Algebra

1. Introduction to Algebra

Algebra is great fun - you get to solve puzzles!
A Puzzle

What is the missing number?

$$\square - 2 = 4$$

OK, the answer is 6, right? Because 6-2=4. Easy stuff.

Well, in Algebra we don't use blank boxes, we use a letter (usually an x or y, but any letter is fine). So we would write:

$$x - 2 = 4$$

It is really that simple. The letter (in this case an x) just means "we don't know this yet", and is often called the unknown or the variable.
And when you solve it you write:

$$x = 6$$

2. Translating verbal phrases into algebraic language.

An inequality means just that the quantities we are comparing are NOT equal.

That means that there are 4 possibilities:

1) One quantity may be *LESS THAN* another

2) One quantity may be GREATER THAN another

3) One quantity may be LESS THAN OR EQUAL TO another

4) One quantity may be *GREATER THAN OR EQUAL TO* another

Each inequality has it's own symbol:
less than $<$
greater than $>$
less than or equal to $\leq$
greater than or equal to $\geq$

Example 1
Sue and David are friends. David is 6 years older than Sue.
If the sum of Sue and David's age is less than 42 years, what is the greatest age each can be?

When translating, you must begin with a "let" statement.
In a let statement you assign a variable to stand for the quantity (ies) you are trying to solve for.
In this problem there are two quantities we are interested in finding.

That's right...Sue's age and David's age.....
Let x = Sue's age
Let x+6 = David's age

Now for the translation into an algebraic inequality:
$(x) + (x + 6) < 42$

Example 2
On a farm, the number of cows is 50 more than twice the number of sheep. If there are at most 250 animals in all, find the greatest number of cows, and the greatest number of sheep there could be on this farm.

(Do you understand that that means the total number of animals must be *less than or equal* to 250)

let x = the number of sheep
let 2x+ 50 = the number of cows $(x) + (2x+50) \leq 250$

Example 3
Bob received grades of 88, 91, 89, and 87 on four science tests.
What is the lowest grade that Bob can receive on the fifth test in order for his average to be greater than 90?

(Remember to find an average add up all the terms and divide by the number of terms)
Let x = the grade he must receive on the fifth test $(88 + 91 + 89 + 87 + x) \div 5 > 90$

Example 4
In Sara's bank, there are twice as many nickels as quarters.
If the value of these coins is at least $8.00, find the smallest possible number of nickels and quarters in her bank.

Let x = the *number* of quarters
Let 2x = the *number* of nickels
Then 25x = the *value* of the quarters in cents
Then 10x = the *value* of the nickels in cents

$10x + 25x \geq 800$ (remember $8 = 800 cents!)

Translating in mathematics usually involves changing a verbal phrase, or sentence, into a mathematical phrase, or sentence.

3. Translating a mathematical phrase, or sentence, into a verbal phrase.

Example 1
Mathematical Sentence: $x + 13 = 20$

Matching Verbal Expression:
"A number increased by thirteen is twenty."

Example 2
Mathematical Sentence: $3y - 7 = 2y + 8$

Matching Verbal Expression:
"When three times a number is decreased by seven,
the result is the same as when two times the same number is increased by eight."

Example 3
Mathematical Phrase: $85 - 3(a + 7)$

Matching Verbal Expression:
"Eighty-five decreased by three times the sum of a number and seven."

Let's try a little multiple choice this time...

Choose the ***mathematical*** sentence which matches the given ***verbal*** sentence.
"When eight is subtracted from five times a number the result is six."
a.) $8 - 5x = 6$
b.) $5x - 6 = 8$
c.) $5x - 8 = 6$
d.) $8 - 6x = 5$

Let's look for the clues.....
In subtraction, the amount you subtract "from" is written first.
In the sentence we are subtracting "from" "five times a number"

That means that "5x" must be written first.
Therefore only choices "b" and "c" could be correct....

