First name: ________________ Last name: _________________ Student ID: __________________

Number Theory Homework

Basic problems

1. Add

1. $(14x + 5) + (10x + 5)$	2. $(10x + 12) + (6x + 20)$
3. $(19x^2 + 12x + 12) + (7x^2 + 10x + 13)$	4. $(17x^2 + 20x + 11) + (15x^2 + 11x + 17)$
5. $(20x^2 + 15x + 13) + (-19x^2 + 17x + 5)$	6. $(-13x^2 - 13x - 10) + (19x^2 - 19x - 5)$

2. Subtract

1. $(6x + 14) - (9x + 5)$	2. $(6x + 19) - (14x + 5)$
3. $(14x^2 + 13x + 12) - (7x^2 + 20x + 4)$	4. $(19x^2 + 9x + 16) - (5x^2 + 12x + 7)$
5. $(-18x^2 + 4x - 16) - (15x^2 + 4x - 13)$	6. $(-9x^2 - 4x - 4) - (-9x^2 - 11x + 12)$

3. Multiply

1. $(12x)(12x + 11)$	2. $(9x)(4x + 2)$
3. $x(9x^2 + 4x + 3)$	4. $(2x + 7)(8x)$
5. $(4x)(-8x - 9)$	6. $5x(-6x - 3)$

4. Divide

1. $(6x^3 + 30x^2 + 24x) \div 12$	2. $(15x^3 + 20x^2 + 5x) \div 5$
3. $(14x^4 + 10x^2 + 6x) \div 6x$	4. $(72x^4 + 81x^2 + 9x) \div 9x$
5. $(60x^{12} + 60x^{11} + 30x^8) \div 30x$	6. $(60x^{12} + 24x^{10} + 24x^3) \div 12x^2$

►Challenge problems

1. Bertha has 6 daughters and no sons. Some of her daughters have 6 daughters, and the rest have none. Bertha has a total of 30 daughters and granddaughters, and no great-granddaughters. How many of Bertha's daughters and grand-daughters have no daughters?

2. Which of the following numbers is a perfect square?
(A) 98! (99!) (B) 98! (100!) (C) 99! (100!) (D) 99! (101!) (E) 100! (101!)

3. The symbolism $\lfloor x \rfloor$ denotes the largest integer not exceeding x. For example, $\lfloor 3 \rfloor = 3$, and $\lfloor 9/2 \rfloor = 4$. Compute $\lfloor \sqrt{1} \rfloor + \lfloor \sqrt{2} \rfloor + \lfloor \sqrt{3} \rfloor + ... + \lfloor \sqrt{16} \rfloor$.

4. A restaurant offers three desserts and exactly twice as many appetizers as main courses. A dinner consists of an appetizer, a main course, and a dessert. What is the least number of main courses that the restaurant should offer so that a customer could have a different dinner each night in the year 2003?

5. Sally has five red cards numbered 1 through 5 and four blue cards numbered 3 through 6. She stacks the cards so that the colors alternate and so that the number on each red card divides evenly into the number on each neighbouring blue card. What is the sum of the numbers on the middle three cards?

6. Let n be a positive integer such that $\frac{1}{2}+\frac{1}{3}+\frac{1}{7}+\frac{1}{n}$ is an integer. Which of the following statements is not true:

(A) 2 divides n (B) 3 divides n (C) 6 divides n (D) 7 divides n (E) n > 84

7. Suppose July of year N has five Mondays. Which of the following must occur five times in August of year N? (Note: Both months have 31 days.)

(A) Monday (B) Tuesday (C) Wednesday (D) Thursday (E) Friday

8. Suppose that n is the product of three consecutive integers and that n is divisible by 7. Which of the following is not necessarily a divisor of n?

(A) 6 (B) 14 (C) 21 (D) 28 (E) 42

9. Consider the set of all fractions $\frac{x}{y}$, where x and y are relatively prime positive integers. How many of these fractions have the property that if both numerator and denominator are increased by 1, the value of the fraction is increased by 10%?

10. Two different prime numbers between 4 and 18 are chosen. When their sum is subtracted from their product, which of the following numbers could be obtained?

(A) 21 (B) 60 (C) 119 (D) 180 (E) 231