

## Error Analysis Exercise

SID #24253445  
Jung Lin (Doris) Lee

1) Since radioactive decay is modelled by a Poisson distribution, its error is  $\sqrt{N}$

$$N=1000 \Rightarrow \text{specific activity} = \lambda = \frac{1000}{5} = 200 \frac{\text{decay}}{\text{min}}$$

$$\text{precision of } 1\% = 0.01 = \frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}} \Rightarrow N=10^4$$

$$\text{time required} = \frac{N}{\lambda} = \frac{10^4 \text{ decays}}{(200 \frac{\text{decay}}{\text{min}})} = \boxed{50 \text{ minutes}}$$

assuming that time measurement doesn't contribute to error.

2) a)  $\sigma = \sqrt{\sigma_A^2 + \sigma_B^2}$

b)  $\sigma = \sqrt{\sigma_A^2 + \sigma_B^2}$

this is basically same as a)  $\because$  error both  $\pm$

c) General function:

$$f(A,B) = 2A + 2B \rightarrow \frac{\partial f}{\partial A} = 2 \quad \frac{\partial f}{\partial B} = 2$$

$$\sigma = \sigma_f = \sqrt{\left(\frac{\partial f}{\partial A} \sigma_A\right)^2 + \left(\frac{\partial f}{\partial B} \sigma_B\right)^2} = \sqrt{(2\sigma_A)^2 + (2\sigma_B)^2} = \boxed{2\sqrt{\sigma_A^2 + \sigma_B^2}}$$

d) for multiplication:  $\frac{\sigma_f}{f} = \sqrt{\left(\frac{\sigma_A}{A}\right)^2 + \left(\frac{\sigma_B}{B}\right)^2}$

$$f(A,B) = A \times B$$

$$\sigma = \sigma_f = f \sqrt{\left(\frac{\sigma_A}{A}\right)^2 + \left(\frac{\sigma_B}{B}\right)^2} = AB \sqrt{\left(\frac{\sigma_A}{A}\right)^2 + \left(\frac{\sigma_B}{B}\right)^2} = \sqrt{A^2 B^2 \frac{\sigma_A^2}{A^2} + A^2 B^2 \frac{\sigma_B^2}{B^2}}$$

$$\sigma = \sqrt{B^2 \sigma_A^2 + A^2 \sigma_B^2} = \boxed{\sqrt{(B\sigma_A)^2 + (A\sigma_B)^2}}$$

3)  $\mu=0$

①  $\sigma=1$

After sampling 5 times,

the expected mean = 0

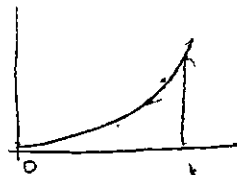
standard deviation = 1

$$\text{error on the mean} = \sqrt{N} = \sqrt{5} = 2.24$$

④ See plot attached

② Using the computer to generate a list of 5 random number we get  $\mu=0.0\%$ ,  $\sigma=0.8418$  but this can vary a lot if a pick another set of 5 normally distributed random numbers

③ The histogram centered around 0 which makes sense, because most experiments should yield something close to  $\mu=0$ , some don't because there is only 5 number in the list which is relatively small. This explains the wide spread of the histogram. Counts of experiment within 1, 2, 3 sigma is 675, 955, 998 respectively. This agree with expected because almost all experiment (99.8%) lie in 3 sigma, less for 1, 2 sigma.



4) ① To generate an exponential distribution use a uniform distribution to draw a random # and put in into  $x = -\log(y-1)$

$$f(x) = Ae^{-x} \xrightarrow{\text{Normalize}} \int_0^{\infty} A f(x) dx = 1 = A [e^{-x}]_0^{\infty} \Rightarrow A = 1$$

$$N = \langle x \rangle = \int_0^{\infty} x e^{-x} dx = 1$$

$$\langle x^2 \rangle = \int_0^{\infty} x^2 e^{-x} dx = 2$$

$$\sigma = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \sqrt{2 - 1^2} = \sqrt{1} = 1$$

so both the mean and the standard deviation should be 1  
The error is the standard deviation so it should be one. The histogram should be centered around one and with a relatively narrow peak because the standard deviation is relatively small

The error is  $\approx \frac{1}{\sqrt{N}}$  so for the  $N=100 \Rightarrow \text{Error} \approx 0.1$

② For  $N=100$ , mean = 1.1717, standard deviation = 1.066

③ The distribution is centered around 1 and the error is 0.007304 which is close to  $\frac{1}{\sqrt{100}} \approx 0.01$  agreeing with what I thought.

