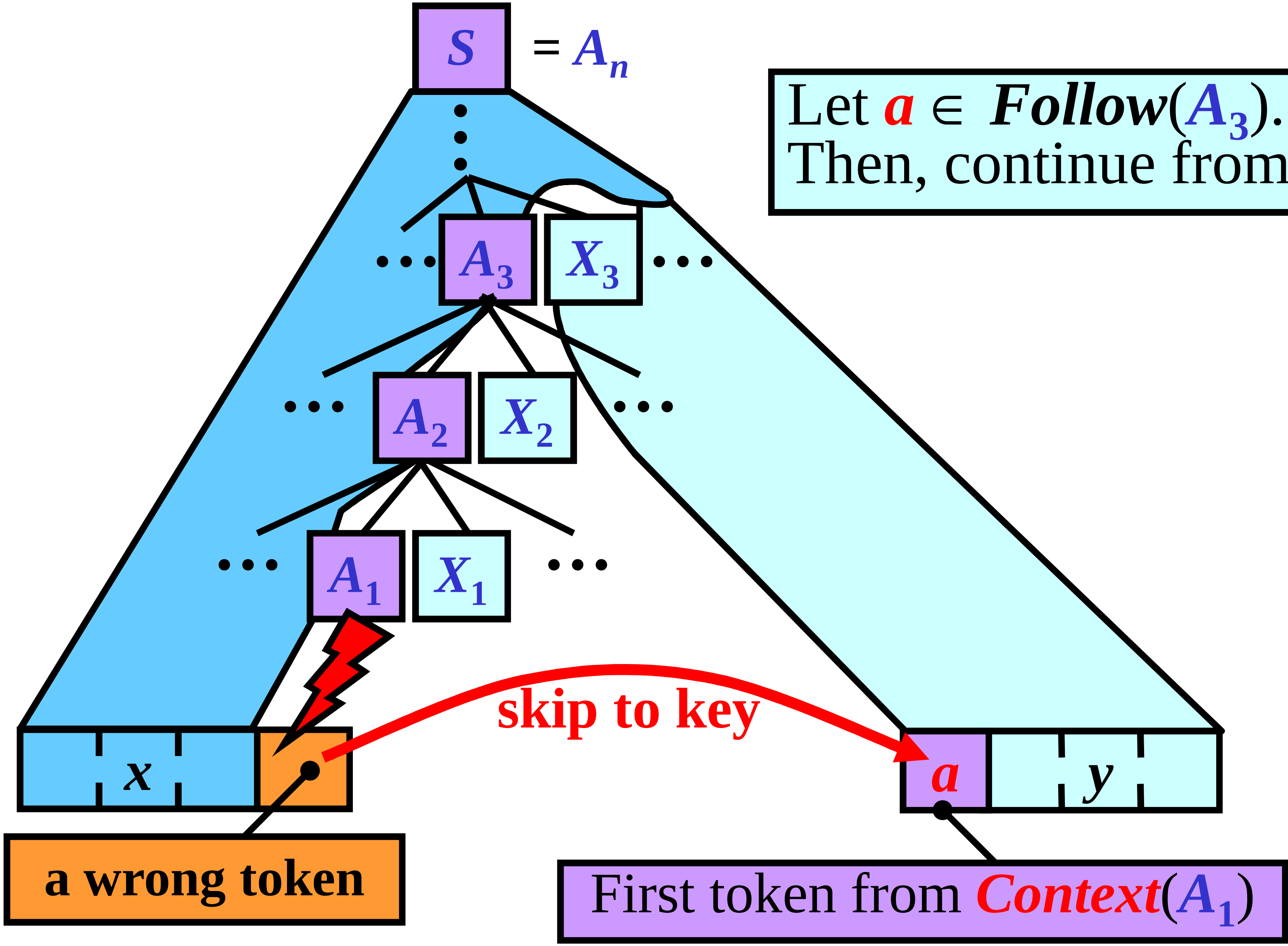


Panic-Mode Recovery: Illustration 2/2



Let $a \in$ ***Follow***(A_3).
Then, continue from X_3

Panic-Mode Recovery: Illustration 2/2



