Calculations and Counting

The notes are same as before. Please review them, then do the <u>Questions in class</u> and <u>homework</u>.

1. The Commutative Properties of Addition and Multiplication

a + b = b + a   and   ab = ba

The Commutative Property, in general, states that changing the ORDER of two numbers either being added or multiplied, does NOT change the value of it.

The two sides are called equivalent expressions because they look different but have the same value.

Example 1: Use the commutative property to write an equivalent expression to 2.5x + 3y.

2.5x + 3y = 3y + 2.5x.

Example 2: Use the commutative property to write an equivalent expression to $\frac{1}{5}\cdot\frac{x}{7}$.

$$\frac{1}{5}\cdot\frac{x}{7}=\frac{x}{7}\cdot\frac{1}{5}$$

2. The Associative Properties of Addition and Multiplication

a + (b + c) = (a + b) + c   and   a(bc) = (ab)c

The Associative property, in general, states that changing the GROUPING of numbers that are either being added or multiplied does NOT change the value of it. Again, the two sides are equivalent to each other.

Example 1: Use the associative property to write an equivalent expression to (a + 5b) + 2c.

(a + 5b) + 2c = a + (5b + 2c).

Example 2: Use the associative property to write an equivalent expression to (1.5x)y.

(1.5x)y = 1.5(xy)

3. Distributive Properties

a(b + c) = ab + ac* or *(b + c)a = ba + ca

In other words, when you have a term being multiplied times two or more terms that are being added (or subtracted) in a ( ), multiply the outside term times EVERY term on the inside.

Remember terms are separated by + and -.

Example 1: Use the distributive property to write $2(x - y)$ without parenthesis.

Multiplying every term on the inside of the ( ) by 2 we get:
$2(x-y)=2(x)-2(y)=2x-2y$

Example 2: Use the distributive property to write - (5x + 7) without parenthesis.
$-(5x+7)=(-1)(5x)+(-1)(7)=-5x-7$

Basically, when you have a negative sign in front of a ( ), like this example, you can think of it as taking a -1 times the ( ). What you end up doing in the end is taking the opposite of every term in the ( ).

Example 3: Use the distributive property to find the product $3(2a + 3b + 4c)$.

As mentioned above, you can extend the distributive property to as many terms as are inside the ( ). The basic idea is that you multiply the outside term times EVERY term on the inside.
$3(2a+3b+4c)=3(2a)+3(3b)+3(4c)=6a+9b+12c$

4. Identity Properties

1) Addition
The additive identity is 0
a + 0 = 0 + a = a
In other words, when you add 0 to any number, you end up with that number as a result.

2) Multiplication
Multiplication identity is 1
a(1) = 1(a) = a
And when you multiply any number by 1, you will end up with that number as your answer.

5. The Inverse Properties

1) Additive Inverse (or negative)
For each real number a, there is a unique real number, denoted -a,
such that a + (-a) = 0.

In other words, when you add a number to its additive inverse, the result is 0. Other terms that are synonymous with additive inverse are negative and opposite.

2) Multiplicative Inverse (or reciprocal)

For each real number a, except 0, there is a unique real number $\frac{1}{a}$ such that

$$a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$$

In other words, when you multiply a number by its multiplicative inverse the result is 1. A more common term used to indicate a multiplicative inverse is the **reciprocal**.

A multiplicative inverse or reciprocal of a real number *a* (except 0) is found by "flipping" *a* upside down. The numerator of *a* becomes the denominator of the reciprocal of *a* and the denominator of *a* becomes the numerator of the reciprocal of *a*.

Example 1: Write the opposite (or additive inverse) and reciprocal (or multiplicative inverse) of -3.

The opposite of -3 is 3, since -3 + 3 = 0.
The reciprocal of -3 is -1/3, since -3(-1/3) = 1.

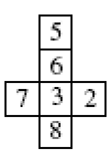
When you take the reciprocal, the sign of the original number stays intact. Remember that you need a number that when you multiply times the given number you get 1. If you change the sign when you take the reciprocal, you would get a -1, instead of 1, and that is a no.

Example 2: Write the opposite (or additive inverse) and reciprocal (or multiplicative inverse) of 1/5.

The opposite of 1/5 is -1/5, since 1/5 + (-1/5) = 0.
The reciprocal of 1/5 is 5, since 5(1/5) = 1.

►Questions in class

1. The figure shown is folded to form a cube. Three faces meet at each corner. If the numbers on the three faces at a corner are multiplied, what is the largest possible product?



2. In a set of five numbers, the average of two of the numbers is 12 and the average of the other three numbers is 7. What is the average of all five numbers?

3. A set of five different positive integers has an average (arithmetic mean) of 11. What is the largest possible number in this set?

4. In a certain month, three of the Sundays have dates that are even numbers. The tenth day of this month is a what?

5. Jim drives 60 km south, 40 km west, 20 km north, and 10 km east. What is the distance from his starting point to his finishing point?

6. The scores of eight students on a quiz are 6, 7, 7, 8, 8, 8, 9, and 10. Which score should be removed to leave seven scores with the same mode and range as the original eight scores, but with a higher average (mean)?

7. In the addition of three-digit numbers shown, the letters x and y represent different digits. What is the value of y – x?

$$\begin{array}{cccc} & 3 & x & y \\ + & y & x & 3 \\ \hline 1 & x & 1 & x \end{array}$$

8. A purse contains a collection of quarters, dimes, nickels, and pennies. The average value of the coins in the purse is 17 cents. If a penny is removed from the purse, the average value of the coins becomes 18 cents. How many nickels are in the purse?

9. The sum of all of the digits of the integers from 98 to 101 is 9 + 8 + 9 + 9 + 1 + 0 + 0 + 1 + 0 + 1 = 38. What is the sum of all of the digits of the integers from 1 to 2008?

10. In the diagram, a positive integer is to be placed in each of the nine boxes so that the products of the numbers in each row, column, and diagonal are equal. Some of the entries are already filled in. what is the number of possible values for *N?*

N		24
	12	
6		