Geometry

1. Parallel Lines and Pairs of Angles

1) Parallel Lines

Lines are parallel if they are always the same distance apart (called "equidistant"), and will never meet. Just remember:
Always the same distance apart and never touching.The red line is parallel to the blue line in both these cases:

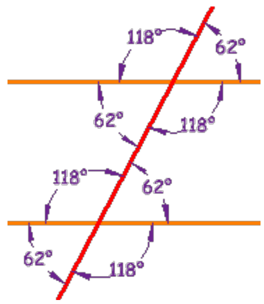
Example 1 Example 2

Parallel lines also point in the same direction.

2) Pairs of Angles

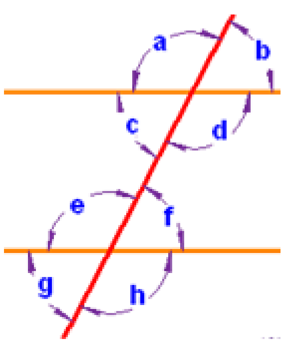
When parallel lines get crossed by another line (which is called a Transversal), you can see that many angles are the same, as in this example:

These angles can be made into pairs of angles which have special names.



3) Testing for Parallel Lines

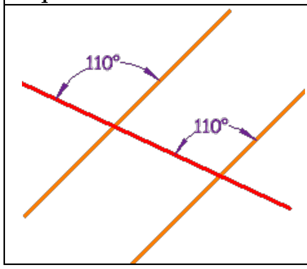
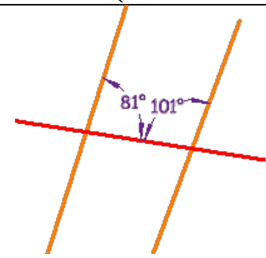
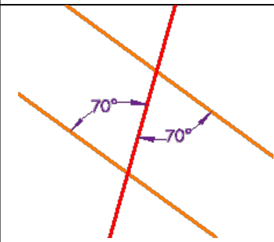
Some of those special pairs of angles can be used to test if lines really are parallel:



If Any Pair Of ...	*Example:*
Corresponding Angles are equal, or	$a = e$
Alternate Interior Angles are equal, or	$c = f$
Alternate Exterior Angles are equal, or	$b = g$
Consecutive Interior Angles add up to 180°	$d + f = 180°$

... then the lines are Parallel

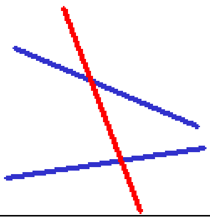
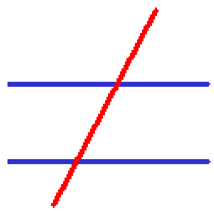
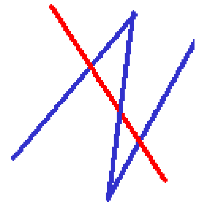
Examples

These lines are parallel, because a pair of Corresponding Angles are equal.	These lines are not parallel, because a pair of Consecutive Interior Angles do not add up to 180° (81° + 101° =182°)	These lines are parallel, because a pair of Alternate Interior Angles are equal
		

4) Transversals

A Transversal is a line that crosses at least two other lines.

The red line is the transversal in each example:

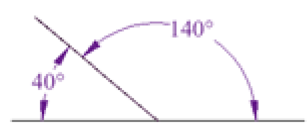
Transversal crossing two lines	this Transversal crosses two parallel lines	... and this one cuts across three lines
		

5) Supplementary Angles

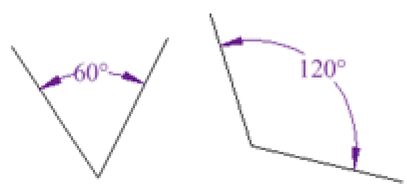
Two Angles are Supplementary if they add up to 180 degrees.

These two angles (140° and 40°) are Supplementary Angles, because they add up to 180°.

Notice that together they make a straight angle.



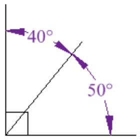
But the angles don't have to be together.
These two are supplementary because 60° + 120° = 180°



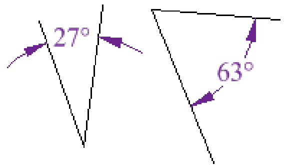
6) Complementary Angles

Two Angles are Complementary if they **add up to 90 degrees** (a Right Angle).

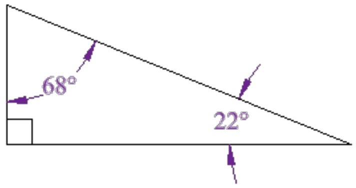
These two angles (40° and 50°) are Complementary Angles, because they add up to 90°.
Notice that together they make a right angle.



