Analysis of the Monetary Policy Impact on Regional Gross Domestic Product: A Regional DSGE Model

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PPGDE-UFPR

Contents

Introduction

Introduction

Brazilian regions have heterogeneous economic matrices that respond in diverse ways to the decisions of the monetary authority. (BERTANHA; HADDAD, 2008).

Introduction

Objectives:

- Develop a NK DSGE model with:
  - two regions with different parameters;
  - monetary-policy shocks.
- Demonstrate that different regions react in distinct ways to the monetary policy.

What is a NK DSGE model?

NK DSGE model is a macroeconomic tool with:

- **New Keynesian**: monopolistic competition, nominal rigidities, short-run non-neutrality of monetary policy.
- **Dynamic**: shows the changes over time.
- **Stochastic**: considers random and uncertainty.
- **General Equilibrium**: agents optimize and markets clear (microfoundations).

Literature Review

Macro modeling

- Costa Junior (2016): presents a RBC model and then adds NK elements in each chapter;
- Galí (2015): discuss monetary policy starting with a RBC model and also adds NK elements in each chapter;
- Bergholt (2012): presents a NK and the method of programming in `Dynare`;
- Solis-Garcia (2022): presents a RBC model and demonstrate the math tools necessary to solve a DSGE model;

Regional Modeling

- Rickman (2010): link between macro and regional modeling.
- Mora e Costa Junior (2019): Effects of foreign direct investment (FDI), taking into consideration where it is applied: DSGE model with two regions (Bogotá and the rest of Colombia).
- Costa Junior et al. (2022): Effects of fiscal policy, considering the federative entities: DSGE model for the State of Goiás and the rest of the country.
- Osterno (2022): Regionalization of SAMBA: SAMBA+REG (Stochastic Analytical Model with Bayesian Approach from the Central Bank of Brazil).

Model

Characteristics

- four agents: households, intermediate and final-goods firms, monetary authority.
- no bonds.
- capital and investment.
- price stickiness of intermediate goods.

Agents

- the representative household maximizes utility;
- firms producing intermediate goods minimize costs and maximize profit flow;
- firms producing final goods maximize profit.
- the monetary authority determines the interest rate, aiming to control inflation and pursuing economic growth.

Model Structure

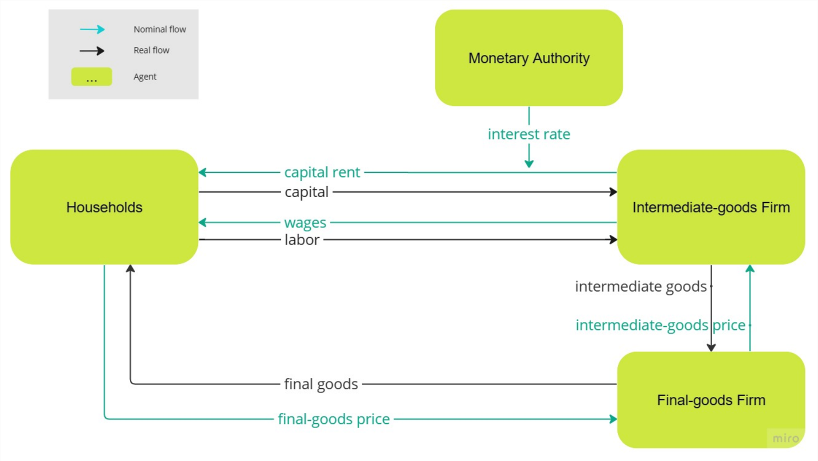


Figure 1: Model Diagram

Household Maximization Problem

$$
\begin{aligned}
\max_{C_t, L_t, K_{t+1}} :\quad & U(C_t, L_t) = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \phi \frac{L_t^{1+\varphi}}{1+\varphi} \right) && (1) \\
\text{s. t.} :\quad & P_t(C_t + I_t) = W_t L_t + R_t K_t + \Pi_t && (2) \\
& K_{t+1} = (1-\delta) K_t + I_t && (3) \\
& C_t, L_t, K_{t+1} \geq 0 \text{ ; } K_0 \text{ given.}
\end{aligned}
$$

Final-goods Firm Maximization Problem

$$\max_{Y_{jt}} : \quad \Pi_t = P_t Y_t - \int_0^1 P_{jt} Y_{jt} \, \mathrm{d}\, j \tag{4}$$

$$\text{s. t.} : \quad Y_t = \left(\int_0^1 Y_{jt}^{\frac{\psi-1}{\psi}} \, \mathrm{d}\, j \right)^{\frac{\psi}{\psi-1}} \tag{5}$$

