6. Wave Transferring Energy

Grade 11 Physics
Olympiads School

Course Overview

1. Introduction & 1D Kinematics
2. Motion in a Plane (two-dimensional kinematics)
3. Newton's Laws of motion
4. Work and mechanical energy
5. Heat and energy transformation
6. **Energy transfer through vibrations and waves**
7. Wave model of sound
8. Electricity and magnetism

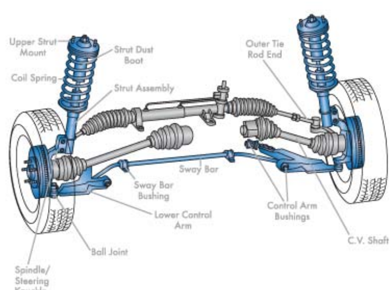
Energy

- So far we have looked at two different forms of mechanical energy:
  - Kinetic energy
    $$E_k = \frac{1}{2}mv^2$$
  - Gravitational potential energy
    $$E_g = mgh$$
- Work can convert kinetic energy E_k of an object into E_g …or something else
- Work can also convert E_g of an object to E_k …or something else
- Heat is one possibility for that "something else"
- Are there other forms of energy?

Vibrations

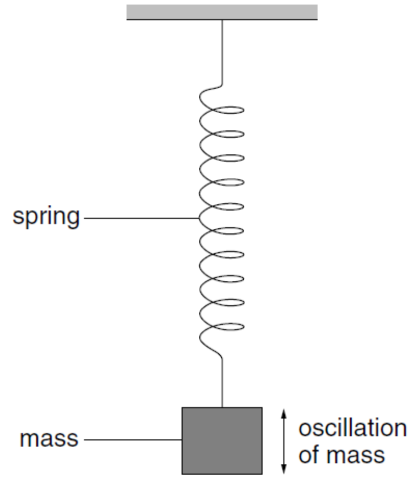
- The suspension in your car
- Bobble-head doll
- Grandfather clock
- Mass on a spring...

(The last one isn't an "every day example",
Instead, it's how physicists model vibrations)



Vibrations

- With a mass on an "ideal spring", if I stretch the spring then let go, the mass will oscillate back and forth forever
  - Real springs aren't ideal, so we have to consider a damper as well



Keywords

Periodic Motion
- Object moves in a repeated pattern over regular time intervals.

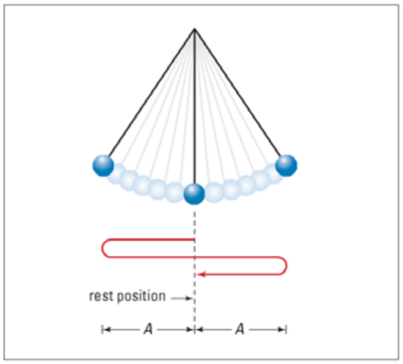
Cycle
- One complete repeat of the pattern

Period (T)
- The time required to complete on cycle.

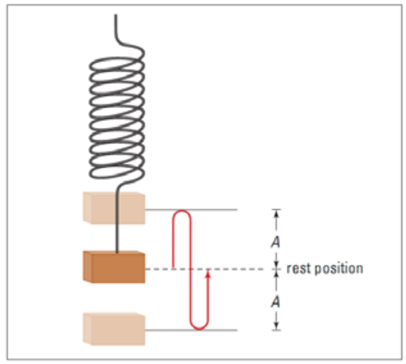
Amplitude (A)
- The distance from the rest position to the maximum displacement when in motion.

Amplitude

Two examples of amplitudes:



Simple Pendulum



Simple spring-mass system

Period and Frequency

Period and **frequency** are reciprocal of each other:

$$T = \frac{\Delta t}{N} \qquad f = \frac{N}{\Delta t} \qquad f = \frac{1}{T}$$

Quantity	Symbol	SI Unit
Period	T	s (seconds)
Frequency	f	Hz (hertz)
time interval	Δt	s (seconds)
Number of cycles	N	pure number, no units

Sample Problem #1

A mass suspended from the end of a spring vibrates up and down 24 times in 36 s. What are the frequency and period of the vibration?