Let $a \in$ ***Follow***(A_3).
Then, continue from X_3

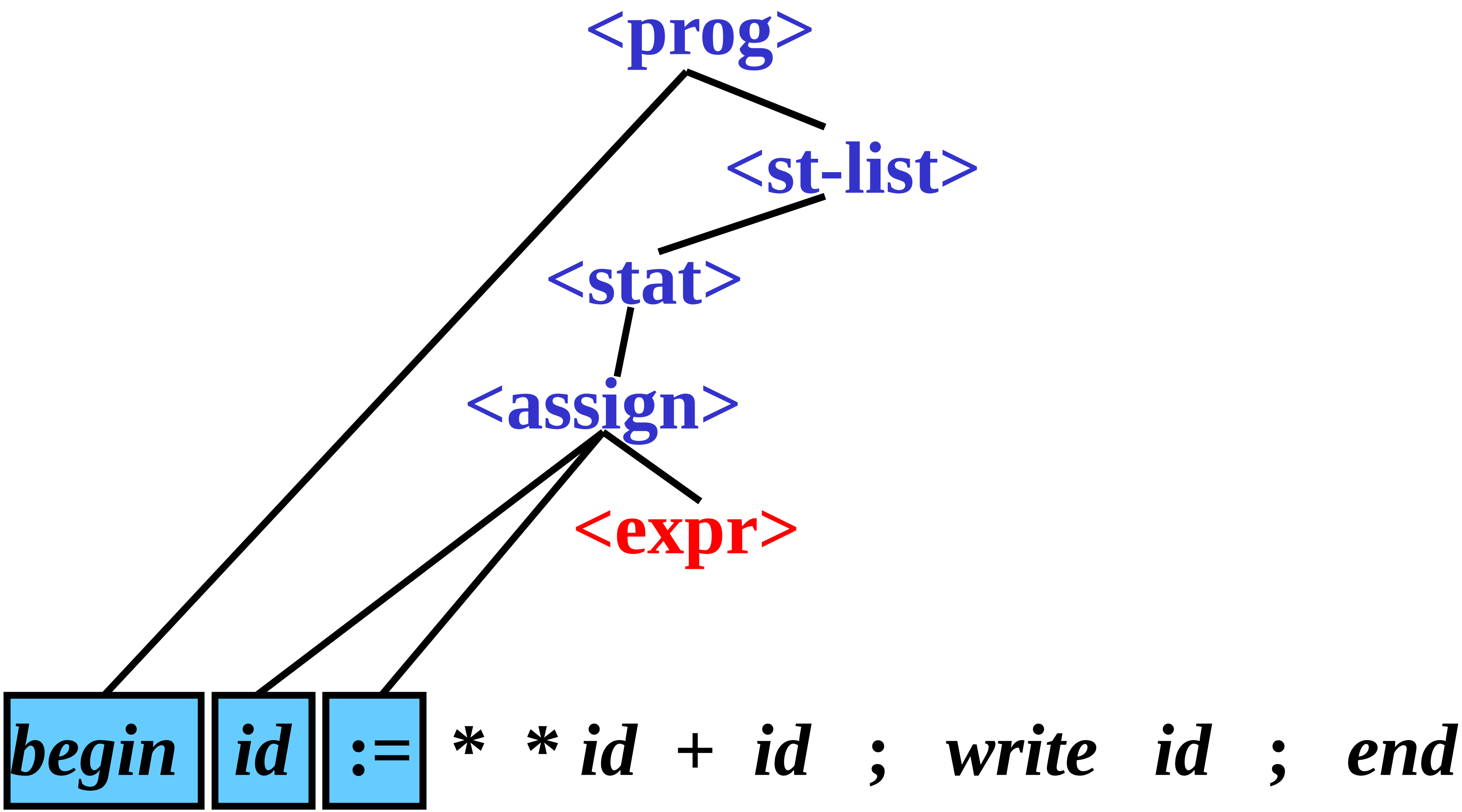
Context(*X*) for Predictive Parser: Variant I

