2. Two-Dimensional Motion

Grade 11 Physics
Olympiads School

A Closer Look at Vectors

- In the first unit, you learned the difference between **vectors** and **scalars**
- Vectors have a **magnitude**, and a **direction**
- **Position** is a place in a specific frame of reference

$$\vec{d}$$

- **Displacement** is the change in position from position 1 to position 2:

$$\Delta\vec{d} = \vec{d}_2 - \vec{d}_1$$

- **Velocity** is how quickly your displacement is changing with time:

$$\vec{v}_{\text{ave}} = \frac{\Delta\vec{d}}{\Delta t}$$

A Closer Look at Vectors

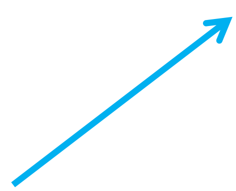
- **Acceleration** how quickly velocity is changing with time:

$$\vec{a}_{\text{ave}} = \frac{\Delta \vec{v}}{\Delta t}$$

- Time interval, Δt, is a scalar quantity.

Vectors

Think of a vector as a line segment with an arrowhead.

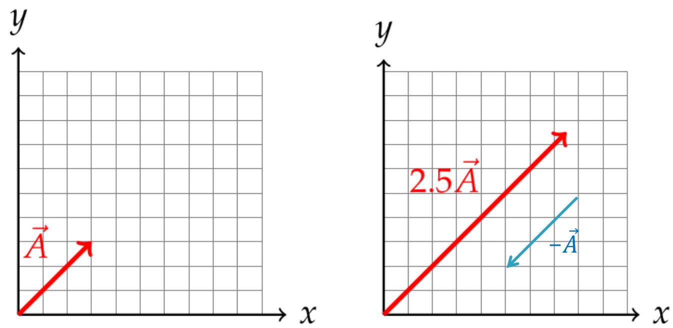


- Magnitude: the length of the line
- Direction: where the arrow is pointing.
- This vector can be any kind of a vector (position, displacement, velocity, or acceleration), but the mathematical properties are the same

Vector Multiplication & Division

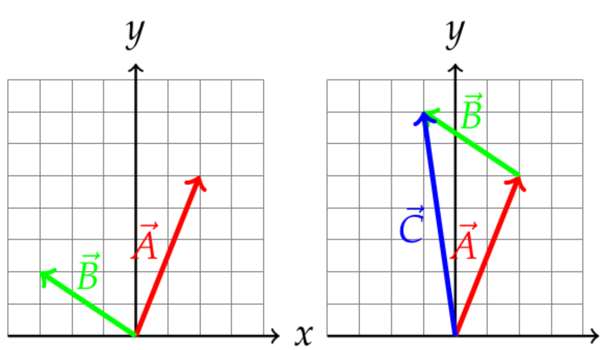
Multiplication and division by a scalar

- Direction remains the same for a positive scalar, or become opposite for a negative scalar
- Magnitude changes.



Vectors Addition: Head to Toe/Tip to Tail

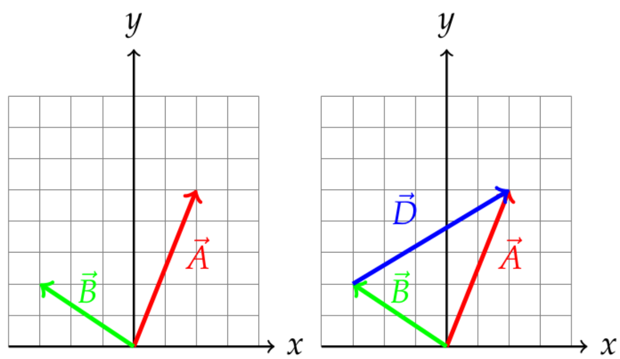
$$\vec{C} = \vec{A} + \vec{B}$$



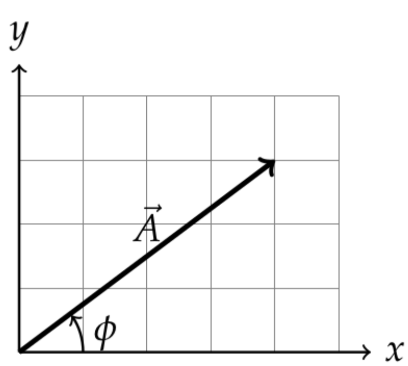
Vector Subtraction:

Just like scalars, you can also subtract vectors

$$\vec{D} = \vec{A} - \vec{B}$$

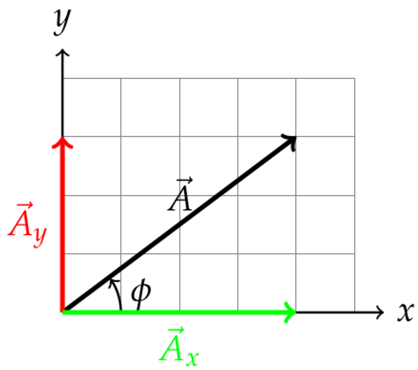


Vectors in 2D



- Length of the arrow is the vector's magnitude
- The angle ϕ tells you the direction with respect to the positive part of x-axis. It is measured counter clockwise (CCW), from 0^0 to 360^0.
- CCW – Positive angles
- CW – Negative angles (clockwise direction)

Components of a Vector

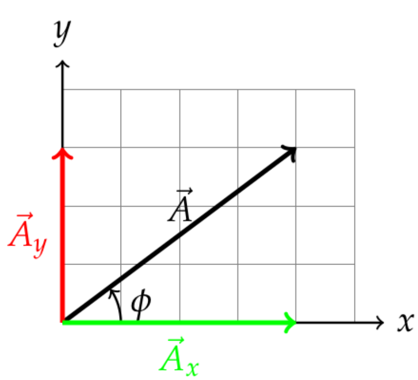


You can “decompose” $\vec{A}$ into two vectors:

- $\vec{A}_x$
- $\vec{A}_y$

Vector components are scalars: $|\vec{A}_x|$, $|\vec{A}_y|$, or A_x and A_y

Components



If you know magnitude and direction, then A_x and A_y are:

$$A_x = A\cos\phi \qquad A_y = A\sin\phi$$

If you know components A_x and A_y, then magnitude A and direction ϕ can be calculated using Pythagorean theorem and trigonometry:

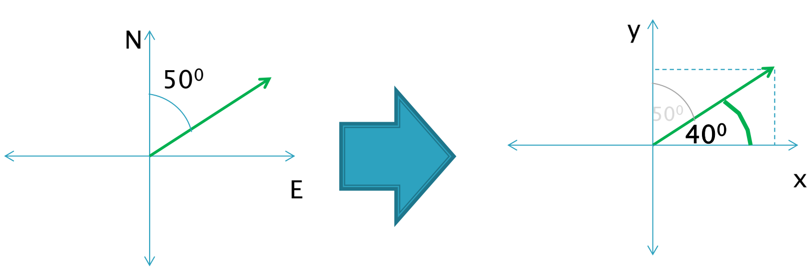
$$A = |\vec{A}| = \sqrt{A_x^2 + A_y^2} \quad \phi = \tan^{-1}\frac{A_y}{A_x}$$

Sample Problem #1

Resolve the vector 56 km/h [N 50° E], into components.

Solution

$\vec{v}$ = 56 km/h [N 50° E],



$v_x = v \cos 40^0$

$v_x = (56 km/h)(\cos 40^0)$

$v_x = 43\ km/h$

$v_y = v \sin 40^0$

$v_y = (56 km/h)(\sin 40^0)$

$v_y = 36\ km/h$

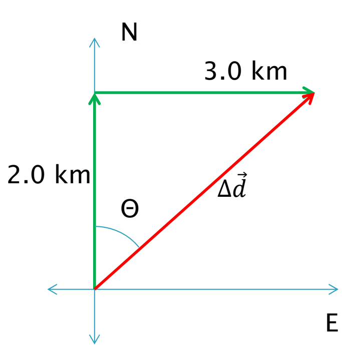
Methods for Adding 2D Vectors

Method 1: Pythagorean theorem + tangent of an angle

- Suitable for adding two vectors that are at right angles of each other

Example 2: Dawn starts from home and bikes 2.0 km north and then 3.0 km east. What is her displacement?

Solution – Right Angle Triangle



Step 1.
Find the magnitude of the total displacement with the Pythagorean Theorem

$\Delta d = \sqrt{2.0^2 + 3.0^2}$ km

$\Delta d = 3.6$ km

Step 2.
Find the direction of the total displacement with the $\tan^{-1}$ function

$\Theta = \tan^{-1}\left(\frac{3.0}{2.0}\right)$

$\Theta = 56^0$

Step 3. Answer

$\overrightarrow{\Delta d} = 3.6\ km[N56^0E], or$

$\overrightarrow{\Delta d} = 3.6\ km[E34^0N]$

Methods for Adding 2D Vectors

Method 2: Cosine + sine laws

- Suitable for adding **two** vectors that are not perpendicular to each other
- Cosine law:

$$C^2 = A^2 + B^2 - 2AB\cos c$$

- Sine law:

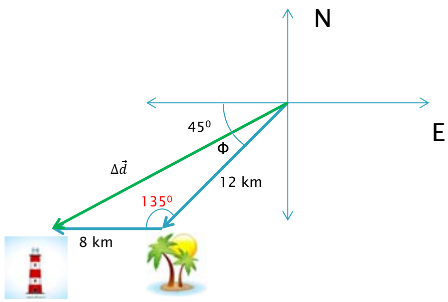
$$\frac{\sin a}{A} = \frac{\sin b}{B} = \frac{\sin c}{C}$$

Sample Problem #3

A kayaker sets out for a paddle on a broad stretch of water. She heads toward the west, but is blown off course by a strong wind. After an hour of hard paddling, she arrives at a lighthouse that is 12 km southwest ([S45^{0}W]) of her starting point. She lands and waits for the wind to die down. She then paddles toward the setting sun and lands on a small island that is 8.0 km west of the lighthouse. In the calm of evening, the kayaker plans to paddle straight back to her starting point.

1. Use a vector diagram to determine her displacement from her starting point to the island.
2. In which direction should she now head and how far will she have to paddle to go directly to the point from which she originally started paddling?

Solution (with trigonometry)



Step 1.

Find the angle Θ between two displacement vectors, using their directions

Θ = 135^0

Step 2.

Find the magnitude of the total displacement, Δd, using the cosine law

$$\Delta d = \sqrt{12^2 + 8.0^2 - 2(12)(8.0)\cos 135^0}\ \text{km}$$

Δd = 18.5 km

Step 3.

Find the angle Φ, using the sine law

$$\frac{\sin\emptyset}{8.0} = \frac{\sin 135}{18.5}$$

Θ = 18^0

Step 4.

Find the direction of $\Delta\vec{d}$

$45^0 - 18^0 = 27^0$

Answer (rounded).

