

Analysis of the Monetary Policy Impact on Regional Gross Domestic Product: A Regional DSGE Model

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PPGDE-UFPR

1. Introduction
2. Literature Review
3. Model
4. Regional Model
5. Expected Results
6. Project Timeline

Introduction



Brazilian regions have heterogeneous economic matrices that respond in diverse ways to the decisions of the monetary authority. (BERTANHA; HADDAD, 2008).

Objectives:

- Develop a NK DSGE model with:
 - two regions with different parameters;
 - monetary-policy shocks.
- Demonstrate that different regions react in distinct ways to the monetary policy.

What is a NK DSGE model?

NK DSGE model is a macroeconomic tool with:

- **New Keynesian:** monopolistic competition, nominal rigidities, short-run non-neutrality of monetary policy.
- **Dynamic:** shows the changes over time.
- **Stochastic:** considers random and uncertainty.
- **General Equilibrium:** agents optimize and markets clear (microfoundations).

Literature Review

- Costa Junior (2016): presents a RBC model and then adds NK elements in each chapter;
- Galí (2015): discuss monetary policy starting with a RBC model and also adds NK elements in each chapter;
- Bergholt (2012): presents a NK and the method of programming in *Dynare*;
- Solis-Garcia (2022): presents a RBC model and demonstrate the math tools necessary to solve a DSGE model;

- Rickman (2010): link between macro and regional modeling.
- Mora e Costa Junior (2019): Effects of foreign direct investment (FDI), taking into consideration where it is applied: DSGE model with two regions (Bogotá and the rest of Colombia).
- Costa Junior et al. (2022): Effects of fiscal policy, considering the federative entities: DSGE model for the State of Goiás and the rest of the country.
- Osterno et al. (2022): Regionalization of SAMBA: SAMBA+REG (Stochastic Analytical Model with Bayesian Approach from the Central Bank of Brazil).

Model



Characteristics

- four agents: households, intermediate and final-goods firms, monetary authority.
- no bonds.
- capital and investment.
- price stickiness of intermediate goods.

- the representative household maximizes utility;
- firms producing intermediate goods minimize costs and maximize profit flow;
- firms producing final goods maximize profit.
- the monetary authority determines the interest rate, aiming to control inflation and pursuing economic growth.

Model Structure

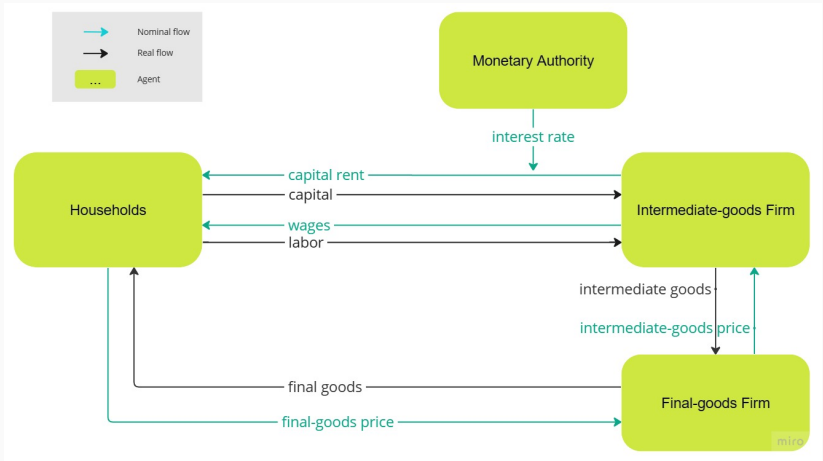


Figure 1: Model Diagram

Household Maximization Problem

$$\max_{C_t, L_t, K_{t+1}} : U(C_t, L_t) = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \phi \frac{L_t^{1+\varphi}}{1+\varphi} \right) \quad (1)$$

$$\text{s. t. : } P_t(C_t + I_t) = W_t L_t + R_t K_t + \Pi_t \quad (2)$$

$$K_{t+1} = (1 - \delta)K_t + I_t \quad (3)$$

$$C_t, L_t, K_{t+1} \geq 0 ; K_0 \text{ given.}$$

Final-goods Firm Maximization Problem

$$\max_{Y_{jt}} : \quad \Pi_t = P_t Y_t - \int_0^1 P_{jt} Y_{jt} dj \quad (4)$$

$$\text{s. t. :} \quad Y_t = \left(\int_0^1 Y_{jt}^{\frac{\psi-1}{\psi}} dj \right)^{\frac{\psi}{\psi-1}} \quad (5)$$

