# Analysis of the Monetary Policy Impact on Regional Gross Domestic Product: A Regional DSGE Model

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PPGDE-UFPR

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# Introduction

# Introduction

Brazilian regions have heterogeneous economic matrices that respond in diverse ways to the decisions of the monetary authority. (BERTANHA; HADDAD, 2008).

# Introduction

Objectives:

- Develop a NK DSGE model with:
  - two regions with different parameters;
  - monetary-policy shocks.
- Demonstrate that different regions react in distinct ways to the monetary policy.

# What is a NK DSGE model?

NK DSGE model is a macroeconomic tool with:

- **New Keynesian**: monopolistic competition, nominal rigidities, short-run non-neutrality of monetary policy.
- **Dynamic**: shows the changes over time.
- **Stochastic**: considers random and uncertainty.
- **General Equilibrium**: agents optimize and markets clear (microfoundations).

# Literature Review

# Macro modeling

- Costa Junior (2016): presents a RBC model and then adds NK elements in each chapter;
- Galí (2015): discuss monetary policy starting with a RBC model and also adds NK elements in each chapter;
- Bergholt (2012): presents a NK and the method of programming in *Dynare*;
- Solis-Garcia (2022): presents a RBC model and demonstrate the math tools necessary to solve a DSGE model;

# Regional Modeling

- Rickman (2010): link between macro and regional modeling.
- Mora e Costa Junior (2019): Effects of foreign direct investment (FDI), taking into consideration where it is applied: DSGE model with two regions (Bogotá and the rest of Colombia).
- Costa Junior et al. (2022): Effects of fiscal policy, considering the federative entities: DSGE model for the State of Goiás and the rest of the country.
- Osterno et al. (2022): Regionalization of SAMBA: SAMBA+REG (Stochastic Analytical Model with Bayesian Approach from the Central Bank of Brazil).

