Analysis of the Monetary Policy Impact on Regional Gross Domestic Product: A Regional DSGE Model

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Contents

Introduction

- **PROBLEM:** heterogeneous economic matrices that respond in diverse ways to the monetary decisions (BERTANHA; HADDAD, 2008).

- **OBJECTIVE:** develop a DSGE model with two regions to verify regional responses to monetary shocks.

- **CONTRIBUTION:** DSGE models demonstrate the relations among variables and determine how the economy respond to the monetary shock.

Literature Review

Macro modeling

- Costa Junior (2016): presents a RBC model and then adds NK elements in each chapter;

- Galí (2015): discuss monetary policy starting with a RBC model and also adds NK elements in each chapter;

- Bergholt (2012): presents a NK and the method of programming in *Dynare*;

- Solis-Garcia (2022): presents a RBC model and demonstrate the math tools necessary to solve a DSGE model;

Regional Modeling

- Rickman (2010): link between macro and regional modeling.

- Tamegawa (2012): two regions and fiscal policy.

- Tamegawa (2013): small regions and crowding in effects.

- Okano et al. (2015): regional economic stagnation.

- Mora e Costa Junior (2019): foreign direct investment (FDI) in Bogotá and the rest of Colombia.

- Costa Junior et al. (2022): fiscal policy, regional and national for the State of Goiás.

- Osterno et al. (2022): Regionalization of SAMBA: SAMBA+REG (Stochastic Analytical Model with Bayesian Approach from the Central Bank of Brazil).

Regional NK DSGE Model

NK DSGE:

- **N**ew **K**eynesian

- **D**ynamic

- **S**tochastic

- **G**eneral **E**quilibrium

Sequential Process:

- Structural Model;

- Steady State;

- Log-linearization;

- Calibration;

- Impulse Response Functions.

- four agents: households, intermediate and final-goods firms, monetary authority.

- no bonds.

- capital and investment.

- price stickiness of intermediate goods.

- two regions: final good is what links both.

- the representative household maximizes utility;

- firms producing intermediate goods minimize costs and maximize profit flow;

- firms producing final goods maximize profit.

- the monetary authority determines the interest rate, aiming to control inflation and pursuing economic growth.

Model Structure
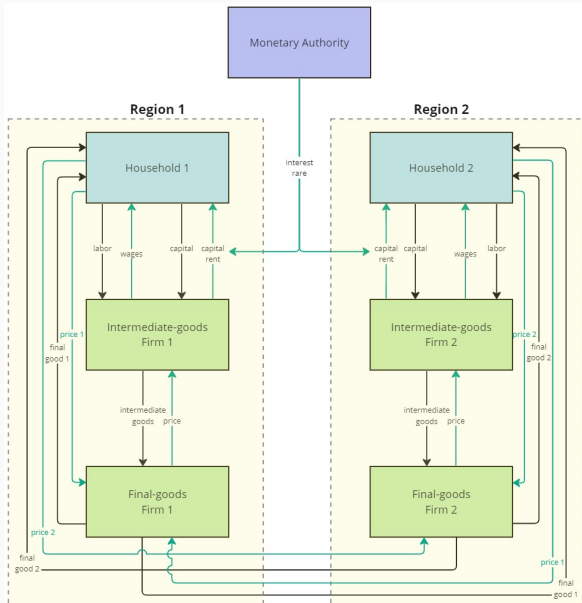
Cost Minimization Problem

$$\min_{C_{\eta 1t}, C_{\eta 2t}} : \quad Q_{\eta t} C_{\eta t} = P_{1t} C_{\eta 1t} + P_{2t} C_{\eta 2t} \tag{1}$$

$$\text{s.t.}: \quad C_{\eta t} = C_{\eta 1t}^{\omega_{\eta 1}} C_{\eta 2t}^{1-\omega_{\eta 1}} \tag{2}$$

$$C_{\eta t} > 0$$

Cost Minimization Problem

Solution:

$$C_{\eta 2t} = C_{\eta 1t} \frac{(1 - \omega_{\eta 1})P_{1t}}{\omega_{\eta 1}P_{2t}} \tag{3}$$

$$C_{\eta 1t} = C_{\eta t} \left(\frac{P_{2t}\omega_{\eta 1}}{P_{1t}(1 - \omega_{\eta 1})} \right)^{1 - \omega_{\eta 1}} \tag{4}$$

$$Q_{\eta t} = \left(\frac{P_{1t}}{\omega_{\eta 1}} \right)^{\omega_{\eta 1}} \left(\frac{P_{2t}}{1 - \omega_{\eta 1}} \right)^{1 - \omega_{\eta 1}} \tag{5}$$