Cost Minimization Problem:

$$\min_{K_{jt}, L_{jt}} : R_t K_{jt} + W_t L_{jt} \quad (6)$$

$$\text{s. t. : } Y_{jt} = Z_{At} K_{jt}^{\alpha} L_{jt}^{1-\alpha} \quad (7)$$

Price Stickiness and Profit Flow, Calvo's Rule (CALVO, 1983):

$$\mathbb{P}(P_t = P_{t-1}) = \theta \quad (8)$$

$$\max_{P_{jt}} : \quad \mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \frac{\theta^s [P_{jt} Y_{j,t+s} - TC_{j,t+s}]}{\prod_{k=0}^{s-1} (1 + R_{t+k})} \right\} \quad (9)$$

$$\text{s. t. :} \quad Y_{jt} = Y_t \left(\frac{P_t}{P_{jt}} \right)^{\psi} \quad (10)$$

Taylor's Rule (TAYLOR, 1993):

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\gamma_R} \left[\left(\frac{\pi_t}{\pi} \right)^{\gamma_\pi} \left(\frac{Y_t}{Y} \right)^{\gamma_Y} \right]^{1-\gamma_R} Z_{Mt} \quad (11)$$

Productivity Shock:

$$\ln Z_{At} = (1 - \rho_A) \ln Z_A + \rho_A \ln Z_{A,t-1} + \varepsilon_{At} \quad (12)$$

Monetary Policy Shock:

$$\ln Z_{Mt} = (1 - \rho_M) \ln Z_M + \rho_M \ln Z_{M,t-1} + \varepsilon_{Mt} \quad (13)$$

Square system of 16 variables and 16 equations:

- from the household problem: C_t, L_t, K_{t+1} ;
- from the final-good firm problem: Y_{jt}, P_t ;
- from the intermediate-good firm problems: K_{jt}, L_{jt}, P_t^* ;
- from the market clearing condition: Y_t, I_t ;
- prices: $W_t, R_t, \Lambda_t, \pi_t$;
- shocks: Z_{At}, Z_{Mt} .

Equations:

1. Labor Supply:

$$\frac{\phi L_t^\varphi}{C_t^{-\sigma}} = \frac{W_t}{P_t} \quad (14)$$

2. Household Euler Equation:

$$\left(\frac{\mathbb{E}_t C_{t+1}}{C_t} \right)^\sigma = \beta \left[(1 - \delta) + \mathbb{E}_t \left(\frac{R_{t+1}}{P_{t+1}} \right) \right] \quad (15)$$

3. Budget Constraint:

$$P_t(C_t + I_t) = W_t L_t + R_t K_t + \Pi_t \quad (16)$$

4. Law of Motion for Capital:

$$K_{t+1} = (1 - \delta)K_t + I_t \quad (17)$$

5. Bundle Technology:

$$Y_t = \left(\int_0^1 Y_{jt}^{\frac{\psi-1}{\psi}} dj \right)^{\frac{\psi}{\psi-1}} \quad (18)$$

6. General Price Level:

$$P_t = \left[\theta P_{t-1}^{1-\psi} + (1 - \theta) P_t^{*1-\psi} \right]^{\frac{1}{1-\psi}} \quad (19)$$

7. Capital Demand:

$$K_{jt} = \alpha Y_{jt} \frac{\Lambda_t}{R_t} \quad (20)$$

8. Labor Demand:

$$L_{jt} = (1 - \alpha) Y_{jt} \frac{\Lambda_t}{W_t} \quad (21)$$

9. Marginal Cost:

$$\Lambda_t = \frac{1}{Z_{At}} \left(\frac{R_t}{\alpha} \right)^\alpha \left(\frac{W_t}{1 - \alpha} \right)^{1 - \alpha} \quad (22)$$

10. Production Function:

$$Y_{jt} = Z_{At} K_{jt}^{\alpha} L_{jt}^{1-\alpha} \quad (23)$$

11. Optimal Price:

$$P_t^* = \frac{\psi}{\psi - 1} \cdot \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \theta^s Y_{j,t+s} \Lambda_{t+s} / \prod_{k=0}^{s-1} (1 + R_{t+k}) \right\}}{\mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \theta^s Y_{j,t+s} / \prod_{k=0}^{s-1} (1 + R_{t+k}) \right\}} \quad (24)$$

12. Market Clearing Condition:

$$Y_t = C_t + I_t \quad (25)$$

13. Monetary Policy:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\gamma_R} \left[\left(\frac{\pi_t}{\pi} \right)^{\gamma_\pi} \left(\frac{Y_t}{Y} \right)^{\gamma_Y} \right]^{1-\gamma_R} Z_{Mt} \quad (26)$$

