Chapter 5 Analytic Geometry (2)

1. Equation of a Straight Line

1) slope-y intercept form

A line with equation $y = mx + c$ has **slope** m and ***y*-intercept** c. We also use y = mx + b where b is the y-intercept. m is also called **gradient**.

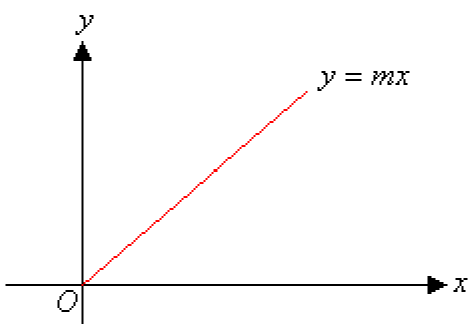
$$y = mx + c$$

y-intercept

Gradient

The slope of a straight line is the coefficient of x.

If a straight line passes through the origin, then its y-intercept is 0. So, the equation of a straight line passing through the origin is $y = mx$, where m is the slope of the line.



Example 1

Write down the gradient and the y-intercept for the following equations:

a. $y = 4x + 3$

b. $6x + 3y = 9$

Solution:

a. Comparing $y = 4x + 3$ with $y = mx + c$ gives $m = 4$, $c = 3$.
   So, the gradient is 4 and the y-intercept is 3.

b. Write $6x + 3y = 9$ in the form $y = mx + c$.

$$
\begin{aligned}
6x + 3y &= 9 && \{\text{Subtract } 6x \text{ from both sides}\} \\
6x + 3y - 6x &= 9 - 6x \\
3y &= -6x + 9 && \{\text{Divide both sides by } 3\} \\
\frac{3y}{3} &= \frac{-6x+9}{3} \\
y &= -2x + 3
\end{aligned}
$$

Comparing $y = -2x + 3$ with $y = mx + c$ gives $m = -2$, $c = 3$.

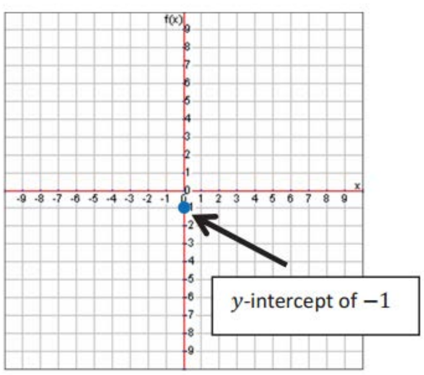
So, the gradient is -2 and the y-intercept is 3.

Since the equation has slope and intercept, we call $y = mx + c$ the slope-intercept form.

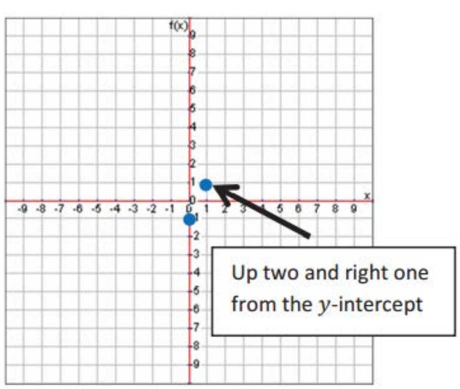
Once you have an equation in slope-intercept form, start by graphing the y-intercept on the coordinate plane. From the y-intercept, move the rise and run of the slope to plot another point. Finally, draw the line that connects the two points.

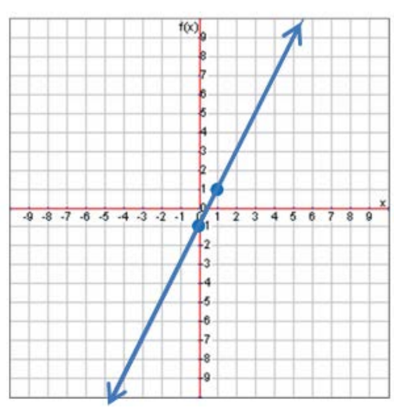
Example 1: Graph y = 2x – 1 using the slope and y-intercept.

The y-intercept is -1, so we plot a point at -1 on the y-axis to start.

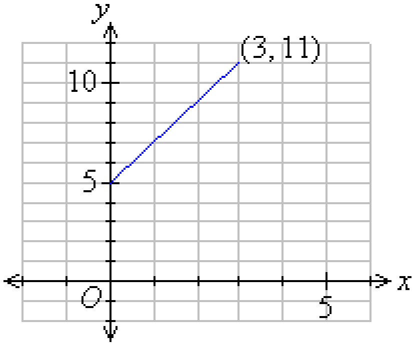


Next we know the slope is 2 which means a rise of 2 and a run of 1. So we will move up two and right one to plot the next point. Lastly, draw the line by connecting the points.





Example 2: Calculate the slope of the straight line given in the following diagram; and find its equation.



Solution:

Let $(x_1, y_1) = (0, 5)$ and $(x_2, y_2) = (3, 11)$.

$$\therefore m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11-5}{3-0} = \frac{6}{3} = 2$$

$c = 5$

The general equation of the straight line is

$$y = mx + c$$

Substituting $m = 2$ and $c = 5$ gives

$$y = 2x + 5$$

Example 3: Write the slope-intercept form of the equation of the line with slope 3 and which passes through the point (1, −4).

