Equations (1)

1. Equation - Two expressions set equal to each other.

Linear Equation in One Variable - An equation that can be written in the form $\boldsymbol{ax + b = c}$
where a, b, and c are constants.

Note that the exponent on the variable of a linear equation is always 1.

The following is an example of a linear equation: $3x - 4 = 5$

Solution - A value, such that, when you replace the variable with it, it makes the equation true.
(The left side comes out equal to the right side)

Solution Set - Set of all solutions

Solving a Linear Equation in General - Get the variable you are solving for alone on one side and everything else on the other side using INVERSE operations.

Addition and Subtraction Properties of Equality

If a = b, then a + c = b + c *If a = b, then a - c = b – c*

In other words, if two expressions are equal to each other and you add or subtract the exact same thing to both sides, the two sides will remain equal.

Note that addition and subtraction are inverse operations of each other.
For example, if you have a number that is being added that you need to move to the other side of the equation, then you would subtract it from both sides of that equation.

Example 1: Solve the equation $x - 5 = 2$.

$$x - 5 = 2$$
$$x - 5 + 5 = 2 + 5$$
$$x = 7$$

Note that if you put 7 back in for *x* in the original problem you will see that 7 is the solution to our problem.

Example 2: Solve the equation $y + \frac{3}{4} = \frac{1}{2}$.

$$y + \frac{3}{4} = \frac{1}{2}$$
$$y + \frac{3}{4} - \frac{3}{4} = \frac{1}{2} - \frac{3}{4}$$
$$y = \frac{2}{4} - \frac{3}{4}$$
$$y = -\frac{1}{4}$$

If you put -1/4 back in for *y* in the original problem you will see that -1/4 is the solution to our problem.

2. Solving Equations Using Division and Multiplication

Multiplication and Division Properties of Equality

If a = b, then a(c) = b(c) *If a = b, then a/c = b/c where c is not equal to 0.*

In other words, if two expressions are equal to each other and you multiply or divide (except for 0) the exact same constant to both sides, the two sides will remain equal.

Note that multiplication and division are inverse operations of each other.
For example, if you have a number that is being multiplied that you need to move to the other side of the equation, then you would divide it from both sides of that equation.

Example 1: Solve the equation $\frac{x}{2} = 5$

$$\frac{x}{2} = 5$$
$$(2)\frac{x}{2} = (2)5$$
$$x = 10$$

If you put 10 back in for *x* in the original problem, you will see that 10 is the solution.

Example 2: Solve the equation $5x = 7$.

$$5x = 7$$
$$\frac{5x}{5} = \frac{7}{5}$$
$$x = \frac{7}{5}$$

If you put 7/5 back in for *x* in the original problem, you will see that 7/5 is the solution.

Example 3: Solve the equation $9 = -\frac{3}{2}a$

$$9 = -\frac{3}{2}a$$
$$\left(-\frac{2}{3}\right)9 = \left(-\frac{2}{3}\right)\left(-\frac{3}{2}a\right)$$
$$-6 = a$$

If you put -6 back in for *a* in the original problem you will see that -6 is the solution.

Note that it doesn't matter what side the variable is on. - 6 = *a* means the same thing as *a* = - 6.
But usually we write the variable on the left hand side, and the solution on the right: *a* = - 6

The examples above were using only one property at a time to help you understand the different properties that we use to solve equations. However, most times, we have to use several properties to get the job done. The following is a strategy that you can use to help you solve linear equations that are a little bit more involved.

Strategy for Solving a Linear Equation

Note that your teacher or the book you are using may have worded these steps a little differently than I do, but it all boils down to the same concept - get your variable on one side and everything else on the other using inverse operations.

Step 1: Simplify each side, if needed.
This would involve things like removing ( ), removing fractions, removing decimals, and adding like terms.
To remove ( ): Just use the distributive property
To remove fractions: Since fractions are another way to write division, and the inverse of divide is to multiply, you remove fractions by multiplying both sides by the LCD of all of your fractions.

