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### **Identities and Equations (1)**

1. Use a counterexample to show that  $\tan(\theta) + \sin(\theta) = \cot(\theta) + \cos(\theta)$  is not an identity.

2. Consider the trigonometric function  $f(x) = \frac{\tan(x) + \sin(x)}{1 + \cos(x)}$ .

- a. Identify the non-permissible values of  $x$ .
- b. Graph  $y = f(x)$  using graphing technology.
- c. Use this graph to help create a possible trigonometric identity involving  $f(x)$ .
- d. Prove your identity from part b) is true for all permissible values of the variable.

3. Prove.

$$\text{a. } \csc^2(x) - \csc(x) \cot(x) = \frac{1}{1+\cos(x)}$$

$$\text{b. } \frac{\sin(\theta)+1}{1-\sin(\theta)} = (\tan(\theta) + \sec(\theta))^2$$

$$\text{c. } \frac{\tan(\beta)-\sin(\beta)}{\sin^3(\beta)} = \frac{\sec(\beta)}{1+\cos(\beta)}$$

$$\text{d. } \frac{\cos^3(\beta)+\sin^3(\beta)}{\sin(\theta)+\cos(\theta)} = 1 - \sin(\theta) \cos(\theta)$$

$$\text{e. } (\csc(\theta) \sec(\theta))^2 - \frac{(1-\tan^2(\theta))^2}{\tan^2(\theta)} = 4$$

$$\text{f. } \frac{\sin(x+y)}{\sin(x-y)} = \frac{\tan(x)+\tan(y)}{\tan(x)-\tan(y)}$$

4. Determine the exact value of each trigonometric ratio. Express answers in simplest form.

a.  $\sin(195^\circ)$

b.  $\cos(\frac{19\pi}{12})$

5. Given  $\sin(\theta) = \frac{15}{17}$  and  $\cos(\beta) = \frac{1}{3}$ , where  $\frac{\pi}{2} < \theta < \pi$  and  $\frac{3\pi}{2} < \beta < 2\pi$ , determine the exact value of  $\cos(\theta + \beta)$ .

6. Simplify each expression to a single trigonometric ratio.

a.  $\sin(3x)\cos(x) - \cos(3x)\sin(x)$

b.  $\sin(\frac{\pi}{5})\sin(\frac{\pi}{3}) - \cos(\frac{\pi}{5})\cos(\frac{\pi}{3})$

c.  $\frac{1 - \tan(80^\circ)\tan(20^\circ)}{\tan(80^\circ) + \tan(20^\circ)}$