Physics 111 : Hall Effect Plasma

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ABSTRACT

In this lab, we perform — Hall effect plasma

INTRODUCTION

Hall effect is the phenomena where a voltage develop across a system with a electric field transverse a magnetic field due to the Lorentz force. While Hall effect is most commonly demonstrated in semiconductors plates, it can be similarly observed in plasma from a gas discharge tube. In addition to studying the science of —–, hall effect has useful applications outside the laboratory as magnetic sensors in vehicle braking systems. [2].

Since charge recombination occurs at the walls of the discharge tube, the electron density in the tube is not uniform. It is distributed linearly with almost zero density at the ends and maximum density at the center. Thus the Hall voltage that we measure from the probe is half of the Hall voltage computed from the uniform density assumption.

THEORY

Hall Effect

Hall effect results from the force balance between magnetic and electric components of the Lorentz force.

$$F = 0 = e(\vec{E_H} + v \times \vec{B}) \tag{1}$$

$$\vec{E_H} = v \times \vec{B} \tag{2}$$

where e is the charge of the electron and E_H is the Hall effect electric field, which gives rise to a measurable Hall effect voltage (V_H). In a plasma, the electrons and ions are not bound as they would be in a semiconductor, so the drift velocity (v) of the free electrons is higher. Since electric field is proportional to voltage, $E = \int V \cdot dl$, this results in a higher measurable V_H in the plasma Hall effect experiment setup[**?**].

Plasma

The plasma results from a helium-argon-nitrogen gas mixture in a discharge tube ionized with high voltage. By adjusting to the right voltage and gas pressure, we can create an electric field inside the tube, resulting in a pink glow. Instabilities can arise in plasma due to perturbations of the physical parameters controlling the plasma (e.g. pressure, discharge voltage)[4]. Many instrumental care has been undertaken in the experimental setup to address this problem: from low pass filters attached to the probes and multi-levelled pressure gauges that keeps the gas from flowing out (minimizing the pressure fluctuation). Voltage oscillation is indicated by the peak-to-peak voltage measured by the oscilloscope, which gives us an idea of whether the plasma is in a good state for taking measurements.

Plasma Parameters

Due to the non-uniform charge distribution in the plasma tube, when we integrate the Hall Voltage from the Hall field, we obtain a factor of 1/2 :

$$V_H = \frac{1}{2}\int E_H \cdot dl \tag{3}$$

The separation between the top and bottom walls of the plasma tube as indicated in Fig.1 is dl = 8mm.

$$V_H = 4mmE_H \tag{4}$$

The field generated from the discharge voltage(E_D) is another quantity that we are interested in. Since this is just a simple ohmic plate setup, there is no additional factor of 1/2 due to the charge distribution. By using the value of 75mm for the width of the tube, we obtain:

$$V_D = 75mmE_D \tag{5}$$

The cross product in Eq.2 simplifies to $E_H = vB$ since $\vec{v} \perp \vec{B}$. Macroscopically, the drift velocity of the electron is related to the number (n) and current(j) density of the electron and its associated charges(e) through $j = env = I/A$. We can approximate the tube with a cylindrical geometric with circular r = 4mm with a cross section of area (A) of $16\pi mm^2$. Combining these relations with Eq.4 yields:

$$n = \frac{BI}{(4\pi mm)eV_H} \tag{6}$$

Microscopically, the collision frequency of the electrons in the plasma (ν) is an average over the product of the number density of the gas species, its cross section and velocity.

$$\nu = < n_{gas}\Sigma v > \tag{7}$$

Assuming that our gas is ideal, we can obtain n_g by the ideal gas law, using 270K as room temperature.

$$n_g = \frac{P}{270Kk_B} \tag{8}$$

The collision frequency can also be written like the cyclotron frequency in terms of the ratio between the E_D and E_H [3].

$$\nu = \frac{eBE_D}{m_eE_H} = \frac{4eBV_0}{75m_eV_H} \tag{9}$$

For the Helium used in our experiment $\sigma = 3.8 \times 10^{-20}m^2$. Combining these relationships yields:

$$< \sigma v > = \frac{4eBk_BV_DB(295K)}{15m_ePV_H} \tag{10}$$

Since n_g and σ is assumed to be constant throughout the tube, we can pull them out of the expectation value product.

