Tensor Network Machine Learning

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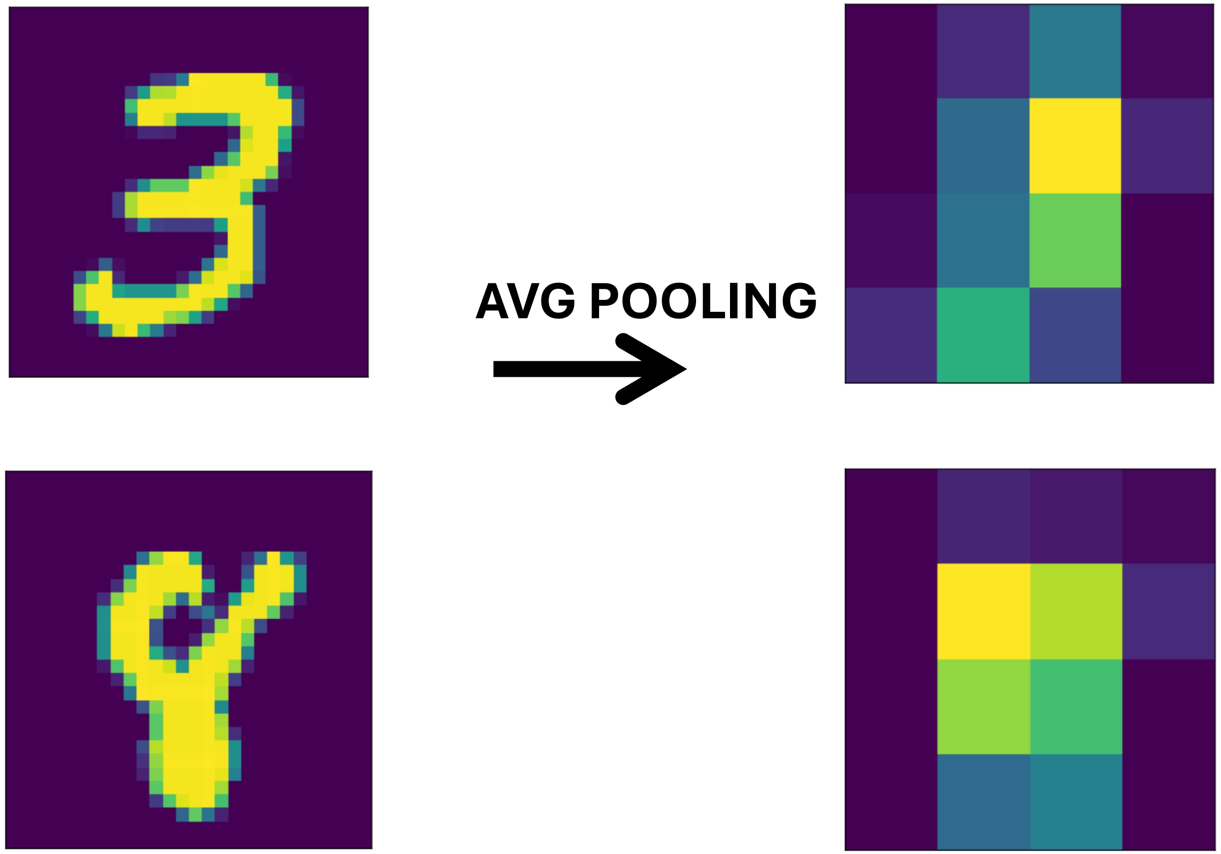
Tasks

1. Successfully classify the digits and of the MNIST dataset using the MPS classifier, using encoding 1.
2. “ “ encoding 2.
3. Analyze the entanglement entropy of the classifier. Study the variation of the entanglement entropy and accuracy with the variation of the max bond dimension.
4. (Optional) Compress a classical NN weights using an MPO.

Data preparation

- Subset of **500** images of size **28x28** from **MNIST** (only 3s & 8s)
- **Normalization** of pixel intensity: $[0, 255] \rightarrow [0,1]$
- **Average pooling** with 7x7 kernel: $28\text{x}28 \rightarrow 4\text{x}4$



Learning process

1. Build another **MPS** (giving a bond dimension χ) that learns how to classify images by contracting with their **MPS** representation
2. Study how the learning is affected by the choice of χ

Note: the choice of the encoding will affect the learning!

Encoding 1	Encoding 2
MPS classifier has 16 qubits	**MPS** classifier has 4 qubits

