Analysis of the Monetary Policy Impact on Regional Gross Domestic Product: A Regional DSGE Model

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Contents

Introduction

Introduction

Brazilian regions have heterogeneous economic matrices that respond in diverse ways to the decisions of the monetary authority. (BERTANHA; HADDAD, 2008).

Introduction

Objectives:

- Develop a NK DSGE model with:
    - two regions with distinct structures of production;
    - monetary-policy shocks.
- Demonstrate that different regions react in distinct ways to the monetary policy.

What is a NK DSGE model?

NK DSGE model is a macroeconomic tool with:

- **New Keynesian**: monopolistic competition, nominal rigidities, short-run non-neutrality of monetary policy.
- **Dynamic**: shows the changes over time.
- **Stochastic**: considers random and uncertainty.
- **General Equilibrium**: agents optimize and markets clear (microfoundations).

Literature Review

Macro modeling

- Costa Junior (2016): presents a RBC model and then adds NK elements in each chapter;
- Galí (2015): discuss monetary policy starting with a RBC model and also adds NK elements in each chapter;
- Bergholt (2012): presents a NK and the method of programming in *Dynare*;
- Solis-Garcia (2022): presents a RBC model and demonstrate the math tools necessary to solve a DSGE model;

Regional Modeling

- Rickman (2010): link between macro and regional modeling.
- Mora e Costa Junior (2019): Effects of foreign direct investment (FDI), taking into consideration where it is applied: DSGE model with two regions (Bogotá and the rest of Colombia).
- Costa Junior et al. (2022): Effects of fiscal policy, considering the federative entities: DSGE model for the State of Goiás and the rest of the country.
- Osterno et al. (2022): Regionalization of SAMBA: SAMBA+REG (Stochastic Analytical Model with Bayesian Approach from the Central Bank of Brazil).

Regional NK Model

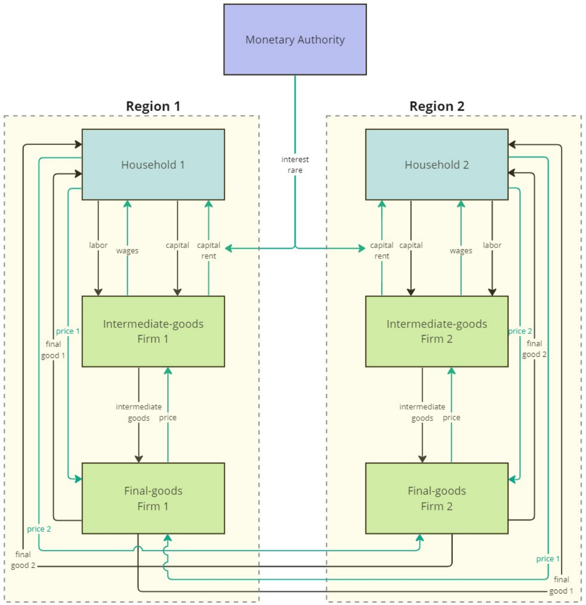
Characteristics

- four agents: households, intermediate and final-goods firms, monetary authority.
- no bonds.
- capital and investment.
- price stickiness of intermediate goods.
- two regions: final good is what links both.

Agents

- the representative household maximizes utility;
- firms producing intermediate goods minimize costs and maximize profit flow;
- firms producing final goods maximize profit.
- the monetary authority determines the interest rate, aiming to control inflation and pursuing economic growth.

Model Structure



Cost Minimization Problem

$$\min_{C_{\eta 1t}, C_{\eta 2t}} : \quad Q_{\eta t} C_{\eta t} = P_{1t} C_{\eta 1t} + P_{2t} C_{\eta 2t} \tag{1}$$

$$\text{s.t.} : \quad C_{\eta t} = C_{\eta 1t}^{\omega_{\eta 1}} C_{\eta 2t}^{1-\omega_{\eta 1}} \tag{2}$$

$$C_{\eta t} > 0$$

Cost Minimization Problem

Solution:

$$C_{\eta 2t} = C_{\eta 1t} \frac{(1-\omega_{\eta 1})P_{1t}}{\omega_{\eta 1} P_{2t}} \tag{3}$$

$$C_{\eta 1t} = C_{\eta t} \left(\frac{P_{2t}\omega_{\eta 1}}{P_{1t}(1-\omega_{\eta 1})} \right)^{1-\omega_{\eta 1}} \tag{4}$$

$$Q_{\eta t} = \left(\frac{P_{1t}}{\omega_{\eta 1}} \right)^{\omega_{\eta 1}} \left(\frac{P_{2t}}{1-\omega_{\eta 1}} \right)^{1-\omega_{\eta 1}} \tag{5}$$

