Thin-Film Interference and EM Waves

Unit 5: The Wave Nature of Light

Dr. Timothy Leung

Grade 12 Physics
Olympiads School

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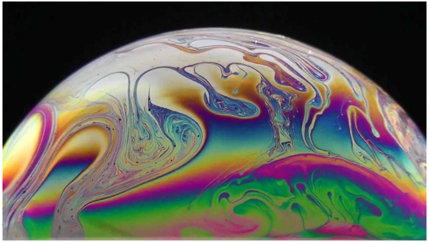
Thin-Film Interference

Thin-film interference occurs when light reflected/refracted at the upper & lower boundaries of a thin film of an indexed material interfere with one another

- “Indexed material” means a material that has a refractive index of $n > 1$
- The film is a few wavelengths in thickness
- The thickness determines whether the interference is constructive or destructive
- When white light (“broadband”, with multiple wavelengths) is incident on the film, some colours are enhanced (constructive) while others are reduced (destructive)

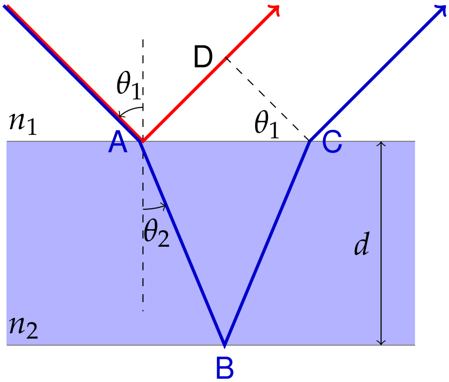
Thin-Film Interference



Examples:

- Soap bubbles
- Oil films on water
- Anti-reflection coatings on glasses and camera lenses

Calculating Path Length Difference



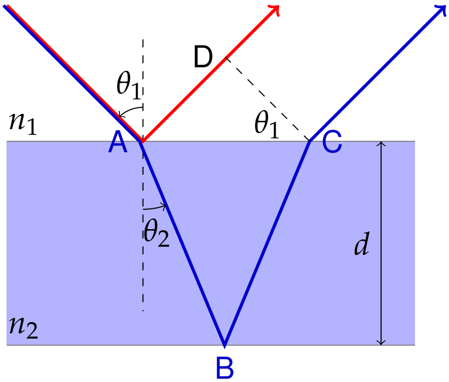
Optical path length difference Γ:

$$\Gamma = n_2(AB + BC) - n_1 AD$$

From basic geometry, we know that

$$AB = BC = \frac{d}{\cos\theta_2}$$

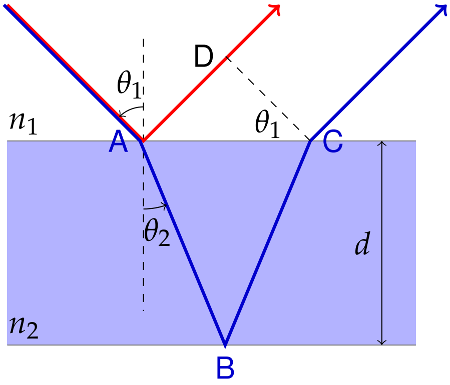
Calculating Path Length Difference



We can also calculate the length AD, and apply Snell's law:

$$\begin{aligned} n_1 AD &= n_1 AC \sin\theta_1 \\ &= (2d\tan\theta_2)(n_2\sin\theta_2) \end{aligned}$$

Calculating Path Length Difference



Collecting terms gives:

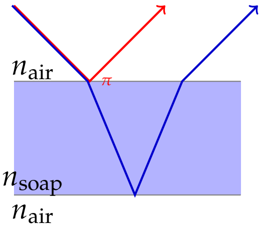
$$\Gamma = 2dn_2\left[\frac{1}{\cos\theta_2} - \tan\theta_2\sin\theta_2\right]$$

$$= 2dn_2\left[\frac{1 - sin^2\theta_2}{\cos\theta_2}\right]$$

$$= 2dn_2\cos\theta_2$$

Path length difference $2dn\cos\theta_2$ determines the condition for constructive and destructive interference.

