Chapter 8 Geometric Relationships (2)

1. Circle Terminology

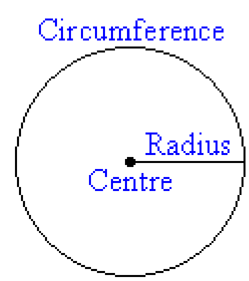
In this section we will consider circles, arcs, subtended angles, the angle at the circumference, and the Angle at Centre Theorem.

Circle

A **circle** is a set of points in a plane that are the same distance from a fixed point (called the centre). This set of points forms the **perimeter** of the circle.

The **radius** is the distance from the centre of the circle to any point on its perimeter.

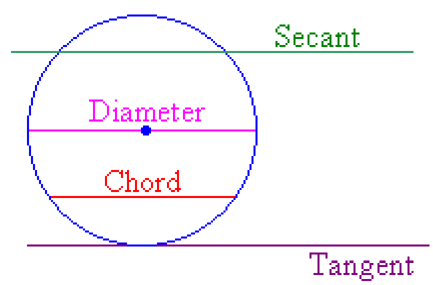
The **circumference** of a circle is the perimeter of the circle. These parts of a circle are indicated in the accompanying diagram.
Note: The plural of radius is **radii**.



2. Lines in a Circle

The name of a line in a circle depends on its position in the circle.

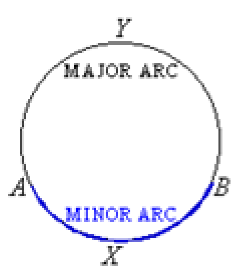
A **secant** is a line that passes through any two points on a circle.
A **chord** is a line that joins two points on the circumference of a circle.
The **diameter** is a chord that passes through the centre of a circle.
A **tangent** is a line that touches the circle at only one point.



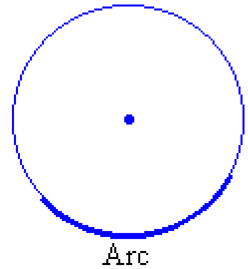
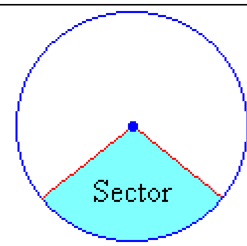
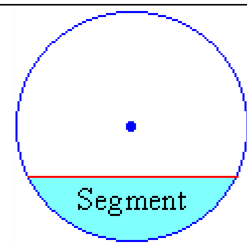
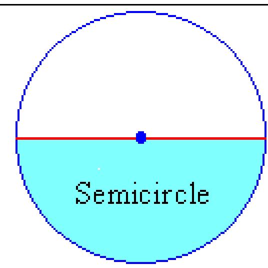
3. Arcs

An **arc** is a curved line along the circumference of a circle.

For example, *AXB* is a **minor arc**, and *AYB* is a **major arc**.

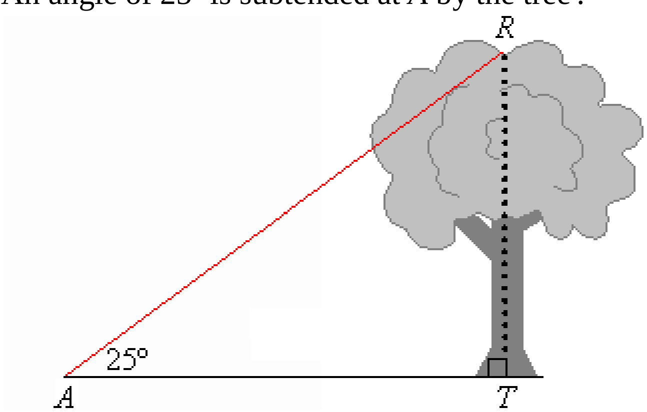


4. Parts of a Circle

An **arc** is a part of the circumference.	
A **sector** is the part of a circle between two radii.	
A **segment** is the part of a circle that is between a chord and the circumference.	
A **semicircle** is a half of a circle.	

