

# **Analysis of the Monetary Policy Impact on Regional Gross Domestic Product: A Regional DSGE Model**

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André Luiz Brito

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PPGDE-UFPR

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# Introduction



Brazilian regions have heterogeneous economic matrices that respond in diverse ways to the decisions of the monetary authority. (BERTANHA; HADDAD, 2008).

## Objectives:

- Develop a NK DSGE model with:
  - two regions with different parameters;
  - monetary-policy shocks.
- Demonstrate that different regions react in distinct ways to the monetary policy.

# What is a NK DSGE model?

NK DSGE model is a macroeconomic tool with:

- **New Keynesian:** monopolistic competition, nominal rigidities, short-run non-neutrality of monetary policy.
- **Dynamic:** shows the changes over time.
- **Stochastic:** considers random and uncertainty.
- **General Equilibrium:** agents optimize and markets clear (microfoundations).

# Literature Review

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- Costa Junior (2016): presents a RBC model and then adds NK elements in each chapter;
- Galí (2015): discuss monetary policy starting with a RBC model and also adds NK elements in each chapter;
- Bergholt (2012): presents a NK and the method of programming in Dynare;
- Solis-Garcia (2022): presents a RBC model and demonstrate the math tools necessary to solve a DSGE model;



- Rickman (2010): link between macro and regional modeling.
- Mora e Costa Junior (2019): Effects of foreign direct investment (FDI), taking into consideration where it is applied: DSGE model with two regions (Bogotá and the rest of Colombia).
- Costa Junior et al. (2022): Effects of fiscal policy, considering the federative entities: DSGE model for the State of Goiás and the rest of the country.
- Osterno (2022): Regionalization of SAMBA: SAMBA+REG (Stochastic Analytical Model with Bayesian Approach from the Central Bank of Brazil).

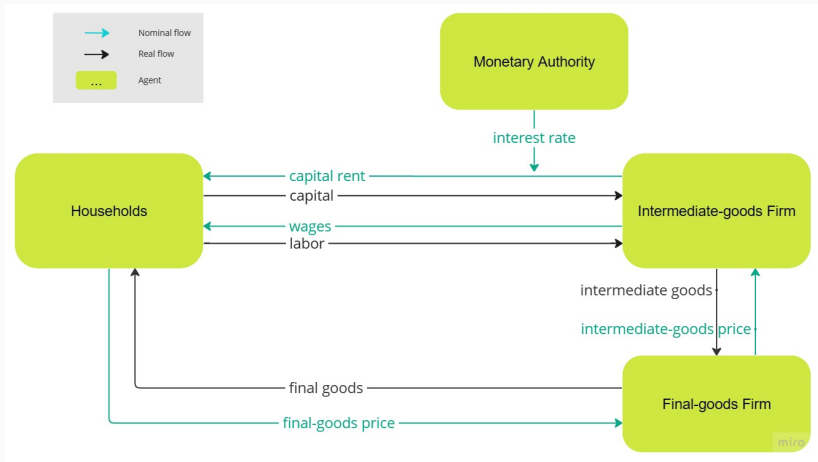
# Model



- four agents: households, intermediate and final-goods firms, monetary authority.
- no bonds.
- capital and investment.
- price stickiness of intermediate goods.

- the representative household maximizes utility;
- firms producing intermediate goods minimize costs and maximize profit flow;
- firms producing final goods maximize profit.
- the monetary authority determines the interest rate, aiming to control inflation and pursuing economic growth.