$n = 24$

$\Delta t = 36s$

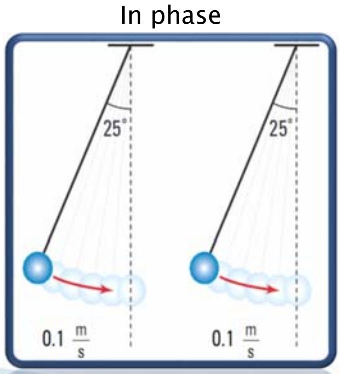
$$f = \frac{n}{\Delta t} = \frac{24}{36\ s} = 0.67\ s^{-1} = 0.67\ Hz$$

$$T = \frac{\Delta t}{n} = \frac{36\ s}{24} = 1.5\ s$$

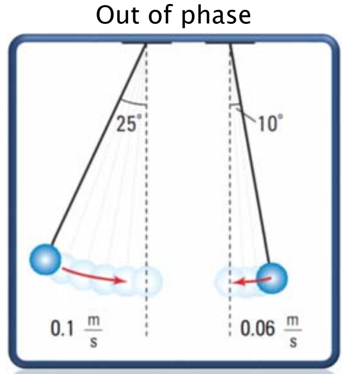
Phase Difference

- Two vibrations are **in phase** if they always move in the same direction
- They are **out of phase** if they are not in phase (even if they're slight out)
- Example: both pendulums have the same amplitude and period:

In phase



Out of phase



Natural Frequency and Resonance

Natural Frequency

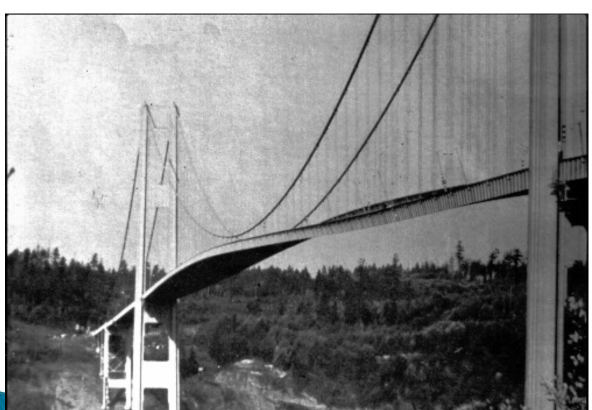
- When an object is allowed to vibrate freely, it will vibrate in its natural frequency
- Examples: mass on a spring, pendulum

Resonance

- Caused by in-phase excitation at the natural frequency
- Amplitude of the vibration becomes very large very quickly
- Think: how to you push a child on a swing?

Example: Tacoma Narrows Bridge

The bridge opened on July 1, 1940, and collapsed on November 7 in the same year.

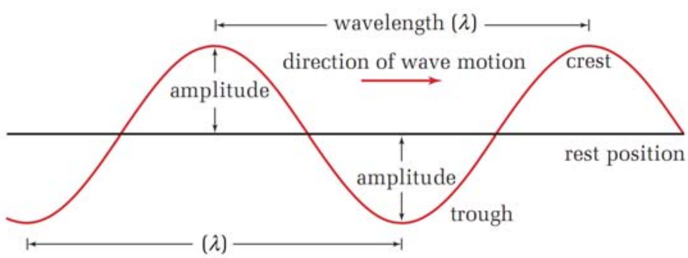


What is a Wave?

A mechanical wave…

- Transports energy through a medium
  - Does not transport matter
- The particles in the medium is excited by the vibration of neighbouring particles
  - The medium has a net displacement of zero
  - The vibration get transferred to the next particle

Describing a Wave



- **Crest:** highest point
- **Trough:** lowest point
- **Wavelength:** shortest distance between two points in the medium that are in phase.

(The easiest way to measure wavelength is from crest to crest, or from trough to trough.)