Analysis

Compute the **entanglement entropy** of the **MPS** classifier

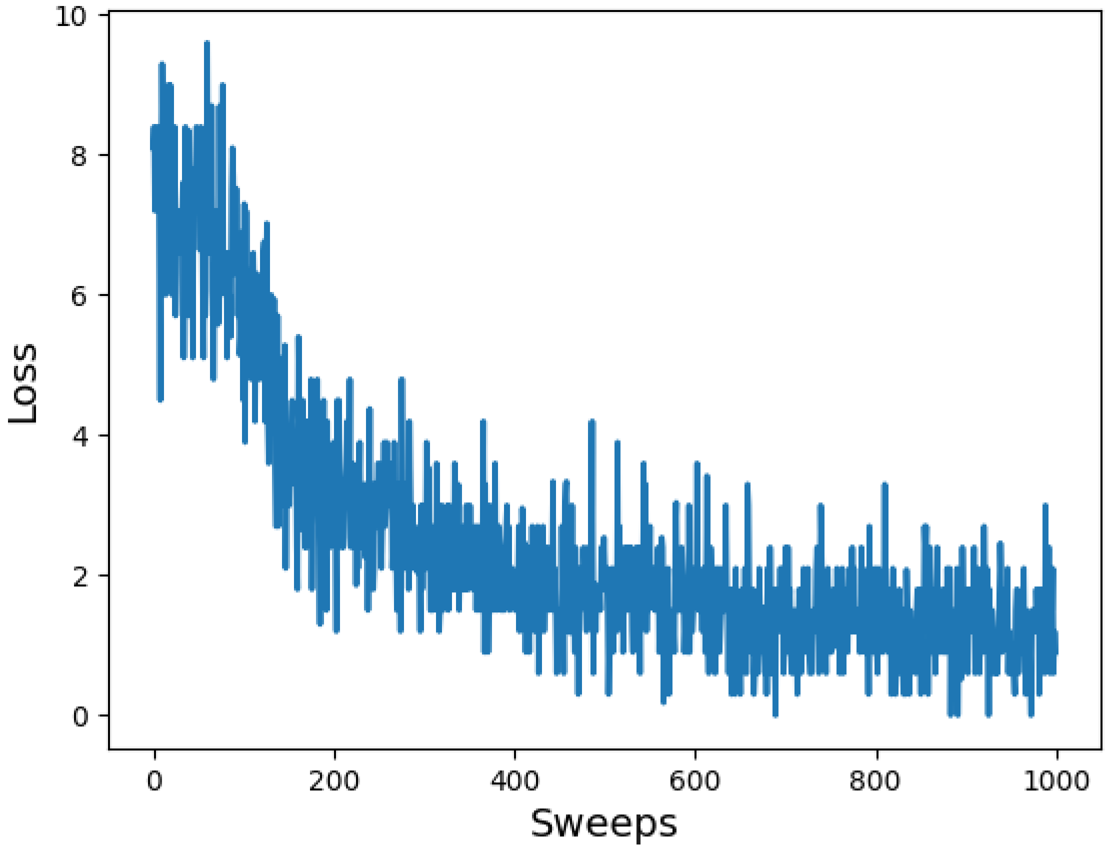
$$S = -\sum_{i=1}^{\min(\chi,N)} s_i^2 \ln(s_i^2)$$

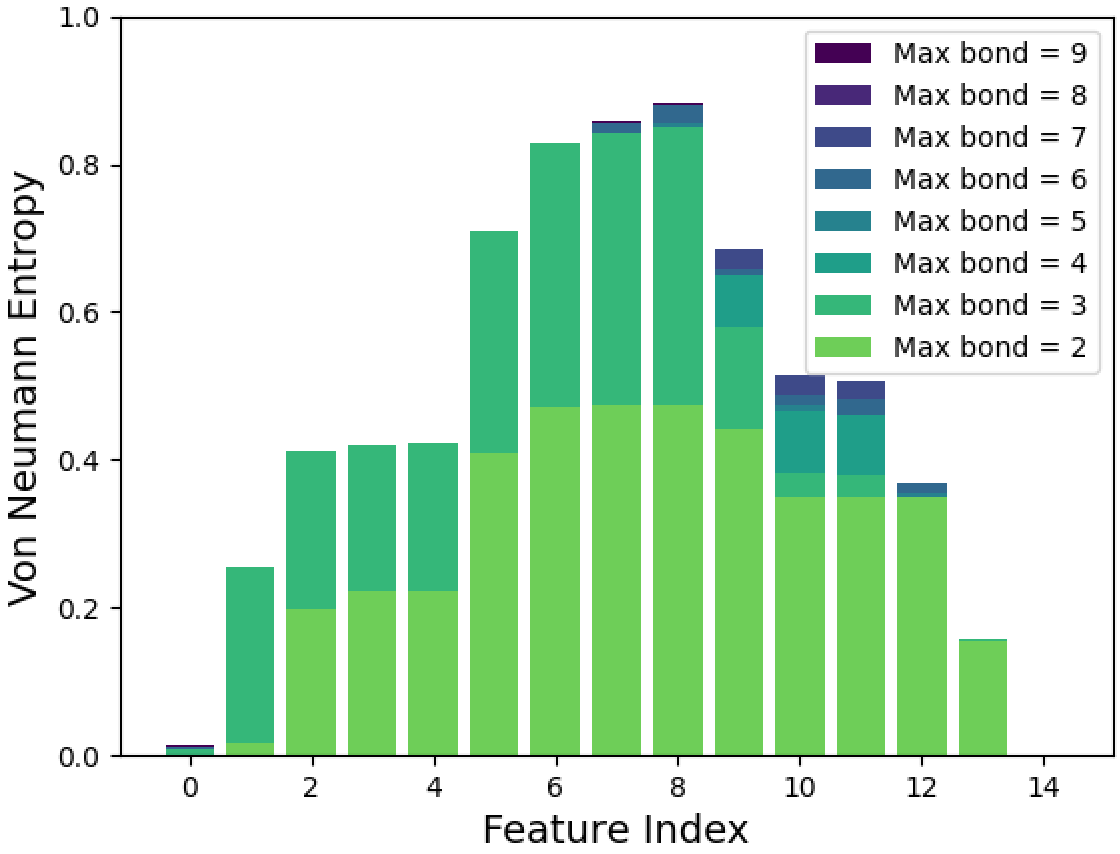
This will be computed between two partitions of the system, the bipartition will be taken along each bond.

Interpretability: we expect Encoding 1 to be easier to interpret since every qubit matches one pixel.