# Model Structure



**Figure 1: Model Diagram**

# Household Maximization Problem

$$\max_{C_t, L_t, K_{t+1}} : U(C_t, L_t) = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma}}{1-\sigma} - \phi \frac{L_t^{1+\varphi}}{1+\varphi} \right) \quad (1)$$

$$\text{s. t. : } P_t(C_t + I_t) = W_t L_t + R_t K_t + \Pi_t \quad (2)$$

$$K_{t+1} = (1 - \delta)K_t + I_t \quad (3)$$

$$C_t, L_t, K_{t+1} \geq 0 ; K_0 \text{ given.}$$

# Final-goods Firm Maximization Problem

$$\max_{Y_{jt}} : \quad \Pi_t = P_t Y_t - \int_0^1 P_{jt} Y_{jt} dj \quad (4)$$

$$\text{s. t. :} \quad Y_t = \left( \int_0^1 Y_{jt}^{\frac{\psi-1}{\psi}} dj \right)^{\frac{\psi}{\psi-1}} \quad (5)$$

Cost Minimization Problem:

$$\min_{K_{jt}, L_{jt}} : R_t K_{jt} + W_t L_{jt} \quad (6)$$

$$\text{s. t. : } Y_{jt} = Z_{At} K_{jt}^{\alpha} L_{jt}^{1-\alpha} \quad (7)$$



# Intermediate-goods Firm Problems

Price Stickiness and Profit Flow, Calvo's Rule (CALVO, 1983):

$$\mathbb{P}(P_t = P_{t-1}) = \theta \quad (8)$$

$$\max_{P_{jt}} : \quad \mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \frac{\theta^s [P_{jt} Y_{j,t+s} - TC_{j,t+s}]}{\prod_{k=0}^{s-1} (1 + R_{t+k})} \right\} \quad (9)$$

$$\text{s. t. :} \quad Y_{jt} = Y_t \left( \frac{P_t}{P_{jt}} \right)^{\psi} \quad (10)$$

Taylor's Rule (TAYLOR, 1993):

$$\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\gamma_R} \left[ \left( \frac{\pi_t}{\pi} \right)^{\gamma_\pi} \left( \frac{Y_t}{Y} \right)^{\gamma_Y} \right]^{1-\gamma_R} Z_{Mt} \quad (11)$$

Productivity Shock:

$$\ln Z_{At} = (1 - \rho_A) \ln Z_A + \rho_A \ln Z_{A,t-1} + \varepsilon_{At} \quad (12)$$

Monetary Policy Shock:

$$\ln Z_{Mt} = (1 - \rho_M) \ln Z_M + \rho_M \ln Z_{M,t-1} + \varepsilon_{Mt} \quad (13)$$

Square system of 16 variables and 16 equations:

- from the household problem:  $C_t, L_t, K_{t+1}$ ;
- from the final-good firm problem:  $Y_{jt}, P_t$ ;
- from the intermediate-good firm problems:  $K_{jt}, L_{jt}, P_t^*$ ;
- from the market clearing condition:  $Y_t, I_t$ ;
- prices:  $W_t, R_t, \Lambda_t, \pi_t$ ;
- shocks:  $Z_{At}, Z_{Mt}$ .

Equations:

1. Labor Supply:

$$\frac{\phi L_t^\varphi}{C_t^{-\sigma}} = \frac{W_t}{P_t} \quad (14)$$

2. Household Euler Equation:

$$\left( \frac{\mathbb{E}_t C_{t+1}}{C_t} \right)^\sigma = \beta \left[ (1 - \delta) + \mathbb{E}_t \left( \frac{R_{t+1}}{P_{t+1}} \right) \right] \quad (15)$$

3. Budget Constraint:

$$P_t(C_t + I_t) = W_t L_t + R_t K_t + \Pi_t \quad (16)$$

### 4. Law of Motion for Capital:

$$K_{t+1} = (1 - \delta)K_t + I_t \quad (17)$$

### 5. Bundle Technology:

$$Y_t = \left( \int_0^1 Y_{jt}^{\frac{\psi-1}{\psi}} dj \right)^{\frac{\psi}{\psi-1}} \quad (18)$$

### 6. General Price Level:

$$P_t = \left[ \theta P_{t-1}^{1-\psi} + (1 - \theta) P_t^{*1-\psi} \right]^{\frac{1}{1-\psi}} \quad (19)$$

