

# **Analysis of the Monetary Policy Impact on Regional Gross Domestic Product: A Regional DSGE Model**

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from March 1st, 2023 to Feb 23rd, 2024

PPGDE-UFPR

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2. Literature Review
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# Introduction



- **PROBLEM:** heterogeneous economic matrices that respond in diverse ways to the monetary decisions (BERTANHA; HADDAD, 2008).
- **OBJECTIVE:** develop a DSGE model with two regions to verify regional responses to monetary shocks.
- **CONTRIBUTION:** DSGE models demonstrate the relations among variables and determine how the economy respond to the monetary shock.

# Literature Review



- Costa Junior (2016): presents a RBC model and then adds NK elements in each chapter;
- Galí (2015): discuss monetary policy starting with a RBC model and also adds NK elements in each chapter;
- Bergholt (2012): presents a NK and the method of programming in *Dynare*;
- Solis-Garcia (2022): presents a RBC model and demonstrate the math tools necessary to solve a DSGE model;

- Rickman (2010): link between macro and regional modeling.
- Tamegawa (2012): two regions and fiscal policy.
- Tamegawa (2013): small regions and crowding in effects.
- Okano et al. (2015): regional economic stagnation.
- Mora e Costa Junior (2019): foreign direct investment (FDI) in Bogotá and the rest of Colombia.
- Costa Junior et al. (2022): fiscal policy, regional and national for the State of Goiás.
- Osterno et al. (2022): Regionalization of SAMBA: SAMBA+REG (Stochastic Analytical Model with Bayesian Approach from the Central Bank of Brazil).

# Regional NK DSGE Model

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## NK DSGE:

- **New Keynesian**
- **Dynamic**
- **Stochastic**
- **General Equilibrium**

## Sequential Process:

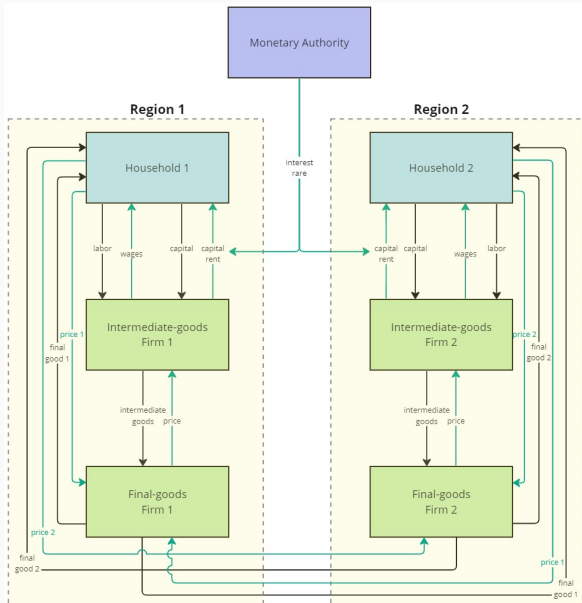
- Structural Model;
- Steady State;
- Log-linearization;
- Calibration;
- Impulse Response Functions.

# Characteristics

- four agents: households, intermediate and final-goods firms, monetary authority.
- no bonds.
- capital and investment.
- price stickiness of intermediate goods.
- two regions: final good is what links both.

- the representative household maximizes utility;
- firms producing intermediate goods minimize costs and maximize profit flow;
- firms producing final goods maximize profit.
- the monetary authority determines the interest rate, aiming to control inflation and pursuing economic growth.

# Model Structure



# Cost Minimization Problem

$$\min_{C_{\eta 1t}, C_{\eta 2t}} : Q_{\eta t} C_{\eta t} = P_{1t} C_{\eta 1t} + P_{2t} C_{\eta 2t} \quad (1)$$

$$\text{s. t. : } C_{\eta t} = C_{\eta 1t}^{\omega_{\eta 1}} C_{\eta 2t}^{1-\omega_{\eta 1}} \quad (2)$$

$$C_{\eta t} > 0$$

# Cost Minimization Problem

Solution:

$$C_{\eta 2t} = C_{\eta 1t} \frac{(1 - \omega_{\eta 1})P_{1t}}{\omega_{\eta 1}P_{2t}} \quad (3)$$

$$C_{\eta 1t} = C_{\eta t} \left( \frac{P_{2t}\omega_{\eta 1}}{P_{1t}(1 - \omega_{\eta 1})} \right)^{1-\omega_{\eta 1}} \quad (4)$$

$$Q_{\eta t} = \left( \frac{P_{1t}}{\omega_{\eta 1}} \right)^{\omega_{\eta 1}} \left( \frac{P_{2t}}{1 - \omega_{\eta 1}} \right)^{1-\omega_{\eta 1}} \quad (5)$$