For $G = (N, T, P, S)$,

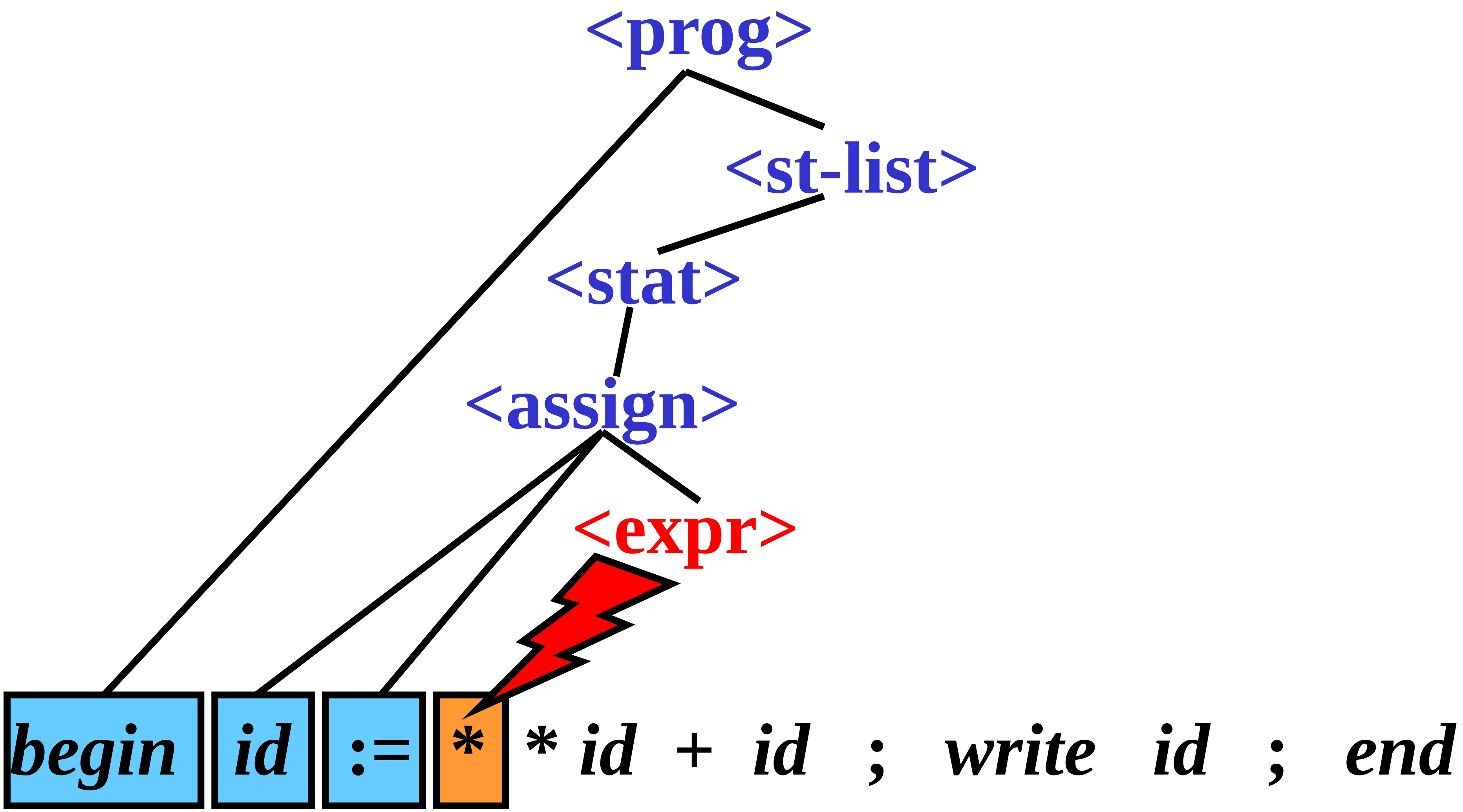
Context*(*A*) = *Follow*(*A*) for every *A* $\in$ *N

- **Method:**
- Let ***A*** be pushdown top & no rule is applicable:
- **repeat**
  a := GetNextToken;
  {These tokens are skipped}
  until ***a*** in ***Context***(***A***)
- pop ***A*** from the pushdown;