5. Subtended Angles

The angle *RAT* is said to be the angle subtended by the tree *TR* at *A*. So, the tree subtends an angle of 25º at *A*. This can be described as follows.
'An angle of 25º is subtended at *A* by the tree'.

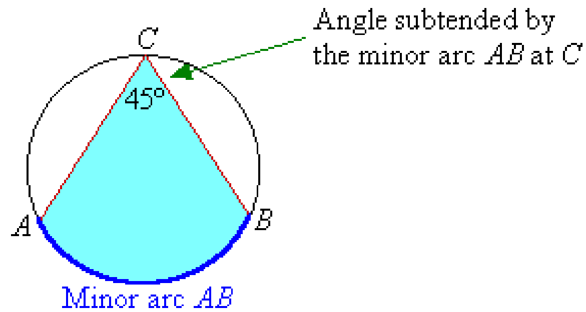


6. Angle at the Circumference

If the end points of an arc are joined to a third point on the circumference of a circle, then an angle is formed. It is called the **inscribed angle**.

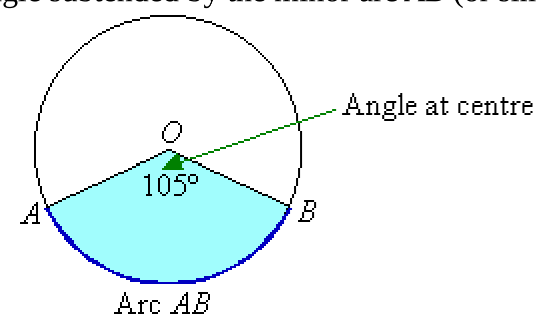
For example, the minor arc *AB* subtends an angle of 45º at *C*. The angle *ACB* is said to be the angle subtended by the minor arc *AB* (or simply arc *AB*) at *C*.

The angle *ACB* is an angle at the circumference standing on the arc *AB*.



7. Angle at the Centre

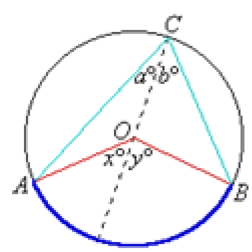
If the end points of an arc are joined to the centre of a circle, then an angle is formed.
For example, the minor arc *AB* subtends an angle of 105º at *O*. The angle *AOB* is said to be the angle subtended by the minor arc *AB* (or simply arc *AB*) at the centre *O*.



The angle *AOB* is an angle at the centre *O* standing on the arc *AB*, it is also called the **central angle**.

8. Angle at Centre Theorem

Theorem
Use the information given in the diagram to prove that the angle at the centre of a circle is twice the angle at the circumference if both angles stand on the same arc.



Given:
$\angle AOB$ and $\angle ACB$ stand on the same arc; and O is the centre of the circle.
To prove:
$\angle AOB = 2\angle ACB$

Proof:

From ΔOAC, $x = a + a$ {Exterior angle of a triangle}

$\therefore x = 2a$ ... (1)

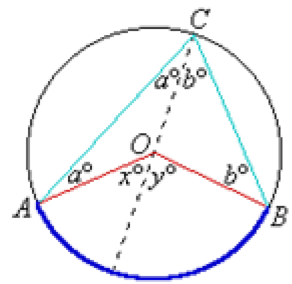
From ΔOBC, $y = b + b$ {Exterior angle of a triangle}

$\therefore y = 2b$ ... (2)

Adding (1) and (2) gives:

$x + y = 2a + 2b = 2(a+b)$ $\therefore \angle AOB = 2\angle ACB$

As required.



In general: The angle at the centre of a circle is twice the angle at the circumference if both angles stand on the same arc. This is called the **Angle at Centre Theorem**.

