First Name:____________________ Last Name: __________________ Student ID: ________

An introduction to calculus (2)

1. Find the following limits.

a. $\lim_{x\to 5}(2x+3)$

b. $\lim_{x\to 2}(-x^2+3x-2)$

c. $\lim_{x\to 1}\frac{x-1}{x+2}$

d. $\lim_{x\to -1}(x^3 + x^2+x+1)$

e. $\lim_{x\to 1}\frac{1-x^3}{x^2-1}$

f. $\lim_{x\to 1}\frac{\sqrt{x}-2}{x-4}$

g. $\lim_{x\to 1}\frac{x^3-x^2-x+1}{x^3-2x^2+x}$

h. $\lim_{x\to \frac{1}{2}}\frac{3-12x^2}{2x^2+x-1}$

i. $\lim_{x\to 3}\frac{x^3-9x^2+27x-27}{x^3-27}$

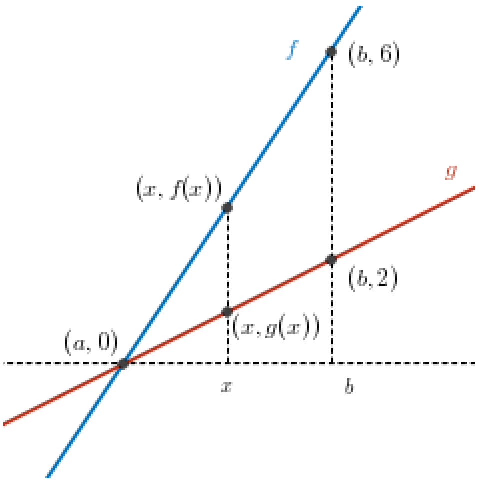
j. $\lim_{x\to -1}\frac{\sqrt{x+9}-2\sqrt{2}}{x+1}$

k. $\lim_{x\to 0}\frac{\sqrt{9-x}-\sqrt{9+x}}{x}$

l. $\lim_{x\to 1}\frac{\sqrt{x}-1}{\sqrt[3]{x}-1}$

2. If $\lim_{x\to 1} f(x) = -2$ and $\lim_{x\to 1} g(x) = 3$, then what is the value of $\lim_{x\to 1} \frac{[f(x)]^3+[g(x)]^2}{5-2g(x)}$?

3. Let a<b be real numbers. Consider two linear functions as shown in the graph.



Evaluate $\lim_{x\to a} \frac{f(x)}{g(x)}$.

4. Consider the piecewise function f(x) defined below, where A is a constant.

$$f(x) = \begin{cases} A^2x - 3A & if\ x \geq 1 \\ -2 & if\ x < 1 \end{cases}$$

Determine all values of A so that $\lim_{x\to 1} f(x)$ exist.

5. The greatest integer function(or the step/floor function) is defined as f(x)=[x]=n, where n is an integer such that

$$n \le x < n+1 .$$

a. Sketch the graph of $f(x)=[x]$.

b. For what values of p do the following one sided limits exist?

i. $\lim_{x \to p^-} f(x)$

ii. $\lim_{x \to p^+} f(x)$

c. For what values of p do the the right and left hand limits exist, but $\lim_{x \to p^-} f(x) \neq \lim_{x \to p^+} f(x)$?

d. For what values of p does $\lim_{x \to p} f(x)$ exist?

6. Analyse the continuity of the functions:

a. $f(x) = \begin{cases} \sin(x) - 1, & x < -\pi \\ x^2 - \pi^2, & -\pi \le x \le 0 \\ -\pi^2 + x, & x > 0 \end{cases}$

b. $f(x) = \begin{cases} \frac{|x|}{x} & if\ x \neq 0 \\ a & if\ x = 0 \end{cases}$ where a is a real number.

7. Give an example of functions *f*(*x*) and *g*(*x*) such that $\lim_{x\to 1}(f(x)+g(x))$ exists but $\lim_{x\to 1} f(x)$ and $\lim_{x\to 1} g(x)$ do not exist.

8. Give an example of function *f*(*x*) such that $\lim_{x\to 3} f^2(\mathrm{x})$ exists but $\lim_{x\to 3} f(x)$ do not exist.

9. For each of the following, sketch the graph of a function *f*(*x*) that satisfies the given description.

a. *f* is continuous for all $x\neq 2$, and has a removable discontinuity at $x=2$.

b. The domain of *f* is $\{x|0\leq x\leq 1, x\in \mathrm{R}\}$, *f* is continuous from the right at $x=0$, continuous on $0<x<1$, and has an infinite discontinuity at $x=1$.

10. Find the value of *a* for which the following limit statement is true:

$$\lim_{x\to 1}\frac{\sqrt{a-x}-\sqrt{8+x}}{x-1} = -2$$

11. Evaluate the following limits, using an answer of $+\infty$ or $-\infty$ whenever appropriate.

a. $\lim\limits_{x\to\infty} \frac{-14x+37}{7x-3}$

b. $\lim\limits_{x\to-\infty} \frac{100-3x^2+7x^5-6x^7}{2x^7-1}$

c. $\lim\limits_{x\to-\infty} \left(x - \frac{x^2-4x+1}{x-3}\right)$

d. $\lim\limits_{x\to-\infty} \frac{\sqrt{x^2+2}}{x+1}$