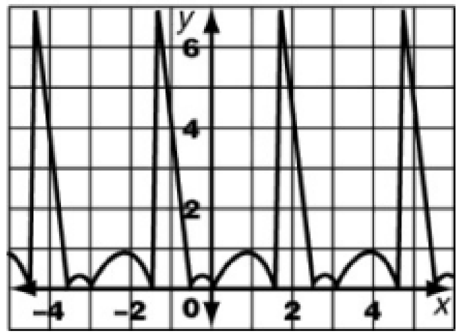
Trigonometric Function (2)

1. Modelling Periodic Behavior



Consider the following graph:
What is unique about this graph?
It has a repeating pattern.
This is a periodic function, because its y - values have a pattern which repeats itself at regular intervals.

Periodic function: A function whose y - values have a pattern which repeats at regular intervals.
A function is periodic if there is a positive number, p, such that **f(x + np) = f(x)** for every x in the domain of f(x) where p is the period and n is any integer. The least value of p that works is the period of the function.

2. Properties of Periodic Functions

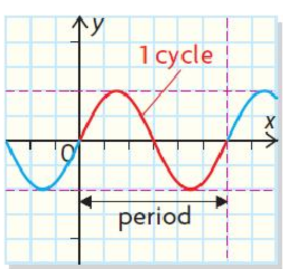
Cycle: One complete pattern.
Period: The horizontal length of one cycle.
(To determine the period, subtract the first x-value of cycle from the last)
Amplitude: Half the distance between the maximum and minimum y - values of a periodic function.

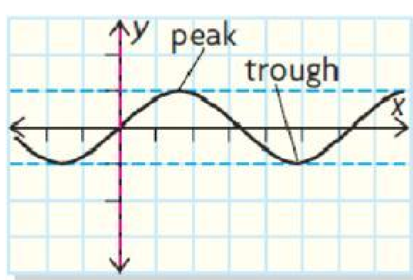
(To determine the amplitude, $a = \frac{\max - \min}{2}$)

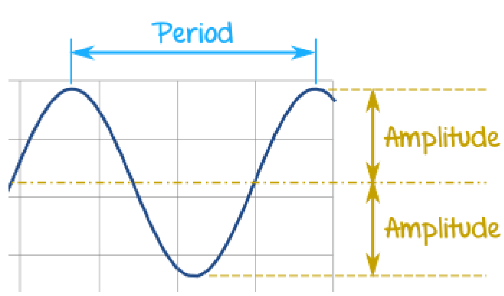
Trough: the minimum point on a graph.
Peak (Crest): the maximum point on a graph

The horizontal line that is halfway between the maximum and the minimum values of a periodic curve is called the **axis of the curve** or the **equilibrium position**. Its equation would be found by finding the average between the maximum and the minimum values

$$y = \frac{\max + \min}{2}$$

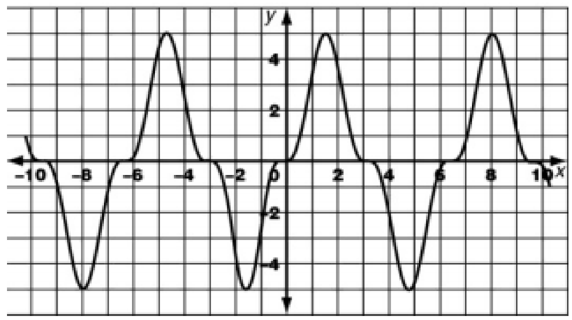






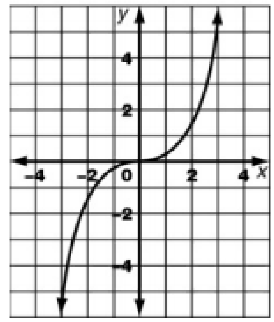
Example 1: For each of the graphs below, state whether or not it is a periodic function. If it is periodic, determine the period and amplitude.

a)



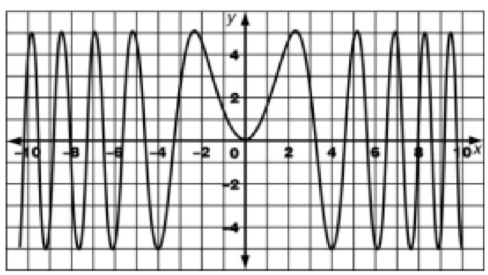
Periodic, p =6.5, a = 5

b)



