0.1 REGIONAL NEW KEYNESIAN MODEL

The model is populated by four agents: (1) a representative household, (2) a continuum of firms producing intermediate-goods, (3) a firm producing final-goods, and (4) the monetary authority.

The representative household maximizes utility based on consumption and labor, subject to a budget constraint composed of wages, capital rental rates, and firm profits.

The final-goods firm produces the final-good consumed by households: it aggregates all intermediate-goods produced by intermediate firms, operates under perfect competition and seeks to maximize profit subject to the bundle technology.

Each intermediate-goods firm produces a single intermediate-good, all exhibiting imperfect substitution, thus operating in monopolistic competition. Intermediate-goods firms have two problems to solve: minimize costs subject to the production technology available and choose an optimal price to maximize the intertemporal profit flow.

Periodically, a portion of intermediate-goods firms have the opportunity to adjust prices, while others miss this chance, following a **calvo_staggered_1983** rule. This mechanism generates nominal price rigidities, altering equilibrium relationships in the system. These rigidities lead to the non-neutrality of money in the short term, as explained by **costa_junior_understanding_2016**.

The monetary authority determines the nominal interest rate in response to fluctuations in previous period's inflation and production, aiming to control price levels and growth, following a **taylor_discretion_1993** rule.

Stochastic shocks will be present in the intermediate-goods firms' productivity and in the monetary policy.

These elements define a canonical NK DSGE model, as presented by (**solis-garcia_ucb_2022**). The model will be adapted to accommodate two distinct regions: the main region and the rest of the country, replacing the single aggregated region. To achieve this, an index will differentiate the studied region from the rest of the country, resulting in separate households, intermediate- and final-goods firms for each region. Households lack mobility between regions. The link connecting the two regions is established through the final-goods, allowing households to consume from both regions.

Then, equilibrium conditions of the system will be determined. Assuming the system tends toward long-term equilibrium, a steady state will be reached where variables cease to change. Thus, for a given $t \to \infty$, there is a $\boldsymbol{X}_t = \boldsymbol{X}_{t+1} = \boldsymbol{X}_{ss} \implies \dot{\boldsymbol{X}} = 0$, where $\boldsymbol{X}$ denotes the vector of system variables, the subscript ss indicates the steady state and $\dot{\boldsymbol{X}} = {\partial X}/{\partial t}$.

After that, the log-linearization method proposed by **uhlig_toolkit_1999** will be employed to convert the system of equations into a linear system, so that this linear system can be solved by the program Dynare, which computes the solution and produces impulse-response graphs based on the stochastic shocks.

0.1.1 Regions

Regions will be indexed by $\eta \in \{1, 2, \ldots, n\}$, representing the variables of each region. Whenever necessary, a second region index, $\nu \in \{1, 2, \ldots, n\}$, will be used. For example, the variable C_t represents the total consumption (the aggregate of all regions), $C_{\eta t}$ represents the consumption composition of region η, and $C_{\eta \nu t}$ represents the consumption of the final good produced in region ν and consumed in region η (with the first index indicating the destination and the second one indicating the origin of the goods). Without loss of generality, the model will consider two regions: the main region labeled as 1 and the rest of the country as 2, so that $\eta, \nu \in \{1, 2\}$.

Figure (1) illustrates the model's mechanics. In this diagram, black arrows depict the real economy, while green arrows represent the nominal economy. The representative household supplies labor and capital to intermediate-goods firms in exchange for wages and capital rent, respectively. Using these resources, intermediate-goods firms produce goods, which are then sold to the final-goods firm. The final-goods firm aggregates all intermediate-goods into a final product, sold back to the household. Operating under a monetary rule, the monetary authority determines the nominal interest rate to achieve output growth and price stability.

Figure 1: Model Diagram

Source: created by the author.

0.1.2 Household

The household problem is divided into two steps: first, the household must minimize the consumption costs, and then maximize the utility, which is subject to a budget constraint.