But the angles don't have to be together.
These two are complementary because
27° + 63° = 90°



Right Angled Triangle
In a right angled triangle, the two acute angles are complementary, because in a triangle the three angles add to 180°, and 90° have been taken by the right angle.



7) Complementary vs Supplementary

A related idea is Complementary Angles, they add up to 90° .

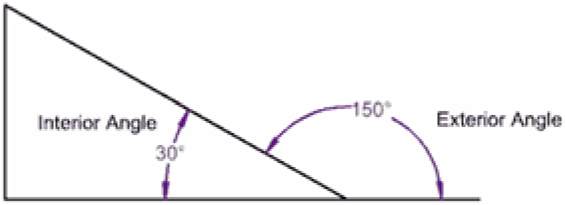
How can you remember which is which? Easy! Think:

- "C" of Complementary stands for "Corner"  (a Right Angle), and
- "S" of Supplementary stands for "Straight" (180 degrees is a straight line)

2. Interior and Exterior Angle

An Interior Angle is an angle inside a shape.

The Exterior Angle is the angle between any side of a shape, and a line extended from the next side.



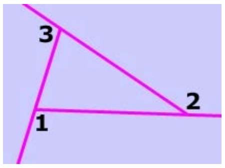
Note: If you add up the Interior Angle and Exterior Angle you get a straight line, 180°.

1) Exterior Angles of Polygons

The Exterior Angle is the angle between any side of a shape, and a line extended from the next side.
The sum of the measures of the exterior angles of a polygon, one at each vertex, is 360°.

For example:

A Polygon is any flat shape with straight sides.



The Exterior Angles of a Polygon add up to 360°.
$\angle 1 + \angle 2 + \angle 3 = 360^0$

2) Interior Angles sum of Polygons

An interior angle of a regular polygon with n sides is $\frac{(n-2)\times 180}{n}$.

Example:

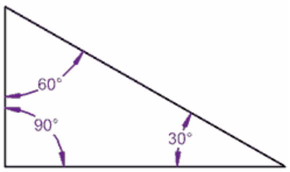
To find the measure of an interior angle of a regular octagon, which has 8 sides, apply the formula above as follows:
((8-2) × 180) /8 = 135°

The sum of the measures of the interior angles of a polygon with *n* sides is (n-2)180.

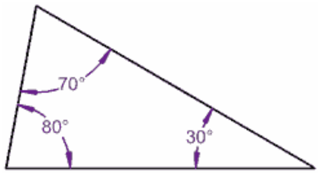
3) Triangles

The Interior Angles of a Triangle add up to 180°

90° + 60° + 30° = 180°

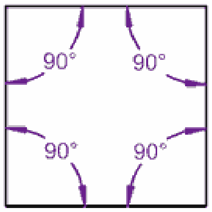


80° + 70° + 30° = 180°



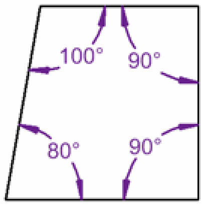
4) Quadrilaterals (Squares, etc)

(A Quadrilateral is any shape with 4 sides)



90° + 90° + 90° + 90° = 360°

A Square adds up to 360°



80° + 100° + 90° + 90° = 360°

Let's tilt a line by 10° ... still adds up to 360°!

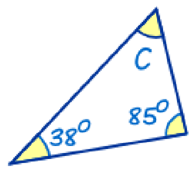
The Interior Angles of a Quadrilateral add up to 360°

2. Triangles

In a triangle, the three angles always add to 180°: **A + B + C = 180°.**

Example: Find the Missing Angle "C"

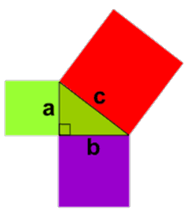
Start With: A + B + C = 180°
Fill in what we know: 38° + 85° + C = 180°
Rearrange: C = 180° - 38° - 85°
Calculate: C = 57°

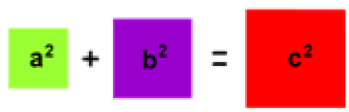


3. Pythagorean Triples

A "Pythagorean Triple" is a set of positive integers, **a**, **b** and **c** that fits the rule:
$a^2 + b^2 = c^2$

Triangles
And when you make a triangle with sides **a**, **b** and **c** it will be a right angled triangle:

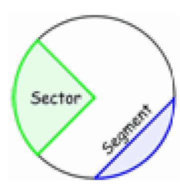




Note:
c is the **longest side** of the triangle, called the "hypotenuse"
a and **b** are the other two sides

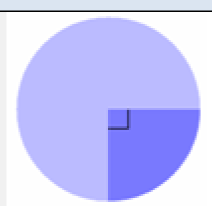
4. Circle Sector and Segment

Slices
There are two main "slices" of a circle:
The "pizza" slice is called a **Sector**.
And the slice made by a chord is called a **Segment**.



