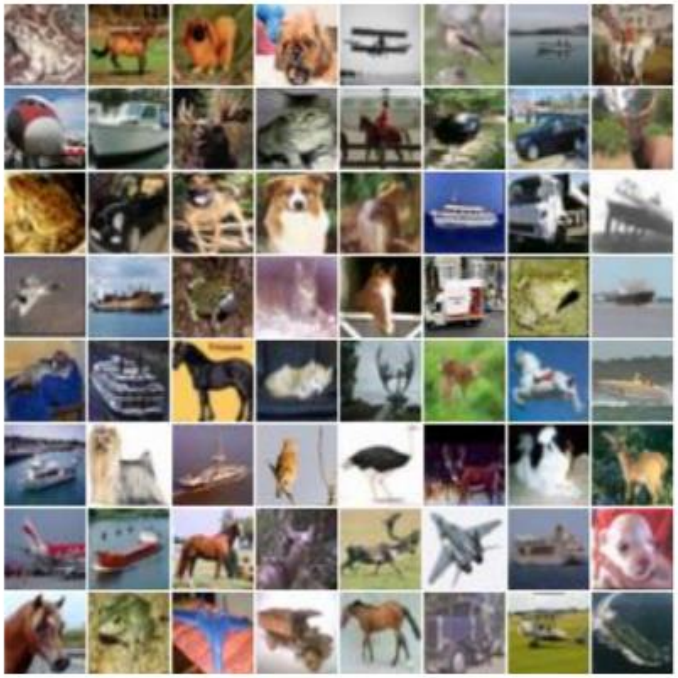
10707
Deep Learning: Spring 2020

Andrej Risteski

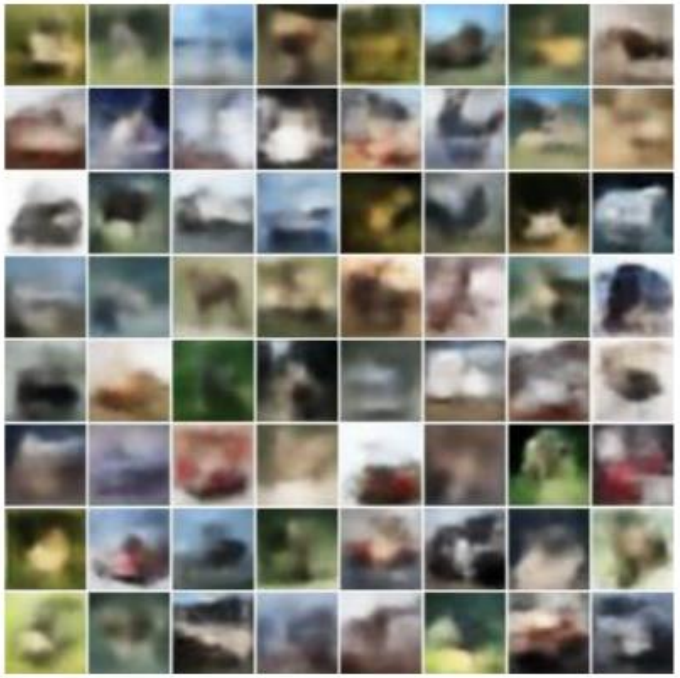
Machine Learning Department

Lecture 16:
Generative adversarial networks
Part I: Introduction

Some samples generated with VAEs and RBMs



Data



VAE samples

Faces generated using a trained VAE, slides from
http://efrosgans.eecs.berkeley.edu/CVPR18_slides/VAE_GANS_by_Rosca.pdf

Some samples generated with VAEs and RBMs

Faces generated using a trained VAE

The problem

Samples are blurry, though they capture some high-level structure.

Some hypotheses for what goes wrong:

Strong metric: VAEs try to match the input distribution in KL divergence, which is quite a strong metric.

Poor posteriors: The posteriors in a VAE are Gaussian – very poor modeling power, e.g. cannot model multimodal distributions.

Max-likelihood encourages averaging: The posteriors in a VAE are Gaussian – very poor modeling power, e.g. cannot model multimodal distributions.

The idea behind GANs

Matching a distribution on images is hard because we don't have good measures of "distance" between images.
(Intuitively, two images could be very different in pixel space, while "semantically" being the same image.)

Why don't we simultaneously train a "distance" metric as we are training the model?

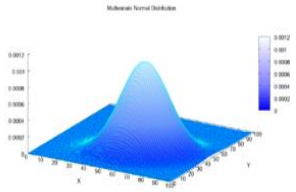
As a result, we will no longer be fitting the "maximum likelihood" model, but instead trying to learn some distribution close to the distribution of the input images in a learned metric.

This is (one of many) models which are "likelihood-free": we won't be able to explicitly write a likelihood for the model, but (importantly) we will efficiently be able to draw samples from the model!

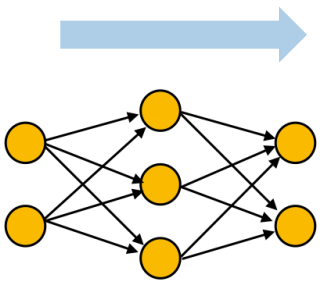
The GAN paradigm (Goodfellow et al. '14)

Goal: **Learn** a distribution close to some distribution we have few samples from. (Additionally, we will be able to sample efficiently from distribution.)

