First Name:_____   Last Name: _____   Student ID: _____

Rational Functions (1)

1. State the domain of each function, then determine the equation of any vertical asymptotes and/or coordinates of any holes in the graph of the function.

a. $f(x) = \dfrac{2x}{x-3}$
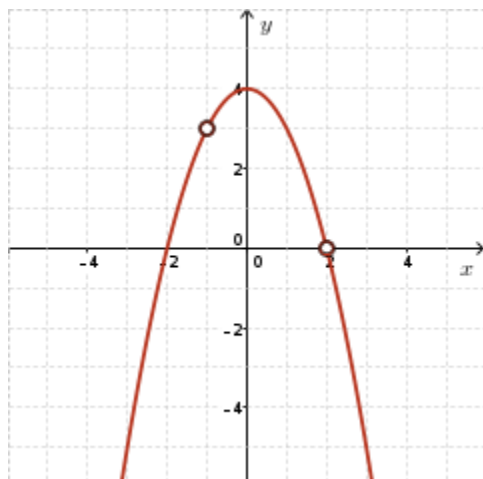
b. $f(x) = \dfrac{2x^2+x}{x^2-5x+6}$

c. $f(x) = \dfrac{3x^2-21x}{6x^2-39x-21}$

d. $f(x) = \dfrac{x^3+x}{6x^3+x^2-x}$

2. Determine, with support, an equation for a rational function of the form $y=\dfrac{g(x)}{h(x)}$ that satisfies the given conditions.

a. Vertical asymptotes of $x=-1$ and $x=3$
b. A hole at $(1/3,-2)$ and a vertical asymptote of $x=1$

3. Determine an equation for the rational function shown in the graph below.

4. a. Under what conditions does a rational function have an oblique asymptote?

b. Explain how to determine the equation of the oblique asymptote of a rational function that satisfies the conditions in part a).

c. Which of these functions has an oblique asymptote? Determine the equation of the oblique asymptote, if it exists.

i. $y = \dfrac{x^2}{x+3}$

ii. $y = \dfrac{3x}{x^2+1}$

iii. $y = \dfrac{x^2+4x+5}{x^2-4}$
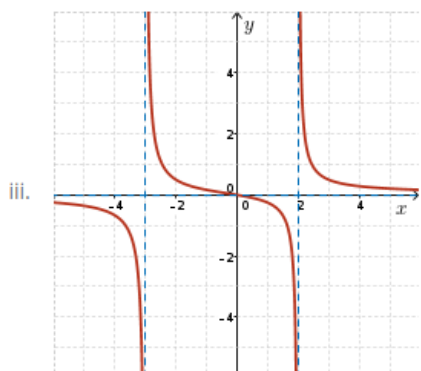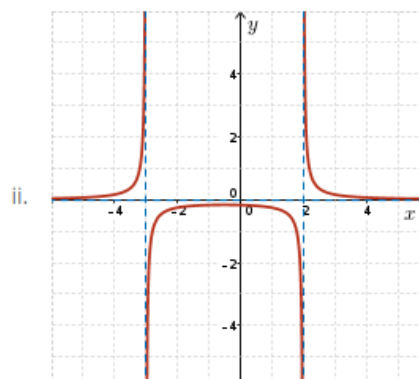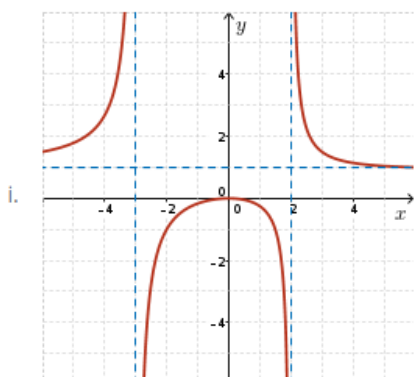
iv. $y = \dfrac{2x^2-3x+5}{x-4}$

v. $y = \dfrac{x^3-1}{x^2-1}$

5. By identifying asymptotes and intercepts, match the equation of each function to the most appropriate graph. Justify your choice.

a. $y = \dfrac{1}{x^2+x-6}$

b. $y = \dfrac{x}{x^2+x-6}$

c. $y = \dfrac{x^2}{x^2+x-6}$

i.

ii.

iii.

6. Complete the comparison table below. Use this information, along with additional points, to sketch the graph of each function.

Function	a. $y = \dfrac{x+1}{x^2+2x-3}$	b. $y = \dfrac{x-1}{x^2+2x-3}$
Domain		

3

Vertical Asymptote(s) and/or Points of Discontinuity		
Horizontal Asymptote(s)		
x-intercepts		
y-intercepts		
Symmetry (Even/Odd)		

7. For the following functions

- Identify all asymptotes, points of discontinuity and intercepts.
- Discuss the behaviour of the graph of the function near its asymptotes.
- Based on your findings from parts a) and b), along with additional points, sketch a graph of the function.

a. $g(x) = \dfrac{x^2+3x-8}{x+2}$:

b) $y = \dfrac{x^3-1}{x^2+2x}$

8. The function $f(x) = \dfrac{2x^2+ax+b}{5x^2-26x+b}$, where a and b are real numbers, has a point of discontinuity (hole) when $x=6$.

a. Determine the values of a and b.
b. Determine the location of the hole, the x- and y-intercepts, and the equations of the asymptotes of the function.