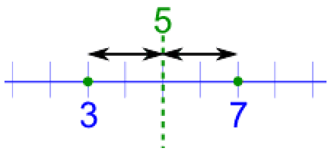
Statistic and Probability

1. Finding a Central Value
When you have two or more numbers it is nice to find a value for the "center".

2 Numbers
With just 2 numbers the answer is easy: go half-way in-between.

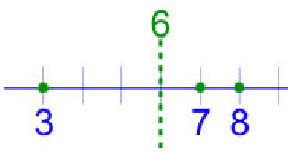
Example: what is the central value for 3 and 7?
Answer: Half-way in-between, which is 5.



You can calculate it by adding 3 and 7 and then dividing the result by 2:
(3+7) / 2 = 10/2 = 5

3 or More Numbers
You can use the same idea when you have 3 or more numbers:

Example: what is the central value of 3, 7 and 8?
Answer: You calculate it by adding 3, 7 and 8 and then dividing the results by 3 (because there are 3 numbers):
(3+7+8) / 3 = 18/3 = 6



Notice that we divided by 3 because we added 3 numbers ... very important!

2. Mean
The mean is just the **average** of the numbers.

It is easy to calculate: Just **add up** all the numbers, then **divide by how many** numbers there are.
In other words it is the ***sum*** divided by the ***count***.

But sometimes the Mean can let you down:

Example: Birthday Activities
Uncle Bob wants to know the average age at the party, to choose an activity.
There will be 6 kids aged 13 and also 5 babies aged 1

(13+13+13+13+13+13+1+1+1+1+1) / 11 = **7.5...**
(We divided by 11 because there were 11 numbers)


The mean age is about **7½**, so he gets a **Jumping Castle**!
The 13 year olds are embarrassed, and the 1-year olds can't jump.

The Mean was accurate, but in this case it was not useful.

3. Median
The Median is the ***"middle number"*** (in a sorted list of numbers).
Half the numbers in the list will be less, and half the numbers will be greater.

How to Find the Median Value
To find the Median, place the numbers you are given in **value order** and find the **middle number**.

Example: What is the Median of 3, 4, 7, 9, 12, 15
There are two numbers in the middle:
3, 4, 7, 9, 12, 15
So we average them:
(7+9) / 2 = 16/2 = 8
The Median is **8**

Two Numbers in the Middle
BUT, if there are an **even amount of numbers** things are slightly different.

In that case we need to find the **middle pair** of numbers, and then find the value that would be half way between them. This is easily done by adding them together and dividing by two.

For example:
3, 13, 7, 5, 21, 23, 23, 40, 23, 14, 12, 56, 23, 29

If we put those numbers in order we have:
3, 5, 7, 12, 13, 14, 21, 23, 23, 23, 23, 29, 40, 56

There are now **fourteen** numbers and so we don't have just one middle number, we have a **pair of middle numbers**:
3, 5, 7, 12, 13, 14, **21, 23**, 23, 23, 23, 29, 40, 56
In this example the middle numbers are **21 and 23**.

To find the value half-way between them, add them together and divide by 2:
21 + 23 = 44
44 ÷ 2 = 22
And, so, the **Median** in this example is **22**.

(Note that 22 was not in the list of numbers ... but it is OK, because "half the numbers in the list are less, and half the numbers are greater.")

4. Mode
The mode is simply the number which appears **most often**.

To find the mode, or modal value, first put the numbers **in order**.
Look at these numbers:
3, 7, 5, 13, 20, 23, 39, 23, 40, 23, 14, 12, 56, 23, 29

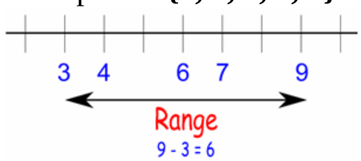
In order these numbers are:
3, 5, 7, 12, 13, 14, 20, 23, 23, 23, 23, 29, 39, 40, 56

This makes it easy to see which numbers appear the most.
In this case the mode is **23**.

Example: What is the Mode of 3, 4, 4, 5, 6, 6, 7
Well ... 4 occurs twice but 6 **also** occurs twice.
So **both 4 and 6** are modes.
When there are two modes it is called "bimodal", when there are three or more modes we call it "multimodal".

5. Range (Statistics)
The Range is the difference between the lowest and highest values.

Example: In **{4, 6, 9, 3, 7}** the lowest value is 3, and the highest is 9. So the range is 9-3 = **6**.



The Range Can Be Misleading
The range can sometimes be misleading when there are extremely high or low values.
Example: In **{8, 11, 5, 9, 7, 6, 3616}**: the lowest value is 5, and the highest is 3616,
So the range is 3616-5 = **3611**.