# Model

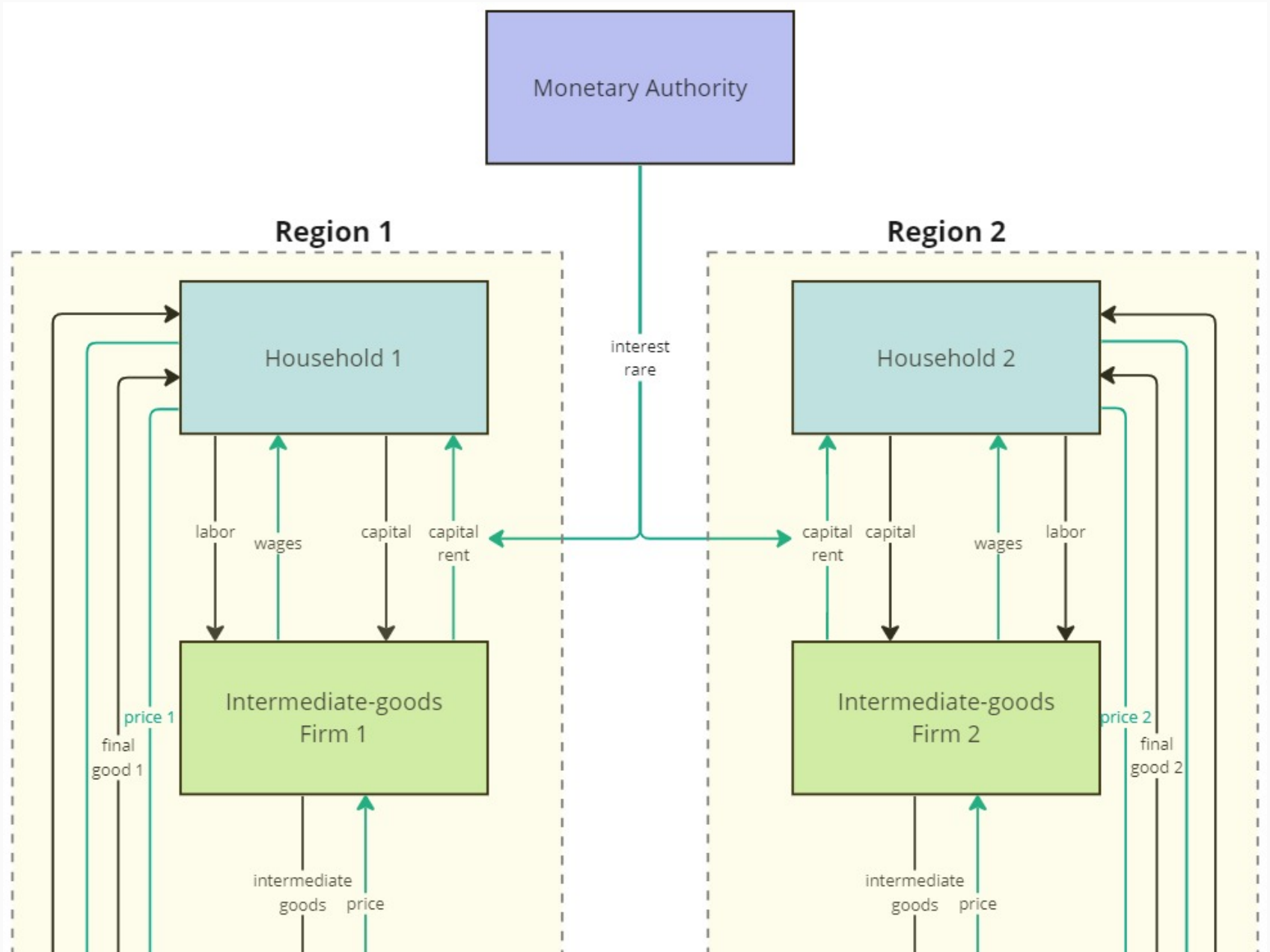
# Characteristics

- four agents: households, intermediate and final-goods firms, monetary authority.
- no bonds.
- capital and investment.
- price stickiness of intermediate goods.

# Agents

- the representative household maximizes utility;
- firms producing intermediate goods minimize costs and maximize profit flow;
- firms producing final goods maximize profit.
- the monetary authority determines the interest rate, aiming to control inflation and pursuing economic growth.

# Model Structure



# Household Maximization Problem

$$\max_{C_t, L_t, K_{t+1}} : \quad U(C_t, L_t) = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma}}{1-\sigma} - \phi \frac{L_t^{1+\varphi}}{1+\varphi} \right) \tag{1}$$

$$\text{s. t. :} \quad P_t(C_t + I_t) = W_t L_t + R_t K_t + \Pi_t \tag{2}$$

$$K_{t+1} = (1-\delta)K_t + I_t \tag{3}$$

$$C_t, L_t, K_{t+1} \geq 0 \; ; \; K_0 \text{ given.}$$

# Final-goods Firm Maximization Problem

$$\max_{Y_{jt}}: \quad \Pi_t = P_t Y_t - \int_0^1 P_{jt} Y_{jt} \, \mathrm{d}\, j \tag{4}$$

$$\text{s. t.}: \quad Y_t = \left( \int_0^1 Y_{jt}^{\frac{\psi - 1}{\psi}} \, \mathrm{d}\, j \right)^{\frac{\psi}{\psi - 1}} \tag{5}$$

# Intermediate-goods Firm Problems

Cost Minimization Problem:

$$\min_{K_{jt}, L_{jt}} : \quad R_t K_{jt} + W_t L_{jt} \tag{6}$$

