Chapter 2 Polynomials (1)

1. Variables in Expression

Variable - A variable is a letter that represents a number.
Don't let the fact that it is a letter throw you. Since it represents a number, you treat it just like you do a number when you do various mathematical operations involving variables.

x is a very common variable that is used in algebra, but you can use any letter (a, b, c, d, ...) to be a variable.

Algebraic Expressions - An algebraic expression is a number, variable or combination of the two connected by some mathematical operation like addition, subtraction, multiplication, division, exponents, and/or roots.

$2x + y$, $a/5$, and $10 - r$ are all examples of algebraic expressions.

Evaluating an Expression - You evaluate an expression by replacing the variable with the given number and performing the indicated operation.

Example 1: Let the value of the variable y depend on the value of the variable x as follows: $y = 3x + 5$. Calculate the value of y that corresponds to each value of x:

When $x = 0$, $y = 3(0) + 5 = 5$ When $x = 1$, $y = 3(1) + 5 = 8$
When $x = 2$, $y = 3(2) + 5 = 11$ When $x = 3$, $y = 3(3) + 5 = 14$

Value of an Expression - When you are asked to find the value of an expression that means you are looking for the result that you get when you evaluate the expression.

Keep in mind that **vary means to change** - a variable allows an expression to take on different values, depending on the situation.

For example, the area of a rectangle is length times width. Well, not every rectangle is going to have the same length and width, so we can use an algebraic expression with variables to represent the area and then plug in the appropriate numbers to evaluate it.

So if we let the length be the variable l and width be w, we can use the expression lw. If a given rectangle has a length of 4 and width of 3, we would evaluate the expression by replacing l with 4 and w with 3 and multiplying to get a value of 4 times 3 or 12.

Example 2: Evaluate the expression $3x + y^2 - z$ when $x = 4$, $y = 6$, $z = 8$.

Plugging in the corresponding value for each variable and then evaluating the expression we get:

$$3x + y^2 - z = 3 \cdot 4 + 6^2 - 8 = 12 + 36 - 8 = 48 - 8 = 40$$

2. Collecting like Terms

When numbers are added or subtracted, they are called ***terms***.

When numbers are multiplied, they are called ***factors***.

Here is a sum of four terms: $a - b + c - d$.

In algebra we speak of a "sum" of several terms, even though there are subtractions. In other words, anything that *looks* like what you see above, we call a sum.

Here is a *product* of four factors: *abcd*.
The word *factors* always signifies multiplication.
And again, we speak of the "product" *abcd*, even though we do not name an answer.

Problem: In the expression $2a + 4ab + 5a(b + c)$, how many *terms* are there? And each term has how many factors?
Solution: There are 3 terms. $2a$ has 2 factors, $4ab$ has 3 factors, and $5a(b + c)$ has 3 factors.

Term - A term is a number, variable or the product of a number and variable(s).

Examples of terms are $3x$, $5y^3$, $2ab$, z

Coefficient - A coefficient is the numeric factor of your term.

Here are the coefficients of the terms listed above:

Term	Coefficient
$3x$	3
$5y^3$	5
$2ab$	2
z	1

Constant Term - A constant term is a term that contains only a number. In other words, there is no variable in a constant term.

Examples of constant terms are 4, 100, and -5.

Like Terms - Like terms are terms that have the exact same variables raised to the exact same exponents.
Some examples of like terms are $3x^2$ and $-5x^2$. Another example is $5ab^2$ and $24ab^2$.
Are these like terms? $-2x^2y$, $8xy^2$, $13x^2y$
NO! They have the same variables but different exponents.

Combining like terms
You can only combine terms that are like terms. You think of it as the reverse of the distributive property.

It is like counting apples and oranges. You just count the number of variables you have the same and write the number in front of the common variable part.

Example 1: Simplify $7a-10a$ by combining like terms.

Are there any like terms that we can combine?
It looks like it. Both terms have the same variable, a.

$$7a-10a=(7-10)a=-3a$$

Example 2: Simplify $15b+9+5b-2$ by combining like terms.

Are there any like terms that we can combine?
Two terms have the same variable part, b. The other pair of terms is constant that can be combined together.

$$15b+9+5b-2=15b+5b+9-2=(15+5)b+9-2=20b+7$$

From here on, the distributive property stop will be ignored when combining like terms, since it has already been shown in the above examples to give you the thought behind combining like terms.

Example 3: Simplify the expression $5x^2+7x-2x^2-10x+5$.

It looks like we have two terms that have an *x* squared that we can combine and we have two terms that have an *x* that we can combine. The poor 5 does not have anything it can combine with so it will have to stay as 5.

Grouping like terms together and combining them we get:

$$5x^2+7x-2x^2-10x+5=5x^2-2x^2+7x-10x+5=3x^2-3x+5$$

3. Adding and Subtracting Polynomials

Adding Polynomials

Step 1: Remove the ( ).
If there is only a + sign in front of ( ), then the terms inside of ( ) remain the same when you remove the ( ).

Step 2: Combine like terms and simplify.

Example 1: Simplify $(5x^2-4x+10)+(3x^2-2x-12)$

$$(5x^2-4x+10)+(3x^2-2x-12)=5x^2-4x+10+3x^2-2x-12=8x^2-6x-2$$

Example 2: Write the following as an algebraic expression and simplify it if possible.
Add $3a + 9$ to $7a - 2$.

Basically we will be adding these two expressions together.
Writing this as an algebraic expression we get: $(3a+9)+(7a-2)$
Regrouping and combining like terms we get: $(3a+9)+(7a-2)=3a+7a+9-2=10a+7$

Subtracting Polynomials

Step 1: Remove the ( ).

If there is - in front of the ( ) then distribute it by multiplying every term in the ( ) by a -1.
Or you can think of it as negating every term in the ( ).
Step 2: Combine like terms and simplify.

Example 1: Simplify: $(5x-2y+1)-(2x-7y+4)$:

$$(5x-2y+1)-(2x-7y+4)=5x-2y+1-2x+7y-4=3x+5y-3$$

Example 2: Subtract $9x^3-6$ from $3x^3-7x+5$:

$$(3x^3-7x+5)-(9x^3-6)=3x^3-7x+5-9x^3+6=-6x^3-7x+11$$

Practice: Simplify.

a) $\frac{3}{4}w^2-\frac{2}{3}w^2+\frac{1}{4}w^2-\frac{4}{3}w^2$ b) $(3x^2-4xy+6y^2)-(6x^2-8xy-3y^2)$

c) $4ab-6ab+3a^2b+4ab^2+5a^2b$ d) $(3x^2+2xy+4y^2)+(6x^2-5xy+3y^2)+(9x^2-25y^2)$