Intermediate-goods Firm Problems

Cost Minimization Problem:

$$\min_{K_{jt}, L_{jt}} : \quad R_t K_{jt} + W_t L_{jt} \tag{6}$$

$$\text{s.t.} : \quad Y_{jt} = Z_{At} K_{jt}^{\alpha} L_{jt}^{1-\alpha} \tag{7}$$

Intermediate-goods Firm Problems

Price Stickiness and Profit Flow, Calvo's Rule (CALVO, 1983):

$$\mathbb{P}(P_t = P_{t-1}) = \theta \tag{8}$$

$$\max_{P_{jt}} : \quad \mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \frac{\theta^s \left[P_{jt} Y_{j,t+s} - TC_{j,t+s} \right]}{\prod_{k=0}^{s-1} (1 + R_{t+k})} \right\} \tag{9}$$

$$\text{s. t.} : \quad Y_{jt} = Y_t \left(\frac{P_t}{P_{jt}} \right)^{\psi} \tag{10}$$

Monetary Authority

Taylor's Rule (TAYLOR, 1993):

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\gamma_\pi} \left(\frac{Y_t}{Y}\right)^{\gamma_Y}\right]^{1-\gamma_R} Z_{Mt} \tag{11}$$

Stochastic Shocks

Productivity Shock:

$$\ln Z_{At} = (1 - \rho_A) \ln Z_A + \rho_A \ln Z_{A,t-1} + \varepsilon_{At} \tag{12}$$

Monetary Policy Shock:

$$\ln Z_{Mt} = (1 - \rho_M) \ln Z_M + \rho_M \ln Z_{M,t-1} + \varepsilon_{Mt} \tag{13}$$

Model Structure i

Square system of 16 variables and 16 equations:

- from the household problem: C_t, L_t, K_{t+1};
- from the final-good firm problem: Y_{jt}, P_t;
- from the intermediate-good firm problems: K_{jt}, L_{jt}, P_t^*;
- from the market clearing condition: Y_t, I_t;
- prices: $W_t, R_t, \Lambda_t, \pi_t$;
- shocks: Z_{At}, Z_{Mt}.

Model Structure i

Equations:

1. Labor Supply:

$$\frac{\phi L_t^{\varphi}}{C_t^{-\sigma}} = \frac{W_t}{P_t} \tag{14}$$

2. Household Euler Equation:

$$\left(\frac{\mathbb{E}_t C_{t+1}}{C_t}\right)^{\sigma} = \beta \left[(1-\delta) + \mathbb{E}_t \left(\frac{R_{t+1}}{P_{t+1}}\right)\right] \tag{15}$$

3. Budget Constraint:

$$P_t(C_t + I_t) = W_t L_t + R_t K_t + \Pi_t \tag{16}$$

Model Structure ii

4. Law of Motion for Capital:

$$K_{t+1} = (1-\delta)K_t + I_t \tag{17}$$

5. Bundle Technology:

$$Y_t = \left(\int_0^1 Y_{jt}^{\frac{\psi-1}{\psi}} \, \mathrm{d}j \right)^{\frac{\psi}{\psi-1}} \tag{18}$$

6. General Price Level:

$$P_t = \left[\theta P_{t-1}^{1-\psi} + (1-\theta) P_t^{*1-\psi} \right]^{\frac{1}{1-\psi}} \tag{19}$$

Model Structure iii

7. Capital Demand:

$$K_{jt} = \alpha Y_{jt} \frac{\Lambda_t}{R_t} \tag{20}$$

8. Labor Demand:

$$L_{jt} = (1-\alpha) Y_{jt} \frac{\Lambda_t}{W_t} \tag{21}$$

9. Marginal Cost:

$$\Lambda_t = \frac{1}{Z_{At}} \left(\frac{R_t}{\alpha}\right)^{\alpha} \left(\frac{W_t}{1-\alpha}\right)^{1-\alpha} \tag{22}$$

Model Structure iv

10. Production Function:

$$Y_{jt} = Z_{At} K_{jt}^{\alpha} L_{jt}^{1-\alpha} \tag{23}$$

11. Optimal Price:

$$P_t^* = \frac{\psi}{\psi - 1} \cdot \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \theta^s Y_{j,t+s} \Lambda_{t+s} / \prod_{k=0}^{s-1} (1 + R_{t+k}) \right\}}{\mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \theta^s Y_{j,t+s} / \prod_{k=0}^{s-1} (1 + R_{t+k}) \right\}} \tag{24}$$

