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Opinion dynamics on networks with community structure

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Introduction

- *Goal:* study opinion evolution on a social network.
- *Our approach:* incorporate random graph theory by letting the opinion process evolve on a directed Stochastic Block Model (dSBM).
- *Key insight:* as the graph gets denser, individuals aren't really interacting but rather update their beliefs based on an "average" opinion.
- *Technical tools:* mean-field approximation, strong couplings

The Network

- dSBM $G(V_n, E_n)$ with n nodes and K communities.
- Each node $i \in V_n$ has a community label $J_i \in \{1, \ldots, K\}$.
- Kernel: $K \times K$ symmetric matrix κ with larger diagonal values
- Connection probability of nodes i, j: $p_{ij}^{(n)} = \frac{\kappa(J_i,J_j)\theta_n}{n} \wedge 1$, $p_{ii}^{(n)} = 0$

The Model

- $\mathbf{R}_i^{(k)} \in [-1,1]^\ell$: the opinion that node i holds at time k on ℓ topics.
- $\mathbf{W}_i^{(k)} \in [-d,d]^\ell$: media signals that node i receives at time k on ℓ topics.
- The weight node i puts in j's opinion:

$$C_{ij} = \frac{B_{ij}1(j \to i)}{\sum_{r=1}^n B_{ir}1(r \to i)} 1\left(D_i^- > 0, i \neq j\right),$$

  where B_{ij}'s distribution depends only on the communities of i and j.
- Update opinions according to the recursion

$$\mathbf{R}_i^{(k)} = c\sum_{j=1}^n C_{ij}\mathbf{R}_j^{(k-1)} + \mathbf{W}_i^{(k)} + (1-c-d)\mathbf{R}_i^{(k-1)}, \tag{1}$$

  where the $\{\mathbf{W}_i^{(k)} : k \geq 0\}$ are i.i.d. and $0 < c + d \leq 1$.
- Our model takes into account two key-elements in opinion formation:
  1. *Confirmation Bias* (people's tendency to listen to similar opinions as their current opinions): the weights B_{ij} depend only on the communities.
  2. *Selective Exposure* (people's tendency to follow targeted media signals): the media distribution depends only on the community belongings.
- Our model can capture both polarization and consensus.