Approach: **Fit** distribution P_g parametrized by **neural network g**



$Z \sim N(0, I_{k\times k})$



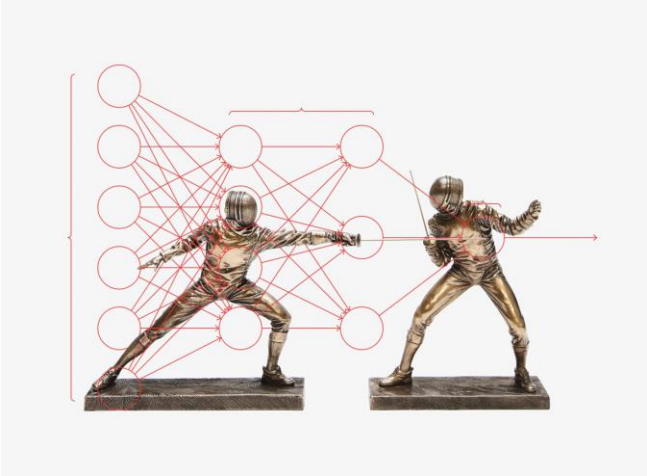
Neural network $g(\cdot)$

$X = g(Z)$

The GAN paradigm (Goodfellow et al. '14)

MIT Technology Review

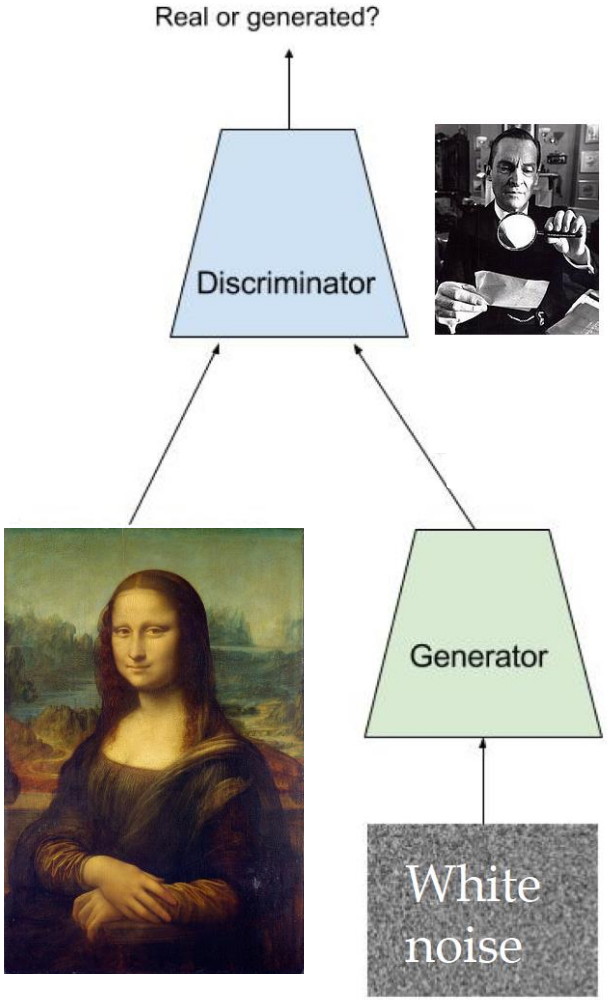
Top 10 Breakthrough Technologies 2018



Photorealistic image/video generation

Extracting complex features

The GAN paradigm (Goodfellow et al. '14)



Game theoretic idea:

Generator trained to **fool** discriminator.

Discriminator trained to **beat** generator.

W-GAN formalization (Arjovsky et al. '17)

Min-max problem:

- Min-player: generators $g \in G$; Max-player: discriminators f $\in F$.
- Samples from image distr. P_{real}. Unif. distribution over samples: $P_{samples}$
- P_g - generator distribution: $Z \sim N(0, I) \rightarrow g(Z)$

Training loss:

$$\min_{g \in G} \max_{f \in F} \left| \mathbb{E}_{P_g}[f] - \mathbb{E}_{P_{\text{samples}}}[f] \right|$$

Difference of expectation of f on **samples vs generated** images

W-GAN formalization (Arjovsky et al. '17)

Min-max problem:

- Min-player: generators $g \in G$; Max-player: discriminators f $\in F$.
- Samples from image distr. P_{real}. Unif. distribution over samples: $P_{samples}$
- P_g - generator distribution: $Z \sim N(0, I) \rightarrow g(Z)$

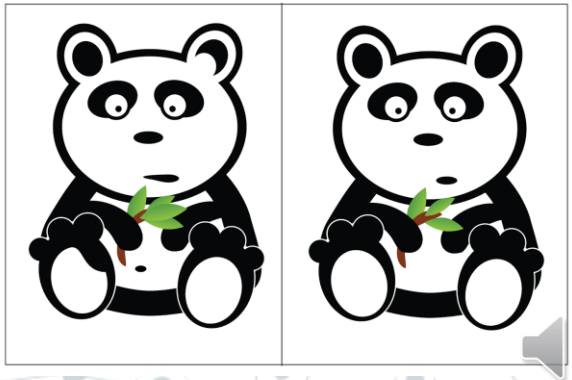
Training loss:

$$\min_{g \in G} \max_{f \in F} \left| \mathbb{E}_{P_g}[f] - \mathbb{E}_{P_{\text{samples}}}[f] \right|$$

Generator g **fools** discriminators F **:**

$$\forall f \in F, \mathbb{E}_{P_g}[f] \approx \mathbb{E}_{P_{\text{samples}}}[f]$$

Equivalently, small training loss!



