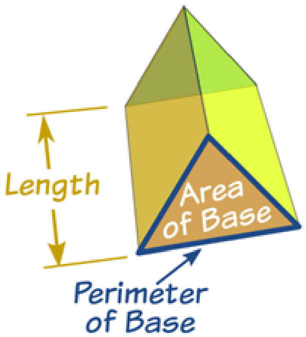
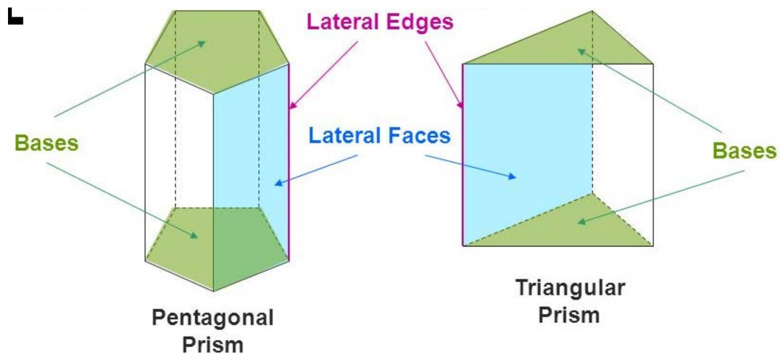
Chapter 8 Geometry 3 (Volume)

1. Volume of 3D Figures - is the amount of space it occupies. The volume of any figure is measured in **cubic units** (or units cubed).

2. Volume of Prisms & Cylinder

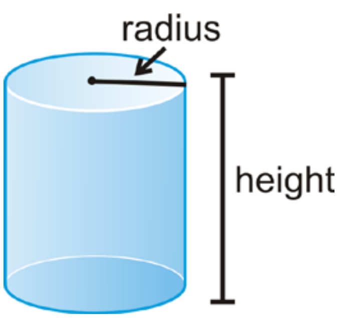
A **prism** is a 3D figure with two parallel, congruent, polygon-shaped faces that are called bases. The remaining faces are either rectangles or parallelograms joining the two bases together. An exception is the cylinder which has circular bases and a smooth surface joining them together.





The formula for finding the volume of any prism is:
V = area of base x height

A **cylinder** has a radius (r) and a height (h) (see picture below).



The base of a cylinder is a circle. The formula for the area of the base of a cylinder is $A = \pi \times r^2$

The volume of a cylinder can be found using the formula: Volume = Area of base × height
$\mathbf{V = (\pi \times r^2) \times h}$

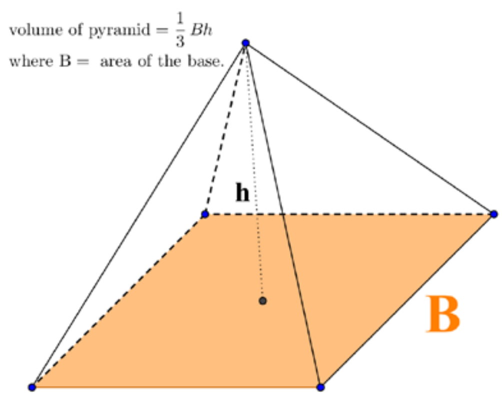
3. Volume of Cones & Pyramids

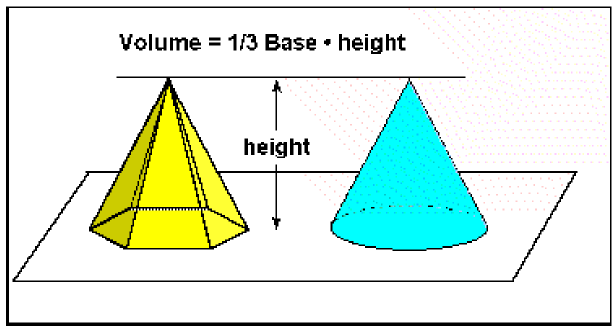
A **pyramid** is a 3D figure with a polygon base. All other faces are triangles that meet at a **common vertex**. An exception is the cone which has a circular base and a smooth surface that meets at a common vertex.

