UNIVERSIDADE DO PARANÁ

1 Methodology

1.1 Regional Model

Regions

For a regional model, it is considered that different representative agents exist in each region and some rules must be applied:

- Consumption $C_{\nu\eta t}$: households from region $\eta \in \{1,2\}$ consume from both regions $\nu \in \{1,2\}$.
- Labor $L_{\eta t}$: there is no mobility in the labor market, so that households will work for firms in the same region they live.
- Investment and Capital $I_{\eta t}$: there is no mobility in investments and capital rent: households will invest and rent capital in their own region.
- Final-good production $Y_{\nu t}$: there is one representative final-good firm in each region that aggregates all intermediate-goods of that region.
- Final-good price $P_{C\nu t}$ and regional inflation $\pi_{\nu t}$: in each region, there is a final-good price and a regional inflation level.
- Intermediate-goods firms $Y_{\nu jt}$: there is a continuum $j \in [0,1]$ for each region and these firms will demand labor and capital from within the region.
- Productivity level $Z_{A\nu t}$ and capital weight in production α_ν: each region has its own characteristics and because of that has a difference productivity level subject to different shock rule and a different capital weight in production.

Regional Model

In this section, the model presented in the previous section will be expanded to encompass two regions: the main region to be studied and the rest of the country.

Regions will have two indexes: $\eta \in \{1,2,\ldots,n\}$ will represent the household variables and $\nu \in \{1,2,\ldots,n\}$ will determine the firm variables. For example, the variable C_t represents the total consumption, $C_{\eta t}$ represents the consumption of region η, the variable Y_t represents the national production and $Y_{\nu t}$ represents the production of region ν. Finally, $C_{\nu\eta t}$ represents the consumption of the final-good produced in region ν by the household in region η. Without loss of generality, the model will have two regions: the main region 1 and the remaining of the country 2, so that $\eta, \nu \in \{1, 2\}$.

1.1.1 Household

Utility Maximization Problem

Following the models presented by Costa Junior (2016) and Solis-Garcia (2022), the representative household problem is to maximize an intertemporal utility function U_η with respect to consumption $C_{\eta t}$ and labor $L_{\eta t}$, subject to a budget constraint, a capital accumulation rule and the non-negativity of real variables:

$$\max_{C_{1\eta t}, C_{2\eta t}, L_{\eta t}, K_{\eta,t+1}} : \quad U_\eta(C_{\eta t}, L_{\eta t}) = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\frac{C_{\eta t}^{1-\sigma}}{1-\sigma} - \phi \frac{L_{\eta t}^{1+\varphi}}{1+\varphi} \right) \tag{1.1}$$

$$\text{s. t.} : \quad P_{C1t}C_{1\eta t} + P_{C2t}C_{2\eta t} + P_{C\eta t}I_{\eta t} = W_t L_{\eta t} + R_t K_{\eta t} + \Pi_{\nu t} \tag{1.2}$$

$$K_{\eta,t+1} = (1-\delta)K_{\eta t} + I_{\eta t} \tag{1.3}$$

$$C_{\eta t} = C_{1\eta t}^{\omega_\eta} C_{2\eta t}^{1-\omega_\eta} \tag{1.4}$$

$$C_{\nu\eta t}, L_{\eta t}, K_{\eta,t+1} > 0 \, ; K_0 \text{ given.}$$

where $\mathbb{E}_t$ is the expectation operator, β is the intertemporal discount factor, σ is the relative risk aversion coefficient, ϕ is the relative labor weight in utility, φ is the marginal disutility of labor supply. In the budget constraint, $P_{C\nu t}$ is the price level, $I_{\eta t}$ is the investment, W_t is the wage level, $K_{\eta t}$ is the capital stock, R_t is the return on capital, and $\Pi_{\nu t}$ is the firm profit. In the capital accumulation rule, δ is the capital depreciation rate. In the consumption aggregation, $C_{\nu\eta t}$ is the good produced in region $\nu \in \{1, 2\}$ with weight ω_η in the consumption bundle $C_{\eta t}$ of region $\eta \in \{1, 2\}$.

Isolate $I_{\eta t}$ in 1.3 and substitute in 1.2:

$$K_{\eta,t+1} = (1-\delta)K_{\eta t} + I_{\eta t} \implies I_{\eta t} = K_{\eta,t+1} - (1-\delta)K_{\eta t} \tag{1.3}$$

$$P_{C1t}C_{1\eta t} + P_{C2t}C_{2\eta t} + P_{C\eta t}I_{\eta t} = W_t L_{\eta t} + R_t K_{\eta t} + \Pi_{\nu t} \implies \tag{1.2}$$

$$P_{C1t}C_{1\eta t} + P_{C2t}C_{2\eta t} + P_{C\eta t}(K_{\eta,t+1} - (1-\delta)K_{\eta t}) = W_t L_{\eta t} + R_t K_{\eta t} + \Pi_{\nu t} \tag{1.5}$$

Substitute 1.4 in 1.1:

$$U_\eta(C_{1\eta t}, C_{2\eta t}, L_{\eta t}) = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\frac{\left[C_{1\eta t}^{\omega_1} C_{2\eta t}^{1-\omega_1}\right]^{1-\sigma}}{1-\sigma} - \phi \frac{L_{\eta t}^{1+\varphi}}{1+\varphi} \right) \tag{1.6}$$

Lagrangian

The maximization problem with restriction can be transformed in one without restriction using the Lagrangian function $\mathscr{L}$ with 1.6 and 1.5:

$$\begin{aligned} \mathscr{L} =& \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left\{ \left(\frac{\left[C_{1\eta t}^{\omega_1} C_{2\eta t}^{1-\omega_1}\right]^{1-\sigma}}{1-\sigma} - \phi \frac{L_{\eta t}^{1+\varphi}}{1+\varphi} \right) - \mu_t \Big[P_{C1t} C_{1\eta t} + \right. \\ & \left. + P_{C2t} C_{2\eta t} + P_{C\eta t}(K_{\eta,t+1} - (1-\delta)K_{\eta t}) - (W_t L_{\eta t} + R_t K_{\eta t} + \Pi_{\nu t}) \Big] \right\} \end{aligned} \tag{1.7}$$