Next clue....
"the result" means "equals"

In the sentence "the result is six", is the same as "equals six"
Only choice "c" equals six....

So the correct translation is choice "c"
"$5x - 8 = 6$"

Challenge Examples
Example 1

At the beginning of a trip, the mileage odometer read 56,200 miles. The driver filled the gas tank with 6 gallons of gasoline. During the trip, the driver filled his tank again with 12 gallons of gasoline when the odometer read 56,560. At the end of the trip, the driver filled the tank again with 20 gallons of gasoline. The odometer read 57,060. To the nearest tenth, what was the car's average miles-per-gallon for the entire trip?

Solution

The trip was 57,060 - 56,200 = 860 miles long and 12 + 20 = 32 gallons of gasoline were used during the trip. Thus the average number of miles per gallon was 860/ 32 = 26.9. Note that the 6 gallons needed to fill the tank at the start of the trip had no effect on the answer. No matter how many gallons were needed to fill the tank at the start, the average miles per gallon for the trip would be 26.9.

Example 2

Which of the following sets of whole numbers has the largest average?
A) multiples of 2 between 1 and 101 B) multiples of 3 between 1 and 101
C) multiples of 4 between 1 and 101 D) multiples of 5 between 1 and 101
E) multiples of 6 between 1 and 101

Solution
In a set of whole numbers which are equally spaced, the average of the numbers in the set is the average of the smallest number and the largest number. For example, the average of { 1,3,5,7,9 } = (1 + 9)/2 = 5, and the average of { 2,5,8,11,14,17 } is (2 + 17)/2 = 9.5. In this problem, then, the averages are:

A: (2+100)/2 = 51, B: (3+99)/2 = 51, C: (4+10)/2 = 52, D: (5+100)/2 = 52.5,
E: (6+96)/2 = 51.

One could "guesstimate" that the set with the "largest" numbers should have the largest average. The numbers 5 and 100 are (overall) larger than the corresponding numbers in the other sets.

Example 3
A calculator has a squaring key [x^2] which replaces the current number displayed with its square. For example, if the display is [3] and the [x^2] key is depressed, then the display becomes [9]. If the display reads [2], how many times must you depress the [x^2] key to produce a displayed number greater than 500?

Solution

The next four numbers displayed are 4, 16, 256, and $256^2 \approx 60{,}000$. Thus 500 is exceeded on the 4th depression of the [x^2] key.

Questions in class

1. The sale ad read: "Buy three tires at the regular price and get the fourth tire for $30." Sam paid $240 for a set of four tires at the sale. What was the regular price of one tire?

2. The arithmetic mean (average) of four numbers is 85. If the largest of these numbers is 97, then what is the mean of the remaining 3 numbers?

3. Each day Maria must work 8 hours. This does not include the 45 minutes she takes for lunch. If she begins working at 7:25 A.M. and takes her lunch break at noon, then her working day will end at
(A) 3:40 P.M. (B) 3:55 P.M. (C) 4:10 P.M. (D) 4:25 P.M. (E) 4:40 P.M.

4. Each principal of Lincoln High School serves exactly one 3-year term. What is the maximum number of principals this school could have during an 8-year period?

5. What is the minimum possible product of three different numbers of the set {-8, -6,-4, 0, 3, 5, 7}?

6. You have nine coins: a collection of pennies, nickels, dimes, and quarters having a total value of $1.02, with at least one coin of each type. How many dimes must you have?

7. Six trees are equally spaced along one side of a straight road. The distance from the first tree to the fourth is 60 feet. What is the distance in feet between the first and last trees?

8. A number is three less than four times another number. Their difference is eighty-seven. What are the numbers?

9. A number is twenty-nine less than five times another number. Their sum is one hundred ninety-nine. What are the numbers?

10. A group of children riding on bicycles and tricycles rode past Billy Bob's house. Billy Bob counted 7 children and 19 wheels. How many tricycles were there?