Algebra

1. Commutative, Associative, and Distributive Laws

1) Commutative Laws

The "Commutative Laws" say you can swap numbers over and still get the same answer.
When you add:
$a + b = b + a$
or when you multiply:
$a \times b = b \times a$

2) Associative Laws

The "Associative Laws" say that it doesn't matter how you group the numbers.
When you add:
$(a + b) + c = a + (b + c)$
or when you multiply:
$(a \times b) \times c = a \times (b \times c)$

Examples:

This:	$(2 + 4) + 5 = 6 + 5 = 11$
Has the same answer as this:	$2 + (4 + 5) = 2 + 9 = 11$
This:	$(3 \times 4) \times 5 = 12 \times 5 = 60$
Has the same answer as this:	$3 \times (4 \times 5) = 3 \times 20 = 60$

Sometimes it is easier to add or multiply in a different order:

What is 19 + 36 + 4?
$19 + 36 + 4 = 19 + (36 + 4) = 19 + 40 = 59$

Or to rearrange a little:
What is $2 \times 16 \times 5$?
$2 \times 16 \times 5 = (2 \times 5) \times 16 = 10 \times 16 = 160$

3) Distributive Law

The "Distributive Law" is the BEST one of all, but needs careful attention.

This is what it lets you do:
3 lots of (2+4) is the same as 3 lots of 2 plus 3 lots of 4
So, the 3× can be "distributed" across the 2+4, into 3×2 and 3×4
Try the calculations yourself:
$3 \times (2 + 4) = 3 \times 6 = 18$

$3 \times 2 + 3 \times 4 = 6 + 12 = 18$

Either way gets the same answer.
So, the "Distributive Law" says:
You get the same answer when you:
Multiply a number by a group of numbers added together, or do each multiply separately then add them like this:
$a \times (b + c) = a \times b + a \times c$

Sometimes it is easier to break up a difficult multiplication:
Example: What is 6×204?
$6 \times 204 = 6 \times 200 + 6 \times 4 = 1{,}200 + 24 = 1{,}224$

Or to combine:
Example: What is $16 \times 6 + 16 \times 4$?
$16 \times 6 + 16 \times 4 = 16 \times (6+4) = 16 \times 10 = 160$

You can use it in subtraction too:
Example: $26 \times 3 - 24 \times 3$
$26 \times 3 - 24 \times 3 = (26 - 24) \times 3 = 2 \times 3 = 6$

You could use it for a long list of additions, too:
Example: $6 \times 7 + 2 \times 7 + 3 \times 7 + 5 \times 7 + 4 \times 7$
$6 \times 7 + 2 \times 7 + 3 \times 7 + 5 \times 7 + 4 \times 7 = (6+2+3+5+4) \times 7 = 20 \times 7 = 140$

Don't go too far! These laws are to do with adding or multiplying, not dividing or subtracting.
The Commutative Law does not work for division:
Example:
$12 / 3 = 4$, but $3 / 12 = \frac{1}{4}$

The Associative Law does not work for subtraction:
Example:
$(9 - 4) - 3 = 5 - 3 = 2$, but $9 - (4 - 3) = 9 - 1 = 8$

The Distributive Law does not work for division:
Example:
$24 / (4 + 8) = 24 / 12 = 2$, but $24 / 4 + 24 / 8 = 6 + 3 = 9$

Summary

Commutative Laws:	$a + b = b + a$ $a \times b = b \times a$
Associative Laws:	$(a + b) + c = a + (b + c)$ $(a \times b) \times c = a \times (b \times c)$
Distributive Law:	$a \times (b + c) = a \times b + a \times c$

2. Exponents

Rule 1: $(x^{m})(x^{n}) = x^{(m+n)}$

Example 1: Simplify $(x^3)(x^4)$

$(x^3)(x^4) = (xxx)(xxxx) = xxxxxxx = \mathbf{x^7}$ or, $(x^3)(x^4) = x^{3+4} = x^7$

Example 2: Simplify $(x^2)^4$

$(x^2)^4 = (x^2)(x^2)(x^2)(x^2) = (xx)(xx)(xx)(xx) = xxxxxxxx = x^8$ or, $(x^2)^4 = x^{(2\times 4)} = x^8$

Rule 2: $(x^{m})^{n} = x^{mn}$

Example 1: $(xy^2)^3 = (xy^2)(xy^2)(xy^2) = (xxx)(y^2y^2y^2) = (xxx)(yyyyyy) = x^3y^6 = (x)^3(y^2)^3$.