1. $\Delta\vec{d}$ = 19 km [W27^0S]

2. In the opposite direction: E27^0N

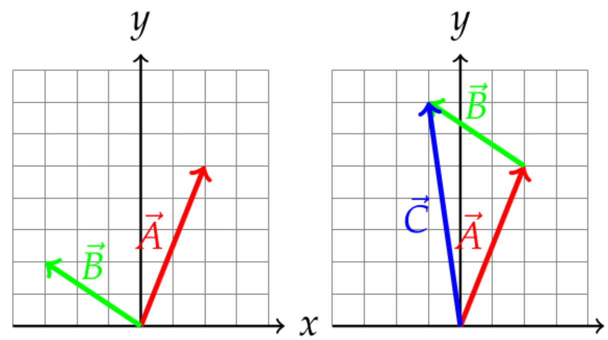
Methods for Adding 2D Vectors

Method 3: Component method

- Decomposing vectors in to their components, then sum each component independently, then reassemble them using Pythagorean theorem and tangent of the angle (Method 1).
- Comparable (time of solving, complexity) for adding 2 vectors using trigonometry, but faster when there are more than 2 vectors.

Vectors Addition Using Components

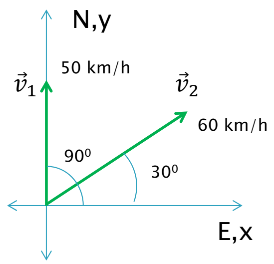
$$\vec{C} = \vec{A} + \vec{B}$$



Sample Problem #4

A water-skier begins his ride by being pulled straight behind the boat. Initially, he has the same velocity as the boat 50 km/h [N]. Once up, the water-skier takes control and cuts out to the side. In cutting out to the side, the water-skier changes his velocity in both magnitude and direction. His new velocity is 60 km/h [N 60° E]. Find the water-skier's change in velocity.

Solution (with components)



Step 1.
Convert vectors from NSWE to x-,y- coordinates
$\vec{v}_1$=50km/h[N]
$\vec{v}_1$=50km/h[90^0]

$\vec{v}_2$=60km/h[N60^0E]
$\vec{v}_2$=60km/h[30^0]

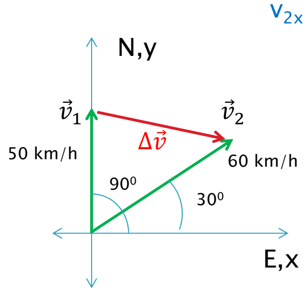
Step 2.
Find the x- and y- components
v_{1x} = (50km/h)(cos90^0)
v_{1x} = 0
v_{2x} = (60km/h)(cos30^0)
v_{2x} = 52 km/h

v_{1y} = (50km/h)(sin90^0)
v_{1y} = 50 km/h
v_{2y} = (60km/h)(sin30^0)
v_{2y} = 30 km/h

Solution (with components)

v_{1x} = 0
v_{2x} = 52 km/h

v_{1y} = 50 km/h
v_{2y} = 30 km/h



Step 3.
Find the components of $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$
Δv_x = 52km/h - 0 km/h
Δv_x = 52km/h

Δv_y = 30km/h - 50 km/h
Δv_y = -20 km/h

Step 4.
Find the magnitude of $\Delta\vec{v}$
$\Delta v = \sqrt{52^2 + (-20)^2}$ km/h

Δv = 56 km/h

Step 5.
Find the direction of $\Delta\vec{v}$
$\theta = \tan^{-1}\frac{\Delta v_y}{\Delta v_x}$
$\theta = \tan^{-1}[\frac{(-20)}{52}]$
$\theta = -21^0$

Step 6.
Return from x-,y- coordinates to NSWE:
$\Delta\vec{v} = 36km/h[-21^0]$

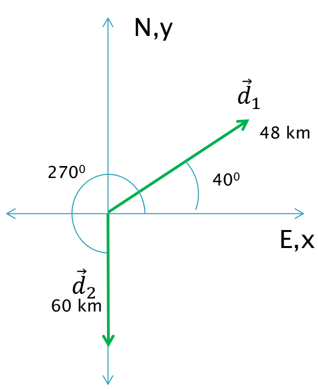
$\underline{\boldsymbol{\Delta\vec{v} = 36km/h[E\ 21^0 S]}}$

Step 7 *(optional).*
Draw $\Delta\vec{v}$

Sample Problem #6

A hot-air balloon rises into the air and drifts with the wind at a rate of 24 km/h [E 40° N] for 2.0 h. The wind shifts, so that balloon changes direction and drifts south at a rate of 40 km/h for 1.5 h before landing. Determine the balloon's displacement for the flight.

Solution (with components)



Step 1.

Convert vectors from NSWE to x-,y- coordinates

$\overrightarrow{\Delta d_1}$=48km[E$40^0$N]

$\overrightarrow{\Delta d_1}$=48km[$40^0$]

$\overrightarrow{\Delta d_2}$=60km[S]

$\overrightarrow{\Delta d_2}$=60km[$270^0$]

Step 2.

Find the x- and y- components

Δd_{1x} = (48km)(cos40^0)

Δd_{1x} = 36.8 km

Δd_{2x} = (60km)(cos270^0)

Δd_{2x} = 0

Δd_{1y} = (48km)(sin40^0)

Δd_{1y} = 30.9 km

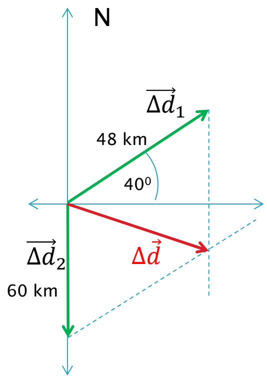
Δd_{2y} = (60km)(sin270^0)

Δd_{2y} = -60 km

Solution (with components)

$\Delta d_{1x} = 36.8$ km
$\Delta d_{2x} = 0$

$\Delta d_{1y} = 30.9$ km
$\Delta d_{2y} = -60$ km



Step 3.
Find the components of
$\Delta\vec{d} = \overrightarrow{\Delta d_2} + \overrightarrow{\Delta d_1}$
$\Delta d_x = 0 + 36.8$ km
$\Delta d_x = +36.8$ km
$\Delta d_y = -60$ km $+ 30.9$ km
$\Delta d_y = -29.1$ km

