Donald A. Glaser
Advanced Lab

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# OPT Pre-Lab

**INCLUDE THIS SHEET AS THE FIRST PAGE OF YOUR REPORT.**

Student's Name: Jung Lin (Doris) Lee

Partner's Name: Xiyue (Sissi) Wang

## Before the 1st Day of Lab [1]

## Pre-lab Questions and Sign Off Sheet

It is your responsibility to discuss this lab with an instructor on the first day of your scheduled laboratory period. This signed sheet must be included as the first page of your report. Without it you will lose 1/3 of a letter grade. You should think about and be prepared to discuss at least the following questions before you come to lab:

1. What is the general principle of optical pumping? Go over your derivation of the Breit-Rabi formula and the values of the Lande g-factors of the hyperfine energy levels of $^{85}$Rb and $^{87}$Rb. Draw qualitative energy-level diagrams for $^{85}$Rb and $^{87}$Rb showing the fine, hyperfine, and Zeeman splittings. How do the Lande g-factors affect the ordering of the Zeman levels? Show the transitions between these levels that are important to this experiment. Include these drawings in your write-up. For our rubidium system, what is the pumping process? Where is the pumped level? Where is the RF transition?
2. Why do we modulate (vary sinusoidally) the external magnetic field? How would we take data if the magnetic field were not modulated?
3. In this experiment, how will you determine the resonance frequency? How can you best estimate the error? Will the modulation amplitude affect your result? What data will you take, and what plots will you make?

Staff Signature ______________________ Date 2/12/16

Completed *before the first* day of lab? (circle) Yes / No

## Mid-lab Questions and Sign Off Sheet

On day 2 of this lab, you should have successfully produced a plot of frequency versus current for at least one rubidium isotope, and have made an estimate of the earth's magnetic field. Show them to an instructor and ask for a signature.

Staff Signature ______________________ Date 2/16/16

Completed on the *second* day of lab? (circle) Yes / No

**Please also fill out the Student Evaluation of Experiment [2].**

# Physics 111 : Optical Pumping

**Jung Lin (Doris) Lee [Partner: Xiyue Wang]**


## ABSTRACT

In this lab, we perform the technique of optical pumping to two species of Rubidium. Based on the ratio of obtained the resonance frequencies of Rb-85 and Rb-87, we determined their respective nuclear moments. Given these current measurements, we get a more precise numerical coefficient in the equation that relates to how the current generate magnetic field in the Helmholtz coil. We conduct a linear regression to fit the data to the Breit-Rabi equation with a p<0.01 goodness of fit. When we turn off the Helmholtz coil fields, we determine the magnetic field of the Earth as $75.354 \pm 9.72\mu T$. This field strength measurement demonstrates the applicability of the optical pumping technique as weak field magnetometers. Sources of error in our experiment include non-uniformity of the magnetic field from the Helmholtz coil and the uncertainty in the range of currents that yields a symmetrical Lissajous figure.


## INTRODUCTION

Optical pumping is a experimental technique that enable us to measure the splitting levels of atoms in a magnetic field. In addition to studying the science of energy levels as determined by degenerate perturbation theory of quantum mechanics, optical pumping has useful applications outside the laboratory as weak-field magnetometers on space satellites and atomic clocks of great precision [3].

In this experiment, we conduct optical pumping to experimentally determine the nuclear moments of the Rubidium isotopes in the gas bulb. This is achieved by sending a circularly polarized radio-frequency field (RF) to excite the atom from the $S_{1/2}$ ground state to the $P_{1/2}$ excited state. Spontaneous emmision causes the atoms to fall back down to the ground state, but with a 50% chance of landing on the $m_F = 1$ splitting level due to Zeeman effect of the magnetic field, due to selection rules in quantum mechanics. As we continue to excite the atoms up to the $P_{1/2}$ states, the atoms that failed to get onto the $m_F = 1$ state gets another chance and eventually the $m_F = 1$ state is populated with all the Rubidium atoms. Then using the obtained nuclear moment, we vary the frequency and the field strength to detect the resonant frequencies of these two samples. Using these field values, we can then obtain the magnitude of the Earth's magnetic field.

In this report, we present the experimental methods and analysis used to obtain the magnitude of the Earth's magnetic field and nuclear moments of Rb-85 and Rb-87. In section 2, I will present the theory behind energy level splitting and the physics of optical pumping. In section 3, I will detail the experimental setup and procedure of the experiment. Finally, in section 4, I will present our analysis of the experimental data and how we obtained the estimates of the Earth's magnetic field and errors propagated from our measurement results.

## THEORY

In degenerate perturbation theory of quantum mechanics, we can find the first order corrections to spin orbit coupling and relativistic correction, which together yields the fine structure correction to the eigenstate energies [2]. In addition, there is hyperfine splitting due to interactions of the nucleus of the Rb atom which generates its own electromagnetic fields that gives rise to the perturbation [1]. Additionally, in this experiment, we also apply an external magnetic field (in addition to the Earth's magnetic field) which results in Zeeman splitting in the case of the weak field. Using these derivation from perturbation theory, we obtain

$$E = h\nu = g_F \mu_B B$$

$$g_F = g_J \frac{F(F+1) + J(J+1) - I(I+1)}{2F(F+1)}$$

$$g_J = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

where g is the Lande g-factor. Plugging in the appropriate values J=1/2, S=1/2,L=0, F=1, we can derive the Breit-Rabi equation

$$\frac{\nu}{B_{ext}} = \frac{2.799}{2I+1} \frac{MHz}{gauss} \tag{1}$$

In Fig. 1, we can see that the fine structure, hyperfine, and Zeeman interactions splits the energy levels into j , f and $m_f$ levels respectively. The goal of the optical pumping is to "pump" the atoms into the m=1/2 state. This is achieved by sending in radio frequency field (RF) to induce transition from the ground state ($S_{1/2}$) to the $P_{1/2}$ state. Compared to the transition level energy difference, the splitting of magnetic field energies is smaller by many orders of magnitude smaller, so the spread of energy of the light is sufficient to bring the atom up to the $m_F$= 1 state [5]. Then the excited atoms undergoes spontaneous emission where they decay down to the $S_{1/2}$ state with a 50%-50% probability of landing on the $m_F$= 0 and 1 levels. The $m_F$= 0 atoms are again pumped up by the RF and then fall down to the $m_F$= 0 and 1 levels. As many iterations of this cycle occurs, the atoms are "stacked" up to the $m_F$= 1 level and we have successfully pumped all the atoms onto that level.

