Uniform Circular Motion

Unit 1: Fundamentals of Dynamics

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Grade 12 Physics
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Summer 2019

Where Are We In the Course

1. Fundamentals of Dynamics
2. Momentum, Impulse and Energy
3. Gravitational, Electric and Magnetic Fields
4. Wave Nature of Light
5. Modern Physics
   - 5.1 Theory of Special Relativity
   - 5.2 Introduction to Quantum Mechanics

Uniform Circular Motion

- Constant speed (magnitude of velocity)
- Changing direction (direction of velocity)
- Changing velocity, i.e. always accelerating
- Examples:
    - Roller coaster rides
    - Motors
    - Swinging a key chain

Centripetal Acceleration

Centripetal acceleration is the acceleration that causes circular motion.

- Its direction is always:
  - Towards the centre of the circular motion
  - Perpendicular to the instantaneous velocity vector
- Its magnitude is given by:

$$\boxed{a_c = \frac{v^2}{r}}$$

Quantity	Symbol	SI Unit
Centripetal acceleration	a_c	$\mathrm{m/s^2}$
Speed (magnitude of velocity)	v	$\mathrm{m/s}$
Radius (of the circular path)	r	m

Centripetal Force

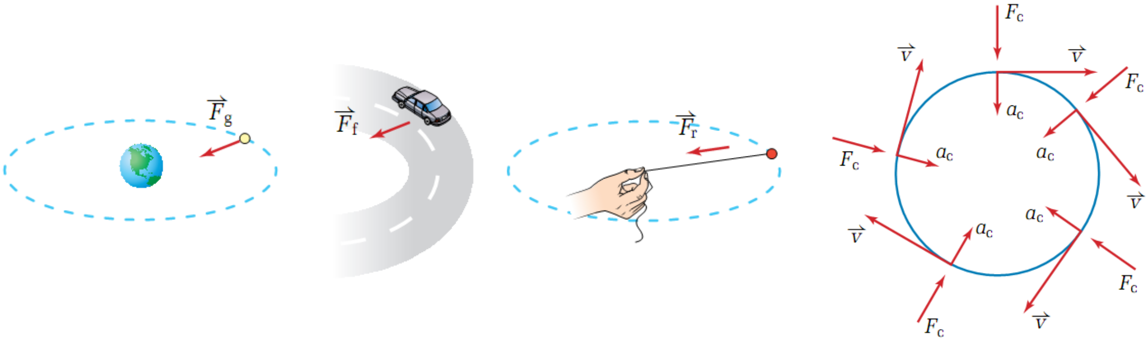
Centripetal force is the force that causes centripetal acceleration

- In the same direction as acceleration (towards the centre of motion)
- Its magnitude is given by:

$$\boxed{F_c = ma_c = \frac{mv^2}{r}}$$

Quantity	Symbol	SI Unit
Centripetal force	F_c	N
Mass	m	kg
Speed (magnitude of velocity)	v	m/s
Radius (of the circular path)	r	m
Centripetal acceleration	a_c	m/s^2

Centripetal Force



How to Solve Circular Motion Problems

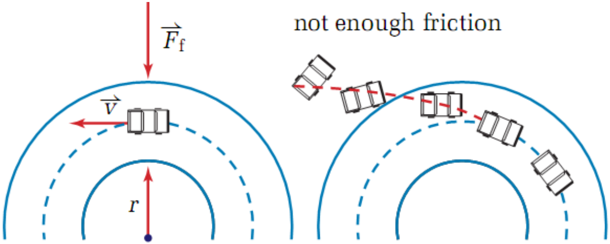
The condition for circular motion is:

$$\mathbf{F}_{\text{provided}} = \mathbf{F}_{\text{required}}$$

- The *provided* force comes from the free-body diagram. It may include:
  - Gravity
  - Friction
  - Normal force
  - Tension
  - Etc.
- The *required* force comes from the centripetal equation we have
- If the provided force is not sufficient, then the motion will change

Example Problem

Example 1: A car with a mass of $m = 2135\,\mathrm{kg}$ is rounding a curve on a level road. If the radius of curvature of the road is $r = 52\,\mathrm{m}$ and the coefficient of friction between the tires and the road is $\mu = 0.70$, what is the maximum speed at which the car can make the curve without skidding off the road?



(Did you spot the error in the diagram? The radius r should be from the centre of the circle to the car, not the edge of the road!)

Example Problem

Example 2: You are playing with a yo-yo with a mass of $225\,\mathrm{g}$. The full length of the string is $1.2\,\mathrm{m}$. You decide to see how slowly you can swing it in a vertical circle while keeping the string fully extended, even when the yo-yo is at the top of its swing.

- Calculate the minimum speed at which you can swing the yo-yo while keeping it on a circular path.
- At the speed that you determined, find the tension in the string when the yo-yo is at the side and at the bottom of its swing.

(Note: The speed will *surely* change when the yo-yo is at the bottom of the swing. We are just simplifying the problem a bit.)

What is Centrifugal Force?

- Objects in non-inertial frames of references (i.e. under acceleration) experience a **fictitious force**, or **pseudo force**
- **Centrifugal force** is one such force
- An object moving in circular motion is not in an inertial frame of reference
- For example, when a car makes a left turn, you feel a push towards the right side of the car
  - You sensation of this fictitious force is indistinguishable from a real force
  - A scale will be able to measure this centrifugal force
- Centrifugal force is equal in magnitude and opposite in direction to the centripetal force (this is *not* an application of Newton's third law!)

