Quantum Entanglement and Interference Oral Presentation

April 12, 2016

1 Background:

- Bell's theorem: any physical theory that incorporates local realism cannot reproduce all the predictions of quantum mechanical theory.
- CHSH inquality: specific case of Bell's inequality
- Historical significance: resolve EPR paradox argument

2 Experiment:

- two beams of light (treat as quantum particles from distant locations)
- measure the parity (S) of each system:

$$E = \frac{N_{vv} - N_{vh} - N_{hv} + N_{hh}}{N_{total}}$$

$$S = E(\alpha, \beta) - E(\alpha, \beta') + E(\alpha', \beta) + E(\alpha', \beta')$$

- Experimental goal : show violation of CHSH inequality:

$$|S| \leq 2$$

  to show that Quantum mechanics (following $|S| \leq 2\sqrt{2}$)is not a local hidden vairable theory.

3 Experimental Setup

4 Calibration

- Laser Power:
  - vary laser input by 10 mW from 0˜100mW
- found that the optimal range is 20˜60mW , the efficiency decreases after we go beyond 70 mW
- $\therefore$ used P $\approx$ 40 mW throughout experiment
- incident power > measured power $\rightarrow$ possibly due to instrumentation/systematics issue
- Setting the 405 half wave plate as 40 deg to let in vertically polarized light.
- Adjust the BBO angle, so that the light directly incident the detector (tested with a red laser backward tracing from detector $\rightarrow$ source)
- Calibrating the second set of waveplates :

  - from varying the angle alpha, fixing beta, we saw a phase shift in the sinusodial
  - send in horizontally polarized light (from setting the first half wave plate at the default) then rotate α and β waveplates so that they each independently attain maximum count rate.
  - half wave plate need to adjust angle as half of the actual since that's the angle that the optical axis is adjusted.
  - Calibration showed that the optical axes is zeroed when $\alpha = 140$, $\beta = 60$.

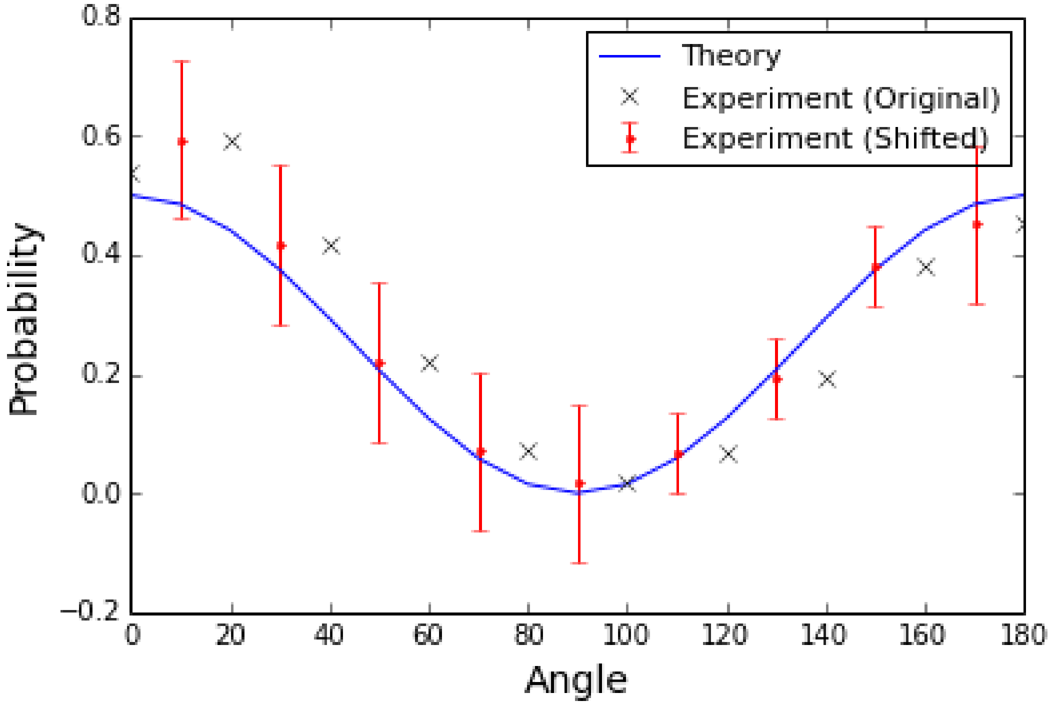
- Calibrating the phasor :

  - rotate until maximize coincidence count
  - obtained values 280 deg

In [45]:
```
prob_list = []
prob_list2 = []
alpha_range=np.arange(0,360,10)
for alpha in alpha_range:
    prob_list.append(compute_prob(1./sqrt(2),1./sqrt(2),alpha,0))
    prob_list2.append(compute_prob(sqrt(3.)/2,1./2,alpha,0))
alpha_range=np.arange(0,360,10)
shift = -10.0
count_exp = np.array([80.76, 89.14,62.5,33.18, 10.59,
                      2.61,10.39,28.79,57.38,67.76])
yerr = np.array([30, 20,20,20, 20, 20,10,10,10,20])
plt.xlabel("Angle",fontsize=14)
plt.ylabel("Probability",fontsize=14)
# plt.plot(alpha_range[:10]*2+shift,count_exp/150.,'.')
plt.plot(alpha_range,prob_list,'-',label="Theory")
plt.plot(alpha_range[:10]*2,count_exp/150.,'x', linewidth=12,
         label= "Experiment (Original)",color="k")
plt.errorbar(alpha_range[:10]*2+shift,count_exp/150.,yerr=yerr/150.,
             fmt='.', label= "Experiment (Shifted)",color="red")

# plt.plot(alpha_range,prob_list2,'-',label="bad")
plt.xlim(0,180)
plt.legend(loc='upper right',prop={'size':11},numpoints=1)
```