To solve the cost minimization problem, consider that the representative household must decide on consuming goods from both regions. For this purpose, there must be a consumption bundle index $C_{\eta t}$ and a consumption price index $Q_{\eta t}$ that minimize the total consumption cost $Q_{\eta t}C_{\eta t}$, as demonstrated by **walsh_monetary_2017**:

$$\min_{C_{\eta 1t}, C_{\eta 2t}} : \quad Q_{\eta t}C_{\eta t} = P_{1t}C_{\eta 1t} + P_{2t}C_{\eta 2t} \tag{0.1}$$

$$\text{s. t.} : \quad C_{\eta t} = C_{\eta 1t}^{\omega_{\eta 1}} C_{\eta 2t}^{1-\omega_{\eta 1}} \tag{0.2}$$

$$C_{\eta t} > 0$$

where P_{1t} and P_{2t} are the prices of goods 1 and 2, respectively, $C_{\eta 1t}$ and $C_{\eta 2t}$ are the goods produced in region 1 and 2, respectively, and consumed in region η. In the consumption aggregation, $\omega_{\eta 1}$ and $(1 - \omega_{\eta 1})$ are the weights of goods $C_{\eta 1t}$ and $C_{\eta 2t}$, respectively, in the consumption bundle $C_{\eta t}$.

The solutions for the household cost minimization problem are:

$$C_{\eta 2t} = C_{\eta 1t} \frac{(1 - \omega_{\eta 1})P_{1t}}{\omega_{\eta 1}P_{2t}} \tag{0.3}$$

$$C_{\eta 1t} = C_{\eta t} \left(\frac{P_{2t}\omega_{\eta 1}}{P_{1t}(1 - \omega_{\eta 1})} \right)^{1-\omega_{\eta 1}} \tag{0.4}$$

$$Q_{\eta t} = \left(\frac{P_{1t}}{\omega_{\eta 1}} \right)^{\omega_{\eta 1}} \left(\frac{P_{2t}}{1 - \omega_{\eta 1}} \right)^{1-\omega_{\eta 1}} \tag{0.5}$$

Therefore, there is a consumption bundle $C_{\eta t}$ and a consumption price index $Q_{\eta t}$ that minimize the total consumption cost $Q_{\eta t}C_{\eta t}$ for the household in region η. Notice that the cost problems of both regions are (must be) related, as the consumption level in one region influences the demand for goods in both regions. Now, this result will be used in the next problem that the household faces.

Following the models presented by **costa_junior_understanding_2016** and **solis-garcia_ucb_2022**, the representative household next problem is to maximize an intertemporal utility function U_η with respect to consumption $C_{\eta t}$ and labor $L_{\eta t}$, subject to a budget constraint, a capital accumulation rule and the non-negativity of real variables:

$$\max_{C_{\eta t},L_{\eta t},K_{\eta,t+1}} : \quad U_\eta(C_{\eta t}, L_{\eta t}) = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\frac{C_{\eta t}^{1-\sigma}}{1-\sigma} - \phi \frac{L_{\eta t}^{1+\varphi}}{1+\varphi} \right) \tag{0.6}$$

$$\text{s. t. :} \quad Q_{\eta t} C_{\eta t} + P_{\eta t} I_{\eta t} = W_{\eta t} L_{\eta t} + R_t K_{\eta t} + \Pi_{\eta t} \tag{0.7}$$

$$K_{\eta,t+1} = (1-\delta) K_{\eta t} + I_{\eta t} \tag{0.8}$$

$$C_{\eta t}, L_{\eta t}, K_{\eta t} > 0$$

where $\mathbb{E}_t$ is the expectation operator, β is the intertemporal discount factor, σ is the relative risk aversion coefficient, ϕ is the relative labor weight in utility, φ is the marginal disutility of labor supply. In the budget constraint, $I_{\eta t}$ is the investment, $W_{\eta t}$ is the wage level, $K_{\eta t}$ is the capital, R_t is the return on capital, and $\Pi_{\eta t}$ is the firm profit. In the capital accumulation rule, δ is the capital depreciation rate.

The solutions for the household utility maximization problem are:

$$\frac{\phi L_{\eta t}^{\varphi}}{C_{\eta t}^{-\sigma}} = \frac{W_{\eta t}}{Q_{\eta t}} \tag{0.9}$$

Equation 0.9 is the Household Labor Supply and shows that the marginal rate of substitution (MRS) of labor for consumption is equal to the real wage, which is the relative price between labor and goods.

$$\frac{\mathbb{E}_t\{Q_{\eta,t+1} C_{\eta,t+1}^{\sigma}\}}{Q_{\eta t} C_{\eta t}^{\sigma}} = \beta \frac{\mathbb{E}_t\{P_{\eta,t+1}(1-\delta) + R_{t+1}\}}{P_{\eta t}} \tag{0.10}$$

Equation 0.10 is the Euler equation for the return on capital.