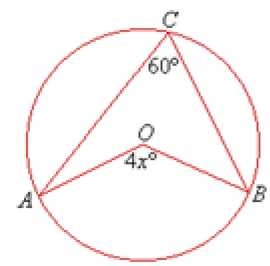
Example 1: Find the value of unknown in the following circle centred at *O*.

Solution:

$4x = 2 \times 60$ {Angle at Centre Theorem}

$4x = 120$

$\frac{4x}{4} = \frac{120}{4}$ $x = 30$



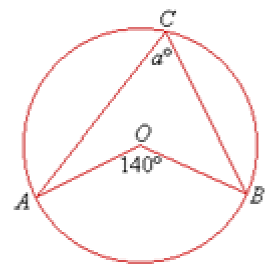
Example 2: Find the value of the unknown in the following circle centred at *O*.

Solution:

$2a = 140$ {Angle at Centre Theorem}

$\frac{2a}{2} = \frac{140}{2}$

$a = 70$



9. Angle in a Semi-Circle

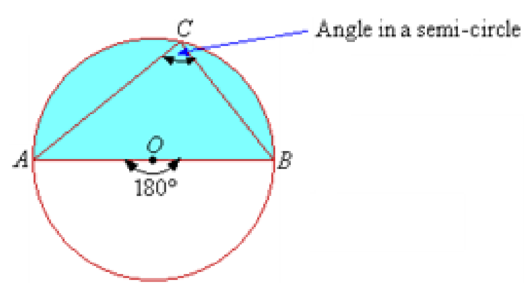
Let $\angle AOB = 180°$.

Then $\angle ACB$ is in a semi-circle.

By the Angle at Centre Theorem, we have:

$\angle AOB = 2\angle C$ $2\angle C = 180°$

$\therefore \angle C = 90°$



In general: The angle in a semi-circle is a right angle.

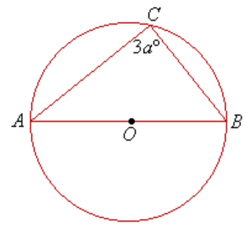
Example 1: Find the value of the unknown in the following circle centred at *O*.

Solution:

$3a = 90$ {Angle in a semi-circle}

$\frac{3a}{3} = \frac{90}{3}$

$a = 30$

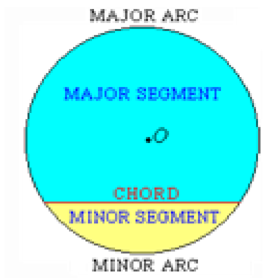


10. Segments of a Circle

A **chord** of a circle divides the circle into two regions, which are called the **segments** of the circle.

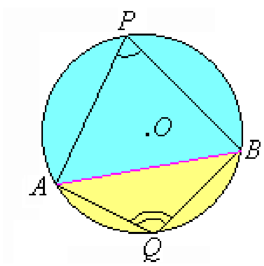
The **minor segment** is the region bounded by the chord and the minor arc intercepted by the chord.

The **major segment** is the region bounded by the chord and the major arc intercepted by the chord.



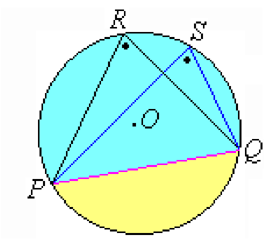
11. Angles in Different Segments

In the left diagram, $\angle APB$ is in the major segment and $\angle AQB$ is in the minor segment. So, we say that angle *APB* and angle *AQB* are in different segments.



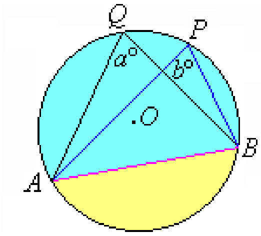
12. Angles in the Same Segment

In the right diagram, $\angle PRQ$ and $\angle PSQ$ are in the major segment. So, we say that angle *PRQ* and angle *PSQ* are in same segments.