Step 4.
Find the magnitude of $\Delta\vec{d}$
$\Delta d = \sqrt{(36.8)^2 + (-29.1)^2}$ km

$\Delta d = 47$ km

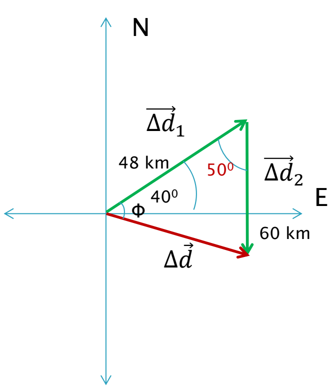
Step 5.
Find the direction of $\Delta\vec{v}$
$\theta = \tan^{-1}\frac{\Delta d_y}{\Delta d_x}$
$\theta = \tan^{-1}[\frac{(-29.1)}{(36.8)}]$
$\theta = -38^0$

$\theta = -38^0$, or E38^0S

Step 6.
Return from x-,y- coordinates to NSWE:
$\boldsymbol{\Delta\vec{d} = 47\ km[E38^0S]}$

Step 7 *(optional).*
Draw $\Delta\vec{d}$

Solution (with trigonometry)



Step 1.

Find the angle Θ between two displacement vectors, linked tip to tail, using their directions
Θ = 50^0

Step 2.

Find the magnitude of the total displacement, Δd, using the cosine law

$\Delta d = \sqrt{48^2 + 60^2 - 2(48)(60)\cos 50^0}$ km

Δd = 46.9 km

Step 3.

Find the angle Φ, at the tail of $\Delta\vec{d}$, using the sine law

$$\frac{\sin \emptyset}{60} = \frac{\sin 50^0}{46.9}$$

Φ = 78^0

Step 4.

Find the direction of $\Delta\vec{d}$

$78^0 - 40^0 = 38^0$

Answer (rounded):
$\Delta\vec{d} = 47$ km [E38^0S]

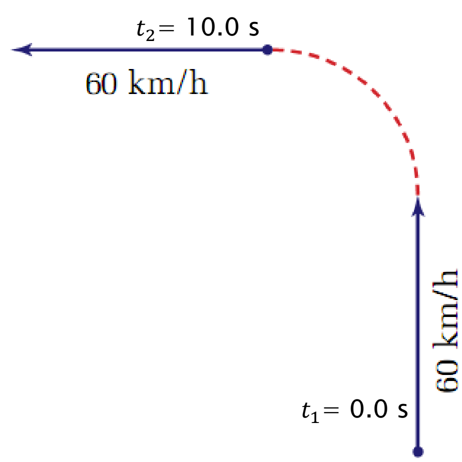
Acceleration in 2D

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} \qquad \text{or} \qquad \vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

Sample Problem #13

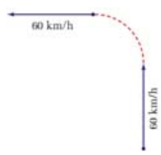
Calculate the acceleration of the jet-ski as described in the following.

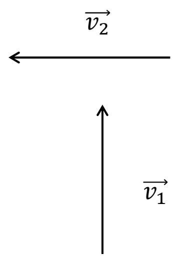
- Is there an acceleration at all?
- If yes, then why?
- If no, then why not?

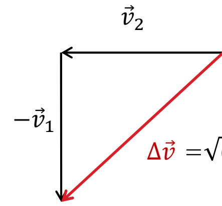


Solution

$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$







$\Delta\vec{v} = \sqrt{60^2 + 60^2}$ km/h [W45^{0}S]

$\Delta\vec{v}$ = 84.9 km/h [W45^{0}S]

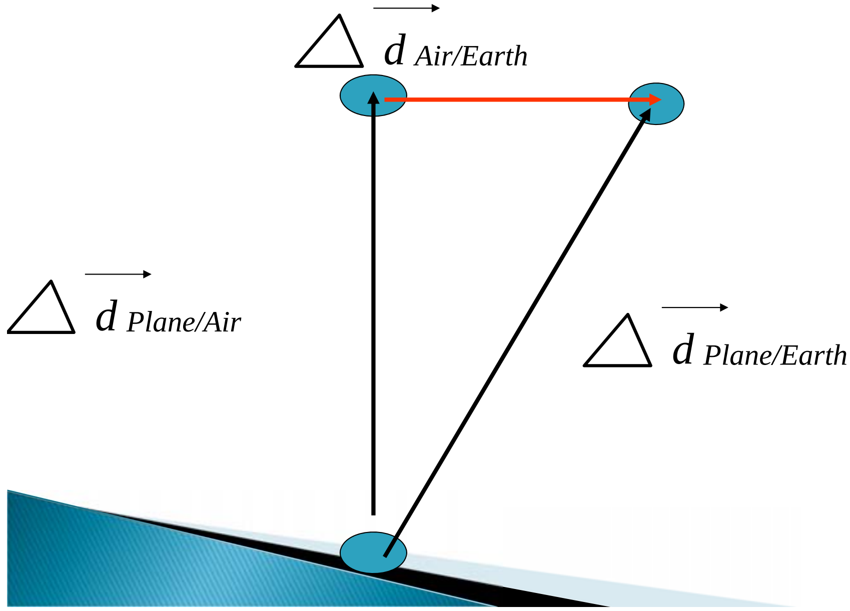
$\Delta\vec{v}$ = 23.6 m/s [W45^{0}S], Δt= 10.0 s

<u>$\vec{a}$ = 2.36 m/s [W45^{0}S]</u>

Relative Velocity

- More than one object is moving in the same frame of reference
- Velocity is always relative (no absolute rest, no absolute motion)

Displacement of the airplane:



Velocity equation

$$\vec{v}_{Plane/Earth} = \vec{v}_{Plane/Air} + \underbrace{\vec{v}_{Air/Earth}}_{\text{wind}}$$

Relative Velocity

Reconciling different frame of reference from different observer's perspectives

Example: Sailing

- Account for current
- Account for wind

Relative Motion

Relative motion notation:

$$\vec{v}_{AB}$$

means: "Velocity of an object A relatively to an object B".

