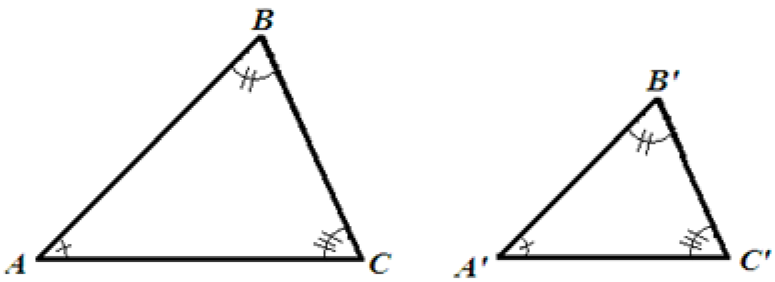
Chapter 11 Similar Triangles 1

1. Definition

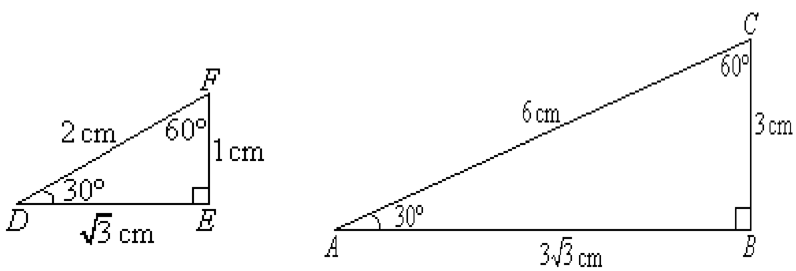
Two triangles ABC and A'B'C' are similar if the three angles of the first triangle are congruent to the corresponding three angles of the second triangle and the lengths of their corresponding sides are proportional as follows.

$\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CA}{C'A'}$ or AB: A'B'= BC: B'C' = CA: C'A'



Equiangular triangles have the same shape but may have different sizes. So, equiangular triangles are also called **similar triangles**.

For example, triangle *DEF* is similar to triangle *ABC* as their three angles are equal (equal angles are marked in the same way in diagrams).



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The length of each side in triangle *DEF* is multiplied by the same number, 3, to give the sides of triangle *ABC*.

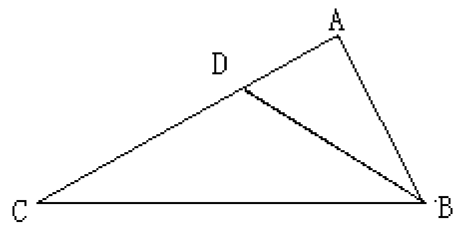
$$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = 3$$

Generally speaking, if two triangles are similar, then the corresponding sides are in the same ratio.

Example: In the diagram, $\triangle ABC \sim \triangle ADB$, and $\angle ABD = \angle C$. List all corresponding sides and angles.

Solution
Corresponding sides: AB and AC, AD and AB, DB and AC

Corresponding angles: $\angle A$ and $\angle A$, $\angle ADB$ and $\angle ABC$, $\angle ABD$ and $\angle C$



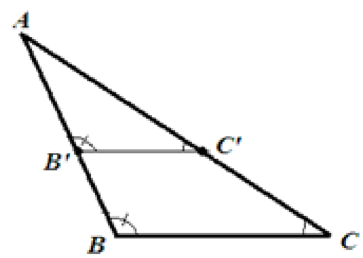
2. Angle-Angle (AA~) Similarity

Theorem
If two angles in a triangle are congruent to the two corresponding angles in a second triangle, then the two triangles are similar. This is because sum of three angles of a triangle equals 180^o which assures the third pair of corresponding angles must be equal.

Example 1: Let ABC be a triangle and B'C' // BC. Prove ΔABC is similar to ΔA'BC'.

Solution

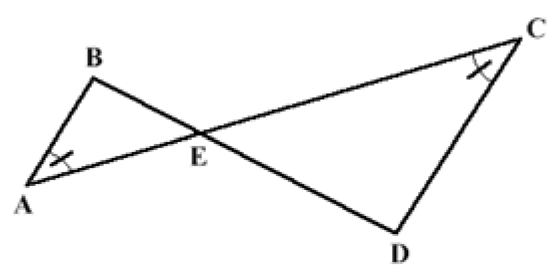
Statement	Reason
B'C' // BC	Given
∠AB'C' = ∠ABC and ∠AC'B' = ∠ACB	Alternate angles
ΔAB'C' ~ ΔABC	AA~



Example 2: If AB//CD, Prove triangle ABE is similar to triangle CDE.

Solution:

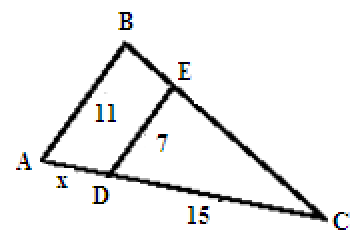
Statement	Reason
AB // CD	Given
∠A= ∠C	Alternate angles
∠AEB = ∠*CED*	Opposite angles
ΔABE ~ ΔCDE	AA~



Example 3: Given that lines DE and AB are parallel in the figure to the right, determine the value of x, the distance between points A and D.