Soap Bubble



Light travels through air ($n_{\text{air}} = 1$) and strikes a soap film ($n_{\text{soap}} > 1$)

- Light is reflected and transmitted at the air-film boundaries
- At the upper boundary:
  - Reflected has a phase shift of $180^\circ(\pi)$, because
  - $n_{\text{air}} < n_{\text{soap}}$, light reflects from a fast to slow medium
- At the lower boundary:
  - The reflection has no phase shift (slow to fast medium)

Soap Bubble

Condition for interference for a soap bubble

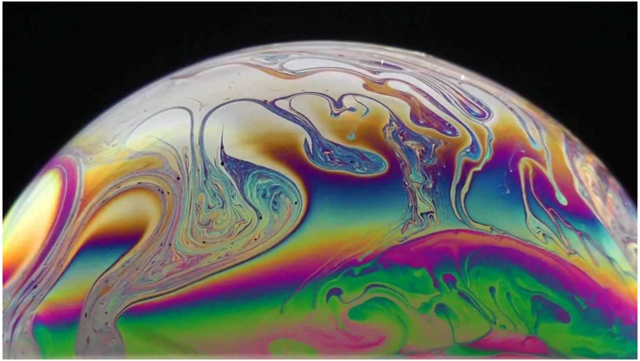
Constructive interference if path length difference is a half-number multiple of wavelength, because of the phase shift:

$$\boxed{2dn_{\text{soap}}\cos\theta_2 = \left(m - \frac{1}{2}\right)\lambda}$$

Destructive interference if the path length difference is a whole-number multiple of wavelength, also because of the phase shift:

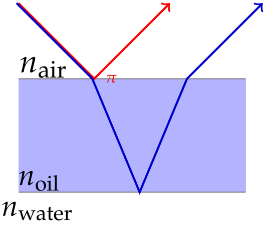
$$\boxed{2dn_{\text{soap}}\cos\theta_2 = m\lambda}$$

Soap Bubble



- When incident light is a white-light, some colours experience constructive interference, while other colours there is destructive interference
- The colour pattern comes from the variations of thickness of the film (it is not constant!)

Oil Film on Water



Light travels through air ($n_{\text{air}} = 1$) and strikes a oil film ($n_{\text{oil}} > 1$). Below the oil film is water, which has a lower refractive index than oil

- Upper boundary: Reflected light has a phase shift of 180° (π) because $n_{\text{air}} < n_{\text{oil}}$ (fast to slow medium)
- Lower boundary: Reflected light has no phase shifts (slow to fast medium)

Oil Film on Water

Like the soap bubble, we observe *constructive interference* if path length difference is a half-number multiple of wavelength:

$$2dn_{\text{oil}}\cos\theta_2 = \left(m - \frac{1}{2}\right)\lambda$$

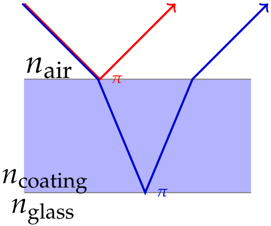
And **destructive interference** if path length difference is a whole-number multiple of wavelength:

$$2dn_{\text{oil}}\cos\theta_2 = m\lambda$$

Assuming that the refractive index of the oil film is *higher* than water:

$$n_{\text{oil}} > n_{\text{water}}$$

Anti-Reflection Coating



The anti-reflective coating on eyeglasses and camera lenses has a refractive index lower than glass (but higher than air), i.e.:

$$n_{\text{air}} < n_{\text{coating}} < n_{\text{glass}}$$

- Upper boundary: Reflected light has a phase shift of π, same as the soap bubble and oil film (fast to slow medium)
- Lower boundary: Reflected light also has a phase shifts of π!

Anti-Reflection Coating

The interference conditions for anti-reflection coating is oppposite to the soap bubble and oil film, because the phase shifts occur on both boundaries.