The second subscript B represents also the observer's perspective ("frame of reference")

Example: if an airplane ("P") is travelling at 251 km/h [N] relative to Earth ("E"), its velocity is expressed a $\vec{v}_{PE} = 251\,\text{km/h}$ [N]

Relative Motion

If the airplane flies windy air ("A") we must consider the velocity of the airplane relative to the air v_{PA} and the velocity of the air relative to Earth v_{AE}. The velocity of the airplane relative to Earth is therefore

$$\vec{v}_{PE} = \vec{v}_{PA} + \vec{v}_{AE}$$

Example: If an airplane is flying at a constant velocity of 253 km/h [S] relative to the air and the air velocity is 24 km/h [N], what is the velocity of the airplane relative to Earth?

Relative Motion

In general, the equation for relative motion takes the form:

$$\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC}$$

$\vec{v}_{AC}$ - velocity of *A* in a frame of reference *C*
$\vec{v}_{AB}$ - velocity of *A* in a frame of reference *B*
$\vec{v}_{BC}$ - velocity of frame of reference *B* relative to frame of reference *C*.

Typically:

A – moving object (plane, boat, swimmer, etc.)
B – moving medium (air, water, boat, etc.)
C – stationary frame of reference (ground, shore, etc.)

Relative Velocity Equation

$$\vec{v}_{os} = \vec{v}_{om} + \vec{v}_{ms}$$

Stationary Relative

$$\vec{v}_{om} = \vec{v}_{os} - \vec{v}_{ms}$$

In other words……..

If two objects, A and B, are moving at velocities, $\vec{v}_A$ and $\vec{v}_B$ respectively, the velocity of the object A, relatively to object B is given by:

$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

Relative Motion: Sample Problem #7

A traveller is walking on a moving walkway at 0.80 m/s in the same direction as the walkway moves at 4.1 m/s.

1. What is the traveller's velocity relative to the ground?
2. If he moved in the opposite direction instead, what would his relative velocity relatively to ground be?

Solution:
A – Traveller (moving object)
B – Walkway (medium)
C– Ground (stationary)

$$\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC}$$

1.

$\vec{v}_{AC} = 0.80\frac{m}{s}[fwd] + 4.1\frac{m}{s}[fwd]$

$\boldsymbol{\vec{v}_{AC} = 4.9\frac{m}{s}[fwd]}$

2.

$\vec{v}_{AC} = 0.80\frac{m}{s}[bckwd] + 4.1\frac{m}{s}[fwd]$

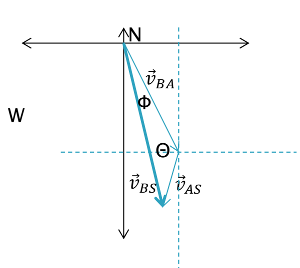
$\vec{v}_{AC} = -0.80\frac{m}{s}[fwd] + 4.1\frac{m}{s}[fwd]$

$\boldsymbol{\vec{v}_{AC} = 3.3\frac{m}{s}[fwd]}$

Relative Motion: Sample Problem #8

A boat is motoring into shore. The boat's velocity relative to the air is 21.0 km/h [S 25° E] and the wind is blowing at 6.5 km/h [S 15° W]. Determine the boat's velocity relative to the shore.

SOLUTION: $\vec{v}_{BS} = \vec{v}_{BA} + \vec{v}_{AS}$, where B –boat, S –shore, A –air (wind)



$\Theta = 65^0 + 75^0$

$\Theta = 140^0$

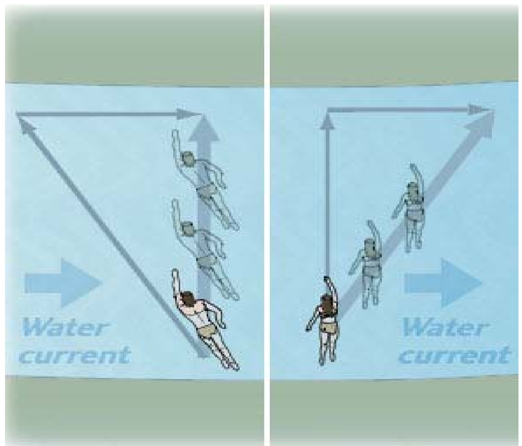
$v_{bs} = \sqrt{21^2 + 6.5^2 - 2(21)(6.5)\cos 140^0}$ km/h

$v_{bs} = 26.3$ km/h

$\frac{\sin \emptyset}{6.5} = \frac{\sin 140^0}{26.3}$, $\Phi = 9.1^0$, $25^0 - 9.1^0 = 15.9^0$

$\underline{\boldsymbol{\vec{v}_{BS}} = 26 \text{ km/h}[S16^0E]}$

Relative velocity in 2 D motion

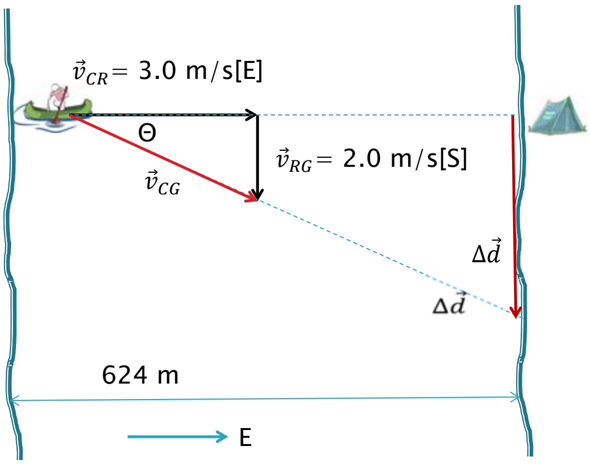


Sample Problem #9

A canoeist is planning to paddle to a campsite directly across a river that is 624 m wide. The velocity of the river is 2.0 m/s [S]. In still water, the canoeist can paddle at a speed of 3.0 m/s. If the canoeist points her canoe straight across the river, toward the east:

- How long will it take her to reach the other river bank?
- Where will she land relative to the campsite?
- What is the velocity of the canoe relative to the river bank?

Solution



1. Time of travel (*x-component*)

$\Delta t = \frac{\Delta d_x}{v_x}$

$\Delta t = \frac{624\ m}{3.0\ m/s}$

$\Delta t = 210\ s$ (*rounded*)

2. Landing spot (*y-component*)

$\Delta\vec{d} = \vec{v}_{RS}\,\Delta t$

$\Delta\vec{d}$ = (2.0 m/s[S])(210s)

$\Delta\vec{d}$ = 420 m[S] of the campsite

3. Ground velocity $\vec{v}_{CG}$, (Method 1.)

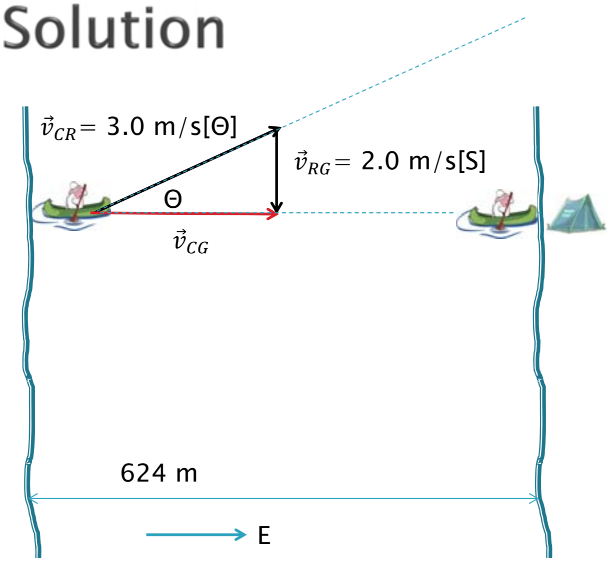
$\vec{v}_{CG}$=3.3 m/s[E37^{0}S]

Sample Problem #10

The canoeist in the previous question want to head her canoe in such a direction that she will actually travel straight across the river to the campsite.

- In what direction must she point the canoe?
- Find the magnitude of her velocity relative to the shore.
- How long will it take the canoeist to paddle to the campsite?

Solution



1. Heading (*from the velocities triangle*)

   $\Theta = \sin^{-1}(\frac{2.0}{3.0})$

   $\Theta = 42^0$ (*from the normal, upstream)*

2. Crossing speed, v_{CG}

$v_{CG} = \sqrt{3.0^2 - 2.0^2}\ m/s$

$\boldsymbol{v_{CG} = 2.2\ m/s}$

3. Time of travel (*x-component*)

$\Delta t = \frac{\Delta d_x}{v_x}$

$\Delta t = \frac{624\ m}{2.2\ m/s}$

$\Delta \boldsymbol{t} = \mathbf{280\ s}$ (*rounded, 2 s.f.*)

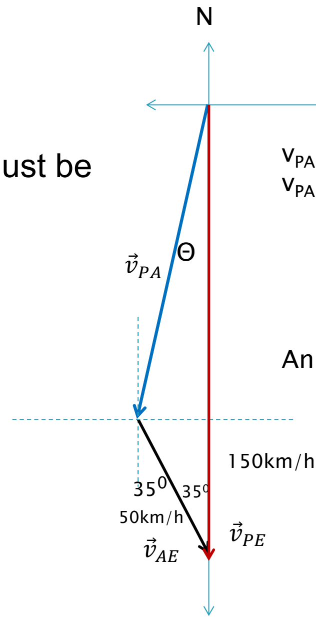
Sample Problem #12

You are the pilot of a small plane and want to reach an airport, 600 km due south, in 4.0 h. A wind is blowing at 50 km/h [S 35° E]. With what heading and airspeed should you fly to reach the airport on time?

Solution

- The ground speed, v_{PE}, must be
  - $v_{PE} = \frac{600\ km}{4.0\ h}$
  - v_{PE} =150 km/h
  - $\vec{v}_{PE}$= 150 km/h[S]

$$\vec{v}_{PE} = \vec{v}_{PA} + \vec{v}_{AE}$$



$v_{PA}=\sqrt{150^2+50^2-2(150)(50)\cos 35^0}$ km/h
v_{PA}=113 km/h

$$\frac{\sin\Theta}{50} = \frac{\sin 35^0}{113}$$

$\Theta = 15^0$

Answer:

Airspeed – 110 km/h (rounded, 2 s.f.)
Heading – 15^0 West of South

Projectile Motion

Recall that a free-falling object:

- Only force acting on a projectile is gravity
- Therefore, only acceleration is towards the ground
- In Physics 11, we ignore air resistance most of the time