12. Market Clearing Condition:

$$Y_t = C_t + I_t \tag{25}$$

Model Structure v

13. Monetary Policy:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\gamma_\pi} \left(\frac{Y_t}{Y}\right)^{\gamma_Y}\right]^{1-\gamma_R} Z_{Mt} \tag{26}$$

14. Gross Inflation Rate:

$$\pi_t = \frac{P_t}{P_{t-1}} \tag{27}$$

15. Productivity Shock:

$$\ln Z_{At} = (1 - \rho_A) \ln Z_A + \rho_A \ln Z_{A,t-1} + \varepsilon_{At} \tag{28}$$

Model Structure vi

16. Monetary Shock:

$$\ln Z_{Mt} = (1 - \rho_M) \ln Z_M + \rho_M \ln Z_{M,t-1} + \varepsilon_{Mt} \tag{29}$$

Steady State

Steady State

Steady state solution (COSTA JUNIOR, 2016, p.41):

$$\mathbb{E}_t X_{t+1} = X_t = X_{t-1} = X_{ss} \tag{30}$$

Log-linearization

Log-linearization

Uhlig's rules for log-linearization (UHLIG, 1999).

Log-linear Model i

Square system of 12 variables and 12 equations:

Variables:

$$\begin{pmatrix} \tilde{\pi} & \hat{P} & \hat{\lambda} & \hat{C} & \hat{L} & \hat{R} & \hat{K} & \hat{I} & \hat{W} & \hat{Z}_A & \hat{Y} & \hat{Z}_M \end{pmatrix} \tag{31}$$

Log-linear Model i

Equations:

1. Gross Inflation Rate:

$$\widetilde{\pi}_t = \hat{P}_t - \hat{P}_{t-1} \tag{32}$$

2. New Keynesian Phillips Curve:

$$\widetilde{\pi}_t = \varrho \mathbb{E}_t \widetilde{\pi}_{t+1} + \frac{(1-\theta)(1-\theta\varrho)}{\theta} \hat{\lambda}_t \tag{33}$$

3. Labor Supply:

$$\varphi \hat{L}_t + \sigma \hat{C}_t = \hat{W}_t + \hat{P}_t \tag{34}$$

Log-linear Model ii

4. Household Euler Equation:

$$\mathbb{E}_t \hat{C}_{t+1} - \hat{C}_t = \frac{\beta R}{\sigma P} \mathbb{E}_t (\hat{R}_{t+1} - \hat{P}_{t+1}) \tag{35}$$

5. Law of Motion for Capital:

$$\hat{K}_{t+1} = (1 - \delta)\hat{K}_t + \delta \hat{I}_t \tag{36}$$

6. Real Marginal Cost:

$$\hat{\lambda}_t = \alpha \hat{R}_t + (1 - \alpha)\hat{W}_t - \hat{Z}_{At} - \hat{P}_t \tag{37}$$

Log-linear Model iii

7. Production Function:

$$\hat{Y}_t = \hat{Z}_{At} + \alpha \hat{K}_t + (1 - \alpha)\hat{L}_t \tag{38}$$

8. Marginal Rates of Substitution of Factors:

$$\hat{K}_t - \hat{L}_t = \hat{W}_t - \hat{R}_t \tag{39}$$

9. Market Clearing Condition:

$$\hat{Y}_t = \theta_C \hat{C}_t + \theta_I \hat{I}_t \tag{40}$$

Log-linear Model iv

10. Monetary Policy:

$$\hat{R}_t = \gamma_R \hat{R}_{t-1} + (1 - \gamma_R)(\gamma_\pi \widetilde{\pi}_t + \gamma_Y \hat{Y}_t) + \hat{Z}_{Mt} \tag{41}$$