Frequency and Speed of A Wave

Frequency of a wave

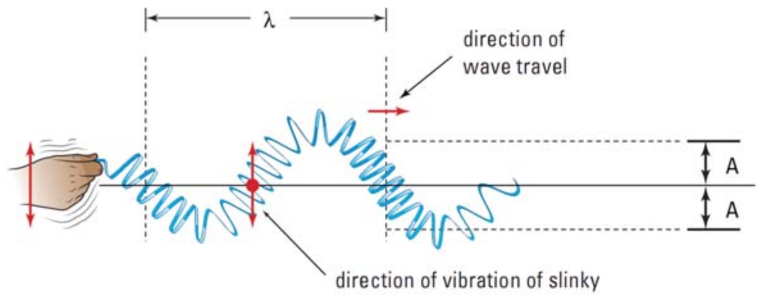
- The number of complete wavelengths that pass a point in a given amount of time.
- Unit: hertz (Hz)
- Same as the frequency of the source vibration that generated the wave
- **Does not depend on the medium**

Speed of a wave

- The speed at which the wave fronts are moving
- **Depends only on the medium**

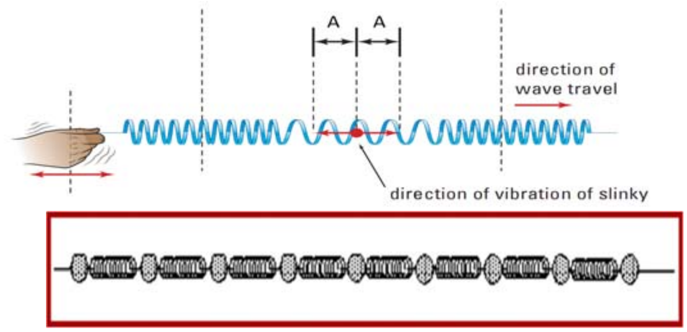
Transverse Wave

- The particles of a medium vibrate at right angles to the direction of the motion
- Creating a wave by waving the slinky up and down
- Examples: ocean waves, wave on a rope, electromagnetic waves (light, x-ray, infrared, ultraviolet, microwaves, radio waves)

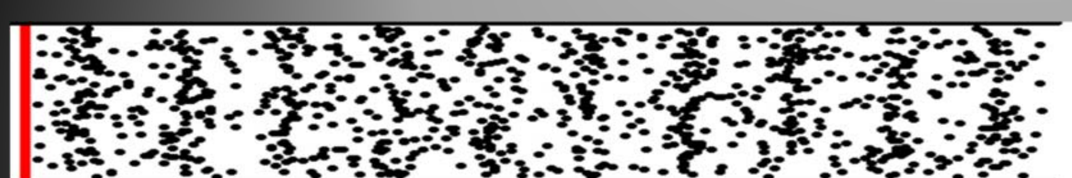


Longitudinal Wave

- The particles of a medium vibrate parallel to the direction of the motion of the wave
- Creating a wave by pushing/pulling the slinky back and forth
- Examples: sound waves

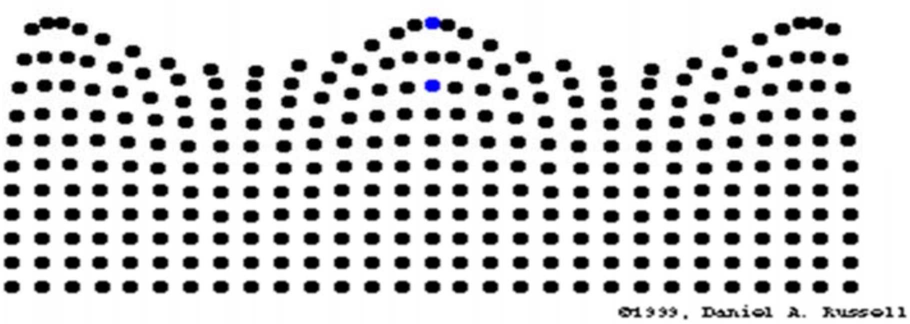


Longitudinal Waves



Longitudal wave is a set of compressions and rarefractions

What are water waves?



They are a circular waves - a superposition of longitudinal and transversal waves

Universal Wave Equation

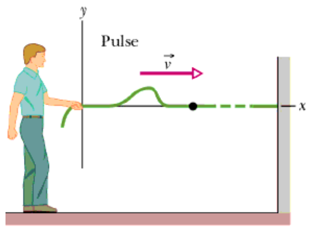
The speed of a wave is the product of the wavelength and the frequency:

$$v = f\lambda$$

Quantity	Symbol	SI Unit
Wave speed	v	m/s (metres per second)
Frequency	f	Hz (hertz)
Wavelength	λ	m (metres)

Sample Problem #2

A physics student vibrates the end of a spring at 2.8 Hz. This produces a wave with a wavelength of 0.36 m. Calculate the speed of the wave.



$v = f\lambda$

$v = (2.8\ Hz)(0.36\ m)$

$v = (2.8\ s - 1)(0.36\ m)$

$\boldsymbol{v = 1.0\ m/s}$

Sample Problem #3

Water waves with wavelength 2.8 m, produced in a wave pool, travel with a speed of 3.80 m/s. What is the frequency of the straight vibrator that produced them?

$v = f\lambda$

$f = \frac{v}{\lambda}$

$f = \frac{3.80\ m/s}{2.8\ m}$

$f = 1.4\ s^{-1}$

$f = 1.4\ Hz$

Wave on a String

The speed of a travelling wave on a string is given by:

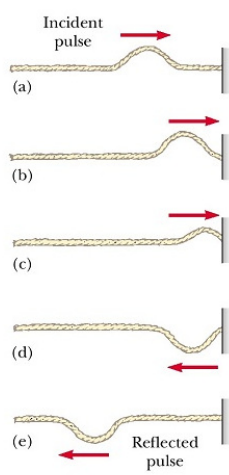
$$v = \sqrt{\frac{F_T}{\mu}} \quad \text{where} \quad \mu = \frac{m}{L}$$

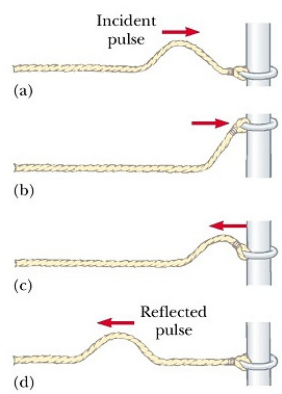
Quantity	Symbol	SI Unit
Wave speed	v	m/s (metres per second)
Tension force in the string	F_T	N (newtons)
Linear mass density	μ	kg/m (kilograms per metre)
Mass of the string	m	kg (kilograms)
Length of the string	L	m (metres)

Reflection of a Wave at a Boundary

When a wave on a string reflects at a boundary, how the reflected wave looks depends on the type of boundary

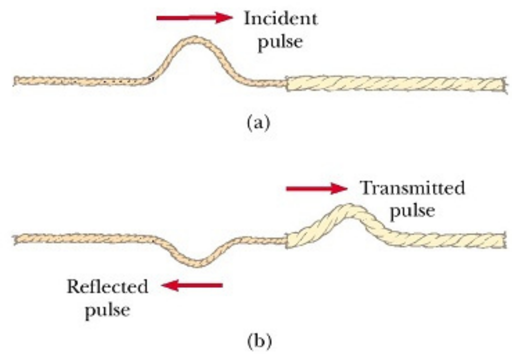
- At a *fixed end* (left), the reflected wave is *inverted*, i.e. a crest becomes a trough
- At a *free end* (right), the reflected wave is *upright*





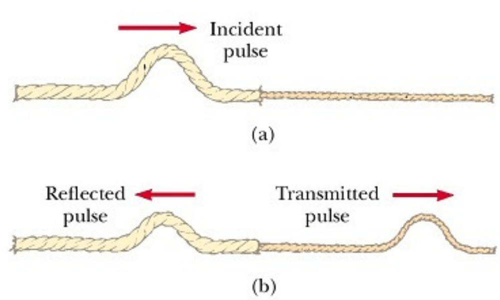
Wave Transmission: Fast to Slow Medium

- Reflected wave:
  - Inverted, like a fixed end
  - Same frequency and wavelength as the incoming wave
  - The amplitude is decreased
- Transmitted wave:
  - Upright
  - Same frequency as incoming wave, but has a shorter wavelength because the wave slowed down



Wave Transmission: Slow to Fast Medium

- Reflected wave:
  - Upright, like a free end
  - Same frequency and wavelength as the incoming wave
  - The amplitude is decreased
- Transmitted wave:
  - Upright
  - Same frequency as incoming wave, but has a longer wavelength because the wave sped up