0.1.3 Firms

Consider two types of firms: (1) a continuum of intermediate-goods firms, which operate in monopolistic competition and each produce one variety with imperfect substitution level between each other and (2) the final-goods firm, which aggregates all these varieties into a final bundle and operates in perfect competition.

0.1.4 Final-Goods Firm

The role of the final-goods firm is to aggregate all the varieties $Y_{\eta jt}$ produced by the intermediate-goods firms in each region $\eta \in \{1, 2\}$, so that the representative consumer can buy only one good $Y_{\eta t}$, the bundle good, from each region.

The final-goods firm problem is to maximize its profit, considering that its output is the bundle $Y_{\eta t}$ formed by a continuum $j \in [0, 1]$ of intermediate-goods $Y_{\eta jt}$, with elasticity of substitution between intermediate-goods ψ:

$$\max_{Y_{\eta jt}} : \quad P_{\eta t} Y_{\eta t} - \int_0^1 P_{\eta jt} Y_{\eta jt} \, \mathrm{d}\, j \tag{0.11}$$

$$\text{s. t.} : \quad Y_{\eta t} = \left(\int_0^1 Y_{\eta jt}^{\frac{\psi - 1}{\psi}} \, \mathrm{d}\, j \right)^{\frac{\psi}{\psi - 1}} \tag{0.12}$$

The solutions for the final-goods firm problem are:

$$Y_{\eta jt} = Y_t \left(\frac{P_{\eta t}}{P_{\eta jt}} \right)^{\psi} \tag{0.13}$$

Equation 0.13 shows that the demand for variety j depends on its relative price.

$$P_{\eta t} = \left[\int_0^1 P_{\eta jt}^{1 - \psi} \, \mathrm{d}\, j \right]^{\frac{1}{1 - \psi}} \tag{0.14}$$

Equation 0.14 is the final-goods firm's markup.

0.1.5 Intermediate-Goods Firms

The intermediate-goods firms, denoted by $j \in [0, 1]$, produce varieties of a representative good with a certain level of substitutability. Each of these firms has to choose labor $L_{\eta jt}$ to minimize production costs, subject to a technology rule.

$$\min_{K_{\eta jt}, L_{\eta jt}} : \quad R_t K_{\eta jt} + W_t L_{\eta jt} \tag{0.15}$$

$$\text{s. t.} : \quad Y_{\eta jt} = Z_{A\eta t} K_{\eta jt}^{\alpha_\eta} L_{\eta jt}^{1 - \alpha_\eta} \tag{0.16}$$

where $Y_{\eta jt}$ is the output obtained by the production technology level $Z_{A\eta t}$ that transforms capital $K_{\eta jt}$ and labor $L_{\eta jt}$ in proportions α_η and $(1-\alpha_\eta)$, respectively, into intermediate goods.[1]

The solutions for the intermediate-goods firm problem are:

$$\frac{K_{\eta jt}}{L_{\eta jt}} = \left(\frac{\alpha_\eta}{1-\alpha_\eta}\right)\frac{W_{\eta t}}{R_t} \tag{0.17}$$

Equation 0.17 demonstrates the relationship between the technical marginal rate of substitution (TMRS) and the economic marginal rate of substitution (EMRS).

$$K_{\eta jt} = \frac{Y_{\eta jt}}{Z_{A\eta t}}\left[\left(\frac{\alpha_\eta}{1-\alpha_\eta}\right)\frac{W_{\eta t}}{R_t}\right]^{1-\alpha_\eta} \tag{0.18}$$

Equation 0.18 is the intermediate-goods firm demand for capital.

$$L_{\eta jt} = \frac{Y_{\eta jt}}{Z_{A\eta t}}\left[\left(\frac{\alpha_\eta}{1-\alpha_\eta}\right)\frac{W_{\eta t}}{R_t}\right]^{-\alpha_\eta} \tag{0.19}$$

Equation 0.19 is the intermediate-goods firm demand for labor.

$$TC_{\eta jt} = \frac{Y_{\eta jt}}{Z_{A\eta t}}\left(\frac{R_t}{\alpha_\eta}\right)^{\alpha_\eta}\left(\frac{W_{\eta t}}{1-\alpha_\eta}\right)^{1-\alpha_\eta} \tag{0.20}$$

Equation 0.20 is the intermediate-goods firm total cost.

$$\Lambda_{\eta t} = \frac{1}{Z_{A\eta t}}\left(\frac{R_t}{\alpha_\eta}\right)^{\alpha_\eta}\left(\frac{W_{\eta t}}{1-\alpha_\eta}\right)^{1-\alpha_\eta} \tag{0.21}$$

Equation 0.21 is the intermediate-goods firm marginal cost.

