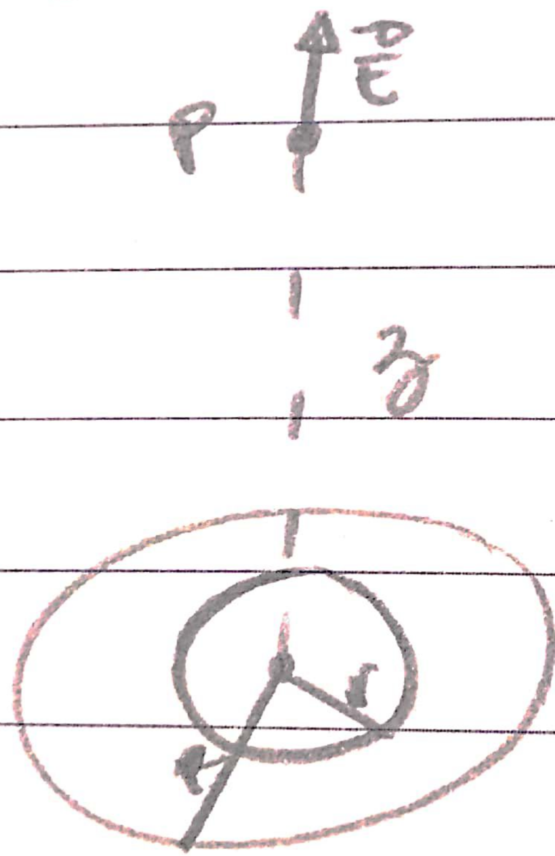
$\boxed{Ex:}$ Calcule o campo elétrico em um ponto ao longo do eixo central à uma distância $z$ do centro



Campo de um anel de raio R e carga Q.

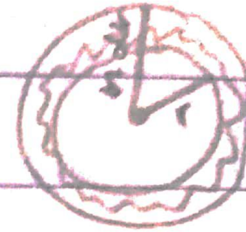
$$\vec{E} = \frac{Q}{4\pi\varepsilon_0} \frac{z}{(z^2+R^2)^{3/2}} \hat{k}$$

no caso do disco, o anel tem raio r e carga dQ

$$dE = \frac{dQ}{4\pi\varepsilon_0} \frac{z}{(z^2+r^2)^{3/2}}$$

$$E_{disco}(z) = \int dE = \int \frac{dQ}{4\pi\varepsilon_0} \frac{z}{(z^2+r^2)^{3/2}}$$

$$\sigma = \frac{Q}{A} = \frac{Q}{\pi R^2} = \frac{dQ}{dA}$$



$$dA = \pi(r+dr)^2 - \pi r^2$$

$$= \pi(r^2 + 2r\,dr + dr^2) - \pi r^2$$

$$dA = 2\pi r\,dr + \pi(dr)^2$$

$$E_{disco}(z) = \int \frac{1}{4\pi\varepsilon_0} \frac{z}{(z^2+r^2)^{3/2}} \sigma\, dA$$

$$E_{disco}(z) = \int_0^R \frac{1}{4\pi\varepsilon_0} \frac{z\sigma}{(z^2+r^2)^{3/2}} 2\pi r\, dr$$

$$E_{disco}(z) = \frac{\pi \cdot \sigma z}{4\pi\varepsilon_0} \int_0^r \frac{2r}{(z^2+r^2)^{3/2}} dr \quad \begin{cases} u = z^2 + r^2 \\ \frac{du}{dr} = 2r \Rightarrow du = 2r\,dr \end{cases}$$

$$\int_0^R \frac{2r}{(z^2+r^2)^{3/2}} = \int \frac{1}{u^{3/2}}\,du = \int u^{-3/2}\,du$$

$$= \frac{1}{-\frac{3}{2}+1} u^{-3/2+1} = \frac{1}{-\frac{1}{2}} u^{-1/2}$$

$$= -2\frac{1}{u^{1/2}} = \frac{-2}{\sqrt{u}} = \frac{-2}{\sqrt{z^2+r^2}}$$

$$E_{disco}(z) = \frac{\sigma z}{4\varepsilon_0} \int_0^R \frac{2r}{(z^2+r^2)^{3/2}}\,dr$$