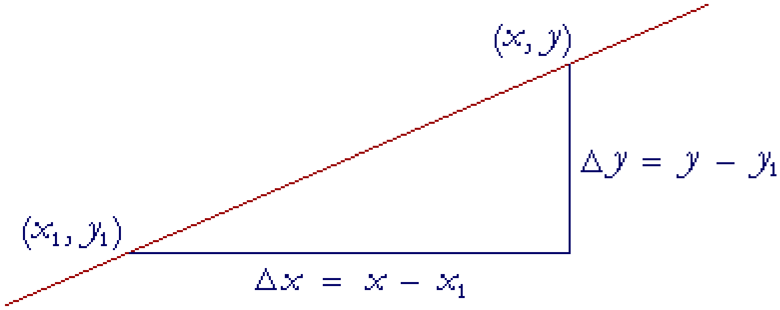
The equation will have the form $y = mx + b$
We are given that $m = 3$. Therefore, the equation is $y = 3x + b$.
Now we must determine b.

We are given that (1, −4) is a point on the line. Those coordinates, then, solve the equation:
$-4 = 3 \cdot 1 + b$
This implies $b = -7$.
The equation of the line is $y = 3x - 7$.

Class practice: Write the slope-intercept form of the equation of the line with slope −1 and which passes through the point (−8, −2).

2) The point-slope form

Let (x, y) be *any* point on the line. And let (x_1, y_1) be a given point on the line.



Then the slope $\frac{\Delta y}{\Delta x}$ is equal to the slope *m* of the line.

$$\frac{y - y_1}{x - x_1} = m \quad \rightarrow \mathbf{y - y_1 = m(x - x_1)}$$

This is called the **point-slope formula** for the equation of a straight line that passes through (x_1, y_1) with slope m. We can use the formula when we know the slope *m* of the line and one point (x_1, y_1) on it.

Example 1: We are given that $m = 2$ and that $(x_1, y_1) = (1, -3)$.

Therefore, according to the point-slope formula: $\frac{y - (-3)}{x - 1} = 2$

We must solve this for *y*:

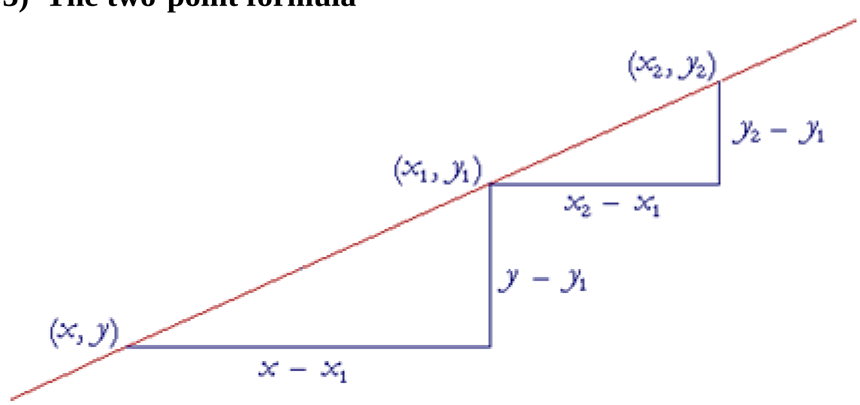
$y + 3 = 2(x - 1)$

$y + 3 = 2x - 2$

This implies y = 2x – 5. This is the equation of the line.

Class practice: Find an equation for the line through (-2, 5) with slope -3 and solve it for y.

3) The two-point formula



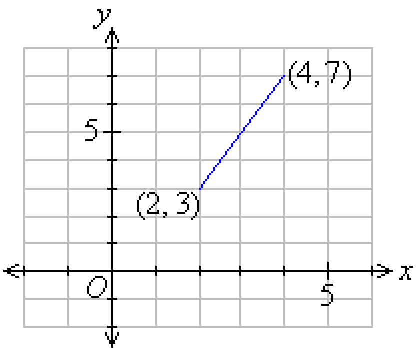
Let $(x_1,\ y_1)$ and $(x_2,\ y_2)$ be two given points, and let (x, y) be *any* point on the line. Then the slope joining one of the given points and any point, is equal to the slope joining the two given points.

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

This is called the two-point formula for the equation of a straight line. We can use it to determine the equation when we know two points. We may choose (x_1, y_1) to be either one of them.

Example 1: Find the equation of the line joining the points (2, 3) and (4, 7).

Solution:



Let $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (4, 7)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-3}{4-2} = \frac{4}{2} = 2$$

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} \rightarrow \frac{y-3}{x-2} = 2 \rightarrow y - 3 = 2(x-2)$$

$$y - 3 = 2x - 4 \rightarrow y = 2x - 1$$

Class practice: Find the equation of the line joining the two points (1, 5) and (4, 17).

4) Linear Equation in Two Variables Standard Form: A*x* + B*y* + C = 0

A linear equation in two variables is an equation that can be written in the form
A*x* + B*y* + C = 0, where A, B, and C are real numbers with A and B are not both 0.

The number A, B, and C in standard form can be any real numbers, but it is a common practice to write standard form using only **integers and a positive coefficient for x.**

This form is called the **standard form** of a linear equation.

Changing to Standard Form

Example: Write the equation $y = \frac{1}{2}x - \frac{3}{4}$ in standard form. Don't forget to use only integer coefficient and a positive coefficient for x.

Subtract y on both sides: $0 = \frac{1}{2}x - y - \frac{3}{4}$
Flip to have everything on the left side: $\frac{1}{2}x - y - \frac{3}{4} = 0$
Times by LCD = 4 on both sides to get rid of the denominators: $2x - 4y - 3 = 0$

Changing to Slope-Intercept Form

Example: Find the slope and y-intercept of the line 3x – 2y = 5.

To find the slope and y-intercept, we need to change to slope-intercept form first.

Isolate y: 2y = 3x – 5
Divide both sides by 2: $y = \frac{3}{2}x - \frac{5}{2}$
Therefore, the slope is $\frac{3}{2}$, and the y-intercept is $-\frac{5}{2}$.