Consider an economy with price stickiness, following a Calvo Rule

[1] the production technology level $Z_{A\eta t}$ will be submitted to a productivity shock, detailed in section **??**.

(**calvo_staggered_1983**): each firm has a probability $(0 < \theta < 1)$ of keeping its price in the next period $(P_{\eta j,t+1} = P_{\eta jt})$, and a probability $(1-\theta)$ of setting a new optimal price $P^*_{\eta jt}$ that maximizes its profits. Therefore, each firm must take this uncertainty into account when deciding the optimal price: the intertemporal profit flow, given the nominal interest rate R_t of each period, is calculated considering the probability θ of keeping the previous price:

$$\max_{P_{\eta jt}} : \quad \mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \frac{\theta^s \left[P_{\eta jt} Y_{\eta j,t+s} - TC_{\eta j,t+s} \right]}{\prod_{k=0}^{s-1} (1 + R_{t+k})} \right\} \tag{0.22}$$

$$\text{s.t.} : \quad Y_{\eta jt} = Y_{\eta t} \left(\frac{P_{\eta t}}{P_{\eta jt}} \right)^{\psi} \tag{0.13}$$

where s is the period in time when the decision must be made; t is the last period in time when the price was updated and k is the period in the future when the interest rate applies.

The solution to the optimal price problem is equation 0.23, which represents the optimal price that firm j will choose. Since all firms that are able to choose will opt for the highest possible price, they will all select the same price and for that the index j can be omitted.

$$P^*_{\eta t} = \frac{\psi}{\psi - 1} \cdot \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \theta^s Y_{\eta j,t+s} \Lambda_{\eta,t+s} / \prod_{k=0}^{s-1} (1 + R_{t+k}) \right\}}{\mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \theta^s Y_{\eta j,t+s} / \prod_{k=0}^{s-1} (1 + R_{t+k}) \right\}} \tag{0.23}$$

For the sake of closure, the intermediate-goods firm profit must be defined:

$$\Pi_{\eta t} = P_{\eta t} \int_0^1 Y_{\eta jt} \, \mathrm{d}\, j - W_{\eta t} L_{\eta t} \tag{0.24}$$

And the household constraint in terms of the intermediate-goods firm profit is:

$$Q_{\eta t} C_{\eta t} + P_{\eta t} I_{\eta t} = R_t K_{\eta t} + P_{\eta t} \int_0^1 Y_{\eta jt} \, \mathrm{d}\, j \tag{0.25}$$

0.1.6 Final-Goods Firm, part II

The process of fixing prices is random: in each period, θ firms will maintain the price from the previous period, while $(1-\theta)$ firms will choose a new optimal price. The price level for each period will be a composition of these two prices. Use this information in 0.14 to determine the aggregate price level:

$$P_{\eta t} = \left[\theta P_{\eta,t-1}^{1-\psi} + (1-\theta) P_{\eta t}^{*1-\psi}\right]^{\frac{1}{1-\psi}} \tag{0.26}$$

Equation 0.26 is the aggregate price level.

0.1.7 Regional Inflation

In each region, the price level $P_{\eta t}$ generates a regional inflation rate:

$$\pi_{\eta t} = \frac{P_{\eta t}}{P_{\eta,t-1}} \tag{0.27}$$

0.1.8 Monetary Authority

The objective of the monetary authority is to conduct the economy to price stability and economic growth, using a Taylor rule (**taylor_discretion_1993**) to determine the nominal interest rate:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\gamma_\pi} \left(\frac{Y_t}{Y}\right)^{\gamma_Y}\right]^{1-\gamma_R} Z_{Mt} \tag{0.28}$$

where R, π, Y are the nominal interest rate, gross inflation rate and the production level in steady state, respectively; γ_R is the smoothing parameter for the interest rate R_t, γ_π and γ_Y are the interest-rate sensitivities in relation to inflation and product, respectively, Z_{Mt} is the monetary shock and π_t is the gross inflation rate, defined by:[2]

$$\pi_t = \pi_{1t}^{\theta_\pi} \pi_{2t}^{1-\theta_\pi} \tag{0.29}$$

$$\text{where: } \theta_\pi = \frac{P_{1t} Y_{1t}}{P_{1t} Y_{1t} + P_{2t} Y_{2t}} \tag{0.30}$$

[2] for the monetary shock definition, see section **??**.