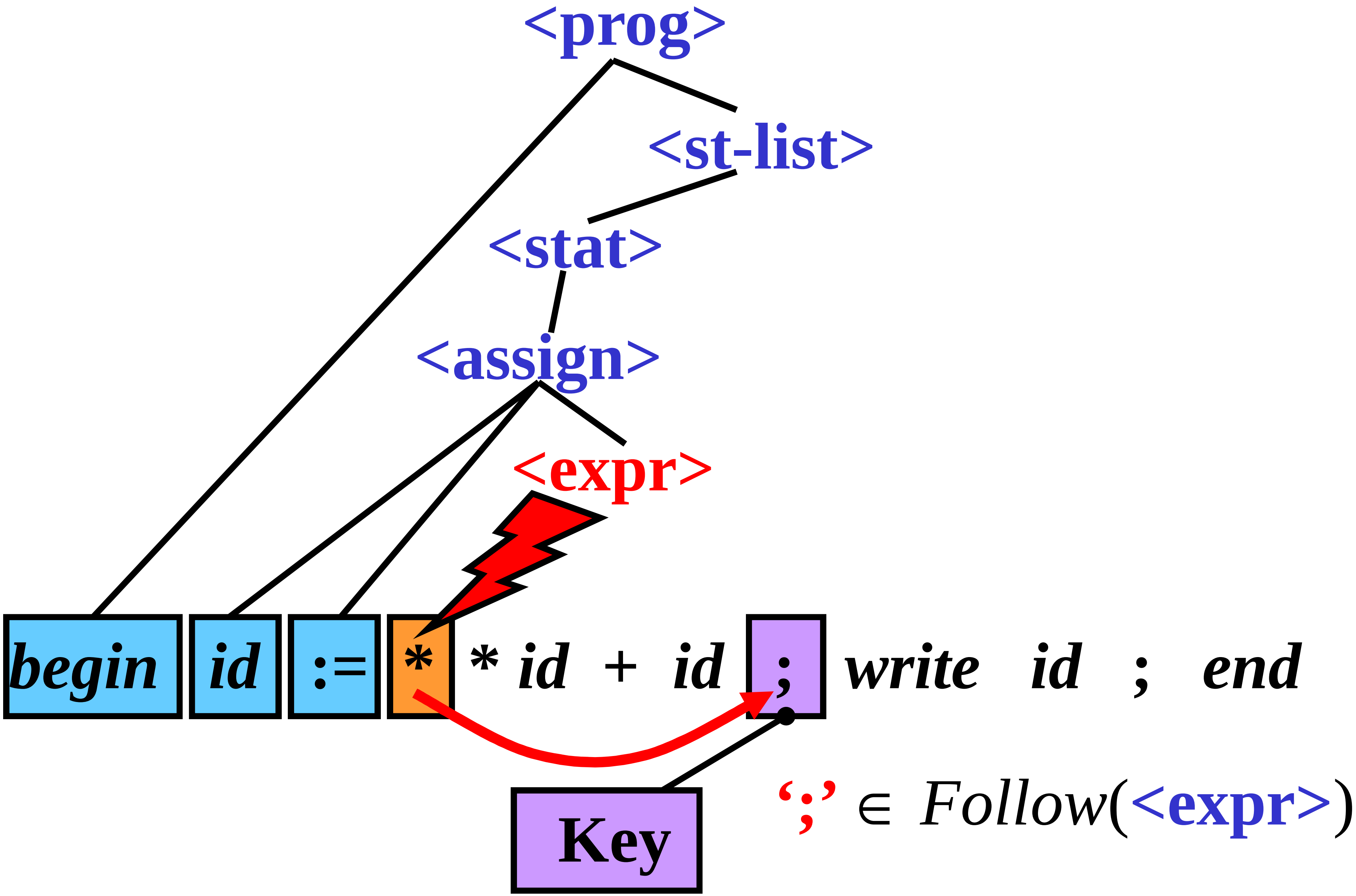
Variant I: Example



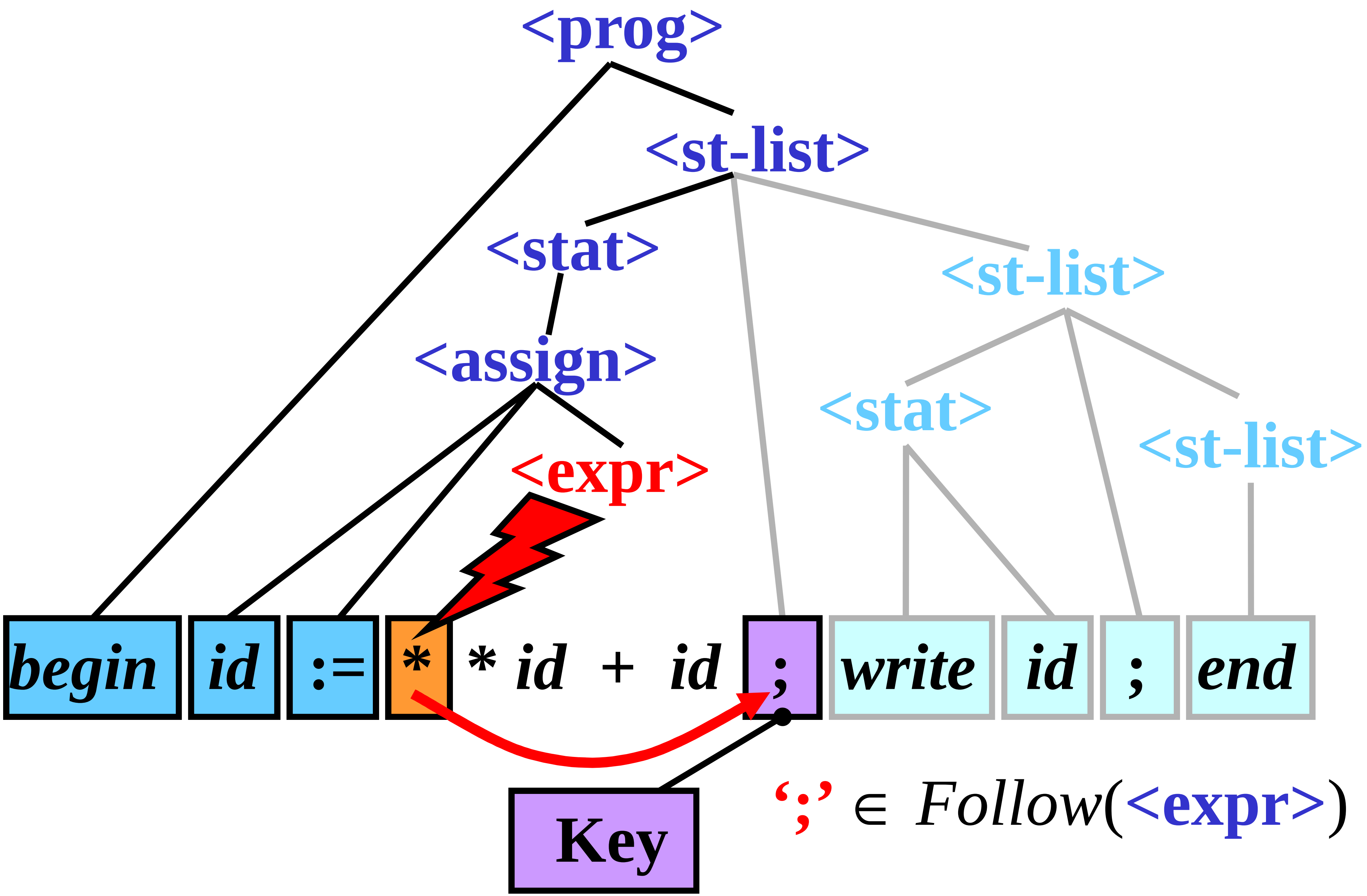
Variant I: Example



Variant I: Example



