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INTELLIGENT SYSTEMS

1º Semestre 2023/2024 MEMec, MEAer

Exercises

Fuzzy sets

1. What is the difference between a characteristic function and a membership function?
2. Consider the fuzzy set young described by:

   $A = 1.0/5 + 1.0/10 + 0.8/20 + 0.5/30 + 0.2/40 + 0.1/50,$

   and the fuzzy set B defined by the membership function:

   $$\mu_B(x) = \frac{1}{1+x^2}$$

   Determine, justifying:

   a) $support(A)$, $support(B)$

   b) $core(A)$, $core(B)$

   c) $A_{\alpha|\alpha=0.7}$, $B_{\alpha|\alpha=0.8}$;

   d) $A^{+}_{\alpha^{+}|\alpha=0.7}$, $B^{+}_{\alpha^{+}|\alpha=0.8}$.

3. Consider the set of pencils given by: X = {P1, P2, P3, P4, P5, P6}, and the fuzzy sets *long pencils* and *short pencils* described by fuzzy sets C and D, respectively:

   C = {0.1/P1, 0.2/P2, 0.4/P3, 0.6/P4, 0.8/P5, 1.0/P6}

   D = {1.0/P1, 0.6/P2, 0.4/P3, 0.3/P4, 0.1/P5}

   a) Determine the union of the two sets using the max and the probabilistic union;

   b) Determine the intersection of the two sets using the min and the product;

   c) Comment the results obtained in a) and b).

4. Consider two fuzzy sets A and B such that $core(A) \cap core(B) = \varnothing$. The fuzzy set $C = A \cap B$ can be normal? What is the necessary condition between the supports of A and B such that # $(C) > 0$? Justify your answers.

5. Consider the two fuzzy sets in the Universe of Discourse X ={−8, −6, −4, −2, 0, 2, 4, 6, 8}:

   $$\mu_A(x) = \frac{1}{1+|x|} \text{ and } \mu_B(x) = 1 - \frac{|x|}{20}$$

   a) Are the membership functions valid in the given Universe?

   b) Compute the α–cuts of A and B for $\alpha = 0.3$;

   c) Define the previous set of α–cuts of A:

      i) by enumerating its elements;

      ii) by a property of its elements;

iii) by using a membership function;

d) Using Zadeh's operators, compute $C = A \cap \bar{B}$ e $D = A \cap B$. Are C and D convex sets? Justify.

6. Prove that Morgan's law $\overline{(A \cup B)} = \bar{A} \cap \bar{B}$ is valid for Zadeh's operators (union, intersection and complement).

7. What is a linguistic modifier? Given a fuzzy set A, how can '*more or less A*' and '*very A*' be computed?

Fuzzy relations

8. Compute the cylindrical extension of the fuzzy set $A = 0.2/x_1 + 0.8/x_2$ to $X \times Y$, with $X = \{x_1, x_2\}$ and $Y = \{y_1, y_2, y_3\}$.

9. Consider the fuzzy relation $\mathcal{R}$ given by:

	y_1	y_2	y_3	y_4
x_1	0.8	0.9	0.6	0.1
x_2	0.2	0.4	0.7	0.8
x_3	0.1	0.2	0.5	0.2

Obtain the projections of $\mathcal{R}$ onto X and Y respectively.

10. Consider that the fuzzy relation in 9 corresponds to "*x* is considerably bigger than *y*". Consider also the fuzzy set expressing "*x* is small" given by $A = \{0.3/x_1, 1/x_2, 0.8/x_3\}$. Obtain the value of the expression "*x* is considerably bigger than *y* **and** *x* is small".

11. Given an n^{th}-order relation, how many projections of this relation exist?

12. Consider the definition of Takagi-Sugeno fuzzy models.

    a) Define a type-zero and a type-one Takagi-Sugeno models.

    b) What is an affine Takagi-Sugeno model?

    c) Describe briefly the necessary steps to derive a Takagi-Sugeno model of a system from numeric data.

