3.3.2 Extended Log-linear Structure

1. Regional Gross Inflation Rate

$$\begin{aligned}\hat{\pi}_{1t} &= \hat{P}_{1t} - \hat{P}_{1,t-1} \\ \hat{\pi}_{2t} &= \hat{P}_{2t} - \hat{P}_{2,t-1}\end{aligned} \tag{3.102}$$

2. New Keynesian Phillips Curve

$$\begin{aligned}\hat{\pi}_{1t} &= \beta \mathbb{E}_t \hat{\pi}_{1,t+1} + \frac{(1-\theta)(1-\theta\beta)}{\theta}\hat{\lambda}_{1t} \\ \hat{\pi}_{2t} &= \beta \mathbb{E}_t \hat{\pi}_{2,t+1} + \frac{(1-\theta)(1-\theta\beta)}{\theta}\hat{\lambda}_{2t}\end{aligned} \tag{3.112}$$

3. Law of Motion for Capital

$$\begin{aligned}\hat{K}_{1,t+1} &= (1-\delta)\hat{K}_{1t} + \delta \hat{I}_{1t} \\ \hat{K}_{2,t+1} &= (1-\delta)\hat{K}_{2t} + \delta \hat{I}_{2t}\end{aligned} \tag{3.113}$$

4. Regional Levels of Consumption and Prices

$$\begin{aligned}\hat{C}_{11t} - \hat{C}_{12t} &= \hat{P}_{2t} - \hat{P}_{1t} \\ \hat{C}_{21t} - \hat{C}_{22t} &= \hat{P}_{2t} - \hat{P}_{1t}\end{aligned} \tag{3.116}$$

5. Total Expenses

$$\begin{aligned}\hat{\mathcal{E}}_{1t} &= \hat{C}_{1t} + \omega_{11}\hat{P}_{1t} + (1-\omega_{11})\hat{P}_{2t} \\ \hat{\mathcal{E}}_{2t} &= \hat{C}_{2t} + \omega_{21}\hat{P}_{1t} + (1-\omega_{21})\hat{P}_{2t}\end{aligned} \tag{3.117}$$

6. Labor Supply

$$\begin{aligned}\varphi \hat{L}_{1t} - (1-\sigma)\hat{C}_{1t} &= \hat{W}_t - \hat{\mathcal{E}}_{1t} \\ \varphi \hat{L}_{2t} - (1-\sigma)\hat{C}_{2t} &= \hat{W}_t - \hat{\mathcal{E}}_{2t}\end{aligned} \tag{3.118}$$

7. Euler equation for the bonds return

$$\begin{aligned}\mathbb{E}_t\left\{\hat{\mathcal{E}}_{1,t+1} - (1-\sigma)\hat{C}_{1,t+1}\right\} - \left[\hat{\mathcal{E}}_{1t} - (1-\sigma)\hat{C}_{1t}\right] &= (1-\beta)\hat{R}_t \\ \mathbb{E}_t\left\{\hat{\mathcal{E}}_{2,t+1} - (1-\sigma)\hat{C}_{2,t+1}\right\} - \left[\hat{\mathcal{E}}_{2t} - (1-\sigma)\hat{C}_{2t}\right] &= (1-\beta)\hat{R}_t\end{aligned} \tag{3.119}$$

8. Euler equation for the capital return

$$\begin{aligned}
&\mathbb{E}_t\left\{\hat{\mathcal{E}}_{1,t+1} - \hat{P}_{1,t+1} - (1-\sigma)\hat{C}_{1,t+1}\right\} - \\
&\quad - (\hat{\mathcal{E}}_{1t} - \hat{P}_{1t} - (1-\sigma)\hat{C}_{1t}) = \beta\frac{R_K}{P_1}\mathbb{E}_t\left\{\hat{R}_{K,t+1} - \hat{P}_{1,t+1}\right\} \\
&\mathbb{E}_t\left\{\hat{\mathcal{E}}_{2,t+1} - \hat{P}_{2,t+1} - (1-\sigma)\hat{C}_{2,t+1}\right\} - \\
&\quad - (\hat{\mathcal{E}}_{2t} - \hat{P}_{2t} - (1-\sigma)\hat{C}_{2t}) = \beta\frac{R_K}{P_2}\mathbb{E}_t\left\{\hat{R}_{K,t+1} - \hat{P}_{2,t+1}\right\}
\end{aligned} \tag{3.120}$$

9. Production Function

$$\begin{aligned}
\hat{Y}_{1t} &= \hat{Z}_{A1t} + \alpha\hat{K}_{1t} + (1-\alpha)\hat{L}_{1t} \\
\hat{Y}_{2t} &= \hat{Z}_{A2t} + \alpha\hat{K}_{2t} + (1-\alpha)\hat{L}_{2t}
\end{aligned} \tag{3.127}$$

10. Marginal Rates of Substitution of Factors

$$\begin{aligned}
\hat{K}_{1t} - \hat{L}_{1t} &= \hat{W}_t - \hat{R}_{Kt} \\
\hat{K}_{2t} - \hat{L}_{2t} &= \hat{W}_t - \hat{R}_{Kt}
\end{aligned} \tag{3.130}$$

11. Marginal Cost

$$\begin{aligned}
\hat{\lambda}_{1t} &= \alpha\hat{R}_{Kt} + (1-\alpha)\hat{W}_t - \hat{Z}_{A1t} - \hat{P}_{1t} \\
\hat{\lambda}_{2t} &= \alpha\hat{R}_{Kt} + (1-\alpha)\hat{W}_t - \hat{Z}_{A2t} - \hat{P}_{2t}
\end{aligned} \tag{3.132}$$

12. Monetary Policy

$$\hat{R}_t = \gamma_R\hat{R}_{t-1} + (1-\gamma_R)(\gamma_\pi\hat{\pi}_t + \gamma_Y\hat{Y}_t) + \hat{Z}_{Mt} \tag{3.133}$$

13. National Gross Inflation Rate

$$\hat{\pi}_t = \hat{P}_t - \hat{P}_{t-1} \tag{3.134}$$

14. National Price Level

$$\hat{P}_t + \hat{Y}_t = \theta_{PY1}(\hat{P}_{1t} + \hat{Y}_{1t}) + (1-\theta_{PY1})(\hat{P}_{2t} + \hat{Y}_{2t}) \tag{3.140}$$

15. Productivity Shock

$$\begin{aligned}
\hat{Z}_{A1t} &= \rho_{A1}\hat{Z}_{A1,t-1} + \varepsilon_{A1} \\
\hat{Z}_{A2t} &= \rho_{A2}\hat{Z}_{A2,t-1} + \varepsilon_{A2}
\end{aligned} \tag{3.141}$$

16. Monetary Shock

$$\hat{Z}_{Mt} = \rho_M\hat{Z}_{M,t-1} + \varepsilon_M \tag{3.142}$$

17. Market Clearing Condition

$$\hat{Y}_t = \theta_{Y1}\hat{Y}_{1t} + (1 - \theta_{Y1})\hat{Y}_{2t} \tag{3.137}$$

18. Regional Market Clearing Condition

$$\begin{aligned}\hat{Y}_{1t} &= \theta_{C11}\hat{C}_{11t} + \theta_{C12}\hat{C}_{12t} + (1 - \theta_{C11} - \theta_{C12})\hat{I}_{1t} \\ \hat{Y}_{2t} &= \theta_{C21}\hat{C}_{21t} + \theta_{C22}\hat{C}_{22t} + (1 - \theta_{C21} - \theta_{C22})\hat{I}_{2t}\end{aligned} \tag{3.145}$$