# Kinematics

## Unit 1: Fundamentals of Dynamics

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Grade 12 Physics
Olympiads School

Summer 2019

# Where Are We In the Course

1. Fundamentals of Dynamics
2. Momentum, Impulse and Energy
3. Gravitational, Electric and Magnetic Fields
4. Wave Nature of Light
5. Modern Physics:
   - 5.1 Special Relativity
   - 5.2 Introduction to Quantum Mechanics

# Files to Download from School Website

- `Phys12-courseOutline.pdf`—The course outline
- `Phys12-equations.pdf`—An equations sheet for this course
- Class slides for Unit 1:
  - `Phys12-1a-kinematics-print.pdf`
  - `Phys12-1b-dynamics.pdf`
  - `Phys12-1c-circMotion-print.pdf`

  If you wish to print the slides, we recommend that you print out 4 slides per page. (For the sake of the environment, please don't print a single colour slide on a sheet of paper.)
- `Phys12-C01-Homework.pdf`—Homework questions for Class 1
- `Phys12-C02-Homework.pdf`—Homework questions for Class 2

# Frame of Reference

Think of a **frame of reference** (or **reference frame**, or just **frame**) as a "hypothetical mobile laboratory" an observer uses to make physical measurements (e.g. mass, lengths, time). At a minimum, it must include:

- A set of rulers (i.e. a coordinate system) to measure lengths
- A clock to measure the passage of time
- A scale to compare forces
- A balance to measure masses

**Note:** High-school textbooks often refer to the frame of reference as a "coordinate system". This incomplete definition makes it difficult to understand special relativity.

# Frame of Reference

We assume that this "hypothetical laboratory" is *perfect*:

- The hypothetical instruments are not subjected to numerical errors
- There is an instrument for whatever you want to measure
- What matters is the *motion* of the frame of reference (at rest, uniform motion, under acceleration etc), and how that motion affects the measurement that you make
- "From the point of view of. . . "

# Inertial Frame of Reference

An **inertial frame of reference** (or **rest frame**) is one that moves in uniform motion (constant velocity, no acceleration)

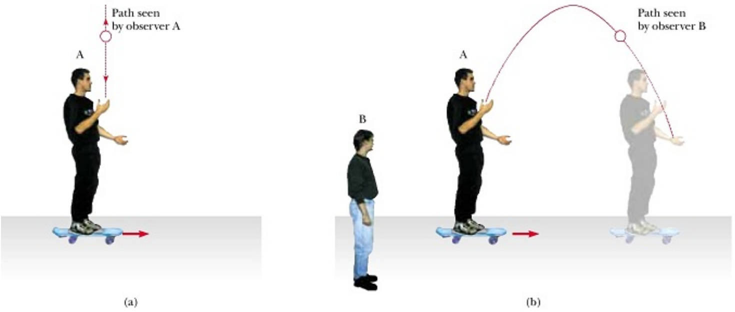
- The frame of reference is not subjected to any net force
- In all inertial frames of reference, the laws of motion are valid
- Since all laws of motion are valid in all inertial frames of reference, *any* inertial frame can be considered to be at rest (stationary)

**The Principle of Relativity:**
The laws of motion must be obeyed in all inertial frames of reference

# Inertial Frame of Reference

Person A is moving uniformly relative to B

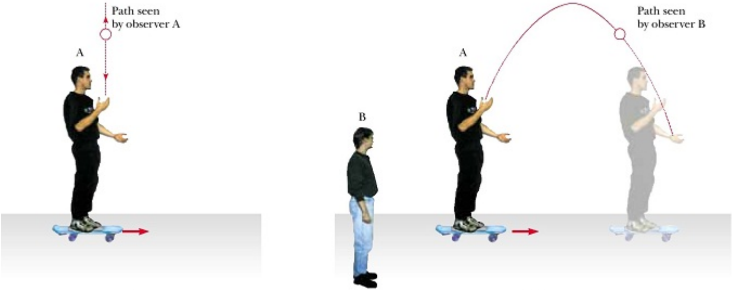
- A observes that the ball that he is throwing only has vertical motion
- B observes that the ball is moving in a parabolic curve
- A & B agree on the *equations* that govern the motion



# Inertial Frame of Reference

Observer A makes the same observation (only vertial motion) regardless of whether he is moving uniformly relative to the ground or not

- Valid for A to think that he is at rest, but B (and the rest of the world) is moving
- Valid for B to think that he is at rest, but A (and his skateboard) is moving