Solution:

Statement	Reason
AB // DE	Given
∠CDE = ∠CAB	Corresponding angles
∠C= ∠C	Shared angle
ΔCDE ~ ΔCAB	AA~



Hence we have $\frac{AC}{DC} = \frac{BC}{EC} = \frac{AB}{DE}$

This gives $\frac{15+x}{15} = \frac{11}{7}$. Solving the equation,

$15 + x = \frac{11}{7}(15)$

$x = 8.57$

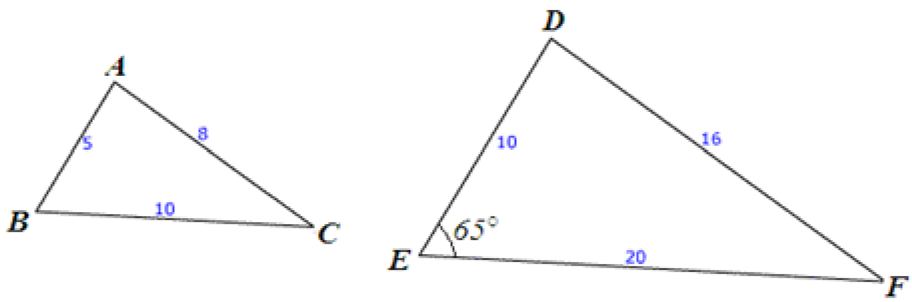
3. Side-Side-Side (SSS~) Similarity

Theorem

If the three sides of a triangle are proportional to the corresponding sides of a second triangle, then the triangles are similar.

Or, the lengths of the corresponding sides are proportional and therefore the two triangles are similar.

Example 4: Prove the two triangles shown below are similar triangles and determine ∠B.



Solution

Since AB : DE = 5:10 = 1: 2

and AC: DF = 8:16 =1:2

and BC: EF = 10:20 =1:2

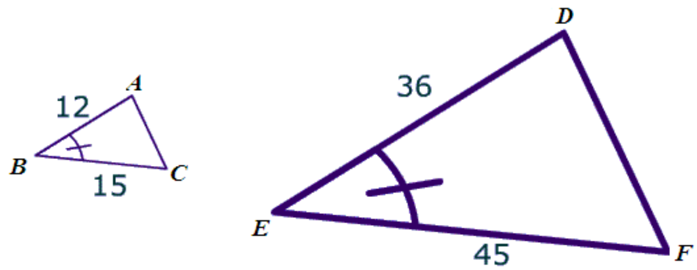
Therefore triangle ABC and triangle DEF are similar triangles because of SSS~.

Since corresponding angles are congruent for similar triangles, and $\angle E$ in triangle DEF is 65^o, then $\angle B = \angle E = 65^o$

4. Side-Angle-Side (SAS~) Similarity

Theorem
If an angle of a triangle is congruent to the corresponding angle of a second triangle, and the lengths of the two sides including the angle in one triangle are proportional to the lengths of the corresponding two sides in the second triangle, then the two triangles are similar.

Example 5: Prove the triangles shown below are similar. If DF = 27, determine the length of AC.



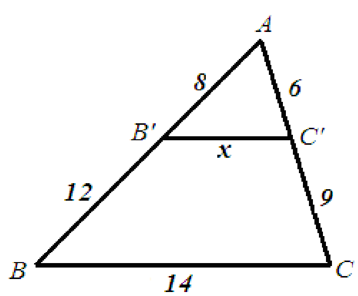
Solution
Since $\angle B = \angle E$,
and BC : EF = 15 : 45 =1:3
and BA : ED=12: 36=1:3

The two triangles have two sides whose lengths are proportional and a congruent angle included between the two sides. Therefore the two triangles are similar (SAS~). We may calculate the ratios of the lengths of the corresponding sides.

Then AB : DE = AC : DF, substitute the given value in written proportion, we have:
12 : 36 = AC : 27
we get AC = 9

Example 6: In the figure shown below, determine the value of x.

Solution
Since $\angle BAC$ and $\angle B'AC'$ are congruent (shared), the lengths of the sides including the congruent angles are given in figure,



and AB : AB' = (12 + 8) : 8 = 20 : 8 = 5 : 2

and AC : AC' = (6 + 9) : 6 = 15 : 6 = 5 : 2

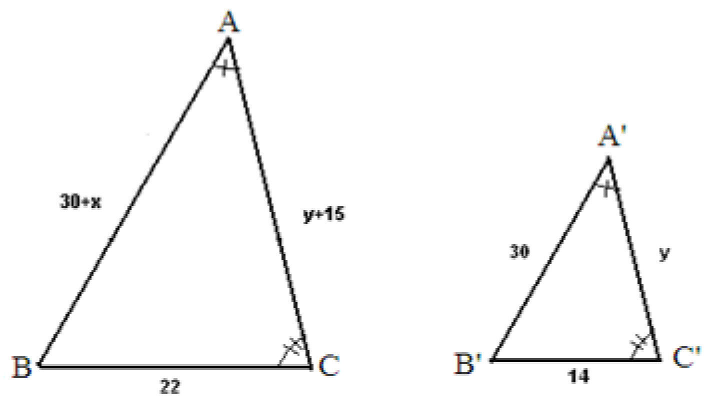
The two triangles have two sides whose lengths are proportional and a congruent angle included between the two sides. Therefore the two triangles are similar (SAS~).