The single value of 3616 makes the range large, but most values are around 10.

6. Probability

Probability is defined as the measure of the *"likelihood"* of an event, or *outcome*.
Another way of saying that is, "What is the *chance* of something happening?"

Probability is all about the *certainty*, the *uncertainty*, and the *prediction* of something happening.
Some events are *impossible*, other events are *certain to occur*, while many are *possible, but not certain* to occur.

Tossing a Coin
When a coin is tossed, there are two possible outcomes:
heads (H) and tails (T).
We say that the probability of the coin landing **H** is 1/2.
Similarly, the probability of the coin landing **T** is 1/2.

Throwing Dice
When a single die is thrown, there are six possible outcomes: 1, 2, 3, 4, 5, 6.
The probability of throwing any one of these numbers is 1/6.

The probability of an event is the chance that it will occur, expressed as a ratio of a specific event to all possible events.

$$probability = \frac{number\ of\ actual\ events}{number\ of\ possible\ events} \quad \text{Or....} \quad probability = \frac{number\ of\ favorable\ outcomes}{number\ of\ possible\ outcomes}$$

The probability of an event that is *certain* to occur is 1.
The probability of an event that is *impossible* to occur is 0.
The probability of an event that is *possible* is > 0 and <1

Before we go any further let's look at a few simple examples of these rules...

For this experiment we have a bag containing 10 cards. Each card is numbered differently from 1 - 10. In each trial we will reach into the bag and pull out exactly 1 card, look at it, and then return it to the bag.

1) What is the probability of choosing a "7"?
Let's see, there is only 1 *favorable outcome*, because there is only 1 - "7" in the bag. There are a total of *10 possible outcomes* because we could have selected any of the 10 cards.
Therefore the probability of choosing the "7" is:

$$P(7) = \frac{1}{10}$$

2) What is the probability of choosing an *<u>even</u>* numbered card?
In the bag there are 5 *even* numbered cards: 2, 4, 6, 8 and 10.
That means there are *5 favorable outcomes* out of the *10 possible outcomes.*
Therefore the probability of choosing an even card is:

$$P(Even) = \frac{5}{10} = \frac{1}{2}$$

3) What is the probability of choosing a card numbered 1 -10?
I think the chances must be pretty good...don't you?
After all, there are 10 cards in the bag, and each card is numbered differently from 1 – 10.

Therefore the probability of choosing a card numbered 1-10:

$$P(10) = \frac{10}{10} = 1$$

4) What is the probability of choosing a "12"?

In the bag there are *no* cards with the number "12" on it. Therefore the number of favorable outcomes is 0.

So, the probability of choosing a "12" is"

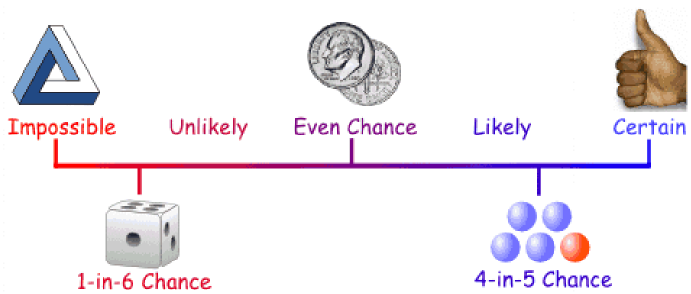
$$P(12) = \frac{0}{10} = 0$$

Before we go to the practice page for this lesson, there is one more very important fact to know about probability.

The sum of the probability of an event happening and the probability of an event not happening is always 1!

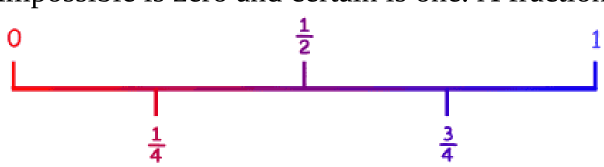
7. Probability Line

Probability is the chance that something will happen. It can be shown on a line.



We can say that the probability of an event occurring will be somewhere between impossible and certain.

As well as words we can use fractions or decimals to show the probability of something happening. Impossible is zero and certain is one. A fraction probability line is shown below.



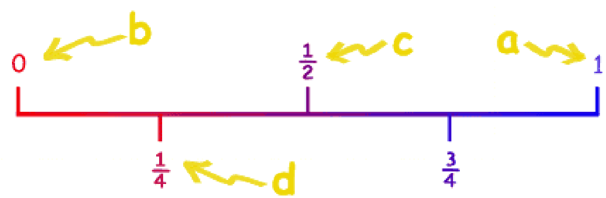
We can show on our line the chance that something will happen:

a) The sun will rise tomorrow.

b) I will not have to learn math at school.

c) If I flip a coin it will land heads up.

d) If you have a choice of red, yellow, blue or green you will choose red.