$$\nu = n_g < \sigma >< v > \tag{11}$$

Assuming that the distribution of electron speeds in the plasma is Maxwellian, $v_{rms} \approx \sqrt{3/2} < v >$ [1]. We can find the temperature of the electron gas by equating the thermal energy and the RMS velocity of the electron:

$$\frac{3}{2}kT_e = \frac{1}{2}m_e(v_{RMS})^2 \tag{12}$$

Putting everything together, we have

$$T_e \approx \frac{1.085^2 m_e}{43.32\times 10^{-40}m^4}\left(\frac{4ekBV_D(295K)}{(15m_e PV_H)}\right)^2 \tag{13}$$

APPARATUS AND PROCEDURE

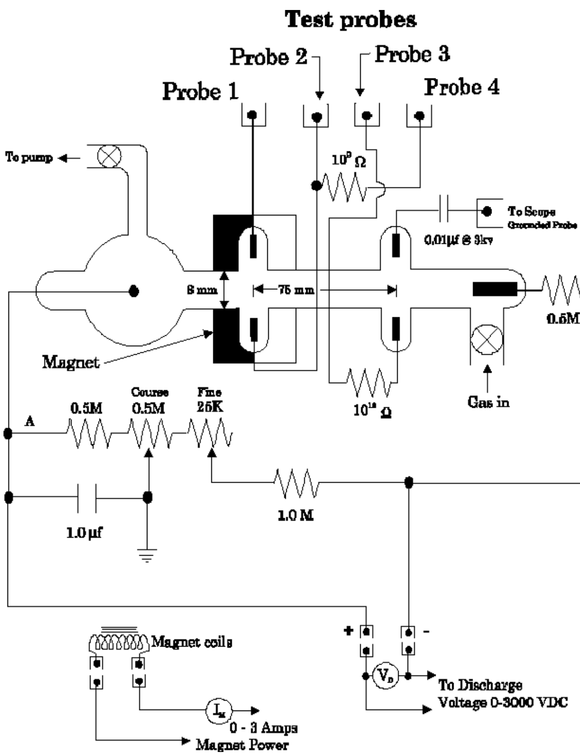

Figure 1. Experimental setup. Probe 1-2 measures the Hall Voltage (V_H) and Probe 2-3 measures the discharge voltage applied to the gas.

Instrumentation

Procedures

ANALYSIS

The discharge voltage has an approximately linear relationship with the discharge current.

Magnetic field as a function of current

Since the magnetic field in this experiment is provided by a Helmholtz coil, there may be hysteresis behavior due to the ferromagnetic nature of the materials [5]. So we measure the magnetic field as a function of current for various gas pressures to make sure that hysteresis is not a huge effect. It is important to distinguish between the magnetic current I_m that we measure in this experiment with the direct current supplied to the Helmholtz coil I_d. By doing linear regression, we can see that the combination of forward/reverse polarity and increasing/decreasing current in the hysteresis loop seems to conform to a linear relationship. Therefore, we conclude that the non-linearity introduced by the hysteresis effect is not significant to our experiment.

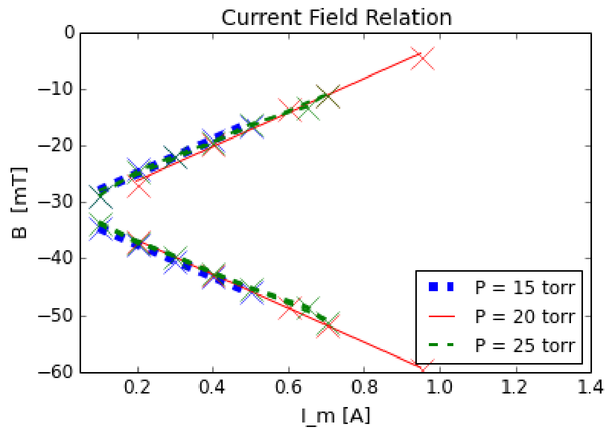

Figure 2. Plot of magnetic field strength as a function of magnetic current for three different pressures. The y-intercept approximately correspond to the Earth's magnetic field, since the "zero"-ing function on our magnetometer seems to be malfunctioned.

Table 1. Table of linear regression coefficient (y=mx+b) where (m,b) are the coefficients for the forward polarity and (m',b') for the reverse polarity.

