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A Decentralized Fault Detection Filter

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Abstract

In this paper, we introduce the decentralized fault detection filter which is the structure that results from merging decentralized estimation theory with the game theoretic fault detection filter. A decentralized approach may be the ideal way to health monitor large-scale systems for faults, since it decomposes the problem down into (potentially smaller) "local" problems and then blends the "local" results into a "global" result that describes the health of the entire systems. The benefits of such an approach include added fault tolerance and easy scalability. An example given at the end of the paper demonstrates the use of this filter for a platoon of cars proposed for advanced vehicle control systems.

1. Introduction

Observers play a central role in an important class of techniques for fault detection and identification (FDI). Since failures, which act as unexpected inputs, drive a system, they will bias the error residuals of any observer designed about the nominal system. Observers designed for FDI take advantage of this fact and provide some structure to this bias so that some interpretation of the health of the underlying system can be made.

One common type of FDI observer is known as the *unknown input observer*. This observer divides the set of modelled faults into two groups: the faults to be detected and the faults which act as disturbances. The sets are made distinguishable by simply making the latter set unobservable to a specially constructed output generated by the observer. Detection is then achieved when this output is nonzero. Identification is trivial because of the way in which the problem has been formulated. A decentralized fault detection filter approximates the actions of an unknown input observer. It is formed by combining the estimates of several "local" estimators (each driven by independent measurement sets). For large-scale systems this approach may be the ideal way to monitor the system for faults for a number of reasons.

The decentralized fault detection filter is the result of combining the game theoretic fault detection filter of [1] with the decentralized filtering algorithm introduced by Speyer in [2] and extended by Willsky *et al.* in [3]. In Section 2, we review the general theory of decentralized estimation and show why it is necessary to use an approximate unknown input observer (i.e. the game theoretic filter of [1]) in order to be able to use the theory for decentralized fault detection and identification. In Section 3 we will provide a brief overview of the fault detection and identification problem and the game theoretic solution. Finally, in Section 5, we present an example which applies our results to a problem in advanced vehicle control systems

2. Decentralized Estimation Theory and its Application to FDI

2.1. The General Solution

In this section, we will pose the genereral decentralized estimation problem and given the solution. A detailed examination of this theory is given in [4]. Consider a system driven by process disturbances, w, and sensor noise, v,

$$\dot{x} = Ax + Bw, \qquad x(0), x \in \mathcal{R}^n, \tag{1}$$

$$y = Cx + v, \qquad y \in \mathcal{R}^m. \tag{2}$$

for which it is desired to derive an estimate of x. The standard approach is a full-order observer,

$$\dot{\hat{x}} = A\hat{x} + L(y - C\hat{x}), \qquad \hat{x}(0) = 0, \tag{3}$$

which we will call a *centralized estimator*. An alternative to this method is to derive the estimate with a *decentralized estimator*. In the decentralized approach, $\hat{x}$ is found by combining estimates based upon "local" models,

$$\dot{x}^j = A^j x^j + B^j w^j, \qquad x^j \in \mathcal{R}^{n^j}, \qquad (j = 1...N), \tag{4}$$

$$y^j = E^j x^j + v^j, \qquad y^j \in \mathcal{R}^{m^j}, \qquad (j = 1...N), \tag{5}$$

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which together provide an alternate representation of the original system, (1, 2), which, as one might guess, is called the "global" system. The vector, x, is likewise called the "global" state. The number of local systems, N, is bounded above by the number of measurements in the system, i.e. $N \leq m$.

The global/local decomposition is really of only secondary we are limited in this paper to the detail with which we can discuss such modelling and instead direct the reader to [4, 5]. The decentralized estimation algorithm is to estimate the local states, x^j with observers designed on the localized model:

$$\dot{\hat{x}}^j = A^j \hat{x}^j + L^j (y^j - E^j \hat{x}^j),\ \hat{x}^j(t_0) = 0,\ (j = 1...N).$$

and then combine these local estimates to form a global estimate, $\hat{x}$,

$$\hat{x} = \sum_{j=1}^{N} \left(G^j \hat{x}^j + h^j \right). \tag{6}$$

The vector, h^j, is a measurement-dependent variable propagated by

$$\dot{h}^j = \Phi h^j + (\Phi G^j - \dot{G}^j - G^j \Phi^j)\hat{x}^j, \qquad h^j(0) = 0. \tag{7}$$

The constituent matrices are defined as

$$\Phi := A - \sum_{j=1}^{N} G^j L^j C^j,$$
$$\Phi^j := A^j - L^j E^j.$$

The G^j matrices are "blending matrices". In [4, 5] we show that these matrices have the effect of blending the local gains together so that the global estimate from (6) is equivalent to an estimate generated by a centralized filter whose gain is found from:

$$L = \begin{bmatrix} G^1 & \dots & G^N \end{bmatrix} \begin{bmatrix} L^1 & 0 & \cdots & 0 \\ 0 & L^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & L^N \end{bmatrix}. \tag{8}$$