0.1.9 Stochastic Shocks

The production technology level $Z_{A\eta t}$ will be submitted to a productivity shock defined by a first-order autoregressive process $AR(1)$:

$$\ln Z_{A\eta t} = (1 - \rho_{A\eta}) \ln Z_{A\eta} + \rho_{A\eta} \ln Z_{A\eta,t-1} + \varepsilon_{A\eta t} \tag{0.31}$$

where $\rho_{A\eta} \in [0, 1]$ and $\varepsilon_{A\eta t} \sim \mathcal{N}(0, \sigma_{A\eta})$.

The monetary policy will also be submitted to a shock, through the variable Z_{Mt}, defined by a first-order autoregressive process $AR(1)$:

$$\ln Z_{Mt} = (1 - \rho_M) \ln Z_M + \rho_M \ln Z_{M,t-1} + \varepsilon_{Mt} \tag{0.32}$$

where $\rho_M \in [0, 1]$ and $\varepsilon_{Mt} \sim \mathcal{N}(0, \sigma_M)$.

0.1.10 Equilibrium Conditions

A Competitive Equilibrium consists of sequences of prices $\{P^*_{\eta t}, R^*_t, W^*_{\eta t}\}$, allocations for households $\boldsymbol{\mathcal{A}}_H := \{C^*_{\eta 1t}, C^*_{\eta 2t}, L^*_{\eta t}, I^*_{\eta t}, K^*_{\eta,t+1}\}$ and allocations for firms $\boldsymbol{\mathcal{A}}_F := \{K^*_{\eta jt}, L^*_{\eta jt}, Y^*_{\eta jt}, Y^*_{\eta t}\}$. In such an equilibrium, given the set of exogenous variables $\{K_0, Z_{A\eta t}, Z_{Mt}\}$, the elements in $\boldsymbol{\mathcal{A}}_H$ solve the household problem, while the elements in $\boldsymbol{\mathcal{A}}_F$ solve the firms' problems, and the markets for goods and labor clear:

$$Y_t = Y_{1t} + Y_{2t} \tag{0.33}$$

$$\text{where:} \quad Y_{\eta t} = C_{\eta t} + I_{\eta t} \tag{0.34}$$

$$L_{\eta t} = \int_0^1 L_{\eta jt} \, \mathrm{d}\, j \tag{0.35}$$

0.2 STEADY STATE

The steady state of a variable is defined by its constancy over time. For any given variable X_t, it is in a steady state if $t \to \infty \implies \mathbb{E}tXt + 1 = X_t = X_{t-1} = X_{ss}$ (**costa_junior_understanding_2016**). For conciseness, the ss index representing the steady state will be omitted, so X is defined as X_{ss}.[3]

0.2.1 Steady State Solution

For the steady-state solution, all endogenous variables are determined with respect to the parameters. It is assumed that the price level P and the productivity level Z_A of region 1 are equal to one. For region 2, it is assumed that these levels are in proportion to the corresponding values of region 1 by factors $\langle \theta_P \quad \theta_Z \rangle$. The solution of the model in steady state is:[4]

$$\vec{\mathbf{1}} = \langle P_1 \quad Z_{A1} \rangle \tag{0.36}$$

$$\langle P_2 \quad Z_{A2} \rangle = \langle P_1 \quad \theta_Z Z_{A1} \rangle \tag{0.37}$$

$$\vec{\mathbf{1}} = \langle Z_M \quad \pi \quad \pi_1 \quad \pi_2 \rangle \tag{0.38}$$

$$\vec{\mathbf{0}} = \langle \varepsilon_{A1} \quad \varepsilon_{A2} \quad \varepsilon_M \rangle \tag{0.39}$$

$$R = P_1 \left[\frac{1}{\beta} - (1 - \delta) \right] \tag{0.40}$$

$$P_\eta^* = P_\eta \tag{0.41}$$

$$Q_\eta = \frac{P_1}{\omega_{\eta 1}^{\omega_{\eta 1}} (1 - \omega_{\eta 1})^{1 - \omega_{\eta 1}}} \tag{0.42}$$

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3 The structural model in the steady state can be consulted in Section (**??**).