# Household Maximization Problem

$$\max_{C_{\eta t}, L_{\eta t}, K_{\eta, t+1}} : U_{\eta}(C_{\eta t}, L_{\eta t}) = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left( \frac{C_{\eta t}^{1-\sigma}}{1-\sigma} - \phi \frac{L_{\eta t}^{1+\varphi}}{1+\varphi} \right) \quad (6)$$

$$\text{s. t. : } Q_{\eta t} C_{\eta t} + P_{\eta t} I_{\eta t} = W_{\eta t} L_{\eta t} + R_t K_{\eta t} + \Pi_{\eta t} \quad (7)$$

$$K_{\eta, t+1} = (1 - \delta) K_{\eta t} + I_{\eta t} \quad (8)$$

$$C_{\eta t}, L_{\eta t}, K_{\eta t} > 0$$



# Household Maximization Problem

Solution:

$$\frac{\phi L_{\eta t}^{\varphi}}{C_{\eta t}^{-\sigma}} = \frac{W_{\eta t}}{Q_{\eta t}} \quad (9)$$

$$\frac{\mathbb{E}_t\{Q_{\eta,t+1}C_{\eta,t+1}^{\sigma}\}}{Q_{\eta t}C_{\eta t}^{\sigma}} = \beta \frac{\mathbb{E}_t\{P_{\eta,t+1}(1-\delta) + R_{t+1}\}}{P_{\eta t}} \quad (10)$$

# Final-goods Firm Maximization Problem

$$\max_{Y_{\eta jt}} : P_{\eta t} Y_{\eta t} - \int_0^1 P_{\eta jt} Y_{\eta jt} dj \quad (11)$$

$$\text{s. t. : } Y_{\eta t} = \left( \int_0^1 Y_{\eta jt}^{\frac{\psi-1}{\psi}} dj \right)^{\frac{\psi}{\psi-1}} \quad (12)$$

# Final-goods Firm Maximization Problem

Solution:

$$Y_{\eta jt} = Y_t \left( \frac{P_{\eta t}}{P_{\eta jt}} \right)^{\psi} \quad (13)$$

$$P_{\eta t} = \left[ \int_0^1 P_{\eta jt}^{1-\psi} dj \right]^{\frac{1}{1-\psi}} \quad (14)$$

# Intermediate-goods Firm Problems

$$\min_{K_{\eta jt}, L_{\eta jt}} : R_t K_{\eta jt} + W_t L_{\eta jt} \quad (15)$$

$$\text{s. t. : } Y_{\eta jt} = Z_{A\eta t} K_{\eta jt}^{\alpha_\eta} L_{\eta jt}^{1-\alpha_\eta} \quad (16)$$

# Intermediate-goods Firm Problems

Solutions:

$$\frac{K_{\eta jt}}{L_{\eta jt}} = \left( \frac{\alpha_{\eta}}{1 - \alpha_{\eta}} \right) \frac{W_{\eta t}}{R_t} \quad (17)$$

$$K_{\eta jt} = \frac{Y_{\eta jt}}{Z_{A\eta t}} \left[ \left( \frac{\alpha_{\eta}}{1 - \alpha_{\eta}} \right) \frac{W_{\eta t}}{R_t} \right]^{1 - \alpha_{\eta}} \quad (18)$$

$$L_{\eta jt} = \frac{Y_{\eta jt}}{Z_{A\eta t}} \left[ \left( \frac{\alpha_{\eta}}{1 - \alpha_{\eta}} \right) \frac{W_{\eta t}}{R_t} \right]^{-\alpha_{\eta}} \quad (19)$$

$$\Lambda_{\eta t} = \frac{1}{Z_{A\eta t}} \left( \frac{R_t}{\alpha_{\eta}} \right)^{\alpha_{\eta}} \left( \frac{W_{\eta t}}{1 - \alpha_{\eta}} \right)^{1 - \alpha_{\eta}} \quad (20)$$