Another example would be:

$$\left(\frac{x}{y}\right)^2 = \frac{x^2}{y^2}$$

Example 2: $(x - 2)^2 = (x - 2)(x - 2) = xx - 2x - 2x + 4 = x^2 - 4x + 4$

Rule 3: Anything to the power zero is just "1"

$x^0 = 1$

Example 1: $(x + y)^0 = 1$

Example 2: Simplify $[(3x^5y^7z^3)^5(\text{-}5xyz)^2]^0$

$[(3x^5y^7z^3)^5(\text{-}5xyz)^2]^0 = 1$

2. Fractions in Algebra

You can add, subtract, multiply and divide fractions in algebra in the same way that you do in simple arithmetic.

1) Adding Fractions

To add fractions there is a simple rule: $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$

Example:

$$\frac{x+4}{3}+\frac{x-3}{4}=\frac{(x+4)(4)+(3)(x-3)}{3\times 4}=\frac{4x+16+3x-9}{12}=\frac{7x+7}{12}$$

2) Subtracting Fractions

Subtracting fractions is very similar to adding, except that the + is now -

$$\frac{a}{b}-\frac{c}{d}=\frac{ad-bc}{bd}$$

Example:

$$\frac{x+2}{x}+\frac{x}{x-2}=\frac{(x+2)(x-2)-(x)(x)}{x(x-2)}=\frac{x^2-2^2-x^2}{x^2-2x}=\frac{-4}{x^2-2x}$$

3) Multiplying Fractions

Multiplying fractions is the easiest one of all, just multiply the tops together, and the bottoms together:

$$\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$$

Example:

$$\frac{3x}{x-2}\times\frac{x}{3}=\frac{(3x)(x)}{3(x-2)}=\frac{x^2}{x-2}$$

4) Dividing Fractions

To divide fractions, first "flip" the fraction you want to divide by, then use the same method as for multiplying:

$$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}=\frac{ad}{bc}$$

Example:

$$\frac{3y^2}{x+1}\div\frac{y}{2}=\frac{3y^2}{x+1}\times\frac{2}{y}=\frac{(3y^2)(2)}{(x+1)(y)}=\frac{6y^2}{(x+1)(y)}=\frac{6y}{x+1}$$

3. Complex Fractions

A complex fraction is a fraction where the numerator, denominator, or both contain a fraction.

- Example 1: $\frac{3}{1/2}$ is a complex fraction. The numerator is 3 and the denominator is 1/2.
- Example 2: $\frac{3/7}{9}$ is a complex fraction. The numerator is 3/7 and the denominator is 9.
- Example 3: $\frac{3/4}{9/10}$ is a complex fraction. The numerator is 3/4 and the denominator is 9/10.

Rule: To manipulate complex fractions, just convert them to simple fractions.

Example: Convert $\frac{3}{1/2}$ to a simple fraction and reduce.

Answer: $\frac{3}{1/2} = 3 \div \frac{1}{2} = \frac{3}{1} \div \frac{1}{2} = \frac{3}{1} \times \frac{2}{1} = 6$

4. Remember these Identities

1) $(x+a)(x+b) = x^2 + (a+b)x + ab$

2) $(a \pm b)^2 = a^2 \pm 2ab + b^2$

3) $(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$

4) $a^2 - b^2 = (a-b)(a+b)$

5) $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

6) $a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \cdots + ab^{n-2} + b^{n-1})$ (n is a positive integer)

7) $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

8) $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$

Questions in class

1. Janet has 10 coins consisting of nickels, dimes, and quarters. Seven of the coins are either dimes or quarters, and eight of the coins are either dimes or nickels. How many dimes does Janet have?

2. The digits 1, 2, 3, 4 can be arranged to form twenty-four different four-digit numbers. If these twenty-four numbers are then listed from the smallest to largest, in what position is 3142?

3. The product of 20^{50} and 50^{20} is written as an integer in expanded form. What is the number of zeros at the end of the resulting integer?

4. If $\frac{97}{19} = w + \cfrac{1}{x + \cfrac{1}{y}}$, where w, x, y are all positive integers, then $w + x + y$ equals what?

5. The sum of the digits of a five-digit positive integer is 2. (A five-digit integer cannot start with zero.) What is the number of such integers?

6. The numbers 123 456 789 and 999 999 999 are multiplied. How many of the digits in the final result are 9's?

7. For how many different values of k is the 4-digit number $7k52$ divisible by 12?

8. In the expression $\frac{a}{b} + \frac{c}{d} + \frac{e}{f}$ each letter is replaced by a different digit from 1, 2, 3, 4, 5, and 6. What is the largest possible value of this expression?

9. Pierre celebrated his birthday on February 2, 2001. On that day, his age equaled the sum of the digits in the year in which he was born. In what year was Pierre born?

10. Five points are located on a line. When the ten distances between pairs of points are listed from smallest to largest, the list reads: 2, 4, 5, 7, 8, k, 13, 15, 17, 19. What is the value of k?