Household Maximization Problem

$$\max_{C_{\eta t}, L_{\eta t}, K_{\eta,t+1}} : \quad U_\eta(C_{\eta t}, L_{\eta t}) = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\frac{C_{\eta t}^{1-\sigma}}{1-\sigma} - \phi \frac{L_{\eta t}^{1+\varphi}}{1+\varphi} \right) \tag{6}$$

$$\text{s. t.} : \quad Q_{\eta t} C_{\eta t} + P_{\eta t} I_{\eta t} = W_{\eta t} L_{\eta t} + R_t K_{\eta t} + \Pi_{\eta t} \tag{7}$$

$$K_{\eta,t+1} = (1-\delta) K_{\eta t} + I_{\eta t} \tag{8}$$

$$C_{\eta t}, L_{\eta t}, K_{\eta t} > 0$$

Household Maximization Problem

Solution:

$$\frac{\phi L_{\eta t}^{\varphi}}{C_{\eta t}^{-\sigma}} = \frac{W_{\eta t}}{Q_{\eta t}} \tag{9}$$

$$\frac{\mathbb{E}_t\{Q_{\eta,t+1} C_{\eta,t+1}^{\sigma}\}}{Q_{\eta t} C_{\eta t}^{\sigma}} = \beta \frac{\mathbb{E}_t\{P_{\eta,t+1}(1-\delta) + R_{t+1}\}}{P_{\eta t}} \tag{10}$$

Final-goods Firm Maximization Problem

$$\max_{Y_{\eta jt}} : \quad P_{\eta t} Y_{\eta t} - \int_0^1 P_{\eta jt} Y_{\eta jt} \, \mathrm{d} j \tag{11}$$

$$\text{s. t.} : \quad Y_{\eta t} = \left(\int_0^1 Y_{\eta jt}^{\frac{\psi - 1}{\psi}} \, \mathrm{d} j \right)^{\frac{\psi}{\psi - 1}} \tag{12}$$

Final-goods Firm Maximization Problem

Solution:

$$Y_{\eta jt} = Y_t \left(\frac{P_{\eta t}}{P_{\eta jt}} \right)^{\psi} \tag{13}$$

$$P_{\eta t} = \left[\int_0^1 P_{\eta jt}^{1-\psi} \, \mathrm{d}\, j \right]^{\frac{1}{1-\psi}} \tag{14}$$

Intermediate-goods Firm Problems

$$\min_{K_{\eta jt}, L_{\eta jt}} : \quad R_t K_{\eta jt} + W_t L_{\eta jt} \tag{15}$$

$$\text{s. t.} : \quad Y_{\eta jt} = Z_{A\eta t} K_{\eta jt}^{\alpha_\eta} L_{\eta jt}^{1-\alpha_\eta} \tag{16}$$

Intermediate-goods Firm Problems

Solutions:

$$\frac{K_{\eta jt}}{L_{\eta jt}} = \left(\frac{\alpha_\eta}{1-\alpha_\eta}\right)\frac{W_{\eta t}}{R_t} \tag{17}$$

$$K_{\eta jt} = \frac{Y_{\eta jt}}{Z_{A\eta t}}\left[\left(\frac{\alpha_\eta}{1-\alpha_\eta}\right)\frac{W_{\eta t}}{R_t}\right]^{1-\alpha_\eta} \tag{18}$$

$$L_{\eta jt} = \frac{Y_{\eta jt}}{Z_{A\eta t}}\left[\left(\frac{\alpha_\eta}{1-\alpha_\eta}\right)\frac{W_{\eta t}}{R_t}\right]^{-\alpha_\eta} \tag{19}$$

$$\Lambda_{\eta t} = \frac{1}{Z_{A\eta t}}\left(\frac{R_t}{\alpha_\eta}\right)^{\alpha_\eta}\left(\frac{W_{\eta t}}{1-\alpha_\eta}\right)^{1-\alpha_\eta} \tag{20}$$

