Fractions and Decimals

1. Fractions

Fractions are expressed as one number over another number, like this: $\frac{1}{2}$

The number on the top is called the *numerator* and the number on the bottom is called the *denominator*.

2. Fractions mixed number

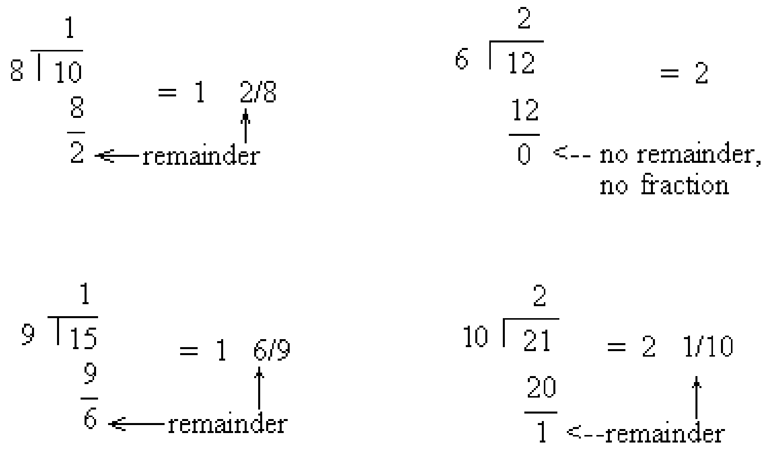
A fraction in which the numerator is larger than the denominator is called an IMPROPER FRACTION. These are improper fractions.

$$\frac{10}{8} \qquad \frac{12}{6} \qquad \frac{15}{9} \qquad \frac{21}{10}$$

Fractions like these can be turned into MIXED NUMBERS. A mixed number is a whole number with a fraction added to it.

You turn an improper fraction into a mixed number by dividing the numerator by the denominator and making a fraction that goes with it (if there is one) by putting the remainder over the denominator.

The above improper fractions change into mixed numbers by dividing the numerator by the denominator, like this:



3. Fractions multiplying mixed number

To multiply mixed numbers by any other number (fraction, whole number or another mixed number) you must first convert the mixed number to an improper fraction.

To convert a mixed number to an improper fraction, you multiply the denominator by the whole number and then add the numerator to get your new numerator. The denominator stays the same. This is just the opposite of what we did in unit 10.

Examples:

$$5\frac{1}{8} = \frac{8\times5+1}{8} \quad = \frac{41}{8} \qquad 2\frac{3}{5} = \frac{5\times2+3}{5} \quad = \frac{13}{5} \qquad 4\frac{1}{3} = \frac{3\times4+1}{3} \quad = \frac{13}{3}$$

4. Decimals

Decimals are numbers that contain a "DECIMAL POINT", like:

1.5 2.8 .2 3.14 66.66

The number to the left of the decimal point is an ordinary whole number.

The number to the right of the decimal is called the "tenths" digit and tells you the number of tenths (1/10's) that are added to the whole number to the left of the decimal point. So...

1.5 is the same as the mixed number $1 \text{ and } \frac{5}{10}$, or $1\frac{5}{10}$

2.8 is the same as the mixed number $2 \text{ and } \frac{8}{10}$, or $2\frac{8}{10}$

0.2 is the same as the mixed number $\frac{2}{10}$

If there are 2 numbers to the right of the decimal point then both numbers to the right of the decimal point are the number of "hundredths" (1/100's) to be added to the whole number, so.....

3.14 is the same as the mixed number $3 \text{ and } \frac{14}{100}$, or $3\frac{14}{100}$

66.66 is the same as the mixed number $66 \text{ and } \frac{66}{100}$, or $66\frac{66}{100}$

5. Decimal to Fraction

The technique I just demonstrated lets you convert any terminating decimal to a fraction. ("Terminating" means "it ends", unlike, say, the decimal for $^{1}/_{3}$, which goes on forever.

A non-terminating AND NON-REPEATING decimal CANNOT be converted to a fraction, because it is an "irrational" (non-fractional) number.

You should probably just memorize some of the more basic repeating decimals, like 0.33333... = $^{1}/_{3}$ and 0.666666... = $^{2}/_{3}$. Check out the table on the last page.)

Any terminating decimal can be converted to a fraction by counting the number of decimal places, and putting the decimal's digits over 1 followed by the appropriate number of zeroes.

For example:

$$0.46 = \frac{46}{100} = \frac{23}{50}, \quad 1.5 = \frac{15}{10} = \frac{3}{2}, \quad 10.2 = \frac{102}{10} = \frac{51}{5}, \quad 0.0003 = \frac{3}{10000}$$

In the case of a repeating decimal, the following procedure is often used. Suppose you have a number like 0.5777777.... This number is equal to some fraction; call this fraction "*x*".

That is:
$x = 0.5777777...$

There is one repeating digit in this decimal, so multiply *x* by "1" followed by one zero; that is, multiply by 10:

$10x = 5.777777...$

Now subtract the former from the latter:

$$\begin{array}{r} 10x = 5.777777\ldots \\ x = 0.577777\ldots \\ \hline 9x = 5.200000\ldots \end{array}$$

That is, $9x = 5.2 = 52/10 = 26/5$. Solving this, we get $x = 26/45$. (You can verify this by plugging "26 ÷ 45" into your calculator and seeing that you get "0.5777777..." for an answer.)