$$\text{s. t. :} \quad Y_{jt} = Z_{At} K_{jt}^{\alpha} L_{jt}^{1-\alpha} \tag{7}$$

# Intermediate-goods Firm Problems

Price Stickiness and Profit Flow, Calvo's Rule (CALVO, 1983):

$$\mathbb{P}(P_t = P_{t-1}) = \theta \tag{8}$$

$$\max_{P_{jt}}: \quad \mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \frac{\theta^s \left[ P_{jt} Y_{j,t+s} - TC_{j,t+s} \right]}{\prod_{k=0}^{s-1} (1 + R_{t+k})} \right\} \tag{9}$$

$$\text{s. t.}: \quad Y_{jt} = Y_t \left( \frac{P_t}{P_{jt}} \right)^{\psi} \tag{10}$$

# Monetary Authority

Taylor's Rule (TAYLOR, 1993):

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\gamma_\pi} \left(\frac{Y_t}{Y}\right)^{\gamma_Y}\right]^{1-\gamma_R} Z_{Mt} \tag{11}$$

# Stochastic Shocks

Productivity Shock:

$$\ln Z_{At} = (1 - \rho_A) \ln Z_A + \rho_A \ln Z_{A,t-1} + \varepsilon_{At} \tag{12}$$

Monetary Policy Shock:

$$\ln Z_{Mt} = (1 - \rho_M) \ln Z_M + \rho_M \ln Z_{M,t-1} + \varepsilon_{Mt} \tag{13}$$

# Model Structure i

Square system of 16 variables and 16 equations:

- from the household problem: $C_t, L_t, K_{t+1}$;
- from the final-good firm problem: $Y_{jt}, P_t$;
- from the intermediate-good firm problems: $K_{jt}, L_{jt}, P_t^*$;
- from the market clearing condition: $Y_t, I_t$;
- prices: $W_t, R_t, \Lambda_t, \pi_t$;
- shocks: $Z_{At}, Z_{Mt}$.

