Chapter 7 Statistics and Probability (2)

1. Probability

Probability is an interesting branch of mathematics that is widely used in genetics, insurance, finance, medicine, sociological surveys, marketing and science. We use probability to measure the **chance** or **likelihood** of an event or events occurring in the future.

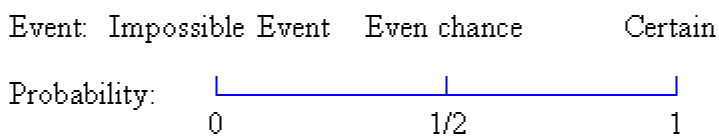
Events
An **event** is something that may or may not occur at some time or during some period in the future.

When we talk about events in terms of chances, we could make statements such as:
"I will **probably** play tennis this summer"
"It isn't **likely** that I will be invited to play in the Australian Open Tennis Tournament"
"My **chances** of winning Tattslotto are not very good"

We could describe the expected occurrence of an event (or events) with the words certain, probable, fifty-fifty, improbable and impossible. These words tell us something about whether or not the event is expected to occur; but the statements are very vague.

We study and apply probability to enable us to better quantify or measure the chance of an event (or events) occurring in the future.

The Probability Scale
If an event is **impossible**, its probability is 0. If an event is **certain** to occur, its probability is 1. The probability of any other event is between these two values.



A result of an event or experiment is called the **outcome.** All the possible outcomes in a probability experiment is called the **sample space.**

How do we calculate probability?

$$\text{Probability of an event happening} = \frac{\text{Number of ways it can happen}}{\text{Total number of outcomes}}$$

Example 1: what is the probability of getting a "Head" when tossing a coin?

Number of ways it can happen: 1 (Head)
Total number of outcomes: 2 (Head and Tail)

So the probability = 1/2 = 0.5

Example 2: what is the probability of getting a "5" or "6" when rolling a die?

Number of ways it can happen: 2 ("5" and "6")
Total number of outcomes: 6 ("1", "2", "3", "4", "5" and "6")

So the probability = 2/6 = 1/3

2. Two or More Independent Events

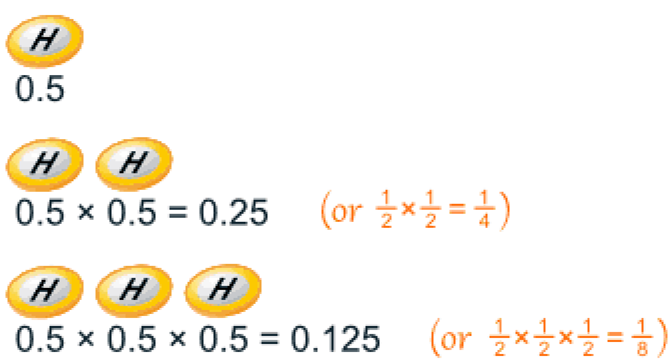
Independent Event is an event occurring does not depend on the results of another event.

We can calculate the chances of two or more **independent** events by **multiplying** the chances.

P(A and B) = P(A) × P(B)

Example 3: Probability of 3 Heads in a Row

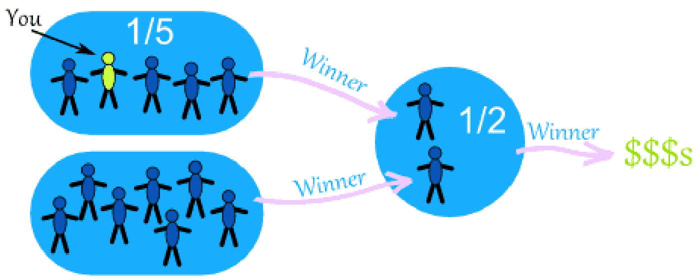
For each toss of a coin a "Head" has a probability of 0.5:



And so the chance of getting 3 Heads in a row is 0.125

So each toss of a coin has a ½ chance of being Heads, but **lots of Heads in a row** is unlikely.

Example 4: Imagine there are two groups. A member of each group gets randomly chosen for the winners circle, **then** one of those gets randomly chosen to get the big money prize:



What is your chance of winning the big prize?

- there is a **1/5 chance** of going to the winners circle
- and a **1/2 chance** of winning the big prize

So you have a 1/5 chance followed by a 1/2 chance ... which makes a 1/10 chance overall:

$$\frac{1}{5} \times \frac{1}{2} = \frac{1}{10}$$

Or we can calculate using decimals (1/5 is 0.2, and 1/2 is 0.5): 0.2 x 0.5 = **0.1**

So your chance of winning the big money is **0.1**

3. Dependent Events

But events can also be "dependent" ... which means they **can be affected by previous events**.

There are 2 blue and 3 red marbles in a bag. What are the chances of getting a blue marble?

The chance is **2 in 5**

But after taking one out the chances change!

So the next time:

- if we got a **red** marble before, then the chance of a blue marble next is **2 in 4**
- if we got a **blue** marble before, then the chance of a blue marble next is **1 in 4**

See how the chances change each time? Each event **depends on** what happened in the previous event, and is called **dependent**.

Example 5: What is the probability of drawing 2 Kings from a deck of card?

Event A is drawing a King first, and **Event B** is drawing a King second.

For the first card the chance of drawing a King is 4 out of 52 (there are 4 Kings in a deck of 52 cards):

P(A) = 4/52

But after removing a King from the deck the probability of the 2nd card drawn is **less** likely to be a King (only 3 of the 51 cards left are Kings):

P(B) = 3/51

P(A and B) = P(A) x P(B) = (4/52) x (3/51) = 12/2652 = **1/221**

So the chance of getting 2 Kings is 1 in 221, or about 0.5%

Practice in class

1. You have a set of 15 pool balls as shown to the right. If you select one ball at random, calculate the probability of selecting the following:

a) P(solid) = b) P(even) =

c) P(solid and even) = d) P(solid or even) =

e) P(even and blue) = f) P(striped and black) =

2. A bag of marbles contains 4 red marbles, 3 green marbles, and 8 blue marbles. What is the probability of selecting the following:

a) P(red) = b) P(red or green) = c) P(2 red marbles) =

d) P(1 green and 1 blue without replacement) =

e) P(2 blue) =

3. If Jeff picks one letter randomly from the alphabet, what is the probability that the letter is in the word ‘probability’?

4. The letters G, A, U, S, and S are written on five tiles, one letter per tile. If Amy selects two tiles at random, what is the probability she gets two S’s?

5. Mark has a bag that contains 3 black marbles, 6 gold marbles, 2 purple marbles, and 6 red marbles. Mark adds a number of white marbles to the bag and tells Susan that if she now draws a marble at random from the bag, the probability of it being black or gold is 3/7. How many white marbles does Mark add to the bag?