Now for a specific class of estimators it is possible to start with local and global gains and solve for the blending matrices. This class is comprised of estimators which take their gains from Riccati solutions, i.e. Kalman Filters [2, 3] or H^∞ filters [6]. In this case, the local gains are found from

$$L^j = P^j (E^j)^T (V^j)^{-1}, \tag{9}$$

where, V_j is the psd of the local measurement noise and P^j is the solution of a Riccati Equation. The global gain is likewise

$$L = PC^T \mathcal{V}^{-1}$$

where $\mathcal{V}$ and P are the corresponding noise psd and Riccati Solution for the global system. The blending matrix solution is then,

$$G^j = P(S^j)^T (P^j)^{-1} \qquad j = 1, \dots, N, \tag{10}$$

where S^j is any matrix such that

$$C^j = E^j S^j. \tag{11}$$

One can, in fact, always take $S^j = (E^j)^\dagger C^j$ where $(E^j)^\dagger$ is the pseudo-inverse of E^j [3]. Note that the solutions for G^j will always exist for Riccati-based observers so long as P^j is invertible or, equivalently, positive-definite. This will always be the case if the triples, (C^j, A^j, B^j), are controllable and observable for each of the local systems.

2.2. Implications for Detection Filters

In the FDI problem, we want our global filter to have a specific structure, i.e. an FDI structure. Thus, we must define the gain independently of the local filter gains. We, likewise, would like the local filters to have specific properties so that we can carry out fault detection or state estimation at the lower levels. In general, however, we can design the global and local filters independently only if all of the pertinent filters are Riccati-based. FDI filter, in general, however are designed via eigenstructure assignment or geometric methods. Thus, we need an alternative and Riccati-based approach to FDI filter design. Such an approach was presented in [1, 5]. We will review the results in the next section.

3. The Approximate Fault Detection and Identification Problem

Consider the following FDI filter design problem. Given

$$\dot{x} = Ax + F_1\mu_1 + F_2\mu_2.$$

It is desired to detect the occurrence of the target fault, μ_1, in spite of the measurement noise, v, and the possible presence of the nuisance faults, μ_2. Assume that our output is a projection of the filter residual

$$z = H(y - C\hat{x}), \tag{12}$$

where H, is chosen so that we only see the orthogonal complement of $(y - C\hat{x})$ on the invariant subspace containing μ_2. In the classic FDI problem, we want to design a filter so that z is non-zero if and only if μ_1 is non-zero. We thus want it to be insensitive to μ_2. In the approximate FDI filter design problem we only require that

$$\frac{\|z\|^2}{\|\mu_2\|^2} \leq \gamma. \tag{13}$$

Equation 13 is clearly a disturbance attenuation problem. We refer to the solution to the approximate detection filter problem as the *game theoretic fault detection filter*. In [1], we describe how to generate the model (3) and the projector, H.

We also show in [1] that the solution to this problem is a game theoretic observer

$$\dot{\hat{x}} = A\hat{x} + \gamma\Pi^{-1}C^T V^{-1}(y - C\hat{x}), \qquad \hat{x}(t_0) = \hat{x}_0, \tag{14}$$

whose gain is taken from the solution to a Riccati Equation:

$$-\dot{\Pi} = A^T\Pi + \Pi A + \frac{1}{\gamma}\Pi F_2 M F_2^T \Pi + C^T(HQH - \gamma V^{-1})C. \tag{15}$$

or in the steady-state case, when it exists,

$$0 = A^T\Pi + \Pi A + \frac{1}{\gamma}\Pi F_2 M F_2^T \Pi + C^T(HQH - \gamma V^{-1})C. \tag{16}$$

4. The Decentralized Fault Detection Filter

Given the results of the previous two sections, we now propose a decentralized fault detection filtering algorithm:

1. Identify the sensors and actuators which must be monitored at the global level, i.e. define the target faults for the global filter.
2. Identify the faults which should be included in the global nuisance set. The remaining faults should be monitored at the local levels.
3. Derive global and local models for the system including failure maps. [1] contains a brief discussion about this process. In Section 5, we will demonstrate one method in which the local models are derived from the global model via a minimum realization.
4. Design game theoretic fault detection filters for the local and global systems. Solve the corresponding Riccati equations and store the solutions for later use.
5. Determine the blending solutions, G^j, from Equation 10.
6. Propagate the local estimates, $\hat{x}^j$, and vectors, h^j, and then use the decentralized estimation algorithm (6) to derive a global estimate, $\hat{x}$.
7. Determine the global failure signal from $(y - C\hat{x})$ where y is the total measurement set, C is the global measurement matrix, and $\hat{x}$ is the global fault detection filter estimate just derived.