### 7. Capital Demand:

$$K_{jt} = \alpha Y_{jt} \frac{\Lambda_t}{R_t} \quad (20)$$

### 8. Labor Demand:

$$L_{jt} = (1 - \alpha) Y_{jt} \frac{\Lambda_t}{W_t} \quad (21)$$

### 9. Marginal Cost:

$$\Lambda_t = \frac{1}{Z_{At}} \left( \frac{R_t}{\alpha} \right)^\alpha \left( \frac{W_t}{1 - \alpha} \right)^{1 - \alpha} \quad (22)$$

## 10. Production Function:

$$Y_{jt} = Z_{At} K_{jt}^{\alpha} L_{jt}^{1-\alpha} \quad (23)$$

## 11. Optimal Price:

$$P_t^* = \frac{\psi}{\psi - 1} \cdot \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \theta^s Y_{j,t+s} \Lambda_{t+s} / \prod_{k=0}^{s-1} (1 + R_{t+k}) \right\}}{\mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \theta^s Y_{j,t+s} / \prod_{k=0}^{s-1} (1 + R_{t+k}) \right\}} \quad (24)$$

## 12. Market Clearing Condition:

$$Y_t = C_t + I_t \quad (25)$$



### 13. Monetary Policy:

$$\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\gamma_R} \left[ \left( \frac{\pi_t}{\pi} \right)^{\gamma_\pi} \left( \frac{Y_t}{Y} \right)^{\gamma_Y} \right]^{1-\gamma_R} Z_{Mt} \quad (26)$$

### 14. Gross Inflation Rate:

$$\pi_t = \frac{P_t}{P_{t-1}} \quad (27)$$

### 15. Productivity Shock:

$$\ln Z_{At} = (1 - \rho_A) \ln Z_A + \rho_A \ln Z_{A,t-1} + \varepsilon_{At} \quad (28)$$

### 16. Monetary Shock:

$$\ln Z_{Mt} = (1 - \rho_M) \ln Z_M + \rho_M \ln Z_{M,t-1} + \varepsilon_{Mt} \quad (29)$$

Steady State

Steady state solution (COSTA JUNIOR, 2016, p.41):

$$\mathbb{E}_t X_{t+1} = X_t = X_{t-1} = X_{ss} \quad (30)$$

# Log-linearization

Uhlig's rules for log-linearization (UHLIG, 1999).

Square system of 12 variables and 12 equations:

Variables:

$$\left( \tilde{\pi} \quad \hat{P} \quad \hat{\lambda} \quad \hat{C} \quad \hat{L} \quad \hat{R} \quad \hat{K} \quad \hat{I} \quad \hat{W} \quad \hat{Z}_A \quad \hat{Y} \quad \hat{Z}_M \right) \quad (31)$$

Equations:

1. Gross Inflation Rate:

$$\tilde{\pi}_t = \hat{P}_t - \hat{P}_{t-1} \quad (32)$$

2. New Keynesian Phillips Curve:

$$\tilde{\pi}_t = \varrho \mathbb{E}_t \tilde{\pi}_{t+1} + \frac{(1-\theta)(1-\theta\varrho)}{\theta} \hat{\lambda}_t \quad (33)$$

3. Labor Supply:

$$\varphi \hat{L}_t + \sigma \hat{C}_t = \hat{W}_t + \hat{P}_t \quad (34)$$



## 4. Household Euler Equation:

$$\mathbb{E}_t \hat{C}_{t+1} - \hat{C}_t = \frac{\beta R}{\sigma P} \mathbb{E}_t (\hat{R}_{t+1} - \hat{P}_{t+1}) \quad (35)$$

## 5. Law of Motion for Capital:

$$\hat{K}_{t+1} = (1 - \delta) \hat{K}_t + \delta \hat{I}_t \quad (36)$$

## 6. Real Marginal Cost:

$$\hat{\lambda}_t = \alpha \hat{R}_t + (1 - \alpha) \hat{W}_t - \hat{Z}_{At} - \hat{P}_t \quad (37)$$

