Ratio, Rate and Proportion

Concepts:

1. Ratio

A ratio is a comparison of two numbers. We generally separate the two numbers in the ratio with a colon (:). Suppose we want to write the ratio of 8 and 12.

We can write this as 8:12 or as a fraction 8/12, and we say the ratio is *eight to twelve*.

Example:
Jeannine has a bag with 3 videocassettes, 4 marbles, 7 books, and 1 orange.

1) What is the ratio of books to marbles?

Expressed as a fraction, with the numerator equal to the first quantity and the denominator equal to the second, the answer would be 7/4.

Two other ways of writing the ratio are 7 to 4, and 7:4.

2) What is the ratio of videocassettes to the total number of items in the bag?

There are 3 videocassettes, and 3 + 4 + 7 + 1 = 15 items total.
The answer can be expressed as 3/15, 3 to 15, or 3:15.

2. Comparing Ratios

To compare ratios, write them as fractions. The ratios are equal if they are equal when written as fractions.

Example:
Are the ratios 3 to 4 and 6:8 equal?
The ratios are equal if 3/4 = 6/8.

These are equal if their cross products are equal; that is, if $3 \times 8 = 4 \times 6$. Since both of these products equal 24, the answer is yes, the ratios are equal.

Remember to be careful! Order matters!
A ratio of 1:7 is not the same as a ratio of 7:1.

Example:
Are the ratios 7:1 and 4:81 equal? No!
$7/1 > 1$, but $4/81 < 1$, so the ratios can't be equal.

Are 7:14 and 36:72 equal?
Notice that 7/14 and 36/72 are both equal to 1/2, so the two ratios are equal.

3. Proportion

A proportion is an equation with a ratio on each side. It is a statement that two ratios are equal.

3/4 = 6/8 is an example of a proportion.

When one of the four numbers in a proportion is unknown, cross products may be used to find the unknown number. This is called solving the proportion. Question marks or letters are frequently used in place of the unknown number.

Example:
Solve for n: $1/2 = n/4$.

Using cross products we see that $2 \times n = 1 \times 4 = 4$, so $2 \times n = 4$. Dividing both sides by 2, $n = 4 \div 2$ so that $n = 2$.

4. Rate

A rate is a ratio that expresses how long it takes to do something, such as traveling a certain distance. To walk 3 kilometers in one hour is to walk at the rate of 3 km/h.

The fraction expressing a rate has units of distance in the numerator and units of time in the denominator.

Problems involving rates typically involve setting two ratios equal to each other and solving for an unknown quantity, that is, solving a proportion.

Example:
Juan runs 4 km in 30 minutes. At that rate, how far could he run in 45 minutes?

Give the unknown quantity the name n. In this case, n is the number of km Juan could run in 45 minutes at the given rate.

We know that running 4 km in 30 minutes is the same as running n km in 45 minutes;

that is, the rates are the same. So we have the proportion
4km/30min = n km/45min, or $4/30 = n/45$.

Finding the cross products and setting them equal, we get $30 \times n = 4 \times 45$, or $30 \times n = 180$.

Dividing both sides by 30, we find that $n = 180 \div 30 = 6$ and the answer is 6 km.

5. Converting rates

We compare rates just as we compare ratios, by cross multiplying. When comparing rates, always check to see which units of measurement are being used.

For instance, 3 kilometers per hour is very different from 3 meters per hour!

3 kilometers/hour = 3 kilometers/hour × 1000 meters/1 kilometer = 3000 meters/hour

because 1 kilometer equals 1000 meters; we "cancel" the kilometers in converting to the units of meters.

Important:

One of the most useful tips in solving any math or science problem is to always write out the units when multiplying, dividing, or converting from one unit to another.

Example:
If Juan runs 4 km in 30 minutes, how many hours will it take him to run 1 km?

Be careful not to confuse the units of measurement. While Juan's rate of speed is given in terms of minutes, the question is posed in terms of hours. Only one of these units may be used in setting up a proportion.

To convert to hours, multiply 4 km/30 minutes × 60 minutes/1 hour = 8 km/1 hour

Now, let n be the number of hours it takes Juan to run 1 km. Then running 8 km in 1 hour is the same as running 1 km in n hours. Solving the proportion,
8 km/1 hour = 1 km/n hours, we have $8 \times n = 1$, so $n = 1/8$.

6. Average Rate of Speed

The average rate of speed for a trip is the total distance traveled divided by the total time of the trip.

Example:
A dog walks 8 km at 4 km per hour, then chases a rabbit for 2 km at 20 km per hour. What is the dog's average rate of speed for the distance he traveled?

The total distance traveled is 8 + 2 = 10 km.
Now we must figure the total time he was traveling.

For the first part of the trip, he walked for 8 ÷ 4 = 2 hours. He chased the rabbit for 2 ÷ 20 = 0.1 hour. The total time for the trip is 2 + 0.1 = 2.1 hours.

The average rate of speed for his trip is 10/2.1 = 100/21 kilometers per hour.

Questions in class (After the class, please do the following questions again)

1. The three angles of a triangle are in the ratio 3 : 4 : 5 . What is the middle angle?

2. At a school, there are 3 boys for every 4 girls. How many girls are at the school if there are 366 boys?

3. The edge of a cube is increased by 50%. What is the ratio of its new surface area to its old?

4. If $x/y = (x + 1)/(y + 1)$, what is the value of x/y?

5. In a class, 2/5 of the boys wear glasses and 1/3 of the girls wear glasses. If the ratio of boys to girls is 10:3, what is the fraction of the class wear glasses?

6. How many seconds will it take for a train 300 meters long travelling at 100 km/hr to pass a man jogging at 10 km/hr in the same direction?

7. Allen and Bill began to walk toward each other along a road from points A and B respectively. The ratio of the speed of Allen to Bill is 5:4. After they meet, Allen's speed is reduced by 20% and Bill's speed is increased by 20%. When Allen reached point B, Bill is still 10 km away from point A. What's the distance between A and B?

8. There are two piles of stones. Pile A has 350 blue stones and 500 white stones. Pile B has 400 blue stones and 100 white stones. How many blue and white stones must be moved from Pile B to Pile A so that blue stones take up 50% of Pile A and 75% of Pile B?

9. There are two rectangles A and B with equal perimeters. The ratio of length to width of rectangle A is 3:2 and ratio of length to width of rectangle B is 7:5. Determine the ratio of the area of A and B.

10. In trapezoid ABCD, E is a midpoint on AD and EC divides the trapezoid into two areas M and N with ratio 10 : 7, (see figure below). Find the ratio of top base and bottom base.