Chapter 2 Polynomials (2)

1. Polynomials

A **polynomial** is a finite sum of terms where the exponents on the variables are non-negative integers. Note that the terms are separated by +'s and -'s.
An example of a polynomial expression is $3x^5 - 5x^3 + x - 10$.

Some **Types of Polynomials**

Type	Definition	Example
Monomial	A polynomial with one term	$5x$
Binomial	A polynomial with two terms	$5x$ - 10
Trinomial	A polynomial with three terms	$7x^4 - 6x^2 + 5$

Also note that a polynomial can be "missing" terms. For example, the polynomial written above starts with a degree of 5, but notice there is not a term that has an exponent of 4. That means the coefficient on it is 0, so we do not write it.

Descending Order
Note that the standard form of a polynomial that is shown above is written in descending order. This means that the term that has the highest degree is written first, the term with the next highest degree is written next, and so forth.

Degree of a Term
The degree of a term is the sum of the exponents on the variables contained in the term.

Example 1: Find the degree of the term 8.

When you have a constant term, its degree is always 0, because there is no variable there.
Since this is a constant term, its degree is 0.

Example 2: Find the degree of the term $4ab^3$.

Since the degree is the sum of the variable exponents and it looks like we have a 1 and a 3 as our exponents, the degree would have to be 1 + 3 = 4.

Degree of the Polynomial
The degree of the polynomial is the largest degree of all its terms.

Example 3: Find the degree of the polynomial and indicate whether the polynomial is a monomial, binomial, trinomial, or none of these. $4x^2 - 7x + 2$

Since the degree of the polynomial is the highest degree of all the terms, the degree is 2.
Since there are three terms, this is a trinomial.

Example 4: How about $7x^3 - 9x^6$?
Since the degree of the polynomial is the highest degree of all the terms, the degree is 6.

Make sure that you don't fall into the trap of thinking it is always the degree of the first term. This polynomial is not written in standard form (descending order). So we had to actually go to the second term to get the highest degree.

Since there are two terms, this is a binomial.

2. The Distributive Property

Distributive Properties
a(b + c) = ab + ac or *(b + c)a = ba + ca*

In other words, when you have a term being multiplied times two or more terms that are being added (or subtracted) in a ( ), multiply the outside term times EVERY term on the inside.

Remember terms are separated by + and -.
This idea can be extended to more than two terms in the ( ).

Example 1: Use the distributive property to write $2(x - y)$ without parenthesis.
Multiplying every term on the inside of the ( ) by 2 we get:

$$2(x-y) = 2(x) - 2(y) = 2x - 2y$$

Example 2: Use the distributive property to find the product of $3(2a + 3b + 4c)$.

As mentioned above, you can extend the distributive property to as many terms as are inside the ( ). The basic idea is that you multiply the outside term times EVERY term on the inside.

$$3(2a+3b+4c) = 3(2a) + 3(3b) + 3(4c) = 6a + 9b + 12c$$

Example 3: Simplify the expression $-(5x+7)$.

$$-(5x+7) = (-1)(5x+7) = (-1)(5x) + (-1)(7) = -5x - 7$$

Basically, when you have a negative sign in front of a ( ), like this example, you can think of it as taking a -1 times the ( ). What you end up doing is taking the opposite of every term in the ( ).

Example 4: Simplify the expression $2(6x-5) - 3(5x+4)$.
Let's first apply the distributive property and see what we get:

$$2(6x-5) - 3(5x+4) = 2(6x) - 2(5) - 3(5x) - 3(4) = 12x - 10 - 15x - 12$$

Regrouping and combining like terms we get: $12x - 10 - 15x - 12 = 12x - 15x - 10 - 12 = -3x - 22$

Example 5: Write the following as an algebraic expression and simplify if possible.
The sum of 5 times a number and 2, subtracted from 12 times a number.

Let x represents the unknown number.
$12x - (5x + 2) = 12x - 5x - 2 = 7x - 2$

3. Multiplying Polynomials

(Monomial)(Monomial)
In this case, there is only one term in each polynomial. You simply multiply the two terms together.

Example 1: Find the following product $(-7x^5)(5x^3)$.

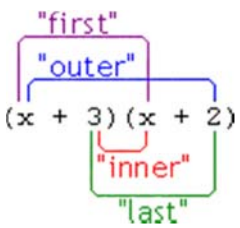
$$(-7x^5)(5x^3) = (-7)(5)(x^5 \cdot x^3) = -35x^8$$

(Monomial)(Polynomial)
In this case, there is only one term in one polynomial and more than one term in the other. You need to distribute the monomial to EVERY term of the other polynomial.