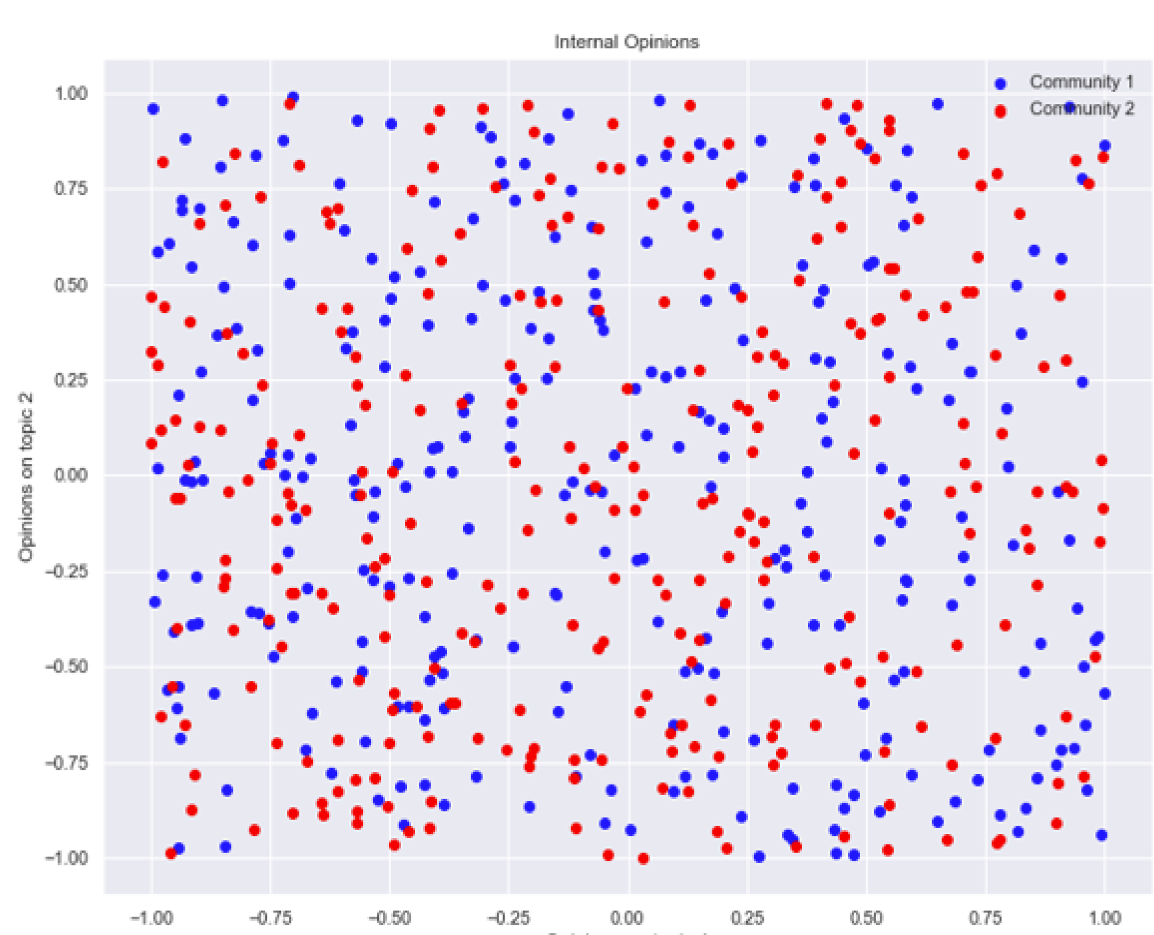

Figure 1: Initial opinions on 2 topics. No clear pattern between nodes from different communities.