4 where $\vec{\mathbf{1}}$ is the unit vector.

$$\Lambda_\eta = P_\eta \frac{\psi - 1}{\psi} \tag{0.43}$$

$$W_\eta = (1 - \alpha_\eta) \left[\Lambda_\eta Z_{A\eta} \left(\frac{\alpha_\eta}{R} \right)^{\alpha_\eta} \right]^{\frac{1}{1-\alpha_\eta}} \tag{0.44}$$

$$a_\eta = \left[\frac{W_\eta}{\phi Q_\eta} \left[Z_{A\eta} \left(\frac{\alpha_\eta W_\eta}{(1 - \alpha_\eta) R} \right)^{\alpha_\eta} \right]^{\varphi} \right]^{\frac{1}{\sigma}} \tag{0.45}$$

$$b_\eta = \frac{\delta}{Z_{A\eta}} \left[\left(\frac{\alpha_\eta}{1 - \alpha_\eta} \right) \frac{W_\eta}{R} \right]^{1-\alpha_\eta} \tag{0.46}$$

$$Y_\eta = \left(\frac{a_\eta}{1 - b_\eta} \right)^{\frac{\sigma}{\sigma + \varphi}} \tag{0.47}$$

$$Y = Y_1 + Y_2 \tag{0.48}$$

$$C_\eta = a_\eta Y_\eta^{\frac{-\varphi}{\sigma}} \tag{0.49}$$

$$I_\eta = b_\eta Y_\eta \tag{0.50}$$

$$K_\eta = \frac{I_\eta}{\delta} \tag{0.51}$$

$$C_{\eta 1} = C_\eta \left(\frac{P_2 \omega_{\eta 1}}{P_1 (1 - \omega_{\eta 1})} \right)^{1-\omega_{\eta 1}} \tag{0.52}$$

$$C_{\eta 2} = C_{\eta 1} \frac{(1 - \omega_{\eta 1}) P_1}{\omega_{\eta 1} P_2} \tag{0.53}$$

$$L_\eta = \frac{Y_\eta}{Z_{A\eta}} \left[\left(\frac{1 - \alpha_\eta}{\alpha_\eta} \right) \frac{R}{W_\eta} \right]^{\alpha_\eta} \tag{0.54}$$

0.3 LOG-LINEARIZATION

Due to the number of variables and equations to be solved, computational brute force will be necessary. `Dynare` is a specialized software for macroeconomic modeling, commonly used for solving DSGE models. Before the model can be processed by the software, it must undergo linearization to eliminate the infinite sum in Equation 0.23. For this purpose, Uhlig's rules of log-linearization (**uhlig_toolkit_1999**) will be applied to all equations in the model. For any given variable X_t, its deviation will be represented with a hat, $\hat{X}_t$.[5] The log-linear model is a square system of 30 variables and equations, summarized as follows:

[5] see Lemma **??** for details.

– Real Variables: $\langle \hat{C}_\eta \quad \hat{L}_\eta \quad \hat{K}_\eta \quad \hat{I}_\eta \quad \hat{C}_{\eta 1} \quad \hat{C}_{\eta 2} \quad \hat{Y}_\eta \quad \hat{Y} \quad \hat{Z}_{A\eta} \quad \hat{Z}_M \rangle$;

– Nominal Variables: $\langle \hat{Q}_\eta \quad \hat{P}_\eta \quad \hat{R} \quad \hat{\pi} \quad \hat{W}_\eta \quad \hat{\lambda}_\eta \quad \hat{\pi}_\eta \rangle$.

– Regional Gross Inflation Rate

$$\hat{\pi}_{\eta t} = \hat{P}_{\eta t} - \hat{P}_{\eta,t-1} \tag{0.55}$$

– New Keynesian Phillips Curve

$$\hat{\pi}_{\eta t} = \beta \mathbb{E}_t \hat{\pi}_{\eta,t+1} + \frac{(1-\theta)(1-\theta\beta)}{\theta} \hat{\lambda}_{\eta t} \tag{0.56}$$

– Regional Consumption Weight

$$\hat{C}_{\eta 2t} - \hat{C}_{\eta 1t} = \hat{P}_{1t} - \hat{P}_{2t} \tag{0.57}$$

