S.A) Se f EL'e a>0, mostre que
S.A) Se $f \in L'$ e $a > 0$, mostre que $E_a = 2x \in X$; $1f(x)1 > a$
tem mededa finita. Alem dino, o conjunto E= 3xeX; f(x) ≠03
e T-finito
RES)
Seja a>0 esuponta que µ(Ea)=+00 Note que aXEa < f , loyo
Note que a X = a < f , loyo
SaxEn < SIfT <+00
Mas SaxEa = a µ(Ea) = + 00, alusurdo.
Loyo, U(Ea) < + 00
Agora seja
$\pm_{n} = \{z \in X \mid f(z) \mid j \neq j \}$
Entro M(En) < +00, Vn e E = Un=1 En
$ = \bigcup_{n=1}^{\infty} E_n $
Loya E e v-finito.
S.B. Deja f mensurancel. De $f(x)=0$ 9.1.P. em X , então $f \in L'$ e $f = 0$
então félie
$\int f = 0$

```
RES)
De l cto, tomos flor=0 q.t.P. ⇒ 1f(00)=0 q.c.P.
                                  > SH < 50-0
        ⇒ SIf1=0 <+∞ => fel'
Alen ono,
            [ Sf[ < S1f[ = 0 => Sf=0 ]
S.C.) De f \in L' o g \in Mensurareel talque <math>f(x) = g(x)

g(t, P), entro g \in L' e

f(x) = f(x)
XES De lato, noteque h (x)= f(x)-g(x) = 0 q. L. P., logo,
               \int h(x) = 0
Mas
          \int h(x) = \int f(x) - g(x) = 0
```

 $-\int g(x) = \int f(x) - g(x) - \int f(x)$

 $\int g(x) < +\infty$

Lugo
$$\int g(x) = \int f(x) < \infty$$
.

(5. D) Ne $\int f \in L^{+}(X) = E > 0$, $\exists \ Q \ function supples mensursuel to lique $\int |f - f| < E$

Exercise $\int f = \int f^{+} - \int f^{-}, \ endo \int f^{+}e \int f^{-} xoo as partes regalizative e positive de f^{-} .

Entos, pela def de integral, clado $E > 0$, $\exists \ Q^{+}$, Q^{-} funcion sumples tois que $\int f^{+} + \int f^{-} + \int f^{-$$$

S. I Sera
$$f:X \to \mathbb{C}$$
. Mostre que f e'integrareel
re e so' sa lf le' integrareel, e
$$\left| \begin{cases} f \ d\mu \\ \end{cases} \leqslant \int |f| \ d\mu \end{aligned} \right|$$

RESP)

Pelato, suponla
$$f$$
 real. Entro

$$\left| \begin{array}{c} \left| \right| \right| = \left| \left| \right| f^{+} - \left| \right| + \left| \right| f^{-} = \left| \right| f \right| \right|$$

De f complexa, suponla $f \neq 0$.

Entro $f = re^{i\theta}$, $r > 0$, $\theta \in E_{0,2}\pi$).

Assum $\int e^{i\theta} f = e^{-i\theta} f = r = |f|$, lego

$$\left| \left| \left| f \right| = e^{-i\theta} f = |f| + |f|$$

$$= \int Re(e^{-i\theta} f) \leq \int |Re(e^{-i\theta} f)|$$

$$\leq \int |e^{-i\theta} f| = |f|$$

5.5)
$$\Lambda_{e,c} = f_n: X \rightarrow C, \forall n, f_n \rightarrow f_{q,t,p}.$$
 $\Lambda_{e} \ni g \in L^{1}(X) + algue |f_n| \leq g, g, t, p. entage$

$$\begin{cases} f \mid d\mu = lim \\ n \Rightarrow \infty \end{cases} f_n$$

KESP: RESP:

To lato, If n | < g q.t.P. => | Re(fn) | < g q.t.P.

| Im(fn) | < g q.t.P.