④ Yes, the error on the mean scales as  $\frac{1}{\sqrt{N}}$  as I thought.

5) ① See attached plots  
Error =  $\sqrt{N}$  ( $\because$  Poisson process)

② Mean =  $1.20268 \pm 10$   $\leftarrow$  # of datapoints = 1000

Standard deviation

$$\sigma = \sqrt{1000} = 10$$

$$= 0.10379 \pm 0.0001$$

$\leftarrow$  peak.dat. given with 5 s.f.

③ we fit  $y = Ae^{-\frac{x-\mu}{2\sigma^2}}$

and get  $\mu = 1.20117$  and  $\sigma = 0.10210$

The difference arise from the fact that the Gaussian assumes a continuous functional form for the fit whereas in ① we computed  $\mu$  &  $\sigma$  based on its discrete definition.

$$\mu = \frac{\sum x_1 \dots x_n}{N} \quad \text{and} \quad \sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{N}}$$

$$\text{④ } \chi^2 = \sum_{\text{data}} \frac{(\text{data} - \text{fit})^2}{N} = 39.2$$

where data is the observed frequency of each bin

The  $\chi^2$  is large which means that the Gaussian distribution may not be a very good fit.

6) ① see attached

②  $\sigma_f = 0.01 \text{ MHz}$

Referring to Numerical recipes 15.2, we analytically maximize the log-likelihood

$y = a + bx$  the uncertainty on the estimated parameters are:

$$\sigma_a^2 = \frac{S_{xx}}{\Delta} = \frac{\sum_{i=1}^N \frac{x_i^2}{\sigma_i^2}}{S_{xx} - (S_x)^2} \quad \sigma_b^2 = \frac{S}{\Delta} = \frac{\sum_{i=1}^N \frac{1}{\sigma_i^2}}{\Delta}$$

$S_x = \sum_{i=1}^N \frac{x_i}{\sigma_i^2}$       $S_{xx} = \sum_{i=1}^N \frac{x_i^2}{\sigma_i^2}$       $\Delta = S_{xx} - (S_x)^2$

Computing this for  $\sigma_f = 0.01 \text{ MHz}$   $\Rightarrow \begin{cases} \sigma_a^2 = -1.314 \times 10^{-5} \\ \sigma_b^2 = -6.49 \times 10^{-7} \end{cases}$

for  $\sigma_f = 1 \text{ MHz}$   $\Rightarrow \begin{cases} \sigma_a^2 = -0.1314 \\ \sigma_b^2 = -0.06494 \end{cases}$

So when the uncertainty on the measured frequency is larger the error propagates to the parameter estimation, yielding a greater uncertainty in the slope and intercept estimated from the fit.

③ Uncertainty measured from  $\sigma^2 = \frac{\sum_{i=1}^N (y_i - f(x_i))^2}{N-1} = \frac{0.1518}{11} = \boxed{0.014}$

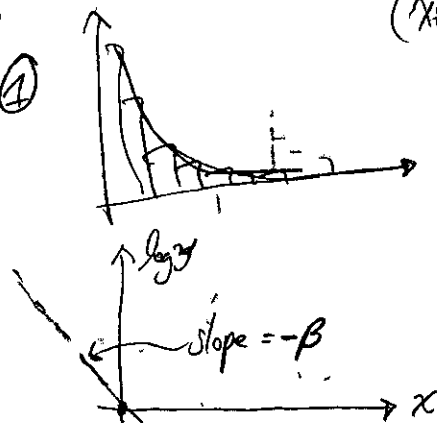
④ Using the weighted least square fit,

I obtained a slope of 3.11287 (rounded up to 3.1) and  
and an intercept of -0.04518 (rounded up to 0.05)