Encoding 1.a

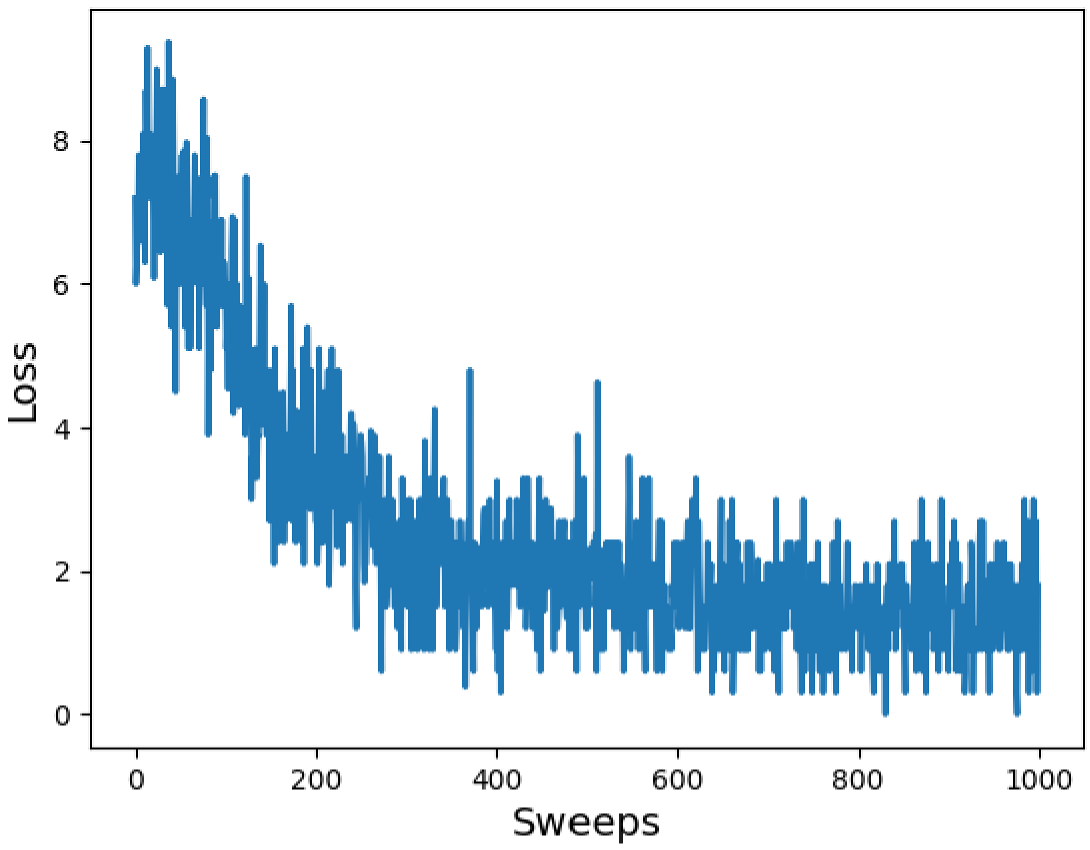
$$|q\rangle = \sqrt{1 - p_i}\,|0\rangle + p_i\,|1\rangle$$

