4. Work and Mechanical Energy

Grade 11 Physics

Olympiads School

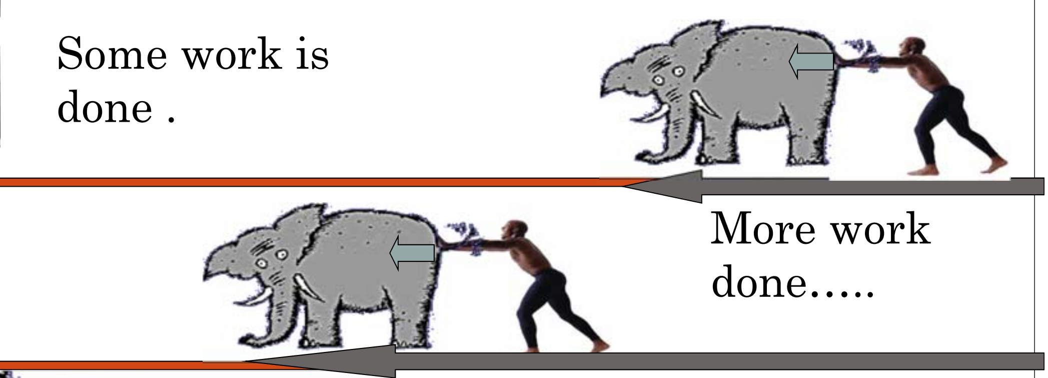
Course Overview

1. Introduction & 1D Kinematics
2. Motion in a Plane (two-dimensional kinematics)
3. Newton's Laws of motion
4. **Work and mechanical energy**
5. Heat and energy transformation
6. Energy transfer through vibrations and waves
7. Wave model of sound
8. Electricity and magnetism

Work

- "Everyday" concept of work:
  - Lifting a box
  - Working out, exercises
  - Doing your Physics 11 homework (gasp!)
- In physics, the definition of work is a lot more precise

Examples of Work



Work depends on:

- Displacement
- Force acting on the object during its motion

The Definition of Work

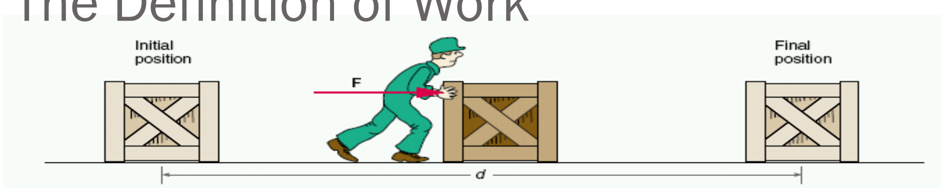


Figure 6.5 A crate is moved a distance *d* across a concrete floor under the influence of a constant horizontal force **F**.

Work = force x displacement

$$W = \vec{F} \bullet \Delta\vec{d}$$

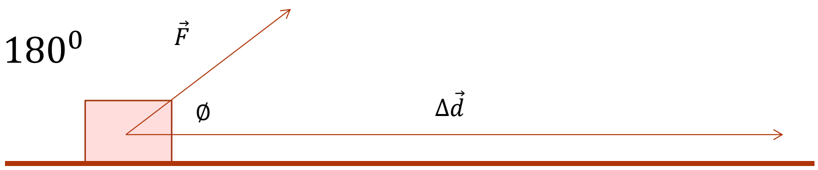
But what does it mean to multiply two vectors?

Work – Definition and Units

$$\boxed{W = \vec{F} \cdot \Delta\vec{d} = F\,\Delta d \cos\Theta}$$

$0^0 \leq \emptyset \leq 180^0$

$F \geq 0$
$\Delta d \geq 0$



Work = force x distance, so units will be Nm in SI system. But this is renamed the joule (abbreviated as J):

1 J = 1 Nm

Work

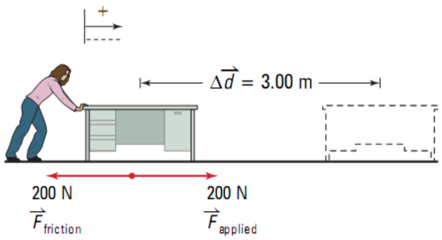
$$\boxed{W = F\,\Delta d \cos\Theta}$$

- Both force and displacement are vector quantities, but work, is a **scalar** quantity. **This equation applies ONLY when $\vec{F}$ is constant**.