Example 2: Find the following product $-2a(5ab + 3a^2b^2 + 7a^3b^3)$.

$$-2a(5ab + 3a^2b^2 + 7a^3b^3) = -2a(5ab) - 2a(3a^2b^2) - 2a(7a^3b^3) = -10a^2b - 6a^3b^2 - 14a^4b^3$$

(Binomial)(Binomial)
In this case, both polynomials have two terms. You need to distribute both terms of one polynomial times both terms of the other polynomial.
One way to keep track of your distributive property is to use the FOIL method.



That is, FOIL tells you to multiply the first terms in each of the parentheses, then multiply the two terms that are on the "outside" (furthest from each other), then the two terms that are on the "inside" (closest to each other), and then the last terms in each of the parentheses.

Note that this method only works on (Binomial)(Binomial).

Example 3: Find the following product $(3x + 5)(2x - 7)$.
$(3x + 5)(2x - 7) =$

$= (3x)(2x) + (3x)(-7) + (5)(2x) + 5(-7) = 6x^2 - 21x + 10x - 35 = 6x^2 - 11x - 35$

(Polynomial)(Polynomial)
As mentioned above, use the distributive property until every term of one polynomial is multiplied times every term of the other polynomial. Make sure that you simplify your answer by combining any like terms.

Example 4: Find the following product $(3y-1)(2y^2+5y-8)$.

$$(3y-1)(2y^2+5y-8) = (3y)(2y^2) + (3y)(5y) + (3y)(-8) - (1)(2y^2) - (1)(5y) - (1)(-8)$$
$$= 6y^3 + 15y^2 - 24y - 2y^2 - 5y + 8$$
$$= 6y^3 + 13y^2 - 29y + 8$$

Practice: Simplify.

a) $2a^3(4a^2 - a)$ b) $(2x - 3)(-5x - 9)$ c) $-(x - 1)(2x^2 - x + 3)$

4. Powers of Binomials

The square of a binomial

$(x + 5)^2 = (x + 5)(x + 5) = x^2 + 10x + 25$.

$x^2 + 10x + 25$ is called a **perfect square trinomial**. It is the square of a binomial.
To see what happens when we square any binomial, let us square $(a + b)$:
$(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$

The square of any binomial produces the following three terms:
1. The square of the first term of the binomial: a^2
2. Twice the product of the two terms: $2ab$
3. The square of the second term: b^2

The square of every binomial -- every perfect square trinomial -- has that form: $a^2 + 2ab + b^2$. To recognize that is to know an important product in the "multiplication table" of algebra.

Note: If the binomial has a minus sign, then the minus sign appears in the middle term of the trinomial.
Therefore, using the double sign ± ("plus or minus"), we can state the rule as follows:

$\boldsymbol{(a \pm b)^2 = a^2 \pm 2ab + b^2}$

This means: If the binomial is $a + b$, then the middle term will be $+2ab$; but if the binomial is $a - b$, then the middle term will be $-2ab$

Example 1: Square the binomial $(x + 6)$.

$(x + 6)^2 = x^2 + 12x + 36$
1. x^2 is the square of x.
2. $12x$ is twice the product of $x \cdot 6$. ($x \cdot 6 = 6x$. Twice that is $12x$.)
3. 36 is the square of 6.

$$
\begin{aligned}
&(a+b)^2 \\
&= (a+b)(a+b) \\
&= a(a+b)+b(a+b) \\
&= a^2+ab+ba+b^2 \\
&= a^2+2ab+b^2
\end{aligned}
$$

Example 2: Find the following product $(3y-2)^2$.

$$
\begin{aligned}
&(3y-2)^2 \rightarrow (3y-2)(3y-2) \\
&= (3y)(3y)+(3y)(-2)+(-2)(3y)+(-2)(-2) \\
&= 9y^2-6y-6y+4 \\
&= 9y^2-12y+4
\end{aligned}
$$

Or use the Perfect Square formula:
$(3y - 2)^2$
$= (3y)^2 - 2(3y)(2) + (2)^2$
$= 9y^2 - 12y + 4$

$$
\begin{aligned}
&(a-b)^2 \\
&= (a-b)(a-b) \\
&= a(a-b)-b(a-b) \\
&= a^2-ab-ba+b^2 \\
&= a^2-2ab+b^2
\end{aligned}
$$

Practice: a) $(x + 8)^2$ b) $(2s - 8t)^2$