Theorem

Use the information given in the diagram to prove that the angles in the same segment of a circle are equal. That is, *a* = *b*.

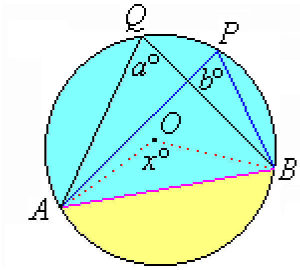


Given: $\angle APB$ and $\angle AQB$ are in the same segment; and *O* is the center of the circle.
To prove: $\angle APB = \angle AQB$.

Construction: Join *O* to *A* and *B*.
Proof:

Let $\angle AOB = x°$.

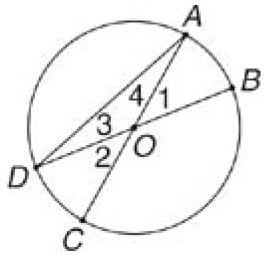
Clearly, $x = 2a$		{Angle at Centre Theorem}
	$x = 2b$	{Angle at Centre Theorem}
	$\therefore\ 2a = 2b$	{Transitive}
	$\therefore\ a = b$	As required.



In general: Angles in the same segment of a circle are equal.

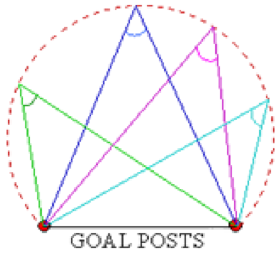
Also, it is true that for all equal inscribed angles in the same circle, subtending arcs must be equal.

Given angle 3 = angle 4, then we can conclude that arc AB = arc CD

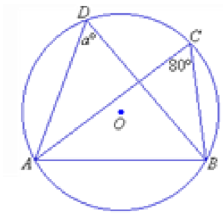


13. Practical applications

Angle for Scoring a Goal in Soccer
All positions on the same arc of a circle give the same angle for scoring a goal in soccer. Note that the distance of the shot changes but the angle of possible shots remains constant.



Example 1: Find the value of the unknown in the following circle centred at *O*.



Solution:

$a = 80$ {Angles in the same segment}

Example 2: Find the value of each of the unknowns in the following circle centred at *O*.

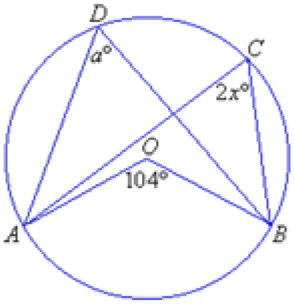
Solution:

$2a = 104$ {Angle at Centre Theorem}

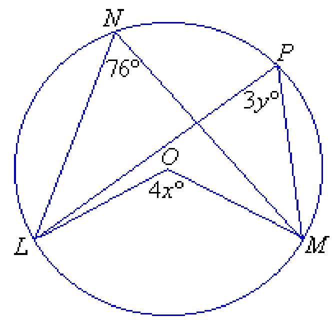
$\frac{2a}{2} = \frac{104}{2}$ $\quad a = 52$

Also, $2x = a$ {Angles in the same segment}

$\therefore\ 2x = 52$ $\quad \frac{2x}{2} = \frac{52}{2}$ $\quad x = 26$



Example 3: Find the value of each of the unknowns in the following circle centred at *O*.



Solution:

$4x = 2 \times 76$ {Angle at Centre Theorem}

$4x = 152$ $\therefore \dfrac{4x}{4} = \dfrac{152}{4}$ $x = 38$

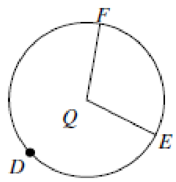
Also, $3y = 76$ {Angles in the same segment}

$\therefore \dfrac{3y}{3} = \dfrac{76}{3}$ $y = 25\dfrac{1}{3}$

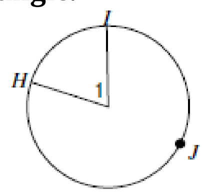
Class Practice:

1. Name the arc made by the given angle.

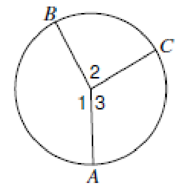
1) $\angle FQE$



2) $\angle 1$

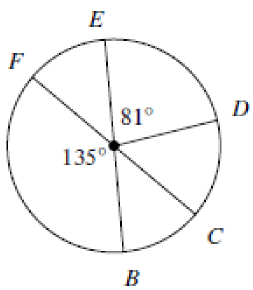


3) Major arc for $\angle 1$

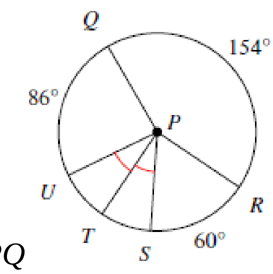


2. Find the measure of the arc or central angle indicated.

1) $\overset{\frown}{CFD}$

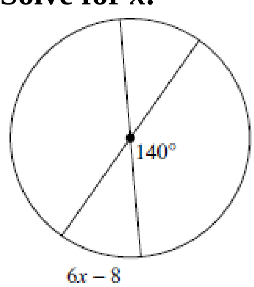


2) $\angle SPQ$

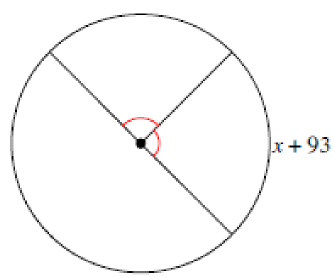


3. Solve for *x*.

1)

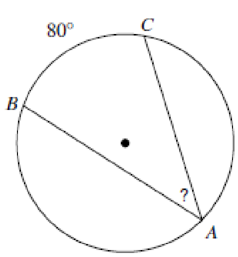


2)

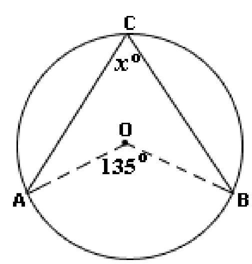


4. Find the measure of the angle indicated.

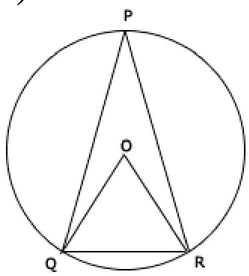
1)



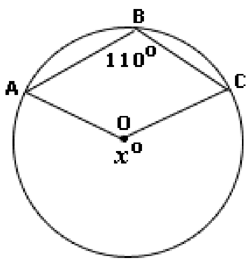
2)



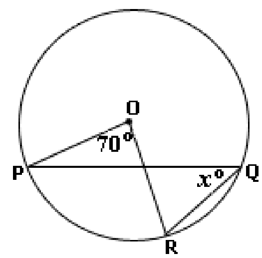
3) O is the center of the circle. What is $m\angle QPR$, if $m\angle OQR = 47$?



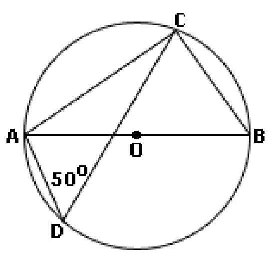
4)



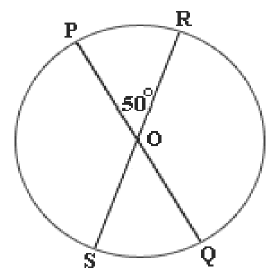
5)



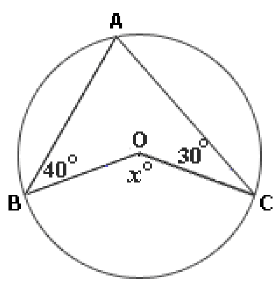
6) Find $\angle CAB$



7) Find $\angle SRQ$



8)



9)