W-GAN formalization (Arjovsky et al. '17)

Min-max problem:

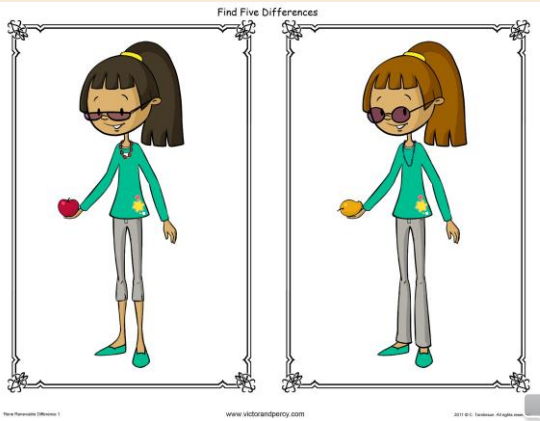
- Min-player: generators $g \in G$; Max-player: discriminators f $\in F$.
- Samples from image distr. P_{real}. Unif. distribution over samples: $P_{samples}$
- P_g - generator distribution: $Z \sim N(0, I) \rightarrow g(Z)$

Training loss:

$$\min_{g \in G} \max_{f \in F} \left| \mathbb{E}_{P_g}[f] - \mathbb{E}_{P_{\text{samples}}}[f] \right|$$

Discriminators F **beat** generators if for all $g \in G$, there is an f $\in F$

$$\mathbb{E}_{P_g}[f] \not\approx \mathbb{E}_{P_{\text{samples}}}[f]$$



W-GAN formalization (Arjovsky et al. '17)

Min-max problem:

- Min-player: generators $g \in G$; Max-player: discriminators f $\in F$.
- Samples from image distr. P_{real}. Unif. distribution over samples: $P_{samples}$
- P_g - generator distribution: $Z \sim N(0, I) \rightarrow g(Z)$

Training loss:

$$\min_{g \in G} \max_{f \in F} \left| \mathbb{E}_{P_g}[f] - \mathbb{E}_{P_{\text{samples}}}[f] \right|$$

$$d_F(P_{\text{samples}}, P_g)$$

"Distance" specified by discriminators F .
Captures how well F's can **distinguish** two distributions

Discriminators F **beat** generators if for all $g \in G$, there is an f $\in F$

$$\mathbb{E}_{P_g}[f] \not\approx \mathbb{E}_{P_{\text{samples}}}[f]$$

W-GAN formalization (Arjovsky et al. '17)

Min-max problem:

- Min-player: generators $g \in G$; Max-player: discriminators f $\in F$.
- Samples from image distr. P_{real}. Unif. distribution over samples: $P_{samples}$
- P_g - generator distribution: $Z \sim N(0, I) \rightarrow g(Z)$

Training loss:

$$\min_{g \in G} \max_{f \in F} \left| \mathbb{E}_{P_g}[f] - \mathbb{E}_{P_{\text{samples}}}[f] \right|$$

$$d_F(P_{\text{samples}}, P_g)$$

Discriminators F **beat** generators if for all $g \in G$, there is an f $\in F$

$$\mathbb{E}_{P_g}[f] \not\approx \mathbb{E}_{P_{\text{samples}}}[f]$$

Training loss $= \min_{g \in G} d_F(P_g, P_{\text{samples}})$

Examples of distances d_F

$$\max_{f \in F} \left| \mathbb{E}_{P_g}[f] - \mathbb{E}_{P_{\text{samples}}}[f] \right| = d_F(P_{\text{samples}}, P_g)$$

$F = \{f : |f|_\infty \leq 1\}$: **Total variation distance**

Measures differences of bounded functions

$F = \{f : \text{Lip}(f) \leq 1\}$: **$W_1$ (Wasserstein, earthmover) distance**

Measures differences of 1-Lipschitz functions

Examples of distances d_F

$$\max_{f \in F} \left| \mathbb{E}_{P_g}[f] - \mathbb{E}_{P_{\text{samples}}}[f] \right|$$

$$d_F(P_{\text{samples}}, P_g)$$

If distance d_F is a **metric**: $d_F(p, q) \geq 0$ and $d_F(p, q) = 0$ only if $p = q$

Hence, if we learn a distribution P_g, s.t. $d_F(P_g, P_{real}) = 0$, and P_{real} is the true data distribution, we have $P_g = P_{real}$.

In the limit of infinite samples, $P_{real} = P_{\text{samples}}$, so if training error is 0, we have learned a distribution $P_g = P_{real}$

What affects our choice of F?

Statistical considerations: very powerful discriminators (e.g. large neural networks) will require a lot of samples. Weak discriminators will specify a very weak metric: very "different" distributions will look very "similar" to metric.

Our understanding here is much better.

Algorithmic considerations: if discriminators are very powerful, gradient information for generator is too weak and can vanish. If they are too weak – metric is weak.

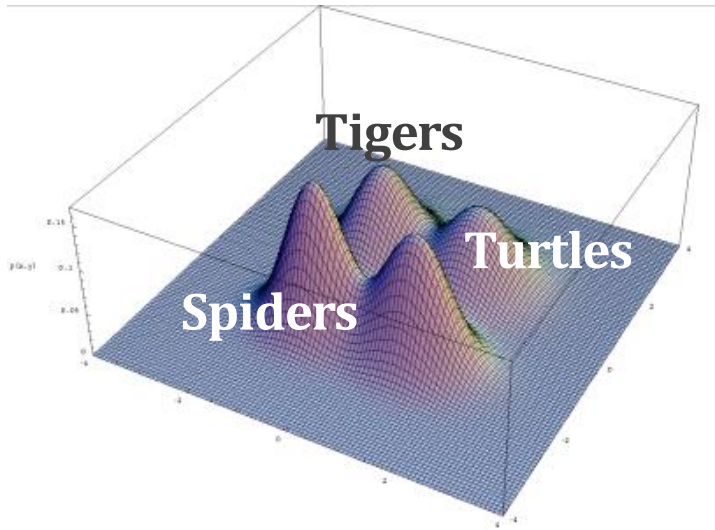
Our understanding of training dynamics is very poor.