# Model Structure i

Equations:

1. Labor Supply:

$$\frac{\phi L_t^{\varphi}}{C_t^{-\sigma}} = \frac{W_t}{P_t} \tag{14}$$

2. Household Euler Equation:

$$\left(\frac{\mathbb{E}_t C_{t+1}}{C_t}\right)^{\sigma} = \beta\left[(1-\delta) + \mathbb{E}_t\left(\frac{R_{t+1}}{P_{t+1}}\right)\right] \tag{15}$$

3. Budget Constraint:

$$P_t(C_t + I_t) = W_t L_t + R_t K_t + \Pi_t \tag{16}$$

# Model Structure ii

4. Law of Motion for Capital:

$$K_{t+1} = (1 - \delta)K_t + I_t \tag{17}$$

5. Bundle Technology:

$$Y_t = \left( \int_0^1 Y_{jt}^{\frac{\psi - 1}{\psi}} \, \mathrm{d}j \right)^{\frac{\psi}{\psi - 1}} \tag{18}$$

6. General Price Level:

$$P_t = \left[ \theta P_{t-1}^{1-\psi} + (1 - \theta) P_t^{*1-\psi} \right]^{\frac{1}{1-\psi}} \tag{19}$$

# Model Structure iii

7. Capital Demand:

$$K_{jt} = \alpha Y_{jt} \frac{\Lambda_t}{R_t} \tag{20}$$

8. Labor Demand:

$$L_{jt} = (1 - \alpha) Y_{jt} \frac{\Lambda_t}{W_t} \tag{21}$$

9. Marginal Cost:

$$\Lambda_t = \frac{1}{Z_{At}} \left( \frac{R_t}{\alpha} \right)^{\alpha} \left( \frac{W_t}{1 - \alpha} \right)^{1-\alpha} \tag{22}$$

# Model Structure iv

10. Production Function:

$$Y_{jt} = Z_{At} K_{jt}^{\alpha} L_{jt}^{1-\alpha} \tag{23}$$

11. Optimal Price:

$$P_t^* = \frac{\psi}{\psi - 1} \cdot \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \theta^s Y_{j,t+s} \Lambda_{t+s} / \prod_{k=0}^{s-1} (1 + R_{t+k}) \right\}}{\mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \theta^s Y_{j,t+s} / \prod_{k=0}^{s-1} (1 + R_{t+k}) \right\}} \tag{24}$$

12. Market Clearing Condition:

$$Y_t = C_t + I_t \tag{25}$$

# Model Structure v

13. Monetary Policy:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\gamma_\pi} \left(\frac{Y_t}{Y}\right)^{\gamma_Y}\right]^{1-\gamma_R} Z_{Mt} \tag{26}$$

14. Gross Inflation Rate:

$$\pi_t = \frac{P_t}{P_{t-1}} \tag{27}$$

15. Productivity Shock:

$$\ln Z_{At} = (1 - \rho_A) \ln Z_A + \rho_A \ln Z_{A,t-1} + \varepsilon_{At} \tag{28}$$

# Model Structure vi

16. Monetary Shock:

$$\ln Z_{Mt} = (1 - \rho_M) \ln Z_M + \rho_M \ln Z_{M,t-1} + \varepsilon_{Mt} \tag{29}$$

# Steady State

# Steady State

Steady state solution (COSTA JUNIOR, 2016, p.41):

$$\mathbb{E}_t X_{t+1} = X_t = X_{t-1} = X_{ss} \tag{30}$$

# Log-linearization

# Log-linearization

Uhlig's rules for log-linearization (UHLIG, 1999).

# Log-linear Model i

Square system of 12 variables and 12 equations:

Variables:

$$\begin{pmatrix} \tilde{\pi} & \hat{P} & \hat{\lambda} & \hat{C} & \hat{L} & \hat{R} & \hat{K} & \hat{I} & \hat{W} & \hat{Z}_A & \hat{Y} & \hat{Z}_M \end{pmatrix} \tag{31}$$

# Log-linear Model i

Equations:

1. Gross Inflation Rate:

$$\widetilde{\pi}_t = \hat{P}_t - \hat{P}_{t-1} \tag{32}$$

2. New Keynesian Phillips Curve:

$$\widetilde{\pi}_t = \varrho \mathbb{E}_t \widetilde{\pi}_{t+1} + \frac{(1-\theta)(1-\theta\varrho)}{\theta} \hat{\lambda}_t \tag{33}$$