We will now apply these steps in an example.

5. Range Sensor Fault Detection in a Platoon of Cars

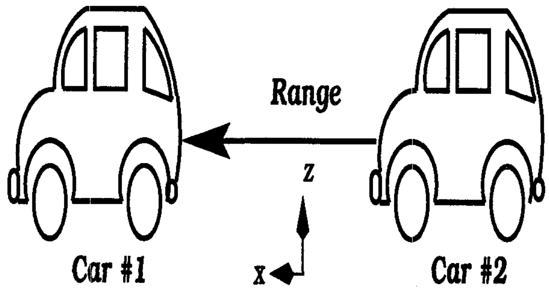

Figure 1: Two-Car Platoon with Range Sensor

5.1. Problem Statement

We will now examine the utility of the decentralized approach to FDI by working through an example. The problem that we will look at involves the detection of failures within a system of two automated cars traveling as a platoon (See Figure 1). Such a system is a simplified model of Integrated Vehicle Highway System program currently under development in California and other places. We are limited by space from describing the problem in detail and instead direct the reader to [5].

The problem objective is to detect the a failure in a range sensor that measures the distance the two cars. Such a sensor is crucial to maintaining separation between the cars. The filter that we design must be sensitive to the range sensor fault while being insensitive to sensor noise and a slew of nuisance fault including faults in the engine sensors, onboard gyroscopes, accelerometers, brake, and throttle.

The dynamics of a single car are given in [7]. By linearizing the full, nonlinear 6 degree of freedom equations of motion of a car travelling in a straightline path at 25 meters/sec, [7] found that the longitudinal dynamics of a car are given by a seven state linear system:

$$\dot{\underline{x}} = A^L\underline{x},$$
$$\underline{y} = C^L\underline{x},$$

with

$$\underline{x} = \left\{ \begin{array}{c} m_a \\ \omega_e \\ v_x \\ v_z \\ z \\ q \\ \theta \end{array} \right\} \begin{array}{l} \textit{engine air mass}\ \text{(kg)} \\ \textit{engine speed}\ \text{(rad/sec)} \\ \textit{long. velocity}\ \text{(m/sec)} \\ \textit{vertical velocity}\ \text{(m/sec)} \\ \textit{vertical position}\ \text{(m)} \\ \textit{pitch rate}\ \text{(rad/sec)} \\ \textit{pitch}\ \text{(rad)} \end{array} \tag{17}$$

and

$$
\underline{y} = \left\{ \begin{array}{c} m_a \\ \omega_e \\ \dot{v}_x \\ \dot{v}_z \\ q \\ \overline{\omega}_f \\ \overline{\omega}_r \end{array} \right\} \begin{array}{l} \textit{engine air mass } (\mathrm{kg}) \\ \textit{engine speed } (\mathrm{rad/sed}) \\ \textit{long. acceleration } (\mathrm{m/sec^2}) \\ \textit{heave acceleration } (\mathrm{m/sec^2}) \\ \textit{pitch rate } (\mathrm{rad/sec}) \\ \textit{front symmetric wheel speed } (\mathrm{rad/sec}) \\ \textit{rear symmetric wheel speed } (\mathrm{rad/sec}) \end{array} \tag{18}
$$

The rear and front symmetric wheel speeds are states that were eliminated when the fast modes were factored out of the linearized system. See [5] or [7] for the state matrices.

In order to build a detection filter for the range sensor, we need to use the single car model to build state space models for the platoon,

$$
\begin{aligned}
\dot{\eta} &= A\eta + F_1\mu_1 + F_2\mu_2, \\
y &= C\eta,
\end{aligned}
$$

and the two individual cars,

$$
\begin{aligned}
\dot{\eta}^1 &= A^1\eta^1 + F_1^1\mu_1^1 + F_2^1\mu_2^1, \\
y^1 &= E^1\eta^1, \\
\dot{\eta}^2 &= A^2\eta^2 + F_1^2\mu_1^2 + F_2^2\mu_2^2, \\
y^2 &= E^2\eta^2.
\end{aligned}
$$

We will build up our models with the following steps:

1. Using single car state matrices A_L and C_L, we will derive the global state matrices, A and C.
2. Using the modelling techniques described in [8, 1], we will determine the failure maps, F_i.
3. We will then obtain the local state matrices, A^i, E^i, and F_j^i, from the minimum realization of the triples (C^1, A, F_2) and (C^2, A, F_2).

Our general strategy is to derive the global equation first and then get the local equations from decompositions based upon observability and controllability. While this is by no means the only way to obtain the global and local representations of a system, it is a logical method that can be applied to any problem.