Statistical questions

Tension: strength of discriminators?

Small (weak) discriminators ⇒ mode collapse:

Neural net discriminators with ≤ **m parameters** **fooled** by generator w/ support size ≈ **m**.
[Arora et al'17, Arora-Risteski-Zhang **ICLR'18**]



Real-life distributions have large support!

Tension: strength of discriminators?

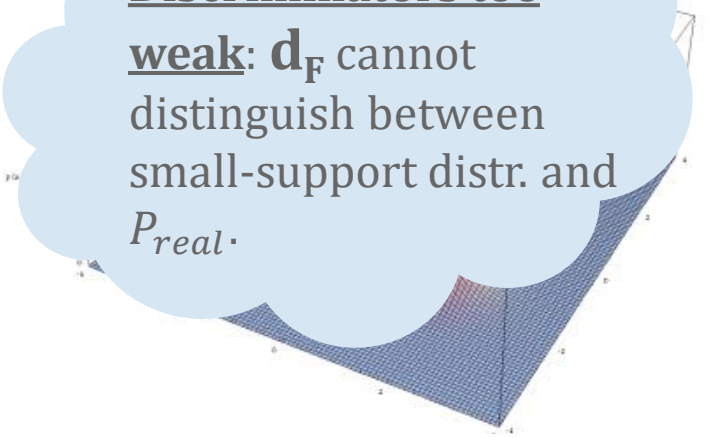
Small (weak) discriminators ⇒ mode collapse.

Happens for any P_{real}

Neural net discriminators with **≤ m parameters fooled** by generator w/ support size **≈ m.**
[Arora et al'17, Arora-Risteski-Zhang **ICLR'18**]

Not memorization!
More training samples **don't help.**

Discriminators too weak: $\mathbf{d_F}$ cannot distinguish between small-support distr. and P_{real}.



Real-life distributions have large support!

Tension: strength of discriminators

Small discriminators ⇒ mode collapse:

Generator w/ support size $\approx m$ **fools** neural net discriminators with **≤ m parameters.** [Arora et al'17, Arora-Risteski-Zhang 'ICLR18]

Large discriminators ⇒ poor generalization:

Loss with small # samples differs a lot from loss with infinite # samples.

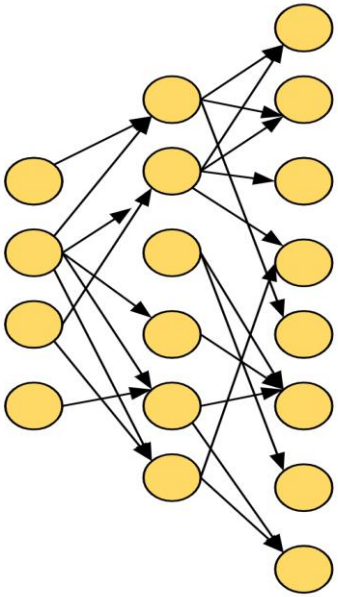
$$d_F(P_{samples}, P_g) \not\approx d_F(P_{real}, P_g)$$

Sweet spot for natural distributions

Let P_{real} **itself** be generated by neural net. ($P_{real} = P_g$, $g \in G$)

Let G = { **1-to-1 neural networks** of bounded size }

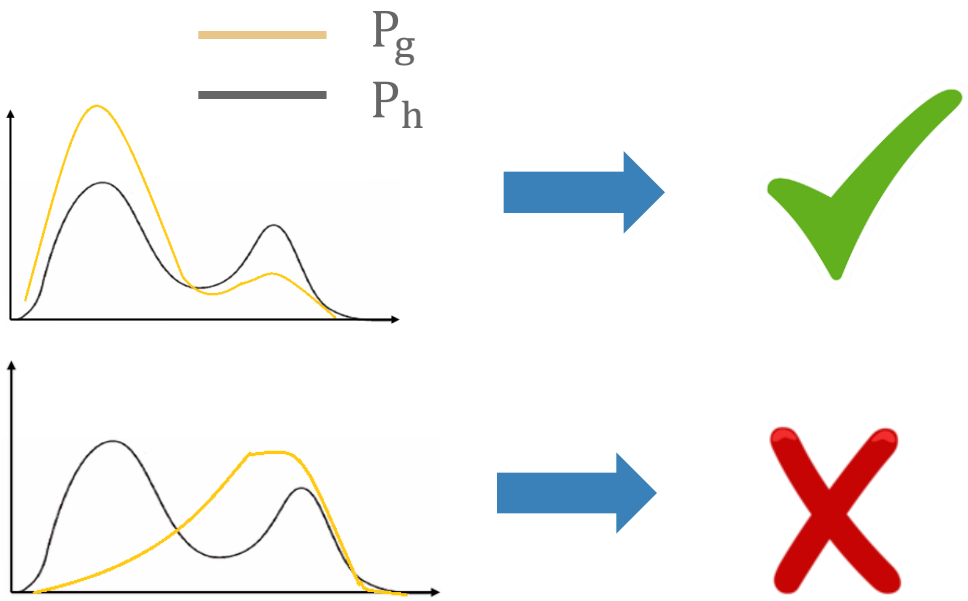
Design **small** discriminators F w/ good distinguishing power.