Variant I: Example



$`;' \in Follow(\text{<expr>})$

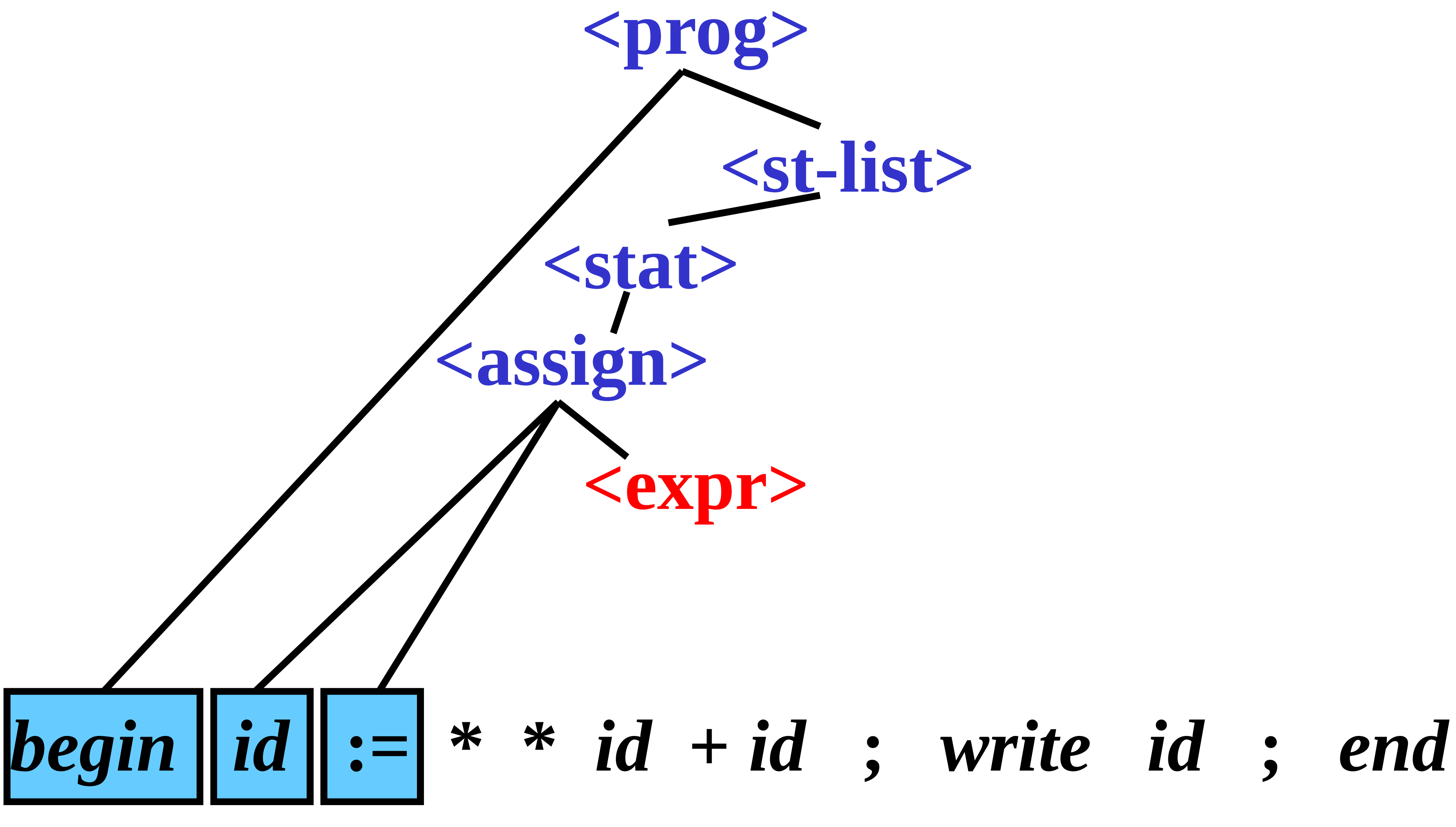
Context(*X*) for Predictive Parser: Variant II