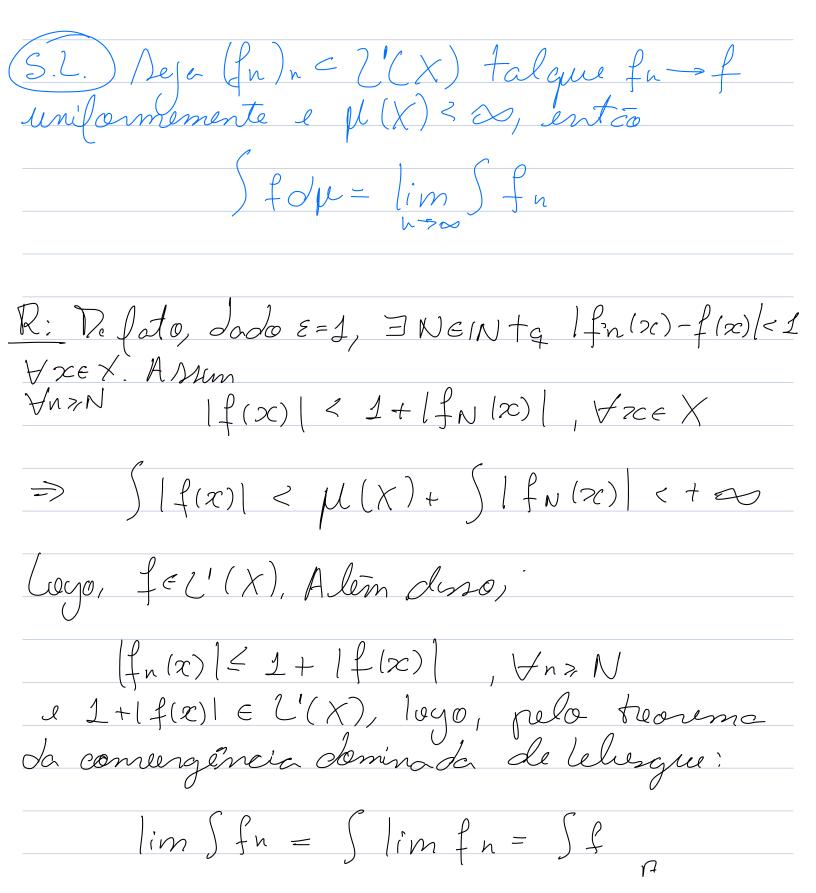
A lim dono | Re(fn) | -> | Re(f) | < g

| Im(fn) | -> | Im(fn) | < g q.t.P.

Lago, pelo teorema da consergência dominada de

Ulesguer

| im | Re(fn) = | Reff) $\lim_{n \to \infty} \operatorname{Im}(f_n) = \int \operatorname{Im}(f)$ Vondo lim fn = lim Re(fn) + i Im(fn) $= \int Re(f) + i \int Im(f)$



5.0. De fre L'1
$\sum_{n=1}^{\infty} \int f_n d\mu < +\infty$
Entoca série D'fn(x) comberge q.t.P. a uma fein Goofenn E'. Além d'uno
$\int \int d\mu = \sum_{n=1}^{\infty} \int \int$
Pens) Note que = Ifn \(\sum_{n=1} \) fu , ago, pelo T-CD.
$\int_{N-2\infty}^{\infty} f_n = \lim_{N\to\infty} \int_{n=1}^{\infty} f_n $ Soma funta
$\lim_{N\to\infty} f_n = \lim_{N\to\infty} f_n = \lim_{N\to\infty} f_n $
Into o' , $\int_{n=1}^{\infty} f_n = \sum_{n=1}^{\infty} \int_{n=1}^{\infty} f_n $
Loyo, a função $g = \sum_{s=1}^{\infty} f_s $ está em l' . Em particular, $\sum_{s=1}^{\infty} f_s(x) < +\infty$ $q.t.P$.
$\sum f_{i}(x) < + \infty q.t. P.$

\sim
Λ Λ - Λ - Λ
Hem and $ f_j(x) \leq q$, $\forall h$
\sim
$\sum_{j=1}^{n} f_j(x) \longrightarrow \sum_{j=1}^{n} f_j(x) q. \tau. \beta.$
J=() J
Leyo, Relo I.C. D.L. temess
$\lim_{n\to\infty}\int_{J=1}^{\infty}f_j(x)=\int_{J=1}^{\infty}\lim_{n\to\infty}\int_{J=1}^{\infty}f_j(x)$
11 Soma finita
$\int_{\overline{z}} f_{j}(z)$
$\lim_{x \to \infty} \int_{j=1}^{\infty} f_j(x)$
$\tilde{J} = (J + J)$
$\sum_{i=1}^{n} \int f_i(x)$
\int_{-1}^{2}
S. P. Nova Pine 21 Pin = Laitin
Mo The City The Transfer of the City Th
5. Pleja (fn)=21, fn=fait.p. Ne lim SIfn-f1=0 entoo SIf1=1m SIfn
RESP