# Intermediate-goods Firm Problems

Price Stickiness and Profit Flow, Calvo's Rule (CALVO, 1983):

$$\max_{P_{\eta jt}} : \mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \frac{\theta^s [P_{\eta jt} Y_{\eta j, t+s} - TC_{\eta j, t+s}]}{\prod_{k=0}^{s-1} (1 + R_{t+k})} \right\} \quad (21)$$

$$\text{s. t. : } Y_{\eta jt} = Y_{\eta t} \left( \frac{P_{\eta t}}{P_{\eta jt}} \right)^{\psi} \quad (13)$$

# Intermediate-goods Firm Problems

Solution:

$$P_{\eta t}^* = \frac{\psi}{\psi - 1} \cdot \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \theta^s Y_{\eta j, t+s} \Lambda_{\eta, t+s} / \prod_{k=0}^{s-1} (1 + R_{t+k}) \right\}}{\mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \theta^s Y_{\eta j, t+s} / \prod_{k=0}^{s-1} (1 + R_{t+k}) \right\}} \quad (22)$$

$$P_{\eta t} = \left[ \theta P_{\eta, t-1}^{1-\psi} + (1 - \theta) P_{\eta t}^{*1-\psi} \right]^{\frac{1}{1-\psi}} \quad (23)$$

$$\pi_{\eta t} = \frac{P_{\eta t}}{P_{\eta, t-1}} \quad (24)$$



Taylor's Rule (TAYLOR, 1993):

$$\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\gamma_R} \left[ \left( \frac{\pi_t}{\pi} \right)^{\gamma_\pi} \left( \frac{Y_t}{Y} \right)^{\gamma_Y} \right]^{1-\gamma_R} Z_{Mt} \quad (25)$$

$$\text{where: } \pi_t = \pi_{1t}^{\theta_\pi} \pi_{2t}^{1-\theta_\pi} \quad (26)$$

$$\text{and: } \theta_\pi = \frac{P_{1t} Y_{1t}}{P_{1t} Y_{1t} + P_{2t} Y_{2t}} \quad (27)$$

Productivity Shock:

$$\ln Z_{At} = (1 - \rho_A) \ln Z_A + \rho_A \ln Z_{A,t-1} + \varepsilon_{At} \quad (28)$$

Monetary Policy Shock:

$$\ln Z_{Mt} = (1 - \rho_M) \ln Z_M + \rho_M \ln Z_{M,t-1} + \varepsilon_{Mt} \quad (29)$$

# Equilibrium Conditions

A Competitive Equilibrium consists of sequences of:

- prices  $\{P_{\eta t}^*, R_t^*, W_{\eta t}^*\}$ ,
- allocations for households  $\mathcal{A}_H := \{C_{\eta 1t}^*, C_{\eta 2t}^*, L_{\eta t}^*, I_{\eta t}^*, K_{\eta, t+1}^*\}$
- allocations for firms  $\mathcal{A}_F := \{K_{\eta jt}^*, L_{\eta jt}^*, Y_{\eta jt}^*, Y_{\eta t}^*\}$ .

In such an equilibrium, given the set of exogenous variables  $\{K_0, Z_{A\eta t}, Z_{Mt}\}$ , the elements in  $\mathcal{A}_H$  solve the household problem, while the elements in  $\mathcal{A}_F$  solve the firms' problems, and the markets for goods and labor clear:

$$Y_t = Y_{1t} + Y_{2t} \quad (30)$$

$$L_{\eta t} = \int_0^1 L_{\eta jt} dj \quad (31)$$

Steady State

Steady state solution (COSTA JUNIOR, 2016, p.41):

$$\mathbb{E}_t X_{t+1} = X_t = X_{t-1} = X_{ss} \quad (32)$$

# Log-linearization

Uhlig's rules for log-linearization (UHLIG, 1999).

$$\hat{x}_t := \frac{x_t - X}{X} \iff x_t = X(1 + \hat{x}_t)$$

Square system of 30 variables and equations:

- Real Variables:  $\langle \hat{C}_\eta \quad \hat{L}_\eta \quad \hat{K}_\eta \quad \hat{I}_\eta \quad \hat{C}_{\eta 1} \quad \hat{C}_{\eta 2} \quad \hat{Y}_\eta \quad \hat{Y} \quad \hat{Z}_{A\eta} \quad \hat{Z}_M \rangle$ ;
- Nominal Variables:  $\langle \hat{Q}_\eta \quad \hat{P}_\eta \quad \hat{R} \quad \hat{\pi} \quad \hat{W}_\eta \quad \hat{\lambda}_\eta \quad \hat{\pi}_\eta \rangle$ .



