0.1 Definitions and Lemmas

The objective of this appendix is to present the definitions and lemmas used throughout the text.

Household

Definition 0.1 (Discount Factor β)**.** Other things the same, a unit of consumption enjoyed tomorrow is less valuable (yields less utility) than a unit of consumption enjoyed today (**solis-garcia_ucb_2022**).

Definition 0.2 (Inada Condition)**.** The Inada conditions (**inada_two-sector_1963**) avoid corner solutions. For this purpose, it is assumed that the partial derivatives u_C and u_L of the function $u(C, L)$ satisfy the following rules:

$$\begin{aligned} &\lim_{C\to 0} u_C(C, L^*) = \infty \quad \text{and} \quad \lim_{C\to\infty} u_C(C, L^*) = 0 \\ &\lim_{L\to 0} u_C(C^*, L) = \infty \quad \text{and} \quad \lim_{L\to\infty} u_C(C^*, L) = 0 \end{aligned} \tag{0.1}$$

where $C^*, L^* \in \mathbb{R}_{++}$ and u_j is the partial derivative of the utility function with respect to $j = C, L$ (**solis-garcia_ucb_2022**)

Definition 0.3 (Transversality Condition)**.** (**solis-garcia_ucb_2022**)

Firms

Lemma 0.1 (Marginal Cost)**.** *The Lagrangian multiplier Λ_t is the nominal marginal cost of the intermediate-good firm:*

$$MC_t := \frac{\partial TC_t}{\partial Y_t} = \Lambda_t \tag{0.2}$$

Proof. **simon_mathematics_1994**. ■

Definition 0.4 (Constant Returns to Scale)**.** (**solis-garcia_ucb_2022**)

Definition 0.5 (Homogeneous Function of Degree k)**.** (**solis-garcia_ucb_2022**)

Monetary Authority

Shocks

Equilibrium Conditions

Definition 0.6 (Competitive Equilibrium)**.** (**solis-garcia_ucb_2022**)

Steady State

Lemma 0.2 (Steady State Inflation)**.** *In steady state, prices are stable $P_t = P_{t-1} = P$ and the gross inflation rate is one.*

Proof. Equation **??**. ■

Corollary 0.2.1. *In steady state, all firms have the same level of production Y and therefore demand the same amount of factors, capital K and labor L.*

$$P_t = P_{t-1} = P \implies \begin{pmatrix} Y_j & K_j & L_j \end{pmatrix} = \begin{pmatrix} Y & K & L \end{pmatrix}$$

Log-linearization

Definition 0.7 (PERCENTAGE DEVIATION)**.** The percentage deviation of a variable x_t from its steady state is given by (**solis-garcia_ucb_2022**):

$$\hat{x}_t := \frac{x_t - x}{x} \tag{0.3}$$

Lemma 0.3 (UHLIG'S RULES)**.** *The Uhlig's rules are a set of approximations used to log-linearize equations (***solis-garcia_ucb_2022***).*

- *Rule 1:*

  $x_t = x(1 + \hat{x}_t)$

- *Rule 2 (Product):*

- *Rule 3 (Exponential):*

Corollary 0.3.1 (Logarithm Rule)**.**

$$\ln x_t \approx \ln x + \hat{x}_t$$

Definition 0.8 (LEVEL DEVIATION)**.** The level deviation of a variable u_t from its steady state is given by: (**solis-garcia_ucb_2022**)

$$\widetilde{u}_t := u_t - u \tag{0.4}$$

Lemma 0.4 (UHLIG'S RULES FOR LEVEL DEVIATIONS)**.** *Uhlig's rules can be applied to level deviations in order to log-linearize equations (***solis-garcia_ucb_2022***).*

- *Rule 1:*

$$u_t = u + \widetilde{u}_t \tag{0.5}$$

$$u_t = u\left(1 + \frac{\widetilde{u}_t}{u}\right) \tag{0.6}$$

- *Rule 2 (Product):*
- *Rule 3 (Exponential):*
- *Rule 4 (Logarithm):*
- *Rule 5 (Percentage and Level Deviations)*

Lemma 0.5 (LEVEL DEVIATION OF THE PRESENT VALUE DISCOUNT FACTOR)**.** *The level deviation of the present value discount factor is equivalent to (***solis-garcia_ucb_2022***):*

$$\prod_{k=0}^{s-1}(1+R_{t+k}) = (1+R)^s\left(1 + \frac{1}{1+R}\sum_{k=0}^{s-1}\widetilde{R}_{t+k}\right) \tag{0.7}$$

Proof. Substitute the interest rate by the gross interest rate $GR_t = 1 + R_t$ and apply rule 0.6:

$$\begin{aligned}
&\prod_{k=0}^{s-1}(1+R_{t+k}) = \prod_{k=0}^{s-1}(GR_{t+k}) && \Longrightarrow \\
&GR \times \cdots \times GR\left(1 + \frac{1}{GR}\widetilde{GR}_t + \frac{1}{GR}\widetilde{GR}_{t+1} + \cdots + \frac{1}{GR}\widetilde{GR}_{t+s-1}\right) && \Longrightarrow \\
&GR^s\left(1 + \frac{1}{GR}\sum_{k=0}^{s-1}\widetilde{GR}_{t+k}\right) && \Longrightarrow \\
&(1+R)^s\left(1 + \frac{1}{1+R}\sum_{k=0}^{s-1}\widetilde{R}_{t+k}\right)
\end{aligned}$$

■

Definition 0.9 (Geometric Series)**.** A geometric series is the sum of the terms of a geometric sequence.

$$S_\infty = \sum_{i=0}^{\infty} ar^i \implies S_\infty = \frac{a}{1-r}\ ,\ |r| < 1$$

$$\sum_{mi=0}^{n} \sum_{g=0}^{m} \text{equação aqui} \tag{0.8}$$

Definition 0.10 (LAG AND LEAD OPERATORS)**.** The lag operator $\mathbb{L}$ is a mathematical operator that represents the backshift or lag of a time series (**solis-garcia_ucb_2022**):

$$\begin{aligned} \mathbb{L}x_t &= x_{t-1} \\ (1 + a\mathbb{L})y_{t+2} &= y_{t+2} + ay_{t+1} \end{aligned}$$

Analogously, the lead operator $\mathbb{L}^{-1}$ (or inverse lag operator) yields a variable's lead (**solis-garcia_ucb_2022**):

$$\begin{aligned} \mathbb{L}^{-1}x_t &= x_{t+1} \\ (1 + a\mathbb{L}^{-1})y_{t+2} &= y_{t+2} + ay_{t+3} \end{aligned}$$

Canonical NK Model

Definition 0.11 (Medium Scale DSGE Model)**.** A Medium Scale DSGE Model has habit formation, capital accumulation, indexation, etc. (**gali_monetary_2015**).

See Galí, Smets, and Wouters (2012) for an analysis of the sources of unemployment fluctuations in an estimated medium-scale version of the present model.

Definition 0.12 (Stochastic Process)**.** (**solis-garcia_ucb_2022**).

Definition 0.13 (Markov Process)**.** (**solis-garcia_ucb_2022**).

Definition 0.14 (first-order autoregressive process $AR(1)$)**.** the first-order autoregressive process $AR(1)$ (**solis-garcia_ucb_2022**).

Definition 0.15 (Blanchard-Kahn Conditions)**.** (**solis-garcia_ucb_2022**).