We may calculate the ratios of the lengths of the corresponding sides.

Then we have AB' : AB = B'C' : BC, substitute the value shown in the figure in proportion,
8 : 20 = x : 14, we get x = 5.6

Example 7: In the figure shown below, if $\angle A = \angle A'$ and $\angle C = \angle C'$ and some sides are given specific value, determine x and y.

Solution
Since the two triangles have two corresponding congruent angles, they are similar by **Angle-Angle**. Now we use the proportionality of the lengths of the side to write equations that help in solving for x and y.



(30 + x) / 30 = 22 / 14 = (y + 15) / y

First, an equation in x may be written as follows.

(30 + x) / 30 = 22 / 14

Solve the above for x.
420 + 14 x = 660
x = 17.1 (rounded to one decimal place).

Second, an equation in y may be written as follows.

22 / 14 = (y + 15) / y

Solve the above for y.
y = 26.25

Example 8: Find the value of the pronumeral in the following diagram.

Solution

ΔADE and ΔABC are similar as they are equiangular.

$$\therefore \frac{AD}{AB} = \frac{DE}{BC}$$

$$\frac{x+4}{x} = \frac{6}{3}$$

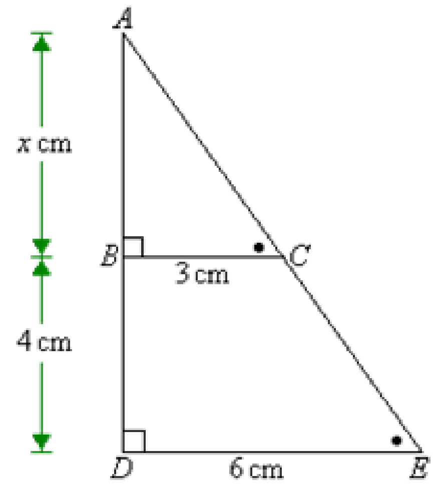
$$\frac{x+4}{x} = 2 \qquad \{\text{Multiply both sides by } x\}$$

$$x\left(\frac{x+4}{x}\right) = x \times 2$$

$$x+4 = 2x \qquad \{\text{Subtract } x \text{ from both sides}\}$$

$$x+4-x = 2x-x$$

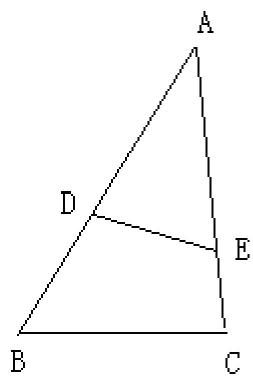
$$4 = x \qquad x = 4$$



Example 9: In triangle ABC, $\angle ADE = \angle C$ Which of the following is true?

A. $\frac{AD}{AB} = \frac{AE}{AC}$ B. $\frac{AE}{BC} = \frac{AD}{BD}$

C. $\frac{DE}{BC} = \frac{AE}{AB}$ D. $\frac{DE}{BC} = \frac{AD}{AB}$



Solution: C

SINCE $\Delta ABC \backsim \Delta AED$ $\quad \therefore \frac{DE}{BC} = \frac{AE}{AB}$

Example 10: In ΔABC, AB = 3AD, DE // BC, EF // AB. If AB = 9, DE = 2, then FC = ?

Solution

DE//BC (Given)

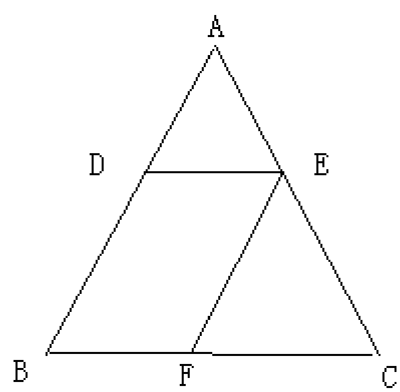
$\angle$ADE = $\angle$B (Corresponding angles)

$\angle$A = $\angle$A (Shared angle)

ΔADE∽ΔABC (AA~)

$\therefore \frac{AB}{AD} = \frac{BC}{DE}$

$\therefore BC = 3DE = 6$



Notice DEBF is a parallelogram, so BF = DE

FC = BC – DE = 6 – 2 = 4