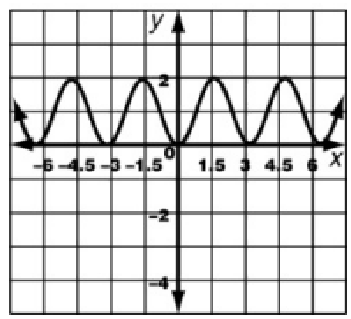
Not periodic

c)



Not periodic

d)



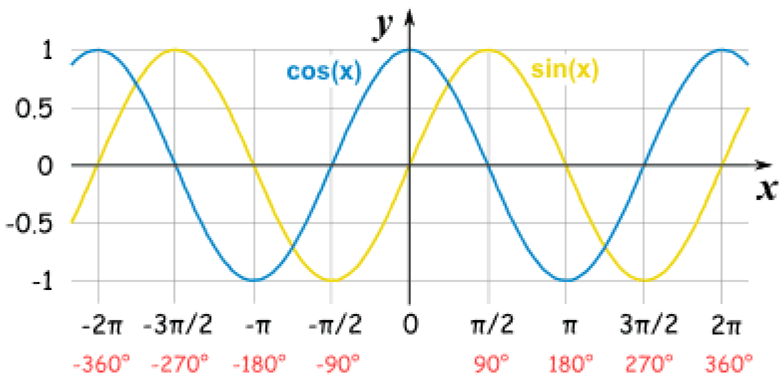
Periodic, p =3, a = 1

3. Graph of Sine and Cosine

Sinusoidal Functions - a periodic function whose graph looks like smooth symmetrical waves, where any portion of the wave can be horizontally translated onto another portion of the curve.

<u>*The difference between Periodic Function and Sinusoidal Function*</u>*:*
A Sinusoidal Function has smooth symmetrical waves and can have any portion of the wave be horizontally translated onto another portion of the curve. A Periodic Function has a repetitive pattern but does not possess all the characteristics that a Sinusoidal Function does.

In other words, a sinusoidal function is a periodic function, but a periodic function is not necessary a sinusoidal function.



Properties

	Sin x	Cos x
Domain	$x \in \mathbb{R}$	$x \in \mathbb{R}$
Range	$y \in \mathbb{R}, -1 \le y \le 1$	$y \in \mathbb{R}, -1 \le y \le 1$
x-int	180^on or $n\pi$, n is any integer	90 + 180^on or $\pi/2 + n\pi$, n is any integer
y-int	0	1
Amplitude	1	1
Period	360^o or 2π	360^o or 2π
Max	1	1
Min	-1	-1
Axis	y = 0	y = 0

4. Transformation

1) Vertical Stretches

Stretching a graph involves introducing a coefficient into the function, whether that coefficient fronts the equation as in $y = 3\sin(x)$ or is acted upon by the trigonometric function, as in $y = \sin(3x)$. Though both of the given examples result in stretches of the graph of $y = \sin(x)$, they are stretches of a certain sort. The first example creates a vertical stretch, the second a horizontal stretch.

To stretch a graph vertically, place a coefficient in front of the function. This transformation is given by the function: f(x) = a sin x. This coefficient "a" is the **amplitude** of the function.

For example, the amplitude of $y = f(x) = \sin(x)$ is one. The amplitude of $y = f(x) = 2\sin(x)$ is two. Note that the period is unchanged and that the curve still passes through the origin. Compare the graphs below.

