## Chapter 8 Geometry 2 (Surface Area)

1. Surface Area of 3D Figures
The **surface area SA** of a 3D figure is the sum of the areas of all of its faces and curved surfaces. The surface area of any figure is measured in square units (or units squared).

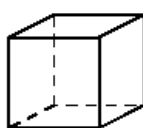**2. Surface Area of a Prism and a Cylinder**
A **prism** is a three-dimensional shape that has non-curved sides. A cube is a prism, but a sphere is not. A prism has a pair of congruent sides, called bases, like the cube, triangular prism and the rectangular prism. Don't confuse a prism with a pyramid, which only has one base.
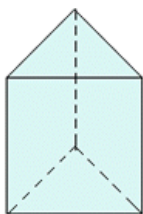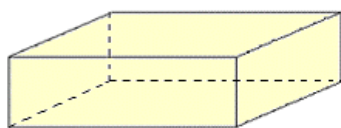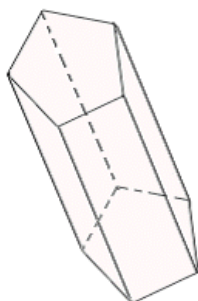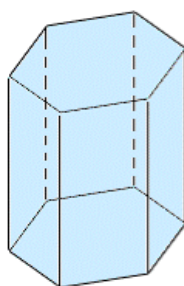
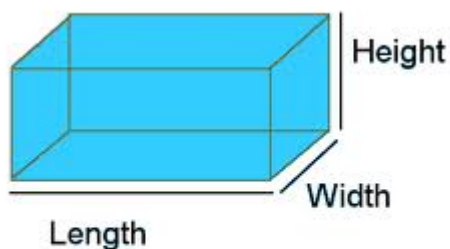Triangular Prism   Rectangular Prism   Cube

Pentagonal Prism   Hexagonal Prism

The **surface area** of a three-dimensional shape is the sum of all of the surface areas of each of the sides. I like to think of the shape as a birthday present and the surface area as the wrapping paper. If we carefully took the wrapping paper off of the present and added up each side, the total would be the surface area of the shape.

Start with a right rectangular prism as shown below and call the length L, the width w, and the height h:

Height

Width

Length

Rectangular prism

In order to make a rectangular prism like the one shown above, you basically use the following rectangular prism template:
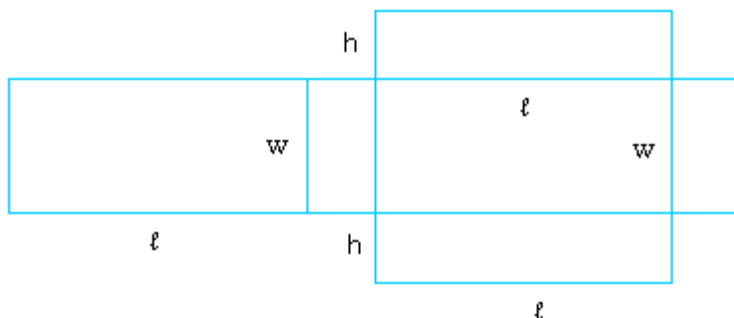
Looking at the rectangular prism template, it is easy to see that the solid has six sides and each side is a rectangle

The bottom side and the top side are equal and have L and w as dimensions

The area for the top and bottom side is Lw + Lw = 2Lw

The front side and the back side are equal and have h and l as dimensions

The area for the front and the back side is Lh + Lh = 2Lh

Then, the last two sides have h and w as its dimensions.

The area for the front and the back side is wh + wh = 2wh

The total surface area, call it SA is: **SA = 2Lw + 2Lh + 2wh**

Example 1: Find the surface area of a rectangular prism with a length of 4 cm, a width of 5 cm, and a height of 10 cm.
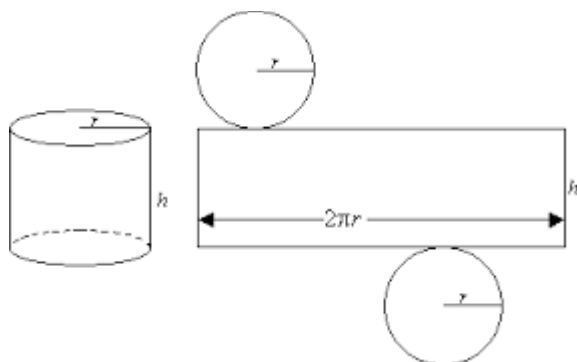
$SA = 2 \times l \times w + 2 \times l \times h + 2 \times w \times h$

$SA = 2 \times 4 \times 5 + 2 \times 4 \times 10 + 2 \times 5 \times 10$

$SA = 40 + 80 + 100$

$SA = 220 \text{ cm}^2$

 A **cylinder** has a total of three surfaces: a top, bottom, and middle. The top and bottom, which are circles, are easy to visualize.