3. Labor Supply:

$$\varphi \hat{L}_t + \sigma \hat{C}_t = \hat{W}_t + \hat{P}_t \tag{34}$$

# Log-linear Model ii

4. Household Euler Equation:

$$\mathbb{E}_t \hat{C}_{t+1} - \hat{C}_t = \frac{\beta R}{\sigma P} \mathbb{E}_t (\hat{R}_{t+1} - \hat{P}_{t+1}) \tag{35}$$

5. Law of Motion for Capital:

$$\hat{K}_{t+1} = (1 - \delta)\hat{K}_t + \delta \hat{I}_t \tag{36}$$

6. Real Marginal Cost:

$$\hat{\lambda}_t = \alpha \hat{R}_t + (1 - \alpha)\hat{W}_t - \hat{Z}_{At} - \hat{P}_t \tag{37}$$

# Log-linear Model iii

7. Production Function:

$$\hat{Y}_t = \hat{Z}_{At} + \alpha \hat{K}_t + (1 - \alpha)\hat{L}_t \tag{38}$$

8. Marginal Rates of Substitution of Factors:

$$\hat{K}_t - \hat{L}_t = \hat{W}_t - \hat{R}_t \tag{39}$$

9. Market Clearing Condition:

$$\hat{Y}_t = \theta_C \hat{C}_t + \theta_I \hat{I}_t \tag{40}$$

# Log-linear Model iv

10. Monetary Policy:

$$\hat{R}_t = \gamma_R \hat{R}_{t-1} + (1 - \gamma_R)(\gamma_\pi \widetilde{\pi}_t + \gamma_Y \hat{Y}_t) + \hat{Z}_{Mt} \tag{41}$$

11. Productivity Shock:

$$\hat{Z}_{At} = \rho_A \hat{Z}_{A,t-1} + \varepsilon_A \tag{42}$$

12. Monetary Shock:

$$\hat{Z}_{Mt} = \rho_M \hat{Z}_{M,t-1} + \varepsilon_M \tag{43}$$

# Matlab and Dynare

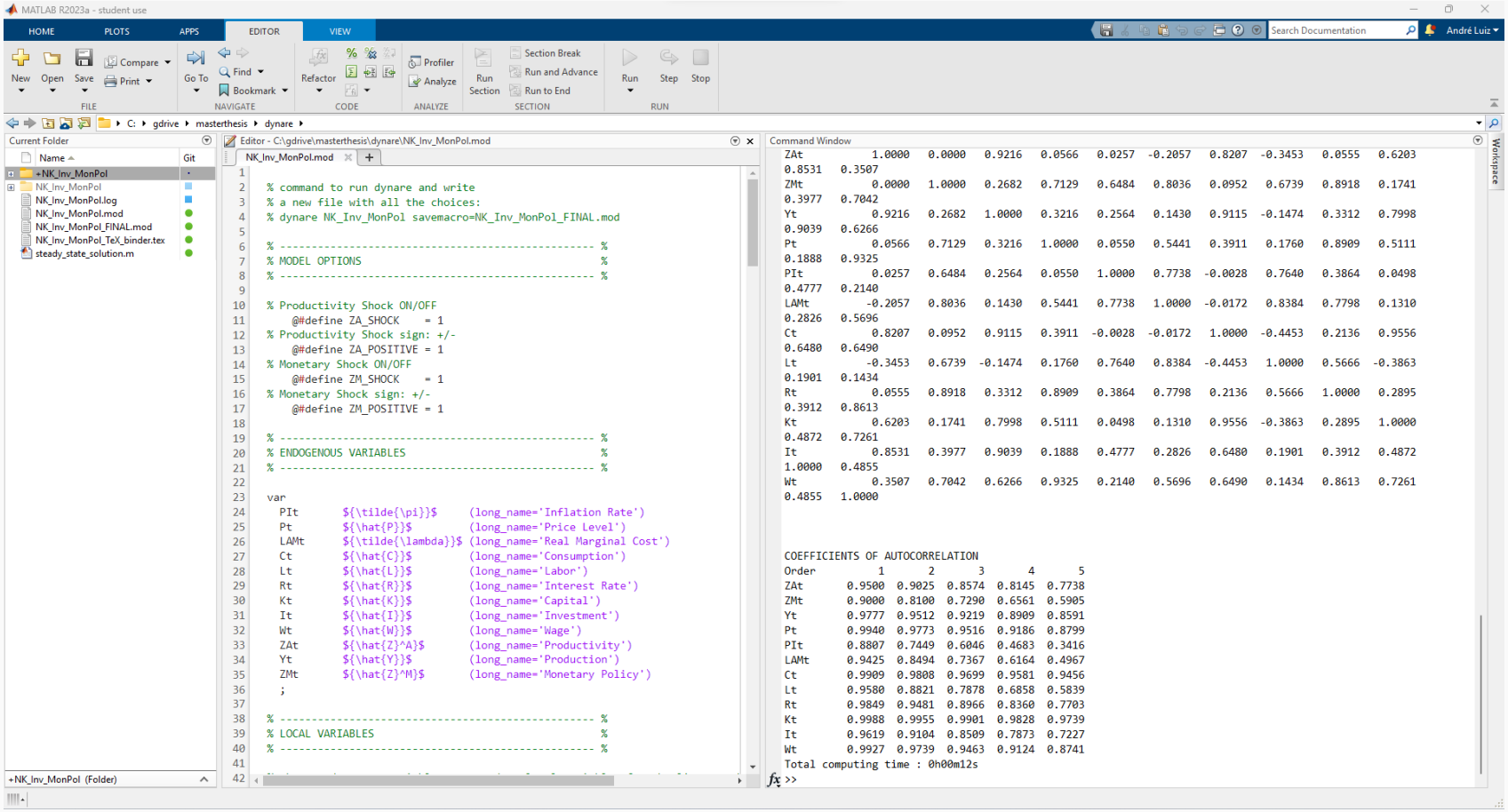
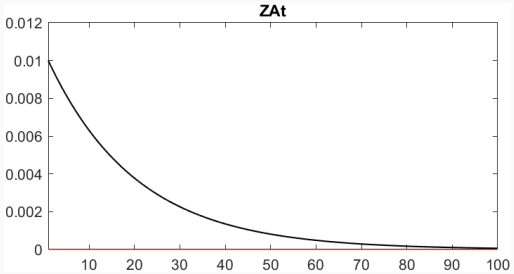
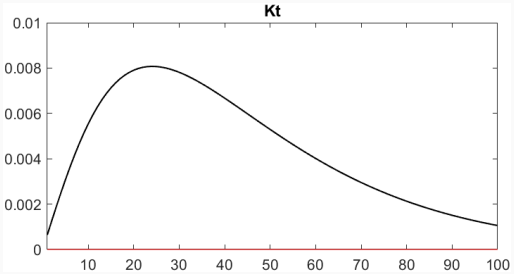



