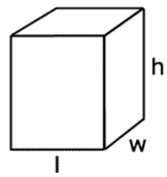
3D Geometry

1. Surface Area and Volume of Prisms

1) Surface Area:



SA = 2 (lw + hl + hw)

<u>Example 1</u>:
Given l = 4 yds, w = 2 yds, and h = 5 yds, the surface area would be

SA = 2[(4 yds)(2 yds) + (5 yds)(4 yds) + (5 yds)(2 yds)]= 2(8 yds^2 + 20 yds^2 + 10 yds^2)
= 2(38 yds^2) = 76 yds^2.

<u>Example 2</u>:
Given l = 6 mm, w = 9 mm, and h = 8 mm, the surface area would be

SA = 2[(6 mm)(9 mm) + (8 mm)(6 mm) + (8 mm)(9 mm)]= 2(54 mm^2 + 48 mm^2 + 72 mm^2)
= 2(174 mm^2) = 348 mm^2.

2) Volume:
V = Bh = lwh

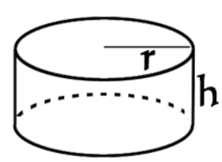
<u>Example 1</u>:
If l = 7 ft and w = 5 ft and h = 2 ft, then the volume would be
V = (7 ft)(5 ft)(2 ft) = 70 ft^3.

<u>Example 2</u>:
If l = 12 yds and w = 10 yds and h = 1/2 yd, then the volume would be
V = (12 yds)(10 yds)(1/2 yd) = 60 yds^3.

2. Surface Area and Volume of Cylinders

1) Surface Area of Cylinders
SA = $2\pi r^2 + 2\pi rh$

A cylinder has a total of three surfaces: a top, bottom, and middle. The top and bottom, which are circles, are easy to visualize.
The area of a circle is πr^2. So, the area of two circles would be
$\pi r^2 + \pi r^2 = 2\pi r^2$.



The third surface is the lateral surface area. The surface being referred to is the curved wall of the cylinder.
The dimensions of the rectangle are the circumference, C = $2\pi r$, and the height of the cylinder, h. So, the area of the rectangle is A = l x w = C x h = $2\pi r$ x h = $2\pi rh$.

<u>Example 1</u>:
If r = 6 ft and h = 4 ft, then the surface area would be

SA = 2(3.14)(6 ft)^2 + 2(3.14)(6 ft)(4 ft) = 2(3.14)(36 ft^2) + 2(3.14)(24 ft^2)
= 226.08 ft^2 + 150.72 ft^2 = 376.8 ft^2.

<u>Example 2</u>:
If r = 5 cm and h = 2 cm, then the surface area would be

SA = 2(3.14)(5 cm)^2 + 2(3.14)(5 cm)(2 cm) = 2(3.14)(25 cm^2) + 2(3.14)(10 cm^2)
= 157 cm^2 + 62.8 cm^2 = 219.8 cm^2.

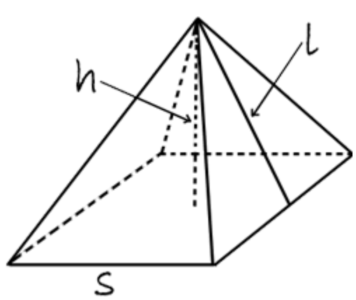
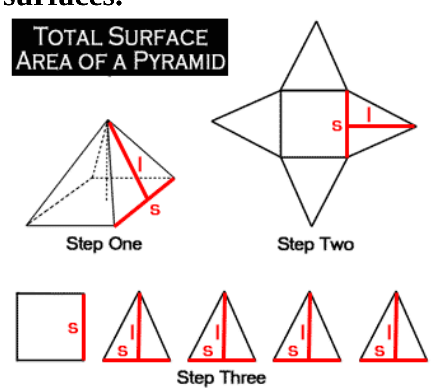
2) Volume of Cylinders
V = Bh = $\pi r^2 h$

<u>Example 1</u>:
If r = 5 cm and h = 6 cm, then the volume would be
V = (3.14)(5)^2(6) = 471 cm^3.

<u>Example 2</u>:
If r = 12 ft and h = 2 ft, then the volume would be
V = (3.14)(12)^2(2) = 904.3 ft^3.