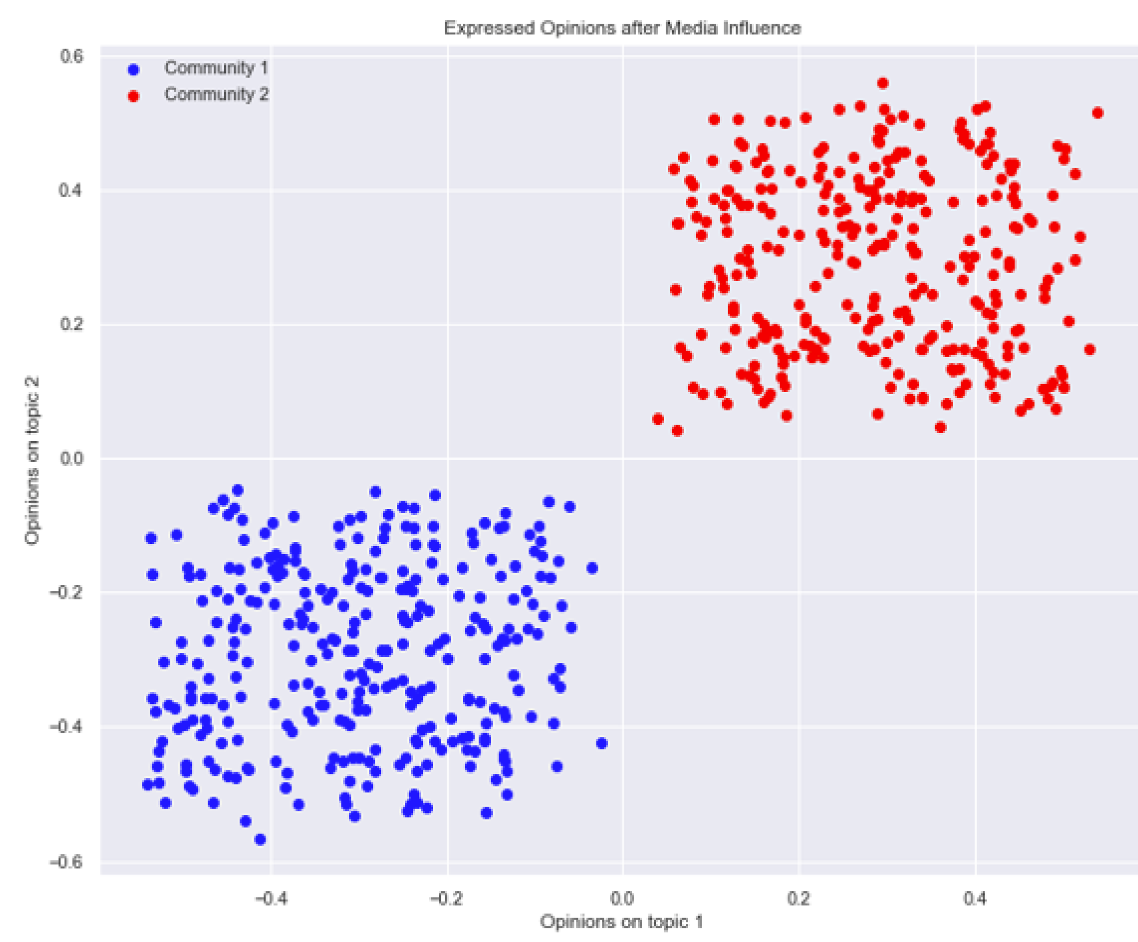

Figure 2: After receiving targeted media signals, the two communities polarize.

Mean-Field Approximation

- *Intuition:* too many individual interactions, so instead update opinions based only on the average interaction.
- In (1), replace the contribution of the neighbors' opinions with their mean:

$$\boldsymbol{\mathcal{R}}_i^{(k)} \approx c\,\mathbb{E}_n\left[\sum_{j=1}^n C_{ij}\mathbb{E}_n\left[\mathbf{R}_j^{(k-1)}\right]\right] + \mathbf{W}_i^{(k)} + (1-c-d)\boldsymbol{\mathcal{R}}_i^{(k-1)}.$$

Main Result

Theorem. Suppose $\theta_n \geq (6H\Lambda_n)^2\Delta_n \log n$. Then, there exists a constant $\Gamma < \infty$ such that

$$\sup_{k\geq 0} \mathbb{E}_n\left[\left\|R^{(k)} - \mathcal{R}^{(k)}\right\|_\infty\right] \leq \Gamma\left(\sqrt{\frac{\log n}{\theta_n}} + \mathcal{E}_n\right), \tag{2}$$

where $\mathcal{E}_n := \max_{1\leq r,s\leq K}\left|\frac{\pi_s^{(n)}\pi_r - \pi_s\pi_r^{(n)}}{\pi_r^{(n)}\pi_s}\right|$. Moreover, for any sequence θ_n satisfying $\theta_n \to \infty$ as $n \to \infty$,

$$\sup_{k\geq 0}\max_{i\in V_n} \mathbb{E}_n\left[\left\|\mathbf{R}_i^{(k)} - \boldsymbol{\mathcal{R}}_i^{(k)}\right\|_1\right] \xrightarrow{P} 0, \tag{3}$$

as $n \to \infty$.

Remarks:

- The components of the limiting process $\{\mathcal{R}^{(k)} : k \geq 1\}$ are independent, so the system exhibits *propagation of chaos.*
- Individuals essentially don't interact, but rather update their opinion based on the average opinion in the system.

Proof ideas:

- Construct an intermediate process $\tilde{R}$ by replacing the random weights C_{ij} with their expectations (loosely speaking).
- Show that the processes $\tilde{R}$ and $\mathcal{R}$ are close regardless of the density regime.
- In the supercritical regime, use concentration inequalities to prove closeness of R and $\tilde{R}$.
- In the subcritical regime, there is not enough averaging to guarantee independence, so use sparse approximation by coupling with a Galton-Watson tree. The branching process gives conditional independence.