- Less general than arbitrary neural-net generators ✗
- Allows data to lie on **low-dim. manifold**. ✓

Distinguishing power

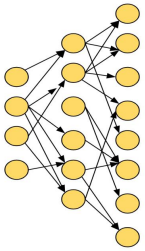
Discriminators F have **distinguishing power** against generators G, if: $\forall \mathrm{g}, \mathrm{h} \in \mathrm{G} : d_F(\mathrm{P_g}, \mathrm{P_h}) \gtrsim W_1(P_g, P_h)$



Distance d_F is not too much weaker than W_1, **only** for distributions in our class.

Mai

Neural nets of **slightly larger** depth/size than generators. (Suggestion for practice!)



Thm (Bai-Ma-Risteski [ICLR 19]**) :** **Small** discriminators F with **distinguishing power** for G = {**1-to-1 neural nets** of bdd size} exist.

So, if P_{real} generated by 1-to-1 neural net with **d** params,

w/ **poly(d)** samples, $d_F(P_{samples}, P_g) \leq \epsilon \Rightarrow W_1(P_{real}, P_g) \leq O(\sqrt{\epsilon})$

Training was successful

True distribution learned.

Algorithmic questions

How to train a GAN

"**Best response dynamics**": fix generator, find best discriminator; then fix discriminator, find best generator. Repeat.

Better in practice: take one gradient step for generator, do a few gradient steps for discriminator. Repeat.

Going with intuition of Wasserstein distance: we'd like the discriminators to be somewhat Lipschitz – clipping weights is a good idea.

How to train a GAN

How many discriminator gradient steps to take for each generator gradient step

Algorithm 1 WGAN, our proposed algorithm. All experiments in the paper used the default values $\alpha = 0.00005$, $c = 0.01$, $m = 64$, $n_{\text{critic}} = 5$.

Require: : α, the learning rate. c, the clipping parameter. m, the batch size. n_{critic}, the number of iterations of the critic per generator iteration.

Require: : w_0, initial critic parameters. θ_0, initial generator's parameters.

1: **while** θ has not converged **do**
2: **for** $t = 0, ..., n_{\text{critic}}$ **do**
3: Sample $\{x^{(i)}\}_{i=1}^{m} \sim \mathbb{P}_r$ a batch from the real data.
4: Sample $\{z^{(i)}\}_{i=1}^{m} \sim p(z)$ a batch of prior samples.
5: $g_w \leftarrow \nabla_w \left[\frac{1}{m}\sum_{i=1}^{m} f_w(x^{(i)}) - \frac{1}{m}\sum_{i=1}^{m} f_w(g_\theta(z^{(i)}))\right]$
6: $w \leftarrow w + \alpha \cdot \text{RMSProp}(w, g_w)$
7: $w \leftarrow \text{clip}(w, -c, c)$
8: **end for**
9: Sample $\{z^{(i)}\}_{i=1}^{m} \sim p(z)$ a batch of prior samples.
10: $g_\theta \leftarrow -\nabla_\theta \frac{1}{m}\sum_{i=1}^{m} f_w(g_\theta(z^{(i)}))$
11: $\theta \leftarrow \theta - \alpha \cdot \text{RMSProp}(\theta, g_\theta)$
12: **end while**

Empirical estimates of expectations to calculate discriminator gradient

Clip

Generator gradient

Common training problems

Unstable training: the problem is a min-max problem (also called saddle point problem) – typically optimization is much less stable than pure minimization.

Particularly common instantiation: **cycling**

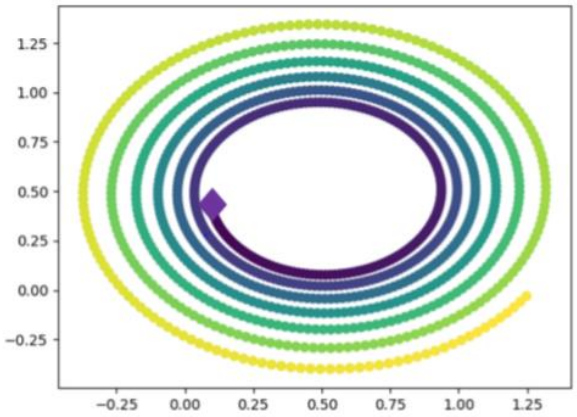


Figure 5: f(x,y)=(x-1/2)(y-1/2). x and y are initialized at the purple diamond. Alternating between gradient ascent/ descent on x and y leads to divergent behavior, spiraling away from the optimum, but the average of the parameters is close to the optimal solution

Figure from https://people.csail.mit.edu/madry/6.883/files/lecture_8.pdf

Common training problems

Unstable training: the problem is a min-max problem (also called saddle point problem) – typically optimization is much less stable than pure minimization.

Vanishing gradient: if the discriminator is too good, the generator gradients have a propensity to be small. (This is concerning, as to be taking gradients of the Wasserstein/JS/… objective, the discriminator needs to be optimal.)

Less of a problem with more modern GANs than with DC-GAN.