The area of a circle is πr^2. So, the area of two circles would be $\pi r^2 + \pi r^2 = 2\pi r^2$.

The perimeter of the base is actually the circumference of the circle, $2\pi r$.

To get the area of the rectangle, multiply h by $2 \times pi \times r$ and that is equal to $2 \times pi \times r \times h$

Therefore, the total surface area of the cylinder, call it SA is:

SA of a cylinder = (2 x Area of Base) + (h x Perimeter of Base) = $2\pi r^2 + 2\pi rh$

Example 2: Find the surface area of a cylinder with a radius of 4 cm, and a height of 3 cm

$SA = 2 \times pi \times r^2 + 2 \times pi \times r \times h$

$SA = 2 \times 3.14 \times 4^2 + 2 \times 3.14 \times 4 \times 3$
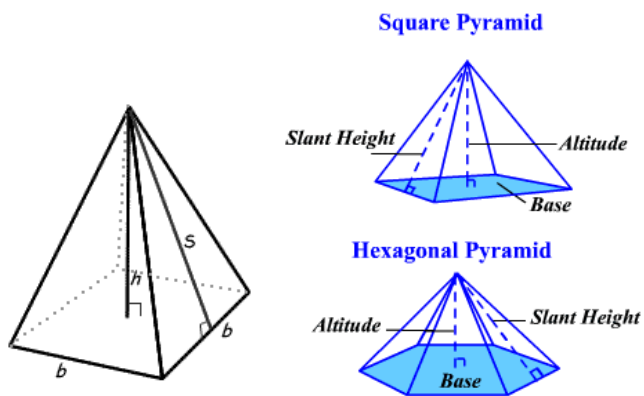
$SA = 6.28 \times 16 + 6.28 \times 12$

$SA = 100.48 + 75.36$

Surface area $= 175.84 \text{ cm}^2$

3. Surface Area of a Pyramid and a Cone

Start with a square pyramid as shown below and call the length of the base b and the height of one triangle h. s is the slant height.

The area of the square is b^2

The area of one triangle is (bs)/2

Since there are 4 triangles, the area is $4 \times (bs)/2 = 2bs$

Therefore, the surface area, call it SA is: **SA = b² + 2bs**

Sometimes the question doesn't give the slant height but gives height instead, we can use the Pythagorean Theorem to find the slant height.

$$s = \sqrt{h^2 + \left(\frac{b}{2}\right)^2}$$

Example 1: Find the surface area of a square pyramid with a base length of 5 cm, and a slant height of 10 cm.

$SA = 5^2 + 2 \times 5 \times 10$

$SA = 25 + 100$

$SA = 125 \text{ cm}^2$

Example 2: Find the surface area with a base length of 6 cm, and a height of 4 cm

$SA = b^2 + 2\,bs$

$$s = \sqrt{h^2 + \left(\frac{b}{2}\right)^2} = \sqrt{4^2 + 3^2} = 5$$

$SA = 6^2 + 2 \times 6 \times 5$

$SA = 36 + 60$

$SA = 96 \text{ cm}^2$

Start with a square pyramid and just keep increasing the number of sides of the base. After a very large number of sides, you can see that the figure will eventually look like a cone. For a **cone**, the **slant height** is the length of a line from the common vertex to any point on the edge of the circular base.

SA = πrs + πr²

Example 3: Find the surface area of a cone with a radius of 4 cm, and a height of 8 cm

$S = \pi \times r^2 + \pi \times r \times \sqrt{(r^2 + h^2)}$

$S = 3.14 \times 4^2 + 3.14 \times 4 \times \sqrt{(4^2 + 8^2)}$

$S = 3.14 \times 16 + 12.56 \times \sqrt{(16 + 64)}$

$S = 50.24 + 12.56 \times \sqrt{(80)}$

$S = 50.24 + 12.56 \times 8.94$

$S = 50.24 + 112.28$

$S = 162.52 \text{ cm}^2$

4. Surface Area of a Sphere
The surface area of a sphere is equal to four times the area of a circle with the same radius.

The surface area of a sphere is:
$SA = 4\pi r^2$

Example 1: Find the surface area of a sphere with a radius of 6 cm.