$$= -\frac{\sigma z}{4\varepsilon_0} \frac{2}{\sqrt{z^2+r^2}}\Bigg|_0^R = -\frac{\sigma \cdot z}{2\varepsilon_0}\left(\frac{1}{\sqrt{z^2+r^2}} - \frac{1}{\sqrt{z^2}}\right)$$

$$\boxed{E_{disco}(z) = \frac{\sigma z}{2\varepsilon_0}\left[\frac{1}{z} - \frac{1}{\sqrt{z^2+r^2}}\right]}$$

Se colocar uma carga no ponto P vai sentir uma força

$$F = q \cdot E_{disco}(z)$$

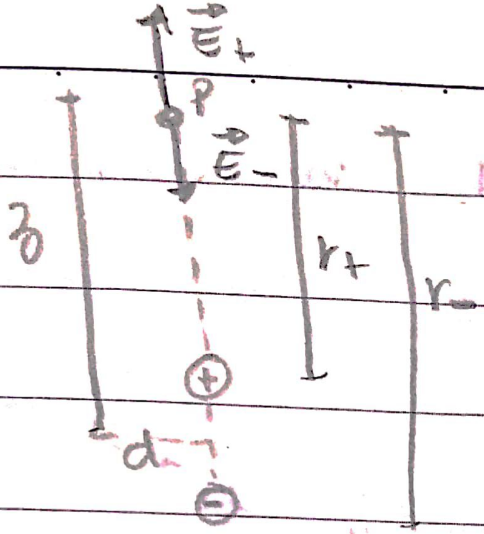
$$\lim_{r \gg z} E_{disco}(z) = \frac{\sigma z}{2\varepsilon_0}\left[\frac{1}{z} - 0\right] = \frac{\sigma}{2\varepsilon_0}$$

$$\sigma = \frac{Q}{\pi R^2}$$

$$\lim_{r \gg z} E_{disco}(z) = \frac{Q}{2\varepsilon_0 \pi R^2}$$

Date. Page.

Ex:




$$\vec{E} = \vec{E}_+ + \vec{E}_-$$

$$\vec{E} = |\vec{E}_+|\hat{k} + (-|\vec{E}_-|\hat{k})$$

$$\vec{E} = (|E_+| - |E_-|)\hat{k}$$

$$|E_+| = \frac{q}{4\pi\varepsilon_0 r_+^2} \qquad |E_-| = \frac{q}{4\pi\varepsilon_0 r_-^2}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_+^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_-^2}$$

$$\begin{cases} r_+ = z - \frac{d}{2} \\ r_- = z + \frac{d}{2} \end{cases}$$

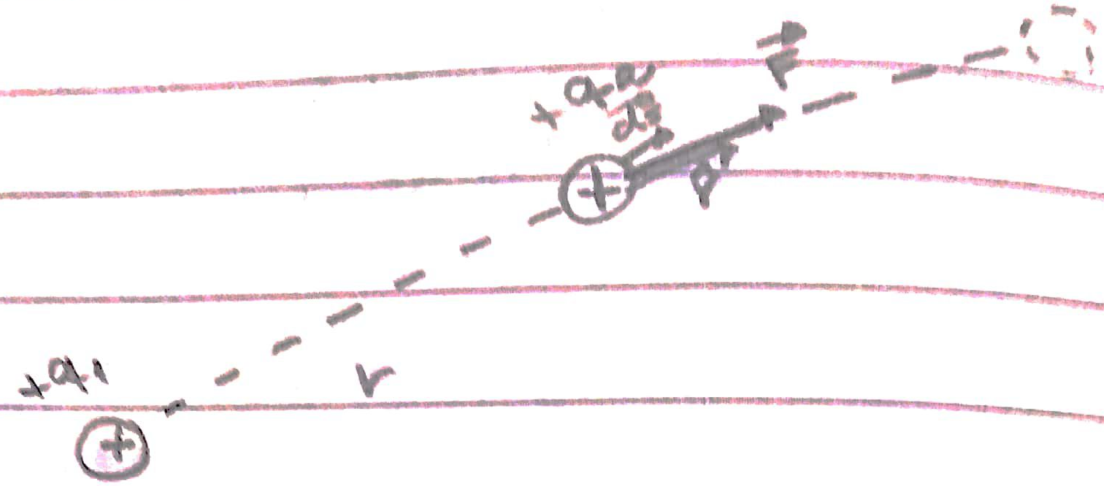
P/ situações físicas de interesse $d \ll z$