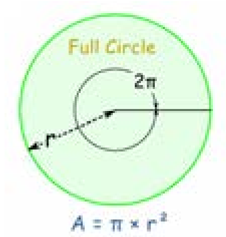
1) Common Sectors

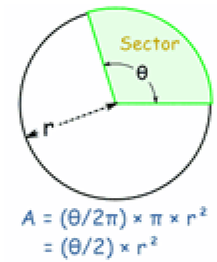
The Quadrant and Semicircle are two special types of Sector:

Quarter of a circle is called a **Quadrant**	Half a circle is called a **Semicircle.**
	

2) Area of a Sector

You can work out the Area of a Sector by comparing its angle to the angle of a full circle.
Note: I am using radians for the angles.





This is the reasoning:

- A circle has an angle of 2π and an Area of: πr^2
- So a Sector with an angle of θ (instead of 2π) must have an area of: $(\theta/2\pi) \times \pi r^2$
- Which can be simplified to: $(\theta/2) \times r^2$

Area of Sector = $\frac{1}{2} \times \theta \times r^2$ *(when θ is in radians)*
Area of Sector = $\frac{1}{2} \times (\theta \times \pi/180) \times r^2$ *(when θ is in degrees)*

Please remember following formulas

1. Area of a triangle $= \frac{1}{2}$(base) (height).

2. Area of a rectangle = (length) (breadth).

3. Perimeter of a rectangle = 2 (length) +2 (breadth).

4. Area of a square = (side)^2.

5. Perimeter of a square = 4 (Side).

6. Volume of a cube = x^3 where (x) is the length of the side.

7. Surface area of a cube = $6x^2$.

8. Area of a trapezium = $\frac{1}{2}$(sum of parallel side) × (distance between the parallel sides).

9. Area of an equilateral triangle $= \frac{\sqrt{3}}{4} a^2$ where (a) is the length of a side.

10. Parallelogram Area = base × height

11. Area of a circle of radius (r) is πr^2.

12. Perimeter (or circumference) of a circle of radius (r) is $2\pi r$.

13. Volume of a sphere of radius (r) is $\frac{4}{3}\pi r^3$.

14. Surface area of a sphere of radius (r) is $4\pi r^2$.

15. Volume of right circular cone is $\frac{1}{3}\pi r^2 h$, where (r) is the radius of the base and (h) is the height of the cone.

16. Curved surface area of a cone is πrl where (l) is the slant height of the cone.

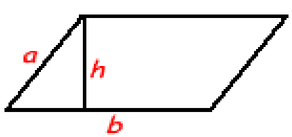
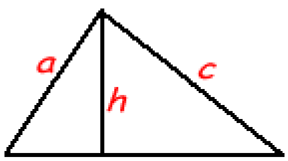
17. Total surface area of a cone is $\pi rl + \pi r^2$.

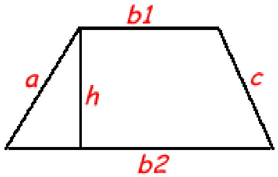
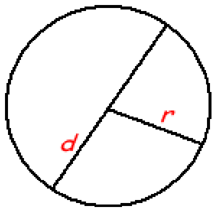
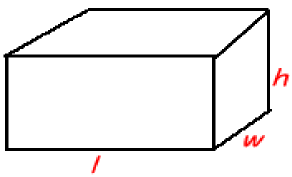
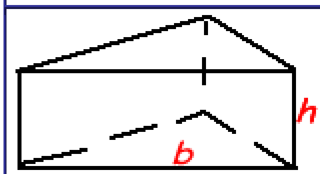
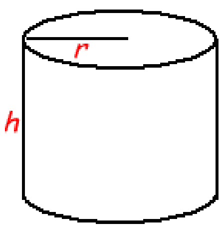
18. Volume of a right circular cylinder is $\pi r^2 h$.