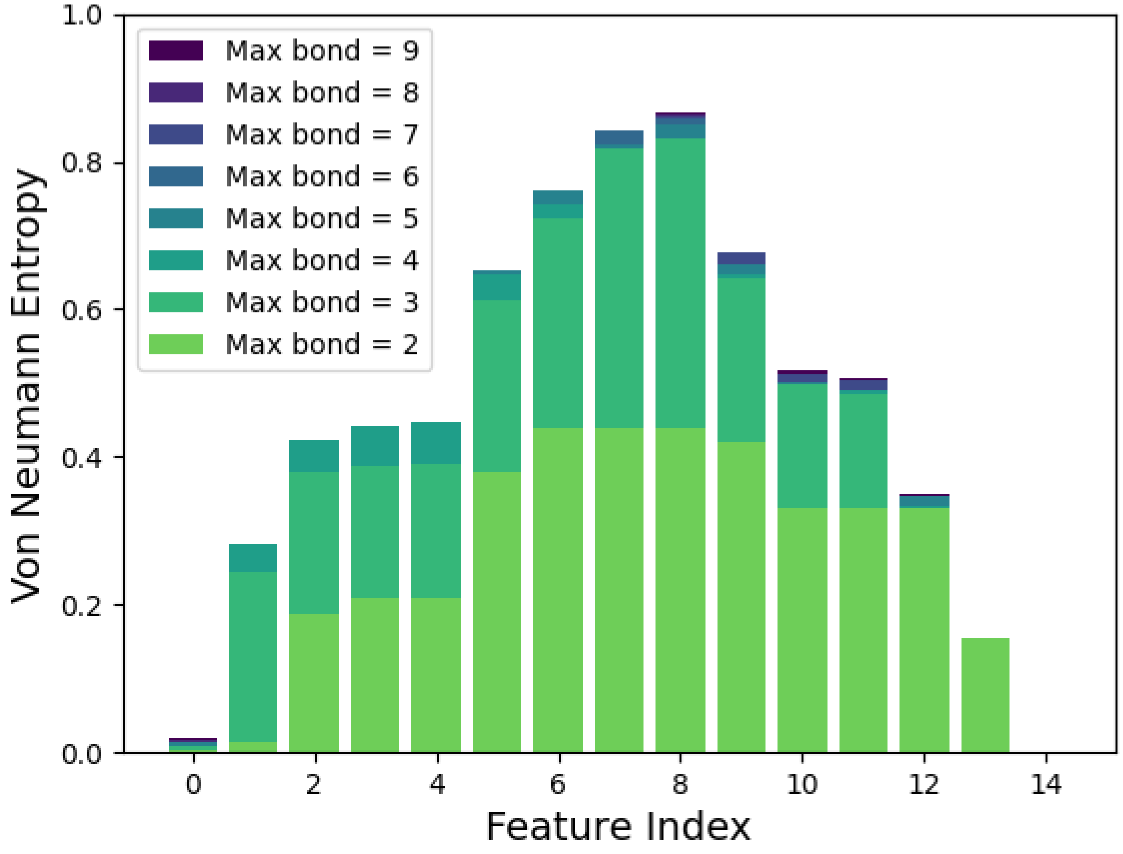




Encoding 1.b

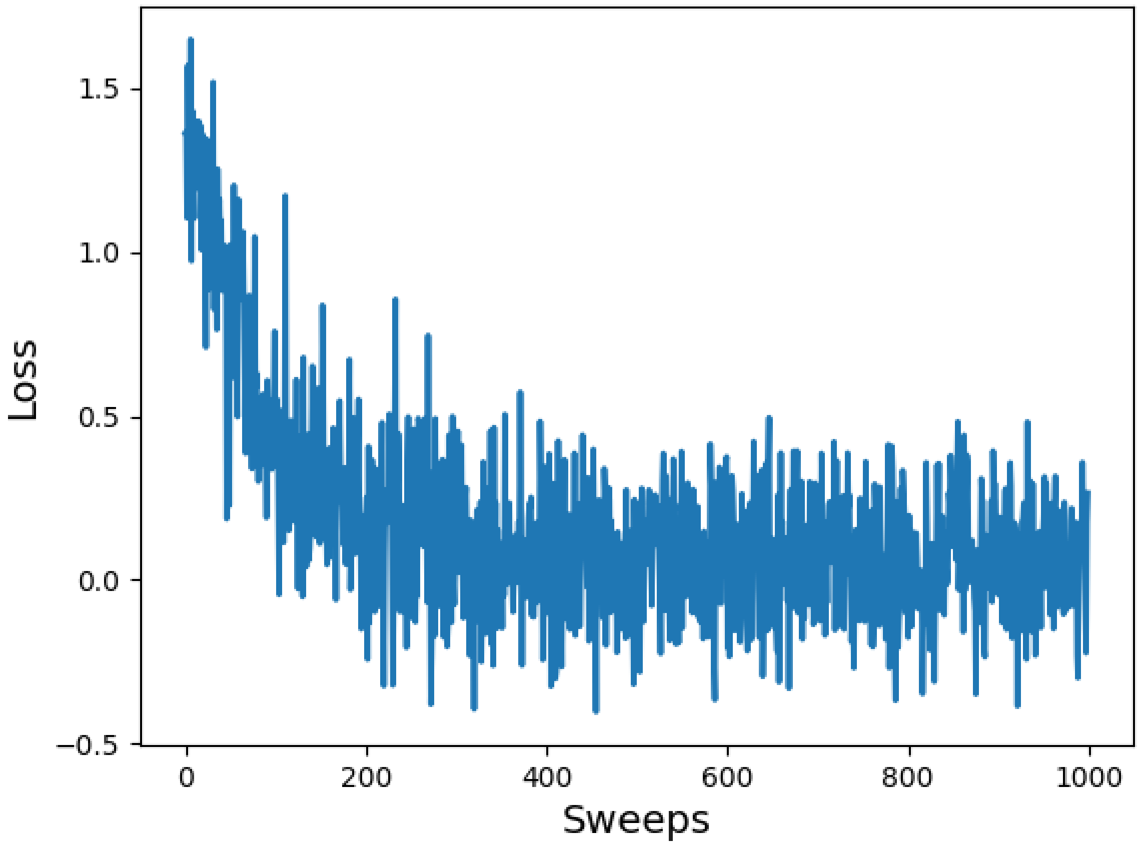
$$|q\rangle = \cos(p_i)|0\rangle + \sin(p_i)|1\rangle$$

