② $\frac{\partial f}{\partial x} = 4x^2 - y^2$, $\frac{\partial f}{\partial f} = -2xy + y + 2y^2$

$f(x,0) = \frac{4x^3}{3}$, $f(0,y) = \frac{y^2}{2} + \frac{2y^3}{3}$

Daí:

$$\begin{cases} 4x^2 - y^2 = 0 \\ -2xy + y + 2y^2 = 0 \end{cases} \quad 4x^2 - y^2 = 0 \quad y^2 = 4x^2 \Rightarrow \begin{matrix} y = 2x \ (1) \\ y = -2x \ (2) \end{matrix}$$

i) $-2xy + y + 2y^2 = 0 \Rightarrow -2x(2x) + 2x + 2(2x)^2 =$
$\Rightarrow -4x^2 + 2x + 8x^2 = 0$
$\Rightarrow 4x^2 + 2x = 2x(2x+1) = 0$
$\Rightarrow x = 0$ ou $x = -\frac{1}{2}$

ii) $-2xy + y + 2y^2 = 0 \Rightarrow -2x(-2x) + (-2x) + 2(-2x)^2 = 0$
$\Rightarrow 4x^2 - 2x + 8x^2 = 0$
$\Rightarrow 12x^2 - 2x = 0$
$\Rightarrow -2x(-6x + 1)$
$\Rightarrow x = 0$ ou $x = \frac{1}{6}$

Logo, temos esse Pontos Críticos
$(0,0)$, $(\frac{1}{6}; \frac{1}{3})$, $(\frac{-1}{2}; 1)$

Det da matriz Hessiana. Temos $\frac{\partial^2 f}{\partial x^2} = 8x$, $\frac{\partial^2 f}{\partial y^2} = 1 - 2x - 4y$,
$\frac{\partial^2 f}{\partial x \partial y} = -2y$, logo:

$$\frac{\partial^2 f}{\partial x^2}(x,y) = 8x, \quad \frac{\partial^2 f}{\partial y^2} = -2x+1-4y, \quad \frac{\partial^2 f}{\partial y \partial x} = -2y$$

Logo, a matriz Hessiana é

$$H(x,y) = \begin{pmatrix} 8x & -2y \\ -2y & -2x+1-4y \end{pmatrix}$$

Aplicando os pontos críticos, teremos

$$H(0,0) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}; \quad H\left(\frac{1}{6},\frac{1}{3}\right) = \begin{pmatrix} 4/3 & -2/3 \\ -2/3 & -2/3 \end{pmatrix}; \quad H\left(-\frac{1}{2},1\right) = \begin{pmatrix} -4 & -2 \\ -2 & -2 \end{pmatrix}$$

$$\det H(0,0) = 0, \quad \det A\left(\frac{1}{6},\frac{1}{3}\right) = \frac{-4}{3}; \quad \operatorname{Det} H\left(\frac{-1}{2},1\right) = 4$$

$$\frac{\partial^2 f}{\partial x^2}\left(\frac{-1}{2},1\right) = -4$$

Logo $\left(\frac{1}{6},\frac{1}{3}\right)$ e $(0,0)$ são pontos de sela e $\left(\frac{-1}{2},1\right)$ é máximo local.