13. Consider the following fuzzy rules:

    1) If x is A_1 and y is B_1 then $z = c_1$.
    2) If x is A_2 and y is B_2 then $z = c_2$.
    3) If x is A_3 and y is B_3 then $z = c_3$.
    4) If x is A_4 and y is B_4 then $z = c_4$.

    How to compute the global output z?

14. Consider the following fuzzy inference system:

    1. If *x* is Small then *y* is Big
    2. If *x* is Medium then *y* is Small
    3. If *x* is Big then *y* is Medium

    a) Describe the basic steps of Mamdani inference.

    b) Use Fig. 1 below to describe these steps for an input $x = 6$.

    c) Compute the defuzzified output using the center of gravity method, when the domain (Universe of Discourse) of the output is = $\{0, 1, 2, 3, 4, 5, 6, 7, 8\}$.

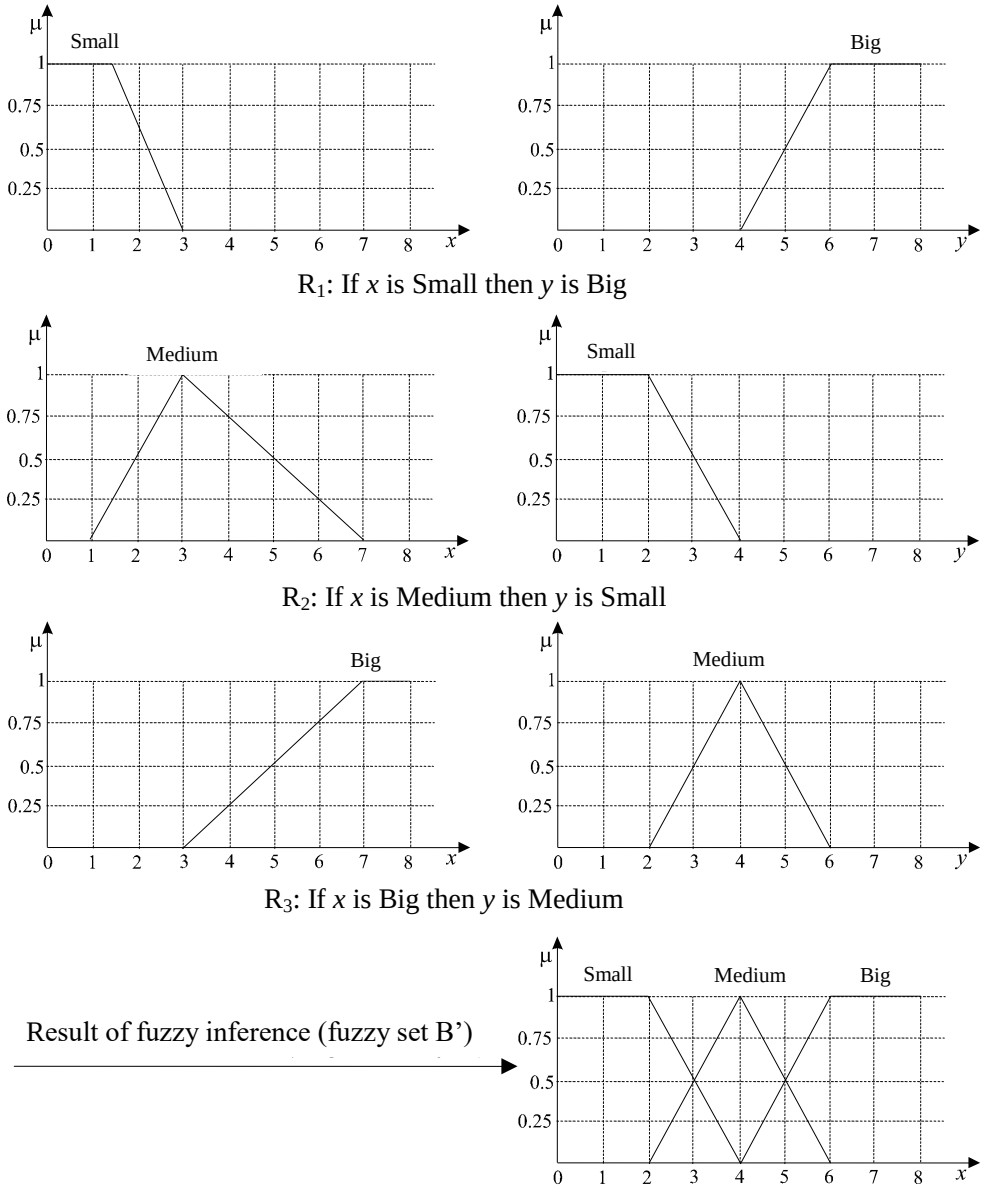

Figure 1. Mamdani inference.

15. What is "fuzzy clustering"? What is the difference from fuzzy clustering to non-fuzzy data clustering? Describe the mechanism of fuzzy clustering, and give examples of some possible applications.

16. What are the similarities and differences between the algorithms: fuzzy c-means, Gustafson-Kessel, and possibilistic clustering?

17. Describe how to use clustering algorithms to derive nonlinear models. Give a simple example on how to identify a model based on input/output data of a given system.

18. Consider a zero-order Takagi Sugeno (singleton) model with the following rules:

    1) If x is *Small* then $y = b_1$,
    2) If x is *Big* then $y = b_2$,

    and the membership functions given in Fig. 2. Consider also that:

    $x_1 = 1,\ y_1 = 3$

$x_2 = 5,\ y_2 = 4.5$

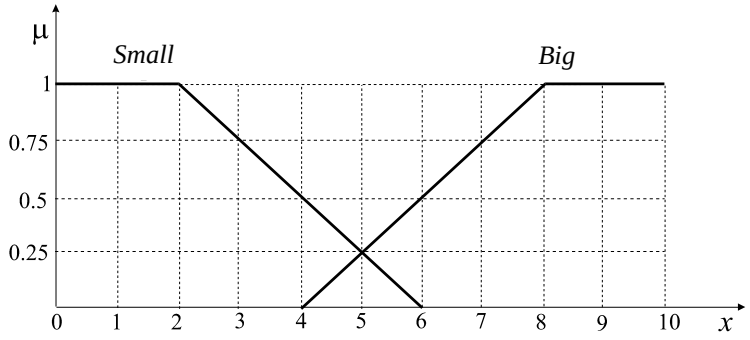

Figure 1. Membership functions.

Compute the consequent parameters b_1 and b_2 such that the model has the minimum squared error using the given data. What is the value of this error?

