Electric Field

Unit 3: Force Fields

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Where Are We In the Course

1. Fundamentals of Dynamics
2. Momentum, Impulse and Energy
3. Gravitational, Electric and Magnetic Fields
4. Wave Nature of Light
5. Theory of Special Relativity
6. Introduction to Quantum Mechanics

The Charges Are

Let's Review Some Basics

We already know quite a bit about charge particles:

- A **proton** carries a **positive** charge
- An **electron** carries a **negative** charge
- A *net charge* of an object means an excess of protons or electrons
- From Physics 11: electrostatic force is one of the fundamental forces in the universe

We will start with electrostatics:

- Charges that are not moving relative to one another

Coulomb's Law for Electrostatic Force

The magnitude of the **electrostatic force** (aka **coulomb force**) between two *point charges* has a very similar form compared to gravitational force:

$$\boxed{F_q = \frac{k\left|q_1 q_2\right|}{r^2}}$$

Quantity	Symbol	SI Unit
Magnitude of electrostatic force	F_q	N (newtons)
Coulomb's constant (electrostatic constant)	k	$\mathrm{N \cdot m^2 / C^2}$
Point charges 1 and 2	q_1, q_2	C (coulombs)
Distance between point charges	r	m (metres)

Coulomb's Constant: $k = \dfrac{1}{4\pi\epsilon_0} = 8.99 \times 10^9\ \mathrm{N \cdot m^2 / C^2}$

Comparing $\mathbf{F}_g$ to $\mathbf{F}_q$

Electrostatic Force (point charge):

$$\boxed{F_q = \frac{k|q_1 q_2|}{r^2}}$$

Gravitational Force (point mass):

$$\boxed{F_g = \frac{G m_1 m_2}{r^2}}$$

- Similarities:
  - Both inversely proportional to r^2 (inverse square law)
  - Both are scaled by a constant
- Difference:
  - For gravity only positive mass (only attraction)
  - Electric charge can be either positive or negative (charges can attract or repel)

Think Electric Field

We can get **electric field** by repeating the same procedure as with gravitational field. Grouping the variables in the equation for electrostatic force:

$$F_q = \underbrace{\left[\frac{k|q_1|}{r^2}\right]}_{E} q_2$$

A source charge q_s creates an "electric field" (E) with an intensity (i.e. magnitude)

$$E(q_s, r) = \frac{k|q_s|}{r^2}$$

Similar to gravitational field, electric field $\mathbf{E}$ created by q is a function ("vector field") that shows how it influences other charged particles around it

Electric Field Intensity Near a Point Charge

The electric field intensity from a point charge is proportional to the charge q_s and inversely proportional for the square of the distance r from the charge.

$$\boxed{E(q_s, r) = \frac{k|q_s|}{r^2}}$$

Quantity	Symbol	SI Unit
Electric field intensity	E	N/C
Coulomb's constant	k	$\mathrm{N \cdot m^2/C^2}$
Source charge	q_s	C
Distance from source charge	r	m

The direction of the field is radially outward from a positive point charge and radially inward towards a negative charge.

Think Electric Field

$\mathbf{E}$ *doesn't do anything* until another charge interacts with it. And when there is a charge q, the electric force $\mathbf{F}_q$ that it experiences in the presence of $\mathbf{E}$ is:

$$\boxed{\mathbf{F}_q = q\mathbf{E}}$$

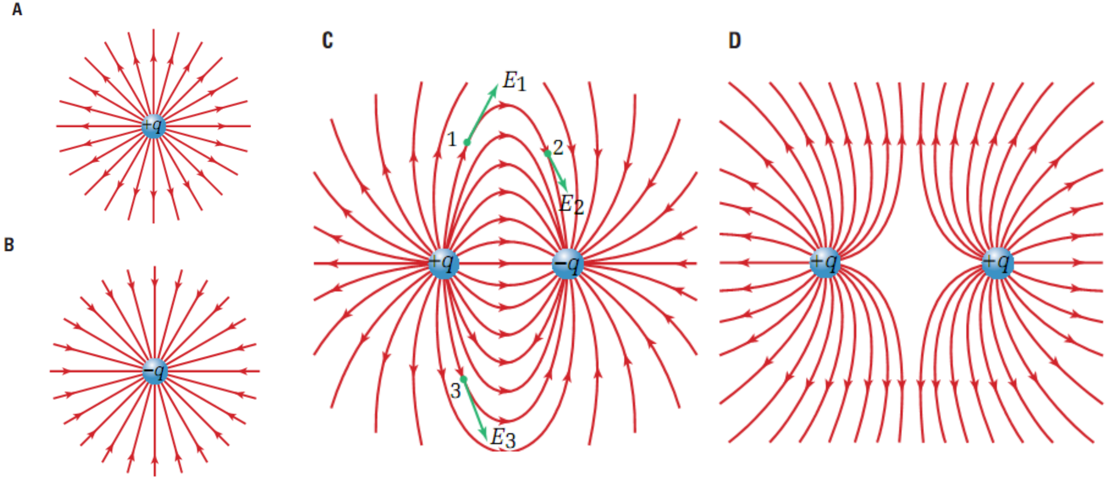
$\mathbf{F}_q$ and $\mathbf{E}$ are vectors, and following the principle of superposition, i.e.

$$\mathbf{F}_q = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4 \ldots$$
$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \mathbf{E}_4 \ldots$$

If there are multiple charges, then the total electric field is the vector sum of all the electric field from each source charge.

Field Lines

Electric field lines: if you place a positive charge in an electric field, the force on the charge will be in the direction of the electric field.

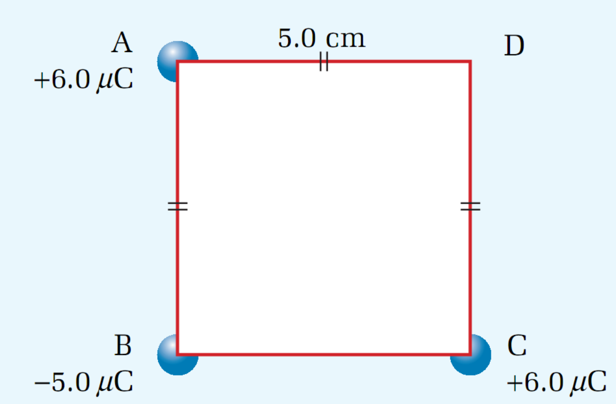


Example Problem

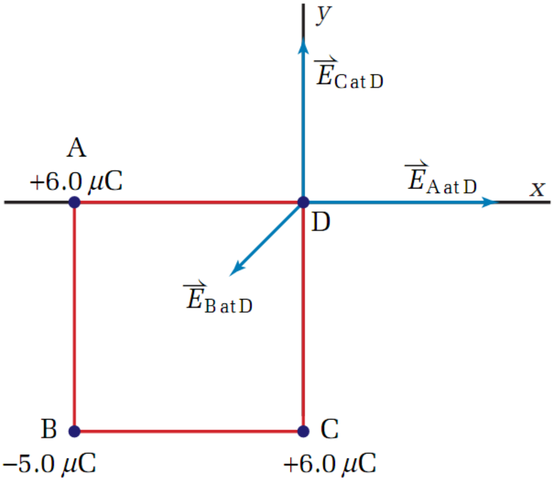
Example 2: What is the electric field intensity at a point $30.0\,\text{cm}$ from the centre of a small sphere that has a positive charge of $2.0 \times 10^{-6}\,\text{C}$?
$(k = 8.99 \times 10^{9}\,\text{N} \cdot \text{m}^2/\text{C}^2)$

Example Problem

Example 3: Three charges, A($6.0\,\mu\text{C}$), B($-5.0\,\mu\text{C}$), and C($6.0\,\mu\text{C}$), are located at the corners of a square with sides that are $5.0\,\text{cm}$ long. What is the electric field intensity at point D?



Example Problem (Cont.)



