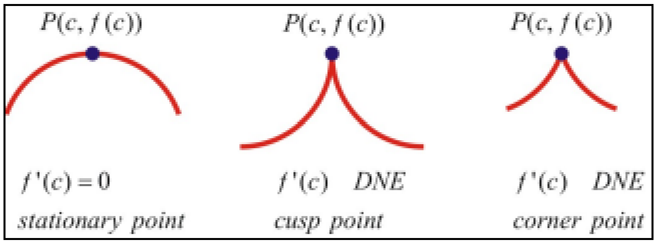
Lesson 6- *Unit 3* – *Derivatives and their applications (1)*

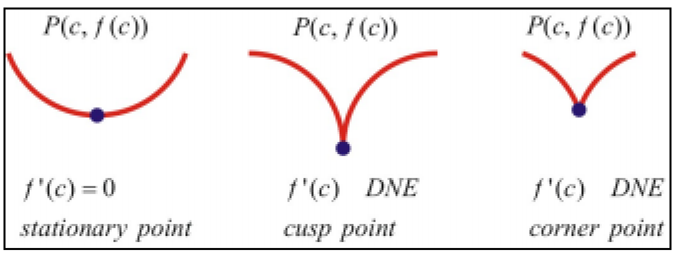
Local Maximum

A function has a local (relative) maximum at x = c if f (x) ≤ f (c) when x is sufficiently close to c(on both sides of c) f(c) is called local (relative) maximum value and (c, f (c)) is called local (relative) maximum point.

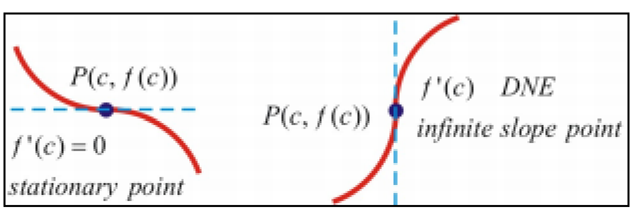


Local Minimum

A function has a local (relative) minimum at f x = c if f (x) ≥ f (c) when x is sufficiently close to c (on both sides of c). f(c) is called local (relative) minimum value and (c, f (c)) is called local (relative) minimum point.

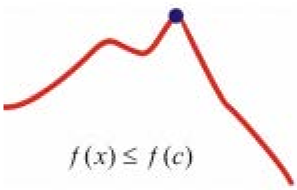


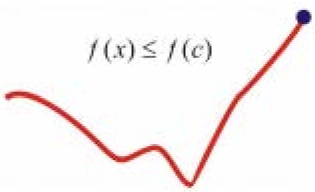
Note: The following points are neither local minimum or maximum points.



Global Maximum

A function f has a global (absolute) maximum at x = c if f (x) ≤ f (c) for all x∈ D_f . f (c) is called the global (absolute) maximum value. (c, f (c)) is called the global (absolute) maximum point.





Global Minimum

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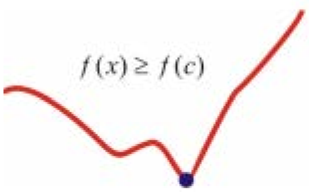
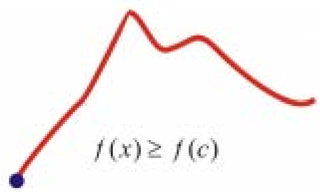
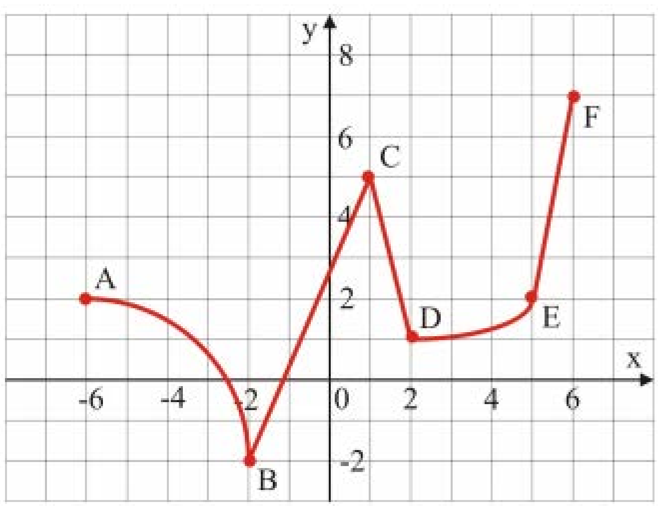




Extremum and Extrema

An extremum is either a minimum or a maximum (value, point, local or global). Extrema is the plural of extremum.