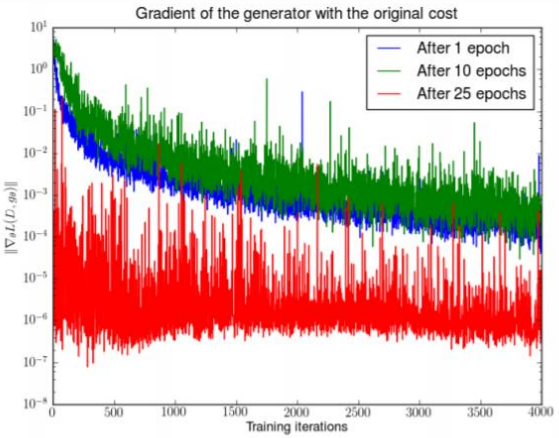


Figure 2: First, we trained a DCGAN for 1, 10 and 25 epochs. Then, with the generator fixed we train a discriminator from scratch and measure the gradients with the original cost function. We see the gradient norms decay quickly, in the best case 5 orders of magnitude after 4000 discriminator iterations. Note the logarithmic scale.

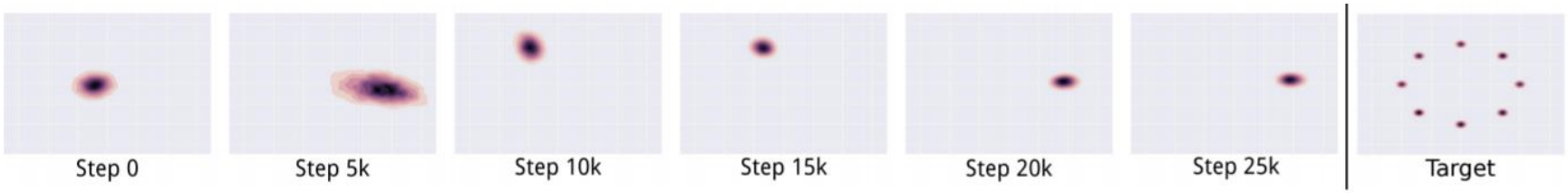
Common training problems

Unstable training: the problem is a min-max problem (also called saddle point problem) – typically optimization is much less stable than pure minimization.

Vanishing gradient: if the discriminator is too good, the generator gradients have a propensity to be small. (This is concerning, as to be taking gradients of the Wasserstein/JS/… objective, the discriminator needs to be optimal.)

Less of a problem with more modern GANs than with DC-GAN.

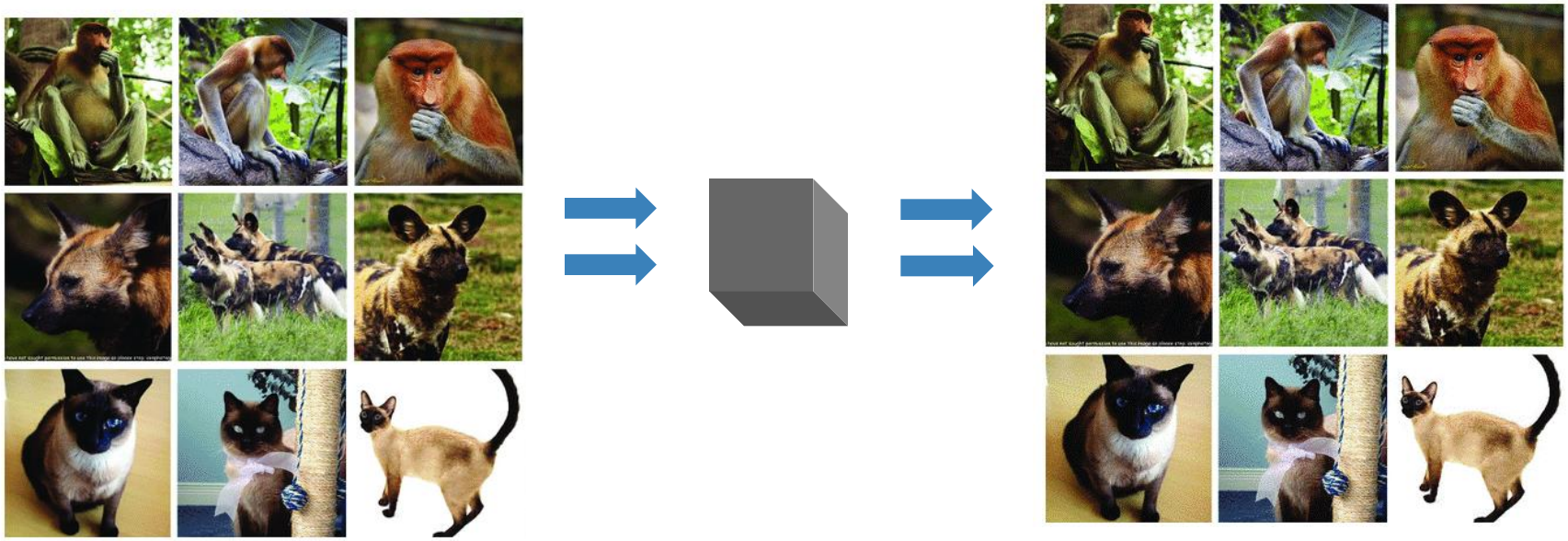
Mode collapse: the training only recovers some of the modes of the underlying distribution. **(NOT** clear if this is a statistical or algorithmic problem.)