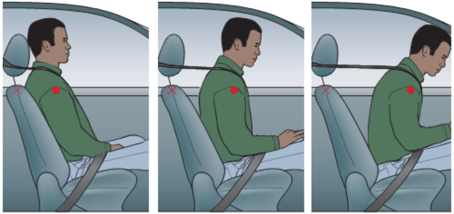



(a) (b)

# Non-inertial Frame of Reference

A **non-inertial frame of reference** is a frame that is undergoing acceleration (non-constant velocity)

- Require **fictitious force** (or **pseudo force**) in the free-body diagram to account for the observations
  - Hypothetical force
  - Does not exist in inertial frame of reference
- Example: A car that is speeding up, slowing down, or turning



# Example: Reference Frame

**Example 1:** Passengers in a high-speed elevator feel as though they are being pressed heavily against the floor when the elevator starts moving up. After the elevator reaches its maximum speed, the feeling disappears.

# Example: Reference Frame

(a) When do the elevator and passengers form
- an inertial frame of reference?
- a non-inertial frame of reference?

(b) Before the elevator starts moving,
- what forces are acting on the passengers?
- how large is the external (unbalanced) force?

(c) Is a person standing outside the elevator in an inertial or non-inertial frame of reference?

# Frame of Reference

**Example 1a:** Given the definition of an inertial frame of reference, is Earth an inertial frame of reference?

**Answer:** No! Earth is rotating on its axis, and also orbiting in an elliptical path around the Sun. However, we can adjust the value of $g$ slightly to account for the acceleration of Earth.

# Kinematics

**Kinematics** is a branch of mechanics that describes the motion of points, bodies (objects), and systems of bodies (groups of objects)

- Mathematical relationship between
  - Position
  - Displacement
  - Distance
  - Velocity
  - Speed
  - Acceleration
- Motion quantities in red are vectors (require a *magnitude* and *direction*)
- Does not deal with what causes the motion

# Position & Displacement

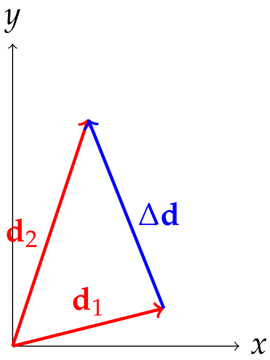
**Position** is the location of an object relative to a "reference point" (origin of the coordinate system). It is a vector:

$$\mathbf{d}$$

**Displacement** is the change in position when an object moves from from 1 to 2. It is also a vector:

$$\Delta\mathbf{d} = \mathbf{d}_2 - \mathbf{d}_1$$

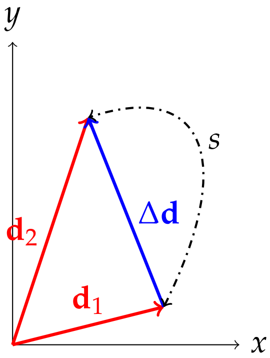
Position of an object is also its displacement from the reference point.



# Distance

**Distance** $s$ is a quantity that is *related* to displacement. It is the length of the path taken when an object moves from $\mathbf{d}_1$ to $\mathbf{d}_2$. It is a scalar quantity.

- The length of any path is always positive, therefore $s > 0$ always
- Although the magnitude of the displacement vector is also a scalar, it is not necessarily the same as distance
- On the diagram on the right: $s > |\Delta\mathbf{d}|$



# Velocity

**Average velocity** is how quickly your position is changing *over a finite time interval*. It is a vector quantity:

$$\mathbf{v}_{\text{ave}} = \frac{\Delta \mathbf{d}}{\Delta t} = \frac{\mathbf{d}_2 - \mathbf{d}_1}{t_2 - t_1}$$

**Instantaneous velocity** is how quickly your displacement is changing *at a specific instnace in time*. It is also called the *rate of change in displacement*.

- Calculating instantaneous velocity may require some calculus
- The direction of the velocity vector is the same as a displacement vector.

# Speed

**Average speed** is similar to average velocity: it is the distance travelled over a finite time interval. Speed is a *scalar*:

$$v_{\text{ave}} = \frac{s}{\Delta t}$$

Similarly, **instantaneous speed** is how quickly distance is changing at a specific instance in time.