Projectile Motion

- Equations are valid for constant acceleration, which is $\vec{g}$
- Valid for 1D problems: the plus/minus signs tell us the direction
- Equations used separately for x-direction and y-direction with common time

$$\Delta d = v_1 \Delta t + \frac{1}{2} a \Delta t^2$$

$$v_2 = v_1 + a\Delta t$$

Projectile Motion

- Break down the problem into its horizontal (*x*) and vertical (*y*) directions, and apply our kinematic equations independently
- No acceleration in the *x* direction, i.e. a_x=0. Kinematic equations in *x* direction reduces to:

$$\Delta x = v_x \Delta t$$

- The only acceleration is in the y direction:

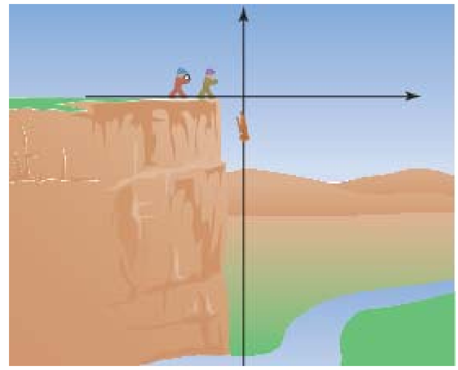
$$a_y = 9.81 \text{ m/s}^2 \text{ [down]}$$

  We *usually* define the (+) direction to be [up], so $a_y = -9.81$ m/s^2, but it can change depending on the problem
- The variable that connects the two directions is the time interval Δt

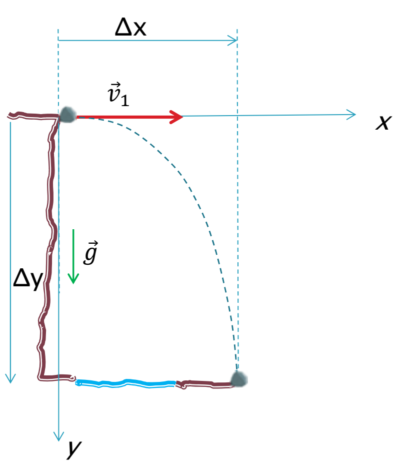
Sample Problem #14

While hiking in the wilderness, you come to a cliff overlooking a river. A topographical map shows that the cliff is 291 m high and the river is 68.5 m wide at that point. You throw a rock directly forward from the top of the cliff, giving the rock a horizontal velocity of 12.8 m/s.

- Does the rock make it across the river?
- With what velocity did the rock hit the ground or water?



Solution



- Down is positive
- $\Delta y = +291$m
- $v_{1x} = 12.8$ s
- $v_{1y} = 0$ s
- $a_x = 0$
- $a_y = g = +9.81$ m/s^2

Step 1.
Find the time of falling down using y-components:

$\Delta y = v_{1y}\,\Delta t + \frac{1}{2}a_y\,\Delta t^2$

$\Delta y = \frac{1}{2}g\Delta t^2$

$$\Delta t = \sqrt{\frac{2\Delta d_y}{g}}$$

$\Delta t = 7.70$ s

Step 2.
Find the horizontal range using x-components:

$\Delta x = v_{1x}\,\Delta t + \frac{1}{2}a_x\,\Delta t^2$

$\Delta x = v_{1x}\,\Delta t$

$\Delta t = 7.70$ s, $v_{1x} = 12.8$ s

$\Delta d_x = 98.6$ m (more than the width of the river)

Solution

Step 3.
Find the components of the final velocity (after 7.70 s):

$v_{2x} = v_{1x} + \cancel{a_x \Delta t}$
$v_{2x} = 12.8$ m/s

$v_{2y} = v_{1y} + a_y \Delta t$
$v_{2y} = g\Delta t = 75.5$ m/s

Step 4.
Find the final velocity (re-assemble the components into the vector)

$$v_2 = \sqrt{(12.8)^2 + (75.5)^2}\, m/s$$

$$v_2 = 76.6\ \frac{m}{s}$$

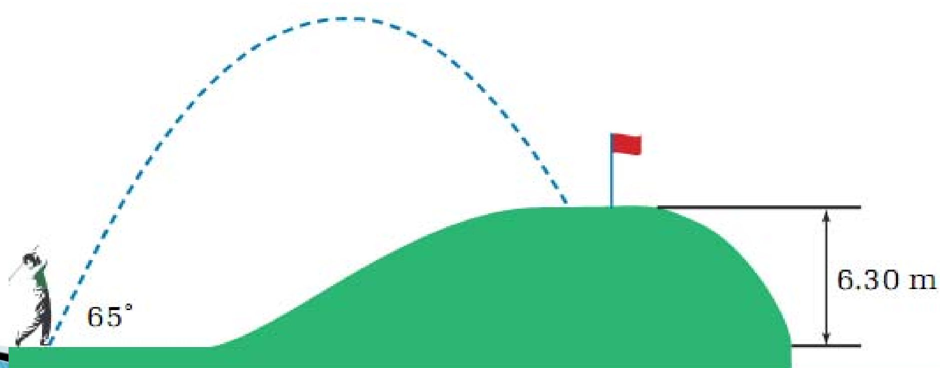
$$\theta = \tan^{-1}\frac{75.5}{12.8}$$

$$\theta = 80.4^0$$

$\vec{v_2}$=76.6m/s[80.4^0 below horizontal]

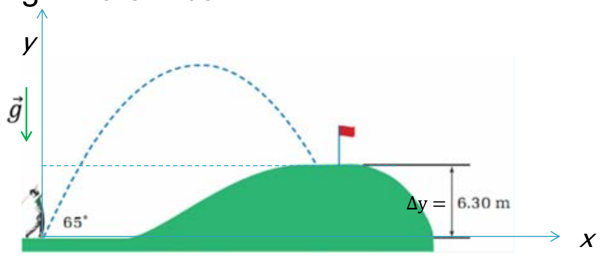
Sample Problem #15

A golfer hits the golf ball off the tee, giving it an initial velocity of 32.6 m/s at an angle of 65° with the horizontal. The green where the golf ball lands is 6.30 m higher than the tee, as shown in the illustration. Find the time interval when the golf ball was in the air.