## APPARATUS AND PROCEDURE

### *Instrumentation*

Most of the experimental apparatus is placed inside a metal box so that the heating can be directly applied to Rubidium atoms. There is a circular polarizer in front of the light source to ensure that the angular momentum of the incident light is $\hbar$, since circularly polarized light can only change the angular momentum by $\Delta m = 1$. This circularly polarized light passes through the Rubidium gas bulb. The Rubidium gas bulb

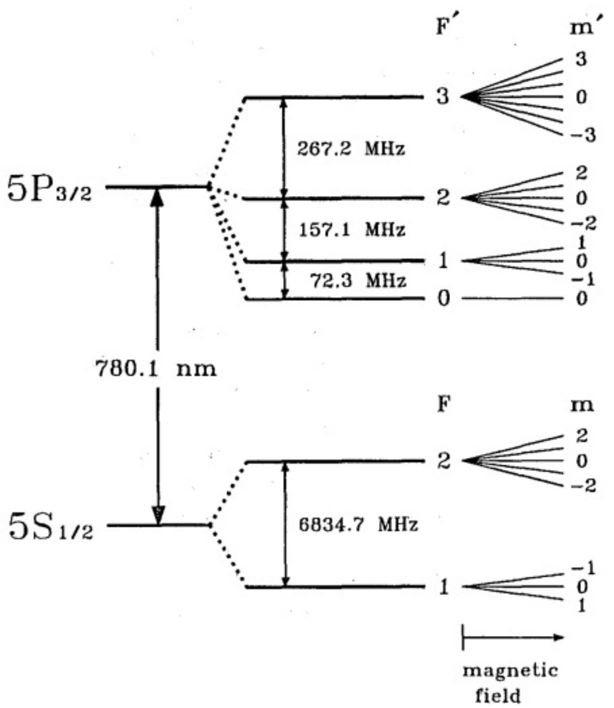



**Figure 1. Energy levels of a Rubidium atom showing the nuclear states, hyperfine and Zeeman splittings. The difference in energy level of each interaction is many orders of magnitude smaller than the energy difference resulting from the interaction on its left. (Image Source: The Optical Society)**

contains both Rb-85 and Rb-87 isotopes which have nuclear moments of I = 5/2 and I=3/2 respectively. The magnetic field given by the Helmholtz coil depends on how much current is passing through the coils, a variable that we are varying in our experiment, as shown by Eq.2.

$$B_H = 0.9 \times 10^{-6} \frac{T \cdot m}{A} \frac{Ni}{a} \tag{2}$$

### *Procedures*

First, we heat up the sample and make sure that it does not exceed [1]. Using the settings given on the lab manual, we additionally sweep through a freqeuncy range adjust the resolution settings until we see a resonance curve on the oscilloscope. Then, we switch the oscilloscope from the default time series mode to the X-Y mode to observe a Lissajous figure. There may be minor fluctuations to the signal due to temperature fluctuations.We measure the minimum and maximum current value ($I^+$, $I^-$) that still yields a symmetric Lissajous curve and make this the error range of our current measurement, since within this range, we can not determine what is the best value corresponding to the resonant frequency. The errors determined by the $I^+$, $I^-$ ranges is propagated through our subsequent analysis in Sec. 4 and to get an error estimate on the final computed value of Earth's magnetic field.

We repeat this procedure to obtain more measurements by setting a different magnetic field strength (by changing the current) and varying the frequency to find the new resonant frequency. Then we change the direction of the magnetic field and take another set of measurements. To find the resonant frequency for the other species, we need to change the frequency and make sure that the resonance occur at a different frequency as the first species that we were measuring and again repeat the data measurement procedure for this species.

[1]Contrary to the temperature sensor's display, the temperature is measured in Fahrenheits rather than Celsius.

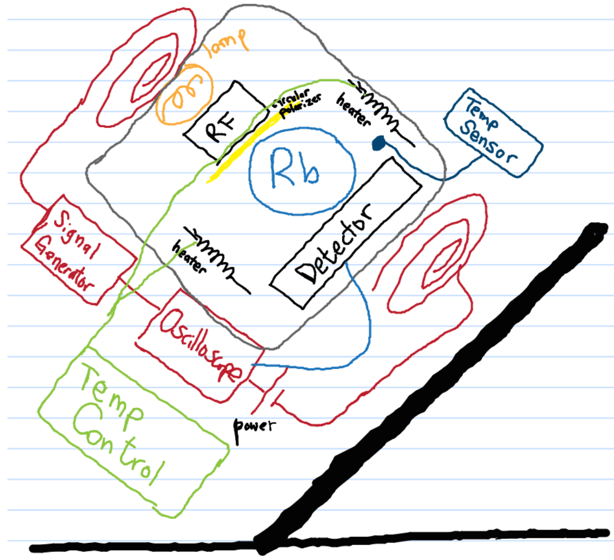



**Figure 2. Optical pumping experimental setup. Helmholtz coil and the bulk of the experiment is tilted so that the magnetic field vector adds with the Earth's magnetic field.**

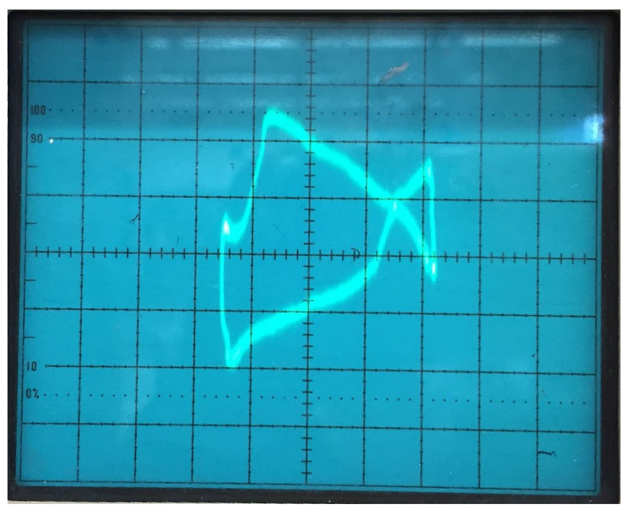


**Figure 3. An example of a assymetrical Lissajous figure measured at zero current.**

## ANALYSIS

### *Determining Nuclear Moments of Observed Species*

By taking the ratio of the Breit-Rabi equation for the two species:

$$\frac{\frac{\nu_1}{B_1} = \frac{2.799}{2I_1+1}}{\frac{\nu_2}{B_2} = \frac{2.799}{2I_2+1}} = \frac{2I_2+1}{2I_1+1}$$

$$\frac{\nu_1}{\nu_2} = \frac{B_1}{B_2}\left(\frac{2I_2+1}{2I_1+1}\right)$$

We chose the “best" value of $\frac{\nu_1}{\nu_2}$ as one and this yielded a 2I+1 ratio of about 2/3. Since we know that nuclear moments must be half integer values, we can confidently round up our result to the closest half integer. We can also deduced that if Species 1 was Rb-85 (I=5/2) and Species 2 was Rb-87 (I = 3/2) [4], then we would get the appropriate 2I+1 ratio. The errors on these deduced values are accurate to 3 significant figures, because the only uncertainty is on the numerical coefficient 2.799.

Knowing the $I_1$ and $I_2$ values, we can use the Breit-Rabi equation (Eq.1) to compute the magnetic field and compare this with the one that we obtain from Eq.2. For species 1, the mean square difference between the two measures of the magnetic field is 0.0915 and for species 2 the mean squared difference is 0.0973. This value gives us an estimate of the systematic error of the possible non-uniformity of the Helmholtz coil number and radius. Using this technique, we were able to obtain a more accurate estimate of the numerical coefficient in Eq.2 of $9.963 \times 10^{-7}$.

*Data Fitting and Transformation*

We perform a linear regression using Python’s `polyfit` function on the data using the model obtained from rearranging the Breit-Rabi equation into the linear form (y = ax+b), where the frequency is the dependent variable and the magnetic field strength is the independent variable.

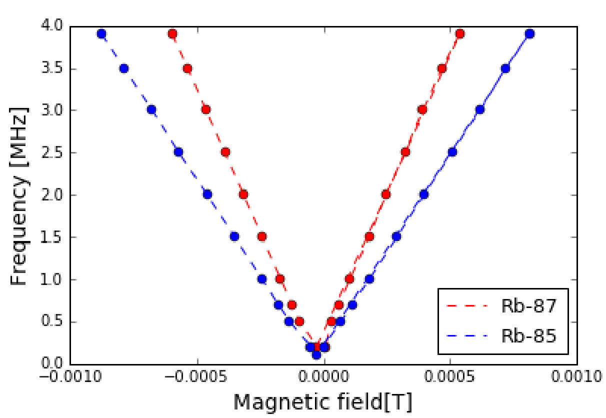



**Figure 4. Experimental data of magnetic field strength of Helmholtz coil versus resonant frequency before data transformation.**

In order to decrease the error on the parameters for linear regression, we flipped the negative data along the x axis so that we could perform two linear fits so that we have double the size of the sample. The error estimates on the parameters can be obtained from the derivation of the maximum log-likelihood estimator on a linear regression model [6]:

$$\sigma_a^2 = S_{xx}/\triangle \; \sigma_b^2 = S/\triangle \qquad (3)$$

where $\triangle = SS_{xx} - (S_x)^2$,$S_{xx} = \sum_{i=1}^{N} \frac{x_i^2}{\sigma_i^2}$, $S_x = \sum_{i=1}^{N} \frac{x_i}{\sigma_i^2}$, $S = \sum_{i=1}^{N} \frac{1}{\sigma_i}$. With this error estimate, we summarize the fitting co-

**Table 1. Fitting coefficients on the experimental data, where a is the slope and b is the y-intercept.**

| Species | a | b | $\sigma_a$ | $\sigma_b$ |
|---|---|---|---|---|
| Rb-85 | 4644.29 | 0.1342 | $1.091 \times 10^{-6}$ | 0.231 |
| Rb-87 | 6983.93 | 0.2069 | $1.201 \times 10^{-6}$ | 0.573 |

efficients and their respective errors in Table. 1. We obtained a chi squared value of 3.571 for Rb-85 and 3.724 for Rb-87. The chi squared goodness-to-fit test shows that the experimental values are very close to the modelled values (p<0.01) and with the values of nuclear spins found in our analysis.

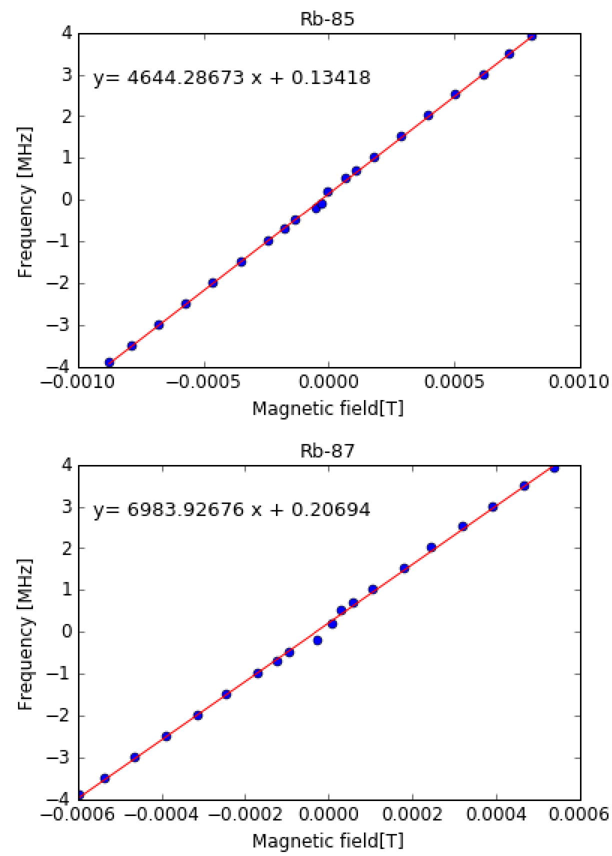



**Figure 5. This is the linear regression on the experimental data for Rb-85 and Rb-87. The error bar is too small compared to the pixels spanned by the datapoint, so it can not be seen on this plot.**

*Estimating Earth’s magnetic field*

At the zero current point, the magnetic field is zero, so there is no magnetic field contribution from the Helmholtz coil so $B_{total} = B_{earth}$. This could also be thought of as the y intercept of the linear regression:

$$B_H = \frac{(2I+1)}{2.799}\nu - B_e$$

. From the two species, we obtained two estimates of the total magnetic field as $59.28 \pm 9.151\mu T$ for species 1 and $91.428 \pm 9.72\mu T$, yielding a average value o f$75.354 \pm 9.72\mu T$ . The actual magnetic field strength in Berkeley is $48.6\mu T$,according to Wolfram Alpha. This discrepancy can not be completely accounted for by the error on our recorded measurement. Possible sources error may include instrumentation systematics, non-uniformity of the magnetic field and electronics reading noise.

## CONCLUSION

In this experiment, we performed the technique of optical pumping to populate $m_F = 1$ level of the Rubidium atom. We obtained the resonance frequencies of the two species and used their ratios to determine the nuclear moments of Rb-85 (I=5/2) and Rb-87(I=7/2). Given these current measurements, we also able to attain a more precise numerical coefficient in the current-field relation of $B_H = 9.963 \times 10^{-7} \frac{T \cdot m}{A} \frac{Ni}{a}$.We perform a linear regression to fit the data to the Breit-Rabi equation with a p<0.01 goodness of fit. Using the y-intercept of the magnetic field versus frequency plot, we can determine the magnetic field of the Earth as $75.354 \pm 9.72 \mu T$. This field strength measurement demonstrates the applicability of the optical pumping technique as weak field magnetometers. Sources of error in our experiment include non-uniformity of the magnetic field from the Helmholtz coil and the uncertainty in the range of currents that yields a symmetrical Lissajous figure. As we can see in Fig. 5, the residual (deviation from the regression) is largest near values close to zero, if we were given more time on this experiment, possible future directions includes taking more measurements in regions that are close to zero current, to better constrain the fitting coefficients and therefore obtain a more precise measurement of the Earth's magnetic field.

### Acknowledgments

I am sincerely thankful for support from Professor Harmut Haeffner, Kam-Biu Luk, Don Orlando, and my lab partner Xiyue Wang for contributing to successful completion of this lab.

## REFERENCES

1. 2016. Hyperfine Structure. *Wikipedia* (2016).
2. 2016. Optical Pumping. *Wikipedia* (2016).
3. Thomas R. Carver. 1963. Optical Pumping. *Science* (1963).
4. S. Millman and M. Fox. 1936. Nuclear Spins and Magnetic Moments of $Rb^{85}$ and $Rb^{87}$. *Phys. Rev.* 50 (Aug 1936), 220–225. Issue 3. DOI: http://dx.doi.org/10.1103/PhysRev.50.220
5. Paschotta. 2016. Encyclopedia of Laser Physics and Technology - optical pumping, pump light, laser crystal, optically pumped lasers, pump absorption, in-band pumping. (2016). https://www.rp-photonics.com/optical_pumping.html
6. William H Press. 1988. *Numerical recipes in C*. Cambridge University Press.

# Optical Pumping Data Analysis

March 12, 2016

```
In [9]: %pylab inline
        def fit_and_plot(x,y,xlabel="",ylabel="",title="",zeroed=False,annotate_fit= True,right_words =
            fig = plt.figure()
            ax1 = fig.add_subplot(111)
            ax1.plot(x,y,'{}'.format(marker))
            z = np.polyfit(x,y, 1)
            p = np.poly1d(z)
            if zeroed :
                a = np.linspace(0,max(x))
            else:
                a = np.linspace(min(x),max(x))
            ax1.plot(a, p(np.linspace(min(x),max(x))),color="red")
            if annotate_fit:
                slope = z[0]
                intercept = z[1]
                if right_words:
                    ax1.text(0.48,0.85,"y= %.5f x + %.5f"%(slope,intercept), fontsize=13,transform=ax1.t
                else:
                    ax1.text(0.03,0.85,"y= %.5f x + %.5f"%(slope,intercept), fontsize=13,transform=ax1.t
            if title !="":
                plt.title(title,fontsize=13 )
            plt.xlabel(xlabel,fontsize=12)
            plt.ylabel(ylabel,fontsize=12)
            if annotate!="":
                if right_annotate:
                    ax1.text(0.48,0.85,annotate, fontsize=13,transform=ax1.transAxes)
            if error_bar!="":
                ax1.errorbar(x, y, yerr=error_bar, fmt='o')
            if sci_lim:
                plt.ticklabel_format(style='sci', axis='x', scilimits=(0,0))
            plt.tick_params(axis='both', which='major', labelsize=12)
            plt.tick_params(axis='both', which='minor', labelsize=12)
            return p
```

```
Populating the interactive namespace from numpy and matplotlib
```

```
In [14]: freq_pos = np.array([3.904,3.500,3.000,2.500,2.000,1.500,1.000,0.7,0.5,0.2])
         I_pos = np.array([1.84,1.63,1.40,1.15,0.9,0.66,0.41,0.26,0.16,0.0])
         freq_neg = np.array([3.900,3.500,3.000,2.500,2.000,1.500,1.000,0.7,0.5,0.2,0.1])
         I_neg = -np.array([1.98,1.78,1.53,1.29,1.04,0.79,0.55,0.40,0.30,0.11,0.06])
         I_mega = list(I_neg)+list(I_pos)
         freq_mega = list(freq_neg)+list(freq_pos)
         freq2_neg = np.array([3.900,3.500,3.000,2.500,2.000,1.500,1.000,0.7,0.5,0.2])
         I2_neg = -np.array([1.35,1.21,1.05,0.88,0.71,0.55,0.38,0.28,0.21,0.06])
```

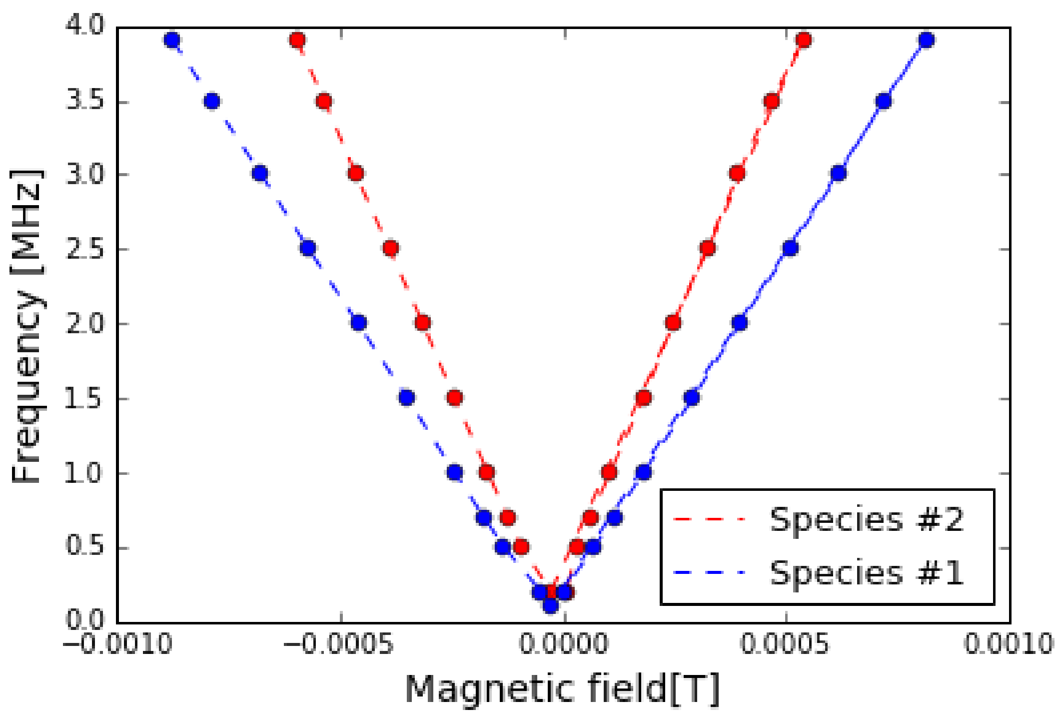
```
freq2_pos = np.array([3.904,3.500,3.000,2.500,2.000,1.500,1.000,0.7,0.5,0.2])
I2_pos = np.array([1.22,1.06,0.89,0.73,0.56,0.41,0.24,0.14,0.07,0.02])
I2_mega = list(I2_neg)+list(I2_pos)
freq2_mega = list(freq2_neg)+list(freq2_pos)
```

In [11]:
```
def I_to_B(I):
    N = 135 #turns
    a = 0.275 #meters
    return 0.9e-6 * N*np.array(I)/a
```

In [12]:
```
from matplotlib.legend_handler import HandlerLine2D
N = 135 #turns
a = 0.275 #meters
B_mega = I_to_B(I_mega)
B2_mega = I_to_B(I2_mega)

plt.plot(B2_mega,freq2_mega,'o',color = "red")
plt.plot(B2_mega,freq2_mega,'--',color = "red",label = "Species #2 ")
plt.plot(B_mega,freq_mega,'o',color = "blue")
plt.plot(B_mega,freq_mega,'--',color = "blue",label = "Species #1 ")
plt.legend(loc='lower right',prop={'size':13},numpoints=1)
plt.title("",fontsize=14)
plt.xlabel("Magnetic field[T]" ,fontsize=14)
plt.ylabel("Frequency [MHz]",fontsize=14)
```

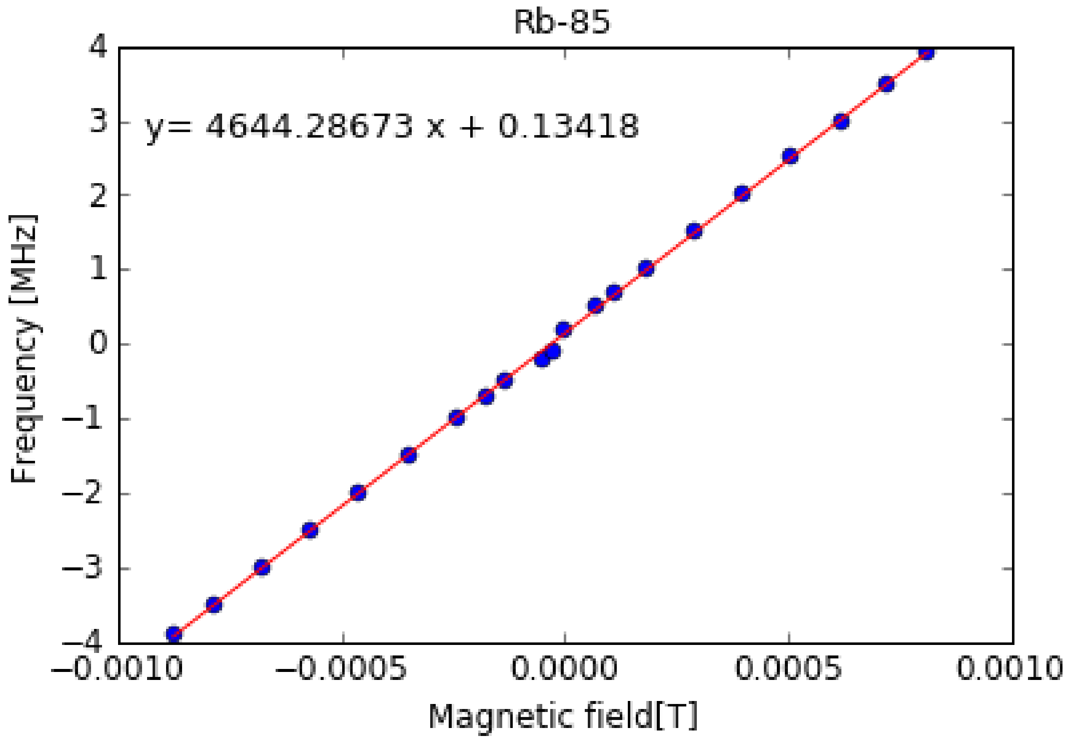
Out[12]: <matplotlib.text.Text at 0x7f1f17b55490>

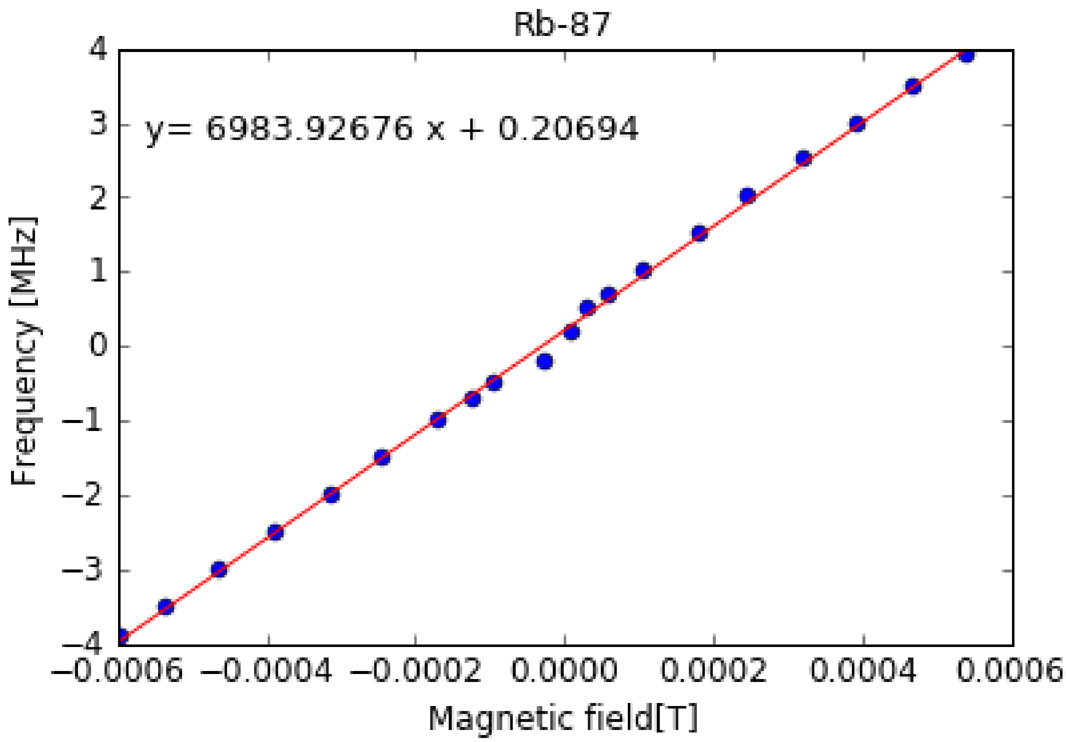



In [88]:
```
I_mega_line = list(I_neg)+list(I_pos)
freq_mega_line = list(-freq_neg)+list(freq_pos)
```

```
I2_mega_line = list(I2_neg)+list(I2_pos)
freq2_mega_line = list(-freq2_neg)+list(freq2_pos)

spec1_fit = fit_and_plot(I_to_B(I_mega_line),freq_mega_line,xlabel="Magnetic field[T]",ylabel=
spec2_fit = fit_and_plot(I_to_B(I2_mega_line),freq2_mega_line,xlabel="Magnetic field[T]",ylabel
```






### 0.0.1 Estimate of the Earth's magnetic field

At the zero current point, the magnetic field is zero and have no magnetic field contribution from the Helmholtz coil so $B_{total} = B_{earth}$. This could also be thought of as the y intercept of the linear regression:

$$B_H = \frac{(2I+1)}{2.799}\nu - B_e$$

.

```
In [21]: print  0.9e-6 * N*spec1_fit[0]/a ,"teslas"
         print  0.9e-6 * N*spec2_fit[0]/a ,"teslas"
```

```
5.92823442548e-05 teslas
9.1428498284e-05 teslas
```

The actual total magnetic field in Berkeley is 48.6 $\mu T$

In order to decrease the error on the parameters for linear regression (y = ax+b), we flipped the negative data along the x axis so that we could perform two linear fits so that we have double the size of the sample.

$$I = \frac{1}{2}\left(\frac{2.799}{a} - 1\right)$$

```
In [25]: spec1_fit[1]
```

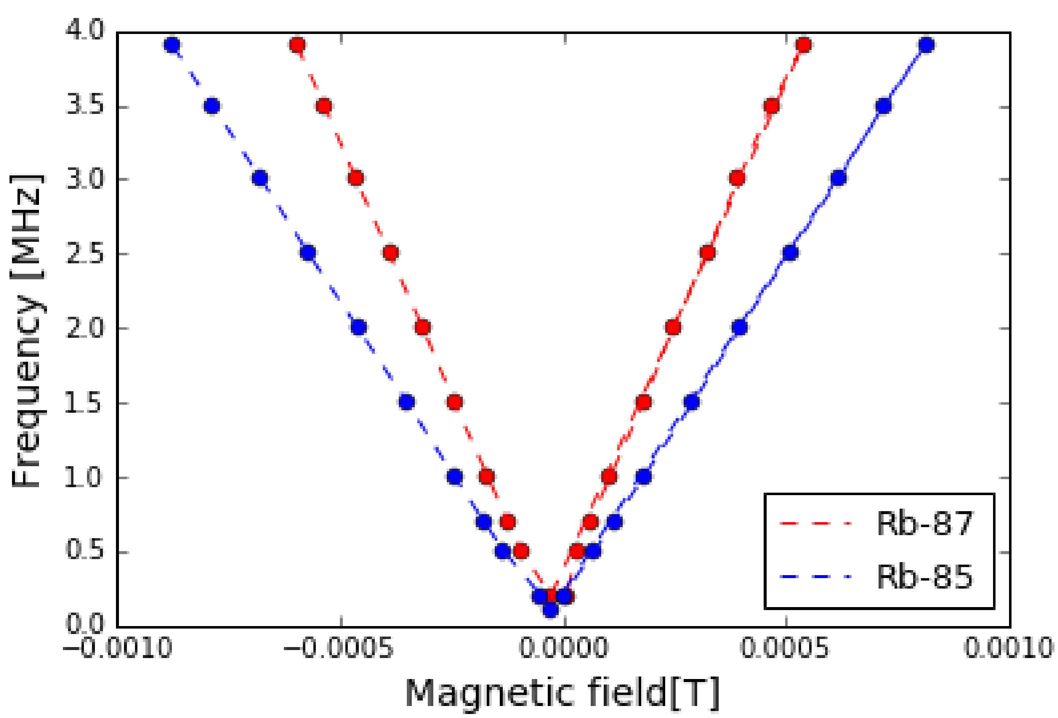
```
Out[25]: 4644.2867320091918
```

```
In [24]: print 0.5*( 2.799/spec1_fit[1] -1 )
         print 0.5*( 2.799/spec2_fit[1] -1 )
```

```
-0.499698662016
-0.499799611301
```

```
In [83]: plt.plot(B2_mega,freq2_mega,'o',color = "red")
         # plt.plot(B2_mega,freq2_mega,'--',color = "red",label = "Species #2 ")
         plt.plot(B2_mega,freq2_mega,'--',color = "red",label = "Rb-87")
         plt.plot(B_mega,freq_mega,'o',color = "blue")
         # plt.plot(B_mega,freq_mega,'--',color = "blue",label = "Species #1 ")
         plt.plot(B_mega,freq_mega,'--',color = "blue",label = "Rb-85")
         plt.legend(loc='lower right',prop={'size':13},numpoints=1)
         plt.title("",fontsize=14)
         plt.xlabel("Magnetic field[T]" ,fontsize=14)
         plt.ylabel("Frequency [MHz]",fontsize=14)
```

```
Out[83]: <matplotlib.text.Text at 0x7f1f175ab8d0>
```




```
In [28]: np.array(freq_mega[:-1])/np.array(freq2_mega)
```

```
Out[28]: array([ 1.        ,  1.        ,  1.        ,  1.        ,  1.        ,
                 1.        ,  1.        ,  1.        ,  1.        ,  1.        ,
                 0.02561475,  1.11542857,  1.16666667,  1.2       ,  1.25      ,
                 1.33333333,  1.5       ,  1.42857143,  1.4       ,  2.5       ])
```

```
In [30]: 1./(B_mega[:-1]/B2_mega)
```

```
Out[30]: array([  0.68181818,   0.67977528,   0.68627451,   0.68217054,
                  0.68269231,   0.69620253,   0.69090909,   0.7       ,
                  0.7       ,   0.54545455, -20.33333333,   0.57608696,
                  0.54601227,   0.52142857,   0.48695652,   0.45555556,
                  0.36363636,   0.34146341,   0.26923077,   0.125     ])
```

```
In [32]: 2/3.
```

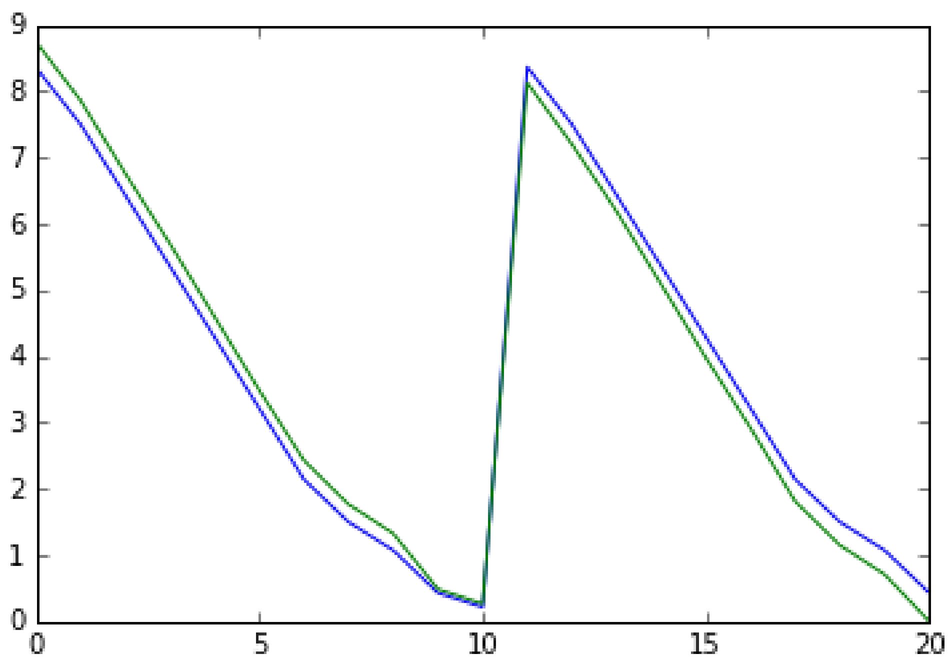
```
Out[32]: 0.6666666666666666
```

For species 1 :

```
In [68]: I = 2.5
         B_guess = (2*I+1)/2.799*np.array(freq_mega)-48.6e-6
         B_actual = abs(B_mega)*1e4
```

```
In [69]: plt.plot(B_guess)
         plt.plot(B_actual)
```

```
Out[69]: [<matplotlib.lines.Line2D at 0x7f1f1773b310>]
```




```
In [70]: mean((B_guess-B_actual)**2)
```

```
Out[70]: 0.091511448991768482
```

For species 2 :

```
In [79]: I = 1.5
         B2_guess = (2*I+1)/2.799*np.array(freq2_mega)-48.6e-6
         B2_actual = abs(B2_mega)*1e4
         # plt.plot(B_guess)
         # plt.plot(B_actual)

         mean((B2_guess-B2_actual)**2)
```

```
Out[79]: 0.097268324274124823
```

```
In [82]: (B_guess*1e-4*a/N/I_mega)[:-1]
```

```
/anaconda/lib/python2.7/site-packages/IPython/kernel/__main__.py:1: RuntimeWarning: divide by zero encoun
  if __name__ == '__main__':
```

```
Out[82]: array([ -8.60090537e-07,  -8.58603207e-07,  -8.56197231e-07,
                 -8.46240170e-07,  -8.39728727e-07,  -8.29096116e-07,
                 -7.93916342e-07,  -7.64137054e-07,  -7.27740146e-07,
                 -7.93844342e-07,  -7.27608146e-07,   9.26481477e-07,
                  9.37615772e-07,   9.35701260e-07,   9.49260713e-07,
                  9.70353195e-07,   9.92402927e-07,   1.06500973e-06,
                  1.17559547e-06,   1.36451277e-06])
```

```
In [80]: mean(B2_guess*1e-4*a/N/I_mega[:-1])
```

```
Out[80]: -9.9626112557009977e-07
```

error on the fit:
$\sigma_a^2 = S_{xx}/\triangle$
$\sigma_b^2 = S/\triangle$

```
In [98]: S_x = sum(B_mega/0.001**2)
         S_xx = sum(B_mega**2/0.001**2)
         S = sum(1/0.001**2)
         delta = S*S_xx-S_x**2
         # print delta
         print "For species 1 : "
         print "error on fitted slope =", S_xx/delta
         print "error on intercept =", S/delta
```

```
For species 1 :
error on fitted slope = 1.09089761621e-06
error on intercept = 0.230934403605
```

```
In [100]: S_x = sum(B2_mega/0.001**2)
          S_xx = sum(B2_mega**2/0.001**2)
          S = sum(1/0.001**2)
          delta = S*S_xx-S_x**2
          # print delta
          print "For species 2 : "
          print "error on fitted slope =", S_xx/delta
          print "error on intercept =", S/delta
```

```
For species 2 :
error on fitted slope = 1.20071091637e-06
error on intercept = 0.572630173934
```

```
In [110]: fc1  = np.poly1d([4644.29, 0.1342])
          fc2  = np.poly1d([ 6983.93,0.2069])
          print sum((fc1(B_mega) - freq_mega)**2/freq_mega)/len(freq_mega)
          print sum((fc2(B2_mega) - freq2_mega)**2/freq2_mega)/len(freq2_mega)
```

```
3.57085081353
3.72395743657
```

```
In [ ]:
```

2/16 OPT

| | | | | |
|---|---|---|---|---|
| rf / MHz | 3.904 | 3.504 | 3.000 | 2.500 |
| Dresonance I/Amp | 1.84 | 1.05 | 1.40 | 1.15 |
| $I_+$ | 1.86 | 1.09 | 1.41 | 1.16 |
| $I_-$ | 1.83 | 1.05 | 1.40 | 1.15 |
| T(F) | 97.0 | 100.6 | 99.8 | 99.4 |

resonance I

| | | | | |
|---|---|---|---|---|
| rf | 2.000 | 1.500 | 1.500 | 1.000 |
| I | 0.90 | 0.66 | 0.65 | 0.41 |
| $I_+$ | 0.91 | 0.66 | 0.66 | 0.41 |
| $I_-$ | 0.89 | 0.65 | 0.65 | 0.40 |
| T | 99.2 | 98.9 | 108.9 | 105.6 |

| | | | | |
|---|---|---|---|---|
| rf | 0.500 | 0.200 | 0.700 | 1.200 |
| I | ~~0.500~~ 0.16 | | 0.14 | 0.31 |
| $I_+$ | 0.16 | zero current | 0.14 | 0.32 |
| $I_-$ | 0.15 | | 0.14 | 0.31 |
| T | 104.1 | 99.9 | 99.3 | 98.1 |

| | | | |
|---|---|---|---|
| rf | 0.700 | 3.500 | ~~4.500~~ |
| I | 0.26 | 1.64 | |
| $I_+$ | 0.26 | 1.64 | |
| $I_-$ | 0.26 | 1.63 | |
| T | 107.1 | 102.2 | |

"reverse" light on
negative B field direction

| rf | I | $I_+$ | $I_-$ | T |
|---|---|---|---|---|
| 3.900 | 1.98 | 1.98 | 1.97 | 100.4 |
| 3.500 | 1.78 | 1.78 | 1.77 | 100.0 |
| 3.000 | 1.53 | 1.53 | 1.52 | 99.8 |
| 2.500 | 1.29 | 1.29 | 1.28 | 106.4 |
| 2.000 | 1.04 | 1.05 | 1.03 | 105.1 |
| 1.500 | 0.79 | 0.80 | 0.79 | 104.0 |
| 1.000 | 0.55 | 0.56 | 0.55 | 101.1 |
| 0.700 | 0.40 | 0.41 | 0.40 | 100.6 |
| 0.500 | 0.30 | 0.31 | 0.30 | 100.3 |
| 0.200 | 0.11 | 0.12 | 0.11 | 99.8 |
| 0.100 | 0.06 | 0.07 | 0.06 | 99.4 |
| 0.050 | | | | |

other species
still negative B field

| I | $I_+$ | $I_-$ | T |
|---|---|---|---|
| 1.35 | 1.36 | 1.35 | 110.6 |
| 1.21 | 1.22 | 1.20 | 107. |
| 1.05 | 1.06 | 1.03 | 105.6 |
| 0.88 | 0.89 | 0.87 | 104.9 |
| 0.71 | 0.72 | 0.71 | 104.1 |
| 0.55 | 0.55 | 0.55 | 103.3 |
| 0.38 | 0.39 | 0.38 | 108.3 |
| 0.28 | 0.29 | 0.28 | 105.0 |
| 0.21 | 0.22 | 0.21 | 103.7 |
| 0.06 | 0.06 | 0.06 | 103.0 |

2nd species, reverse light off.

| rf | I | $I_+$ | $I_-$ | T |
|---|---|---|---|---|
| 3.900 | 1.22 | 1.22 | 1.20 | 106.3 |
| 3.500 | 1.06 | 1.07 | 1.05 | 101.7 |
| 3.0 | 0.89 | 0.9 | 0.89 | 108.2 |
| 2.5 | 0.73 | 0.74 | 0.73 | 111.5 |
| 2.0 | 0.56 | 0.57 | 0.56 | 110.0 |
| 1.5 | 0.40 | 0.41 | 0.40 | 107.8 |
| 1.0 | 0.24 | 0.25 | 0.24 | 105.5 |
| 0.7 | 0.14 | 0.15 | 0.14 | 104.5 |
| 0.5 | 0.07 | 0.08 | 0.07 | 103.2 |
| 0.2 | 0.02 | 0.02 | 0.01 | 109.3 |

lamp output at 25 milliamperes.