11. Productivity Shock:

$$\hat{Z}_{At} = \rho_A \hat{Z}_{A,t-1} + \varepsilon_A \tag{42}$$

12. Monetary Shock:

$$\hat{Z}_{Mt} = \rho_M \hat{Z}_{M,t-1} + \varepsilon_M \tag{43}$$

Matlab and Dynare

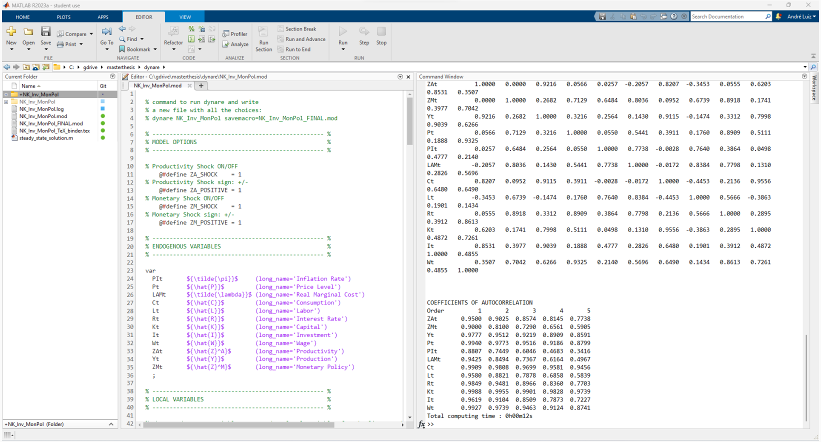
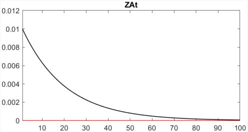
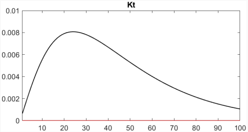


Figure 2: Matlab and Dynare

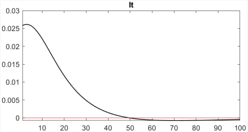
Productivity Shock



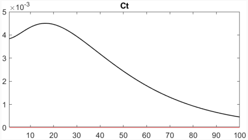
(a) Productivity Shock



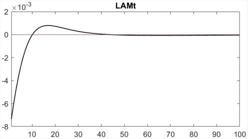
(b) Capital



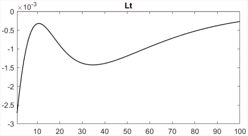
(c) Investment



(d) Consumption

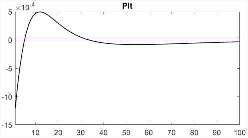


(e) Marginal Cost

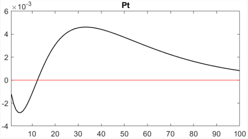


(f) Labor

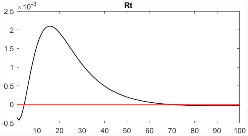
Productivity Shock



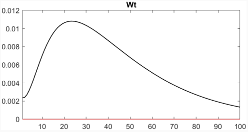
(a) Inflation



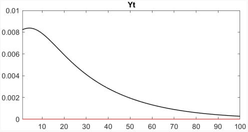
(b) Price Level



(c) Interest Rate

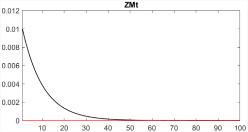


(d) Wage

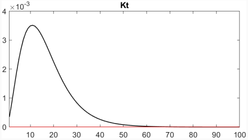


(e) Production

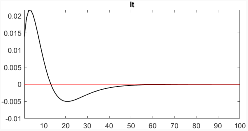
Monetary Shock



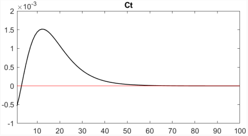
(a) Monetary Shock



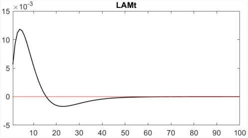
(b) Capital



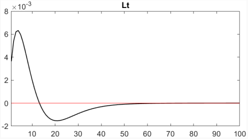
(c) Investment



(d) Consumption

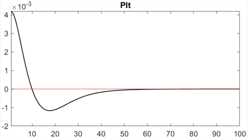


(e) Marginal Cost

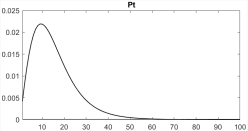


(f) Labor

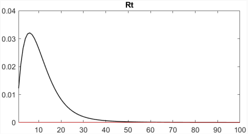
Monetary Shock



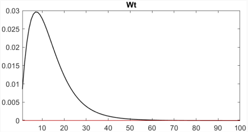
(a) Inflation



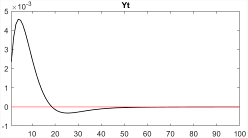
(b) Price Level



(c) Interest Rate



(d) Wage



(e) Production

Figure 6: Monetary Shock Impulse Response Functions

Regional Model

Characteristics

- two regions.
- mobility for final-goods.
- household η and firm ν indexes.
- regional inflation π_ν.

Household Maximization Problem

$$\max_{C_{1\eta t}, C_{2\eta t}, L_{\eta t}, K_{\eta,t+1}} : \quad U_\eta(C_{\eta t}, L_{\eta t}) = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\frac{C_{\eta t}^{1-\sigma}}{1-\sigma} - \phi \frac{L_{\eta t}^{1+\varphi}}{1+\varphi} \right) \tag{44}$$

$$\text{s. t.} : \quad P_{C1t} C_{1\eta t} + P_{C2t} C_{2\eta t} + P_{C\eta t} I_{\eta t} = W_t L_{\eta t} + R_t K_{\eta t} + \Pi_{\nu t} \tag{45}$$

$$K_{\eta,t+1} = (1-\delta) K_{\eta t} + I_{\eta t} \tag{46}$$

$$C_{\eta t} = C_{1\eta t}^{\omega_\eta} C_{2\eta t}^{1-\omega_\eta} \tag{47}$$

$C_{\nu\eta t}, L_{\eta t}, K_{\eta,t+1} > 0$; K_0 given.

Final-goods Firm Maximization Problem

$$\max_{Y_{\nu jt}} : \quad \Pi_{\nu t} = P_{C\nu t} Y_{\nu t} - \int_0^1 P_{C\nu jt} Y_{\nu jt} \, \mathrm{d} j \tag{48}$$

$$\text{s. t.} : \quad Y_{\nu t} = \left(\int_0^1 Y_{\nu jt}^{\frac{\psi - 1}{\psi}} \, \mathrm{d} j \right)^{\frac{\psi}{\psi - 1}} \tag{49}$$

Intermediate-goods Firm Problems

Cost Minimization Problem:

$$\min_{K_{\eta jt}, L_{\eta jt}} : \quad R_t K_{\eta jt} + W_t L_{\eta jt} \tag{50}$$

$$\text{s.t.} : \quad Y_{\nu jt} = Z_{A\nu t} K_{\eta jt}^{\alpha_\nu} L_{\eta jt}^{1-\alpha_\nu} \tag{51}$$

Intermediate-goods Firm Problems

Price Stickiness and Profit Flow, Calvo's Rule (CALVO, 1983):

$$\mathbb{P}(P_t = P_{t-1}) = \theta \tag{52}$$

$$\max_{P_{C\nu jt}} : \quad \mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \frac{\theta^s \left[P_{C\nu jt} Y_{\nu j,t+s} - TC_{\nu j,t+s} \right]}{\prod_{k=0}^{s-1}(1 + R_{t+k})} \right\} \tag{53}$$

$$\text{s. t.} : \quad Y_{\nu jt} = Y_{\nu t} \left(\frac{P_{C\nu t}}{P_{C\nu jt}} \right)^{\psi} \tag{54}$$

Monetary Authority

Taylor's Rule (TAYLOR, 1993):

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\gamma_\pi} \left(\frac{Y_t}{Y}\right)^{\gamma_Y}\right]^{1-\gamma_R} Z_{Mt} \tag{55}$$

Regional Price Level and Inflation

Regional price level $P_{C\nu t}$ and regional inflation rate:

$$\pi_{\nu t} = \frac{P_{C\nu t}}{P_{C\nu,t-1}} \tag{56}$$

National price level:

$$P_t = \vartheta_1 P_{C1t} + (1 - \vartheta_1) P_{C2t} \tag{57}$$

Model Structure

The model is a square system of 27 variables and 27 equations.

- from the household problem: $C_{\eta t}, L_{\eta t}, K_{\eta, t+1}$;
- from the final-goods firm problem: $Y_{\nu t}, Y_{\nu j t}, P_{C \nu t}$;
- from the intermediate-goods firm problems: $K_{\eta j t}, L_{\eta j t}, P^*_{C \nu t}$;
- from the market clearing condition: $Y_t, I_{\eta t}$;
- prices: $W_t, R_t, \Lambda_{\nu t}, \pi_t$;
- shocks: $Z_{A \nu t}, Z_{M t}$.

Next Steps

- Steady State
- Parameter Calibration
- Steady State Solution
- Log-linearization
- Dynare Programming
- Impulse Response Functions

Expected Results

Expected Results

- monetary Policy Shock: 1%, posivite and negative.
- Regional technology level $Z_{A\nu t}$, capital weight in production α_{ν} implies regional price levels $P_{\nu t}$ and inflation rates $\pi_{\nu t}$.
- for different parameters, the reaction of each region shall have a significant change.

Project Timeline

Project Timeline

Activity	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Literature Review	x	x								
Project & Seminar			x	x						
Modeling			x	x	x					
LaTeX			x	x	x					
`Dynare`					x	x				
Qualification						x				
Regional Model						x	☐			
Data Treatment							☐			
Parametrization								☐		
Results								☐		
Systematic Review									☐	
Revision & Edition									☐	
Thesis Defense										☐

Obrigado!

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Dúvidas e Sugestões