- If $\vec{F}$ and $\Delta\vec{d}$ are in the same direction, then work is *positive*
- If $\vec{F}$ and $\Delta\vec{d}$ are in the opposite direction, then work is *negative*

Sample Problem #1

A Physics student is rearranging her room. She decides to move her desk across the room, a total distance of 3.00 m. She moves the desk at a constant velocity by exerting a horizontal force of 200 N. Calculate the amount of work the student did on the desk in moving it across the room.



Solution:

$W = F_{applied} \times \Delta d\ (\cos \varnothing)$

$= (200N)(3.00\ m)(\cos 0^0)$

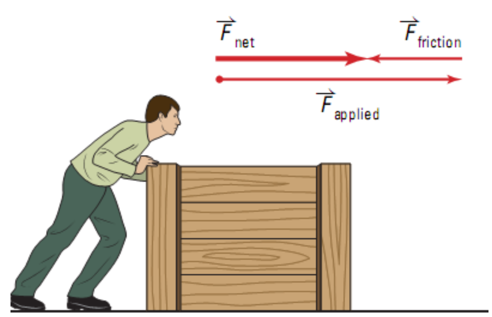
$=600\ Nm$

$= 600\ J$

Comment: Friction is NOT included in the solution

Work Done By a Force

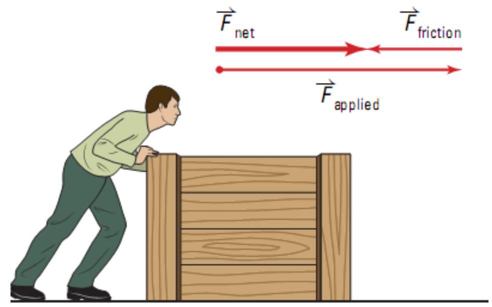
When calculating work, we determine the work done by **a specific force.**



Work Done By a Force

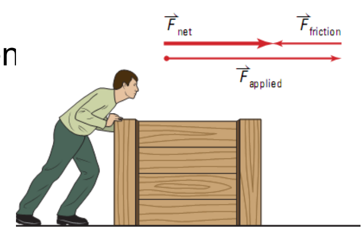
Let's break down the work done by each force.

- Applied force $F_{applied}$ from the man
- Friction force $F_{friction}$ at the contact between the crate and the floor



Work on the object

If the crate moves a distance of Δd to the right, then



- **Work done** by applied force is positive:

$$W_a = F_a \Delta d$$

- **Work done** by friction is negative:

$$W_f = F_f \Delta d$$

- **Work done by gravity and normal force are zero**
- **Total (Net) work** is the sum of the work done by all forces. It is also called the work done on the object:

$$Wnet = Fnet\ \Delta d$$

Zero Work

$$W = F \Delta d \cos \Theta$$

Work is zero when:

- Object does not move (no displacement)

or

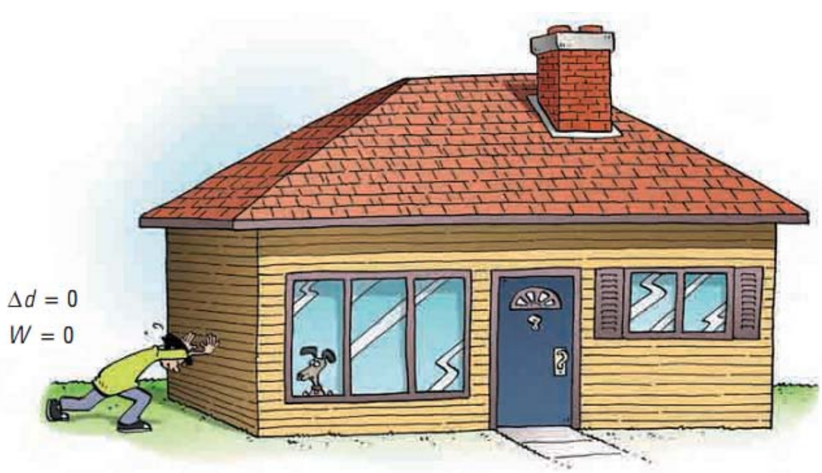
- There are no forces acting on the object