The volume of pyramids and cones are related to their corresponding prisms. For example, the volume of a triangular prism is related to the volume of a triangular pyramid with the same base and height.

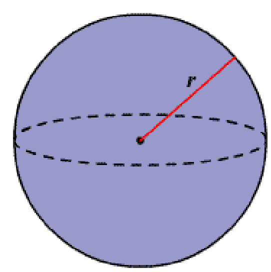
It takes 3 cones to fill the cylinder, meaning the volume of the cylinder is 3 times the volume of the cone. This will be the same for any pyramid and the corresponding prism with the same base and height.

So, the volume of a cone or pyramid is:
V = 1/3 x volume of corresponding prism





4. Volume of Spheres



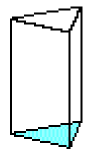
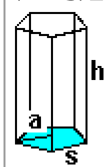
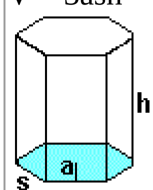
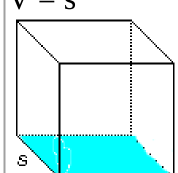
A **sphere** is a 3D figure that is symmetrical around its center, where all the points on its surface are the same distance from the center point. That distance is called the radius.

The formula for finding the volume of a sphere is:
V = 4/3 πr^3

The volume of a hemisphere is one-half the volume of the related sphere.

5. Extra Formulas for Volumes

Geometric Shape	Volume **B** = area of the **b**ase **P** = **p**erimeter of the base

triangular prism **a** = altitude **b** = base **h** = #height	V = Bh V = 1/2 abh 
regular pentagonal prism **a** = apothem length **s** = side length **n** = #sides **h** = height	V = Bh V = 1/2ansh V = 1/2a(5)sh V = 5/2ash 
regular hexagonal prism **a** = apothem length **s** = side length **n** = #sides **h** = height	V = Bh V = 1/2ansh V = 1/2a(6)sh V = 3ash 
cube **s** = side length	V = Bh V = s^3 

Example 1: Suppose prism A has dimensions 5 x 4 x 7 feet. What is the volume?

The volume is 5 x 4 x 7 = 140 cubic feet.

Example 2: Find the volume of the following triangular prism:

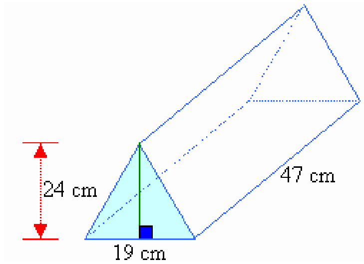
Solution:

a = 19 cm, b =24 cm, h =47 cm.

$$V = Al = \frac{1}{2}abh$$

$$= \frac{1}{2}\times 19\times 24\times 47 = 10716$$

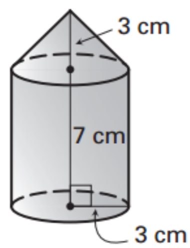
So, the volume is 10,716 cm^3.



Example 3: A tin can has a radius of 2 inches and a height of 3 inches. What is the volume?

The volume is $\pi r^2 h = \pi \bullet 2^2 \bullet 3 = 24\pi \approx 75.4$ in^3.

Example 4: Find the volume of the solid shown. The cone and the cylinder are right.



Solution:

Volume of solid = Volume of cylinder + Volume of cone

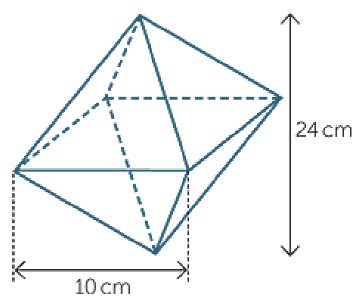
$= \pi r^2 h + \frac{1}{3}\pi r^2 h$

$= \pi(3)^2(7) + \frac{1}{3}\pi(3)^2(3)$

= 63π + 9π

= 226.19 cm^3

Example 5: Find the volume of the 'diamond', with height 24 cm and side length 10 cm as shown.



Solution:
Height of the pyramid is 24 / 2 = 12 cm
Volume of solid = 2 x Volume of pyramid

$= 2\times\frac{1}{3}b^2 h$

$= \frac{2}{3}(10)^2(12)$

= 800 cm^3

Example 6: Find the volume of the sphere with radius 8 m.