3. Surface Area and Volume of Square Pyramids

1) Surface Area of Square Pyramids

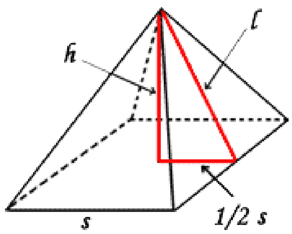
 SA = $s^2 + 2sl$	**Pyramids that have a square base have a total of five surfaces.** 

The square (or base of the solid) has an area that can be calculated by multiplying its length times its width. Since those dimensions are equal, the area is s x s = s^2.

The area formula for a triangle is its base times its height divided by two. In the case of one of the triangles above it would be s x l ÷ 2. However, there are four triangles.

This would make the total lateral surface area equal to four times the area of one triangle, or 4 x s x l ÷ 2. Upon simplifying the expression, we get 2sl.

The total surface area of the pyramid is equal to the area of the base plus its lateral surface area or $s^2 + 2sl$.



A right triangle, which rests internally within the pyramid, has been highlighted.

The hypotenuse (longest side) is the slant height of the pyramid, l.

This is the length we need to know in order to calculate the surface area of the pyramid.

The height of the pyramid is a leg of the right triangle.

The base of the right triangle is half the length of the base edge of the pyramid, s.

At this point, we would use **The Pythagorean Theorem** to calculate the slant height.

The Pythagorean Theorem

$leg^2 + leg^2 = hypotenuse^2$. In the case of our right triangle, we would get $(1/2\ s)^2 + h^2 = l^2$. Solving for the slant height would then depend on our ability to use the Pythagorean Theorem.

<u>Example 1</u>:
Given: s = 8 m and h = 3 m. First we calculate the slant height using **The Pythagorean Theorem** and half the base length. $3^2 + 4^2 = l^2 \implies 9 + 16 = l^2 \implies 25 = l^2$, or l = 5 m.

The surface area would then require use of the formula
$SA = (8\text{ m})^2 + 2(8\text{ m})(5\text{ m}) = 64\text{ m}^2 + 80\text{ m}^2 = 144\text{ m}^2$.

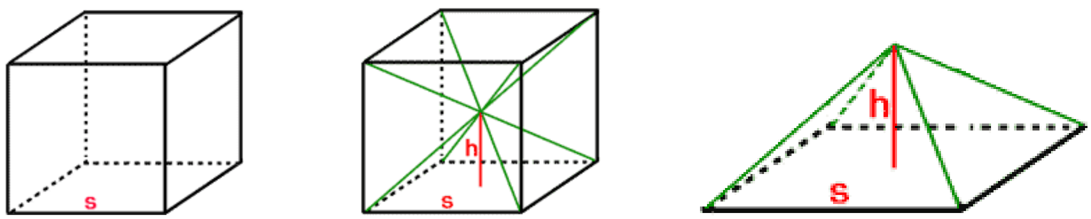
<u>Example 2</u>:
Given: s = 7 in and h = 6 in. First we calculate the slant height using **The Pythagorean Theorem** and half the base length.

$(3.5)^2 + 6^2 = l^2 \implies 12.25 + 36 = l^2 \implies 48.25 = l^2$, or l = 6.95 in.

The surface area would then require use of the formula
SA = $(7\text{ in})^2 + 2(7\text{ in})(6.95\text{ in}) = 49\text{ in}^2 + 97.3\text{ in}^2 = 146.3\text{ in}^2$.

2) Volume of Square Pyramids

$V = Bh/3 = s^2h/3$



The volume of the cube can be found by using the formula for a **prism**, namely V = Bh.
So, V = s^2 times s = s^3.

The volume of each of the pyramids are equal; therefore, their volumes must be 1/6 the original cube's volume. This means each pyramid has a volume, $V = s^3/6$.

By examining the cube with its diagonals, we can see that it takes two pyramid heights to be equal to the base length of the cube, or 2h = s.

By substitution, we get V = $s^3/6$ = (s^2 times s)/6 = (s^2 times 2h)/6.

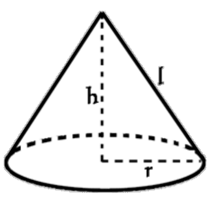
By simplifying the expression, we get V = 2(s^2 times h)/6, which is the same as
V = 2Bh/6 or V = Bh/3.

Example 1:
If s = 3 in and h = 5 in, then the volume would be
$V = Bh/3 = (3\text{ in})^2(5\text{ in})/3 = (9\text{ in}^2)(5\text{ in})/3 = 45\text{ in}^3/3 = 15\text{ in}^3$.

Example 2:
If s = 7 m and h = 12 m, then the volume would be
$V = Bh/3 = (7\text{ m})^2(12\text{ m})/3 = (49\text{ m}^2)(12\text{ m})/3 = 588\text{ m}^3/3 = 196\text{ m}^3$.

4. Surface Area and Volume of Cones

1) Surface Area of Cones
$SA = \pi r^2 + \pi rl$



Example 1:

Given r = 4 m and h = 3 m. First we must calculate the slant height using The Pythagorean Theorem.
$(4\text{ m})^2 + (3\text{ m})^2 = l^2 \Longrightarrow 16\text{ m}^2 + 9\text{ m}^2 = l^2 \Longrightarrow 25\text{ m}^2 = l^2$, or l = 5 m.
The surface area would then require use of the formula
$SA = (\pi)(4\text{ m})^2 + (\pi)(4\text{ m})(5\text{ m}) = (16\text{ m}^2)(\pi) + (20\text{ m}^2)(\pi) = 36\pi\text{ m}^2$ (in terms of π) or 113.0 m^2.

Example 2:

Given r = 7 in and h = 10 in. First we must calculate the slant height using The Pythagorean Theorem.

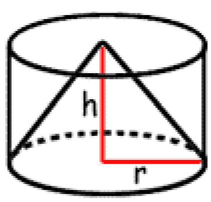
$(7\text{ in})^2 + (10\text{ in})^2 = l^2 \Longrightarrow 49\text{ in}^2 + 100\text{ in}^2 = l^2 \Longrightarrow 149\text{ in}^2 = l^2$, or l = 12.21 in. The surface area would then require use of the formula

$SA = (\pi)(7\text{ in})^2 + (\pi)(7\text{ in})(12.21\text{ in}) = (49\text{ in}^2)(\pi) + (85.47\text{ in}^2)(\pi) = 134.47(\pi)\text{ in}^2 = 422.2\text{ in}^2$.

2) Volume of Cones

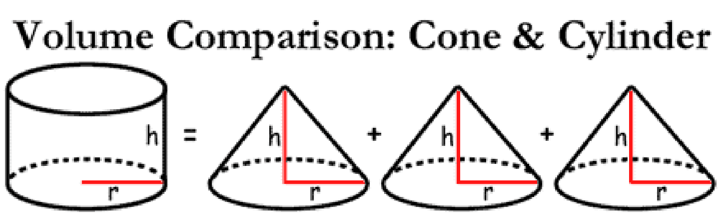
$V = Bh/3 = \pi r^2h/3$

The formula for the volume of a cone can be determined from the volume formula for a cylinder. We must start with a cylinder and a cone that have equal heights and radii, as in the diagram below.



Looking at this in reverse, each cone is one-third the volume of a cylinder. Since a cylinder's volume formula is V = Bh, then the volume of a cone is one-third that formula, or V = Bh/3.

Specifically, the cylinder's volume formula is $V = \pi r^2h$ and the cone's volume formula is $V = \pi r^2h/3$.



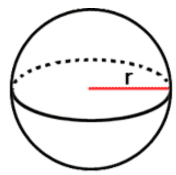
For prisms and cylinders, their volumes are V = Bh.
For pyramids and cones, their volumes are V = Bh/3.

<u>Example 1</u>:
If r = 4 ft and h = 5 ft, then the volume would be
$V = Bh/3 = \pi r^2 h/3 = (3.14)(4\text{ ft})^2(5\text{ ft})/3 = (3.14)(16\text{ ft}^2)(5\text{ ft})/3 = 83.7\text{ ft}^3$.

<u>Example 2</u>:
If r = 6 cm and h = 2 cm, then the volume would be
$V = Bh/3 = \pi r^2 h/3 = (3.14)(6\text{ cm})^2(2\text{ cm})/3 = (3.14)(36\text{ cm}^2)(2\text{ cm})/3 = 75.4\text{ cm}^3$.

5. Surface Area of Spheres

1) Surface Area of Spheres



$SA = 4\pi r^2$

<u>Example 1</u>:
Given r = 3 ft. The surface area requires the use of the formula
$SA = 4(\pi)(3\text{ ft})^2 = 4(\pi)(9\text{ ft}^2) = 36\pi\text{ ft}^2$ (in terms of π) or 113.0 ft^2.

<u>Example 2</u>:
Given diameter = 20 m or r = 10 m. The surface area requires the use of the formula
$SA = 4(\pi)(10\text{ m})^2 = 4(\pi)(100\text{ m}^2) = 400\pi\text{ m}^2$ (in terms of π) or 1256 m^2.

2) Volume of Spheres

$V = 4\pi r^3/3$

The volume of the sphere would then be the sum of the volumes of all the pyramids. To calculate this, we would use the formula for volume of a pyramid, namely V = Bh/3. We would take the sum of all the pyramid bases, multiply by their height, and divide by 3.

First, the sum of the pyramid bases would be the surface area of a sphere, $SA = 4\pi r^2$. Second, the height of each of the pyramids is the radius of the sphere, r. Third, we divide by three. The result of these three actions is $SA = (4\pi r^2)(r)/3 = 4\pi r^3/3$.

<u>Example 1</u>:

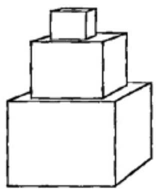
If r = 300 mi (a moon), then the volume would be
$V = 4\pi r^3/3 = 4(3.14)(300\text{ mi})^3/3 = 4(3.14)(27{,}000{,}000\text{ mi}^3)/3 = 113{,}040{,}000\text{ mi}^3$.

<u>Example 2</u>:

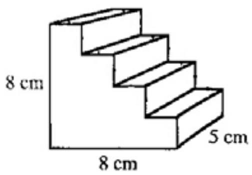
If r = 4 mm (a large marble), then the volume would be
$V = 4\pi r^3/3 = 4(3.14)(4\text{ mm})^3/3 = 4(3.14)(64\text{ mm}^3)/3 = 267.9\text{ mm}^3$.

Questions in class

1. A 3× 3 × 3 cube, a 2 × 2 × 2 cube, and a 1×1×1 cube are glued together as shown. What is the total surface area of this object, including the bottom?



2. In the illustrated solid, the steps are of equal width and equal height. What is the volume of this solid, in cm^3?

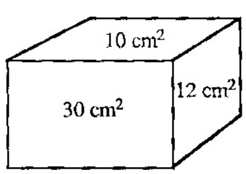


3. Sixty-four white 1×1×1 cubes are used to form a 4×4×4 cube, which is then painted red on each of its six faces. This large cube is then broken up into its 64 unit cubes. Each unit cube is given a score as follows:

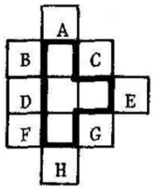
Exact number of faces painted red	Score
3	3
2	2
1	1
0	−7

What is the total score for the 4 × 4 × 4 cube?

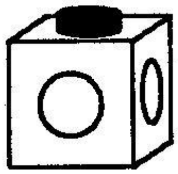
4. The areas of three of the faces of the rectangular box shown are 10 cm^2, 12 cm^2, and 30 cm^2. What is the volume of the box, in cm^3?



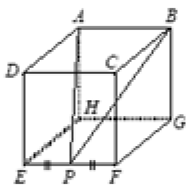
5. Suppose one of the eight lettered identical squares is included with the four squares in the T-shaped figure outlined. How many of the resulting figures can be folded into a topless cubical box?



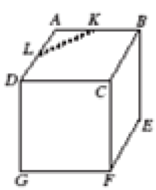
6. A plastic snap-together cube has a protruding snap on one side and receptacle holes on the other five sides as shown. What is the smallest number of these cubes that can be snapped together so that only receptacle holes are showing?



7. ABCDEFGH is a cube having a side length of 2. P is the midpoint of EF, as shown. What is the area of Δ APB?



8. In the cube shown, *L* and *K* are midpoints of adjacent edges *AD* and *AB*. The perpendicular distance from *F* to the line segment *LK* is 10. What is the volume of the cube, to the nearest integer?



9. A cube with 3-inch edges is made using 27 cubes with 1-inch edges. Nineteen of the smaller cubes are white and eight are black. If the eight black cubes are placed at corners of the larger cube, what fraction of the surface area of the larger cube is white?

10. An 8 cm cube has a 4 cm square hole cut through its centre, as shown. What is the remaining volume, in cm^3?