or

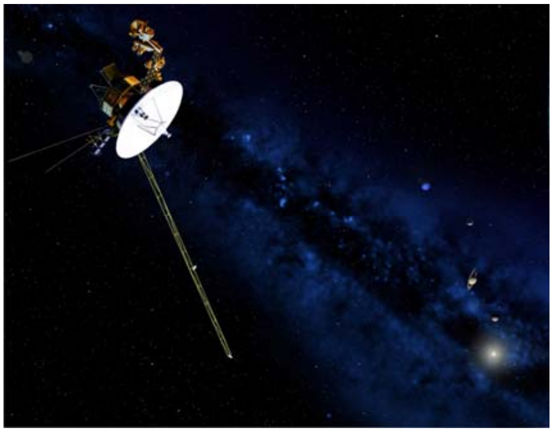
- The angle between the force and displacement is 90 degrees ($\cos \Theta = 0$)

No Work is Done

Applying a force that does not cause motion. $\Delta d=0$



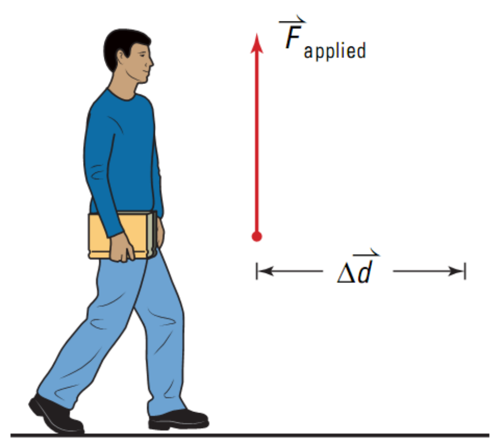
No Work is Done



The Voyager space probe has left our solar system and is travelling through deep space. The gravitational and frictional effects may be considered negligible now. No work is required to keep the probe moving. It is moving because of its inertia.

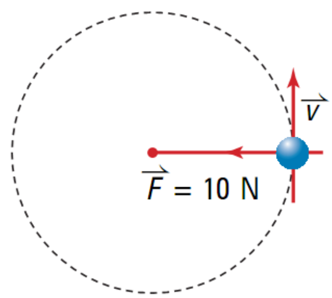
No Work is Done

The work done by a force that is perpendicular to the motion is zero ($\cos 90^0 = 0$)



Sample Problem #2

A child ties a ball to the end of a 1 m string and swings the ball in a circle. If the string exerts a 10 N force on the ball, how much work does the string do on the ball during a swing of one complete circle?



Solution:

$W = F_T \times \Delta d \quad (\Delta d \neq 0!)$

but $\vec{F}_T \perp \vec{v}$, therefore $\vec{F}_T \perp \overrightarrow{\Delta d}$ at any instant