Household Maximization Problem

$$\max_{C_{\eta t}, L_{\eta t}, K_{\eta, t+1}} : \quad U_\eta(C_{\eta t}, L_{\eta t}) = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\frac{C_{\eta t}^{1-\sigma}}{1-\sigma} - \phi \frac{L_{\eta t}^{1+\varphi}}{1+\varphi} \right) \tag{6}$$

$$\text{s.t.} : \quad Q_{\eta t} C_{\eta t} + P_{\eta t} I_{\eta t} = W_{\eta t} L_{\eta t} + R_t K_{\eta t} + \Pi_{\eta t} \tag{7}$$

$$K_{\eta, t+1} = (1-\delta) K_{\eta t} + I_{\eta t} \tag{8}$$

$$C_{\eta t}, L_{\eta t}, K_{\eta t} > 0$$

Solution:

$$\frac{\phi L_{\eta t}^{\varphi}}{C_{\eta t}^{-\sigma}} = \frac{W_{\eta t}}{Q_{\eta t}} \tag{9}$$

$$\frac{\mathbb{E}_t\{Q_{\eta,t+1}C_{\eta,t+1}^{\sigma}\}}{Q_{\eta t}C_{\eta t}^{\sigma}} = \beta \frac{\mathbb{E}_t\{P_{\eta,t+1}(1-\delta) + R_{t+1}\}}{P_{\eta t}} \tag{10}$$

Final-goods Firm Maximization Problem

$$\max_{Y_{\eta j t}} : \quad P_{\eta t} Y_{\eta t} - \int_0^1 P_{\eta j t} Y_{\eta j t} \, \mathrm{d}j \tag{11}$$

$$\mathrm{s.\,t.} : \quad Y_{\eta t} = \left(\int_0^1 Y_{\eta j t}^{\frac{\psi-1}{\psi}} \, \mathrm{d}j \right)^{\frac{\psi}{\psi-1}} \tag{12}$$

Final-goods Firm Maximization Problem

Solution:

$$Y_{\eta jt} = Y_t \left(\frac{P_{\eta t}}{P_{\eta jt}} \right)^{\psi} \tag{13}$$

$$P_{\eta t} = \left[\int_0^1 P_{\eta jt}^{1-\psi} \, \mathrm{d}j \right]^{\frac{1}{1-\psi}} \tag{14}$$

Intermediate-goods Firm Problems

$$\min_{K_{\eta jt}, L_{\eta jt}} : \quad R_t K_{\eta jt} + W_t L_{\eta jt} \tag{15}$$

$$\text{s.t.} : \quad Y_{\eta jt} = Z_{A\eta t} K_{\eta jt}^{\alpha_\eta} L_{\eta jt}^{1-\alpha_\eta} \tag{16}$$

Intermediate-goods Firm Problems

Solutions:

$$\frac{K_{\eta j t}}{L_{\eta j t}} = \left(\frac{\alpha_\eta}{1 - \alpha_\eta} \right) \frac{W_{\eta t}}{R_t} \tag{17}$$

$$K_{\eta j t} = \frac{Y_{\eta j t}}{Z_{A\eta t}} \left[\left(\frac{\alpha_\eta}{1 - \alpha_\eta} \right) \frac{W_{\eta t}}{R_t} \right]^{1 - \alpha_\eta} \tag{18}$$

$$L_{\eta j t} = \frac{Y_{\eta j t}}{Z_{A\eta t}} \left[\left(\frac{\alpha_\eta}{1 - \alpha_\eta} \right) \frac{W_{\eta t}}{R_t} \right]^{-\alpha_\eta} \tag{19}$$

$$\Lambda_{\eta t} = \frac{1}{Z_{A\eta t}} \left(\frac{R_t}{\alpha_\eta} \right)^{\alpha_\eta} \left(\frac{W_{\eta t}}{1 - \alpha_\eta} \right)^{1 - \alpha_\eta} \tag{20}$$