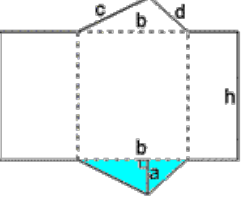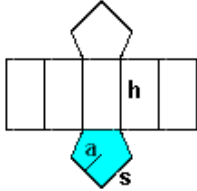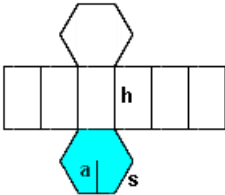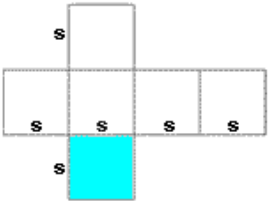$SA = 4 \times pi \times r^2$

$SA = 4 \times 3.14 \times 6^2$

$SA = 12.56 \times 36$

$SA = 452.16$

Surface area $= 452.16 \text{ cm}^2$

5. Extra Formulas for Surface Area

Geometric Shape	Surface Area **B** = area of the **b**ase **P** = **p**erimeter of the base

triangular prism **a** = altitude **b** = base **h** = #height	SA = 2B + Ph SA = 2(1/2ab) + (b + c + d)h SA = ab + (b + c + d)h
regular pentagonal prism **a** = apothem length **s** = side length **n** = #sides **h** = height	SA = 2B + Ph SA = 2(1/2ans) + nsh SA = 2(1/2a)(5)s + 5sh SA = 5as + 5sh
regular hexagonal prism **a** = apothem length **s** = side length **n** = #sides **h** = height	SA = 2B + Ph SA = 2(1/2ans) + nsh SA = 2(1/2a)(6)s + 6sh SA =6as + 6sh
cube **s** = side length	SA = 2B + Ph SA = 2(s^2) + (4s)s = 6s^2

Example 1: Find the surface area of a cube if the length of one side is 5 cm

Surface area = 6a^2
Surface area = 6 × 5^2 = 150 cm^2

Example 2: Suppose a rectangular prism has dimensions 5 x 4 x 7 feet. What are the surface area?

Two sides are 5 x 4, two are 4 x 7 and two are 5 x 7. The areas of these sides are 20, 28, and 35.

Thus the total surface area is twice the sum of these or 166 square feet.

Example 3: A tin can has a radius of 2 inches and a height of 3 inches. What are the surface area?

The surface area is $S = 2\pi r^2 + 2\pi rh = 2\pi 2^2 + 2\pi 2 \bullet 3 = 8\pi + 12\pi = 20\pi \approx 62.8$ in².

Example 4: A ping pong ball has a radius of 2 cm. What was the area of plastic required to create the ball?

A ping pong ball is a sphere, and the plastic is a thin layer forming the surface, so we want to find the surface area of a sphere with radius 2 cm.

SA = 4π (2 cm)² = 50.3 cm²
So 50.3 cm² of plastic was required to make the ball.

Example 5: Suppose that the height of a cylinder is 30 cm and its volume is 750π cm³. Find its radius and surface area.

To get started, we need to organize as much of the given information as possible into a known formula. Since the volume (750π cm³) and height (30 cm) of the cylinder are given, we will start with the equation for volume.

V = πr²h = 30πr²,   V = 750π,   750π = 30πr² ,   r = 5cm.

Now we will find the surface area of the cylinder using our values for the radius and height.

SA = 2πr² + 2πrh,   SA = 2π(25) + 2π(5)(30),   SA = 50 π+ 300π,   SA = 350π cm² = 1099 cm³

In Class Practice:

1. Determine the surface area for each of the following. Round your answer to one decimal place. (use π = 3.14)

a. A cube with side length 4.5 cm.

b. A rectangular prism measures 1.5 m by 2 m by 3 m.

c. A cylinder with radius 12.7 mm and height 35 mm.

d. A right triangular prism with base 7.8 m, height 9.5 m, length 11.2 m and slant height 10.3 m.

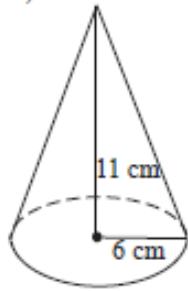2. Find the surface area of the following prisms.

1)

17 cm

1 cm

2.5 cm

2)

8 cm

13 cm
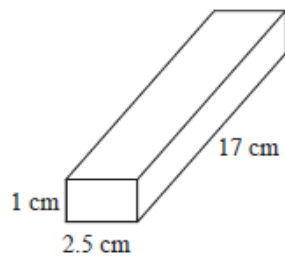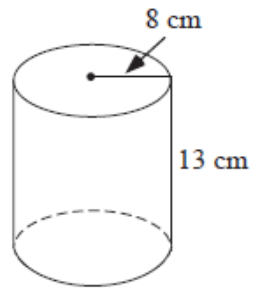
3. Find the surface area of the following cone and pyramid.

1)

11 cm

6 cm

2)

7 cm

5 cm

4 cm