- Regional Gross Inflation Rate

$$\hat{\pi}_{\eta t} = \hat{P}_{\eta t} - \hat{P}_{\eta, t-1} \quad (33)$$

- New Keynesian Phillips Curve

$$\hat{\pi}_{\eta t} = \beta \mathbb{E}_t \hat{\pi}_{\eta, t+1} + \frac{(1-\theta)(1-\theta\beta)}{\theta} \hat{\lambda}_{\eta t} \quad (34)$$

- Regional Consumption Weight

$$\hat{C}_{\eta 2t} - \hat{C}_{\eta 1t} = \hat{P}_{1t} - \hat{P}_{2t} \quad (35)$$

- Regional Consumption of Good 1

$$\hat{C}_{\eta t} - \hat{C}_{\eta 1t} = (1 - \omega_{\eta 1})(\hat{P}_{1t} - \hat{P}_{2t}) \quad (36)$$

- Regional Price Index

$$\hat{Q}_{\eta t} = \omega_{\eta 1}\hat{P}_{1t} + (1 - \omega_{\eta 1})\hat{P}_{2t} \quad (37)$$

- Labor Supply

$$\varphi\hat{L}_{\eta t} + \sigma\hat{C}_{\eta t} = \hat{W}_{\eta t} - \hat{Q}_{\eta t} \quad (38)$$

- Law of Motion for Capital

$$\hat{K}_{\eta,t+1} = (1 - \delta)\hat{K}_{\eta t} + \delta\hat{I}_{\eta t} \quad (39)$$

- Euler equation for capital return

$$\begin{aligned}(\hat{Q}_{\eta,t+1} - \hat{Q}_{\eta t}) + \sigma(\hat{C}_{\eta,t+1} - \hat{C}_{\eta t}) - (\hat{P}_{\eta,t+1} - \hat{P}_{\eta t}) &= \\ &= \beta r(\hat{R}_{\eta,t+1} - \hat{P}_{\eta,t+1})\end{aligned}\tag{40}$$

- Production Function

$$\hat{Y}_{\eta t} = \hat{Z}_{A\eta t} + \alpha_{\eta} \hat{K}_{\eta t} + (1 - \alpha_{\eta}) \hat{L}_{\eta t}\tag{41}$$

- Technical and Economic Marginal Rates of Substitution

$$\hat{K}_{\eta t} - \hat{L}_{\eta t} = \hat{W}_{\eta t} - \hat{R}_{Kt}\tag{42}$$

- Marginal Cost

$$\hat{\lambda}_{\eta t} = \alpha_{\eta} \hat{R}_{Kt} + (1 - \alpha_{\eta}) \hat{W}_{\eta t} - \hat{Z}_{A\eta t} - \hat{P}_{\eta t}\tag{43}$$

- Monetary Policy

$$\hat{R}_t = \gamma_R \hat{R}_{t-1} + (1 - \gamma_R)(\gamma_\pi \hat{\pi}_t + \gamma_Y \hat{Y}_t) + \hat{Z}_{Mt} \quad (44)$$

- National Gross Inflation Rate

$$\hat{\pi}_t = \theta_\pi \hat{\pi}_{1t} + (1 - \theta_\pi) \hat{\pi}_{2t} \quad (45)$$

- Productivity Shock

$$\hat{Z}_{A\eta t} = \rho_{A\eta} \hat{Z}_{A\eta, t-1} + \varepsilon_{A\eta} \quad (46)$$