Evaluating GANs

How do we evaluate GANs

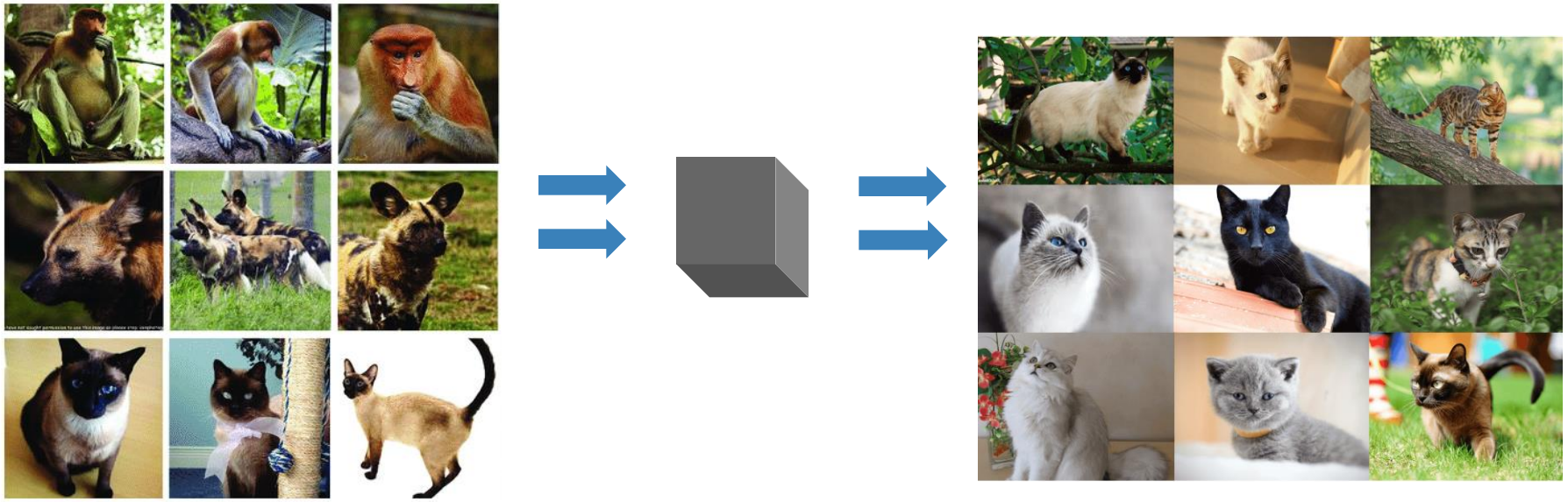
Wins beauty contest, but does the GAN **really** learn distribution?



No, just **memorizing** training samples.

How do we evaluate GANs

Wins beauty contest, but does the GAN **really** learn distribution?



No, **mode collapse:** missing regions (modes) of input distribution.

How do we evaluate GANs

Since we cannot evaluate the likelihood of the input data under a generator, evaluation is hard.

(Disproportionately) frequently, the evaluation is done by visually comparing samples – this cannot exclude issues like memorization, mode collapse, etc.

*Can we test for some common **failure modes**?*

Diagnosing small support size: bday paradox

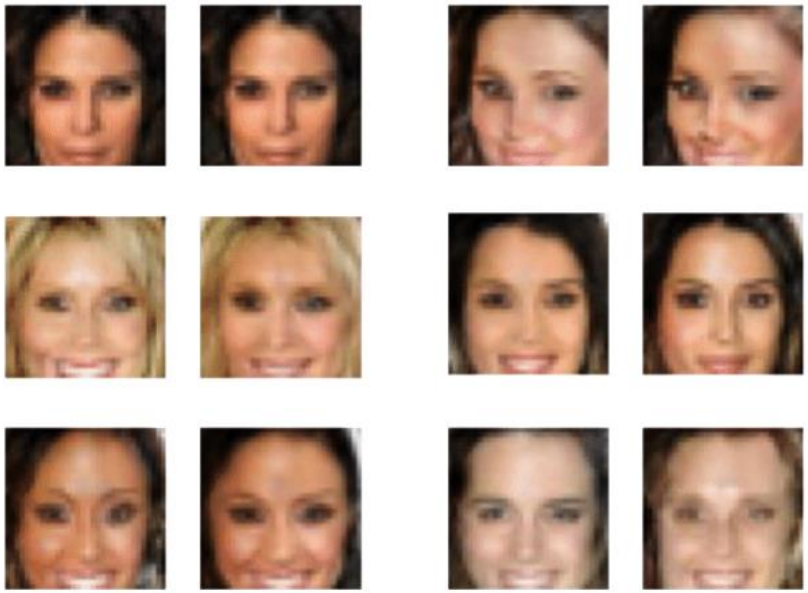
Birthday Paradox:
If there are **23** people in a group, **>** ½ chance that two of them share a birthday.

General version: Suppose a distribution is uniform over N images. Then
Pr[sample of size $\sqrt{N}$ has a duplicate image] > ½.

Birthday paradox test* [Arora-**Risteski**-Zhang **ICLR'18] :** If a sample of size **s** has **duplicate** images with prob. > ½, then distribution essentially* has only **s^2 distinct images**.

Implementation: Draw sample of size **s**; heuristically flag possible near-duplicates. Use human in the loop to verify duplicates.