Intermediate-goods Firm Problems

Price Stickiness and Profit Flow, Calvo's Rule (CALVO, 1983):

$$\max_{P_{\eta jt}} : \quad \mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \frac{\theta^s \left[P_{\eta jt} Y_{\eta j,t+s} - TC_{\eta j,t+s} \right]}{\prod_{k=0}^{s-1} (1 + R_{t+k})} \right\} \tag{21}$$

$$\text{s. t.} : \quad Y_{\eta jt} = Y_{\eta t} \left(\frac{P_{\eta t}}{P_{\eta jt}} \right)^{\psi} \tag{13}$$

Intermediate-goods Firm Problems

Solution:

$$P^*_{\eta t} = \frac{\psi}{\psi - 1} \cdot \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \left\{\theta^s Y_{\eta j,t+s} \Lambda_{\eta,t+s} / \prod_{k=0}^{s-1}(1 + R_{t+k})\right\}}{\mathbb{E}_t \sum_{s=0}^{\infty} \left\{\theta^s Y_{\eta j,t+s} / \prod_{k=0}^{s-1}(1 + R_{t+k})\right\}} \tag{22}$$

$$P_{\eta t} = \left[\theta P^{1-\psi}_{\eta,t-1} + (1 - \theta) P^{*1-\psi}_{\eta t}\right]^{\frac{1}{1-\psi}} \tag{23}$$

Regional Inflation

$$\pi_{\eta t} = \frac{P_{\eta t}}{P_{\eta,t-1}} \tag{24}$$

Monetary Authority

Taylor's Rule (TAYLOR, 1993):

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\gamma_\pi} \left(\frac{Y_t}{Y}\right)^{\gamma_Y}\right]^{1-\gamma_R} Z_{Mt} \tag{25}$$

$$\text{where: } \pi_t = \pi_{1t}^{\theta_\pi} \pi_{2t}^{1-\theta_\pi} \tag{26}$$

$$\text{and: } \theta_\pi = \frac{P_{1t} Y_{1t}}{P_{1t} Y_{1t} + P_{2t} Y_{2t}} \tag{27}$$

Stochastic Shocks

Productivity Shock:

$$\ln Z_{At} = (1 - \rho_A) \ln Z_A + \rho_A \ln Z_{A,t-1} + \varepsilon_{At} \tag{28}$$

Monetary Policy Shock:

$$\ln Z_{Mt} = (1 - \rho_M) \ln Z_M + \rho_M \ln Z_{M,t-1} + \varepsilon_{Mt} \tag{29}$$

Equilibrium Conditions

A Competitive Equilibrium consists of sequences of:

- prices $\{P^*_{\eta t}, R^*_t, W^*_{\eta t}\}$,

- allocations for households $\mathcal{A}_H := \{C^*_{\eta 1t}, C^*_{\eta 2t}, L^*_{\eta t}, I^*_{\eta t}, K^*_{\eta, t+1}\}$

- allocations for firms $\mathcal{A}_F := \{K^*_{\eta jt}, L^*_{\eta jt}, Y^*_{\eta jt}, Y^*_{\eta t}\}$.

In such an equilibrium, given the set of exogenous variables $\{K_0, Z_{A\eta t}, Z_{Mt}\}$, the elements in $\mathcal{A}_H$ solve the household problem, while the elements in $\mathcal{A}_F$ solve the firms' problems, and the markets for goods and labor clear:

$$Y_t = Y_{1t} + Y_{2t} \tag{30}$$

$$L_{\eta t} = \int_0^1 L_{\eta jt} \, \mathrm{d}\, j \tag{31}$$

Steady State

Steady State

Steady state solution (COSTA JUNIOR, 2016, p.41):

$$\mathbb{E}_t X_{t+1} = X_t = X_{t-1} = X_{ss} \tag{32}$$

Log-linearization

Log-linearization

Uhlig's rules for log-linearization (UHLIG, 1999).