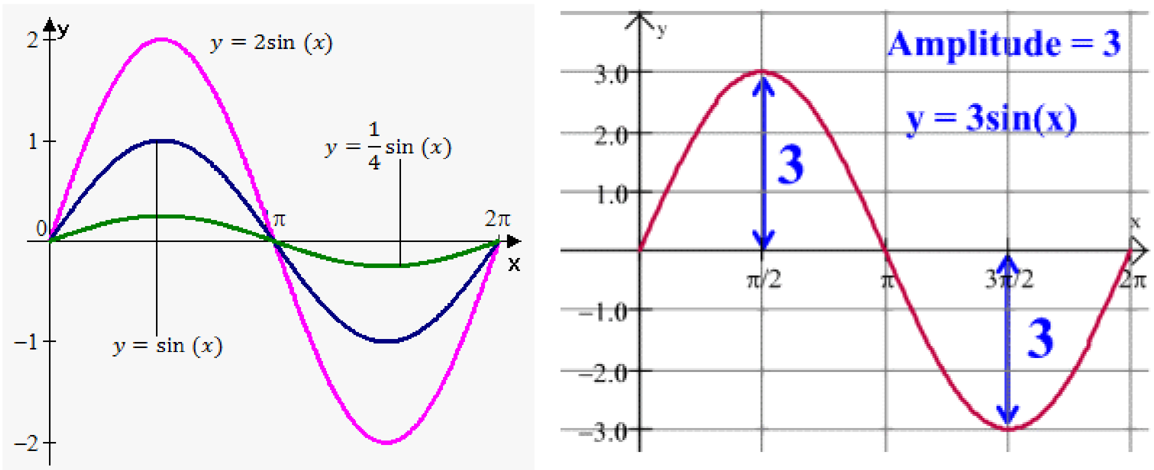


Figure 1: The sine curve is stretched vertically when multiplied by a coefficient

2) Horizontal Stretches

To horizontally stretch the sine function by a factor of k, the function must be altered this way: $y = f(x) = \sin(kx)$. Such an alteration changes the **period** of the function. The **period of a sine function is** $\frac{180°}{k}$**,** where k is the coefficient of the angle. Usually $k = 1$, so the period of the sine function is 180^0. Compare the graphs below. Note that the amplitude has not changed and that the curve still goes through the origin.

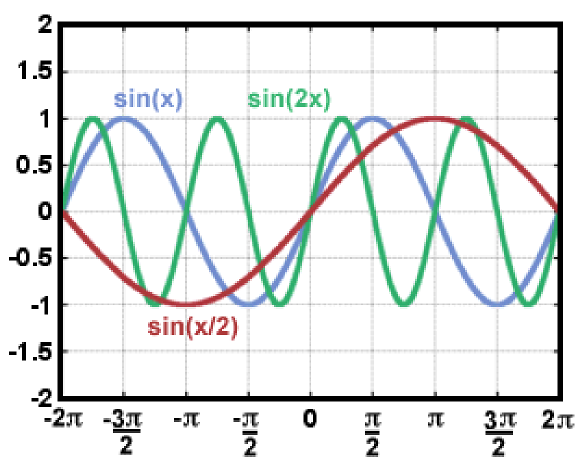
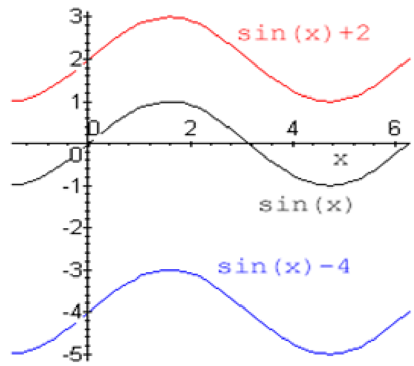


Figure 2: The sine function is stretched horizontally when the angle is multiplied by a scalar

3) Vertical and Horizontal Translation/Shift

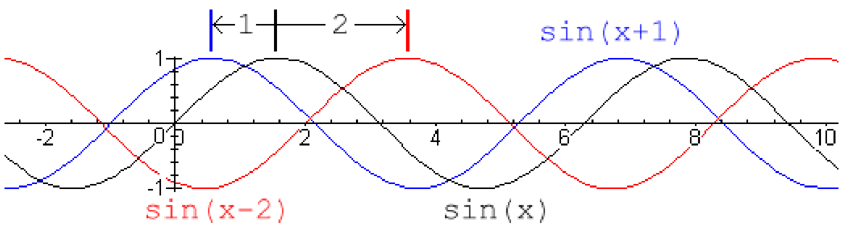
Vertical shift: This transformation is given by the function **f(x) = sin x + d**

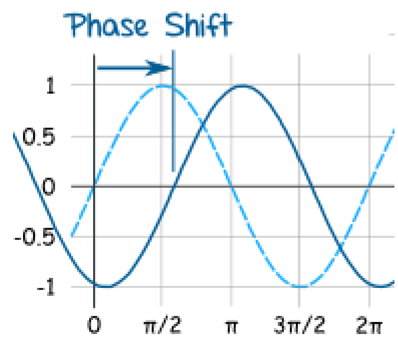
The two curves are identical with the same periods and same amplitudes. However, the parameter d has had the effect of shifting the curve by a constant value d in a vertical direction. Note how the line **y = d** has become the horizontal **axis** which the curve oscillates and from where we measure the amplitude.