19. Curved surface area of a right circular cylinder is $2\pi rh$.

20. Total surface area of a right circular cylinder is $2\pi rh + 2\pi r^2$.

►**Geometric formula**

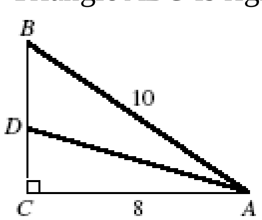
Shapes	**Formula**
	Rectangle: Area = Length X Width A = lw Perimeter = 2 X Lengths + 2 X Widths P = 2l + 2w
	Parallelogram Area = Base X Height a = bh
	Triangle Area = 1/2 of the base X the height a = 1/2 bh Perimeter = a + b + c (add the length of the three sides)

	Trapezoid area $A=\left(\frac{b_1+b_2}{2}\right)h$ Perimeter = a + b1 + b2 + c
	Circle Try the Online tool. The distance around the circle is a circumference. The distance across the circle is the diameter (d). The radius (r) is the distance from the center to a point on the circle. (Pi = 3.14) More about circles. d = 2r c = d = 2π r A = πr^2 (π=3.14)
	Rectangular Solid Volume = Length X Width X Height V = lwh Surface = 2lw + 2lh + 2wh
	Prisms Volume = Base X Height v=bh Surface = 2b + Ph *(b is the area of the base P is the perimeter of the base)*
	Cylinder Volume = πr^2 x height V = πr^2 h Surface = 2π radius x height S = $2\pi rh + 2\pi r^2$

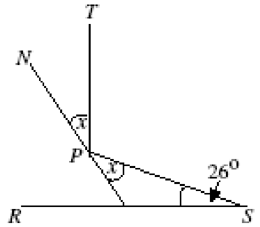
These Geometric formulae come from TDSB website

Questions in class

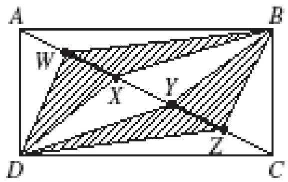
1. Triangle *ABC* is right-angled with *AB* = 10 and *AC* = 8. If *BC* = 3*DC*, then what is the *AD?*



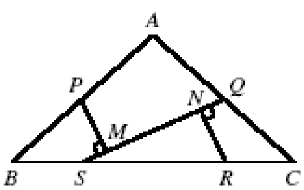
2. A beam of light shines from point *S*, reflects off a reflector at point *P*, and reaches point *T* so that *PT* is perpendicular to *RS*. Find the x.



3. Rectangle *ABCD* has length 9 and width 5. Diagonal *AC* is divided into 5 equal parts at *W*, *X*, *Y*, and *Z*. Determine the area of the shaded region.



4. Triangle ABC is an isosceles triangle in which AB=AC=10 and BC =12. The points S and R are on BC such that BS: SR: RC =1:2:1. The midpoints of AB and AC are P and Q respectively. Perpendiculars are drawn from P and R to SQ meeting at M and N respectively. What is the length of MN?



5. In the diagram, AB and BD are radii of a circle with centre B. The area of sector ABD is 2π, which is one-eighth of the area of the circle. Find the area of the shaded region.

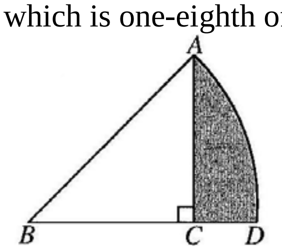


fig. 5

6. ABCD is a rectangle with AD = 10. If the shaded area is 100, what is the shortest distance between the semicircles?

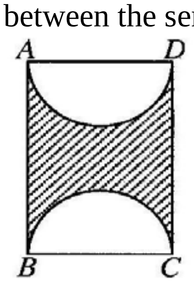


fig. 6

7. Two circles with equal radii are enclosed by rectangle ABCD, as shown. The distance between their centers is 2x / 3. What is the value of x?

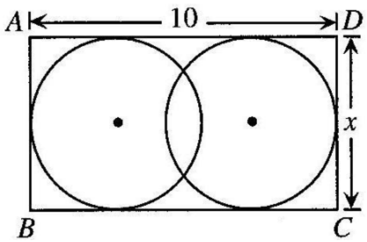


fig. 7

8. Three circles, each with a radius of 10 cm, are drawn tangent to each other so that their centres are all in a straight line. These circles are inscribed in a rectangle which is inscribed in another circle. What is the area of the largest circle?

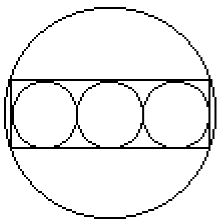


fig.8

9. Three rugs have a combined area of 200 m^2. By overlapping the rugs to cover a floor area of 140 m^2, the area which is covered by exactly two layers of rug is 24 m^2. What area of floor is covered by three layers of rug?