Diagnosing small support size: bday paradox



CelebA (faces): 200k training images

DC-GAN [Radford et al.'15]:
Support size ≈ 250K

BiGAN [Donohue et al.'17] and **ALI [Dumoulin et al.'17]**:
Support size ≈ 1M

A lot of **followup** and **parallel** work about diagnosing mode collapse.
[Borji '18: 24 quantitative, 5 qualitative measures]

Interpolation

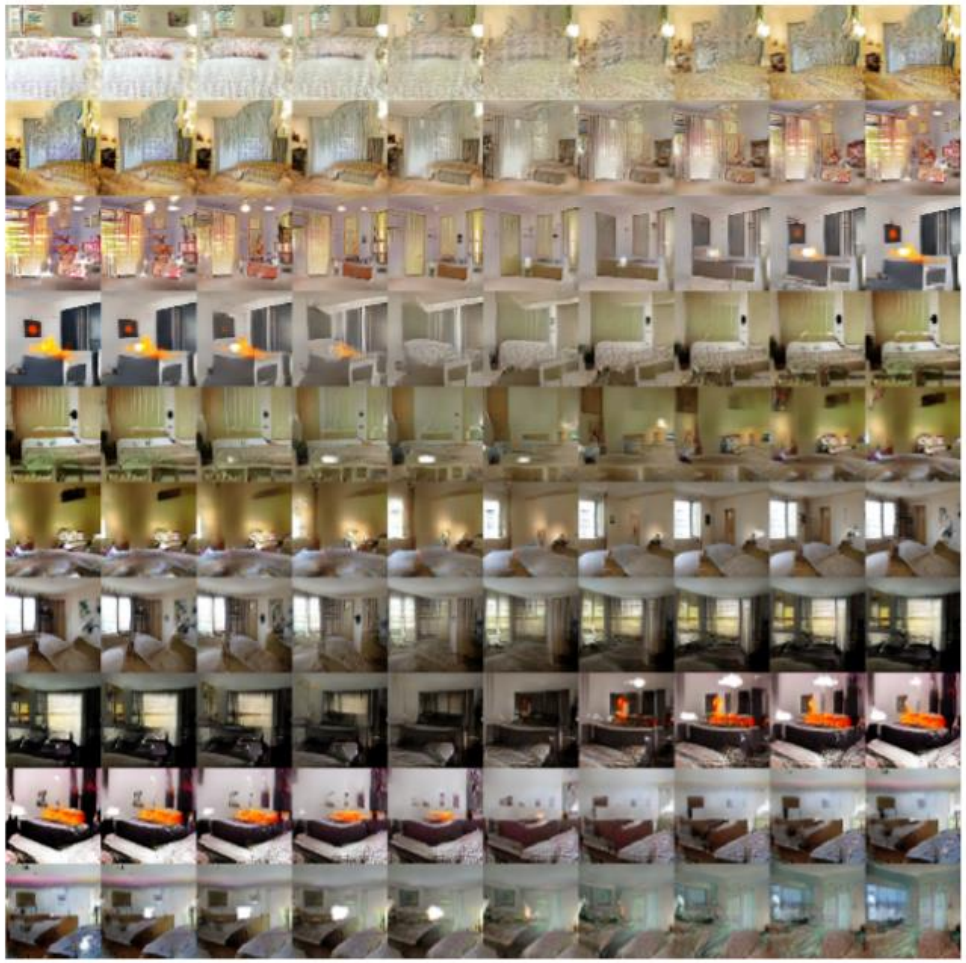


Figure 4: Top rows: Interpolation between a series of 9 random points in Z show that the space learned has smooth transitions, with every image in the space plausibly looking like a bedroom. In the 6th row, you see a room without a window slowly transforming into a room with a giant window. In the 10th row, you see what appears to be a TV slowly being transformed into a window.

If linearly interpolating in latent space gives rise to meaningful images (without sharp transitions), unlikely GAN is just memorizing.

Figure from
Radford, Metz, Chintala '16.

Inception score

Suppose we use trained network – the *Inception* architecture as a **labeler** for images. Inception gives probability over labels y for sample x: p(y|x).

Desirable features of generator: the Inception classifier should be "sure" about the label for most images (p(y|x) should have *low entropy*), and the classes it generates should be diverse (p(y) = $\mathbb{E}_{x\sim P_g} p(y|x)$ should have *high entropy*)

Thus, we want $H\big(p(y|x)\big)$ to be **low**, $H(p(y))$ is **high**.

Consider the expression: $\mathbb{E}_{x\sim P_g} KL(p(y|x)||p(y))$

$$= \mathbb{E}_{x\sim P_g} \mathbb{E}_{y\sim p(y|x)} \log p(y|x) - \log p(y)$$

$$= -\mathbb{E}_{x\sim P_g} H\big(p(y|x)\big) + H\big(p(y)\big)$$

Inception score

Suppose we use trained network – the *Inception* architecture as a **labeler** for images. Inception gives probability over labels y for sample x: p(y|x).

Desirable features of generator: the Inception classifier should be "sure" about the label for most images (p(y|x) should have *low entropy*), and the classes it generates should be diverse (p(y) = $\mathbb{E}_{x\sim P_g} p(y|x)$ should have *high entropy*)

Thus, we want $H\big(p(y|x)\big)$ to be **low**, $H(p(y))$ is **high**.

Consider the expression: $\mathbb{E}_{x\sim P_g} KL(p(y|x)||p(y))$

Inception score: $\exp\big(\mathbb{E}_{x\sim P_g} KL\big(p(y|x)\big|\big|p(y)\big)\big)$

Inception score

Suppose we use trained network – the *Inception* architecture as a **labeler** for images. Inception gives probability over labels y for sample x: p(y|x).

Desirable features of generator: the Inception classifier should be "sure" about the label for most images (p(y|x) should have *low entropy*), and the classes it generates should be diverse (p(y) = $\mathbb{E}_{x\sim P_g} p(y|x)$ should have *high entropy*)

Many follow ups, e.g. Frechet inception distance, modified inception score, …

Check for many other metrics: Borji '18