For $G = (N, T, P, S)$,

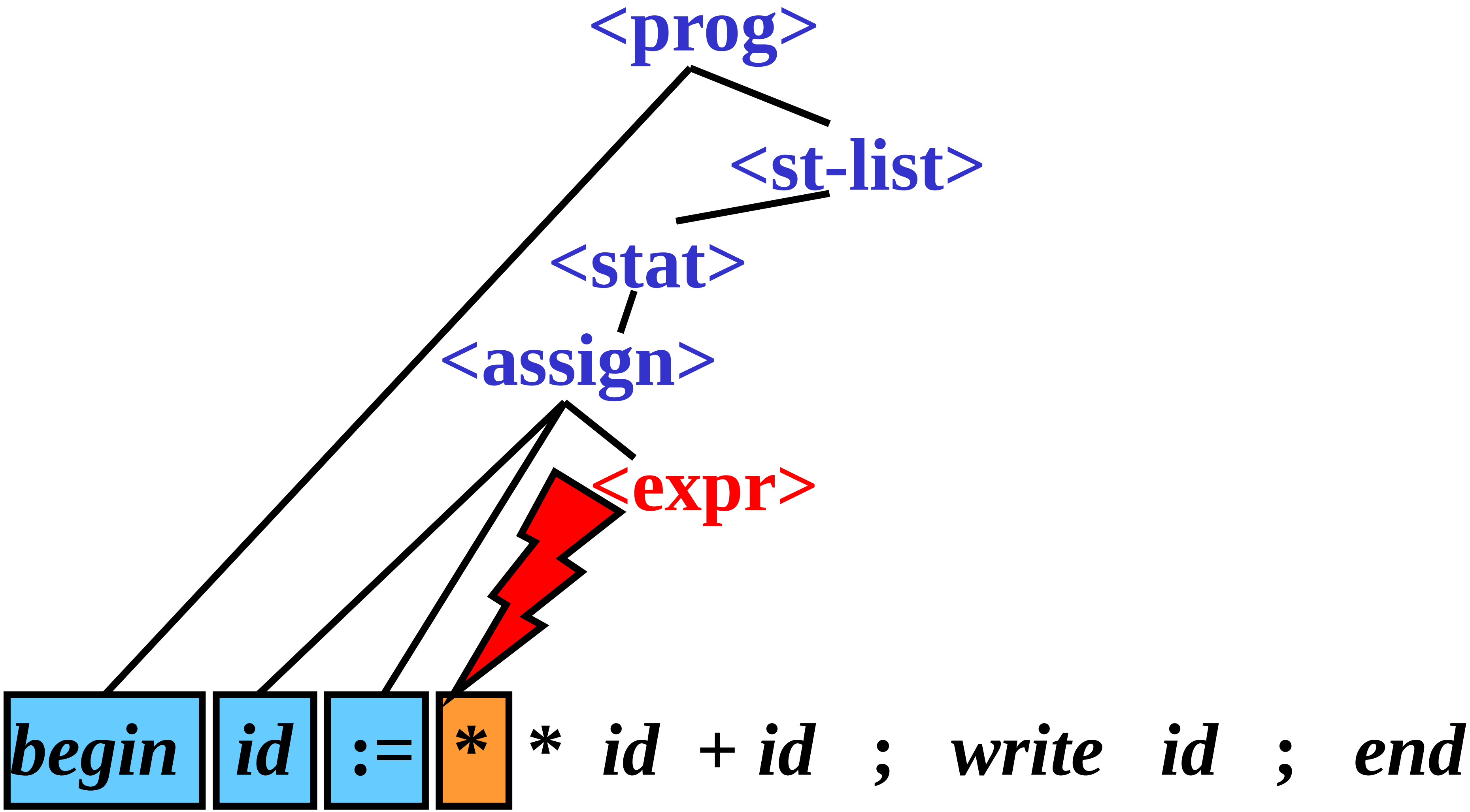
Context*(*A*) = *First*(*A*) $\cup$ *Follow*(*A*) for every *A* $\in$ *N