- Monetary Shock

$$\hat{Z}_{Mt} = \rho_M \hat{Z}_{M, t-1} + \varepsilon_M \quad (47)$$

- Goods-Market Clearing Condition

$$\hat{Y}_t = \theta_Y \hat{Y}_{1t} + (1 - \theta_Y) \hat{Y}_{2t} \quad (48)$$

- Regional Goods-Market Clearing Condition

$$\hat{Y}_{\eta t} = \theta_{C\eta} \hat{C}_{\eta t} + (1 - \theta_{C\eta}) \hat{I}_{\eta t} \quad (49)$$

# Results

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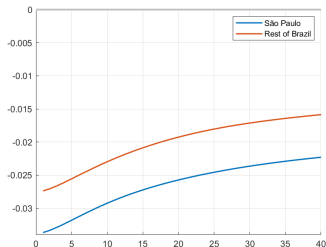
- Given the Regional DSGE model, a region more capital intensive ( $\alpha_\eta$ ) and with a higher production technology level ( $Z_{A\eta}$ ) is more sensitive to a monetary shock:

$$\alpha_1 > \alpha_2 \wedge Z_{A1} > Z_{A2} \implies \Delta \mathbf{X}_1 > \Delta \mathbf{X}_2 \quad (50)$$

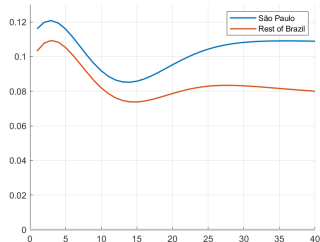
- The responses are different in intensity, but not in direction.

# Impulse Response Functions

## Positive Monetary Policy Shock of 1%



**(a)** Consumption

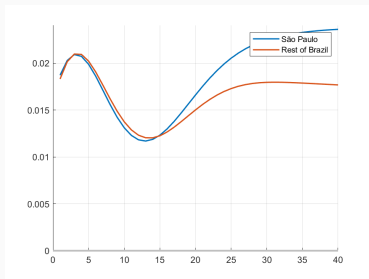


**(b)** Investment

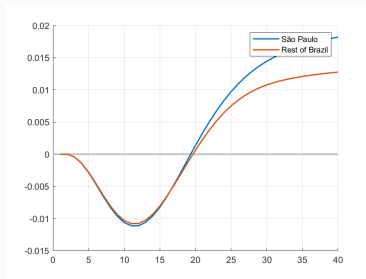


# Impulse Response Functions

## Positive Monetary Policy Shock of 1%



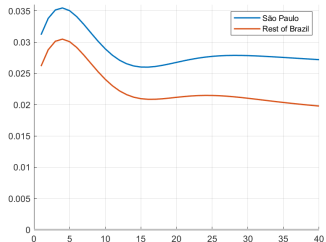
**(a)** Production



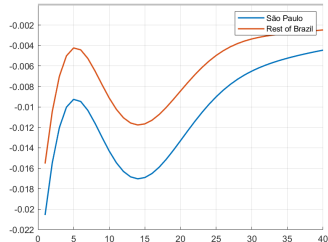
**(b)** Capital

# Impulse Response Functions

## Positive Monetary Policy Shock of 1%



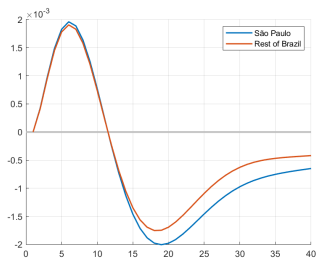
**(a)** Labor



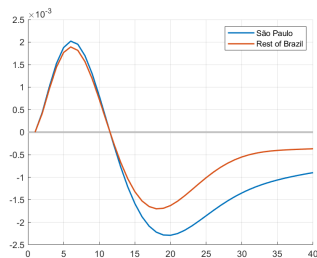
**(b)** Wages

# Impulse Response Functions

## Positive Monetary Policy Shock of 1%



**(a)** Consumer Price Level



**(b)** Price Level

## Final Remarks

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- The national wide interest rate may not have the same effectiveness across all regions of a country with different economic structures.
- Other instruments may be applied in conjunction to achieve the desired objectives of public policies.
- Future challenges include augmenting the model with additional elements to better describe Brazilian regions.

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**Obrigado!**

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**<https://github.com/andrlb/mastersthesis>**

## **Dúvidas e Sugestões**