P [torr]	15	20	25
m	29.90	30.02	28.23
b	-31.03	-32.29	-30.86
m'	-28.70	-30.00	-27.85
b'	-31.69	-30.74	-31.19

Magnetic Field as a function of Hall Voltage

We measured the Hall Voltage (V_H) by varying the magnetic field and conducted this experiment at three different pressures. The Hall field is related to the Hall Voltage that we measure by Eq. 4. As we expect, the relationship between Hall Voltage and magnetic field is approximately linear, with the exception of P=15 torr dataset, which was affected significantly by non-linearity and oscillations. The fitting coefficients are described in Table. 2 Using Eq.2,4, we can obtain a linear relationship between V_H and B, assuming that electrons are travelling perpendicular to the direction of B:

$$V_H = (4mmv)B \tag{14}$$

We show the linear plots for the magnetic field measured with the forward and reverse polarities combined. The non-zero value of the intercept compared to Eq. 14 results from the fact that measurements of the potential is relative.

Table 2. The linear regression coefficients for the hall voltage v.s. magnetic field relationship. Note that the P=15 torr dataset was measured when there was oscillations and striation in the plasma. Therefore there was a significant amount of nonlinearity in the B-V_H relationship, which suggests that the m,b values for P=15 torr is not a very accurate summary of the data.

P [torr]	15	20	25
m	-1.64	-3.70	-0.31
b	-31.21	-31.66	-9.45

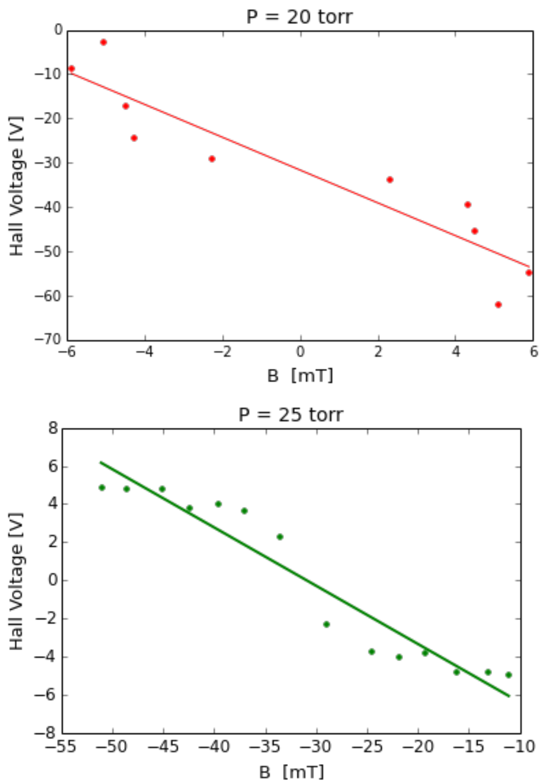

Figure 3. Plot of Hall Voltage versus Magnetic Field at P = 20 and 25 torr with the forward and reverse polarities combined. The Hall voltage of the reverse field is flipped to a positive sign.

Plasma Parameters

Using the relations derived in Sec. , we are able to compute the plasma parameters (v, n_e, ν, $<\sigma v>$,v_{rms}, $<v>$, T_e) based on the experimental parameters P, B, I_D and the experimental measurements of V_D, V_H. The plasma parameters that we obtained are summarized in Table. **??**

CONCLUSION

Acknowledgments

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4. Raoul Franklin. 1976. *Plasma Phenomena in Gas Discharges*.
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Table 3. The— of three selected datasets.

P [torr]	25	20	15
I_D[mA]	0.57±0.02	0.9±0.02	1.61±0.02
B [mT]	-16.3±0.1	-28.9±0.1	-21.8±0.1
V_H [V]	-4.8±0.2	-2.3±0.2	-4.8±0.2
V_D[V]	-62±0.2	-52±0.2	-39±0.2
$n_e[m^{-3}]$	9.6×10^{14}	5.6×10^{15}	3.63×10^{15}
$\nu[s^{-1}]$	1.97×10^{9}	5.89×10^{9}	1.66×10^{9}
$v_{rms}[m/s]$	7.81×10^{5}	2.86×10^{5}	2.61×10^{5}
$T_e[K]$	21100	16300	13400