- **Method:**
- Let ***A*** be pushdown top & no rule is applicable:
- **repeat**
  a := GetNextToken;
  {These tokens are skipped}
  until *a* in ***Context***(***A***)
- **if *a*** $\in$ *First*(***A***) **then** resume according to ***A***
  else pop ***A*** from the pushdown // ***a*** $\in$ *Follow*(***A***)

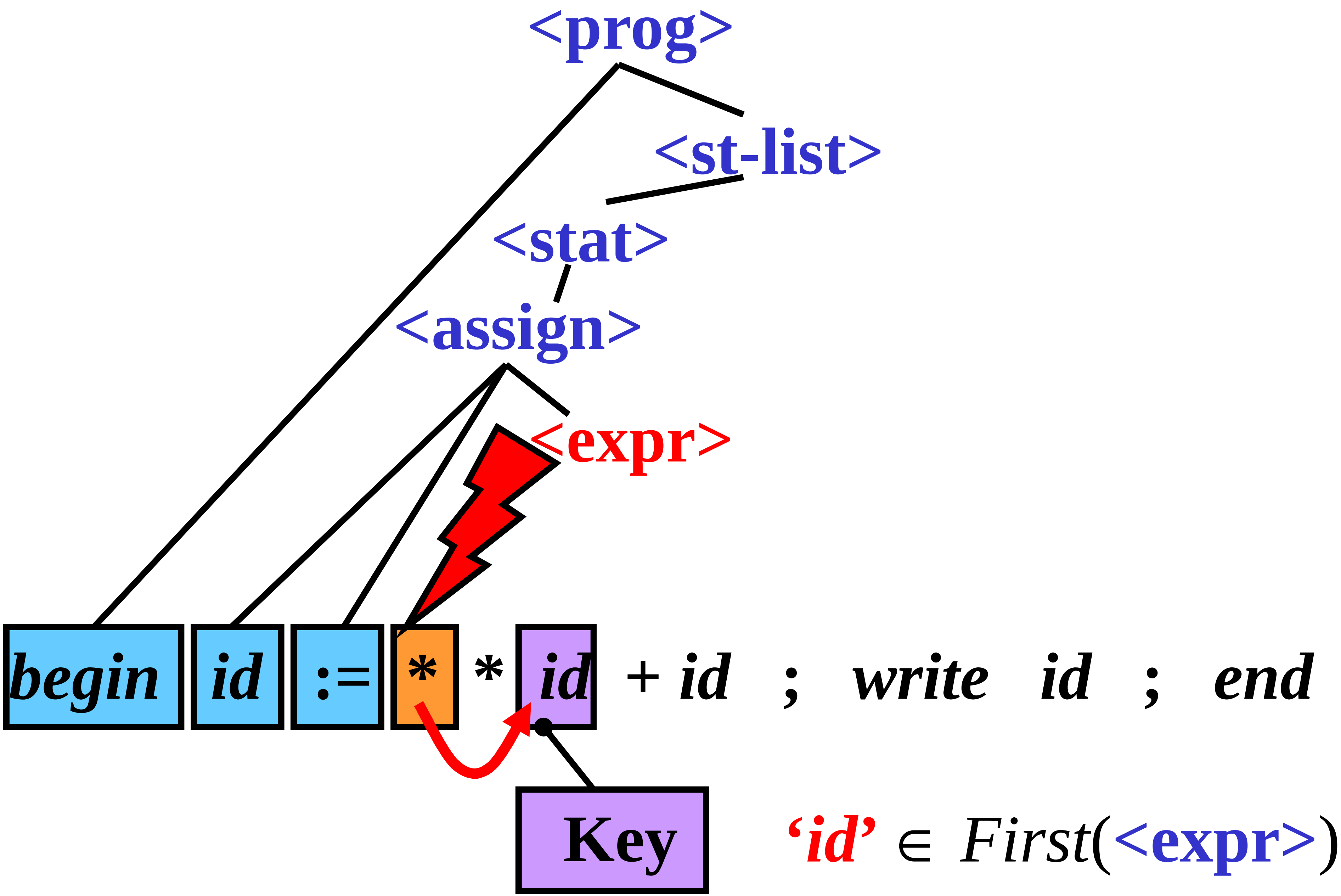
Variant II: Example



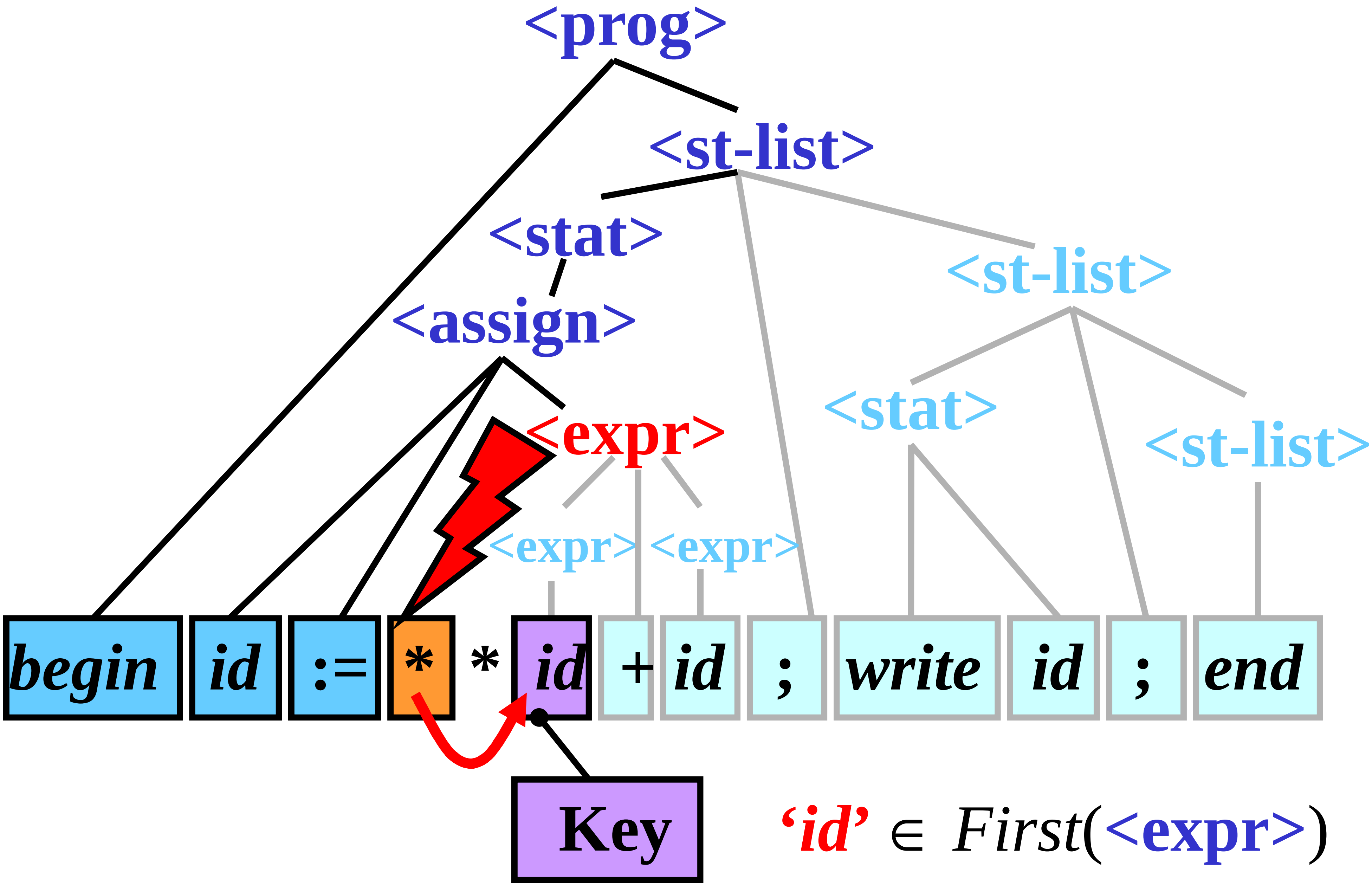
Variant II: Example



Variant II: Example



Variant II: Example



‘*id*’ $\in$ *First*(**<expr>**)