In fact, transmitted wave is always upright

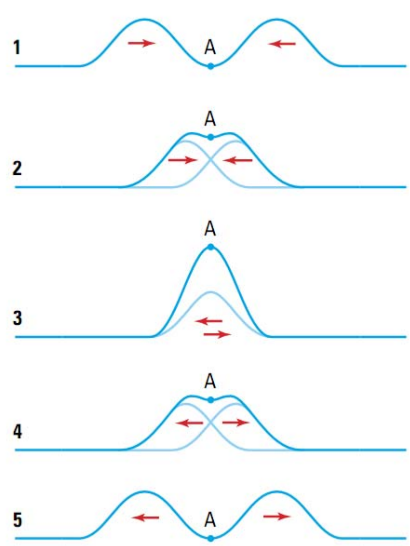
Superposition of Waves

- **Principle of Superposition:** When multiple waves pass through the same point, the resultant wave is the **sum** of the waves
  - A fancy way of saying that waves add together
- The consequence of the principle of superposition is **interference** of waves. There are two kinds of interference:
  - **Constructive interference:** Two wave fronts (crests) passing through creates a wave front with greater amplitude
  - **Destructive interference:** A crest and trough will cancel each other

Superposition of Waves

Constructive interference: In-phase wave fronts sum together

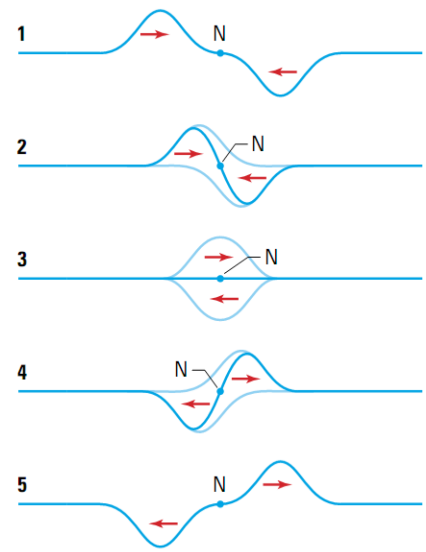
- In this example, two identical pulses move towards each other
- Their crests pass through A at the same time
- The amplitude at A when the waves pass through is higher



Superposition of Waves

Destructive interference: Out-of-phase wave fronts shows the difference of the wave fronts

- Two pulses move towards each other, one a crest, the other a trough
- They both pass through A at the same time
- Two waves cancel each other at A



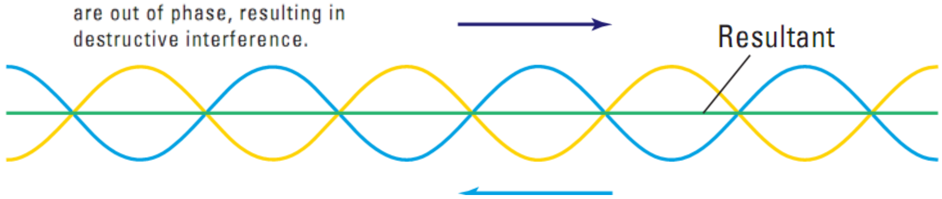
Standing Waves

If two waves of the same frequency meet up under the right conditions, they may appear to be “standing still”. This is called a standing wave

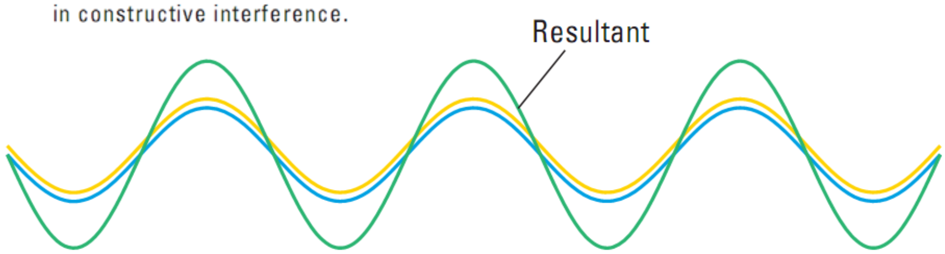
- Node: a point that never moves
- Anti node: a point which moves/vibrates maximally

Standing Waves (Cont.)

A Two waves are travelling in the same medium. Here, the waves are out of phase, resulting in destructive interference.



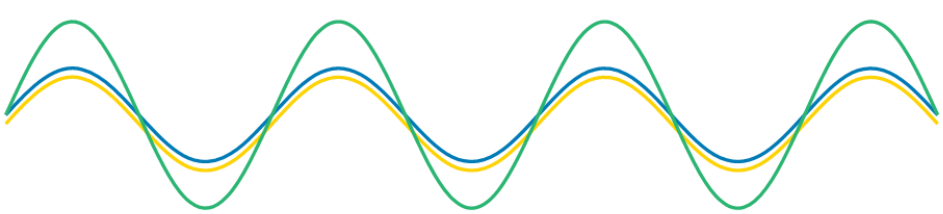
B One quarter of a period after (A), the waves are in phase, resulting in constructive interference.