Price Stickiness and Profit Flow, Calvo's Rule (CALVO, 1983):

$$\max_{P_{\eta jt}} : \quad \mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \frac{\theta^s \left[P_{\eta jt} Y_{\eta j, t+s} - TC_{\eta j, t+s} \right]}{\prod_{k=0}^{s-1}(1 + R_{t+k})} \right\} \tag{21}$$

$$\text{s.t.} : \quad Y_{\eta jt} = Y_{\eta t} \left(\frac{P_{\eta t}}{P_{\eta jt}} \right)^{\psi} \tag{13}$$

Solution:

$$P^*_{\eta t} = \frac{\psi}{\psi - 1} \cdot \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \theta^s Y_{\eta j, t+s} \Lambda_{\eta, t+s} / \prod_{k=0}^{s-1} (1 + R_{t+k}) \right\}}{\mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \theta^s Y_{\eta j, t+s} / \prod_{k=0}^{s-1} (1 + R_{t+k}) \right\}} \tag{22}$$

$$P_{\eta t} = \left[\theta P_{\eta, t-1}^{1-\psi} + (1 - \theta) P_{\eta t}^{*1-\psi} \right]^{\frac{1}{1-\psi}} \tag{23}$$

$$\pi_{\eta t} = \frac{P_{\eta t}}{P_{\eta, t-1}} \tag{24}$$

Taylor's Rule (TAYLOR, 1993):

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\gamma_\pi} \left(\frac{Y_t}{Y}\right)^{\gamma_Y}\right]^{1-\gamma_R} Z_{Mt} \qquad (25)$$

$$\text{where: } \pi_t = \pi_{1t}^{\theta_\pi} \pi_{2t}^{1-\theta_\pi} \qquad (26)$$

$$\text{and: } \theta_\pi = \frac{P_{1t}Y_{1t}}{P_{1t}Y_{1t} + P_{2t}Y_{2t}} \qquad (27)$$

Stochastic Shocks

Productivity Shock:

$$\ln Z_{At} = (1 - \rho_A) \ln Z_A + \rho_A \ln Z_{A,t-1} + \varepsilon_{At} \tag{28}$$

Monetary Policy Shock:

$$\ln Z_{Mt} = (1 - \rho_M) \ln Z_M + \rho_M \ln Z_{M,t-1} + \varepsilon_{Mt} \tag{29}$$

Equilibrium Conditions

A Competitive Equilibrium consists of sequences of:

- prices $\{P_{\eta t}^*, R_t^*, W_{\eta t}^*\}$,

- allocations for households $\mathcal{A}_H := \{C_{\eta 1t}^*, C_{\eta 2t}^*, L_{\eta t}^*, I_{\eta t}^*, K_{\eta, t+1}^*\}$

- allocations for firms $\mathcal{A}_F := \{K_{\eta jt}^*, L_{\eta jt}^*, Y_{\eta jt}^*, Y_{\eta t}^*\}$.

In such an equilibrium, given the set of exogenous variables $\{K_0, Z_{A\eta t}, Z_{Mt}\}$, the elements in $\mathcal{A}_H$ solve the household problem, while the elements in $\mathcal{A}_F$ solve the firms' problems, and the markets for goods and labor clear:

$$Y_t = Y_{1t} + Y_{2t} \tag{30}$$

$$L_{\eta t} = \int_0^1 L_{\eta jt} \, \mathrm{d}j \tag{31}$$

Steady State

Steady state solution (COSTA JUNIOR, 2016, p.41):

$$\mathbb{E}_t X_{t+1} = X_t = X_{t-1} = X_{ss} \tag{32}$$

Log-linearization

Log-linearization

Uhlig's rules for log-linearization (UHLIG, 1999).

$$\hat{x}_t := \frac{x_t - x}{x} \iff x_t = x(1 + \hat{x}_t)$$

Square system of 30 variables and equations:

- Real Variables: $\langle \hat{C}_\eta \quad \hat{L}_\eta \quad \hat{K}_\eta \quad \hat{I}_\eta \quad \hat{C}_{\eta 1} \quad \hat{C}_{\eta 2} \quad \hat{Y}_\eta \quad \hat{Y} \quad \hat{Z}_{A\eta} \quad \hat{Z}_M \rangle$;

- Nominal Variables: $\langle \hat{Q}_\eta \quad \hat{P}_\eta \quad \hat{R} \quad \hat{\pi} \quad \hat{W}_\eta \quad \hat{\lambda}_\eta \quad \hat{\pi}_\eta \rangle$.

- Regional Gross Inflation Rate

$$\hat{\pi}_{\eta t} = \hat{P}_{\eta t} - \hat{P}_{\eta, t-1} \tag{33}$$

- New Keynesian Phillips Curve

$$\hat{\pi}_{\eta t} = \beta \mathbb{E}_t \hat{\pi}_{\eta, t+1} + \frac{(1-\theta)(1-\theta\beta)}{\theta} \hat{\lambda}_{\eta t} \tag{34}$$

- Regional Consumption Weight

$$\hat{C}_{\eta 2t} - \hat{C}_{\eta 1t} = \hat{P}_{1t} - \hat{P}_{2t} \tag{35}$$

- Regional Consumption of Good 1

$$\hat{C}_{\eta t} - \hat{C}_{\eta 1 t} = (1 - \omega_{\eta 1})(\hat{P}_{1t} - \hat{P}_{2t}) \qquad (36)$$

- Regional Price Index

$$\hat{Q}_{\eta t} = \omega_{\eta 1}\hat{P}_{1t} + (1 - \omega_{\eta 1})\hat{P}_{2t} \qquad (37)$$

- Labor Supply

$$\varphi\hat{L}_{\eta t} + \sigma\hat{C}_{\eta t} = \hat{W}_{\eta t} - \hat{Q}_{\eta t} \qquad (38)$$

- Law of Motion for Capital

$$\hat{K}_{\eta,t+1} = (1 - \delta)\hat{K}_{\eta t} + \delta\hat{I}_{\eta t} \qquad (39)$$

- Euler equation for capital return

$$(\hat{Q}_{\eta,t+1} - \hat{Q}_{\eta t}) + \sigma(\hat{C}_{\eta,t+1} - \hat{C}_{\eta t}) - (\hat{P}_{\eta,t+1} - \hat{P}_{\eta,t}) =$$
$$= \beta r(\hat{R}_{\eta,t+1} - \hat{P}_{\eta,t+1}) \qquad (40)$$

- Production Function

$$\hat{Y}_{\eta t} = \hat{Z}_{A\eta t} + \alpha_\eta \hat{K}_{\eta t} + (1 - \alpha_\eta)\hat{L}_{\eta t} \qquad (41)$$

- Technical and Economic Marginal Rates of Substitution

$$\hat{K}_{\eta t} - \hat{L}_{\eta t} = \hat{W}_{\eta t} - \hat{R}_t \qquad (42)$$

- Marginal Cost

$$\hat{\lambda}_{\eta t} = \alpha_\eta \hat{R}_t + (1 - \alpha_\eta)\hat{W}_{\eta t} - \hat{Z}_{A\eta t} - \hat{P}_{\eta t} \qquad (43)$$

- Monetary Policy

$$\hat{R}_t = \gamma_R \hat{R}_{t-1} + (1 - \gamma_R)(\gamma_\pi \hat{\pi}_t + \gamma_Y \hat{Y}_t) + \hat{Z}_{Mt} \tag{44}$$

- National Gross Inflation Rate

$$\hat{\pi}_t = \theta_\pi \hat{\pi}_{1t} + (1 - \theta_\pi)\hat{\pi}_{2t} \tag{45}$$

- Productivity Shock

$$\hat{Z}_{A\eta t} = \rho_{A\eta} \hat{Z}_{A\eta,t-1} + \varepsilon_{A\eta} \tag{46}$$