### 7. Production Function:

$$\hat{Y}_t = \hat{Z}_{At} + \alpha \hat{K}_t + (1 - \alpha) \hat{L}_t \quad (38)$$

### 8. Marginal Rates of Substitution of Factors:

$$\hat{K}_t - \hat{L}_t = \hat{W}_t - \hat{R}_t \quad (39)$$

### 9. Market Clearing Condition:

$$\hat{Y}_t = \theta_C \hat{C}_t + \theta_I \hat{I}_t \quad (40)$$

### 10. Monetary Policy:

$$\hat{R}_t = \gamma_R \hat{R}_{t-1} + (1 - \gamma_R)(\gamma_\pi \tilde{\pi}_t + \gamma_Y \hat{Y}_t) + \hat{Z}_{Mt} \quad (41)$$

### 11. Productivity Shock:

$$\hat{Z}_{At} = \rho_A \hat{Z}_{A,t-1} + \varepsilon_A \quad (42)$$

### 12. Monetary Shock:

$$\hat{Z}_{Mt} = \rho_M \hat{Z}_{M,t-1} + \varepsilon_M \quad (43)$$

# Matlab and Dynare

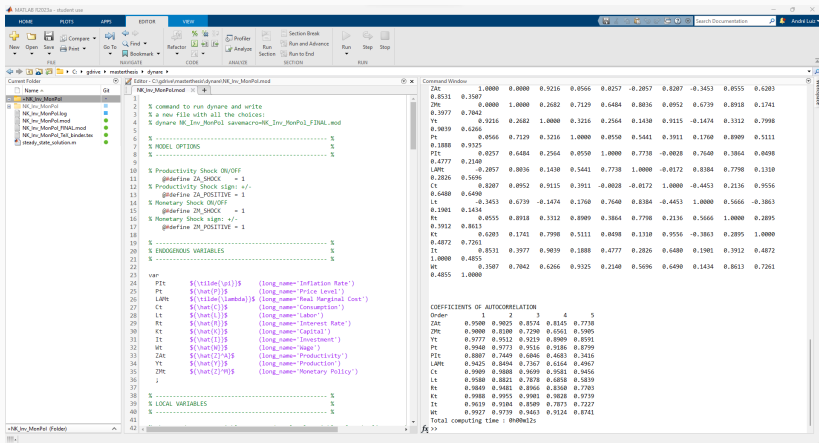
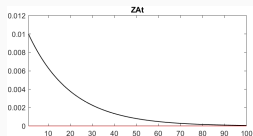
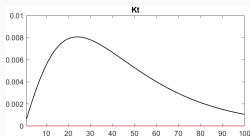