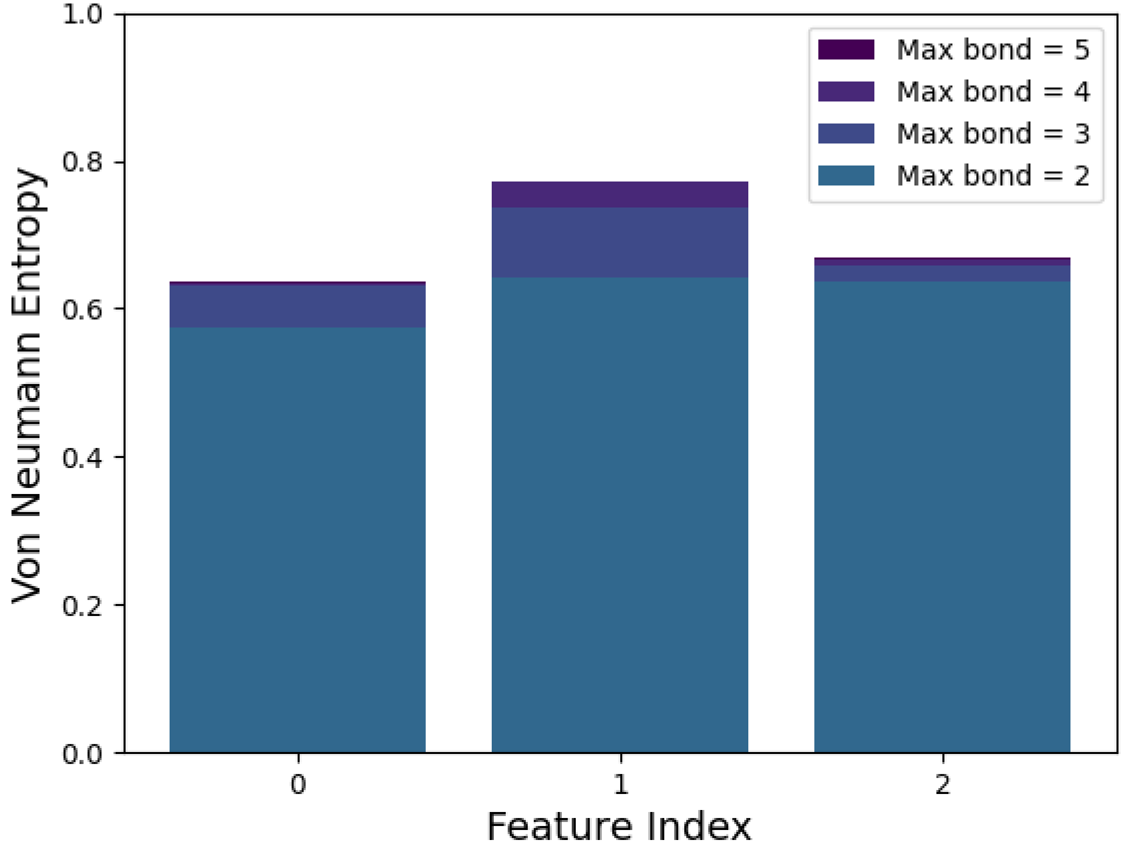




Encoding 2

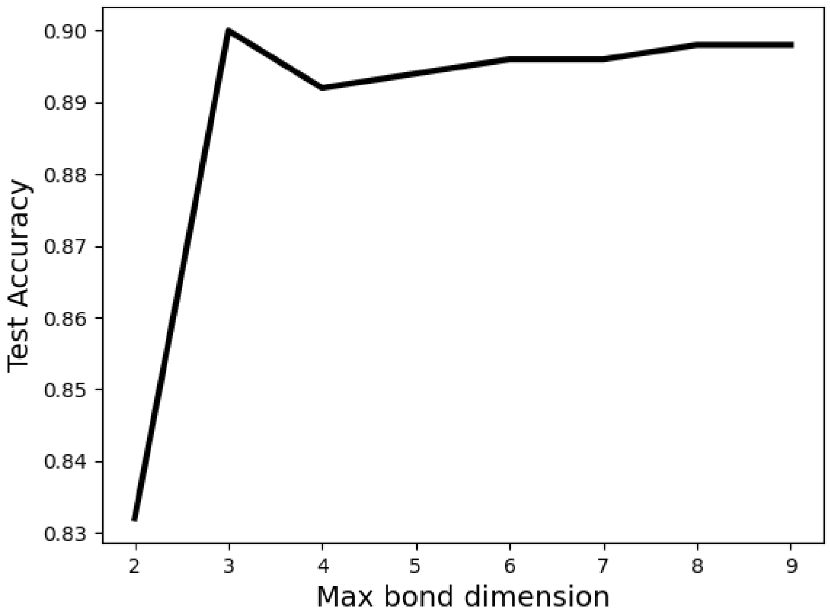
$$|\psi\rangle = \sum_i p_i |i\rangle$$



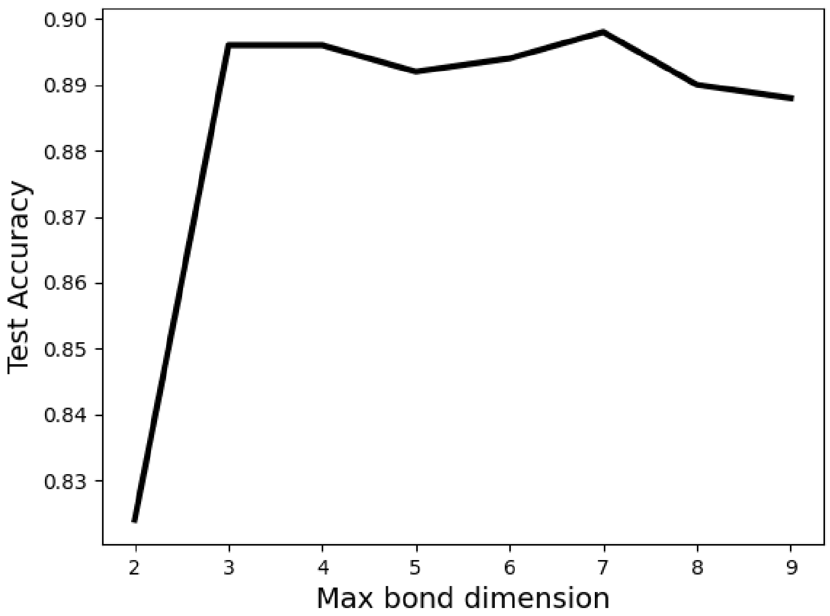


Test accuracy VS max bond dimension

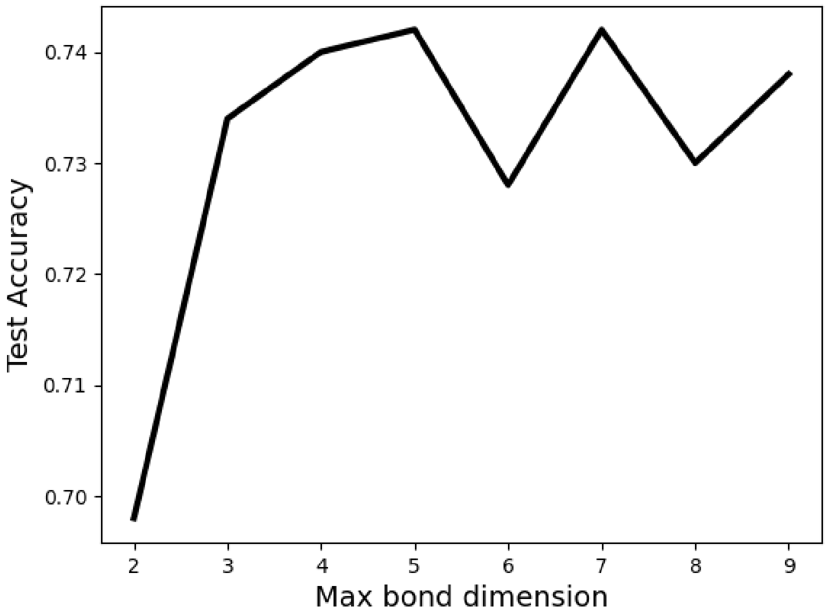
Encoding 1.a



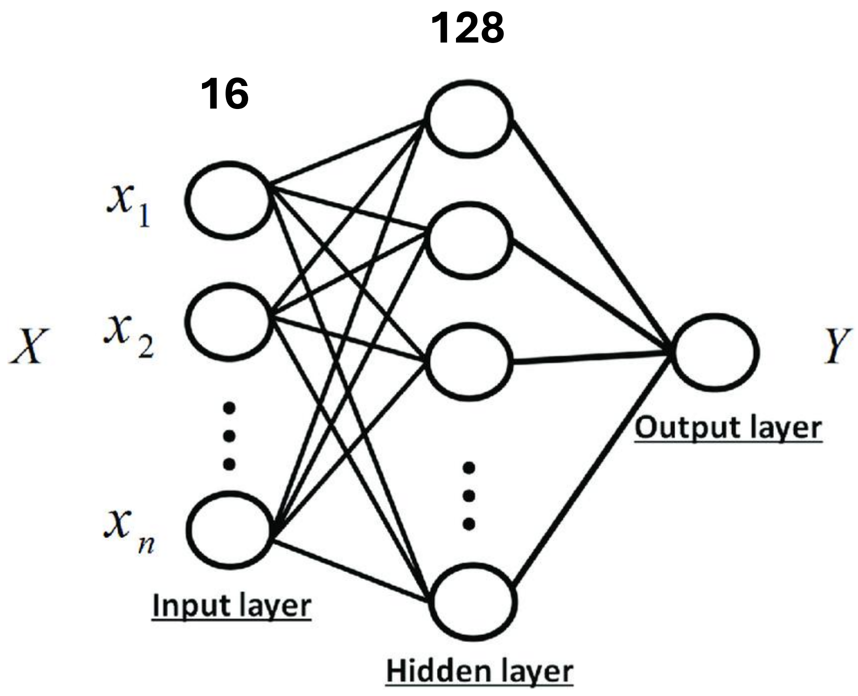
Encoding 1.b

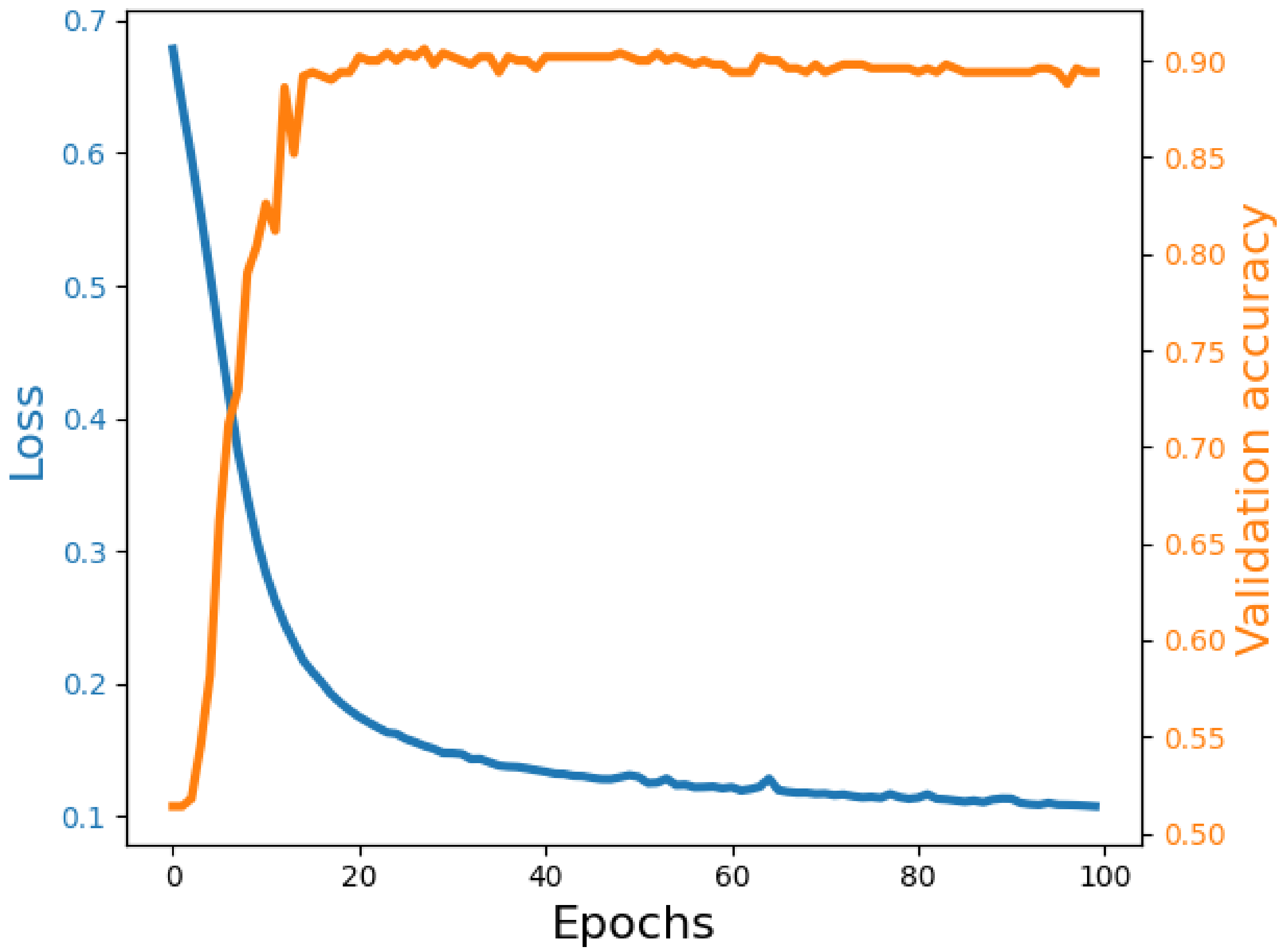


Encoding 2