- Since distance is always positive $s > 0$, both average and instantaneous speed must also be positive

# Acceleration

**Average acceleration** is how quickly the instantaneous velocity vector is changing *over a finite time interval*:

$$\mathbf{a} = \frac{\Delta \mathbf{v}}{\Delta t} = \frac{\mathbf{v}_2 - \mathbf{v}_1}{t_2 - t_1}$$

**Instantaneous acceleration** is how quickly the velocity vector is changing at a *specific instance in time*, i.e. *the rate of change of instantaneous velocity*

- A change in a vector can mean a change in magnitude and/or direction
- There can be acceleration without any speeding up or slowing down!
- Think about what happens if a car is turning at constant speed

# Working with Vectors

Vectors obey the *principle of superposition*, which means that they *add* together. Methods for adding vectors include:

- Using **Pythagorean theorem** (for vectors at right angles to each other)
- Using **cosine and sine laws**
- Decomposing vectors into **components**, then reassemble them using Pythagorean theorem

For 1D problems, (+) and (-) signs are sufficient to indicate direction

- Remember to indicate which way is positive though!

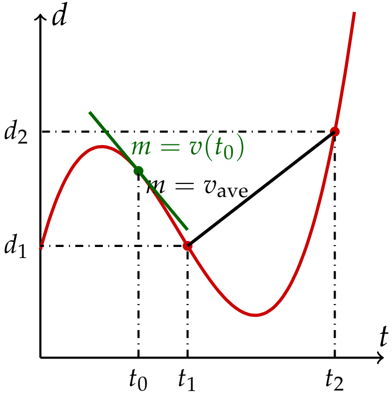
# Motion Graphs

We can describe *one-dimensional* motion graphically using **motion graphs**, by plotting

- Position vs. time ($d$-$t$)
- Instantaneous velocity vs. time ($v$-$t$)
- Instantaneous acceleration vs. time ($a$-$t$)

# Position–Time Graph

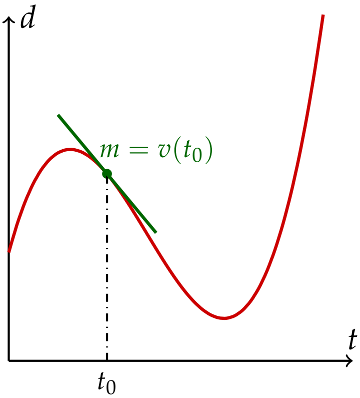
The obvious choice for expressing 1-D motion of an object graphically is by plotting its position as a function of time:

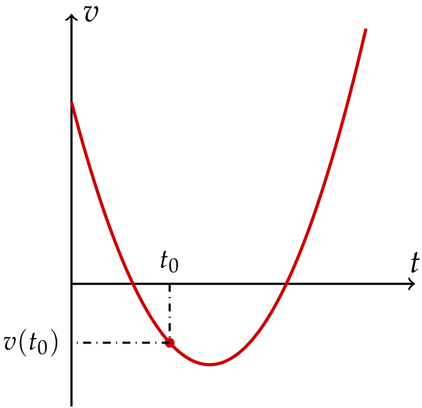



- Motion (usually) begins at $t = 0$
  - Time only moves forward, but the graph does not explicit tell you so
- Read the position straight from the graph
- Slope of the secant: average velocity $v_{\text{ave}}$
- Slope of the tangent: instantaneous velocity
- (+) slope: moving in the positive direction
- (-) slope: moving in the negative direction
- Zero slope: the object is at rest, not moving

# Veocity–Time Graph

A less obvious choice for expressing 1-D motion is by plotting its *instantaneous* velocity as a function of time:

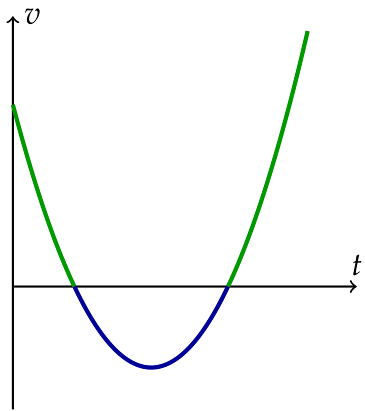


# Veocity–Time Graph

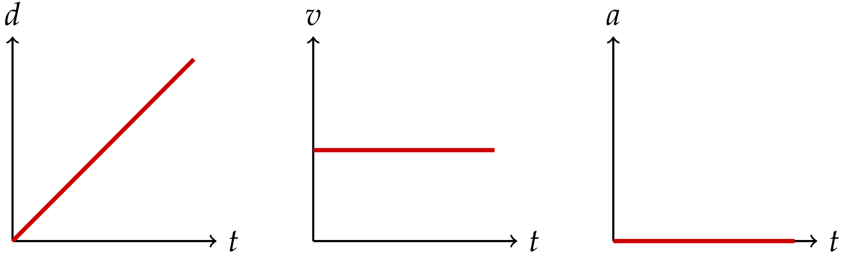
A less obvious choice for expressing 1-D motion is by plotting its *instantaneous* velocity as a function of time:

- Shows how velocity changes with time
- Above the $x$-axis (time axis), velocity is (+)
- Below the $x$-axis, velocity is (-)
- Slope of the secant: average acceleration
- Slope of the tangent: instantaneous acceleration
- In the same way that we converted from a $d$–$t$ graph to a $v$–$t$ graph, we can do the same to convert $v$–$t$ graphs to $a$–$t$ graphs



# Uniform Motion

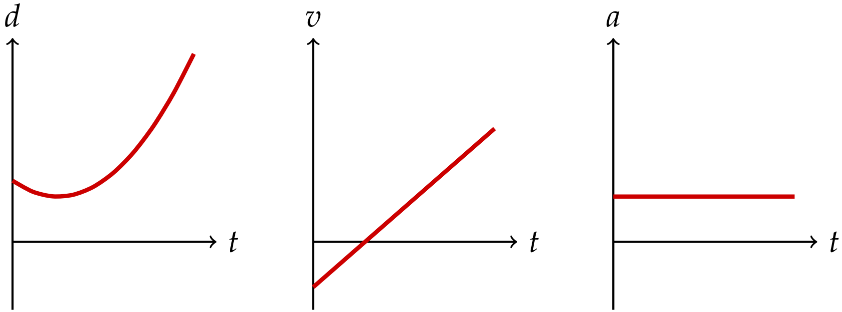
**Uniform motion** is when the velocity vector is constant, and neither its magnitude nor direction changes. In 1D, the motion graphs look like this:




- $d$–$t$ graph is a straight line
- The slope of the $d$–$t$ graph is constant
- There is no acceleration, so $a = 0$ for all $t$

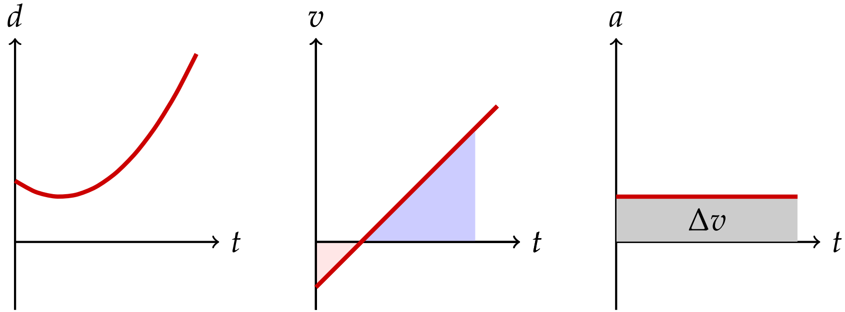
# Uniform Acceleration

When a constant net force acts on an object, it's moves with constant acceleration, or **uniform acceleration**.




- The $d$-$t$ graph is part of a *parabola*
  - If the parabola is *convex*, then acceleration is positive
  - If the parabola is *concave*, then acceleration is negative
- The $v$-$t$ graph is a straight line; the slope is the acceleration

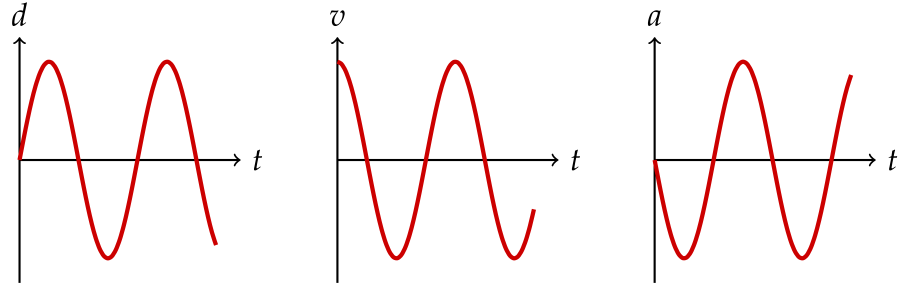
# Area Under Motion Graphs




- The area under the $a$–$t$ graph is the change in velocity $\Delta v$
  - If initial velocity is known, then we can plot $v$–$t$ graph based on this graph
- The area under the $v$–$t$ graph is the displacement $\Delta d$
  - If the area is below the $x$-axis (time axis), then displacement is negative;
  - If the area is above the time axis, then displacement is positive
- The area under the $d$–$t$ graph has no physical meaning