Figure 2: Matlab and Dynare

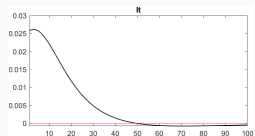
# Productivity Shock



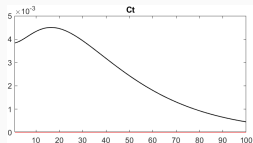
**(a) Productivity Shock**



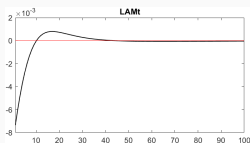
**(b) Capital**



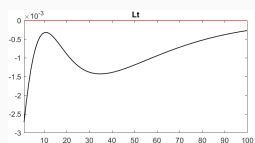
**(c) Investment**



**(d) Consumption**

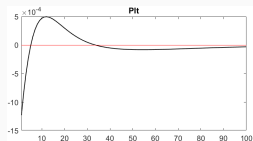


**(e) Marginal Cost**

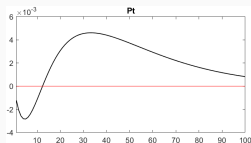


**(f) Labor**

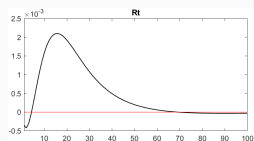
# Productivity Shock



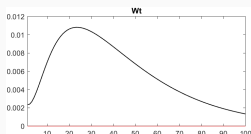
**(a) Inflation**



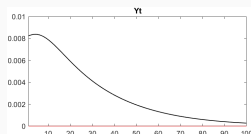
**(b) Price Level**



**(c) Interest Rate**

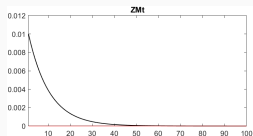


**(d) Wage**

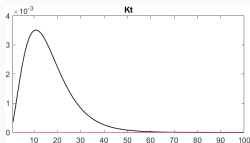


**(e) Production**

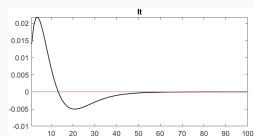
# Monetary Shock



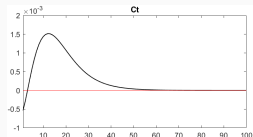
**(a) Monetary Shock**



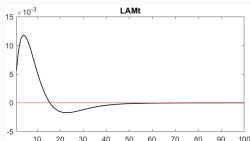
**(b) Capital**



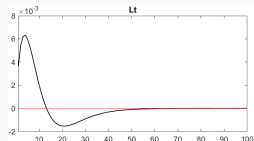
**(c) Investment**



**(d) Consumption**

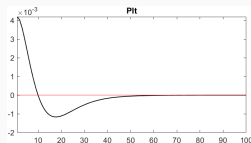


**(e) Marginal Cost**

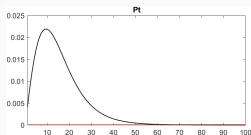


**(f) Labor**

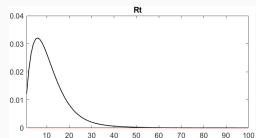
# Monetary Shock



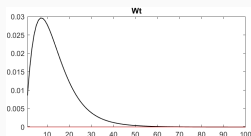
(a) Inflation



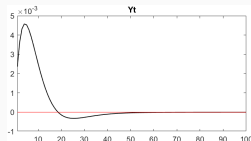
(b) Price Level



(c) Interest Rate



(d) Wage



(e) Production

**Figure 6:** Monetary Shock Impulse Response Functions



# Regional Model

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- two regions.
- mobility for final-goods.
- household  $\eta$  and firm  $\nu$  indexes.
- regional inflation  $\pi_\nu$ .

# Household Maximization Problem

$$\max_{C_{1\eta t}, C_{2\eta t}, L_{\eta t}, K_{\eta, t+1}} : U_{\eta}(C_{\eta t}, L_{\eta t}) = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left( \frac{C_{\eta t}^{1-\sigma}}{1-\sigma} - \phi \frac{L_{\eta t}^{1+\varphi}}{1+\varphi} \right) \quad (44)$$

$$\text{s. t. : } P_{C1t}C_{1\eta t} + P_{C2t}C_{2\eta t} + P_{C\eta t}I_{\eta t} = W_tL_{\eta t} + R_tK_{\eta t} + \Pi_{\nu t} \quad (45)$$

$$K_{\eta, t+1} = (1 - \delta)K_{\eta t} + I_{\eta t} \quad (46)$$

$$C_{\eta t} = C_{1\eta t}^{\omega_{\eta}} C_{2\eta t}^{1-\omega_{\eta}} \quad (47)$$

$$C_{\nu \eta t}, L_{\eta t}, K_{\eta, t+1} > 0 ; K_0 \text{ given.}$$

# Final-goods Firm Maximization Problem

$$\max_{Y_{\nu jt}} : \quad \Pi_{\nu t} = P_{C\nu t} Y_{\nu t} - \int_0^1 P_{C\nu jt} Y_{\nu jt} dj \quad (48)$$

$$\text{s. t. :} \quad Y_{\nu t} = \left( \int_0^1 Y_{\nu jt}^{\frac{\psi-1}{\psi}} dj \right)^{\frac{\psi}{\psi-1}} \quad (49)$$