See attached plot of the fit

7) (x<sub>i</sub>, y<sub>i</sub>)

①



$$y = e^{-\beta x}$$

$$\log y = \log e^{-\beta x} = -\beta x$$

$$\log y = \underbrace{-\beta x}_{m = -\beta} \underbrace{+ 0}_{b = 0} \quad \text{linear form}$$

Using the function plot. semilog y this can be easily plotted

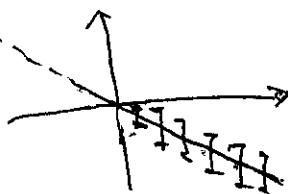
Muon lifetime is governed by a Poisson distribution.

error =  $\frac{1}{\sqrt{N}}$  as N increases the error bar shrinks!

only N > 0 events!

$$\log y = -\beta x \Rightarrow y = e^{-\beta x} = f(x)$$

$$\delta f = \sqrt{\left(\frac{\partial f}{\partial x} \delta x\right)^2} = -\beta e^{-\beta x} \frac{1}{\sqrt{N}}$$



as N increase, error bar of the semi-logged plot  
H. with dependence on  $\frac{\beta e^{-\beta x}}{\sqrt{N}}$

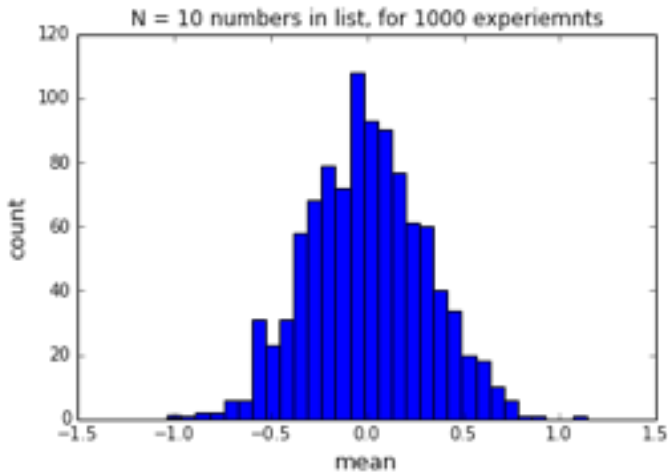
②  $\log E_0 = 2.1 \pm 0.5$

for error propagation on a general function  $y = \log x = f(x)$

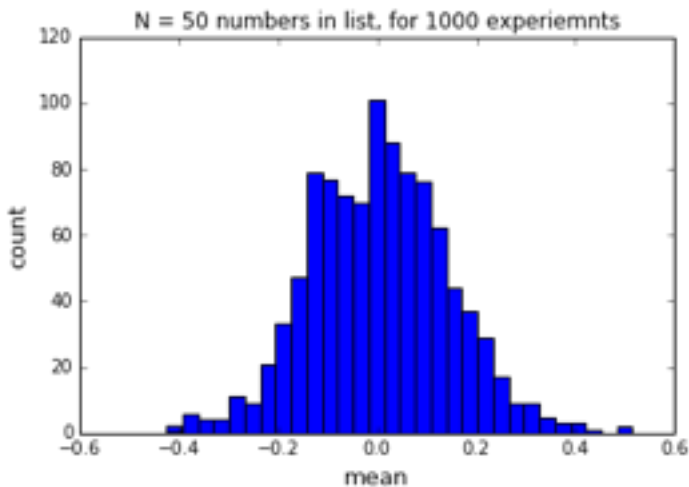
$$\delta f = \sqrt{\left(\frac{\partial f}{\partial x} \delta x\right)^2} = \left(\frac{1}{x} 0.5\right) = \frac{0.5}{2.1} = \frac{5}{21} = 0.238$$