$$\boxed{E \approx \frac{2qd}{4\pi\varepsilon_0 z^3}}$$

$p = q \cdot d$
momento de dipolo

$$\boxed{E \approx \frac{p}{4\pi\varepsilon_0 z^3}}$$

- Energia potencial elétrica




$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r}$$

$$W = \int_r^{r_f} \vec{F} d\vec{s} \quad , \quad d\vec{s} = dr \cdot \hat{r}$$

$$= \int_r^{r_f} \left( \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r} \right) (dr \cdot \hat{r})$$

$$\hat{r} \cdot \hat{r} = 1$$

$$W = \int_r^{r_f} \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} dr$$

$$W = \frac{1}{4\pi\varepsilon_0} q_1 q_2 \left( -\frac{1}{r} \right) \Big|_r^{r_f} \equiv -\Delta U$$

$$W_{r \to r_f} = -(U(r_f) - U(r))$$

$$W_{r \to r_f} = U(r) - U(r_f)$$

Vamos considerar que $U(r \to \infty) = 0$

então $r_f \to \infty$

$$W_{r \to \infty} = U(r) - \overset{0}{\cancel{U(r \to \infty)}}$$

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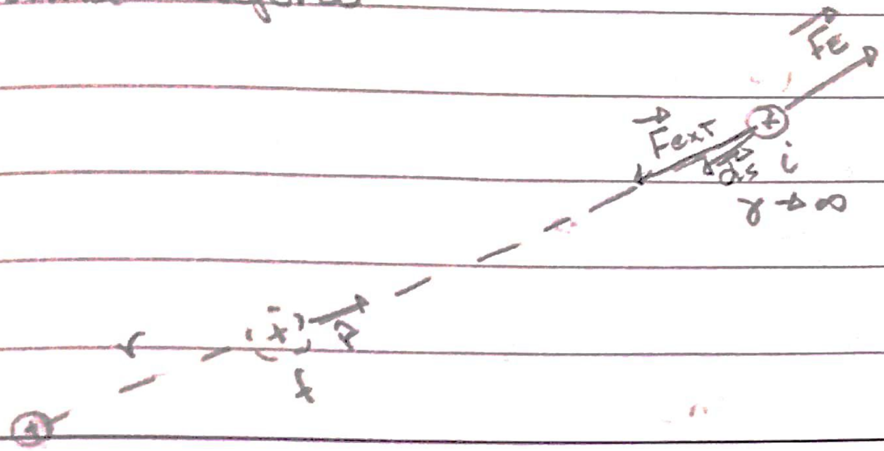
$$U(r) = W(r \to \infty)$$

$$W_{r \to \infty} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$$

$$\boxed{U(r) = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}}$$

Suponha agora

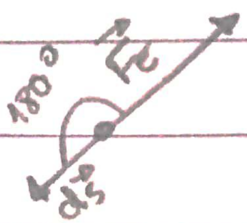



$$W_T = W_{Fext} + W_E \quad \} \; W_e = \text{trab da } \vec{F} \text{ elétrica}$$

$$\cancel{\Delta K}^{\,0} = W_{Fext} + W_E$$

$$\boxed{W_{Fext} = -W_E}$$

$$W_{Fext} = -\int_i^f \vec{F_E}\, \vec{ds} = -\int_i^f |\vec{F_E}|\,|\vec{ds}| \cos\theta$$