Remember the probability of an event will not be more than 1.
This is because 1 is certain that something will happen.

And the probability of an event will not be less than 0.
This is because 0 is impossible (sure that something will not happen).

8. The Probability of Compound Events:
To find the probability of two independent* events "E" and "F", simply multiply the probabilities of the two independent events:

P(E and F) = P(E) × P(F)

Example 1: On Saturday night Lisa can either go to the movies, go out for pizza, or go to the dance. She can do this with any one of these four friends; Mary, Sue, Tina or Julie.
Find the probability that Lisa went out for pizza with either Tina or Sue.

Solution:
P(pizza) = 1/3, P(Tina or Sue) = 2/4 (or 1/2)
Therefore: P(pizza with Tina or Sue) = 1/3 ×1/2 = 1/6

Example 2: A school survey found that 9 out of 10 students like pizza. If three students are chosen at random with replacement, what is the probability that all three students like pizza?

Solution:

$$\text{P(student 1 likes pizza)} = \frac{9}{10}$$

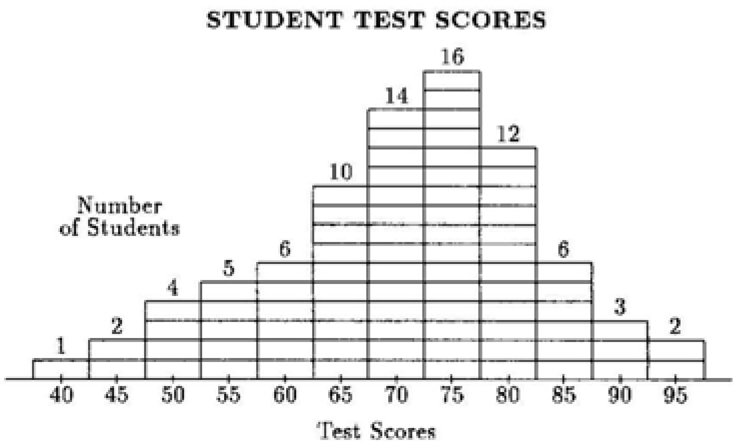
$$\text{P(student 2 likes pizza)} = \frac{9}{10}$$

$$\text{P(student 3 likes pizza)} = \frac{9}{10}$$

$$\text{P(student 1 and student 2 and student 3 like pizza)} = \frac{9}{10} \cdot \frac{9}{10} \cdot \frac{9}{10} = \frac{729}{1000}$$

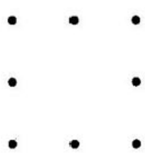
Questions in class

1. Consider this histogram of the scores for 81 students taking a test:

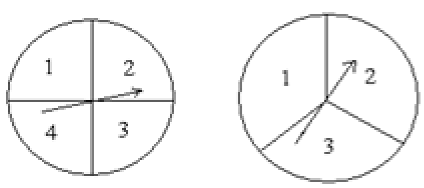


The median is in the interval labeled
(A) 60   (B) 65   (C) 70   (D) 75   (E) 80

2. Eight points are spaced at intervals of one unit around a 2×2 square, as shown. Two of the 8 points are chosen at random. What is the probability that the points are one unit apart?



3. Spinners A and B are spun, in each spinner, the arrow is equally likely to land on each number. What is the probability that the product of the two spinners' numbers is even?



4. A fifth number, n, is added to the set of numbers {3, 6, 9, 10} to make the mean of the set of five numbers equal to its median. What is the number of possible values for n?

5. At a party there are only single women and married men with their wives. The probability that a randomly selected woman is single is 2/5. What fraction of the people in the room are marries men?

6. Harold tosses a nickel four times. What is the probability that he gets at least as many heads as tails?

7. Keiko tosses one penny and Ephraim tosses two pennies. What is the probability that Ephraim gets the same number of heads that Keiko gets?

8. Diana and Apollo each roll a standard die obtaining a number at random from 1 to 6. What is the probability that Diana's number is larger than Apollo's number?

9. If two dice are tossed, the probability that the product of the numbers showing on the tops of the dice is greater than 10 is

(A) $\frac{3}{7}$ (B) $\frac{17}{36}$ (C) $\frac{1}{2}$ (D) $\frac{5}{8}$ (E) $\frac{11}{12}$

10. Every time these two wheels are spun, two numbers are selected by the pointers. What is the probability that the sum of the two selected numbers is even?