Solution: Substitute 8 for r in the formula.

V = 4/3π(8)^3
= 4/3π(512)
≈ 2145 m^3

Therefore, the volume of the sphere is about 2145 m^3.

6. Euler's Formula

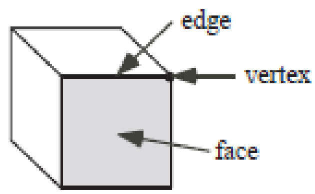
A **polyhedron** is a 3D shape with at, polygon-shaped faces. This means that cones, cylinders and spheres are not polyhedra. A convex polyhedron is a polyhedron where you can take any two points on the polyhedron, and the line joining the points will be inside the polyhedron.

Every polyhedron has faces, vertices and edges.

A **face** is one of the at polygon shaped surfaces.

An **edge** is the line segment where two faces meet.

A **vertex** is a point where 3 or more edges meet. Below is a diagram displaying a face, an edge and a vertex of a cube.



A famous mathematician, Euler, discovered a relationship involving faces, edges and vertices in the form of an equation. This equation works for all convex polyhedral and most non-convex polyhedra.

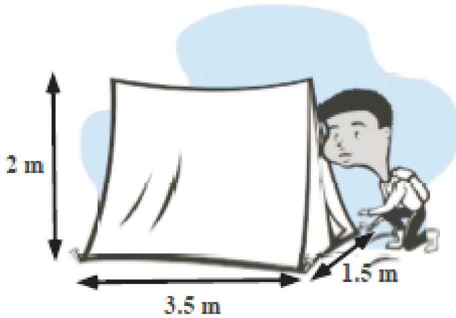
Instead of just stating Euler's formula, we will do an activity that will hopefully let you see the relationship. In the table below, fill out the name of the polyhedron you are studying, and the number of faces, edges and vertices that figure has. Continue doing this until your table is full with different polyhedra. Then, using just addition and subtraction, try to figure out the relationship.

Name of 3D Figure	Faces	Edges	Vertices
Cube	6	12	8
Rectangular Prism	6	12	8
Pentagonal Prism	7	15	10
Triangular Prism	5	9	6
Pyramid	5	8	5
Cuboctahedron	14	24	12
Icosahedron	20	30	12
Tetrahedron	4	6	4
Octahedron	8	12	6
Dodecahedron	12	30	20

Euler's formula is: F + V = E + 2

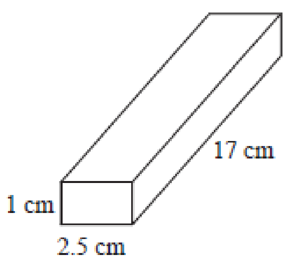
In Class Practice:

1. Jerome is building a tent with his father for the first time and wants to fill the entire tent with sleeping bags for fun. The dimensions of his tent are shown below. How many sleeping bags will he be able to fit in his tent if each sleeping bag has a volume of 0.75 m^3?

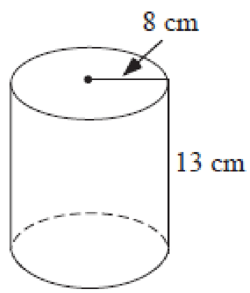


2. Find the volume of the following shapes.

1)

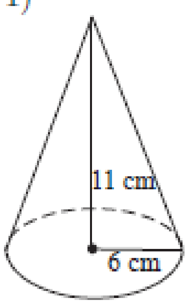


2)



3. Find the volume of the following cone and pyramid.

1)



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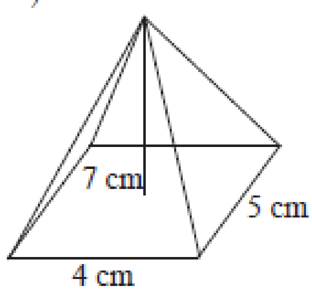

4. Sarah is planning a birthday party for her friend's 12th birthday. She wants to blow up 12 spherical balloons. If each balloon will have a radius of 14 cm, how much air would Sarah need to fill all 12 balloons?