First Order Conditions

The first order conditions are:

$$C_{1\eta t}: \quad \mu_t = \frac{\omega_1 C_{1\eta t}^{\omega_1(1-\sigma)-1} C_{2\eta t}^{(1-\omega_1)(1-\sigma)}}{P_{C1t}} = \frac{\omega_1}{P_{C1t} C_{1\eta t}} C_{\eta t}^{1-\sigma} \tag{1.8}$$

$$C_{2\eta t}: \quad \mu_t = \frac{(1-\omega_1) C_{1\eta t}^{\omega_1(1-\sigma)} C_{2\eta t}^{(1-\omega_1)(1-\sigma)-1}}{P_{C2t}} = \frac{(1-\omega_1)}{P_{C2t} C_{2\eta t}} C_{\eta t}^{1-\sigma} \tag{1.9}$$

$$L_{\eta t}: \quad -\phi L_{\eta t}^{\varphi} + \mu_t W_t = 0 \implies \mu_t = \frac{\phi L_{\eta t}^{\varphi}}{W_t} \tag{1.10}$$

$$\begin{aligned} K_{\eta,t+1}: \quad & -\mu_t P_{C\nu t} + \beta \mathbb{E}_t \mu_{t+1}[(1-\delta)P_{C\eta t+1} + R_{t+1}] = 0 \implies \\ & \mu_t P_{C\nu t} = \beta \mathbb{E}_t \mu_{t+1}[(1-\delta)P_{C\eta t+1} + R_{t+1}] \end{aligned} \tag{1.11}$$

$$\begin{aligned} \mu_t: \quad & P_{C1t} C_{1\eta t} + P_{C2t} C_{2\eta t} + P_{C\nu t}(K_{\eta,t+1} - (1-\delta)K_{\eta t}) = \\ & = W_t L_{\eta t} + R_t K_{\eta t} + \Pi_{\nu t} \end{aligned} \tag{1.5}$$

Solutions

Match 1.8 and 1.9:

$$\mu_t = \frac{\omega_1}{P_{C1t}C_{1\eta t}} C_{\eta t}^{1-\sigma} = \frac{(1-\omega_1)}{P_{C2t}C_{2\eta t}} C_{\eta t}^{1-\sigma} \implies$$

$$C_{1\eta t} = \frac{P_{C2t}C_{2\eta t}}{P_{C1t}} \cdot \frac{\omega_1}{1-\omega_1} \tag{1.12}$$

Substitute 1.12 in 1.4:

$$C_{\eta t} = C_{1\eta t}^{\omega_1} C_{2\eta t}^{1-\omega_1} \tag{1.4}$$

$$C_{\eta t} = \left[\frac{P_{C2t}C_{2\eta t}}{P_{C1t}} \cdot \frac{\omega_1}{1-\omega_1}\right]^{\omega_1} C_{2\eta t}^{1-\omega_1} \implies$$

$$C_{2\eta t} = C_{\eta t}\left[\frac{P_{C1t}}{P_{C2t}} \cdot \frac{1-\omega_1}{\omega_1}\right]^{\omega_1} \tag{1.13}$$

Substitute 1.13 in 1.12:

$$C_{1\eta t} = \frac{P_{C2t}C_{2\eta t}}{P_{C1t}} \cdot \frac{\omega_1}{1-\omega_1} \implies \tag{1.12}$$

$$C_{1\eta t} = \left[C_{\eta t}\left[\frac{P_{C1t}}{P_{C2t}} \cdot \frac{1-\omega_1}{\omega_1}\right]^{\omega_1}\right] \frac{P_{C2t}}{P_{C1t}} \cdot \frac{\omega_1}{1-\omega_1} \implies$$

$$C_{1\eta t} = C_{\eta t}\left[\frac{P_{C1t}}{P_{C2t}} \cdot \frac{1-\omega_1}{\omega_1}\right]^{\omega_1 - 1} \tag{1.14}$$

Define the total goods expense $P_t^{\eta} C_{\eta t}$ $P_{\eta t}C_{\eta t}$ of household η in time t:

$$P_{\eta t}C_{\eta t} = P_{C1t}C_{1\eta t} + P_{C2t}C_{2\eta t} \tag{1.15}$$

Substitute 1.14 and 1.13 in 1.15:

$$P_{\eta t}C_{\eta t} = P_{C1t}C_{\eta t}\left[\frac{P_{C1t}}{P_{C2t}} \cdot \frac{1-\omega_1}{\omega_1}\right]^{\omega_1 - 1} + P_{C2t}C_{\eta t}\left[\frac{P_{C1t}}{P_{C2t}} \cdot \frac{1-\omega_1}{\omega_1}\right]^{\omega_1} \implies$$

$$P_{\eta t} = \left[\frac{P_{C2t}}{1-\omega_1}\right]^{1-\omega_1} \left[\frac{P_{C1t}}{\omega_1}\right]^{\omega_1} \tag{1.16}$$

Equation 1.16 is the price composition of consumption bundle $C_{\eta t}$.

Match 1.8 in 1.10:

$$\mu_t = \frac{\omega_1}{P_{C1t}C_{1\eta t}}C_{\eta t}^{1-\sigma} = \frac{\phi L_{\eta t}^{\varphi}}{W_t} \tag{1.17}$$

Isolate $\frac{\omega_1}{P_{C1t}C_{1\eta t}} = \frac{C_{\eta t}^{-1}}{P_{\eta t}}$ in 1.14 and substitute in 1.17:

$$\mu_t = \frac{C_{\eta t}^{-\sigma}}{P_{\eta t}} = \frac{\phi L_{\eta t}^{\varphi}}{W_t} \implies \tag{1.18}$$

$$\frac{\phi L_{\eta t}^{\varphi}}{C_{\eta t}^{-\sigma}} = \frac{W_t}{P_{\eta t}} \tag{1.19}$$

Equation 1.19 is the Household Labor Supply and shows that the marginal rate of substitution (MRS) of labor for consumption is equal to the real wage, which is the relative price between labor and goods.

Substitute μ_t and μ_{t+1} from equation 1.18 in 1.11:

$$\mu_t P_{\eta t} = \beta \mathbb{E}_t \mu_{t+1}[(1-\delta)P_{C\eta t+1} + R_{t+1}] \implies$$

$$\frac{C_{\eta t}^{-\sigma}}{P_{\eta t}} P_{\eta t} = \beta \mathbb{E}_t \frac{C_{\eta,t+1}^{-\sigma}}{P_{\eta t+1}}[(1-\delta)P_{C\eta t+1} + R_{t+1}] \implies$$

$$\left(\frac{\mathbb{E}_t C_{\eta,t+1}}{C_{\eta t}}\right)^{\sigma} = \beta\left[(1-\delta) + \mathbb{E}_t\left(\frac{R_{t+1}}{P_{C\eta t+1}}\right)\right] \tag{1.20}$$

Equation 1.20 is the Household Euler equation.