$$E_0 = 8.2 \pm 0.2$$

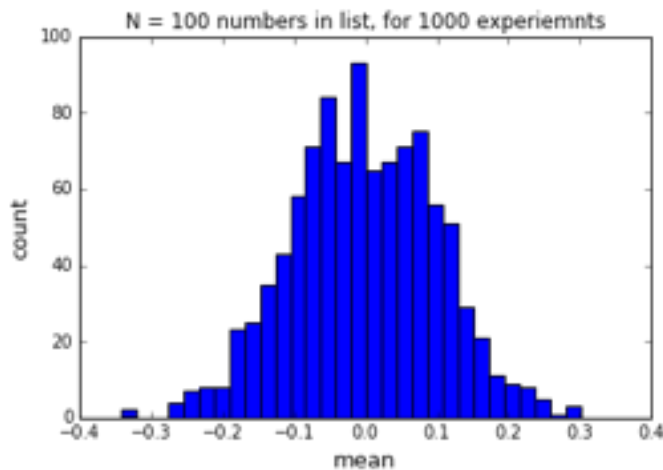
### Question 3 :



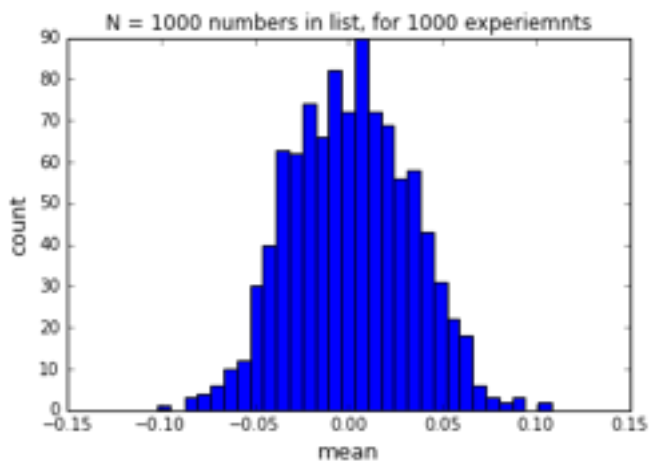
N=10  
mean :-0.00948400059966  
standard deviation: 0.323160945998  
Number of experiments within 1 sigma: 685  
Number of experiments within 2 sigma: 954  
Number of experiments within 3 sigma: 998



N=50  
mean :0.00157182550714  
standard deviation: 0.141286741226  
Number of experiments within 1 sigma: 671  
Number of experiments within 2 sigma: 959  
Number of experiments within 3 sigma: 997

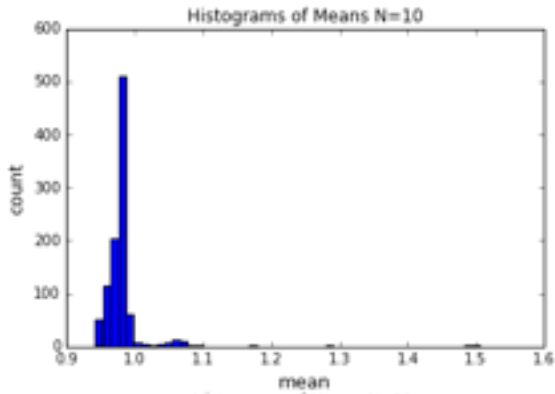


N=100  
mean :-0.0022772056286  
standard deviation: 0.100830644397  
Number of experiments within 1 sigma: 693  
Number of experiments within 2 sigma: 958  
Number of experiments within 3 sigma: 997



N=1000  
mean :-0.0016694934232  
standard deviation: 0.0323115057255  
Number of experiments within 1 sigma: 693  
Number of experiments within 2 sigma: 948  
Number of experiments within 3 sigma: 1000

