Algebra 2

1. Perfect square formula

1) $(a \pm b)^2 = a^2 \pm 2ab + b^2$
2) $(x+a)(x+b) = x^2 + (a+b)x + ab$

2. Factoring perfect square trinomials

Whenever you multiply a binomial by itself twice, the resulting trinomial is called a perfect square trinomial

For example, $(x + 1) \times (x + 1) = x^2 + x + x + 1 = x^2 + 2x + 1$ and $x^2 + 2x + 1$ is a perfect square trinomial

Another example is $(\boldsymbol{x} - 5) \times (x - 5)$

$(x - 5) \times (x - 5) = x^2 + -5x + -5x + 25 = x^2 + (-10x) + 25$ and $x^2 + (-10x) + 25$ is a perfect square trinomial

Now, we are ready to start factoring perfect square trinomials

The model to remember when factoring perfect square trinomials is the following:

$a^2 + 2ab + b^2 = (a+b)^2$ and $(a + b)^2$ is the factorization form for $a^2 + 2ab + b^2$

Notice that all you have to do is to use the base of the first term and the last term

In the model just described, the first term is a^2 and the base is a, the last term is b^2 and the base is b

Put the bases inside parentheses with a plus between them $(a+b)$
Raise everything to the second power $(a+b)^2$ and you are done

Notice that I put a plus between a and b. **You will put a minus if the second term is negative**!

$a^2 + (-2ab) + b^2 = (a-b)^2$

Remember that $a^2 - 2ab + b^2 = a^2 + (-2ab) + b^2 = (a-b)^2$ because a minus is the same thing as adding the negative ($-$ = + -) So, $a^2 - 2ab + b^2$ is also equal to $(a-b)^2$

Example 1

Factor $x^2 + 2x + 1$
Notice that $x^2 + 2x + 1 = x^2 + 2x + 1^2$

Using $x^2 + 2x + 1^2$, we see that... the first term is x^2 and the base is x,

the last term is 1^2 and the base is 1

Put the bases inside parentheses with a plus between them $(x+1)$.

Raise everything to the second power $(x+1)^2$ and you are done

Example 2

Factor $x^2 + 24x + 144$

But wait before we continue, we need to establish something important when factoring perfect square trinomials.

How do we know when a trinomial is a perfect square trinomial?

This is important to check this because if it is not, we cannot use the model described above

Think of checking this as part of the process when factoring perfect square trinomials

We will use Example 2 to show you how to check this

Start the same way you started Example 1:

Notice that $x^2 - 24x = x^2 + 2 \cdot 12x + 12^2$

Using $x^2 + 2 \cdot 12x + 12^2$, we see that...

the first term is x^2 and the base is x

the last term is 12^2 and the base is 12

Now, this is how you check if $x^2 + 2 \cdot 12x + 12^2$ is a perfect square

If 2 times (base of first term) times (base of last term) = second term, the trinomial is a perfect square

If the second term is negative, check using the following instead

-2 times (base of first term) times (base of last term) = second term

Since the second term is 24x and 2 × x × 12 = 24x, $x^2+2\cdot 12x+12^2$ is perfect and we factor like this:

Put the bases inside parentheses with a plus between them $(x+12)$

Raise everything to the second power $(x+12)^2$ and you are done

Example 3

Factor $x^2-18x+81$

Notice that $x^2-18x+81=x^2-2\cdot 9x+9^2$

Using $x^2-2\cdot 9x+9^2$, we see that...

the first term is x^2 and the base is x

the last term is 9^2 and the base is 9

Since the second term is -18x and -2 × p × 9 = -18x, $x^2-2\cdot 9x+9^2$ is a perfect square and we factor like this:

Put the bases inside parentheses with a minus between them $(x-9)$

Raise everything to the second power $(x-9)^2$ and you are done

Example 4

Factor $4x^2+48x+144$

Notice that $4x^2+48x+144=(2x)^2+2\cdot 2\cdot 12x+12^2$

$(2x)^2+2\cdot 2\cdot 12x+12^2$, we see that...

the first term is $(2x)^2$ and the base is 2x, the last term is 12^2 and the base is 12

Since the second term is 48x and 2 × 2x × 12 = 48x, $(2x)^2+48x+12^2$ is a perfect square and we factor like this:

Put the bases inside parentheses with a plus between them $(2x+12)$.

Raise everything to the second power $(2x+12)^2$ and you are done

Questions in class

1. Solve by factoring.
a) $n^2 + 3n - 12 = 6$ b) $5r^2 - 44r + 120 = -30 + 11r$ c) $35k^2 - 22k + 7 = 4$

2. How many integers between 2004 and 4002 are perfect squares?

3. If $a \neq b$ and $\left(\frac{a(1-b)}{b(1-a)}\right)^2 = 1$, what is the value of $(a + b)/(ab)$?

4. Find all ordered triples of positive integers (a, b, c) for which $a^3 - b^3 - c^3 = 3abc$ and $a^2 = 2(b + c)$.

5. What is the largest prime p such that there is a prime q for which p + q and p + 7q are both squares?

6. What is the sum of all positive integers x for which there exists a positive integer y with $x^2 - y^2 = 1001$?

7. What is the sum of all 2-digit positive integers which exceed the product of their digits by 12?

8. Suppose Briley has 10 coins in quarters and dimes and has a total of $1.45. How many of each coin does she have?