Horizontal shift: This transformation is given by the function **f(x) = sin(x − c)**

Note that the curve has been shifted to the right by c. We call c the **phase shift**. The **Phase Shift** is how far the function is **horizontally** to the right of the usual position. The period and amplitude of the curve has not been changed.





4) Combination of Transformations

Transformations can be combined to produce more complex sinusoidal functions.

General Form: y = a sin [k(x – d)] + c   and   y = a cos [k(x – d)] + c

Summary: Given the graph of $y = \sin x$ or $y = \cos x$, the graph of $y = a\sin[k(x-d)]+c$ or $y = a\cos[k(x-d)]+c$ represents

1. a
   - a vertical stretch (compression) by a factor of a
     - changes maximum and minimum values
   - $|a| =$ amplitude
   - if $a < 0$, then there is a reflection in the x-axis
     - the locations of the maximum and minimum values are switched

2. k
   - a horizontal stretch by a factor of $\frac{1}{k}$
   - changes the period of the graph
     - period $= \frac{360^\circ}{k}$

3. d
   - a phase shift (horizontal translation)
     - left if $d < 0$
     - right if $d > 0$

4. c
   - a vertical translation
     - up if $c > 0$
     - down if $c < 0$
   - changes the equation of the axis ("middle" of the graph)
     - equation of the axis becomes $y = c$

Example 1: Sketch $y = 3 \sin [2 (x - \frac{\pi}{4})]$

Identify the properties:

Amplitude = 3

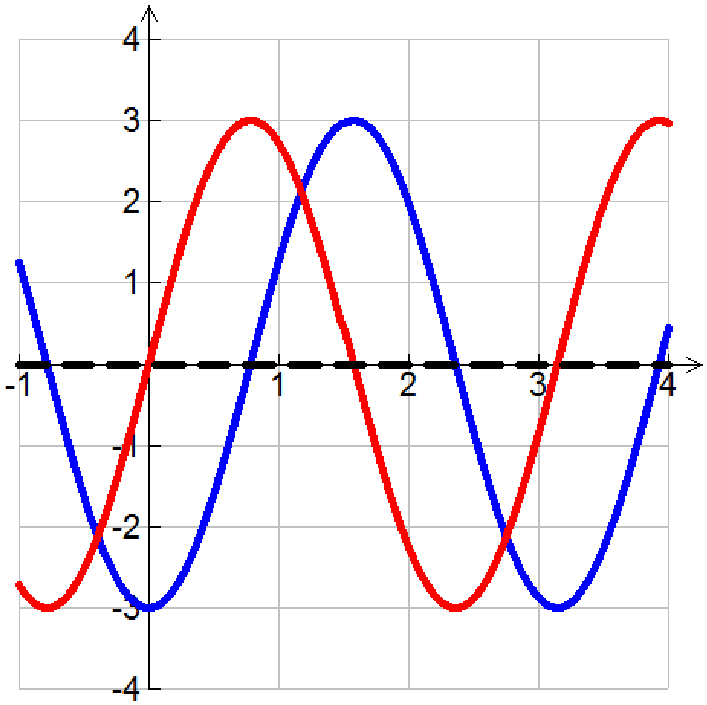
Maximum = 3 and Minimum = -3 and Axis: y = 0 because there is no vertical shift.

Period = $2\pi / 2 = \pi$ or 180°

Horizontal shift $\frac{\pi}{4}$ to the right.

Now, we can graph!

We will graph y = 3sin(2x) first (red). Then we will shift $\frac{\pi}{4}$ = 0.7854 to the right to produce the final graph (blue).



Example 2: Sketch $y = 4\cos\left(\frac{1}{2}x + \frac{\pi}{2}\right) - 1$

Rearrange the function to the standard form: $y = 4\cos\left(\frac{1}{2}(x + \pi)\right) - 1$

Identify the properties:

Amplitude = 4

Maximum = 4-1 = 3 and Minimum = -4-1 = -5 and Axis: y = -1 because there is a vertical shift 1 unit down.

Period = 2π / (1/2) = 4π

Horizontal shift π to the left.

Now, we can graph!

We will graph $y = 4\cos\left(\frac{1}{2}x\right) - 1$ first (red). Then we will shift $\pi = 3.1416$ to the left to produce the final graph (blue).