Prova Cálculo II

1-

2- $\frac{\partial f}{\partial x} = 4x^2 - y^2$, $\frac{\partial f}{\partial y} = -2xy + y - 2y^2$

$f(x,0) = \frac{4x^3}{3}$, $f(0,y) = \frac{y^2}{2} - \frac{2y^3}{3}$

Daí:

$$\begin{cases} 4x^2 - y^2 = 0 \\ -2xy + y - 2y^2 = 0 \end{cases}$$

$$4x^2 - y^2 = 0$$
$$y^2 = 4x^2 \Rightarrow \begin{array}{ll} y = 2x & (i) \\ y = -2x & (ii) \end{array}$$

i) $-2xy + y - 2y^2 = 0 \Rightarrow -2x(2x) + 2x - 2(2x)^2 = 0$

$\Rightarrow -4x^2 + 2x - 8x^2 = 0$

$\Rightarrow -12x^2 + 2x = 2x(-6x + 1) = 0$

$\Rightarrow x = 0$ ou $x = \frac{1}{6}$

ii) $-2xy + y - 2y^2 = 0 \Rightarrow -2x(-2x) + (-2x) - 2(-2x)^2 = 0$

$\Rightarrow 4x^2 - 2x - 8x^2 = 0$

$\Rightarrow -4x^2 - 2x = -2x(2x + 1) = 0$

$\Rightarrow x = 0$ ou $x = -\frac{1}{2}$

Logo, temos os seguintes pontos críticos:

$(0,0)$, $\left(\frac{1}{6}, \frac{1}{3}\right)$, $\left(\frac{-1}{2}, 1\right)$

Vamos analisar com o determinante da matriz Hessiana. Temos $\frac{\partial^2 f}{\partial x^2} = 8x$, $\frac{\partial^2 f}{\partial y^2} = 1 - 2x - 4y$, $\frac{\partial^2 f}{\partial x \partial y} = -2y$, logo:

$$H(x,y) = \begin{vmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial y^2} \end{vmatrix} = \begin{vmatrix} 8x & 1-2x-4y \\ 1-2x-4y & -2y \end{vmatrix}$$

$$= -4x^2 - 16y^2 - 32xy + 8y + 4x - 1$$

$$H(x,y) = -4x^2 - 16y^2 - 32xy + 8y + 4x - 1$$

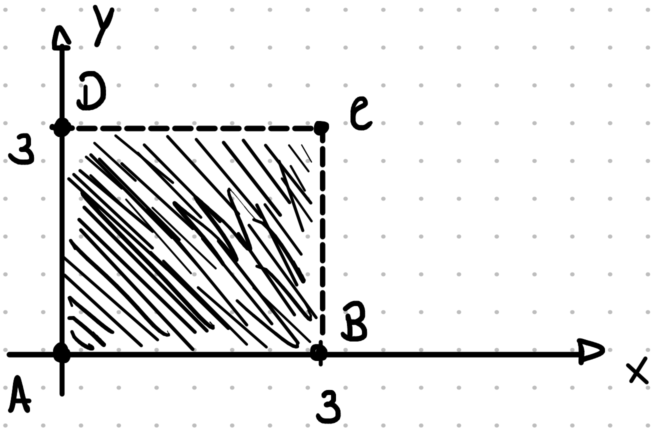
* $(0,0)$: $H(0,0) = -1 \Rightarrow$ Ponto de sela

* $\left(\frac{1}{6}, \frac{1}{3}\right)$: $H\left(\frac{1}{6}, \frac{1}{3}\right) = -\frac{4}{3} \Rightarrow$ Ponto de sela

* $\left(\frac{-1}{2}, 1\right)$: $H\left(\frac{-1}{2}, 1\right) = 4$ e $\frac{\partial^2 f}{\partial x^2}\left(\frac{-1}{2}, 1\right) = -4 \Rightarrow$ Máximo Local

h

3 -



$f(x,y) = 2 + 2x + 2y - x^2 - y^2$

$* \ \overline{AB} : y = 0 \Rightarrow f(x,0) = 2 + 2x - x^2$

$\Rightarrow f'(x,0) = -2x + 2 \Rightarrow f'(x,0) = 0$

$\Rightarrow x = 1, \ x = 0, \ x = 3$

$* \ \overline{BC} : x = 3 \Rightarrow f(3,y) = 2y - y^2 - 1$

$\Rightarrow f'(3,y) = -2y + 2 \Rightarrow f'(3,y) = 0$

$\Rightarrow y = 1, \ y = 0, \ y = 3$

$* \ \overline{CD} : y = 3 \Rightarrow f(x,3) = 2x - x^2 - 1$

$\Rightarrow f'(x,3) = -2x + 2 \Rightarrow f'(x,3) = 0$

$\Rightarrow x = 1, \ x = 0, \ x = 3$

$* \ \overline{DA} : x = 0 \Rightarrow f(0,y) = 2y - y^2 + 2$

$\Rightarrow f'(0,y) = -2y + 2 \Rightarrow f'(0,y) = 0$

$\Rightarrow y = 1, \ y = 0, \ y = 3$

Pontos	Localização	Imagem
(0, 0)	Fronteira	2
(3,0)	Fronteira	-1
(0,3)	Fronteira	-1
(3,3)	Fronteira	-4
(1, 1)	Interior	4
(0, 1)	Fronteira	3
(1,0)	Fronteira	3
(1,3)	Interior	0
(3, 1)	Interior	0

Logo, o valor máximo absoluto é 4 e o valor mínimo absoluto é -4

d

4-