**Figure 2:** Matlab and Dynare

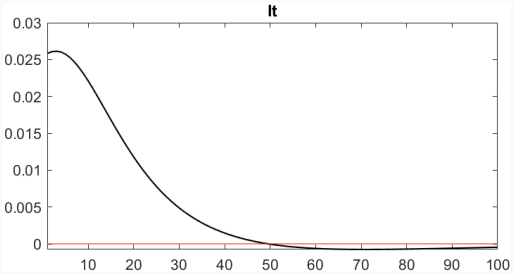
# Productivity Shock



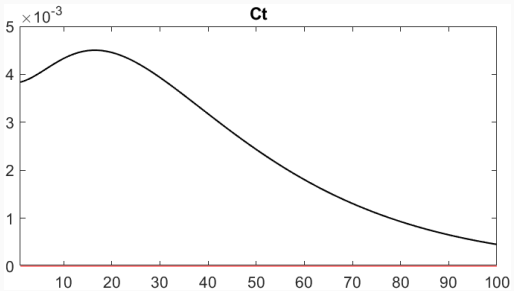

**(a)** Productivity Shock



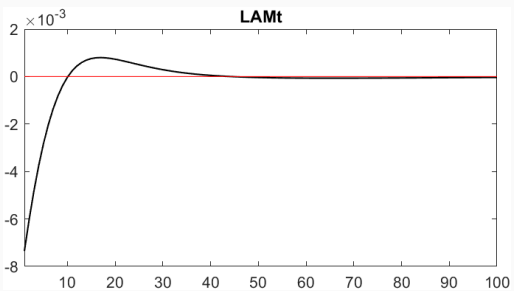

**(b)** Capital



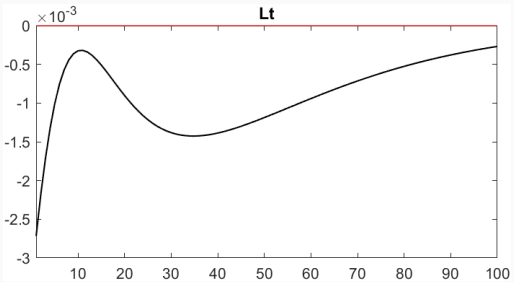

**(c)** Investment




**(d)** Consumption

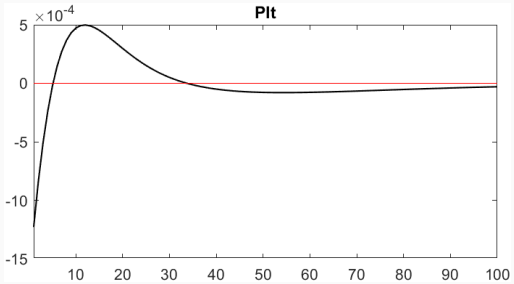



**(e)** Marginal Cost

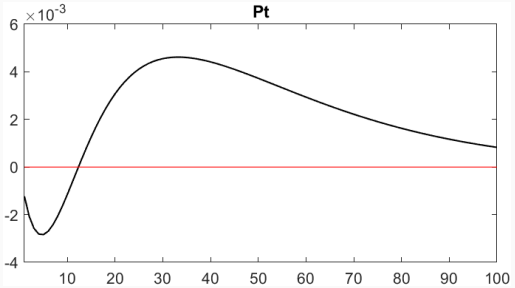



**(f)** Labor

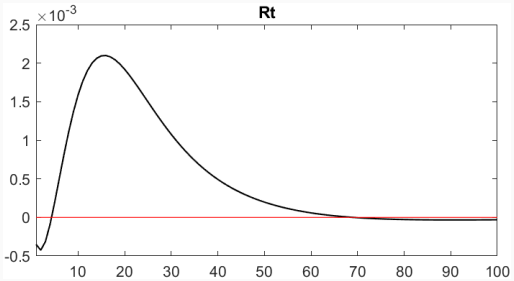
# Productivity Shock



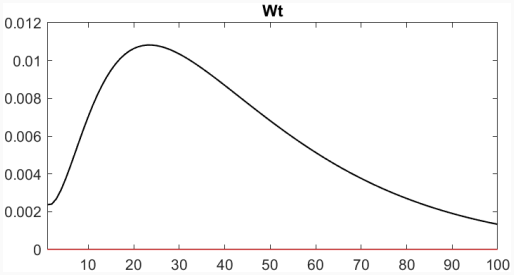

**(a)** Inflation



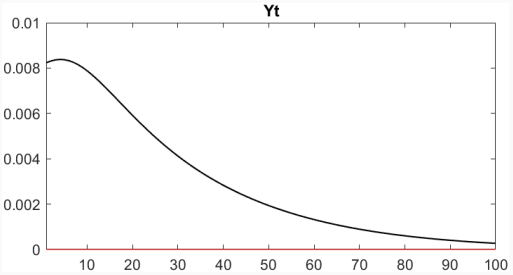

**(b)** Price Level




**(c)** Interest Rate

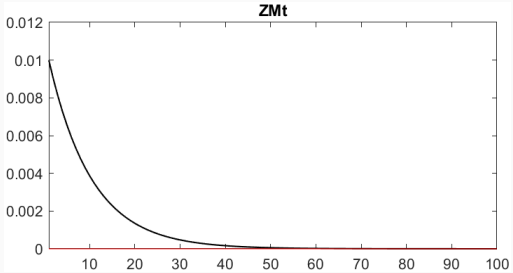



**(d)** Wage

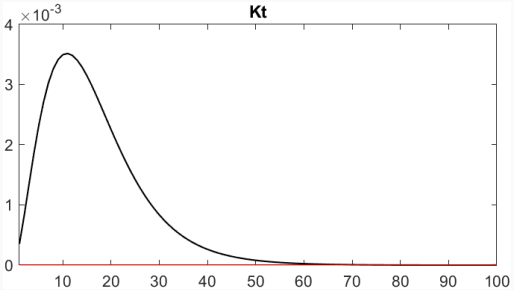



**(e)** Production

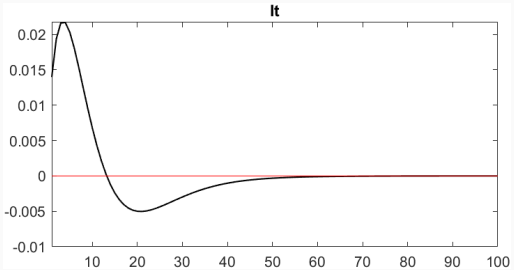
# Monetary Shock



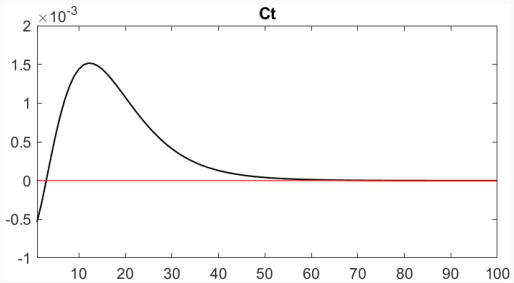

**(a)** Monetary Shock



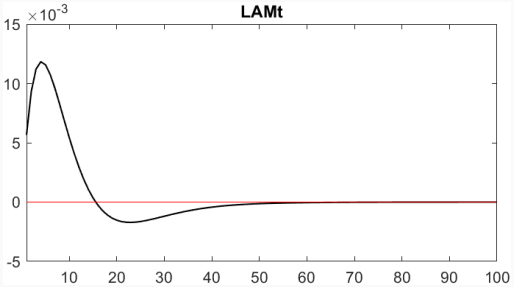

**(b)** Capital



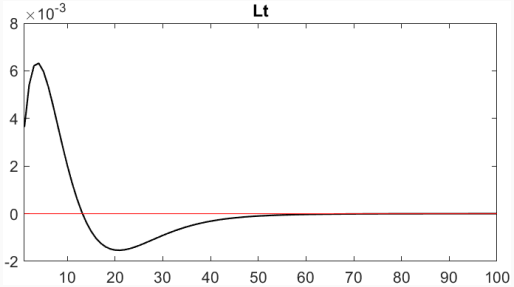