14. Gross Inflation Rate:

$$\pi_t = \frac{P_t}{P_{t-1}} \quad (27)$$

15. Productivity Shock:

$$\ln Z_{At} = (1 - \rho_A) \ln Z_A + \rho_A \ln Z_{A,t-1} + \varepsilon_{At} \quad (28)$$

16. Monetary Shock:

$$\ln Z_{Mt} = (1 - \rho_M) \ln Z_M + \rho_M \ln Z_{M,t-1} + \varepsilon_{Mt} \quad (29)$$

Steady State

Steady state solution (COSTA JUNIOR, 2016, p.41):

$$\mathbb{E}_t X_{t+1} = X_t = X_{t-1} = X_{ss} \quad (30)$$

Log-linearization

Uhlig's rules for log-linearization (UHLIG, 1999).

Square system of 12 variables and 12 equations:

Variables:

$$\left(\tilde{\pi} \quad \hat{P} \quad \hat{\lambda} \quad \hat{C} \quad \hat{L} \quad \hat{R} \quad \hat{K} \quad \hat{I} \quad \hat{W} \quad \hat{Z}_A \quad \hat{Y} \quad \hat{Z}_M \right) \quad (31)$$

Equations:

1. Gross Inflation Rate:

$$\tilde{\pi}_t = \hat{P}_t - \hat{P}_{t-1} \quad (32)$$

2. New Keynesian Phillips Curve:

$$\tilde{\pi}_t = \varrho \mathbb{E}_t \tilde{\pi}_{t+1} + \frac{(1-\theta)(1-\theta\varrho)}{\theta} \hat{\lambda}_t \quad (33)$$

3. Labor Supply:

$$\varphi \hat{L}_t + \sigma \hat{C}_t = \hat{W}_t + \hat{P}_t \quad (34)$$

4. Household Euler Equation:

$$\mathbb{E}_t \hat{C}_{t+1} - \hat{C}_t = \frac{\beta R}{\sigma P} \mathbb{E}_t (\hat{R}_{t+1} - \hat{P}_{t+1}) \quad (35)$$

5. Law of Motion for Capital:

$$\hat{K}_{t+1} = (1 - \delta) \hat{K}_t + \delta \hat{I}_t \quad (36)$$

6. Real Marginal Cost:

$$\hat{\lambda}_t = \alpha \hat{R}_t + (1 - \alpha) \hat{W}_t - \hat{Z}_{At} - \hat{P}_t \quad (37)$$

7. Production Function:

$$\hat{Y}_t = \hat{Z}_{At} + \alpha \hat{K}_t + (1 - \alpha) \hat{L}_t \quad (38)$$

8. Marginal Rates of Substitution of Factors:

$$\hat{K}_t - \hat{L}_t = \hat{W}_t - \hat{R}_t \quad (39)$$

9. Market Clearing Condition:

$$\hat{Y}_t = \theta_C \hat{C}_t + \theta_I \hat{I}_t \quad (40)$$

10. Monetary Policy:

$$\hat{R}_t = \gamma_R \hat{R}_{t-1} + (1 - \gamma_R)(\gamma_\pi \tilde{\pi}_t + \gamma_Y \hat{Y}_t) + \hat{Z}_{Mt} \quad (41)$$

11. Productivity Shock:

$$\hat{Z}_{At} = \rho_A \hat{Z}_{A,t-1} + \varepsilon_A \quad (42)$$

12. Monetary Shock:

$$\hat{Z}_{Mt} = \rho_M \hat{Z}_{M,t-1} + \varepsilon_M \quad (43)$$

Matlab and Dynare

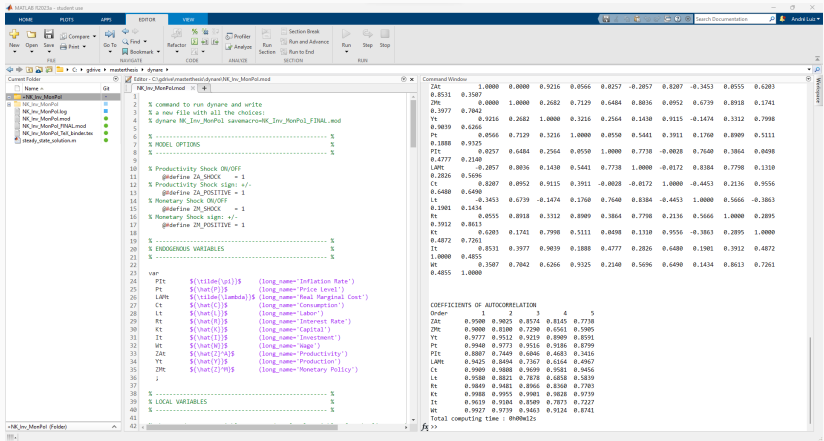
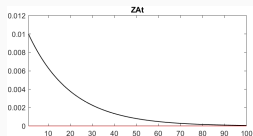
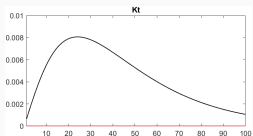