## Question 4:



N=10

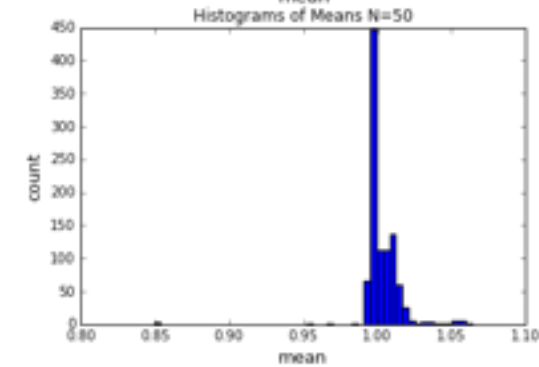
mean :0.980666752016

standard deviation: 0.0326604314666

Number of experiments within 1 sigma: 931

Number of experiments within 2 sigma: 964

Number of experiments within 3 sigma: 993



N=50

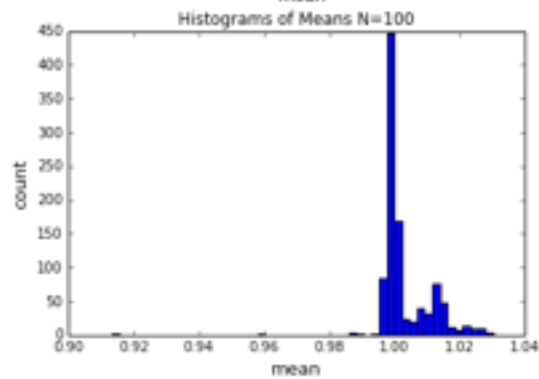
mean :1.00322330781

standard deviation: 0.0120208329751

Number of experiments within 1 sigma: 928

Number of experiments within 2 sigma: 970

Number of experiments within 3 sigma: 980



N=100

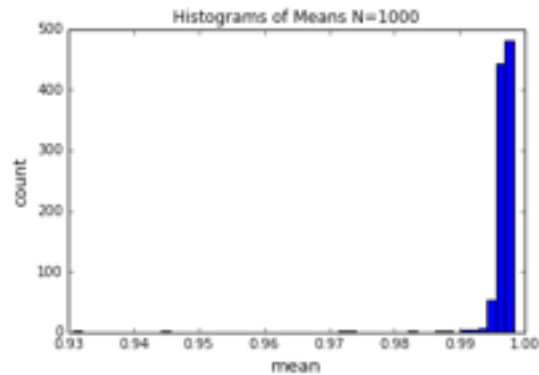
mean :1.00284570038

standard deviation: 0.00779803403721

Number of experiments within 1 sigma: 799

Number of experiments within 2 sigma: 955