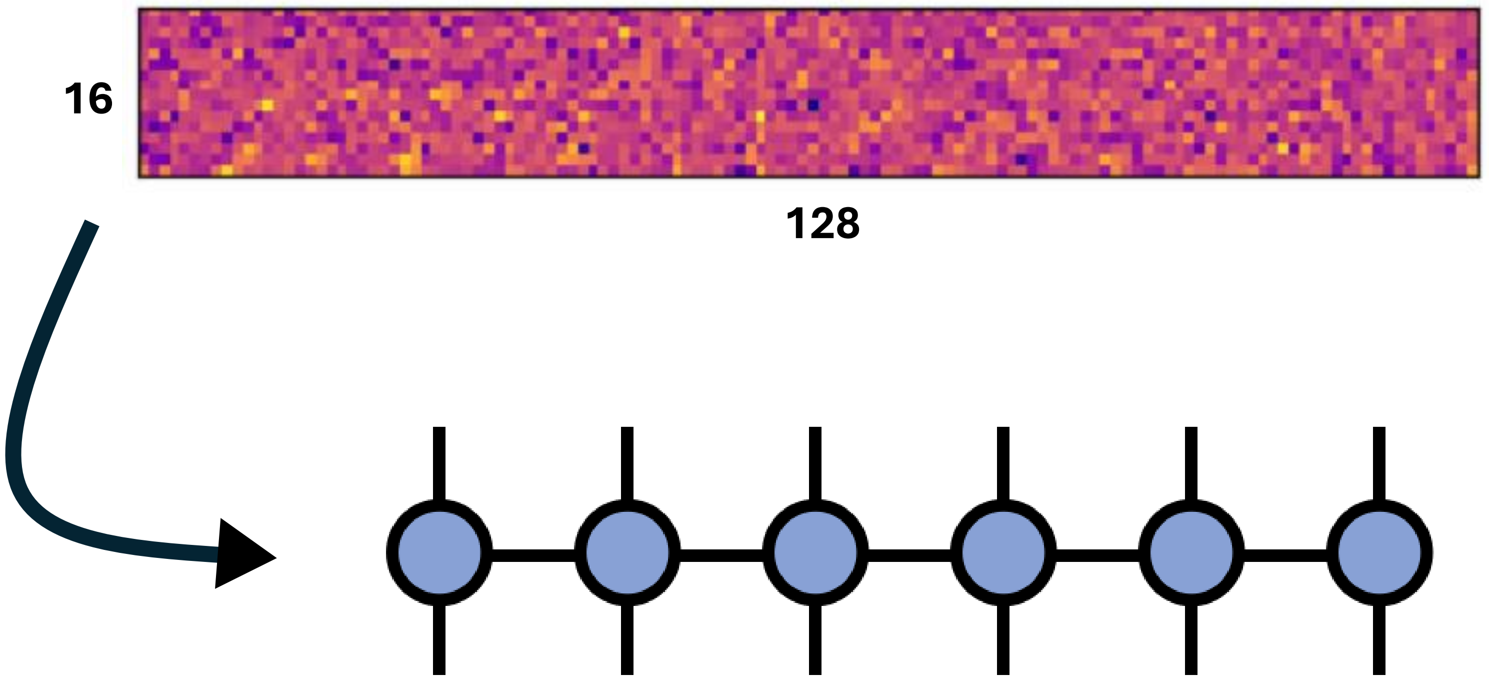


Neural Network



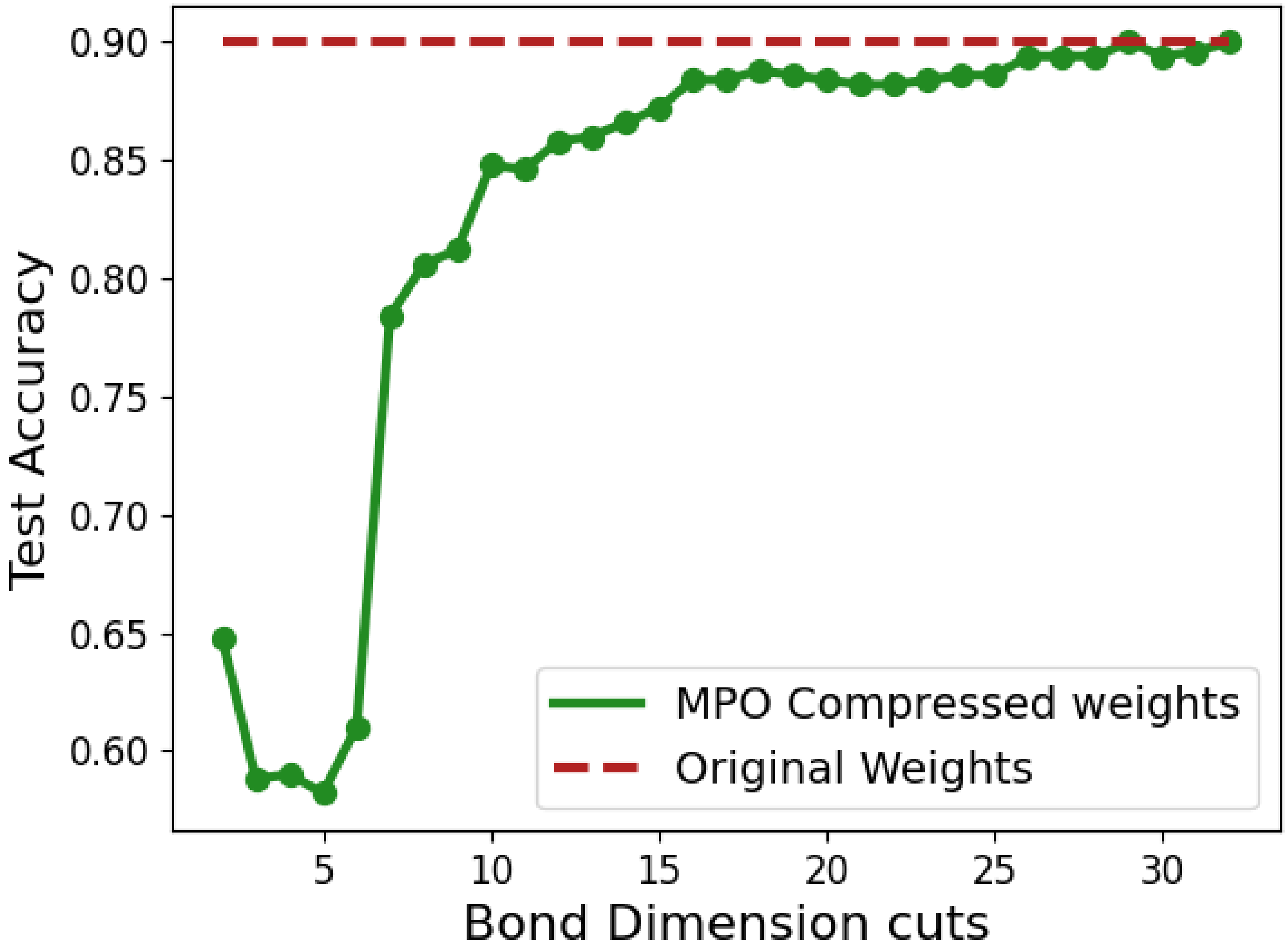


Neural Network – weights compression



SVD, contraction and repeat (with many reshapes and permutations in the middle 😅)

Neural Network – weights compression



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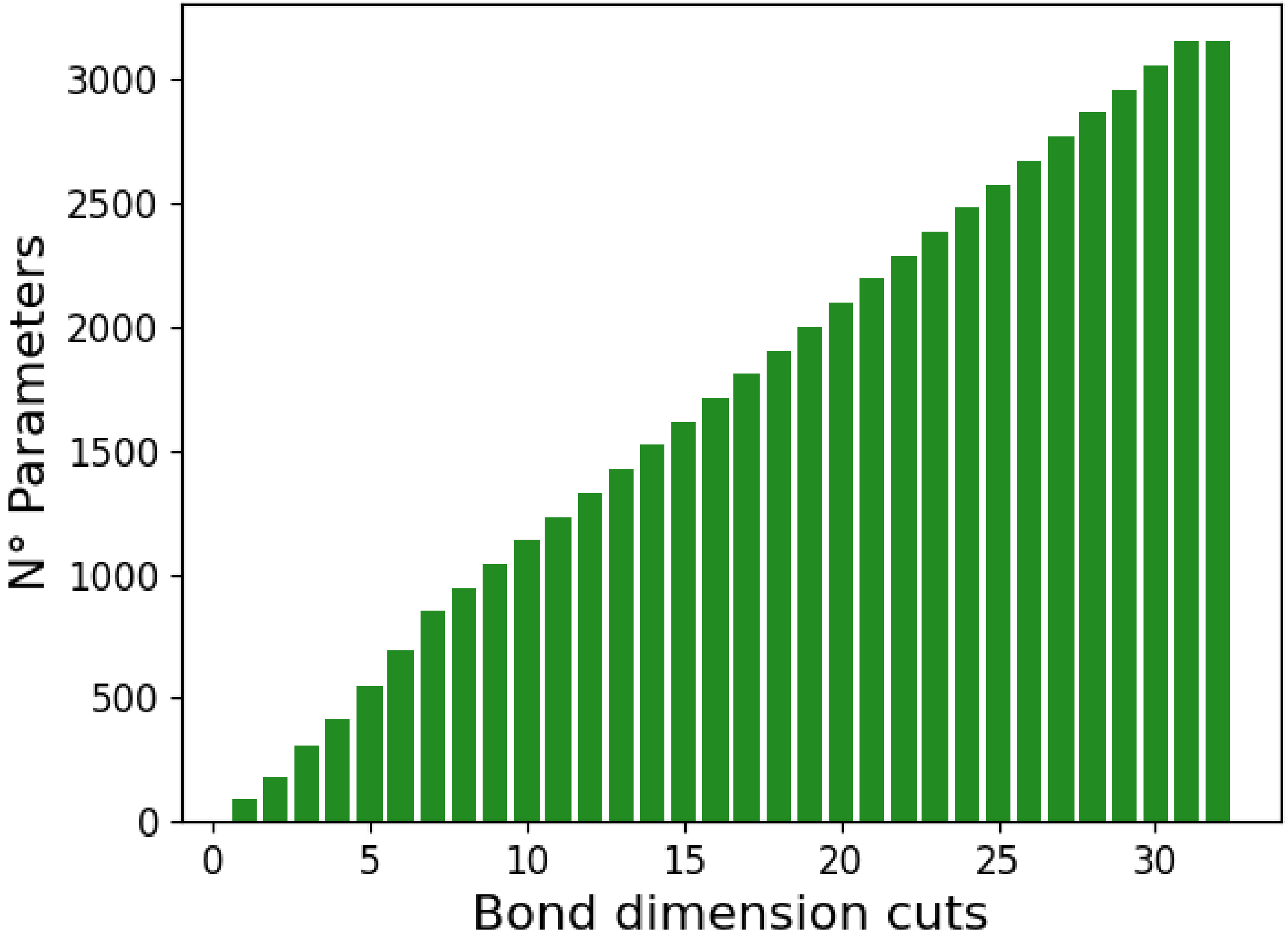

Conclusions

- MPS can be used to learn to classify MNIST digits. Performance similar to classical FCNN, although with significantly higher training cost.
- The type of encoding of the input is important to maximize performance.
- Interpretability is the strong point of using MPS versus NNs. Lack of non-linearities make interpretation easier.
- MPO can be used to compress information in the weights of a NN in an interpretable way.

References

[1] Stoudenmire, Schwab. Supervised Learning with Quantum-Inspired Tensor Networks. *Advanced in Neural Information Processing Systems (NIPS)* (2016)

[2] Qing et al.. Compressing neural network by tensor network with exponentially fewer variational parameters. *arXiv (2024)*

Thank you!

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Funded by the European Union