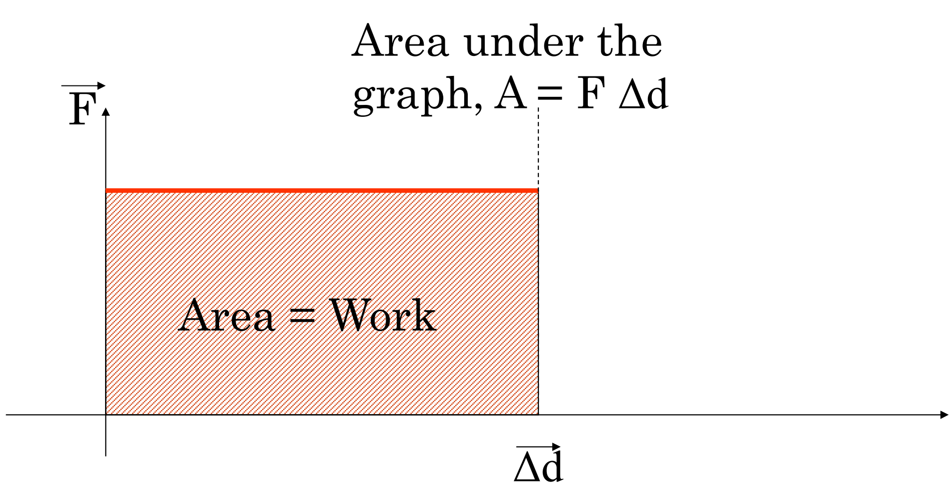
$W = 0$

Comment:

Work done by friction is $2\pi r F_k$

Graphs of Work

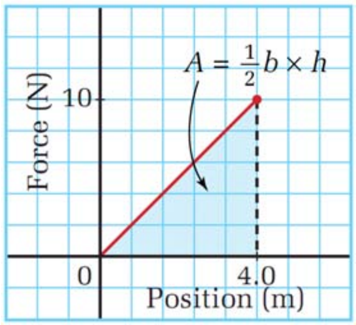
- $\vec{F}$ = constant, $\vec{F} \parallel \overrightarrow{\Delta d}$

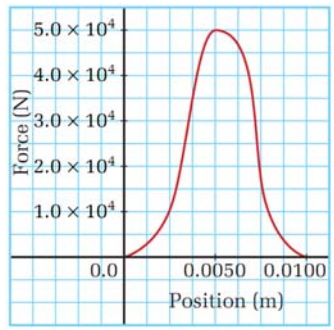


Work by Non-Constant Force

When the force is not constant, it may be useful to plot a force vs. displacement graph. Work is the area under the curve.

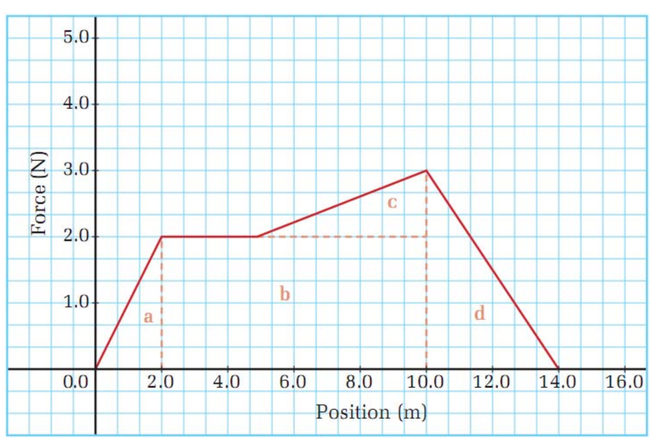
- Work is positive if the area is above the *x* (displacement) axis.
- Work is negative if it is below the *x* axis



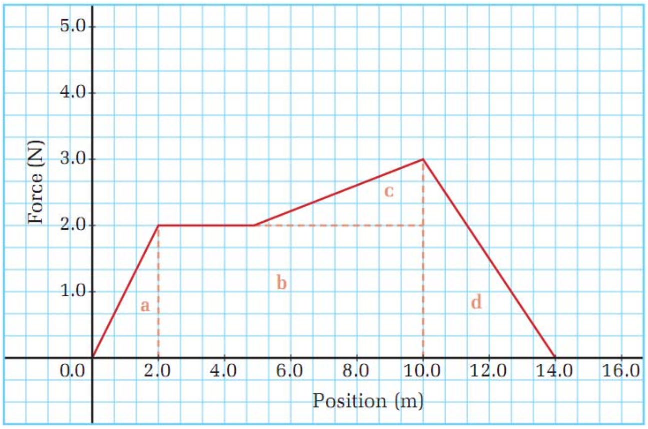


Sample Problem #4

Determine the amount of work done by the changing force represented in the force vs displacement plot shown here.