Number of experiments within 3 sigma: 988



N=1000

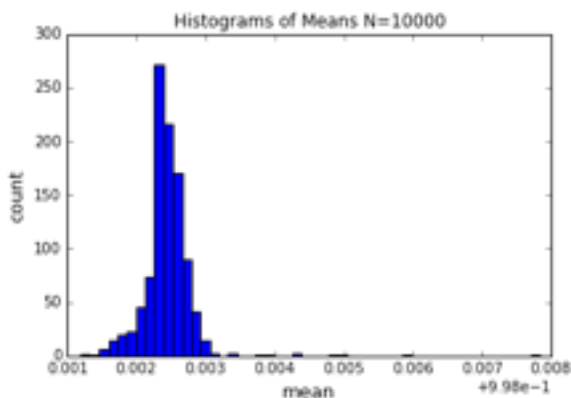
mean :0.996700301988

standard deviation: 0.0031155053156

Number of experiments within 1 sigma: 983

Number of experiments within 2 sigma: 990

Number of experiments within 3 sigma: 994



N=10000

mean :1.00043030173

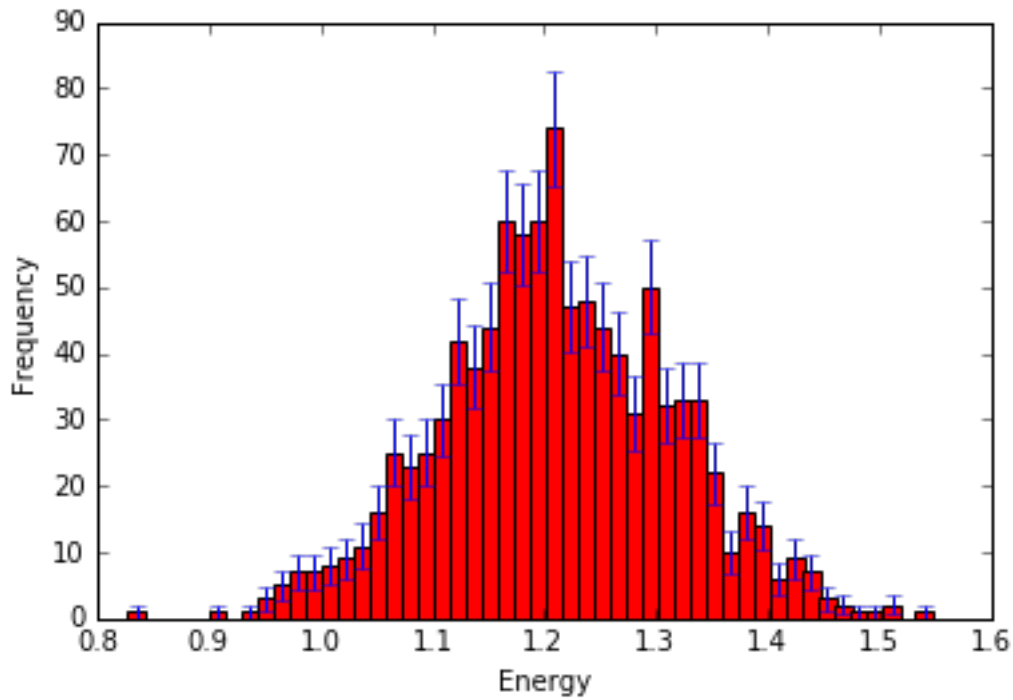
standard deviation: 0.000366673046101

Number of experiments within 1 sigma: 847

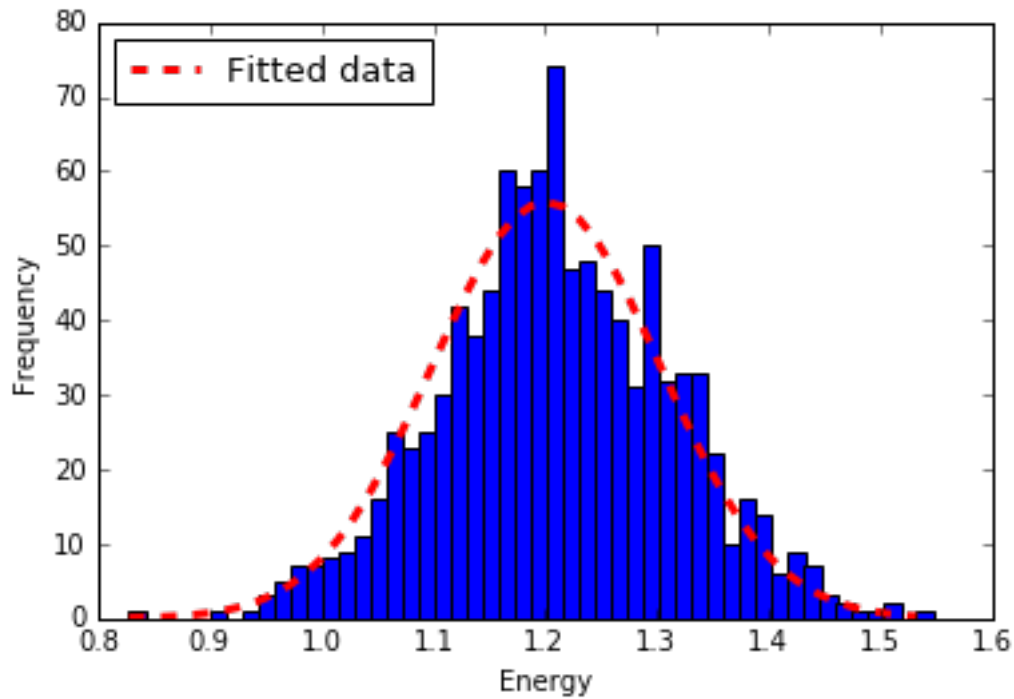
Number of experiments within 2 sigma: 972

Number of experiments within 3 sigma: 991

### Question 5:



Fitted mean = 1.20117112779  
Fitted standard deviation = 0.102097467081  
chi squared = 39.204431231256812





## Question 6:

