BROWNIAN MOTION IN CELLS

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ABSTRACT

In this lab, we investigate Einstein's prediction

1. INTRODUCTION

Brownian motion describes the small random motion of small particles due to their kinetic motion, —- Einstein's —- work was significant because it served as the one of the first relatively-macroscopic experiment that proved the atomic nature of matter.The theory of statistical random walk is also important because —- purely a statistical result— motion from matter composed of atmons to collide with each other Jean Perrin's experiment particle tracking script Then we observe an onion cell under the microscope to observe intracellular transport. We use the —- compare with the random walk Brownian motion.

2. THEORY

2.1. *Basics*

In Einstein's original derivation, he combined the equations that describes osmotic pressure with a series of equations that describes how particles moves through viscous fluids to calculate the theoretical value of D, where D is a dimensionless number called the diffusion coefficient, which describes the extent to which a particle is freely diffusing through the medium without any obstruction. (Einstein 1905)[1]

$$D = \frac{k_b T}{3\eta r} \tag{1}$$

where k_b is the Boltzman's constant, T is temperature, η is the viscosity of the solution and r is the radius of the particle. Despite the complex derivations, this formula makes intuitive sense in terms of the familiar kinetic molecular theory(KMT): as the temperature is higher or if the particle size is small, the velocity of the particles are higher, thus the particles collides more rigorously and frequently and more diffusion occurs; as the solutions gets more viscous, the solution effectively slows the particles down so less diffusion occurs.

Since Brownian motion is a series of random walks, it can be modelled as a normal distribution, which means that we can look at the denominator of its exponent to get the variance of the distribution.(Siegrist 2016) This mean squared displacement for a list of particle positions, either obtained through simulation or experiment, can also enable us to compute the value of D.

$$D = \sqrt{\langle |\vec{r}(t+\tau) - \vec{r}(t)| \rangle^2 / 2d\tau} \tag{2}$$

where d is the number of dimensions, τ is the sampling time (0.1 s)

2.2. *Simulation*

For the particle simulation, (using code provided in the worksheet) we first create a vector of random steps. We then compute the theoretical D using 1 to obtain an amplitude (k=$\sqrt{2dD\tau}$, where d = dimensions) for scaling up the random steps. Then using these simulated particle positions, we estimate the value of D using 2. Becuase this is a stochastic process, the error depends on $1/\sqrt{N}$, a more detailed error propogation yields the uncertainty quoted in the simulated D values We used a temperature of 20 degrees Celsius, comparable to our experimental conditions. There is some uncertainty in the temperature because we record only the ambient room temperature and neglect any external heating of the sample due to the microscope light source. However, effects of heating from the microscope should be minimized by our Kohler Illumination setup.

2.3. *Hypothesis*

We chose to conduct our experiment on 5 different samples, summarized in Table —- . Sample #2 and #3 enable us to test the effect of viscosity on the Brownian motion, while keeping the particle type and size the same. According to Eq.2 and our intuition from KMT, as the viscosity gets larger, there is less random motion and thus Sample #3 should have a lower value of D. Sample #2 and #4 enable us to compare the particle sizes, and we could similarly infer that Sample #4 will have a lower value of D. Sample #2 and #4 enable us to compare the particle type. While the particle type is not explicitly a variable inside Eq.2, we believe that since glycerol particles are more massive than PVP particle, it will result in slower random walk and therefore a lower value of D. The reasoning behind these experimental choices is that we are only changing one variable per comparison so that we know that the Sample #2 serve as a standard for comparison. We also chose particle sizes small enough in the range suggested by the lab worksheet so that Brownian motion can actually be observed.

3. APPARATUS AND PROCEDURE

In this experiment we use an inverted compound microscope that is connected to a CCD camera where the image processing is done by a given particle tracking script. Following the procedures:

1. We adjust our condenser irises and depth of field to achieve Kohler Illumination to control the amount of light and the incident angle that the light shines on the sample. This setup ensures that the image of the light source is not superposed onto the actual image of the specimen and provides uniform illumination and heating to the sample.

2. Then we adjust the condenser so that so that all the illuminating light is blocked off and we only see the light that scatters off the specimen. This technique is called darkfield illumination and it enables us to look at small particles ($\leq 1\mu m$ with the 40x objective) that would otherwise be hard to see with direct illumination.

[1] A more simplified derivation can be found in Newburgh et al. (2013).

3. To calibrate the microscope, we make a solution of 10μm microbeads and water to obtain a pixel-to-meters ratio used in the image-processing script. Since the pixel counting is an error-prone task, we counted the pixel of 3 particles and had both lab members count it, which yields a total of six values, then we averaged these to get an pixel to ratio conversion of 0.48 μm/pixel for the 20x objective and $0.28\mu m$/pixel for the 40x objective. (This makes intuitive sense because twice the magnification approximately yields half the pixel ratio.)

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4. ANALYSIS

5. CONCLUSION

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REFERENCES

A. Einstein, *Investigations on the theory of Brownian movement* (Dover, 1905).

K. Siegrist, *Standard brownian motion* (2016), URL http://www.math.uah.edu/stat/brown/Standard.html.

R. Newburgh, J. Peidle, and W. Rueckner, American Association of Physics Teachers **74**, 478 (2013).

M. Davidson, *Zeiss microscopy online campus microscopy basics kohler illumination* (2015), URL http://zeiss-campus.magnet.fsu.edu/articles/basics/kohler.html.

Wikipedia, *Kohler illumination* (2015).

Table 1
Experimental condition and results of all trials.

Trial	Solution Type	Solution viscocity [cp]	Microbead diameter [μm]	Simulation D	Experimental D
1	H_2O	1.00	0.47	$1.8184 \times 10^{-12} \pm 7.7701 \times 10^{-15}$	1.9464×10^{-12}
2	PVP	2.50	0.47	$7.2988 \times 10^{-13} \pm 3.2730 \times 10^{-15}$	1.1515×10^{-12}
3	PVP	4.65	0.47	$3.9260 \times 10^{-13} \pm 1.6801 \times 10^{-15}$	4.0377×10^{-13}
4	PVP	2.50	1.01	$3.3975 \times 10^{-13} \pm 1.4979 \times 10^{-15}$	4.3144×10^{-13}
5	glycerol	2.50	0.47	$7.3291 \times 10^{-13} \pm 3.2063 \times 10^{-15}$	5.0073×10^{-13}