First name: _____ Last Name: _____ Student ID:_____

Number Theory Homework

Basic problems:

1. Complete each divisibility table. Write <u>yes</u> if the number is divisible by the given number. Write <u>no</u> if it is not divisible by the given number.

1. 2,520 by 2 _____ by 3 _____ by 4 _____ by 6 _____ by 9 _____	2. 66,736 by 2 _____ by 3 _____ by 5 _____ by 7 _____ by10 _____	3. 900 by 3 _____ by 4 _____ by 7 _____ by 8 _____ by 9 _____	4. 48,949 by 2 _____ by 6 _____ by 7 _____ by 9 _____ by13 _____

2. Find the prime factorization of each number.

1. 14	2. 27	3. 36	4. 8
5. 45	6. 28	7. 21	8. 15

3. Find the greatest common factor of each set of numbers.

1. 60, 12, and 36	2. 84, 48, and 66	3. 25, 12, and 10
4. 80, 96, and 32	5. 66, 36, and 32	6. 42, 91, and 7

4. Find the least common multiple.

1. 3, 6, and 11	2. 2, 4, and 14	3. 6, 24, and 54
4. 4, 8, and 32	5. 6, 18, and 36	6. 7, 11, and 14

Challenge problems:

1. Suppose M is a two digit number and that N is obtained by reversing the digits of M. If M + N = 132, how many different values can M have?

2. How many ways can the number 12 be expressed as a sum of exactly three distinct positive integers taken in increasing order? (e.g. 12 = 1 + 3 + 8)

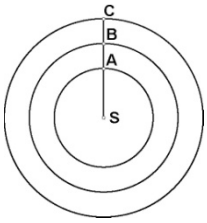
3. How many integers between 1 and 101 are multiples of either 3 or 5 but not the multiples of both?

4. How many of the integers between 31 and 131 are divisible by 7 but not divisible by 6?

5. Which of the following is the largest? *No Calculator!*
(A) $2^{10}3^5$ (B) 2^{17} (C) 4^8 (D) 6^7 (E) 3^9

6. The least common multiple of two numbers is 105 and the greatest common divisor is 5. Which of the following could be the sum of the numbers?
(A) 21 (B) 25 (C) 49 (D) 50 (E) 105

7. Three planets are in straight line as in the diagram below. Planet A makes a complete revolution around the Sun S in 2 years. Planet B makes its revolution in 4 years and Planet C in 6 years. What is the least number of years before all three planets will once again be on the same line?



8. Find the largest number among these:
(A) 2^{2004} (B) 2004^2 (C) 2000^4 (D) 4^{2000} (E) $2\times4\times2000\times2004$

9. Which digit appears most often when writing the integers form 1 to 100?
(A) 0 (B) 1 (C) 3 (D) 9 (E) All appear equally often

10. A number is said to be prime if it is evenly divisible only by itself and by one. Among the numbers given below, which one is not a prime number?
(A) 107 (B) 109 (C) 111 (D) 113 (E) All are prime

11. Three squares have sides of different integer lengths, a, b, and c. The total area of the three squares cannot be
(A) 14 (B) 29 (C) 50 (D) 88 (E) 101

12. How many zeros appear at the end of the product of $1 \times 2 \times 3 \times \ldots \times 50$?