19. Fuzzy system can be interpretable. However, data based models can have redundancy problems. How can these problems be solved?

20. a) Draw an artificial neuron (perceptron) with three inputs. Define the meaning of all the variables. What is the input/output relation of the neuron?

    b) Draw a multilayer perceptron with three inputs, one output and a hidden layer with three neurons. What are the parameters that are adjusted during learning? Describe two learning methods in neural networks.

21. a) Draw a Radial Basis Functions Network. What is the relation between input and outputs?

    b) Explain the differences between a multilayer perceptron and a radial basis functions network.

22. Explain the differences between the optimization methods based on the gradient and the ones based on Newton methods?

23. Draw one of the most common neuro-fuzzy network, explaining the meaning of each layer.

24. Consider the three application examples of Adaptive Neuro-Fuzzy Inference Systems (ANFIS) given in the lectures. What are the advantages and drawbacks of this type of inference systems?

25. Describe one application of fuzzy decision making. Justify using the most important equations, fuzzy goals, fuzzy constraints and the respective aggregation method. Describe how to compute the optimal value.

26. Consider the application of fuzzy decision making. In Linear Programming problems, such as the maximization of profit given in the lectures. What are the advantages and drawbacks of applying fuzzy decision making?

27. Describe generically how to “fuzzify” a decision making problem. What are the advantages and drawbacks of this approach?

28. What is feature selection? Explain its importance in modeling.

29. Explain the phases of knowledge data discovery.