Electric Potential Energy

To bring two charges together, I have to do work against electrostatic force, therefore, there is a gain/lost in **electric potential energy**

$$\boxed{U_q = \frac{kq_1q_2}{r}}$$

Quantity	Symbol	SI Unit
Electric potential energy	U_q	J
Coulomb's constant	k	$\mathrm{N \cdot m^2 / C^2}$
Electric charges	q_1, q_2	C
Distance	r	m

U_q can be positive or negative, because charges can be positive or negative

How it Differs from Gravitational Potential

Two positive charges:

$$U_q > 0$$

Two negative charges:

$$U_q > 0$$

One positive and one negative charge:

$$U_q < 0$$

- $U_q > 0$ means that work is done to bring two charges together (true if both charges have the same sign)
- $U_q < 0$ means that work is done to pull two charges apart (true if the charges have opposite signs)
- For gravitational potential U_g is always < 0

Electric Potential

Start with an Analogy

Think back to gravitational potential energy: Whenever an object with mass m is in a gravitational field, its potential energy is always proportional to its mass, i.e. gravitational potential energy is a "constant" times *any* mass:

$$U_g = Km$$

If the mass is doubled, the stored gravitational potential energy will also double. In the simple case (where g is constant), that constant is just:

$$K = \frac{U_g}{m} = gh$$

That constant is called the **gravitational potential**, which is the *gravitaitonal potential energy per unit mass*.

Electric Potential

This is also true for moving a charged particle against an electric force, and the constant is called the **electric potential**, which is also called “electric potential energy per unit charge”. For a point charge, it is defined as

$$\boxed{V = \frac{U_q}{q} = \frac{kq_s}{r}}$$

The unit for electric potential is a *volt* which is *one joule per coulomb*:

$$1\,\mathrm{V} = 1\,\mathrm{J/C}$$

Electric Potential Difference (Voltage)

The change in electric potential is called the **electric potential difference** or **voltage**:

$$\boxed{\Delta V = \frac{\Delta U_q}{q}}$$

Here, we can relate ΔV to an equation that we knew from Grade 11 Physics, which related to the energy dissipated in a resistor in a circuit ΔU_q to the voltage drop ΔV:

$$\Delta U = q\Delta V$$

Electric potential difference also has the unit *volts* (V)

Getting Those Names Right

Remember that these three quantities are all scalars, as opposed to electric force $\mathbf{F}_q$ and electric field $\mathbf{E}$ which are vectors

- Electric potential energy:

$$U = \frac{kq_1q_2}{r}$$

- Electric potential:

$$V = \frac{kq}{r}$$

- Electric potential difference (voltage):

$$\Delta V = \frac{\Delta U_q}{q}$$

Relating U_q, $\mathbf{F}_q$ and $\mathbf{E}$

Our Integrals In Reverse

Using vector calculus, we find the relationsdhip between electric force ($\mathbf{F}_q$) and electric potential energy (U_q), and electric field ($\mathbf{E}$) to the electric potential (V):

$$\mathbf{F}_q = -\nabla U_q = -\frac{dU_q}{dr}\hat{\mathbf{r}} \qquad \mathbf{E} = -\nabla V = -\frac{dV}{dr}\hat{\mathbf{r}}$$

From the work-energy theorem:

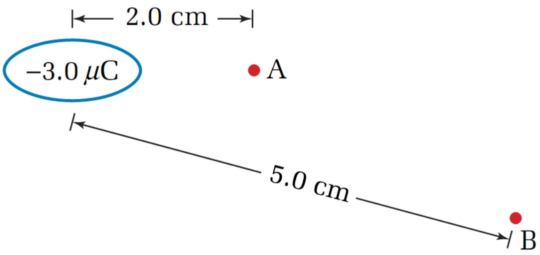
- Electric force $\mathbf{F}_q$ always points from high potential to low potential energy
- Electric field can also be expressed as the change of electric potential per unit distance, which has the unit

$$1\,\mathrm{N/C} = 1\,\mathrm{V/m}$$

- Electric field is also called “electric potential gradient”

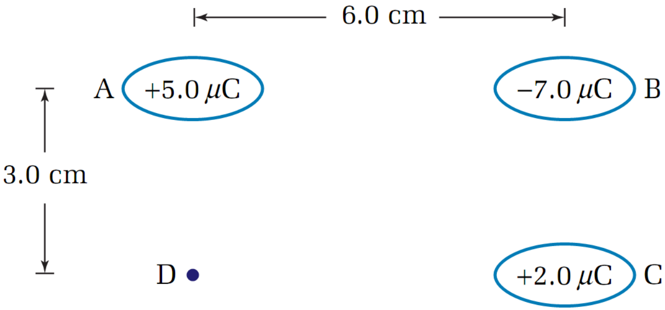
Example Problem

Example 4: A small sphere with a charge of $-3.0\,\mu\text{C}$ creates an electric field. Calculate the electric potential at point A, located $2.0\,\text{cm}$ from the source charge, and at point B, located $5.0\,\text{cm}$ from the same source charge. Which point is at higher potential?