**(c)** Investment




**(d)** Consumption

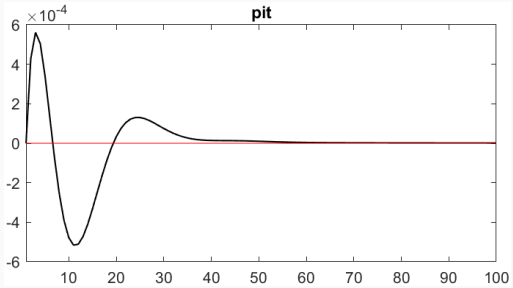



**(e)** Marginal Cost

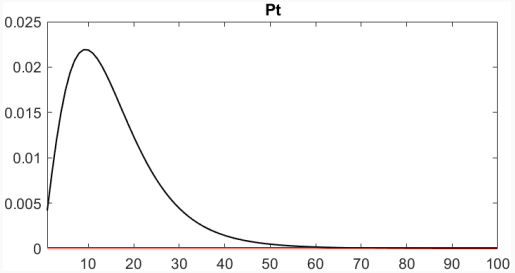



**(f)** Labor

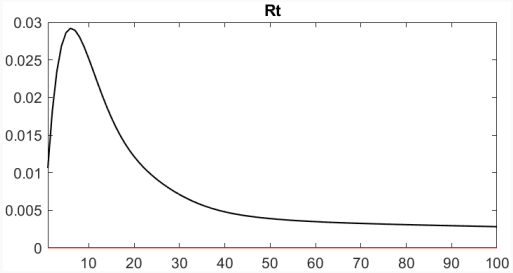
# Monetary Shock



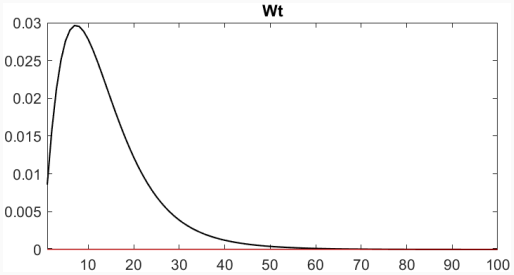

**(a)** Inflation



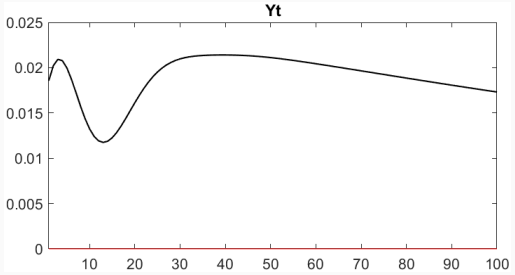

**(b)** Price Level




**(c)** Interest Rate




**(d)** Wage




**(e)** Production

**Figure 6:** Monetary Shock Impulse Response Functions

# Regional Model

# Characteristics

- two regions.
- mobility for final-goods.
- household $\eta$ and firm $\nu$ indexes.
- regional inflation $\pi_\nu$.

# Household Maximization Problem

$$\max_{C_{1\eta t}, C_{2\eta t}, L_{\eta t}, K_{\eta, t+1}} : \quad U_\eta(C_{\eta t}, L_{\eta t}) = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left( \frac{C_{\eta t}^{1-\sigma}}{1-\sigma} - \phi \frac{L_{\eta t}^{1+\varphi}}{1+\varphi} \right) \tag{44}$$

$$\text{s. t. :} \quad P_{C1t} C_{1\eta t} + P_{C2t} C_{2\eta t} + P_{C\eta t} I_{\eta t} = W_t L_{\eta t} + R_t K_{\eta t} + \Pi_{\nu t} \tag{45}$$

