

### 3.3.2 Extended Log-linear Structure

#### 1. Regional Gross Inflation Rate

$$\begin{aligned}\hat{\pi}_{1t} &= \hat{P}_{1t} - \hat{P}_{1,t-1} \\ \hat{\pi}_{2t} &= \hat{P}_{2t} - \hat{P}_{2,t-1}\end{aligned}\tag{3.102}$$

#### 2. New Keynesian Phillips Curve

$$\begin{aligned}\hat{\pi}_{1t} &= \beta \mathbb{E}_t \hat{\pi}_{1,t+1} + \frac{(1-\theta)(1-\theta\beta)}{\theta} \hat{\lambda}_{1t} \\ \hat{\pi}_{2t} &= \beta \mathbb{E}_t \hat{\pi}_{2,t+1} + \frac{(1-\theta)(1-\theta\beta)}{\theta} \hat{\lambda}_{2t}\end{aligned}\tag{3.112}$$

#### 3. Law of Motion for Capital

$$\begin{aligned}\hat{K}_{1,t+1} &= (1-\delta)\hat{K}_{1t} + \delta\hat{I}_{1t} \\ \hat{K}_{2,t+1} &= (1-\delta)\hat{K}_{2t} + \delta\hat{I}_{2t}\end{aligned}\tag{3.113}$$

#### 4. Regional Levels of Consumption and Prices

$$\begin{aligned}\hat{C}_{11t} - \hat{C}_{12t} &= \hat{P}_{2t} - \hat{P}_{1t} \\ \hat{C}_{21t} - \hat{C}_{22t} &= \hat{P}_{2t} - \hat{P}_{1t}\end{aligned}\tag{3.116}$$

#### 5. Total Expenses

$$\begin{aligned}\hat{\mathcal{E}}_{1t} &= \hat{C}_{1t} + \omega_{11}\hat{P}_{1t} + (1-\omega_{11})\hat{P}_{2t} \\ \hat{\mathcal{E}}_{2t} &= \hat{C}_{2t} + \omega_{21}\hat{P}_{1t} + (1-\omega_{21})\hat{P}_{2t}\end{aligned}\tag{3.117}$$

#### 6. Labor Supply

$$\begin{aligned}\varphi\hat{L}_{1t} - (1-\sigma)\hat{C}_{1t} &= \hat{W}_t - \hat{\mathcal{E}}_{1t} \\ \varphi\hat{L}_{2t} - (1-\sigma)\hat{C}_{2t} &= \hat{W}_t - \hat{\mathcal{E}}_{2t}\end{aligned}\tag{3.118}$$

#### 7. Euler equation for the bonds return

$$\begin{aligned}\mathbb{E}_t \{ \hat{\mathcal{E}}_{1,t+1} - (1-\sigma)\hat{C}_{1,t+1} \} - [\hat{\mathcal{E}}_{1t} - (1-\sigma)\hat{C}_{1t}] &= (1-\beta)\hat{R}_t \\ \mathbb{E}_t \{ \hat{\mathcal{E}}_{2,t+1} - (1-\sigma)\hat{C}_{2,t+1} \} - [\hat{\mathcal{E}}_{2t} - (1-\sigma)\hat{C}_{2t}] &= (1-\beta)\hat{R}_t\end{aligned}\tag{3.119}$$

8. Euler equation for the capital return

$$\begin{aligned} \mathbb{E}_t \{ \hat{\mathcal{E}}_{1,t+1} - \hat{P}_{1,t+1} - (1 - \sigma) \hat{C}_{1,t+1} \} - \\ - (\hat{\mathcal{E}}_{1t} - \hat{P}_{1t} - (1 - \sigma) \hat{C}_{1t}) = \beta \frac{R_K}{P_1} \mathbb{E}_t \{ \hat{R}_{K,t+1} - \hat{P}_{1,t+1} \} \\ \mathbb{E}_t \{ \hat{\mathcal{E}}_{2,t+1} - \hat{P}_{2,t+1} - (1 - \sigma) \hat{C}_{2,t+1} \} - \\ - (\hat{\mathcal{E}}_{2t} - \hat{P}_{2t} - (1 - \sigma) \hat{C}_{2t}) = \beta \frac{R_K}{P_2} \mathbb{E}_t \{ \hat{R}_{K,t+1} - \hat{P}_{2,t+1} \} \end{aligned} \quad (3.120)$$

9. Production Function

$$\begin{aligned} \hat{Y}_{1t} &= \hat{Z}_{A1t} + \alpha \hat{K}_{1t} + (1 - \alpha) \hat{L}_{1t} \\ \hat{Y}_{2t} &= \hat{Z}_{A2t} + \alpha \hat{K}_{2t} + (1 - \alpha) \hat{L}_{2t} \end{aligned} \quad (3.127)$$

10. Marginal Rates of Substitution of Factors

$$\begin{aligned} \hat{K}_{1t} - \hat{L}_{1t} &= \hat{W}_t - \hat{R}_{Kt} \\ \hat{K}_{2t} - \hat{L}_{2t} &= \hat{W}_t - \hat{R}_{Kt} \end{aligned} \quad (3.130)$$

11. Marginal Cost

$$\begin{aligned} \hat{\lambda}_{1t} &= \alpha \hat{R}_{Kt} + (1 - \alpha) \hat{W}_t - \hat{Z}_{A1t} - \hat{P}_{1t} \\ \hat{\lambda}_{2t} &= \alpha \hat{R}_{Kt} + (1 - \alpha) \hat{W}_t - \hat{Z}_{A2t} - \hat{P}_{2t} \end{aligned} \quad (3.132)$$

12. Monetary Policy

$$\hat{R}_t = \gamma_R \hat{R}_{t-1} + (1 - \gamma_R)(\gamma_\pi \hat{\pi}_t + \gamma_Y \hat{Y}_t) + \hat{Z}_{Mt} \quad (3.133)$$

13. National Gross Inflation Rate

$$\hat{\pi}_t = \hat{P}_t - \hat{P}_{t-1} \quad (3.134)$$

14. National Price Level

$$\hat{P}_t + \hat{Y}_t = \theta_{PY1}(\hat{P}_{1t} + \hat{Y}_{1t}) + (1 - \theta_{PY1})(\hat{P}_{2t} + \hat{Y}_{2t}) \quad (3.140)$$

15. Productivity Shock

$$\begin{aligned} \hat{Z}_{A1t} &= \rho_{A1} \hat{Z}_{A1,t-1} + \varepsilon_{A1} \\ \hat{Z}_{A2t} &= \rho_{A2} \hat{Z}_{A2,t-1} + \varepsilon_{A2} \end{aligned} \quad (3.141)$$

16. Monetary Shock

$$\hat{Z}_{Mt} = \rho_M \hat{Z}_{M,t-1} + \varepsilon_M \quad (3.142)$$

17. Market Clearing Condition

$$\hat{Y}_t = \theta_{Y1} \hat{Y}_{1t} + (1 - \theta_{Y1}) \hat{Y}_{2t} \quad (3.137)$$

18. Regional Market Clearing Condition

$$\begin{aligned} \hat{Y}_{1t} &= \theta_{C11} \hat{C}_{11t} + \theta_{C12} \hat{C}_{12t} + (1 - \theta_{C11} - \theta_{C12}) \hat{I}_{1t} \\ \hat{Y}_{2t} &= \theta_{C21} \hat{C}_{21t} + \theta_{C22} \hat{C}_{22t} + (1 - \theta_{C21} - \theta_{C22}) \hat{I}_{2t} \end{aligned} \quad (3.145)$$