Figure 2: Matlab and Dynare

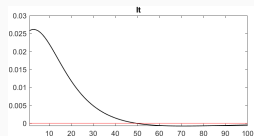
Productivity Shock



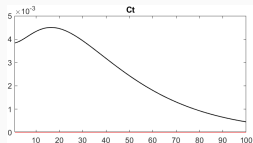
(a) Productivity Shock



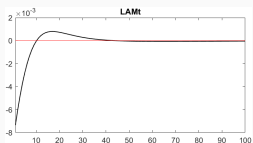
(b) Capital



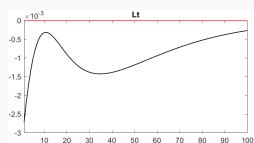
(c) Investment



(d) Consumption

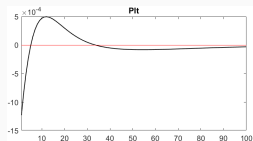


(e) Marginal Cost

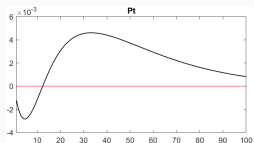


(f) Labor

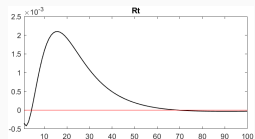
Productivity Shock



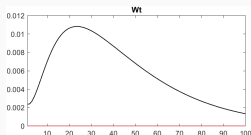
(a) Inflation



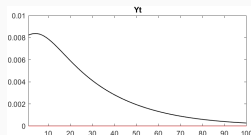
(b) Price Level



(c) Interest Rate

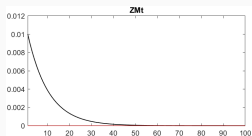


(d) Wage

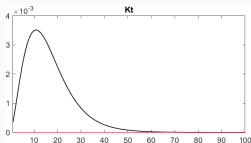


(e) Production

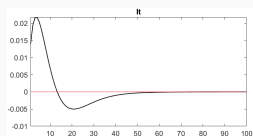
Monetary Shock



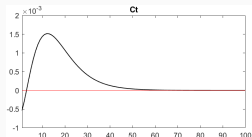
(a) Monetary Shock



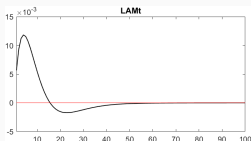
(b) Capital



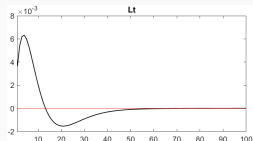
(c) Investment



(d) Consumption

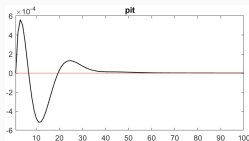


(e) Marginal Cost

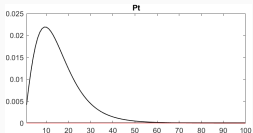


(f) Labor

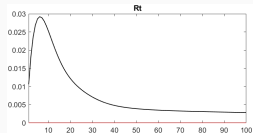
Monetary Shock



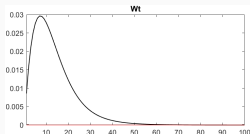
(a) Inflation



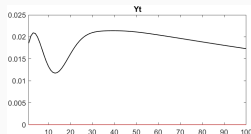
(b) Price Level



(c) Interest Rate



(d) Wage



(e) Production

Figure 6: Monetary Shock Impulse Response Functions

Regional Model

- two regions.
- mobility for final-goods.
- household η and firm ν indexes.
- regional inflation π_ν .