$$\hat{x}_t := \frac{x_t - x}{x} \iff x_t = x(1 + \hat{x}_t)$$

Log-linear Model i

Square system of 30 variables and equations:

- Real Variables: $\langle \hat{C}_\eta \quad \hat{L}_\eta \quad \hat{K}_\eta \quad \hat{I}_\eta \quad \hat{C}_{\eta 1} \quad \hat{C}_{\eta 2} \quad \hat{Y}_\eta \quad \hat{Y} \quad \hat{Z}_{A\eta} \quad \hat{Z}_M \rangle$;
- Nominal Variables: $\langle \hat{Q}_\eta \quad \hat{P}_\eta \quad \hat{R} \quad \hat{\pi} \quad \hat{W}_\eta \quad \hat{\lambda}_\eta \quad \hat{\pi}_\eta \rangle$.

Log-linear Model i

- Regional Gross Inflation Rate

$$\hat{\pi}_{\eta t} = \hat{P}_{\eta t} - \hat{P}_{\eta,t-1} \tag{33}$$

- New Keynesian Phillips Curve

$$\hat{\pi}_{\eta t} = \beta \mathbb{E}_t \hat{\pi}_{\eta,t+1} + \frac{(1-\theta)(1-\theta\beta)}{\theta} \hat{\lambda}_{\eta t} \tag{34}$$

- Regional Consumption Weight

$$\hat{C}_{\eta 2t} - \hat{C}_{\eta 1t} = \hat{P}_{1t} - \hat{P}_{2t} \tag{35}$$

Log-linear Model ii

- Regional Consumption of Good 1

$$\hat{C}_{\eta t} - \hat{C}_{\eta 1t} = (1 - \omega_{\eta 1})(\hat{P}_{1t} - \hat{P}_{2t}) \tag{36}$$

- Regional Price Index

$$\hat{Q}_{\eta t} = \omega_{\eta 1}\hat{P}_{1t} + (1 - \omega_{\eta 1})\hat{P}_{2t} \tag{37}$$

- Labor Supply

$$\varphi \hat{L}_{\eta t} + \sigma \hat{C}_{\eta t} = \hat{W}_{\eta t} - \hat{Q}_{\eta t} \tag{38}$$

- Law of Motion for Capital

$$\hat{K}_{\eta,t+1} = (1 - \delta)\hat{K}_{\eta t} + \delta \hat{I}_{\eta t} \tag{39}$$

Log-linear Model iii

- Euler equation for capital return

$$
\begin{aligned}
(\hat{Q}_{\eta,t+1} - \hat{Q}_{\eta t}) + \sigma(\hat{C}_{\eta,t+1} - \hat{C}_{\eta t}) - (\hat{P}_{\eta,t+1} - \hat{P}_{\eta,t}) = \\
= \beta r(\hat{R}_{\eta,t+1} - \hat{P}_{\eta,t+1})
\end{aligned} \tag{40}
$$

- Production Function

$$
\hat{Y}_{\eta t} = \hat{Z}_{A\eta t} + \alpha_\eta \hat{K}_{\eta t} + (1 - \alpha_\eta)\hat{L}_{\eta t} \tag{41}
$$

- Technical and Economic Marginal Rates of Substitution

$$
\hat{K}_{\eta t} - \hat{L}_{\eta t} = \hat{W}_{\eta t} - \hat{R}_{Kt} \tag{42}
$$

- Marginal Cost

$$
\hat{\lambda}_{\eta t} = \alpha_\eta \hat{R}_{Kt} + (1 - \alpha_\eta)\hat{W}_{\eta t} - \hat{Z}_{A\eta t} - \hat{P}_{\eta t} \tag{43}
$$

Log-linear Model iv

- Monetary Policy

$$\hat{R}_t = \gamma_R \hat{R}_{t-1} + (1 - \gamma_R)(\gamma_\pi \hat{\pi}_t + \gamma_Y \hat{Y}_t) + \hat{Z}_{Mt} \tag{44}$$

- National Gross Inflation Rate

$$\hat{\pi}_t = \theta_\pi \hat{\pi}_{1t} + (1 - \theta_\pi)\hat{\pi}_{2t} \tag{45}$$