- Monetary Shock

$$\hat{Z}_{Mt} = \rho_M \hat{Z}_{M,t-1} + \varepsilon_M \tag{47}$$

- Goods-Market Clearing Condition

$$\hat{Y}_t = \theta_Y \hat{Y}_{1t} + (1 - \theta_Y)\hat{Y}_{2t} \tag{48}$$

- Regional Goods-Market Clearing Condition

$$\hat{Y}_{\eta t} = \theta_{C\eta}\hat{C}_{\eta t} + (1 - \theta_{C\eta})\hat{I}_{\eta t} \tag{49}$$

Results

- Given the Regional DSGE model, a region more capital intensive (α_η) and with a higher production technology level (Z_{A_η}) is more sensitive to a monetary shock:

$$\alpha_1 > \alpha_2 \wedge Z_{A1} > Z_{A2} \implies \Delta\mathbf{X}_1 > \Delta\mathbf{X}_2 \qquad (50)$$

- The responses are different in intensity, bot not in direction.

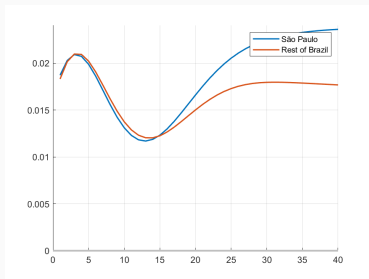Positive Monetary Policy Shock of 1%
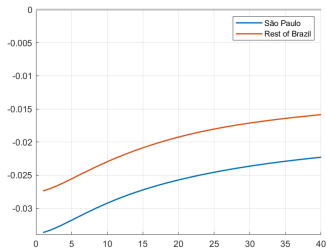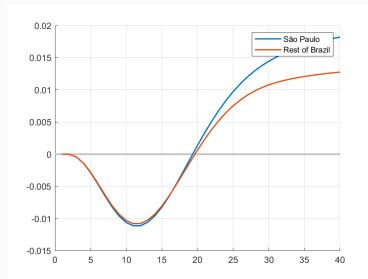

(a) Consumption
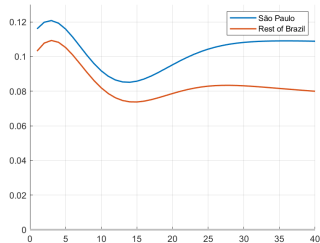
(b) Investment

Positive Monetary Policy Shock of 1%

(a) Production

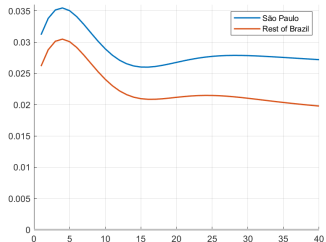**(b)** Capital

Positive Monetary Policy Shock of 1%
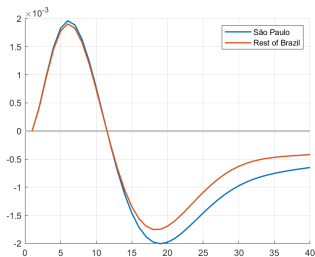

(a) Labor
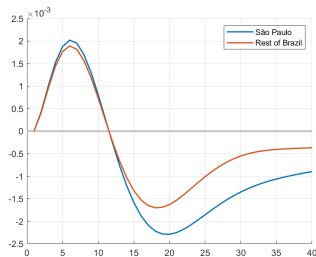

(b) Wages
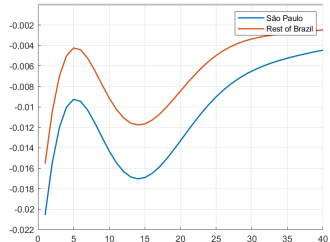
Positive Monetary Policy Shock of 1%

(a) Consumer Price Level

(b) Price Level

Final Remarks

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- The national wide interest rate may not have the same effectiveness across all regions of a country with different economic structures.

- Other instruments may be applied in conjunction to achieve the desired objectives of public policies.

- Future challenges include augmenting the model with additional elements to better describe Brazilian regions.

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Obrigado!
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https://github.com/andrlb/mastersthesis

Dúvidas e Sugestões