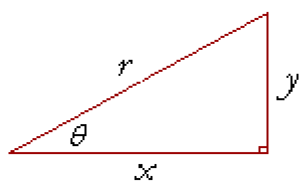
Trigonometric Function (1)

Trigonometry Identities – an equation involving trig. ratios that are true for all values of the variable.

1. Reciprocal Identities

$\sin\theta = \dfrac{1}{\csc\theta}$ $\csc\theta = \dfrac{1}{\sin\theta}$	$\cos\theta = \dfrac{1}{\sec\theta}$ $\sec\theta = \dfrac{1}{\cos\theta}$	$\tan\theta = \dfrac{1}{\cot\theta}$ $\cot\theta = \dfrac{1}{\tan\theta}$

Proof:



We know:

$\sin\theta = \dfrac{y}{r}$ $\qquad$ $\csc\theta = \dfrac{r}{y}$ $\qquad$ So, $\sin\theta = \dfrac{1}{\csc\theta}$

Similarly for the remaining functions.

2. Tangent and cotangent Identities

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$	$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Proof:

$$\tan\theta = \frac{y}{x}$$

Therefore, on dividing both numerator and denominator by *r*,

$$\tan\theta = \frac{y/r}{x/r} = \frac{\sin\theta}{\cos\theta}, \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

3. Pythagorean Identities

$\sin^2\theta+\cos^2\theta=1$	$1+\tan^2\theta=\sec^2\theta$	$1+\cot^2\theta=csc^2\theta$

Proof: According to the Pythagorean Theorem, $x^2+y^2=r^2$

Therefore, on dividing both sides by r^2, $\frac{x^2}{r^2}+\frac{y^2}{r^2}=\frac{r^2}{r^2}=1$

That is, according to the definitions, $\cos^2\theta+\sin^2\theta=1$
Similarly for the remaining functions.

In Summary:

Reciprocal Identities	Quotient Identities	Pythagorean Identities
$\csc\theta=\frac{1}{\sin\theta}$ $\sec\theta=\frac{1}{\cos\theta}$ $\cot\theta=\frac{1}{\tan\theta}$	$\tan\theta=\frac{\sin\theta}{\cos\theta}$ $\cot\theta=\frac{\cos\theta}{\sin\theta}$	$\sin^2\theta+\cos^2\theta=1$ $1+\cot^2\theta=\csc^2\theta$ $1+\tan^2\theta=\sec^2\theta$

To prove a trig identity, you must show that each side of the identity is equivalent.

Some Suggestions….
1. Start with the most complicated side and work on it until it looks like the other side.
2. Rewrite all trig ratios in terms of sine and cosine.
3. If there are fractions, try a common denominator.
4. Try factoring and cancelling.

Example 1: Show $\tan x+\frac{1}{\tan x}=\frac{1}{\sin x\cos x}$

$$LS = \tan x + \frac{1}{\tan x}$$
$$LS = \frac{\sin x}{\cos x} + \frac{1}{\frac{\sin x}{\cos x}}$$
$$LS = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$$

$$LS = \frac{\sin x \sin x + \cos x \cos x}{\cos x \sin x}$$
$$LS = \frac{\sin^2 x + \cos^2 x}{\cos x \sin x}$$
$$LS = \frac{1}{\cos x \sin x}$$
$$\therefore LS = RS$$

Example 2: Show: $\sec^2 x + \csc^2 x = \sec^2 x \cdot \csc^2 x$

We begin with the left side:

LS = $\sec^2 x + \csc^2 x = \dfrac{1}{\cos^2 x} + \dfrac{1}{\sin^2 x}$

$$= \frac{\sin^2 x + \cos^2 x}{\cos^2 x \cdot \sin^2 x} = \frac{1}{\cos^2 x \cdot \sin^2 x}$$

$$= \frac{1}{\cos^2 x} \cdot \frac{1}{\sin^2 x} = \sec^2 x \cdot \csc^2 x$$

= RS

Therefore, LS = RS

Example 3: Prove $\boldsymbol{\frac{sin^2 x}{1-cosx} = 1 + cosx}$

$LS: \dfrac{\sin^2 x}{1-\cos x}$ $\qquad$ $RS: 1+\cos x$

$$= \frac{1-\cos^2 x}{1-\cos x}$$
$$= \frac{(1-\cos x)(1+\cos x)}{1-\cos x}$$
$$= 1+\cos x$$

Therefore, LS = RS

4. Trigonometric Quadratic Equation

- Gather terms to create a quadratic equation.
- Use identities to create a quadratic equation that contains only one trigonometric ratio.
- Factor the quadratic equation and solve the resulting linear equations.

- If the equation cannot be factored, use the quadratic formula to simplify.
- A quadratic equation will have multiple solutions. Consider the context and parameters of the problem carefully as some solutions are inadmissible.

Example 1: Solve the equation $2\cos^2 x - \cos x - 1 = 0$ on the interval $0 \le x \le 360°$.

$2\cos^2 x - \cos x - 1 = 0$
$(2\cos x + 1)(\cos x - 1) = 0$

$2\cos x + 1 = 0$	**OR**	$\cos x - 1 = 0$
$\cos x = -1/2$		$\cos x = 1$
$\cos \beta = ½ \rightarrow \beta = 60°$		$x = 0°, 360°$
By CAST rule:		
$x = 180 - 60 = 120°$		
or $x = 180 + 60 = 240°$		

Therefore, $x = 0°, 120°, 240°, 360°$

Example 2: Solve the equation $6\cos^2 x - \sin x - 5 = 0$ on the interval $0 \le x \le 360°$.

$6\cos^2 x - \sin x - 5 = 0$
$6(1 - \sin^2 x) - \sin x - 5 = 0$
$6\sin 2x + \sin x - 1 = 0$
$(2\sin x + 1)(3\sin x - 1) = 0$

$2\sin x + 1 = 0$	**OR**	$3\sin x - 1 = 0$
$\sin x = -1/2$		$\sin x = 1/3$
$\sin \beta = ½ \rightarrow \beta = 30°$		$x = 19.5°, 180 - 19.5 = 160.5°$
By CAST rule:		
$x = 180 + 30 = 210°$		
or $x = 360 - 30 = 330°$		

Therefore, $x = 19.5°, 160.5°, 210°, 330°$.

Practice in class: Solve each equation such that $0 \le x \le 360°$.

a) $\sin^2(x) - \sin(x) = 2$

b) $2\cos^2(x) - 3\cos(x) + 1 = 0$