# Simple Harmonic Motion

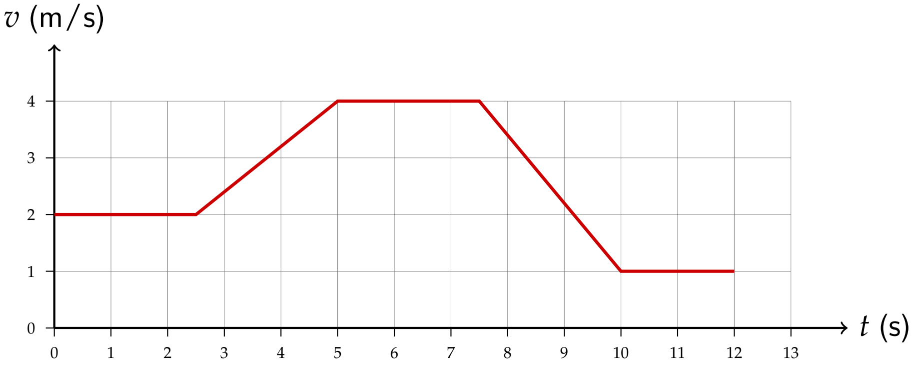
In **simple harmonic motion** (**vibration** or **oscillatory motion**) displacement, velocity and acceleration are all periodic functions, and none of them are constant!




We will discuss this topic later, in the next unit.

# Example Problem

**Example 2:** Change the following graph to a **position-time** graph and an **acceleration-time** graph. Assume displacement starts at zero.



# 1D Kinematic Equations

$$\Delta d = v_1 \Delta t + \frac{1}{2} a \Delta t^2$$

$$\Delta d = v_2 \Delta t - \frac{1}{2} a \Delta t^2$$

$$\Delta d = \frac{v_1 + v_2}{2} \Delta t$$

$$v_2 = v_1 + a \Delta t$$

$$v_2^2 = v_1^2 + 2a \Delta d$$

- Only valid for **constant acceleration**
- Variables are:

$$\Delta d \quad v_1 \quad v_2 \quad \Delta t \quad a$$

- For 1-object problems, you are usually given 3 of the 5 variables, and you are asked to find a 4th one
- For 2-object problems, the motion of the two objects are connected by time interval $\Delta t$ and displacement $\Delta d$

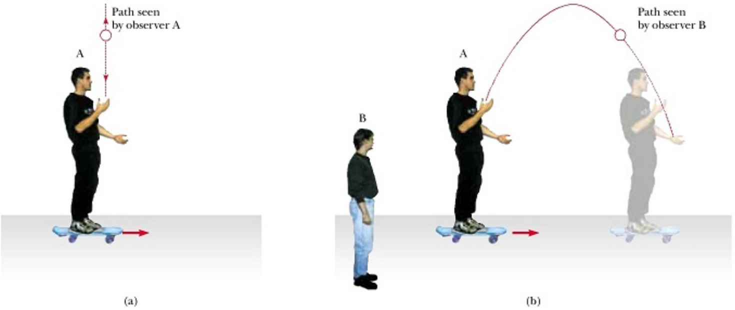
# 1D Kinematic Equations

Kinematic equations *cannot* be used when acceleration is non-uniform (when non-constant forces act on an object):

- Aerodynamic forces
  - Lift and drag forces
  - proportional to $v^2$
- Spring force
  - The force that a compressed/stretched spring applies on objects connected to it
  - Proportional to amount of compression/extension (displacement of the spring $x$)
- Dampers in springs
  - Dampers are used to slow down the vibration of an object
  - Generally proportional to $v$

We will discuss more about forces later in this unit.

# Relative Motion




- Observers (frames of reference) A and B measures different motion of the ball because A and B a moving relative to each other.
- The instantaneous velocity of the ball at any time $t$, as measured by A and B, is related by the instantanenous velocities of A and B relative to each other.

