Algebra 2

1. How to do word problems

The first step to effectively translating and solving word problems is to read the problem entirely. Don't start trying to solve anything when you've only read half a sentence.

Try first to get a feel for the whole problem; try first to see what information you have, and what you still need.

The second step is to work in an organized manner. Figure out what you need but don't have, and name things. Pick variables to stand for the unknowns, clearly labelling these variables with what they stand for. Draw and label pictures neatly. Explain your reasoning as you go along. And make sure you know just exactly what the problem is actually asking for.

You need to do this for two reasons:

- Working clearly will help you think clearly, and
- Figuring out what you need will help you translate your final answer back into English.

The third step is to look for "key" words. Certain words indicate certain mathematical operations. Below is a partial list.

Addition	increased by more than combined, together total of sum added to
Subtraction	decreased by minus, less difference between/of less than, fewer than
Multiplication	of times, multiplied by product of increased/decreased by a factor of (this type can involve both addition or subtraction *and* multiplication!)
Division	per, a out of ratio of, quotient of percent (divide by 100)
Equals	is, are, was, were, will be gives, yields sold for

Note that "per" means "divided by", as in "I drove 90 miles on three gallons of gas, so I got 30 miles per gallon". Also, "a" sometimes means "divided by", as in "When I tanked up, I paid $12.36 for three gallons, so the gas was $4.12 a gallon".

Warning: The "less than" construction is backwards in the English from what it is in the math. If you need to translate "1.5 less than x", the temptation is to write "$1.5 - x$". *Do not do this!*

You can see how this is wrong by using this construction in a "real world" situation: Consider the statement, "He makes $1.50 an hour less than me." You do not figure his wage by subtracting your wage from $1.50. Instead, you subtract $1.50 from your wage. So remember; the "less than" construction is backwards.

Also note that order is important in the "quotient/ratio of" and "difference between/of" constructions. If a problems says "the ratio of x and y", it means "x divided by y", not "y divided by x". If the problem says "the difference of x and y", it means "$x - y$", not "$y - x$".

2. How to do translate

- Translate "the sum of 8 and y" into an algebraic expression.

  This translates to "$8 + y$"

- Translate "4 less than x" into an algebraic expression.

  This translates to "$x - 4$"

Remember? "Less than" is *backwards* in the math from how you say it in words!

- Translate "x multiplied by 13" into an algebraic expression.

  This translates to "$13x$"

- Translate "the quotient of x and 3" into an algebraic expression.

  This translates to "$x/3$"

- Translate "the difference of 5 and y" into an algebraic expression.

  This translates to "$5 - y$"

- Translate "the ratio of 9 more than x to x" into an algebraic expression.

  This translates to "$(x + 9) / x$"

- Translate "nine less than the total of a number and two" into an algebraic expression, and simplify.

  This translates to "$(n + 2) - 9$", which then simplifies to "$n - 7$"

3. Here are some more wordy examples:

1) The length of a football field is 30 yards more than its width. Express the length of the field in terms of its width *w*.

Whatever the width *w* is, the length is 30 more than this. Recall that "more than" means "plus that much", so you'll be adding 30 to *w*.

The expression they're looking for is "*w* + 30".

2) Twenty gallons of crude oil were poured into two containers of different size. Express the amount of crude oil poured into the smaller container in terms of the amount *g* poured into the larger container.

The expression they're looking for is found by this reasoning: There are twenty gallons total, and we've already poured *g* gallons of it. How many gallons are left? There are 20 – *g* gallons left. They want the answer "20 – *g*".

This is the "how much is left" construction: You will be given some total amount. Smaller amounts, of unspecified sizes, are added (combined, mixed, etc) to create this total amount.

You will pick a variable to stand for one of these unknown amounts. After having thus accounted for one of the amounts, the remaining amount is whatever is left after deducting this named amount from the total.

► They may tell you that a trip took ten hours, and that the trip had two legs. You might name the time for the first leg as *"t"*, with the *remaining* time for the second leg being *10 – t*.

► They may tell you that a hundred-pound order of animal feed was filled by mixing products from Bins A, B, and C, and that twice as much was added from Bin C as from Bin A. Let *"a"* stand for the amount from Bin A. Then the amount from Bin C was *"2a"*, and the amount taken from Bin B was the remaining portion of the hundred pounds: *100 – a – 2a*.