- Productivity Shock

$$\hat{Z}_{A\eta t} = \rho_{A\eta} \hat{Z}_{A\eta, t-1} + \varepsilon_{A\eta} \tag{46}$$

- Monetary Shock

$$\hat{Z}_{Mt} = \rho_M \hat{Z}_{M, t-1} + \varepsilon_M \tag{47}$$

Log-linear Model v

- Goods-Market Clearing Condition

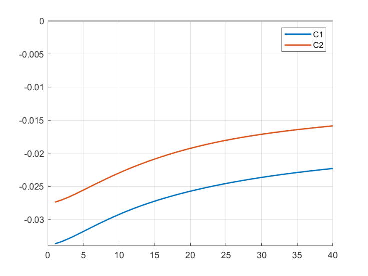
$$\hat{Y}_t = \theta_Y \hat{Y}_{1t} + (1 - \theta_Y)\hat{Y}_{2t} \tag{48}$$

- Regional Goods-Market Clearing Condition

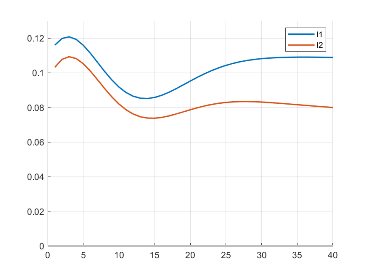
$$\hat{Y}_{\eta t} = \theta_{C\eta} \hat{C}_{\eta t} + (1 - \theta_{C\eta})\hat{I}_{\eta t} \tag{49}$$

Results

Impulse Response Functions

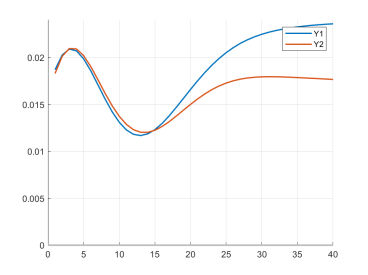


(a) Consumption

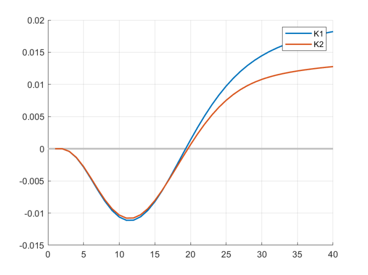


(b) Investment

Impulse Response Functions

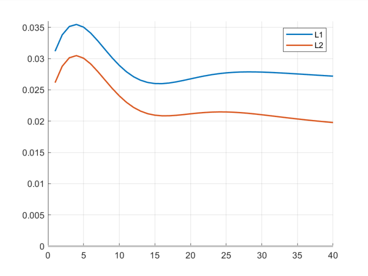


(a) Production

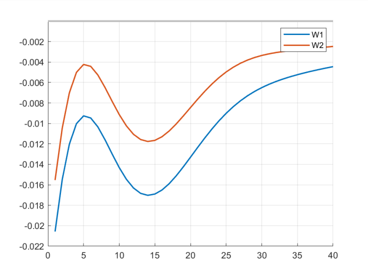


(b) Capital

Impulse Response Functions

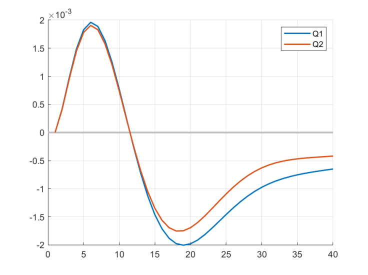


(a) Labor

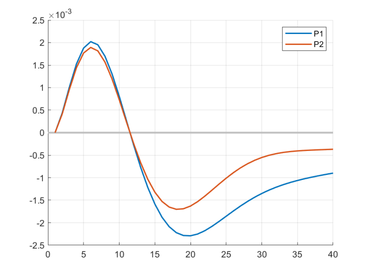


(b) Wages

Impulse Response Functions



(a) Consumer Price Level



(b) Price Level

Obrigado!

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https://github.com/andrlb/mastersthesis

Dúvidas e Sugestões