De foto,

temos gul
$$\lim \sup \int |f_n| \leq \lim \sup \int |f_n - f| + \int |f|$$

$$= \lim \int |f_n - f| + \int |f|$$

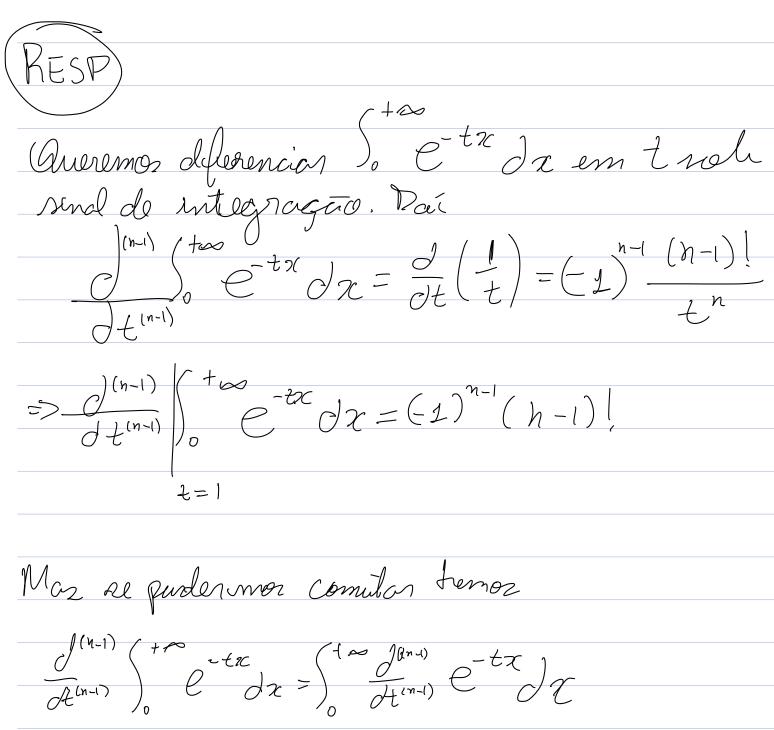
$$= \lim \int |f_n - f| + \int |f|$$

$$= \int |f| \leq \int |f| + \int |f|$$

$$\Rightarrow \int |f| \leq \lim \int |f| + \int |f|$$

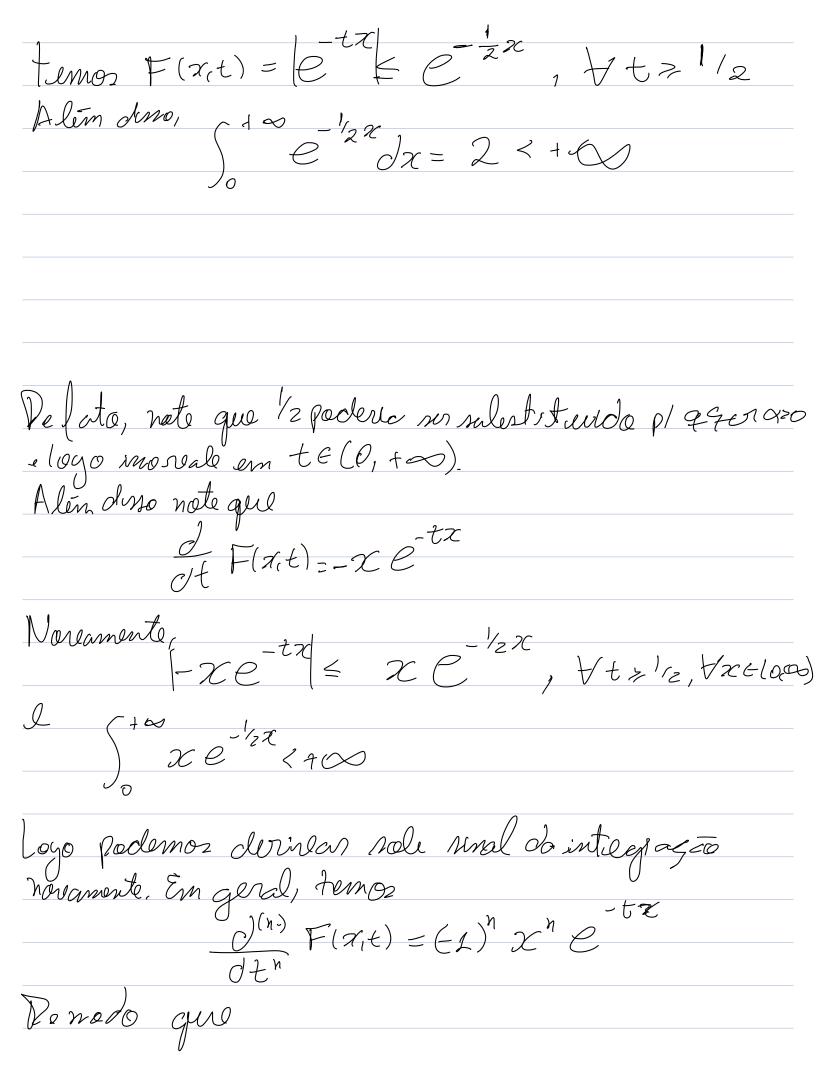
SQ Se
$$t>0$$
, ent $t = \frac{1}{t}$

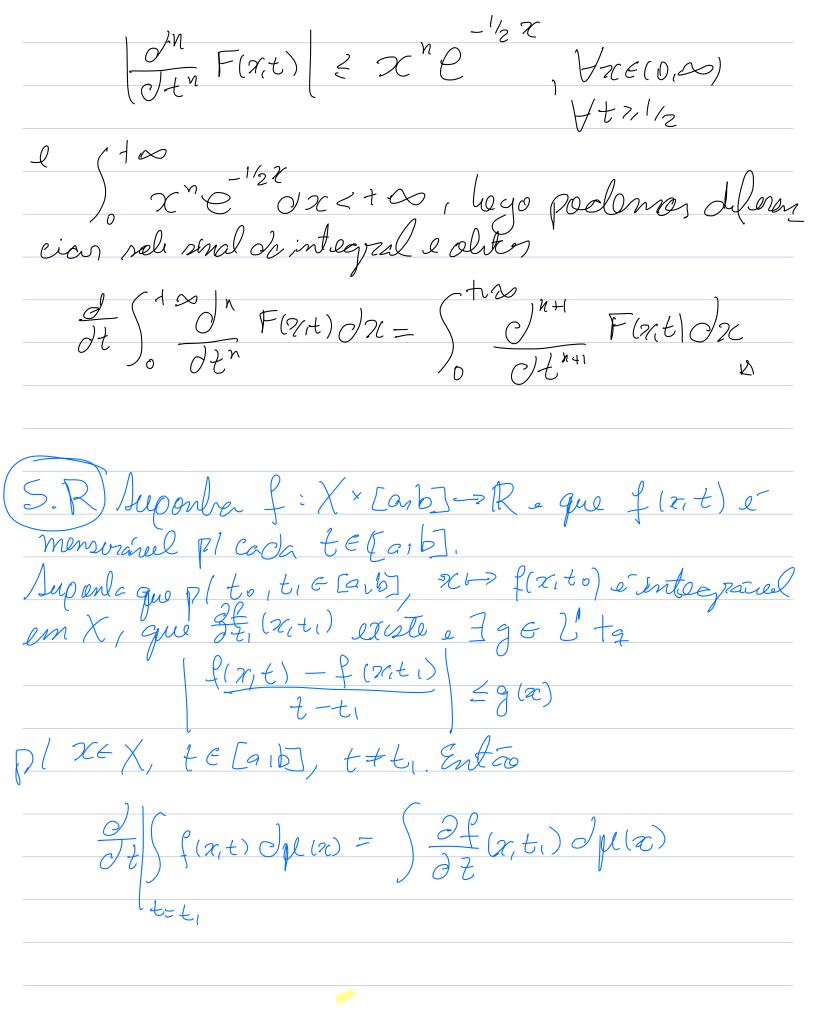
Além duno, re t > a > 0, entro $e^{-tx} = ax$ Use ino p/ provon que $\int_{0}^{+\infty} x^{n}e^{-x} dx = n!$



$$\frac{\int_{0}^{(N-1)} \left(\frac{1}{2} + \frac{1}{2} + \frac{1}{2$$

$$\log C = (n-1)!$$





Pens) seja tr C {a,b], tr-st,.
Panla $h_n(z) = \frac{f(x,t_n) - f(x,t_i)}{t_n - t_i}$
$\frac{\text{Entro}}{\left(h_n(x)\right) \leq g(x) \in \mathcal{U}}$
$h_{\mu}(x) \longrightarrow \mathcal{J}_{t} f(x,t_{i})$
Parmodo que pello t. C.D.L. tunca que Shulx) -> (2 fecti) Mas lim Shulx) de = of (fixiti) de l'ego regue a l'eli
$\lim_{n \to \infty} \int \underbrace{\frac{1x_i + n}{-f(x_i + i)}} dx$
$=\lim_{n\to\infty}\frac{\int f(x_it_n)\delta_z-\int f(x_it_i)dx}{t_n-t_i}$

Mjaf: X-> lR mensurairel.
S. 1) Plade nell, reja
$f_{u}(x) = \int f(x), \ ne f(x) \leq n$
$f_{u(x)} = \int f(x), ne f(x) \le n$ $\int sign(f(x)) n, ne f(x) > n$
Entro, re sup [fn < 100 Entre f e'integrand.
Entre félintegrand.
,
Resp.,
Respi. De loto, temorque f (x) = lim fu (2c), Hz e logo
logo
$\int f = \int \lim_{n \to \infty} f_n \leq \lim_{n \to \infty} f_n $
)(+1=)(lim fn = 1im in f)(fn)
250p∫[fn] < +∞