$$= \int_\infty^r F_E(-dr)\underbrace{\cos 180}_{-1} = -\int_\infty^r [\vec{F_E}\, dr]$$

$$W_{Fext} = -\int_\infty^r F_E\, dr = -\frac{q_1 q_2}{4\pi\varepsilon_0}\int_\infty^r \frac{1}{r^2}\, dr$$

$$= -\frac{q_1 q_2}{4\pi\varepsilon_0}\,\frac{1}{-2+1}\, r^{-2+1}\Big|_\infty^r$$

$$W_{F_{ext}} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r} \Big|_{\infty}^{r}$$

$$\boxed{W_{F_{ext}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r}}$$

$$\vec{F_E} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r}$$

$$d\vec{s} = dr \cdot \hat{r}$$

$$\vec{F_E} \cdot d\vec{s} = \left( \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r} \right) \cdot (dr \cdot \hat{r})$$

$$\vec{F_E} \cdot d\vec{s} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} (\hat{r} \cdot \hat{r})\, dr$$

$$\vec{F_E} \cdot d\vec{s} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}\, dr = \vec{F_E} \cdot dr$$

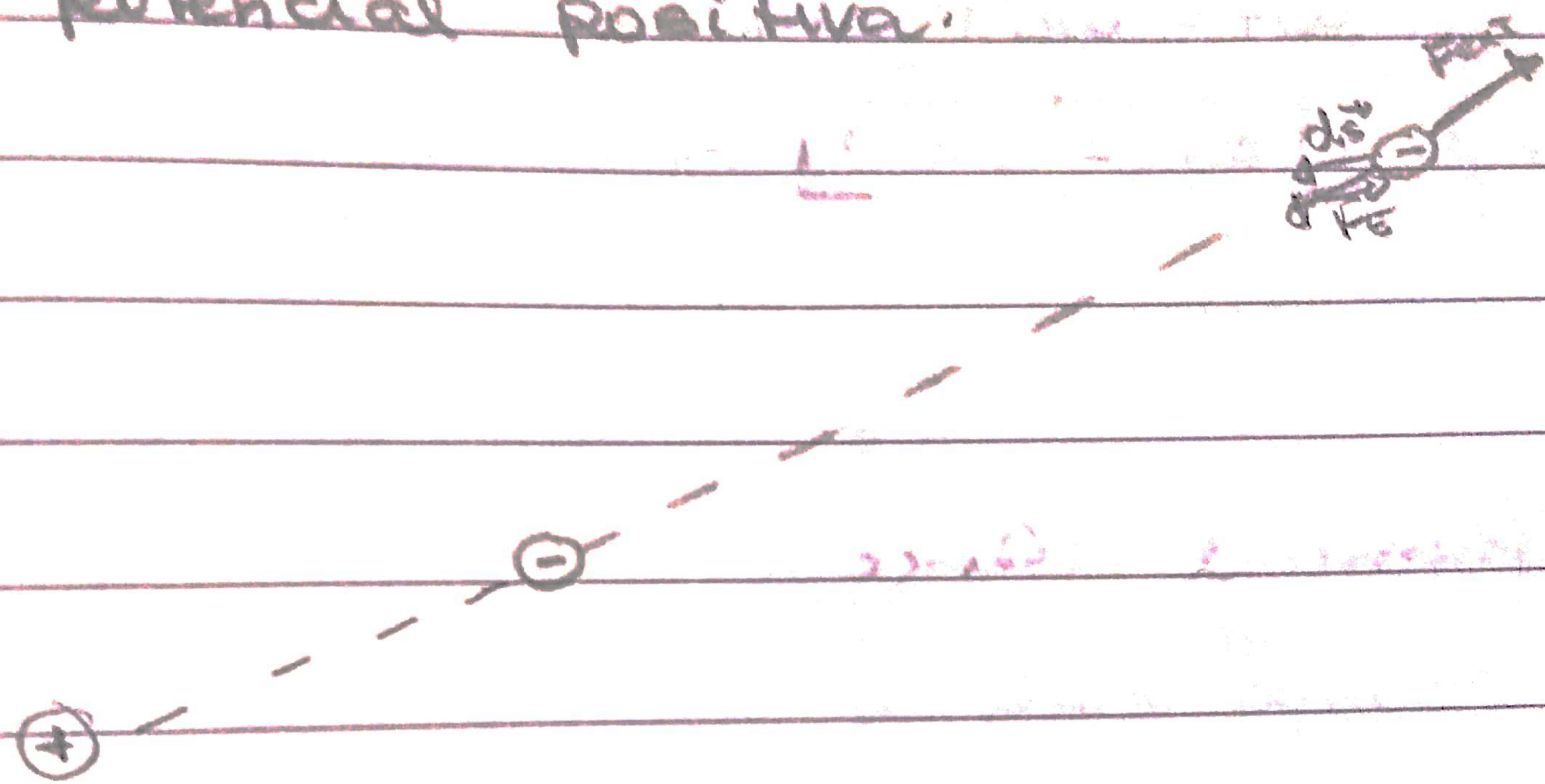
* O que é a energia potencial?