Household Maximization Problem

$$\max_{C_{1\eta t}, C_{2\eta t}, L_{\eta t}, K_{\eta, t+1}} : U_{\eta}(C_{\eta t}, L_{\eta t}) = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\frac{C_{\eta t}^{1-\sigma}}{1-\sigma} - \phi \frac{L_{\eta t}^{1+\varphi}}{1+\varphi} \right) \quad (44)$$

$$\text{s. t. : } P_{C1t}C_{1\eta t} + P_{C2t}C_{2\eta t} + P_{C\eta t}I_{\eta t} = W_tL_{\eta t} + R_tK_{\eta t} + \Pi_{\nu t} \quad (45)$$

$$K_{\eta, t+1} = (1 - \delta)K_{\eta t} + I_{\eta t} \quad (46)$$

$$C_{\eta t} = C_{1\eta t}^{\omega_{\eta}} C_{2\eta t}^{1-\omega_{\eta}} \quad (47)$$

$$C_{\nu\eta t}, L_{\eta t}, K_{\eta, t+1} > 0 ; K_0 \text{ given.}$$

Final-goods Firm Maximization Problem

$$\max_{Y_{\nu jt}} : \quad \Pi_{\nu t} = P_{C\nu t} Y_{\nu t} - \int_0^1 P_{C\nu jt} Y_{\nu jt} dj \quad (48)$$

$$\text{s. t. :} \quad Y_{\nu t} = \left(\int_0^1 Y_{\nu jt}^{\frac{\psi-1}{\psi}} dj \right)^{\frac{\psi}{\psi-1}} \quad (49)$$

Cost Minimization Problem:

$$\min_{K_{\eta jt}, L_{\eta jt}} : R_t K_{\eta jt} + W_t L_{\eta jt} \quad (50)$$

$$\text{s. t. : } Y_{\nu jt} = Z_{A\nu t} K_{\eta jt}^{\alpha_\nu} L_{\eta jt}^{1-\alpha_\nu} \quad (51)$$

Price Stickiness and Profit Flow, Calvo's Rule (CALVO, 1983):

$$\mathbb{P}(P_t = P_{t-1}) = \theta \quad (52)$$

$$\max_{P_{C\nu jt}} : \quad \mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \frac{\theta^s [P_{C\nu jt} Y_{\nu j, t+s} - TC_{\nu j, t+s}]}{\prod_{k=0}^{s-1} (1 + R_{t+k})} \right\} \quad (53)$$

$$\text{s. t. :} \quad Y_{\nu jt} = Y_{\nu t} \left(\frac{P_{C\nu t}}{P_{C\nu jt}} \right)^{\psi} \quad (54)$$

Taylor's Rule (TAYLOR, 1993):

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\gamma_R} \left[\left(\frac{\pi_t}{\pi} \right)^{\gamma_\pi} \left(\frac{Y_t}{Y} \right)^{\gamma_Y} \right]^{1-\gamma_R} Z_{Mt} \quad (55)$$

Regional Price Level and Inflation

Regional price level $P_{C\nu t}$ and regional inflation rate:

$$\pi_{\nu t} = \frac{P_{C\nu t}}{P_{C\nu, t-1}} \quad (56)$$

National price level:

$$P_t = \vartheta_1 P_{C1t} + (1 - \vartheta_1) P_{C2t} \quad (57)$$

The model is a square system of 27 variables and 27 equations.

- from the household problem: $C_{\eta t}, L_{\eta t}, K_{\eta, t+1}$;
- from the final-goods firm problem: $Y_{\nu t}, Y_{\nu jt}, P_{C\nu t}$;
- from the intermediate-goods firm problems: $K_{\eta jt}, L_{\eta jt}, P_{C\nu t}^*$;
- from the market clearing condition: $Y_t, I_{\eta t}$;
- prices: $W_t, R_t, \Lambda_{\nu t}, \pi_t$;
- shocks: $Z_{A\nu t}, Z_{Mt}$.

Next Steps

- Steady State
- Parameter Calibration
- Steady State Solution
- Log-linearization
- Dynare Programming
- Impulse Response Functions

Expected Results

Expected Results

- monetary Policy Shock: 1%, positive and negative.
- Regional technology level $Z_{A\nu t}$, capital weight in production α_ν implies regional price levels $P_{\nu t}$ and inflation rates $\pi_{\nu t}$.
- for different parameters, the reaction of each region shall have a significant change.

Project Timeline

Project Timeline

Activity	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Literature Review	x	x								
Project & Seminar			x	x						
Modeling			x	x	x					
L ^A T _E X			x	x	x					
<i>Dynare</i>					x	x				
Qualification						x				
Regional Model						x	<input type="checkbox"/>			
Data Treatment							<input type="checkbox"/>			
Parametrization								<input type="checkbox"/>		
Results								<input type="checkbox"/>		
Systematic Review									<input type="checkbox"/>	
Revision & Edition									<input type="checkbox"/>	
Thesis Defense										<input type="checkbox"/>

Obrigado!
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Dúvidas e Sugestões