Ex. Find extrema for the function represented in the figure below by its graph.



Ans:

Local minimum points are B(−2,−2) and D(2,1) .

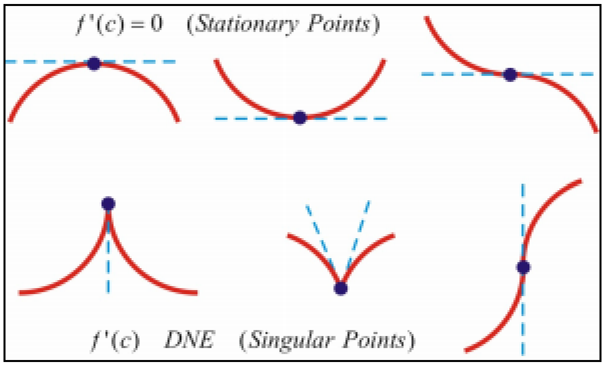
Local maximum points is C(1,5) .

Global minimum point is B(−2,−2) .

Global maximum point is F(6,7) .

Critical Points (Critical Number)

A critical number c is a number in the domain of f where either f'(c) = 0 or f'(c) does not exist. The point (c, f (c)) is called a critical point. If f '(c) = 0 , the critical point is called *stationary point*. If f '(c) does not exist, the critical point is called point of nondifferentiability.



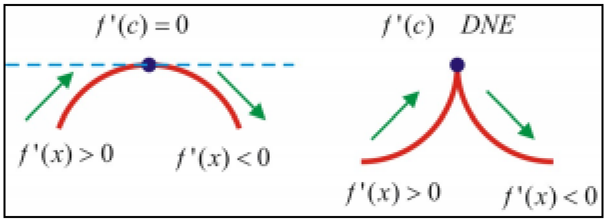
Fermat's Theorem

If f has a local extremum (minimum or maximum) at x = c , then c is a critical number (f '(c) = 0 or f '(c) does not exist).

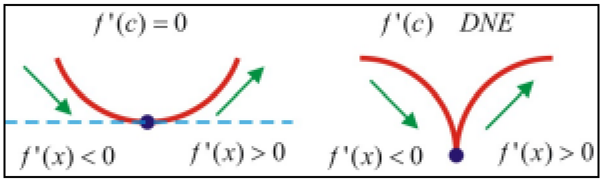
First Derivative Test

Let c be a critical point of a continuous function f.

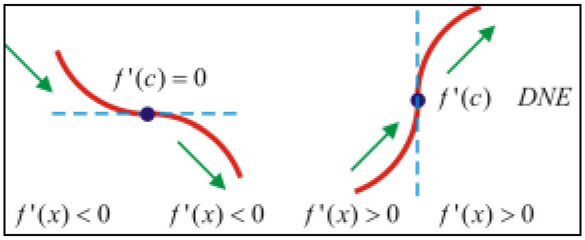
a. If *f'* changes from positive to negative at c, then *f* has a local maximum at c.



b. If *f'* changes from negative to positive at c, then *f* has a local minimum at c.



c. If *f′* does not change sign at c, then has no maximum or minimum at c.



Absolute Extrema Algorithm

To find the absolute extrema for a continuous function f on [a,b] :

a) identify the critical numbers on [a,b]

b) find the values of f at each critical number

c) find the values of at the endpoints of the interval f (a) and f (b)

d) from the values obtained at steps b) and c) the largest value represents the global maximum and the least value represents the global minimum.

Ex. Find the absolute maximum and minimum values of the function *f*(*x*)=3x^4−4x^3 on the interval [−1,2].

Solution

To find the extreme values of *f*(*x*), we first determine the values (if they exist) of the function at the interval's endpoints, the points where (*f′*(*x*)=0), and where the function is undefined.

Note that *f*(*x*) = 3x^4−4x^3 is continuous on the entire interval, and *f′*(*x*)=12x^3 - 12x^2 exists at all values in the interval, so there are no discontinuities, corners, or cusps on *y*=*f*(*x*).

Considering the endpoints: f(-1) = 7 and f(2) = 16.