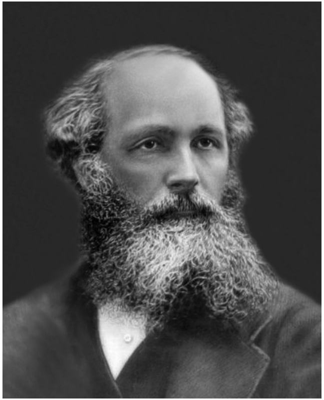
- *Constructive interference* if path length difference is a whole-number multiple of wavelength:

$$\boxed{2dn_{\text{film}}\cos\theta_2 = m\lambda}$$

- *Destructive interference* if path length difference is a half-number multiple of wavelength:

$$\boxed{2dn_{\text{film}}\cos\theta_2 = \left(m - \frac{1}{2}\right)\lambda}$$

New Physics: Maxwell's Equations



James Clerk Maxwell

- Classical laws of electrodynamics
- Published in 1861 and 1862
- Explains the relationship between
  - Electricity
  - Electric Circuits
  - Magnetism
  - Optics
- Previously these disciplines are thought to be separate and not related

Maxwell's Equations

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_o}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = -\mu_o \mathbf{J} + \mu_o \varepsilon_o \frac{\partial \mathbf{E}}{\partial t}$$

That's a lot of symbols that you won't recognize. Solving them require *a lot* of difficult calculus that even most science students in university don't need to learn. (**i.e. you don't need to learn this**)

Maxwell's Equations

Major Findings

- Electric fields starts/ends at a charge
- Magnetic fields runs in a loop, and has no beginning or ends
- A changing electric field creates a magnetic field
- A changing magnetic field creates an electric field
- Disturbances in the electric and magnetic fields propagate as a wave with speed

$$\boxed{c = \frac{1}{\sqrt{\varepsilon_o \mu_o}} = 2.998 \times 10^8\,\mathrm{m/s}}$$

...the speed of light!

Speed of Electromagnetic Radiation

Electric Permittivity ε_0

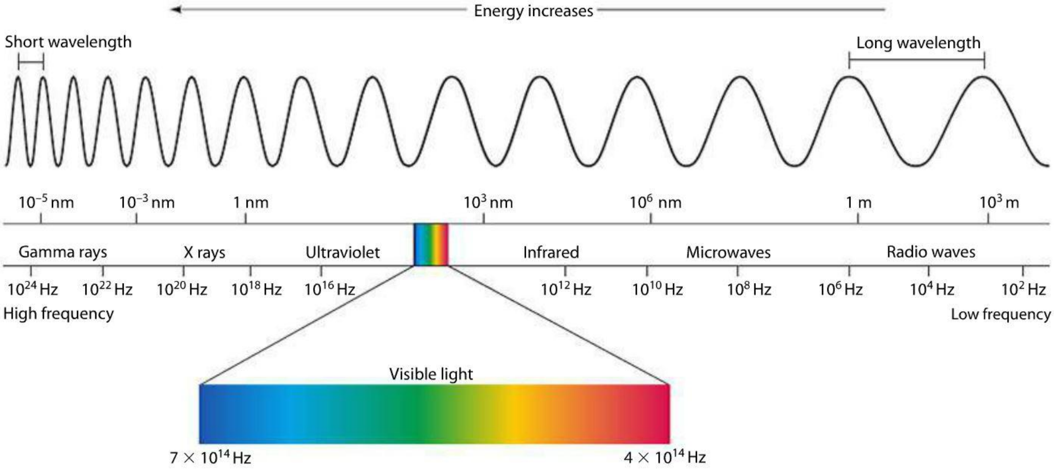
The ability of a medium to resist the formation of an electric field within it. The constant is directly related to the Coulomb constant in Coulomb's law.

Magnetic Permeability μ_o

A measure of the ability of the medium to become magnetized.