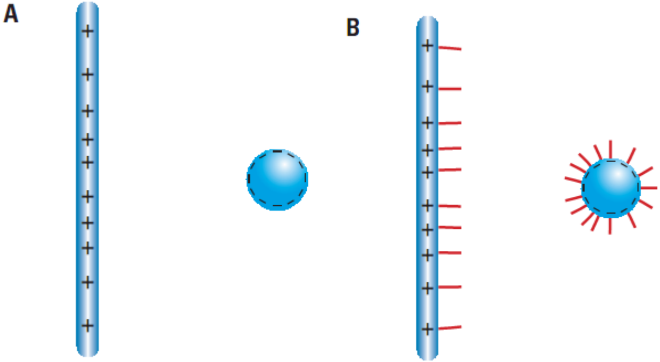
Example Problem

Example 5: The diagram shows three charges, A($5.0\,\mu$C), B($-7.0\,\mu$C), and C($2.0\,\mu$C), placed at three corners of a rectangle. Point D is the fourth corner. What is the electric potential at point D?

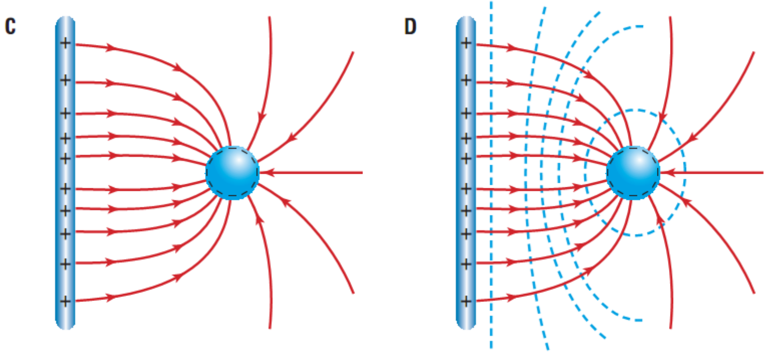


Field Structure

How should we draw the field lines?

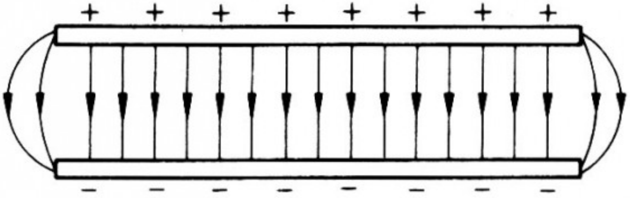


Field Structure



The dotted blue lines are called "equipotential lines". They're always *perpendicular* to the electric field lines. Charges moving in the direction of the equipotential lines do not lose potential energy

Electric Field between Two Parallel Plates



- $\mathbf{E}$ is uniform at all points between the parallel plates, independent of position
- E is proportional to the charge density (charge per unit area) on the plates:

$$E \propto \sigma \quad \text{where} \quad \sigma = \frac{q}{A}$$

- E outside the plates is very low (close to zero), except for the fringe effects at the edges of the plates.

Example Problem

Example 6: An identical pair of metal plates is mounted parallel on insulating stands $20\,\mathrm{cm}$ apart and equal amounts of opposite charges are placed on the plates. The electric field intensity at the midpoint between the plates is $400\,\mathrm{N/C}$.

1. What is the electric field intensity at a point $5.0\,\mathrm{cm}$ from the positive plate?
2. If the same amount of charge was placed on plates that have twice the area and are $20\,\mathrm{cm}$ apart, what would be the electric field intensity at the point $5.0\,\mathrm{cm}$ from the positive plate?
3. What would be the electric field intensity of the original plates if the distance of separation of the plates was doubled?

Electric Field and Potential Difference

A few slides ago we introduced (using some calculus) the relationship between electric field ($\mathbf{E}$) and electric potential difference (V). In a uniform electric field (e.g. parallel plate) the relationship simplifies to a very simple equation:

$$\boxed{E = \frac{\Delta V}{d}}$$

Quantity	Symbol	SI Unit
Electric field intensity	E	N/C (newtons per coulomb)
Electric potential difference	ΔV	V (volts)
Distance between parallel plates	d	m (metres)