Out[45]: <matplotlib.legend.Legend at 0x1132d4e50>




```
In [18]: def compute_prob(c_h, c_v, alpha_deg, beta_deg):
             beta = np.deg2rad(beta_deg)
             alpha = np.deg2rad(alpha_deg)
             P =(c_h*cos(alpha)*cos(beta)+c_v*sin(alpha)*sin(beta) )**2
             return P
         def E(a,b):
             ap = a+pi/2.
             bp = b+pi/2.
         #    print "a,b: ",a,b
             E = P(a,b)+P(ap,bp)-P(ap,b)-P(a,bp)
             return E
         def P(alpha, beta):
             P =0.5*(cos(beta-alpha))**2
             return P
```

4.1 Theoretical Values

- assuming we have a perfect Bell state (i.e. C1 = C2= $\frac{1}{\sqrt{2}}$)

$$S = E(\alpha,\beta) - E(\alpha,\beta') + E(\alpha',\beta) + E(\alpha',\beta') = 2\sqrt{2} \approx 2.828$$

```
In [32]: a1= -pi/4
         a2 = 0.
         b1 = -pi/8
         b2 =  pi/8
         print E(a1,b1)-E(a1,b2)+ E(a2,b1)+E(a2,b2)
```

```
2.82842712475
```

4.2 Experimental Values

E(a1,b2) = E(-45,22.5)

```
In [35]: N = np.array([163.5,251.4,36,21.4])
         print float(N[0]+N[1]-N[2]-N[3])/(sum(N))
```

```
0.756934152022
```

E(a1,b1)=E(-45,-22.5)

```
In [36]: N = np.array([168.0,249.7,27.5,26.9])
         print float(N[0]+N[1]-N[2]-N[3])/(sum(N))
```

```
0.76954035162
```

E(0,22.5)

```
In [37]: N = np.array([29.8,20.9,168.6,265.8])
         print float(N[0]+N[1]-N[2]-N[3])/(sum(N))
```

```
-0.790970933828
```

E(0,-22.5)

```
In [38]: N = np.array([249,182.1,23.9,31.2 ])
         print float(N[0]+N[1]-N[2]-N[3])/(sum(N))
```

```
0.773344302756
```

```
In [54]: val = 0.76954035162+0.756934152022+0.790970933828+0.773344302756
         print "Percent Error: ", abs(2*sqrt(2) - val)/2*sqrt(2) *100. , "% "
```

```
Percent Error:  18.5518384536 %
```

```
In [47]: 0.756934152022 -- 0.790970933828+ 0.76954035162+ 0.773344302756
```

```
Out[47]: 3.090789740226
```

5 Error Propagation

The equations that we are using in this experiment is : $P_{vv} = \frac{1}{2}cos^2(\beta - \alpha)$

Using error propagation, we can derive the errors for P, E, which gives us the measure of uncertianty on S:

$$\delta P(\alpha, \beta) = \delta\alpha\sqrt{\beta^2 + \alpha^2}cos(\beta - \alpha)sin(\beta - \alpha)$$

$$\delta E(\alpha, \beta) = \sqrt{\delta P_{vv}^2 + \delta P_{HH}^2 + \delta P_{Hv}^2 + \delta P_{vH}^2}$$

$$\delta S(\alpha, \beta) = \sqrt{\delta E_{ab}^2 + \delta E_{ab'}^2 + \delta E_{a'b}^2 + \delta E_{a'b'}^2}$$

$$\delta S = 0.0137$$

```
In [4]: d_alpha =np.deg2rad(1)
        d_beta =np.deg2rad(1)
        a = -45.
        a1 = 0.
        b = -22.5
        b1=22.5
        def dP(alpha,beta):
            alpha = np.deg2rad(alpha)
            beta = np.deg2rad(beta)
            return d_alpha*sqrt(beta**2+alpha**2)*cos(beta-alpha)*sin(beta-alpha)
        def dE(a,b):
            return sqrt(dP(a,b)**2+dP(a,b1)**2+dP(a1,b)**2+dP(a1,b1)**2)
        def dS(a,b):
            return sqrt(dE(a,b)**2+dE(a,b1)**2+dE(a1,b)**2+dE(a1,b1)**2)
```

```
In [11]: sqrt(dE(a,b)**2+dE(a,b1)**2+dE(a1,b)**2+dE(a1,b1)**2)
```

```
Out[11]: 0.013707783890401889
```

6 Conclusion

We obtained a value of $S = 3.09 \pm 0.0137$ from the 16 coincident rate measurement. Even though we showed that $S \geq 2$ which disproves local hidden variable theory, according to the CHSH inequality. We also were unable to satisfy the quantum mechanics condition, based on calculating the probability of a pure Bell-state, of S$\leq 2\sqrt{2}$.

Other possible sources of error includes δN which is the uncertainty in the number of counts from the avalanche photodiode and detector apparatus. By looking at the coincidence counts on the LabView program we estimate $\delta N \approx 10$, which may add to compensate for the error.