- Scientist have previously measured the speed of light to good accuracy
- Maxwell's equations show that light is (probably) an electromagnetic ("EM") wave
- *Proving* that, though, brought up a lot of new insights into physics

The Electromagnetic Spectrum



On Polarization of Light

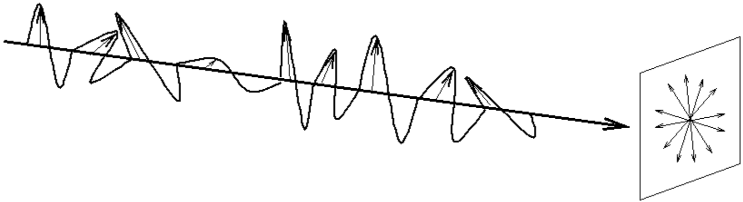
Let's Combine Everything We Know

- Light is an electromagnetic wave, generated by
  - An oscillating charged particle (e.g. shaking an electron violently)
  - An alternating ("A/C") current (i.e. lots of oscillating charged particles)
  - Through black-body radiation
- EM waves have both an oscillating electric field ($\mathbf{E}$) and magnetic field ($\mathbf{B}$), because
  - A charged particle creates an electric field, and
  - A moving charged particle creates a magnetic field
- $\mathbf{E}$ and $\mathbf{B}$ are always perpendicular to one another, according to Maxwell's equations

On Polarizaion of Light

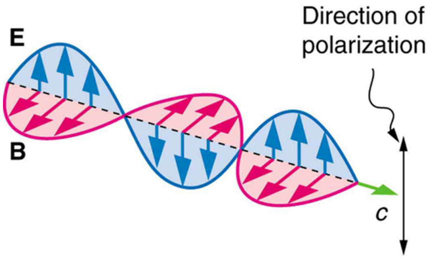
Charged particles can vibrate in any direction, so the oscillating $\mathbf{E}$ and $\mathbf{B}$ can look quite chaotic. We can only guarantee that, $\mathbf{E}$ and $\mathbf{B}$ are:

- Always perpendicular to each other
- Always perpendicular to the direction of wave travel
- This kind of light (or general EM wave) is “unpolarized”
- Most EM waves you experience in life are this kind:



On Polarization of Light

But if we can confine $\mathbf{E}$ and $\mathbf{B}$ to one plane, then we have a “polarized” light:

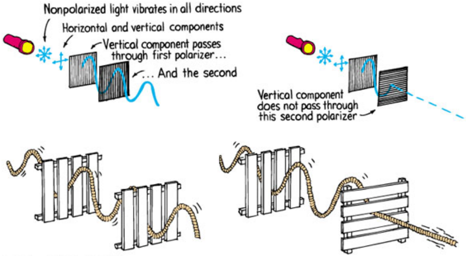


There are a few ways to do this. . .

On Polarization of Light

Using Polarizer

- A polarizer is really just a grill that only lets in vibration in one direction through:

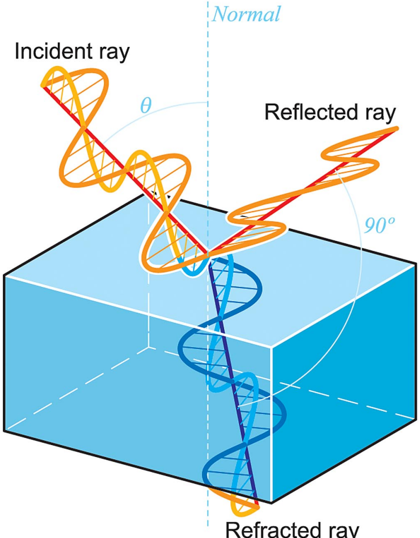


Hewitt, *Conceptual Physics*, Ninth Edition.

- The incoming wave can be vibrating in any direction, but outgoing wave only vibrates in one direction.
- Sunglasses with polarizing lens
- Polarizer filters on cameras

On Polarization of Light

Polarization by Reflection



At **Brewster's angle**, the light reflected off a medium (e.g. glass, water) is also polarized

$$\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

- Incident light is non-polarized
- Reflected light is polarized
- Refracted light is partially polarized
- For water ($n = 1.33$), $\theta_B = 53^\circ$
- For glass ($n = 1.5$), $\theta_B = 56^\circ$