Firms

Consider two types of firms: (1) a continuum of intermediate-goods firms, which operate in monopolistic competition and each produce one variety with imperfect substitution level between each other and (2) the final-goods firm, which aggregates all the varieties into a final bundle and operates in perfect competition.

1.1.2 Final-Goods Firm

Profit Maximization Problem

The role of the final-goods firm is to aggregate all the varieties produced by the intermediate-goods firms in region $\nu \in \{1, 2\}$, so that the representative consumer can buy only one good $Y_{\nu t}$, the bundle good.

The final-goods firm problem is to maximize its profit, considering that its output is the bundle $Y_{\nu t}$ formed by the continuum of intermediate goods $Y_{\nu jt}$, where $j \in [0, 1]$ and ψ is the elasticity of substitution between intermediate goods:

$$\max_{Y_{\nu jt}} : \quad \Pi_{\nu t} = P_{C\nu t} Y_{\nu t} - \int_0^1 P_{C\nu jt} Y_{\nu jt} \, \mathrm{d}\, j \tag{1.21}$$

$$\text{s. t.} : \quad Y_{\nu t} = \left(\int_0^1 Y_{\nu jt}^{\frac{\psi - 1}{\psi}} \, \mathrm{d}\, j \right)^{\frac{\psi}{\psi - 1}} \tag{1.22}$$

Substitute 1.22 in 1.21:

$$\max_{Y_{\nu jt}} : \quad \Pi_{\nu t} = P_{C\nu t} \left(\int_0^1 Y_{\nu jt}^{\frac{\psi - 1}{\psi}} \, \mathrm{d}\, j \right)^{\frac{\psi}{\psi - 1}} - \int_0^1 P_{C\nu jt} Y_{\nu jt} \, \mathrm{d}\, j \tag{1.23}$$

First Order Condition and Solutions

The first order condition is:

$$Y_{\nu jt} : \quad P_{C\nu t} \left(\frac{\psi}{\psi - 1} \right) \left(\int_0^1 Y_{\nu jt}^{\frac{\psi - 1}{\psi}} \, \mathrm{d}\, j \right)^{\frac{\psi}{\psi - 1} - 1} \left(\frac{\psi - 1}{\psi} \right) Y_{\nu jt}^{\frac{\psi - 1}{\psi} - 1} - P_{C\nu jt} = 0 \implies$$

$$Y_{\nu jt} = Y_t \left(\frac{P_{C\nu t}}{P_{C\nu jt}} \right)^{\psi} \tag{1.24}$$

Equation 1.24 shows that the demand for variety j depends on its relative price.

Substitute 1.24 in 1.22:

$$Y_{\nu t} = \left(\int_0^1 Y_{\nu jt}^{\frac{\psi-1}{\psi}} \,\mathrm{d}\, j \right)^{\frac{\psi}{\psi-1}} \qquad \Longrightarrow$$

$$Y_{\nu t} = \left(\int_0^1 \left[Y_{\nu t} \left(\frac{P_{C\nu t}}{P_{C\nu jt}} \right)^{\psi} \right]^{\frac{\psi-1}{\psi}} \,\mathrm{d}\, j \right)^{\frac{\psi}{\psi-1}} \qquad \Longrightarrow$$

$$P_{C\nu t} = \left[\int_0^1 P_{C\nu jt}^{1-\psi} \,\mathrm{d}\, j \right]^{\frac{1}{1-\psi}} \tag{1.25}$$

Equation 1.25 is the final-goods firm's markup.

1.1.3 Intermediate-Goods Firms

Cost Minimization Problem

The intermediate-goods firms, denoted by $j \in [0, 1]$, produce varieties of a representative good with a certain level of substitutability. Each of these firms has to choose capital $K_{\eta jt}$ and labor $L_{\eta jt}$ to minimize production costs, subject to a technology rule.

$$\min_{K_{\eta jt}, L_{\eta jt}} : \quad R_t K_{\eta jt} + W_t L_{\eta jt} \tag{1.26}$$

$$\text{s. t.} : \quad Y_{\nu jt} = Z_{A\nu t} K_{\eta jt}^{\alpha_\nu} L_{\eta jt}^{1-\alpha_\nu} \tag{1.27}$$

where $Y_{\nu jt}$ is the output obtained by the production technology level $Z_{A\nu t}$[1] that transforms capital $K_{\eta jt}$ and labor $L_{\eta jt}$ in proportions α_ν and $(1 - \alpha_\nu)$, respectively, into intermediate goods.

[1] the production technology level $Z_{A\nu t}$ will be submitted to a productivity shock, detailed in section 1.1.5.

Lagrangian

Applying the Lagrangian:

$$\mathscr{L} = (R_t K_{\eta jt} + W_t L_{\eta jt}) - \Lambda_{\nu jt}(Z_{A\nu t} K_{\eta jt}^{\alpha_\nu} L_{\eta jt}^{1-\alpha_\nu} - Y_{\nu jt}) \tag{1.28}$$

where the Lagrangian multiplier $\Lambda_{\nu jt}$ is the marginal cost.[2]

First Order Conditions

The first-order conditions are:

$$\begin{aligned}
K_{\eta jt}: &\quad R_t - \Lambda_{\nu jt} Z_{A\nu t} \alpha_\nu K_{\eta jt}^{\alpha_\nu - 1} L_{\eta jt}^{1-\alpha_\nu} = 0 \qquad \Longrightarrow \\
&\quad K_{\eta jt} = \alpha_\nu Y_{\nu jt} \frac{\Lambda_{\nu jt}}{R_t} &(1.29)\\
L_{\eta jt}: &\quad W_t - \Lambda_{\nu jt} Z_{A\nu t} K_{\eta jt}^{\alpha_\nu} (1-\alpha_\nu) L_{\eta jt}^{-\alpha_\nu} = 0 \qquad \Longrightarrow \\
&\quad L_{\eta jt} = (1-\alpha_\nu) Y_{\nu jt} \frac{\Lambda_{\nu jt}}{W_t} &(1.30)\\
\Lambda_{\nu jt}: &\quad Y_{\nu jt} = Z_{A\nu t} K_{\eta jt}^{\alpha_\nu} L_{\eta jt}^{1-\alpha_\nu} &(1.27)
\end{aligned}$$

Solutions

Divide equation 1.29 by 1.30:

$$\frac{K_{\eta jt}}{L_{\eta jt}} = \frac{\alpha_\nu Y_{\nu jt} \Lambda_{\nu jt} / R_t}{(1-\alpha_\nu) Y_{\nu jt} \Lambda_{\nu jt} / W_t} \Longrightarrow \frac{K_{\eta jt}}{L_{\eta jt}} = \left(\frac{\alpha_\nu}{1-\alpha_\nu}\right) \frac{W_t}{R_t} \tag{1.31}$$

Equation 1.31 demonstrates the relationship between the technical marginal rate of substitution (TMRS) and the economical marginal rate of substitution (EMRS).

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[2] see Lemma **??**

Substitute $L_{\eta jt}$ from equation 1.31 in 1.27:

$$\begin{aligned}
Y_{\nu jt} &= Z_{A\nu t} K_{\eta jt}^{\alpha_\nu} L_{\eta jt}^{1-\alpha_\nu} && \Longrightarrow \\
Y_{\nu jt} &= Z_{A\nu t} K_{\eta jt}^{\alpha_\nu} \left[\left(\frac{1-\alpha_\nu}{\alpha_\nu} \right) \frac{R_t K_{\eta jt}}{W_t} \right]^{1-\alpha_\nu} && \Longrightarrow \\
K_{\eta jt} &= \frac{Y_{\nu jt}}{Z_{A\nu t}} \left[\left(\frac{\alpha_\nu}{1-\alpha_\nu} \right) \frac{W_t}{R_t} \right]^{1-\alpha_\nu} && \qquad (1.32)
\end{aligned}$$

Equation 1.32 is the intermediate-goods firm demand for capital.

Substitute 1.32 in 1.31:

$$\begin{aligned}
L_{\eta jt} &= \left(\frac{1-\alpha_\nu}{\alpha_\nu} \right) \frac{R_t K_{\eta jt}}{W_t} && \Longrightarrow \\
L_{\eta jt} &= \left(\frac{1-\alpha_\nu}{\alpha_\nu} \right) \frac{R_t}{W_t} \frac{Y_{\nu jt}}{Z_{A\nu t}} \left[\left(\frac{\alpha_\nu}{1-\alpha_\nu} \right) \frac{W_t}{R_t} \right]^{1-\alpha_\nu} && \Longrightarrow \\
L_{\eta jt} &= \frac{Y_{\nu jt}}{Z_{A\nu t}} \left[\left(\frac{\alpha_\nu}{1-\alpha_\nu} \right) \frac{W_t}{R_t} \right]^{-\alpha_\nu} && \qquad (1.33)
\end{aligned}$$

Equation 1.33 is the intermediate-goods firm demand for labor.

Total and Marginal Costs

Calculate the total cost TC using 1.32 and 1.33:

$$\begin{aligned}
TC_{\nu jt} &= W_t L_{\eta jt} + R_t K_{\eta jt} && \Longrightarrow \\
TC_{\nu jt} &= W_t \frac{Y_{\nu jt}}{Z_{A\nu t}} \left[\left(\frac{\alpha_\nu}{1-\alpha_\nu} \right) \frac{W_t}{R_t} \right]^{-\alpha_\nu} + R_t \frac{Y_{\nu jt}}{Z_{A\nu t}} \left[\left(\frac{\alpha_\nu}{1-\alpha_\nu} \right) \frac{W_t}{R_t} \right]^{1-\alpha_\nu} && \Longrightarrow \\
TC_{\nu jt} &= \frac{Y_{\nu jt}}{Z_{A\nu t}} \left(\frac{R_t}{\alpha_\nu} \right)^{\alpha_\nu} \left(\frac{W_t}{1-\alpha_\nu} \right)^{1-\alpha_\nu} && \qquad (1.34)
\end{aligned}$$

Calculate the marginal cost Λ using 1.34:

$$\Lambda_{\nu jt} = \frac{\partial TC_{\nu jt}}{\partial Y_{\nu jt}} \Longrightarrow \Lambda_{\nu jt} = \frac{1}{Z_{A\nu t}} \left(\frac{R_t}{\alpha_\nu} \right)^{\alpha_\nu} \left(\frac{W_t}{1-\alpha_\nu} \right)^{1-\alpha_\nu} \qquad (1.35)$$

The marginal cost depends on the technological level $Z_{A\nu t}$, the nominal interest rate R_t and the nominal wage level W_t, which are the same for all intermediate-goods firms, and because of that, the index j may be dropped:

$$\Lambda_{vt} = \frac{1}{Z_{Avt}} \left(\frac{R_t}{\alpha_v}\right)^{\alpha_v} \left(\frac{W_t}{1-\alpha_v}\right)^{1-\alpha_v} \tag{1.36}$$

notice that:

$$\Lambda_{vt} = \frac{TC_{vjt}}{Y_{vjt}} \implies TC_{vjt} = \Lambda_{vt} Y_{vjt} \tag{1.37}$$

Optimal Price Problem

Consider an economy with price stickiness, following the Calvo Rule (CALVO, 1983): each firm has a probability $(0 < \theta < 1)$ of keeping its price in the next period $(P_{Cvj,t+1} = P_{Cvjt})$, and a probability of $(1-\theta)$ of setting a new optimal price P^*_{Cvjt} that maximizes its profits. Therefore, each firm must take this uncertainty into account when deciding the optimal price: the intertemporal profit flow, given the nominal interest rate R_t of each period, is calculated considering the probability θ of keeping the previous price.

$$\max_{P_{Cvjt}} : \quad \mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \frac{\theta^s \left[P_{Cvjt} Y_{vj,t+s} - TC_{vj,t+s}\right]}{\prod_{k=0}^{s-1}(1+R_{t+k})} \right\} \tag{1.38}$$

$$\text{s. t.} : \quad Y_{vjt} = Y_{vt} \left(\frac{P_{Cvt}}{P_{Cvjt}}\right)^{\psi} \tag{1.24}$$

Substitute 1.37 in 1.38:

$$\max_{P_{Cvjt}} : \quad \mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \frac{\theta^s \left[P_{Cvjt} Y_{vj,t+s} - \Lambda_{vt+s} Y_{vj,t+s}\right]}{\prod_{k=0}^{s-1}(1+R_{t+k})} \right\} \tag{1.39}$$

Substitute 1.24 in 1.39 and rearrange the variables:

$$\max_{P_{Cvjt}} : \quad \mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \frac{\theta^s \left[P_{Cvjt} Y_{vt+s} \left(\frac{P_{Cv,t+s}}{P_{Cvjt}}\right)^{\psi} - \Lambda_{vt+s} Y_{vt+s} \left(\frac{P_{Cv,t+s}}{P_{Cvjt}}\right)^{\psi} \right]}{\prod_{k=0}^{s-1}(1+R_{t+k})} \right\} \Longrightarrow$$

$$\max_{P_{Cvjt}} : \quad \mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \frac{\theta^s \left[P_{Cvjt}^{1-\psi} P_{Cv,t+s}^{\psi} Y_{vt+s} - P_{Cvjt}^{-\psi} P_{Cv,t+s}^{\psi} Y_{vt+s} \Lambda_{vt+s} \right]}{\prod_{k=0}^{s-1}(1+R_{t+k})} \right\}$$

First Order Condition

The first order condition with respect to P_{Cvjt} is:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \frac{\theta^s \left[(1-\psi) P_{Cvjt}^{-\psi} P_{Cv,t+s}^{\psi} Y_{vt+s} - (-\psi) P_{Cvjt}^{-\psi-1} P_{Cv,t+s}^{\psi} Y_{vt+s} \Lambda_{vt+s} \right]}{\prod_{k=0}^{s-1}(1+R_{t+k})} \right\} = 0$$

Separate the summations and rearrange the variables:

$$\begin{aligned} \mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \frac{\theta^s (\psi-1) \left(\frac{P_{Cv,t+s}}{P_{Cvjt}}\right)^{\psi} Y_{vt+s}}{\prod_{k=0}^{s-1}(1+R_{t+k})} \right\} &= \\ = \mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \frac{\theta^s \psi P_{Cvjt}^{-1} \left(\frac{P_{Cv,t+s}}{P_{Cvjt}}\right)^{\psi} Y_{vt+s} \Lambda_{vt+s}}{\prod_{k=0}^{s-1}(1+R_{t+k})} \right\} & \end{aligned} \tag{1.40}$$

Substitute 1.24 in 1.40:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \frac{\theta^s (\psi-1) Y_{vj,t+s}}{\prod_{k=0}^{s-1}(1+R_{t+k})} \right\} = \mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \frac{\theta^s \psi P_{Cvjt}^{-1} Y_{vj,t+s} \Lambda_{vt+s}}{\prod_{k=0}^{s-1}(1+R_{t+k})} \right\} \Longrightarrow$$

$$(\psi-1)\mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \frac{\theta^s Y_{vj,t+s}}{\prod_{k=0}^{s-1}(1+R_{t+k})} \right\} = \psi P_{Cvjt}^{-1} \mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \frac{\theta^s Y_{vj,t+s} \Lambda_{vt+s}}{\prod_{k=0}^{s-1}(1+R_{t+k})} \right\} \Longrightarrow$$

$$P_{Cvjt} \mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \frac{\theta^s Y_{vj,t+s}}{\prod_{k=0}^{s-1}(1+R_{t+k})} \right\} = \frac{\psi}{\psi-1} \mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \frac{\theta^s Y_{vj,t+s} \Lambda_{vt+s}}{\prod_{k=0}^{s-1}(1+R_{t+k})} \right\} \Longrightarrow$$

$$P_{Cvjt}^{*} = \frac{\psi}{\psi-1} \cdot \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \theta^s Y_{vj,t+s} \Lambda_{vt+s} / \prod_{k=0}^{s-1}(1+R_{t+k}) \right\}}{\mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \theta^s Y_{vj,t+s} / \prod_{k=0}^{s-1}(1+R_{t+k}) \right\}} \tag{1.41}$$

Equation 1.41 represents the optimal price that firm j will choose. Since all firms that are able to choose will opt for the highest possible price, they will all select the same price. As a result, the index j can be omitted:

$$P^*_{Cvt} = \frac{\psi}{\psi - 1} \cdot \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \theta^s Y_{vj,t+s} \Lambda_{vt+s} / \prod_{k=0}^{s-1} (1 + R_{t+k}) \right\}}{\mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \theta^s Y_{vj,t+s} / \prod_{k=0}^{s-1} (1 + R_{t+k}) \right\}} \tag{1.42}$$

Final-Goods Firm, part II

The process of fixing prices is random: in each period, θ firms will maintain the price from the previous period, while $(1 - \theta)$ firms will choose a new optimal price. The price level for each period will be a composition of these two prices. Use this information in 1.25 to determine the aggregate price level:

$$P_{Cvt} = \left[\int_0^{\theta} P_{Cv,t-1}^{1-\psi} \, \mathrm{d}\, j + \int_{\theta}^{1} P_{Cvt}^{*1-\psi} \, \mathrm{d}\, j \right]^{\frac{1}{1-\psi}} \implies$$

$$P_{Cvt} = \left[\theta P_{Cv,t-1}^{1-\psi} + (1 - \theta) P_{Cvt}^{*1-\psi} \right]^{\frac{1}{1-\psi}} \tag{1.43}$$

Equation 1.43 is the aggregate price level.

1.1.4 Monetary Authority

The objective of the monetary authority is to conduct the economy to price stability and economic growth, using a Taylor rule (TAYLOR, 1993) to determine the nominal interest rate:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\gamma_R} \left[\left(\frac{\pi_t}{\pi} \right)^{\gamma_\pi} \left(\frac{Y_t}{Y} \right)^{\gamma_Y} \right]^{1-\gamma_R} Z_{Mt} \tag{1.44}$$

where R, π, Y are the variables in steady state, γ_R is the smoothing parameter for the interest rate R_t, while γ_π and γ_Y are the interest-rate sensitivities in relation to inflation and product, respectively and Z_{Mt} is the monetary shock.[3]

[3] for the monetary shock definition, see section 1.1.5.

and π_t is the gross inflation rate, defined by:

$$\pi_t = \frac{P_t}{P_{t-1}} \tag{1.45}$$

where P_t is the national price level, defined by:

$$P_t = \vartheta_1 P_{C1t} + (1 - \vartheta_1) P_{C2t} \tag{1.46}$$

where ϑ_1 is the relative weight of regional price level in the national price level.

Regional Inflation

There is one price level $P_{C\nu t}$ in each region, generating an regional inflation rate:

$$\pi_{\nu t} = \frac{P_{C\nu t}}{P_{C\nu,t-1}} \tag{1.47}$$

1.1.5 Stochastic Shocks

Productivity Shock

The production technology level $Z_{A\nu t}$ will be submitted to a productivity shock defined by a first-order autoregressive process $AR(1)$:

$$\ln Z_{A\nu t} = (1 - \rho_{A\nu}) \ln Z_{A\nu} + \rho_{A\nu} \ln Z_{A\nu,t-1} + \varepsilon_{A\nu t} \tag{1.48}$$

where $\rho_{A\nu} \in [0, 1]$ and $\varepsilon_{A\nu t} \sim \mathcal{N}(0, \sigma_{A\nu})$.

Monetary Shock

The monetary policy will also be submitted to a shock, through the variable Z_{Mt}, defined by a first-order autoregressive process $AR(1)$:

$$\ln Z_{Mt} = (1 - \rho_M) \ln Z_M + \rho_M \ln Z_{M,t-1} + \varepsilon_{Mt} \tag{1.49}$$

where $\rho_M \in [0, 1]$ and $\varepsilon_{Mt} \sim \mathcal{N}(0, \sigma_M)$.

1.1.6 Equilibrium Conditions

A Competitive Equilibrium consists of sequences of prices $\{P^*_{C\nu t}, R^*_t, W^*_t\}$, allocations for households $\boldsymbol{\mathcal{A}}_H := \{C^*_{\eta t}, L^*_{\eta t}, K^*_{\eta,t+1}\}$ and for firms $\boldsymbol{\mathcal{A}}_F := \{K^*_{\eta jt}, L^*_{\eta jt}, Y^*_{\nu jt}, Y^*_{\nu t}\}$. In such an equilibrium, given the set of exogenous variables $\{K_0, Z_{A\nu t}, Z_{Mt}\}$, the elements in $\boldsymbol{\mathcal{A}}_H$ solve the household problem, while the elements in $\boldsymbol{\mathcal{A}}_F$ solve the firms' problems, and the markets for goods and labor clear:

$$Y_t = \sum_{\nu=1}^{n} Y_{\nu t} \tag{1.50}$$

$$L_{\eta t} = \int_0^1 L_{\eta jt} \, \mathrm{d}\, j \tag{1.51}$$

Model Structure

The model is composed of the preview solutions, forming a square system of 33 variables and 33 equations, summarized as follows:

- Variables:
  - from the household problem: $C_{\eta t}, C_{\nu\eta t}, L_{\eta t}, K_{\eta,t+1}$;
  - from the final-goods firm problem: $Y_{\nu t}, Y_{\nu jt}, P_{C\nu t}$;
  - from the intermediate-goods firm problems: $K_{\eta jt}, L_{\eta jt}, P^*_{C\nu t}$;
  - from the market clearing condition: $Y_t, I_{\eta t}$;
  - prices: $W_t, R_t, \Lambda_{\nu t}, \pi_{\nu t}, \pi_t$;
  - shocks: $Z_{A\nu t}, Z_{Mt}$.
- Equations:
  1. Labor Supply:
  $$\frac{\phi L^{\varphi}_{\eta t}}{C^{-\sigma}_{\eta t}} = \frac{W_t}{P_{\eta t}} \tag{1.19}$$
  2. Household Euler Equation:
  $$\left(\frac{\mathbb{E}_t C_{\eta,t+1}}{C_{\eta t}}\right)^{\sigma} = \beta\left[(1-\delta) + \mathbb{E}_t\left(\frac{R_{t+1}}{P_{C\eta t+1}}\right)\right] \tag{1.20}$$
  3. Budget Constraint:
  $$P_{C1t}C_{1\eta t} + P_{C2t}C_{2\eta t} + P_{C\eta t}I_{\eta t} = W_t L_{\eta t} + R_t K_{\eta t} + \Pi_{\nu t} \tag{1.2}$$

4. Law of Motion for Capital:

$$K_{\eta,t+1} = (1-\delta)K_{\eta t} + I_{\eta t} \tag{1.3}$$

5. Regional Consumption:

$$C_{\eta t} = C_{1\eta t}^{\omega_\eta} C_{2\eta t}^{1-\omega_\eta} \tag{1.4}$$

6. Bundle Technology:

$$Y_{\nu t} = \left(\int_0^1 Y_{\nu j t}^{\frac{\psi-1}{\psi}} \, \mathrm{d}\, j \right)^{\frac{\psi}{\psi-1}} \tag{1.22}$$

7. Regional Price Level:

$$P_{C\nu t} = \left[\theta P_{C\nu,t-1}^{1-\psi} + (1-\theta) P_{C\nu t}^{*1-\psi} \right]^{\frac{1}{1-\psi}} \tag{1.43}$$

8. Capital Demand:

$$K_{\eta j t} = \alpha_\nu Y_{\nu j t} \frac{\Lambda_{\nu t}}{R_t} \tag{1.29}$$

9. Labor Demand:

$$L_{\eta j t} = (1-\alpha_\nu) Y_{\nu j t} \frac{\Lambda_{\nu t}}{W_t} \tag{1.30}$$

10. Marginal Cost:

$$\Lambda_{\nu t} = \frac{1}{Z_{A\nu t}} \left(\frac{R_t}{\alpha_\nu} \right)^{\alpha_\nu} \left(\frac{W_t}{1-\alpha_\nu} \right)^{1-\alpha_\nu} \tag{1.36}$$

11. Production Function:

$$Y_{\nu j t} = Z_{A\nu t} K_{\eta j t}^{\alpha_\nu} L_{\eta j t}^{1-\alpha_\nu} \tag{1.27}$$

12. Optimal Price:

$$P_{C\nu t}^{*} = \frac{\psi}{\psi-1} \cdot \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \theta^s Y_{\nu j,t+s} \Lambda_{\nu t+s} / \prod_{k=0}^{s-1} (1+R_{t+k}) \right\}}{\mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \theta^s Y_{\nu j,t+s} / \prod_{k=0}^{s-1} (1+R_{t+k}) \right\}} \tag{1.42}$$

13. Market Clearing Condition:

$$Y_t = \sum_{\nu=1}^{n} Y_{\nu t} \tag{1.50}$$

14. Monetary Policy:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\gamma_\pi} \left(\frac{Y_t}{Y}\right)^{\gamma_Y}\right]^{1-\gamma_R} Z_{Mt} \tag{1.44}$$

15. National Gross Inflation Rate:

$$\pi_t = \frac{P_t}{P_{t-1}} \tag{1.45}$$

16. National Price Level:

$$P_t = \vartheta_1 P_{C1t} + (1-\vartheta_1) P_{C2t} \tag{1.46}$$

17. Regional Gross Inflation Rate:

$$\pi_{vt} = \frac{P_{Cvt}}{P_{Cv,t-1}} \tag{1.47}$$

18. Productivity Shock:

$$\ln Z_{Avt} = (1-\rho_{Av}) \ln Z_{Av} + \rho_{Av} \ln Z_{Av,t-1} + \varepsilon_{Avt} \tag{1.48}$$

19. Monetary Shock:

$$\ln Z_{Mt} = (1-\rho_M) \ln Z_M + \rho_M \ln Z_{M,t-1} + \varepsilon_{Mt} \tag{1.49}$$

1.1.7 Steady State

The steady state of a variable is defined by its constancy through time. For any given variable X_t, it is in steady state if $\mathbb{E}_t X_{t+1} = X_t = X_{t-1} = X_{ss}$ (COSTA JUNIOR, 2016, p.41). For conciseness, the ss index representing the steady state will be omitted, so that $X := X_{ss}$. The model steady state is:

1. Labor Supply:

$$\frac{\phi L_{\eta t}^{\varphi}}{C_{\eta t}^{-\sigma}} = \frac{W_t}{P_{\eta t}} \implies \frac{\phi L_{\eta}^{\varphi}}{C_{\eta}^{-\sigma}} = \frac{W}{P_\eta} \tag{1.52}$$

2. Household Euler Equation:

$$\left(\frac{\mathbb{E}_t C_{\eta,t+1}}{C_{\eta t}}\right)^{\sigma} = \beta\left[(1-\delta) + \mathbb{E}_t\left(\frac{R_{t+1}}{P_{C\eta t+1}}\right)\right] \implies$$
$$1 = \beta\left[(1-\delta) + \frac{R}{P_{C\eta}}\right] \tag{1.53}$$

3. Budget Constraint:

$$P_{C1t}C_{1\eta t} + P_{C2t}C_{2\eta t} + P_{C\eta t}I_{\eta t} = W_t L_{\eta t} + R_t K_{\eta t} + \Pi_{\nu t} \implies$$
$$P_{C1}C_{1\eta} + P_{C2}C_{2\eta} + P_{C\eta}I_{\eta} = WL_{\eta} + RK_{\eta} + \Pi_{\nu} \tag{1.54}$$

4. Law of Motion for Capital:

$$K_{\eta,t+1} = (1-\delta)K_{\eta t} + I_{\eta t} \implies K_{\eta} = (1-\delta)K_{\eta} + I_{\eta} \implies$$
$$I_{\eta} = \delta K_{\eta} \tag{1.55}$$

5. Regional Consumption:

$$C_{\eta t} = C_{1\eta t}^{\omega_\eta} C_{2\eta t}^{1-\omega_\eta} \implies C_{\eta} = C_{1\eta}^{\omega_\eta} C_{2\eta}^{1-\omega_\eta} \tag{1.56}$$

6. Bundle Technology:

$$Y_{\nu t} = \left(\int_0^1 Y_{\nu jt}^{\frac{\psi-1}{\psi}}\,\mathrm{d}\,j\right)^{\frac{\psi}{\psi-1}} \implies Y_{\nu} = \left(\int_0^1 Y_{\nu j}^{\frac{\psi-1}{\psi}}\,\mathrm{d}\,j\right)^{\frac{\psi}{\psi-1}} \tag{1.57}$$

7. Regional Price Level:

$$P_{C\nu t} = \left[\theta P_{C\nu t-1}^{1-\psi} + (1-\theta)P_{C\nu t}^{*1-\psi}\right]^{\frac{1}{1-\psi}} \implies$$
$$P_{C\nu}^{1-\psi} = \theta P_{C\nu}^{1-\psi} + (1-\theta)P_{C\nu}^{*1-\psi} \implies$$
$$(1-\theta)P_{C\nu}^{1-\psi} = (1-\theta)P_{C\nu}^{*1-\psi} \implies P_{C\nu} = P_{C\nu}^{*} \tag{1.58}$$

8. Capital Demand:

$$K_{\eta jt} = \alpha_\nu Y_{\nu jt}\frac{\Lambda_{\nu t}}{R_t} \implies K_{\eta j} = \alpha_\nu Y_{\nu j}\frac{\Lambda_\nu}{R} \tag{1.59}$$

9. Labor Demand:

$$L_{\eta jt} = (1-\alpha_\nu) Y_{\nu jt} \frac{\Lambda_{\nu t}}{W_t} \implies L_{\eta j} = (1-\alpha_\nu) Y_{\nu j} \frac{\Lambda_\nu}{W} \tag{1.60}$$

10. Marginal Cost:

$$\begin{aligned}
\Lambda_{\nu t} &= \frac{1}{Z_{A\nu t}} \left(\frac{R_t}{\alpha_\nu}\right)^{\alpha_\nu} \left(\frac{W_t}{1-\alpha_\nu}\right)^{1-\alpha_\nu} \implies \\
\Lambda_\nu &= \frac{1}{Z_{A\nu}} \left(\frac{R}{\alpha_\nu}\right)^{\alpha_\nu} \left(\frac{W}{1-\alpha_\nu}\right)^{1-\alpha_\nu}
\end{aligned} \tag{1.61}$$

11. Production Technology:

$$Y_{\nu jt} = Z_{A\nu t} K_{\eta jt}^{\alpha_\nu} L_{\eta jt}^{1-\alpha_\nu} \implies Y_{\nu j} = Z_{A\nu} K_{\eta j}^{\alpha_\nu} L_{\eta j}^{1-\alpha_\nu} \tag{1.62}$$

12. Optimal Price:

$$P^*_{C\nu t} = \frac{\psi}{\psi-1} \cdot \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \left\{\theta^s Y_{\nu j,t+s} \Lambda_{\nu t+s} / \prod_{k=0}^{s-1} (1+R_{t+k})\right\}}{\mathbb{E}_t \sum_{s=0}^{\infty} \left\{\theta^s Y_{\nu j,t+s} / \prod_{k=0}^{s-1} (1+R_{t+k})\right\}} \tag{1.42}$$

$$\begin{aligned}
P^*_{C\nu} &= \frac{\psi}{\psi-1} \cdot \frac{Y_{\nu j} \Lambda_\nu / [1-\theta(1-R)]}{Y_{\nu j} / [1-\theta(1-R)]} \implies \\
P^*_{C\nu} &= \frac{\psi}{\psi-1} \Lambda_\nu
\end{aligned} \tag{1.63}$$

13. Market Clearing Condition:

$$Y_t = \sum_{\nu=1}^{n} Y_{\nu t} \implies Y = \sum_{\nu=1}^{n} Y_\nu \tag{1.64}$$

14. Monetary Policy:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\gamma_\pi} \left(\frac{Y_t}{Y}\right)^{\gamma_Y}\right]^{1-\gamma_R} Z_{Mt} \implies Z_M = 1 \tag{1.65}$$

15. National Gross Inflation Rate:

$$\pi_t = \frac{P_t}{P_{t-1}} \implies \pi = 1 \tag{1.66}$$

16. National Price Level:

$$P_t = \vartheta_1 P_{C1t} + (1-\vartheta_1) P_{C2t} \implies P = \vartheta_1 P_{C1} + (1-\vartheta_1) P_{C2} \tag{1.67}$$

17. Regional Gross Inflation Rate:

$$\pi_{vt} = \frac{P_{Cvt}}{P_{Cv,t-1}} \implies \pi_v = 1 \tag{1.68}$$

18. Productivity Shock:

$$\begin{aligned} \ln Z_{Avt} &= (1-\rho_{Av}) \ln Z_{Av} + \rho_{Av} \ln Z_{Av,t-1} + \varepsilon_{Avt} && \implies \\ \ln Z_{Av} &= (1-\rho_{Av}) \ln Z_{Av} + \rho_{Av} \ln Z_{Av} + \varepsilon_{Av} && \implies \\ \varepsilon_{Av} &= 0 \end{aligned} \tag{1.69}$$

19. Monetary Shock:

$$\begin{aligned} \ln Z_{Mt} &= (1-\rho_{M}) \ln Z_{M} + \rho_{M} \ln Z_{M,t-1} + \varepsilon_{Mt} && \implies \\ \ln Z_{M} &= (1-\rho_{M}) \ln Z_{M} + \rho_{M} \ln Z_{M} + \varepsilon_{M} && \implies \\ \varepsilon_{M} &= 0 \end{aligned} \tag{1.70}$$

Variables in Steady State

For the steady state solution, all endogenous variables will be determined with respect to the parameters. It's assumed that the productivity and the price level are normalized to one: $[\, P \;\, Z_{Av} \,] = \vec{\mathbf{1}}$.[4]

From 1.58, the regional optimal price P^*_{Cv} is:

$$P_{Cv} = P^*_{Cv} \tag{1.71}$$

From 1.66 and 1.68, the national and regional gross inflation rates are:

$$\begin{bmatrix} \pi & \pi_v \end{bmatrix} = \vec{\mathbf{1}} \tag{1.72}$$

[4] where $\vec{\mathbf{1}}$ is the unit vector.

From 1.65, the monetary shock is:

$$Z_M = 1 \tag{1.73}$$

From 1.69 and 1.70, the productivity and monetary shocks are:

$$\begin{bmatrix} \varepsilon_{A\nu} & \varepsilon_M \end{bmatrix} = \vec{0} \tag{1.74}$$

From 1.53, the return on capital R is:

$$1 = \beta \left[(1-\delta) + \frac{R}{P_{C\eta}} \right] \implies R = P_{C\eta} \left[\frac{1}{\beta} - (1-\delta) \right] \tag{1.75}$$

From 1.63 and 1.58, the marginal cost Λ_ν is:

$$P^*_{C\nu} = \frac{\psi}{\psi - 1} \Lambda_\nu \implies \Lambda_\nu = P_{C\nu} \frac{\psi - 1}{\psi} \tag{1.76}$$

From equation 1.61, the nominal wage W is:

$$\begin{aligned} \Lambda_\nu &= \frac{1}{Z_{A\nu}} \left(\frac{R}{\alpha_\nu} \right)^{\alpha_\nu} \left(\frac{W}{1-\alpha_\nu} \right)^{1-\alpha_\nu} \implies \\ W &= (1-\alpha_\nu) \left[\Lambda_\nu Z_{A\nu} \left(\frac{\alpha_\nu}{R} \right)^{\alpha_\nu} \right]^{\frac{1}{1-\alpha_\nu}} \end{aligned} \tag{1.77}$$

In steady state, prices are the same ($P_{C\nu} = P^*_{C\nu}$), resulting in a gross inflation level of one ($\pi_\nu = 1$), and all firms producing the same output level ($Y_{\nu j} = Y_\nu$) due to the price parity (SOLIS-GARCIA, 2022, Lecture 13, p.12). For this reason, they all demand the same amount of factors (K_η, L_η), and equations 1.59, 1.60, and 1.62 become:

$$K_\eta = \alpha_\nu Y_\nu \frac{\Lambda_\nu}{R} \tag{1.78}$$

$$L_\eta = (1-\alpha_\nu) Y_\nu \frac{\Lambda_\nu}{W} \tag{1.79}$$

$$Y_\nu = Z_{A\nu} K_\eta^{\alpha_\nu} L_\eta^{1-\alpha_\nu} \tag{1.80}$$

here, there is no Y = C + I to substitute...

Substitute 1.78 in 1.55:

$$I_\eta = \delta K_\eta \implies I_\eta = \delta \alpha_\nu Y_\eta \frac{\Lambda_\nu}{R} \tag{1.81}$$

Substitute 1.79 in 1.52:

$$\frac{\phi L_\eta^\varphi}{C_\eta^{-\sigma}} = \frac{W}{P_\eta} \implies C_\eta = \left[L_\eta^{-\varphi} \frac{W}{\phi P_\eta} \right]^{\frac{1}{\sigma}} \implies$$

$$C_\eta = \left[\left((1-\alpha_\nu) Y_\nu \frac{\Lambda_\nu}{W} \right)^{-\varphi} \frac{W}{\phi P_\eta} \right]^{\frac{1}{\sigma}} \tag{1.82}$$

Substitute 1.81 and 1.82 in 1.64:

$$Y_\nu = \sum_{\eta=1}^{n} (C_\eta + I_\eta) \implies$$

$$Y = \left[\left((1-\alpha_\nu) Y \frac{\Lambda}{W} \right)^{-\varphi} \frac{W}{\phi P} \right]^{\frac{1}{\sigma}} + \left[\delta \alpha_\nu Y \frac{\Lambda}{R} \right] \implies$$

$$Y = \left[\left(\frac{W}{\phi P} \right) \left(\frac{W}{(1-\alpha_\nu)\Lambda} \right)^{\varphi} \left(\frac{R}{R - \delta \alpha_\nu \Lambda} \right)^{\sigma} \right]^{\frac{1}{\varphi+\sigma}} \tag{1.83}$$