Solution

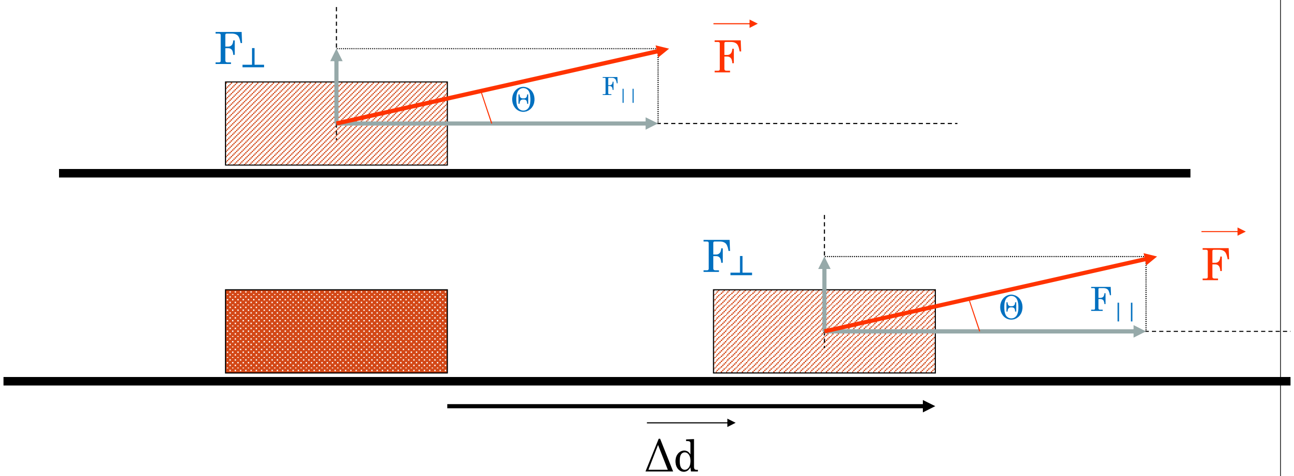


$$\mathbf{W = W_a + W_b + W_c + W_d}$$

W_a = ½ x 2.0 m x 2.0 N = 2.0 J
W_b = 8.0 m x 2.0 N = 16 J
W_c = ½ x 5.0 m x 1.0 N = 2.5 J
W_d = ½ x 4.0 m x 3.0 N = 6.0 J

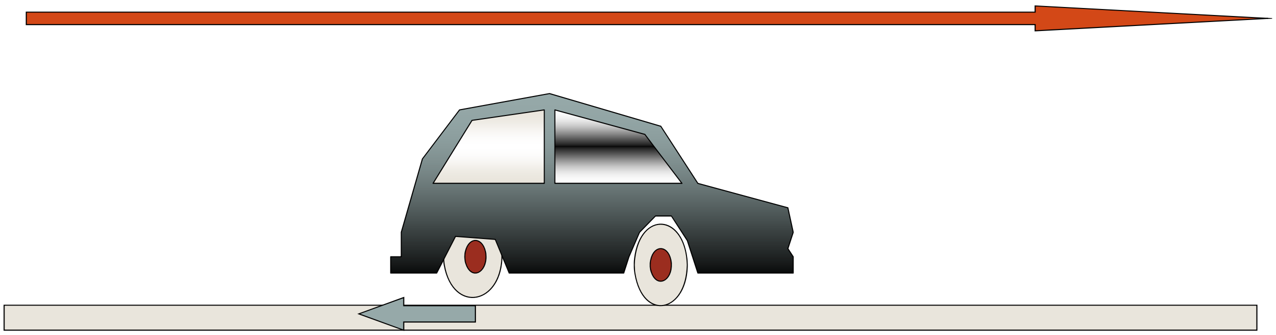
W = 26.5 J

Work Done By a Parallel component



$$W = F \Delta d \cos \Theta = (F \cos \Theta) \Delta d = F_{||} \Delta d$$

NEGATIVE WORK



$$W = F\,\Delta d \cos\Theta = -F\,\Delta d$$

Challenge Questions:

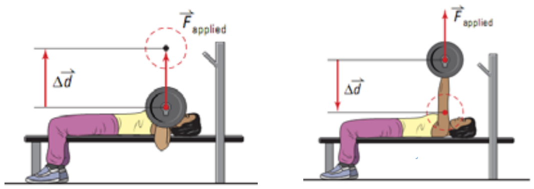
- Can work done by kinetic friction be positive?
- Usually $F_s \geq F_k$. Does it mean that work done by static friction is always greater than the work done by kinetic friction?