The obvious way the get the global matrices, A and C, is to form block diagonal composite matrices with A^L and C^L repeated on the diagonal, i.e.

$$
A' = \begin{bmatrix} A^L & 0 \\ 0 & A^L \end{bmatrix}, \qquad C' = \begin{bmatrix} C^L & 0 \\ 0 & C^L \end{bmatrix}.
$$

This, however, is not sufficient, since there is no way to describe the range, R, between the two vehicles with the given states, (17). Range is the relative distance between the cars,

$$
R = x^1 - x^2,
$$

where x^i is the longitudinal displacement of car i. Displacement, however, is not a state of the vehicle (17). We must, therefore, add a range state to the platoon dynamics, using the equation,

$$
\dot{R} = v_x^1 - v_x^2.
$$

The end result is that the platoon will be a fifteen-state system,

$$
\eta = \left\{ \begin{array}{c} \underline{x}^1 \\ \underline{x}^2 \\ R \end{array} \right\} \begin{array}{l} \text{Longitudinal Dynamic States} - Car\#1 \\ \text{Longitudinal Dynamic States} - Car\#2 \\ \text{Range} \end{array}
$$

The corresponding state matrix is

$$
A = \begin{bmatrix} A^L & 0 \\ 0 & A^L \\ E_1 & -E_1 \end{bmatrix}, \tag{19}
$$

$$
E_1 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}.
$$

The measurement matrix is

$$
C = \left[\begin{array}{c|cc} C^L & \multicolumn{2}{c}{0} \\ \hline 0 & C^L & 0 \\ & 0 & 1 \end{array} \right] = \begin{bmatrix} C^1 & 0 \\ 0 & C^2 \end{bmatrix}, \tag{20}
$$

The local measurement sets are such that y^1 is simply (18) with the measurements specific to car # 1 and y^2 is likewise the measurements given in (18) specific to car # 2 with the range measurement appended to the end. Thus, y^2 is an 8 element vector instead of a 7 element vector.

Using the algorithm given in Section 4, we designed a decentralized fault detection filter with global transmission properties as depicted in Figure 2 (See [5]) for details). As we can see, we get fairly good separation between the range sensor fault and the nuisance faults. This is confirmed by Figure 3 which shows the response of the filter to step inputs in either failure channel. As this figure shows, the target fault is transmitted fairly well, while the nuisance fault is largely attenuated.

6. Conclusions

In this paper, we have introduced a decentralized fault detection filter which provides an alternative way to monitor large-scale systems for faults. The resulting filter has additional fault tolerance because it can check the health of its contituent sensors prior to deriving the top level estimate and it is easily scalable for problems which are varying in size such as collections of systems.

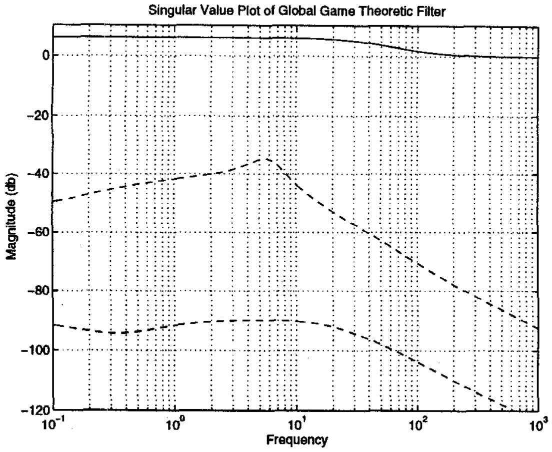

Figure 2: Platoon Example - Signal Transmission in the Global Detection Filter (range sensor fault transmission shown with solid line, nuisance fault transmission shown with dashed line)

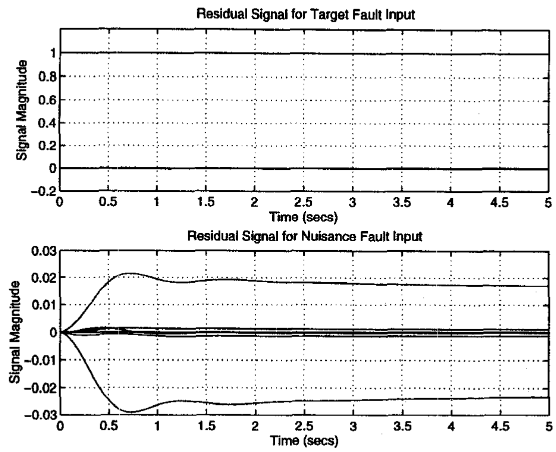

Figure 3: Platoon Example - Failure Signal Response of the Decentralized Fault Detection Filter (Nuisance Fault is a Step Failure in the Longitudinal Accelerometer on Car # 1)

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