Solution

- *Up is positive*
- Δd_y= +6.30 m
- v_1 =32.6 m/s
- $\Theta = 65^0$
- $a_y = -g = -9.81 m/s^2$



Step 1.
Find the y-component of the initial velocity:

$v_{1y} = v_1 \sin\Theta$

$v_{1y} = (32.6)(\sin 65^0) m/s$

$v_{1y} = 29.5\ m/s$

Step 2.
Write displacement equation in y-direction

$\Delta y = v_{1y}\Delta t + \frac{1}{2}a_y \Delta t^2$

Plug into it the given data:

$6.3 = (29.5)\Delta t - (0.5)(9.81)\Delta t^2$

Solve the quad. eq. for Δt:

$4.905\Delta t^2 - 29.5\Delta t + 6.3 = 0$

Both solutions are positive:

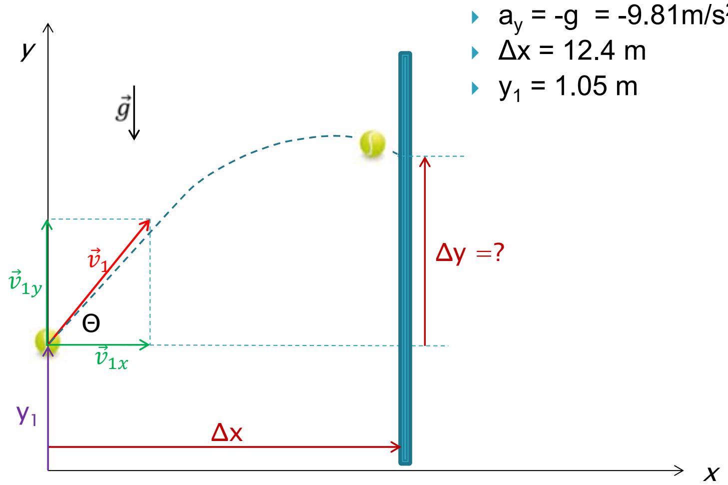
$\Delta t = 0.222\ s$, (moving up) $\Delta t = 5.79\ s$ (moving down)

$\underline{\Delta t = 5.79\ s}$

Sample Problem #16

You are playing tennis with a friend on tennis courts that are surrounded by a 4.8 m - tall fence. You opponent hits the ball over the fence and you offer to retrieve it. You find the ball at a distance of 12.4 m on the other side of the fence. You throw the ball at an angle of 55.0° with the horizontal, giving it an initial velocity of 12.1 m/s. The ball is 1.05 m above the ground when you release it. Did the ball go over the fence, hit the fence, or hit the ground before it reached the fence?

Solution



- *Up is positive*
- v_1 =12.1 m/s
- Θ =55^0
- a_y = -g = -9.81m/s^2
- Δx = 12.4 m
- y_1 = 1.05 m

Step 1.
Find the component of the initial velocity:

$v_{1y} = v_1 \sin\Theta^0$
$v_{1y} = (12.1)\,(\sin 55^0)\,m/s$
$v_{1y} = 9.91\ m/s$
$v_{1x} = v_1 \cos\Theta^0$
$v_{1x} = (12.1)\,(\cos 55^0)\,m/s$
$v_{1x} = 6.94\ m/s$

Step 2.
Find the time of travel of the ball, from you to the fence, using x-direction:

$\Delta x = v_{1x}\,\Delta t$

$\Delta t = \frac{12.4\ m}{6.94\ m/s}$

$\Delta t = 1.79\ s$

Solution

Step 3.
Find the vertical displacement after **Δt** = 1.79 s

$\Delta y = v_{1y}\,\Delta t + \frac{1}{2} a_y\,\Delta t^2$

$\Delta y = (9.91)(1.79) - \frac{1}{2}(9.81)\,(1.79)^2$ *(in metres)*

$\Delta y = +2.02\ m$

Step 4.
Find the vertical position of the ball after **Δt** = 1.79 s

$y_2 = y_1 + \Delta d_y$

$y_2 = 1.05\ m + 2.02\ m$

$y_2 = 3.07\ m$.

Answer: The ball hit the 4.8-m tall fence

Unit 2 Homework Answers

- 2. (A) 34 m; (b) 67 m; (c) 9.4 m/s^2 [S]
- 3. 4.0 s
- 4. 1.8 m/s [downstream]
- 5. 43 m [S36^{0}W]
- 6. 1.6 m/s [E18^{0}S]
- 7. 30 km[E] and 52 km [N]
- 8. 13 km [N23^{0}E]
- 9. 314 km/h
- 10. (a) W43^{0}S; (b) 494 s
- 11. (a) 1.25 m; (b) 0.50 s; (c) 9.81 m/s^2; (d) 25.5 m
- 12. 0.9 m above Kevin's reach; yes, the ball will be 2.7 m above
- the ground
- 13. (a) 014 m [W45^{0}S]; (b)5.0 m [53^0 to the right from the initial direction]; (c) 483 km [S12^{0}E] ; (d) 2070 km [N18^{0}W]
- 14. c
- 15. c
- 16.d
- 17. d
- 18. a
- 19. b
- 20. d
- 21. c
- 22. b
- 23. d

- <u>Disclaimer</u>: *I do not guarantee that all of above answers are correct, they are for a reference purpose only. Please, let me know if you have found any errors here. Good luck with your homework.*