Sample Problem #5

Consider a weight lifter bench-pressing a barbell weighting 6.50x10^2 N through a height of 0.55 m. There are two distinct uniform motions:

1. when the barbell is lifted up and
2. when barbell is lowered back down

Calculate the work done on the barbell during each of the two motions.



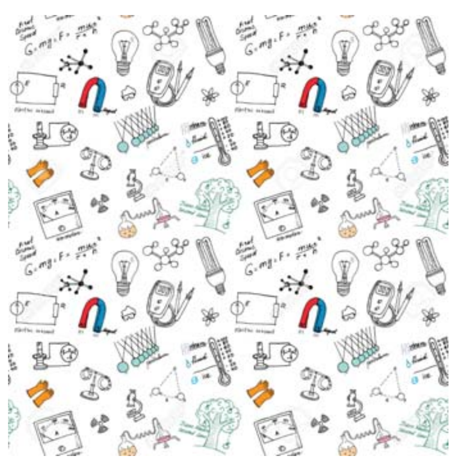
Solution:

$F_{NET} = 0$ (both up and down)

$W_{NET} = 0$

Energy

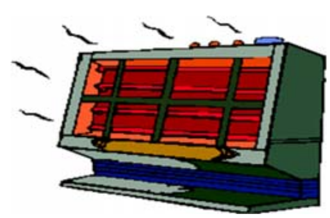
- Energy is the crown for physics. It is found in every branch of physics.
- Energy is the capacity for doing work.
- Energy is measured in joules.

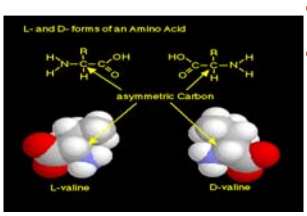


Energy and Work

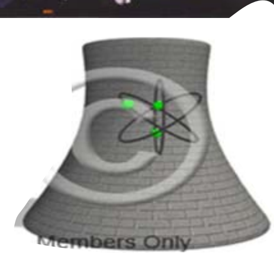
- Positive Work
  - Transfers energy from the environment to the system
  - Increases the total energy of the system
- Negative Work
  - Transfers energy from the system to the environment in a variety of forms, like heat, light, or sound
  - Decreases the total energy of the system
- Energy is neither created nor destroyed, can be only transferred or changed into different forms

Forms of Energy





- Energy appears in many forms. There are five main forms of energy
  - **Mechanical**
  - **Thermal**
  - **Chemical**
  - **Electromagnetic**
  - **Nuclear**



Energy

There are two types of energy:

- **Kinetic energy** is the energy of things that are in motion
- **Potential energy** is the energy of position (Stored energy)
- All forms of energies (e.g. sound, light, electrical, etc.) all fit into one of these two types.

Kinetic Energy

Kinetic energy is the energy of things that are in motion, defined as:

$$E_k = \frac{1}{2}mv^2$$

Quantity	Symbol	SI Unit
Kinetic energy	E_k	J (joules)
Mass	m	kg (kilograms)
Velocity	v	m/s (metres per second)

- Depends on the frame of reference

Sample Problem #6

A 0.200 kg hockey puck, initially at rest, is accelerated to 27.0 m/s. Calculate the kinetic energy of the hockey puck

1. At rest
2. In motion

Solution:

1. $v_1 = 0$, therefore $E_k = 0$ at rest

2. $v_2 = 27$ m/s

$$E_k = \frac{1}{2}mv^2 = 0.5(0.200)(27.0)^2 \text{ kg m}^2/\text{s}^2 = 72.9 \text{ Nm} = 72.9 \text{ J}$$