# Relative Motion

Velocities are always measured *relative* to a frame of reference. Therefore, when expressing relative motion, we can use two subscripts:

$$\mathbf{v}_{AB}$$

$A$ represents the object, and $B$ represents the frame of reference

The velocity of an airplane ($P$) travelling at $251\,\text{km/h}$ [N] relative to Earth ($E$) is expressed as:

$$\mathbf{v}_{PE} = 251\,\text{km/h [N]}$$

# Relative Motion

- Different observers make different observations because they (their frames of reference) are moving relative to each other.
- In *classical* mechanics, the different velocity measurements are related by the **Galilean velocity addition rule**[1]:

$$\boxed{\mathbf{v}_{AC} = \mathbf{v}_{AB} + \mathbf{v}_{BC}}$$

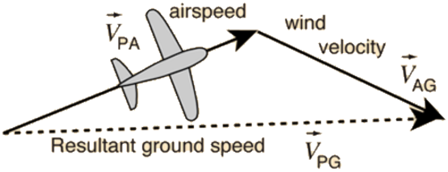
  The velocity of $A$ relative to reference frame $C$ is the velocity of $A$ relative to reference frame $B$, plus the velocity of $B$ relative to $C$.
- Can only be used when velocity $v$ is small compared to the speed of light $c$

[1] This equation was thought to be so obvious that no one bothered to give it a name until Einstein showed that it is not valid near the speed of light

# Relative Motion Example

## Airplane in Air

The velocity of the plane relative to the ground (“ground speed”) is the velocity of the plane relative to the air (“air speed”) plus the velocity of the air relative to the ground (“wind speed”).




The addition of velocities is exactly the same as any vector addition.

# Projectile Motion

- For projectile motion problems, resolve the problem into horizontal ($x$) and vertical ($y$) directions, and apply kinematic equations independently
- No horizontal acceleration ($\mathbf{a}_x = 0$), therefore kinematic equations reduce to a single equation:

$$\Delta x = v_x \Delta t$$

- Acceleration due to gravity in the vertical ($y$) direction:

$$\mathbf{a}_y = \mathbf{g} = 9.81\,\mathrm{m/s^2}\ \text{[down]}$$

We *usually* define the (+) direction to be [up], so $a_y = -g = -9.81\,\mathrm{m/s^2}$

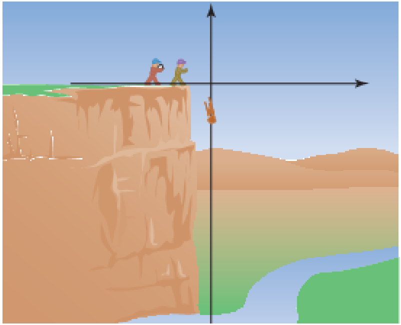
- The variable that connects the two directions is the time interval $\Delta t$

# Projectile Motion Example

**Example 3:** While hiking in the wilderness, you come to a cliff overlooking a river. A topographical map shows that the cliff is $291\ \mathrm{m}$ high and the river is $68.5\ \mathrm{m}$ wide at that point. You throw a rock directly forward from the top of the cliff, giving the rock a horizontal velocity of $12.8\ \mathrm{m/s}$.

(a) Did the rock make it across the river?

(b) With what velocity did the rock hit the ground or water?



# Projectile Motion Example: Tossing a Ball

**Example 5:** You are playing tennis with a friend on tennis courts that are surrounded by a $4.8\,\mathrm{m}$ fence. You opponent hits the ball over the fence and you offer to retrieve it. You find the ball at a distance of $12.4\,\mathrm{m}$ on the other side of the fence. You throw the ball at an angle of $55.0^\circ$ with the horizontal, giving it an initial velocity of $12.1\,\mathrm{m/s}$. The ball is $1.05\,\mathrm{m}$ above the ground when you release it. Did the ball go over the fence, hit the fence, or hit the ground before it reached the fence?

# Symmetrical Trajectory

Trajectory is symmetrical if the object lands at the same height as when it started.

- Time of flight

$$t_{\max} = \frac{2v_i \sin\theta}{g}$$

- Range

$$R = \frac{v_i^2 \sin(2\theta)}{g}$$

- Maximum height

$$h_{\max} = \frac{v_i^2 \sin^2\theta}{2g}$$

The angle $\theta$ is the **above the the horizontal**

# Example Problem

**Example 6:** A player kicks a football for the opening kickoff. He gives the ball an initial velocity of $29\,\mathrm{m/s}$ at an angle of $69^\circ$ with the horizontal. Neglecting friction, determine the ball's maximum height, hang time and range?