$$K_{\eta, t+1} = (1-\delta) K_{\eta t} + I_{\eta t} \tag{46}$$

$$C_{\eta t} = C_{1\eta t}^{\omega_\eta} C_{2\eta t}^{1-\omega_\eta} \tag{47}$$

$$C_{\nu\eta t}, L_{\eta t}, K_{\eta, t+1} > 0 \; ; \; K_0 \text{ given.}$$

# Final-goods Firm Maximization Problem

$$\max_{Y_{\nu jt}}: \quad \Pi_{\nu t} = P_{C\nu t} Y_{\nu t} - \int_0^1 P_{C\nu jt} Y_{\nu jt} \, \mathrm{d}\, j \tag{48}$$

$$\text{s. t.}: \quad Y_{\nu t} = \left( \int_0^1 Y_{\nu jt}^{\frac{\psi-1}{\psi}} \, \mathrm{d}\, j \right)^{\frac{\psi}{\psi-1}} \tag{49}$$

# Intermediate-goods Firm Problems

Cost Minimization Problem:

$$\min_{K_{\eta jt}, L_{\eta jt}} : \quad R_t K_{\eta jt} + W_t L_{\eta jt} \tag{50}$$

$$\text{s. t. :} \quad Y_{\nu jt} = Z_{A\nu t} K_{\eta jt}^{\alpha_\nu} L_{\eta jt}^{1-\alpha_\nu} \tag{51}$$

# Intermediate-goods Firm Problems

Price Stickiness and Profit Flow, Calvo's Rule (CALVO, 1983):

$$\mathbb{P}(P_t = P_{t-1}) = \theta \tag{52}$$

$$\max_{P_{C\nu jt}} : \quad \mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \frac{\theta^s \left[ P_{C\nu jt} Y_{\nu j,t+s} - TC_{\nu j,t+s} \right]}{\prod_{k=0}^{s-1}(1 + R_{t+k})} \right\} \tag{53}$$

$$\text{s. t. :} \quad Y_{\nu jt} = Y_{\nu t} \left( \frac{P_{C\nu t}}{P_{C\nu jt}} \right)^{\psi} \tag{54}$$

# Monetary Authority

Taylor's Rule (TAYLOR, 1993):

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\gamma_\pi} \left(\frac{Y_t}{Y}\right)^{\gamma_Y}\right]^{1-\gamma_R} Z_{Mt} \tag{55}$$

# Regional Price Level and Inflation

Regional price level $P_{C\nu t}$ and regional inflation rate:

$$\pi_{\nu t} = \frac{P_{C\nu t}}{P_{C\nu,t-1}} \tag{56}$$

National price level:

$$P_t = \vartheta_1 P_{C1t} + (1 - \vartheta_1) P_{C2t} \tag{57}$$

# Model Structure

The model is a square system of 27 variables and 27 equations.

- from the household problem: $C_{\eta t}, L_{\eta t}, K_{\eta,t+1}$;
- from the final-goods firm problem: $Y_{\nu t}, Y_{\nu jt}, P_{C\nu t}$;
- from the intermediate-goods firm problems: $K_{\eta jt}, L_{\eta jt}, P^*_{C\nu t}$;
- from the market clearing condition: $Y_t, I_{\eta t}$;
- prices: $W_t, R_t, \Lambda_{\nu t}, \pi_t$;
- shocks: $Z_{A\nu t}, Z_{Mt}$.

# Next Steps

- Steady State
- Parameter Calibration
- Steady State Solution
- Log-linearization
- Dynare Programming
- Impulse Response Functions

# Expected Results

# Expected Results

- monetary Policy Shock: 1%, posivite and negative.
- Regional technology level $Z_{A\nu t}$, capital weight in production $\alpha_\nu$ implies regional price levels $P_{\nu t}$ and inflation rates $\pi_{\nu t}$.
- for different parameters, the reaction of each region shall have a significant change.

# Project Timeline

# Project Timeline

| Activity | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|
| Literature Review | x | x | | | | | | | | |
| Project & Seminar | | | x | x | | | | | | |
| Modeling | | | x | x | x | | | | | |
| LaTeX | | | x | x | x | | | | | |
| *Dynare* | | | | | x | x | | | | |
| Qualification | | | | | | x | | | | |
| Regional Model | | | | | | x | ☐ | | | |
| Data Treatment | | | | | | | ☐ | | | |
| Parametrization | | | | | | | | ☐ | | |
| Results | | | | | | | | ☐ | | |
| Systematic Review | | | | | | | | | ☐ | |
| Revision & Edition | | | | | | | | | ☐ | |
| Thesis Defense | | | | | | | | | | ☐ |

**Obrigado!**

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# Dúvidas e Sugestões