Period and Frequency

Period T is the time it takes to do one complete revolution (distance $2\pi r$ at speed v), and **frequency** f shows how many revolutions per second:

$$T = \frac{2\pi r}{v} = \frac{1}{f} \qquad f = \frac{1}{T} = \frac{v}{2\pi r}$$

We can write centripetal acceleration in terms of frequency or period:

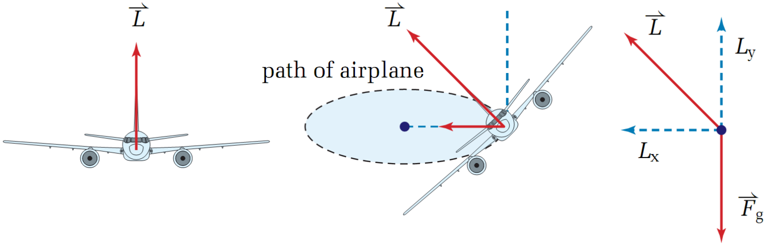
$$a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} = 4\pi^2 r f^2$$

and centripetal force is just mass times acceleration:

$$F_c = m a_c = 4\pi^2 m r f^2$$

How Does A Airplane Turn

Short Answer: Centripetal Force!

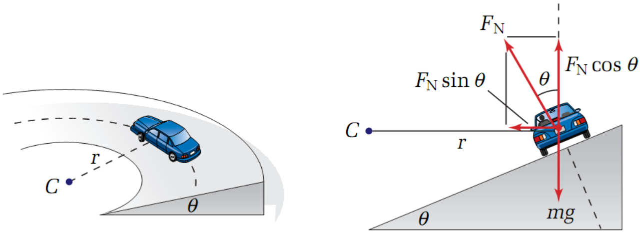


- Lift is generated by the wings
- When the airplane turns, it “rolls” to one side, causing the lift force vector to change direction
- The component of lift that is perpendicular to the velocity vector is now the centripetal force that turns the airplane

How Does An Airplane Roll Then?

The “aileron” is deflected, so that one wing generates a bit more lift than the other, and this rotates (“rolls”) the aircraft to the left or right.

Banked Curves on Highways and Racetracks



y-direction (vertical)

$$F_N \cos\theta = F_g = mg$$

x-direction (towards C)

$$F_N \sin\theta = F_c = \frac{mv^2}{r}$$

Combine the two equations together:

$$\frac{F_N \sin\theta}{F_N \cos\theta} = \frac{\frac{mv^2}{r}}{mg}$$

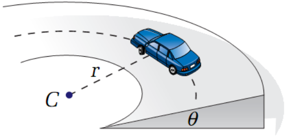
Cancel out F_N and m terms:

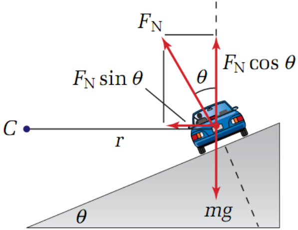
$$\frac{\sin\theta}{\cos\theta} = \frac{v^2}{rg} \longrightarrow \boxed{\tan\theta = \frac{v^2}{rg}}$$

Banked Curves on Highways and Racetracks

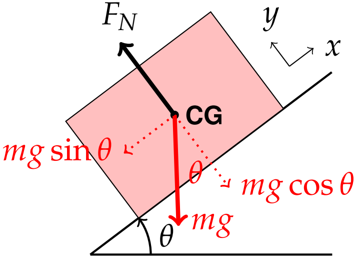
Whoa! Wait a minute! That FBD doesn't look right!

Circular motion:





Mass sliding on a slope:



Banked Curves on Highways and Racetracks

Whoa! Wait a minute! That FBD doesn't look right!

Normal forces are different in both cases because the nature of the motion is different. Block sliding on a ramp is a 2D problem, but the race car around the race track is a pseudo-2D problem (really a slice of a 3D problem). **You'll just have to remember when to use which one.**

Example Problem

Example 3: Canadian Indy racing car driver Paul Tracy set the speed record for time trials at the Michigan International Speedway (MIS) in 2000. Tracy averaged $378.11\,\mathrm{km/h}$ in qualifying. The ends of the $3.0\,\mathrm{km}$ oval track at MIS are banked at 18.0° and the radius of curvature is $382\,\mathrm{m}$.

- At what speed can the cars round the curves without needing to rely on friction to provide a centripetal force?
- Did Tracy rely on friction for some of his required centripetal force?

Making Sense of The Last Example Problem

In real life:

- 18° banked curve is very high for a road!
- Without relying on friction, max speed is only $126\,\mathrm{km/h}$

Normal highways:

- Maximum banking is a few degrees
- Speed limit on on/off ramps are lower than the posted $100\,\mathrm{km/h}$ limit
- Large radius

One Last Example

Example 4: A car exits a highway on a ramp that is banked at 15° to the horizontal. The exit ramp has a radius of curvature of $65\,\mathrm{m}$. If the conditions are extremely icy and the driver cannot depend on any friction to help make the turn, at what speed should the driver travel so that the car will not skid off the ramp?