– Regional Consumption of Good 1

$$\hat{C}_{\eta t} - \hat{C}_{\eta 1t} = (1 - \omega_{\eta 1})(\hat{P}_{1t} - \hat{P}_{2t}) \tag{0.58}$$

– Regional Price Index

$$\hat{Q}_{\eta t} = \omega_{\eta 1} \hat{P}_{1t} + (1 - \omega_{\eta 1}) \hat{P}_{2t} \tag{0.59}$$

– Labor Supply

$$\varphi \hat{L}_{\eta t} + \sigma \hat{C}_{\eta t} = \hat{W}_{\eta t} - \hat{Q}_{\eta t} \tag{0.60}$$

– Law of Motion for Capital

$$\hat{K}_{\eta,t+1} = (1-\delta)\hat{K}_{\eta t} + \delta \hat{I}_{\eta t} \tag{0.61}$$

– Euler equation for capital return

$$\begin{aligned} (\hat{Q}_{\eta,t+1} - \hat{Q}_{\eta t}) + \sigma(\hat{C}_{\eta,t+1} - \hat{C}_{\eta t}) - (\hat{P}_{\eta,t+1} - \hat{P}_{\eta,t}) = \\ = \beta r (\hat{R}_{\eta,t+1} - \hat{P}_{\eta,t+1}) \end{aligned} \tag{0.62}$$

– Production Function

$$\hat{Y}_{\eta t} = \hat{Z}_{A\eta t} + \alpha_\eta \hat{K}_{\eta t} + (1 - \alpha_\eta) \hat{L}_{\eta t} \tag{0.63}$$

– Technical and Economic Marginal Rates of Substitution

$$\hat{K}_{\eta t} - \hat{L}_{\eta t} = \hat{W}_{\eta t} - \hat{R}_t \tag{0.64}$$

– Marginal Cost

$$\hat{\lambda}_{\eta t} = \alpha_\eta \hat{R}_t + (1 - \alpha_\eta)\hat{W}_{\eta t} - \hat{Z}_{A\eta t} - \hat{P}_{\eta t} \tag{0.65}$$

– Monetary Policy

$$\hat{R}_t = \gamma_R \hat{R}_{t-1} + (1 - \gamma_R)(\gamma_\pi \hat{\pi}_t + \gamma_Y \hat{Y}_t) + \hat{Z}_{Mt} \tag{0.66}$$

– National Gross Inflation Rate

$$\hat{\pi}_t = \theta_\pi \hat{\pi}_{1t} + (1 - \theta_\pi)\hat{\pi}_{2t} \tag{0.67}$$

– Productivity Shock

$$\hat{Z}_{A\eta t} = \rho_{A\eta} \hat{Z}_{A\eta,t-1} + \varepsilon_{A\eta} \tag{0.68}$$

– Monetary Shock

$$\hat{Z}_{Mt} = \rho_M \hat{Z}_{M,t-1} + \varepsilon_M \tag{0.69}$$

– Goods-Market Clearing Condition

$$\hat{Y}_t = \theta_Y \hat{Y}_{1t} + (1 - \theta_Y)\hat{Y}_{2t} \tag{0.70}$$

– Regional Goods-Market Clearing Condition

$$\hat{Y}_{\eta t} = \theta_{C\eta} \hat{C}_{\eta t} + (1 - \theta_{C\eta})\hat{I}_{\eta t} \tag{0.71}$$

0.3.1 Eigenvalues and Forward Looking Variables

As it stands, the model has more forward-looking variables than eigenvalues greater than one, indicating that the model is indeterminate. To transform the model into one with a single solution, the number of eigenvalues and forward-looking variables must be equal. To address this, **farmer_solving_2015** employs a method where excess forward-looking variables are substituted with an expectational variable at time t, along with a expectational shock $sunspot_\eta$, representing the deviation between the expected and the realized values. For the present model, the variables created are the expected regional gross inflation rates $\pi^X_{\eta t}$ and the expected capital deviation $K^X_{\eta t}$:

$$\pi^X_{\eta t} = \mathbb{E}_t \hat{\pi}_{\eta,t+1} \tag{0.72}$$

$$sunspot_{\eta} = \hat{\pi}_{\eta t} - \pi^X_{\eta,t-1} \tag{0.73}$$

$$K^X_{\eta t} = \hat{K}_{\eta,t+1} \tag{0.74}$$

$$sunspot_{K\eta} = \hat{K}_{\eta t} - K^X_{\eta,t-1} \tag{0.75}$$