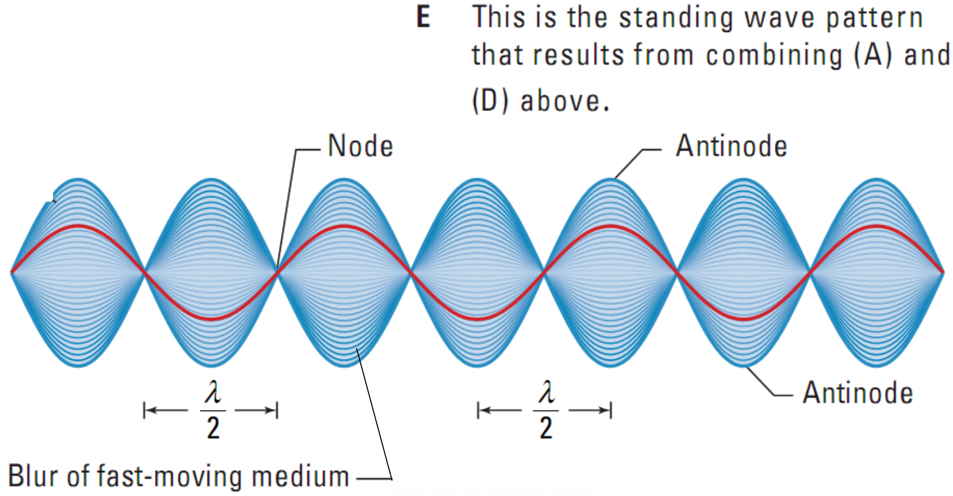
Standing Waves (Cont.)

C One half of a period after (A), the waves are again out of phase.

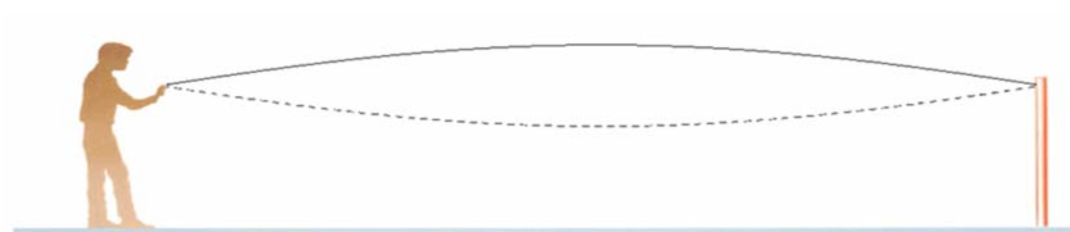
D Three quarters of a period after (A).



Standing Waves

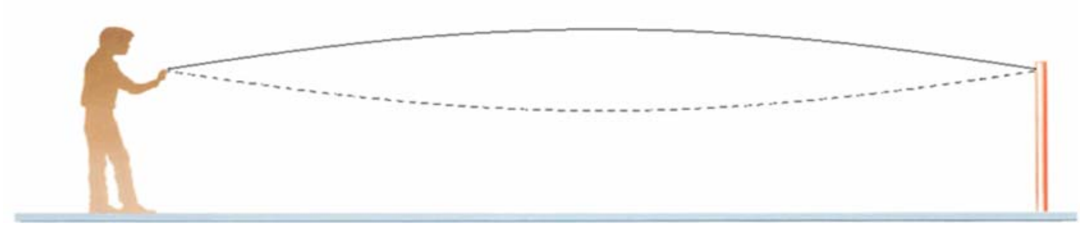


STANDING WAVES PATTERN



- The standing waves are formed, when the distance between two nodes L = n $\lambda/2$, where n = 1, 2, 3,…

STANDING WAVES PATTERN



- $L = n\,\lambda/2$, where n = 1, 2, 3,…
- $\lambda = v/f$, thus finally
- $L = n\,v/(2f)$

FUNDAMENTAL FREQUENCY AND HARMONICS