Trabalho de desconfigurar ou trabalho de gerar a configuração.

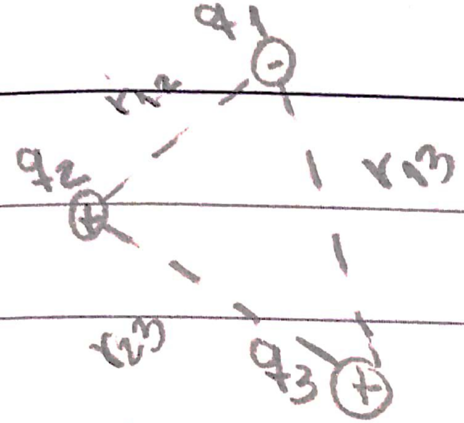
$$\boxed{W_{Fext} = U(r)}$$

Att. Cargas diferentes: trabalho negativo, logo energia potencial negativa. (para evitar o choque)

Cargas iguais: trabalho positivo, logo energia potencial positiva.




$$W_{F_{ext}} = U(r) = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} < 0$$

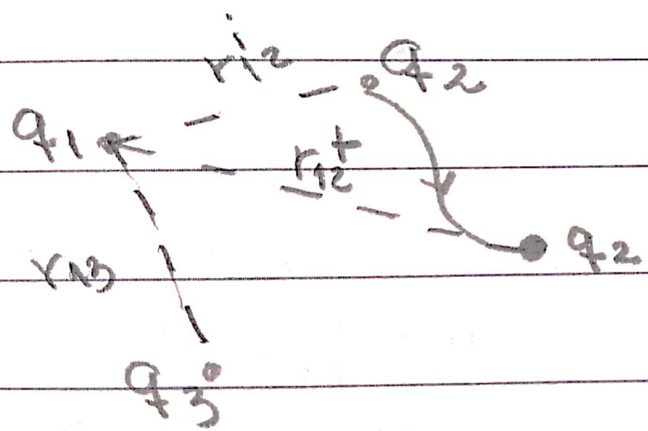



$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

→ energia potencial do sistema total

$$W = -\Delta U$$

$$W = -\Delta U = U_i - U_f$$




$$W = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}^{i}} + \frac{q_1 q_3}{r_{13}} + \frac{q_3 q_2}{r_{23}^{i}}\right) - \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}^{f}} + \frac{q_2 q_3}{r_{23}^{f}} + \frac{q_1 q_3}{r_{13}}\right)$$

$$\boxed{W = \frac{1}{4\pi\varepsilon_0} q_1 q_2\left(\frac{1}{r_{12}^{i}} - \frac{1}{r_{12}^{f}}\right) + \frac{1}{4\pi\varepsilon_0} q_2 q_3\left(\frac{1}{r_{23}^{i}} - \frac{1}{r_{23}^{f}}\right)}$$