If there had been, say, three repeating digits (such as in 0.4123123123...), then you would multiply the *x* by "1" followed by three zeroes; that is, you would multiply by 1000. Then subtract and solve, as in the above example. And don't worry if you have leading zeroes, as in "0.004444..."; the procedure will still work.

6. Fraction to Decimal

If you remember that fractions are division, then this is easy. The calculator can do the work for you, because you can just have it do the division.

For example:

$$\frac{3}{5} = 3 \div 5 = 0.6 \qquad \frac{1}{3} = 1 \div 3 = 0.333333\ldots = 0.\overline{3}$$

The bar is placed over the repeating digits, for convenience sake.

$$\frac{3}{4} = 3 \div 4 = 0.75 \qquad \frac{2}{7} = 0.285714285714\ldots = 0.\overline{285714}$$

When converting fractions to decimals, you may be told to round to a certain place or to a certain number of decimal places.

For instance, looking at that last example, $^{2}/_{7}$ as a decimal rounded to the nearest tenth (rounded to one decimal place) is 0.3; to the nearest hundredth (to two decimal places) is 0.29; to the nearest thousandths (to three decimal places) is 0.286; to the nearest ten-thousandths (to four decimal places) is 0.2857; et cetera.

If you're not sure how you should format your answer, then give the "exact" form and the rounded form:

$$\frac{2}{7} = 0.\overline{285714} \approx 0.286$$

Note that the rounded form can be useful for word problems, where a final answer in rounded form may be more practical than a repeating decimal.

5. Table of Common Fractions and Their Decimal Equivalents or Approximations

$^{1}/_{2} = 0.5$			
$^{1}/_{3} = 0.3333...$	$^{2}/_{3} = 0.6666...$		
$^{1}/_{4} = 0.25$	$^{3}/_{4} = 0.75$		
$^{1}/_{5} = 0.2$	$^{2}/_{5} = 0.4$	$^{3}/_{5} = 0.6$	$^{4}/_{5} = 0.8$
$^{1}/_{6} = 0.1666...$	$^{5}/_{6} = 0.8333...$		
$^{1}/_{7} = 0.142857142857...$		$^{2}/_{7} = 0.285714285714...$	
$^{3}/_{7} = 0.428571428571...$		$^{4}/_{7} = 0.571428571428...$	
$^{5}/_{7} = 0.714285714285...$		$^{6}/_{7} = 0.8571428571428...$	
$^{1}/_{8} = 0.125$	$^{3}/_{8} = 0.375$	$^{5}/_{8} = 0.625$	$^{7}/_{8} = 0.875$
$^{1}/_{9} = 0.111...$	$^{2}/_{9} = 0.222...$	$^{4}/_{9} = 0.444...$	$^{5}/_{9} = 0.555...$
		$^{7}/_{9} = 0.777...$	$^{8}/_{9} = 0.888...$
$^{1}/_{10} = 0.1$	$^{3}/_{10} = 0.3$	$^{7}/_{10} = 0.7$	$^{9}/_{10} = 0.9$
$^{1}/_{12} = 0.08333...$			

Questions in class

1. If $a \otimes b = \dfrac{a+b}{a-b}$ then $(6 \otimes 4) \otimes 3 = ?$

2. The operation $\otimes$ is defined for all nonzero numbers by $a \otimes b = \dfrac{a^2}{b}$.
Determine $[(1 \otimes 2) \otimes 3] - [1 \otimes (2 \otimes 3)]$.

3. If the product $\dfrac{3}{2} \cdot \dfrac{4}{3} \cdot \dfrac{5}{4} \cdot \dfrac{6}{5} \cdot \ldots \cdot \dfrac{a}{b} = 9$, what is the sum of a and b?

4. $\dfrac{1}{1+\dfrac{1}{2+\dfrac{1}{3}}} = ?$

5. When four gallons are added to a tank that is one-third full, the tank is then one-half full. What is the capacity of the tank in gallons?

6. What is the reciprocal of $\left(\dfrac{1}{2}+\dfrac{1}{3}\right)$?

7. The sum $2\dfrac{1}{7}+3\dfrac{1}{2}+5\dfrac{1}{19}$ is between
   A) 10 and 10.5 B) 10.5 and 11 C) 11.5 and 12 D) 12 and 12.5

8. Which of the following fractions has the largest value?
   A) $\dfrac{3}{7}$ B) $\dfrac{4}{9}$ C) $\dfrac{17}{35}$ D) $\dfrac{100}{201}$ E) $\dfrac{151}{301}$

9. Suppose n* means $\dfrac{1}{n}$, the reciprocal of n. For example, $5^* = \dfrac{1}{5}$. How many of the following are true?
i) $3^* + 6^* = 9^*$ ii) $6^* - 4^* = 2^*$ iii) $2^* \cdot 6^* = 12^*$ iv) $10^* \div 2^* = 5^*$
   A) 0 B) 1 C) 2 D) 3 E) 4

10. The value of the expression $\dfrac{(304)^5}{(29.7)(399)^4}$ is closest to
   A) .003 B) .03 C) .3 D) 3 E) 30