Step 2: Use Add./Sub. Properties to move the variable term to one side and all other terms to the other side.

Step 3: Use Mult./Div. Properties to remove any values that are in front of the variable.

Step 4: Check your answer.
What it boils down to is that you want to get the variable you are solving for alone on one side and everything else on the other side using INVERSE operations.

Example 4: Solve the equation $10-3x=7$.

$$10-3x=7$$
$$10-3x-10=7-10$$
$$-3x=-3$$
$$\frac{-3x}{-3}=\frac{-3}{-3}$$
$$x=1$$

Be careful going from line 4 to line 5. Yes, there is a negative sign. But, the operation between the -3 and *x* is multiplication not subtraction. So if you were to add 3 to both sides you would have ended up with -3*x* + 3 instead of the desired *x*.
If you put 1 back in for x in the original problem you will see that 1 is the solution.

Example 5: Solve the equation $3x-4x-5=-3$.

$$3x-4x-5=-3$$
$$-x-5=-3$$
$$-x-5+5=-3+5$$
$$-x=2$$
$$\frac{-x}{-1}=\frac{2}{-1}$$
$$x=-2$$

If you put -2 back in for *x* in the original problem you will see that -2 is the solution we are looking for.

Example 6: Solve the equation $5x - 2 + 4x = 2x + 12$.

$$5x - 2 + 4x = 2x + 12$$
$$9x - 2 = 2x + 12$$
$$9x - 2 - 2x = 2x + 12 - 2x$$
$$7x - 2 = 12$$
$$7x - 2 + 2 = 12 + 2$$
$$7x = 14$$
$$\frac{7x}{7} = \frac{14}{7}$$
$$x = 2$$

If you put 2 back in for *x* in the original problem you will see that 2 is the solution.

Perform a formal check for Example 6:

LS = 5x – 2 + 4x = 5(2) – 2 + 4(2) = 16 Therefore LS = RS.	RS = 2x + 12 = 2(2) + 12 = 16

Example 7: Solve the equation $.35y - .2 = .15y + .1$.

$$(100)(.35y - .2) = (100)(.15y + .1)$$
$$35y - 20 = 15y + 10$$
$$35y - 20 - 15y = 15y + 10 - 15y$$
$$20y - 20 = 10$$
$$20y - 20 + 20 = 10 + 20$$
$$20y = 30$$
$$\frac{20y}{20} = \frac{30}{20}$$
$$y = \frac{3}{2}$$

If you put 3/2 back in for *y* in the original problem you will see that 3/2 is the solution we are looking for.

Example 8: Solve y + 6(y – 3) = 2(3y – 2)

y + 6(y – 3) = 2(3y – 2)
y + 6y – 18 = 6y – 4
7y – 18 = 6y – 4
y = 14

Practice in class:

1. Solve.

(a) $3y + 5 = 11$ (b) $4x - 3 = -11$ (c) $17 = 4c - 3$

(d) $6x + 8 = 4x - 10$ (e) $9p - 10 = 6 + p$ (f) $2m + 6.1 = 16.5$

(g) $4a - 2.8 = 6.8$ (h) $15.8 - 6m = 3.8$ (i) $8y - 6.9 = 3y + 3.6$

(j) $12.8 - 3m = 8m - 33.4$

2. Find the root of each equation.

(a) $3(n + 4) = 5n$ (b) $3x - 10 = 2(x - 3)$ (c) $2(x - 2) = 2(3 - x)$

(d) $4(c - 2) = 3(c + 1)$ (e) $8(m - 1) = 4(m + 4)$ (f) $4(3 - r) = 5(2r + 1)$

(g) $12(2m - 3) = 2(m + 4)$ (h) $0.5(x + 2) = 0.1x + 0.6(x - 3)$

(i) $6.5(x - 3) = 2.4(3 - x)$