Cost Minimization Problem:

$$\min_{K_{\eta jt}, L_{\eta jt}} : R_t K_{\eta jt} + W_t L_{\eta jt} \quad (50)$$

$$\text{s. t. : } Y_{\nu jt} = Z_{A\nu t} K_{\eta jt}^{\alpha_\nu} L_{\eta jt}^{1-\alpha_\nu} \quad (51)$$

Price Stickiness and Profit Flow, Calvo's Rule (CALVO, 1983):

$$\mathbb{P}(P_t = P_{t-1}) = \theta \quad (52)$$

$$\max_{P_{C\nu jt}} : \mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \frac{\theta^s [P_{C\nu jt} Y_{\nu j, t+s} - TC_{\nu j, t+s}]}{\prod_{k=0}^{s-1} (1 + R_{t+k})} \right\} \quad (53)$$

$$\text{s. t. : } Y_{\nu jt} = Y_{\nu t} \left( \frac{P_{C\nu t}}{P_{C\nu jt}} \right)^{\psi} \quad (54)$$

Taylor's Rule (TAYLOR, 1993):

$$\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\gamma_R} \left[ \left( \frac{\pi_t}{\pi} \right)^{\gamma_\pi} \left( \frac{Y_t}{Y} \right)^{\gamma_Y} \right]^{1-\gamma_R} Z_{Mt} \quad (55)$$

# Regional Price Level and Inflation

Regional price level  $P_{C\nu t}$  and regional inflation rate:

$$\pi_{\nu t} = \frac{P_{C\nu t}}{P_{C\nu, t-1}} \quad (56)$$

National price level:

$$P_t = \vartheta_1 P_{C1t} + (1 - \vartheta_1) P_{C2t} \quad (57)$$



The model is a square system of 27 variables and 27 equations.

- from the household problem:  $C_{\eta t}, L_{\eta t}, K_{\eta, t+1}$ ;
- from the final-goods firm problem:  $Y_{\nu t}, Y_{\nu jt}, P_{C\nu t}$ ;
- from the intermediate-goods firm problems:  $K_{\eta jt}, L_{\eta jt}, P_{C\nu t}^*$ ;
- from the market clearing condition:  $Y_t, I_{\eta t}$ ;
- prices:  $W_t, R_t, \Lambda_{\nu t}, \pi_t$ ;
- shocks:  $Z_{A\nu t}, Z_{Mt}$ .

# Next Steps

- Steady State
- Parameter Calibration
- Steady State Solution
- Log-linearization
- Dynare Programming
- Impulse Response Functions

## **Expected Results**

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# Expected Results

- monetary Policy Shock: 1%, positive and negative.
- Regional technology level  $Z_{A\nu t}$ , capital weight in production  $\alpha_\nu$  implies regional price levels  $P_{\nu t}$  and inflation rates  $\pi_{\nu t}$ .
- for different parameters, the reaction of each region shall have a significant change.

# Project Timeline

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# Project Timeline

Activity	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Literature Review	x	x								
Project & Seminar			x	x						
Modeling			x	x	x					
LaTeX			x	x	x					
Dynare					x	x				
Qualification						x				
Regional Model						x	<input type="checkbox"/>			
Data Treatment							<input type="checkbox"/>			
Parametrization								<input type="checkbox"/>		
Results								<input type="checkbox"/>		
Systematic Review									<input type="checkbox"/>	
Revision & Edition									<input type="checkbox"/>	
Thesis Defense										<input type="checkbox"/>

**Obrigado!**  
**andreluizmtg@gmail.com**  
**41.98460.2209**

## **Dúvidas e Sugestões**