Ratio, Rate, and Percent

1. Ratio
A ratio is a comparison of two numbers. We generally separate the two numbers in the ratio with a colon (:). Suppose we want to write the ratio of 8 and 12.

We can write this as 8:12 or as a fraction 8/12, and we say the ratio is *eight to twelve*.

Example: Jeannine has a bag with 3 videocassettes, 4 marbles, 7 books, and 1 orange.

1) What is the ratio of books to marbles?
Expressed as a fraction, with the numerator equal to the first quantity and the denominator equal to the second, the answer would be 7/4.
Two other ways of writing the ratio are 7 to 4, and 7:4.

2) What is the ratio of videocassettes to the total number of items in the bag?
There are 3 videocassettes, and 3 + 4 + 7 + 1 = 15 items total.
The answer can be expressed as 3/15, 3 to 15, or 3:15.

2. Comparing Ratios
To compare ratios, write them as fractions. The ratios are equal if they are equal when written as fractions.

Example: Are the ratios 3 to 4 and 6:8 equal?
The ratios are equal if 3/4 = 6/8.

These are equal if their cross products are equal; that is, if $3 \times 8 = 4 \times 6$. Since both of these products equal 24, the answer is yes, the ratios are equal.

Remember to be careful! Order matters!
A ratio of 1:7 is not the same as a ratio of 7:1.

Examples: Are the ratios 7:1 and 4:81 equal? No!
$7/1 > 1$, but $4/81 < 1$, so the ratios can't be equal.

Are 7:14 and 36:72 equal?
Notice that 7/14 and 36/72 are both equal to 1/2, so the two ratios are equal.

3. Proportion
A proportion is an equation with a ratio on each side. It is a statement that two ratios are equal.
3/4 = 6/8 is an example of a proportion.

When one of the four numbers in a proportion is unknown, cross products may be used to find the unknown number. This is called solving the proportion. Question marks or letters are frequently used in place of the unknown number.

Example: Solve for n: $1/2 = n/4$.
Using cross products we see that $2 \times n = 1 \times 4 = 4$, so $2 \times n = 4$. Dividing both sides by 2, $n = 4 \div 2$ so that $n = 2$.

4. Rate

A rate is a ratio that expresses how long it takes to do something, such as traveling a certain distance. To walk 3 kilometers in one hour is to walk at the rate of 3 km/h. The fraction expressing a rate has units of distance in the numerator and units of time in the denominator.

Problems involving rates typically involve setting two ratios equal to each other and solving for an unknown quantity, that is, solving a proportion.

Example: Juan runs 4 km in 30 minutes. At that rate, how far could he run in 45 minutes?

Give the unknown quantity the name n. In this case, n is the number of km Juan could run in 45 minutes at the given rate. We know that running 4 km in 30 minutes is the same as running n km in 45 minutes; that is, the rates are the same. So we have the proportion
4km/30min = n km/45min, or $4/30 = n/45$.

Finding the cross products and setting them equal, we get $30 \times n = 4 \times 45$, or $30 \times n = 180$. Dividing both sides by 30, we find that $n = 180 \div 30 = 6$ and the answer is 6 km.

5. Converting rates

We compare rates just as we compare ratios, by cross multiplying. When comparing rates, always check to see which units of measurement are being used. For instance, 3 kilometers per hour is very different from 3 meters per hour!

3 kilometers/hour = 3 kilometers/hour × 1000 meters/1 kilometer = 3000 meters/hour
because 1 kilometer equals 1000 meters; we "cancel" the kilometers in converting to the units of meters.

Important: One of the most useful tips in solving any math or science problem is to always write out the units when multiplying, dividing, or converting from one unit to another.

Example: If Juan runs 4 km in 30 minutes, how many hours will it take him to run 1 km?

Be careful not to confuse the units of measurement. While Juan's rate of speed is given in terms of minutes, the question is posed in terms of hours. Only one of these units may be used in setting up a proportion. To convert to hours, multiply
4 km/30 minutes × 60 minutes/1 hour = 8 km/1 hour

Now, let n be the number of hours it takes Juan to run 1 km. Then running 8 km in 1 hour is the same as running 1 km in n hours. Solving the proportion,
8 km/1 hour = 1 km/n hours, we have $8 \times n = 1$, so $n = 1/8$.

6. Average Rate of Speed

The average rate of speed for a trip is the total distance traveled divided by the total time of the trip.

Example: A dog walks 8 km at 4 km per hour, then chases a rabbit for 2 km at 20 km per hour. What is the dog's average rate of speed for the distance he traveled?

The total distance traveled is 8 + 2 = 10 km.
Now we must figure the total time he was traveling.

For the first part of the trip, he walked for 8 ÷ 4 = 2 hours. He chased the rabbit for 2 ÷ 20 = 0.1 hour.
The total time for the trip is 2 + 0.1 = 2.1 hours.
The average rate of speed for his trip is 10/2.1 = 100/21 kilometers per hour.

5. Percent

A percent is a ratio whose second term is 100. Percent means parts per hundred. The word percent comes from the Latin phrase *per centum*, which means per hundred. In mathematics, we use the symbol % for percent.

Let's look at our comparison table again. This time the table includes percent.

Comparing Shaded Boxes to Total Boxes			
Grid	**Ratio**	**Fraction**	**Percent**
1	96 to 100	$\frac{96}{100}$	96%
2	9 to 100	$\frac{9}{100}$	9%
3	77 to 100	$\frac{77}{100}$	77%

►Questions in class

1. Find x/y if (3x + 2) / (4y + 2) =1.

2. The angles of a triangle are in the ratio $2:3:4$. What is the largest angle in the triangle?

3. Laura earns $10/hour and works 8 hours per day for 10 days. She first spends 25% of her pay on food and clothing, and then pays $350 in rent. How much of her pay does she have left?

4. A map is drawn to a scale of 1:10 000. On the map, the Gauss Forest occupies a rectangular region measuring 10 cm by 100 cm. What is the actual area of the Gauss Forest, in km^2?

5. A photo measuring 20 cm by 25 cm is enlarged to make a photo measuring 25 cm by 30 cm. what is the percentage increase in area?

6. On Tony's map, the distance from Saint John, NB to St. John's, NL is 21 cm. The actual distance between these two cities is 1050 km. What is the scale of Tony's map?

7. Water is poured from a full 1.5 L bottle into an empty glass until both the glass and the bottle are 3/4 full. What is the volume of the glass?

8. Lily is 90 cm tall. If Anika is 4/3 of the height of Lily, and Sadaf is 5/4 of the height of Anika, what is the ratio of heights of Sadaf to Lily?

9. When it is 3:00 p.m. in Victoria, it is 6:00 p.m. in Timmins. Stefan's flight departed at 6:00 a.m. local Victoria time and arrived at 4:00 p.m. local Timmins time. How long, in hours, was his flight?

10. Flora had an average of 56% on her first 7 exams. What would she have to make on her eighth exam to obtain an average of 60% on 8 exams?