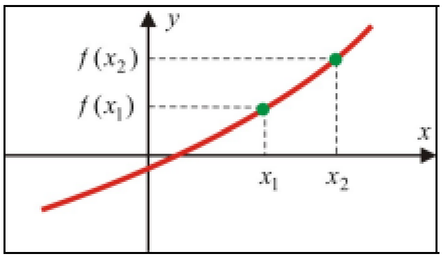
Determining the points where *f′*(*x*)=0:

12x^3 - 12x^2 = 0 ⇨ 12x^2(x-1) = 0 ⇨ x = 0, 1 ⇨ f(0) = 0 and f(1) = -1

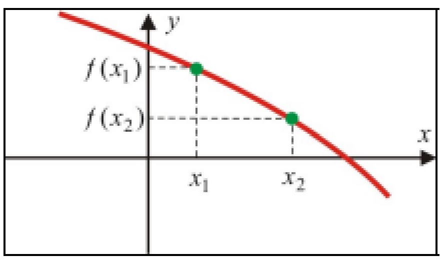
Considering all of the calculated values, the absolute maximum of *f*(*x*) is 16, when *x*=2. The absolute minimum is −1, which occurs when *x*=1.

Increasing and Decreasing Functions

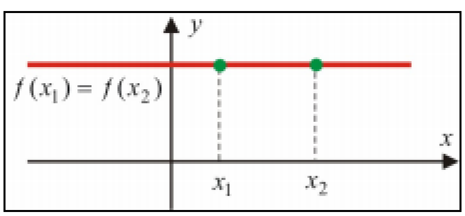
A function f is *increasing* over the interval (a,b) if f (x_1) < f(x_2) whenever x_1 < x_2 in the interval (a,b) .



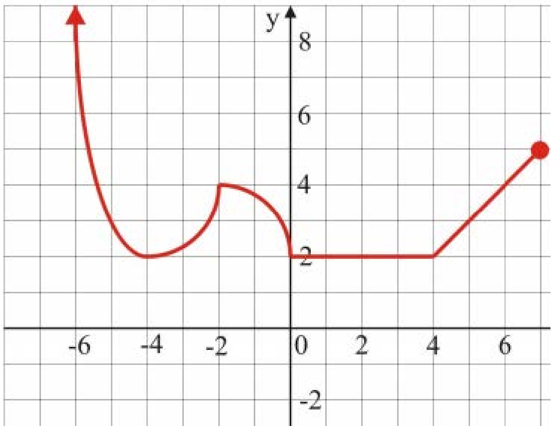
A function f is *decreasing* over the interval (a,b) if f $(x_1) >$ f(x_2) whenever x_1 < x_2 in the interval (a,b) .



A function f is *constant* over the interval (a,b) if f (x_1) = f(x_2) whenever x_1 < x_2 in the interval (a, b).



Ex. Find the intervals where the function y = f (x) is increasing, decreasing, or is constant.



Solution

f is increasing over (−4,−2) and (4,7) .

f is decreasing over (−6,−4) and over (−2,0) .

f is constant over (0,4) .

Test for Intervals of Increase or Decrease

Let y = f (x) be a differentiable function over (a,b) . Then:

1. If f '(x) > 0 for all x∈ (a, b) then f is increasing over (a, b).
2. If f '(x) < 0 for all x∈ (a, b) then f is decreasing over (a, b).
3. If f '(x) = 0 for all x∈(a, b) then f is constant over (a, b).

Ex. Find the intervals of increase or decrease for

f(x) = $2x^3 + 3x^2 - 12x$

Solution

$f'(x) = 6x^2+6x -12$

$f'(x) = 0$ ⇨ $6(x^2+x-2)=0$ ⇨ $x=-2$ or $x=1$

$f(-2) = 20$ and $f(1) = -7$

Sign Chart for f ' :

x		-2		1	
$f(x)$	↗	20	↘	-7	↗
$f'(x)$	+	0	-	0	+

f is increasing over (−∞,−2) and over (1,∞) and is decreasing over (−2,1) .

Ex. Find the intervals of increase or decrease for f (x) = (x − 2) $\sqrt[3]{x^2}$.

Solution

$f(x) = (x-2)x^{\frac{2}{3}}$

$f'(x)= \frac{5x-4}{3\sqrt[3]{x}}$

$f'(x) = 0 \quad \Rightarrow \quad x= \frac{4}{5}$

$f(4/5) \approx -0.345$

$f'(0)$ DNE

$f(0)=0$

Sign Chart for f ' :

x		0		4/5	
$f(x)$	↗	0	↘	−0.345	↗
$f'(x)$	+	DNE	-	0	+

f is increasing over $(-\infty,0)$ and over $(4/5,\infty)$ and is decreasing over $(0,4/5)$.