I'm making a big deal about this "how much is left" construction because it comes up a lot and tends to cause a lot of confusion. Make sure you understand this one!

Example: Silver's Cleaners decided to raise the price of dry cleaning a sports coat from $4.00 to $5.00. The same percentage increase was applied to dry cleaning a jacket. The old cost of dry cleaning a jacket was $10.00. What is the new cost of dry cleaning a jacket?
Answer:

Let x = percent increase
4.00 * x = 5.00
x = 5.00/4.00 * 100% = 125%
10.00 * 125% = $12.50 is the new cost of cleaning a jacket

Solving Systems of Equations

Example 1: Amanda has two fair options at the toy shop.

(1) She can buy one deck of Pok´emon cards and one Rubik's Cube for $20.

(2) Or for $7, she can buy two decks of Pok´emon cards but she also has to trade in the Rubik's Cube she has at home.

What are the prices of a deck of Pok´emon cards and a Rubik's Cube?

How can you solve this problem?

Notice that we have two unknowns: the price of a deck of Pok´emon cards (x) and the price of a Rubik's Cube (y). We also have two different options that can be represented by two different equations:

(1) one deck + one cube = $20. Mathematically, $1x + 1y = 20$ …(1)

(2) two decks = $7 + one cube. Mathematically, $2x = 7 + 1y$ or $2x - 1y = 7$ …(2)

The goal in this situation is to find x and y such that both equations are satisfied.

A System of Equations is a set of multiple equations dealing with multiple variables. You have to work with all of the equations at the same time in order to solve for the variables.

Observe if you add the two equations. What will happen to y? Yes, y will be cancelled. This method of solving is called **elimination**. By adding or subtracting the equations, the goal is to eliminate a variable.

We are left with a single variable x:

$3x = 27$

$x = 9$

Therefore, the price of a deck of Pok´emon cards is $9. How do we find y? We need to substitute $x = 9$ into either equation (1) or (2). Let us use equation (1) since it looks simpler.

$1(9) + 1y = 20$

$y = 20 – 9 = 11$

Therefore, the price of a Rubik's Cube is $11.

Example 2: A box contains 14 disks, each coloured red, blue or green. There are twice as many red disks as green disks, and half as many blue as green. How many disks are green?

Solution: Let R be the number of red disks. Let B be the number of blue disks. Let G be the number of green disks.

We then have the following equations:

$R + B + G = 14$ …(1)

$R = 2G$ …(2)

$B = G/2$ …(3)

Substitute the values of R and B from equations (2) and (3) into equation (1). This method of solving is called **substitution.**

$2G + G/2 + G = 14$

$(2G + G/2 + G)(2) = 14(2)$

$4G + G + 2G = 28$

$7G = 28$

$G = 4$

Therefore, 4 of the disks are green.

Questions in class (After the class, please do the following questions again)

1. Mathilda's age is one fourth that of Mathusalem's. If Mathusalem is 24 years older, what is Mathilda's age?

2. What is the value of the numerator in the second fraction shown in the diagram that will yield an equation that is true? $\frac{n}{n+1} + \frac{?}{n+1} = 2$

3. If 4 apples, 6 oranges and 4 plums cost $6, and 8 apples, 12 oranges and 6 plums cost $11, how many plums can you buy for $10?

4. Luc, Chantal and Rachelle want to purchase a bicycle which costs $90. Luc can pay twice as much as Chantal while Rachelle can pay the average amount payed by Luc and Chantal. What is the amount paid by Chantal?

5. Marina has a bank containing only pennies and nickels. If the pennies were nickels and the nickels were pennies, she would have exactly $1.00 more. If the total value of the money in her bank is $1.75, how many pennies does Marina have?

6. A cinema complex has 800 seats divided into 3 theatres. There are 270 seats in Theatre 1, and there are 150 more seats in Theatre 2 than in Theatre 3. How many seats are in Theatre 2?

7. Last summer Sam worked for a cycle dealer. The dealer agreed to pay him $210 and a new bike for seven weeks of work. But Sam didn't enjoy the job and quit after four weeks. The dealer gave him $21 and the bike. How much was the bike worth?

8. Claude has in his hand 15 coins (either pennies, nickels or dimes) worth 79 cents in total. How many dimes does he have?