Work-Kinetic Energy Theorem

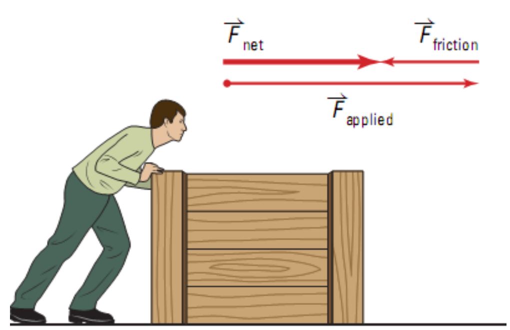
- The crate accelerates to the right with constant acceleration:

$$\vec{a} = \frac{\vec{F}_{net}}{m}$$

- As it speeds up towards the right, velocity increases from initial value v_1 to final value v_2:

$$v_2^2 = v_1^2 + 2a\Delta d$$

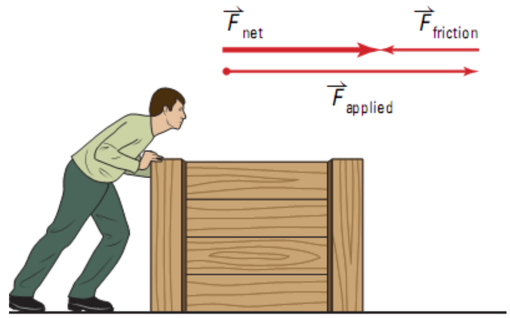
$$= v_1{}^2 + 2\frac{F_{net}}{m}\Delta d$$



Work-Kinetic Energy Theorem

As v increases, E_k increases by the same amount as the work done by F_{net}. We can show this by rearranging the kinematic equation above:

$$F_a \Delta d = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \Delta E_k$$



E_k only changes when $F_{net} \neq 0$.

The Work-Kinetic Energy Theorem

Total work equals to the change in kinetic energy

$$W_{\text{net}} = \Delta E_k$$

- Positive work increases the kinetic energy
- Negative work decreases kinetic energy

Sample Problem #7

A curler does work on a 2.5 kg curling stone by exerting 40 N of force horizontally over a distance of 1.5 m.

- Calculate the work done by the curler on the curling stone.
- Assuming that the stone started from rest, calculate the velocity of the stone at the point of release. (Assuming the ice surface to be frictionless.)

Solution

- $W = F_A \times \Delta d = 40\text{ N} \times 1.5\text{ m} = 60\text{ Nm} = 60\text{ J}$
- $F_A = F_{net}$ (no friction)
- $\mathbf{W} = \Delta \boldsymbol{E_k} = E_{k2} - E_{k1} = E_{k2} - 0$ $(starting\ from\ the\ rest)$
- $W = \frac{1}{2} m v_2^2$
- $v_2^2 = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2 \times 60 J}{2.5\ kg}} = 6.9\text{ m/s}$
- *(you could apply kinematics equations as well)*

Sample Problem #8

A 75 kg skateboarder (including the board), initially moving at 8.0 m/s, exerts an average force of 2.0×10^2 N by pushing on the ground, over a distance of 5.0 m. Find the new kinetic energy of skateboarder if the trip is completely horizontal.

Solution:

- This problem cannot be solved without the assumption, that friction is negligible.
- Without friction, the net force equals the applied force

$$F_{net} = F_{applied}$$

- Work – Kinetic Energy Theorem:

$$\mathbf{W_{net}} = \Delta \boldsymbol{E_k} = E_{k2} - E_{k1}; \quad E_{k1} = (0.5)(75 \text{ kg})(8.0 \text{ m/s}^2)^2 = 4.8 \times 10^3 \text{ J}$$

$$W_{net} = W_{applied} = F_{applied}\Delta d = (2.0 \times 10^2 \text{ N})(5.0 \text{ m}) = 1.0 \times 10^3 \text{ J}$$

$$E_{k2} = E_{k1} + W = 4.8 \times 10^3 \text{ J} + 1.0 \times 10^3 \text{ J} = 5.8 \times 10^3 \text{ J}$$

$$E_{k2} = 5.8 \times 10^3 \text{ J}$$

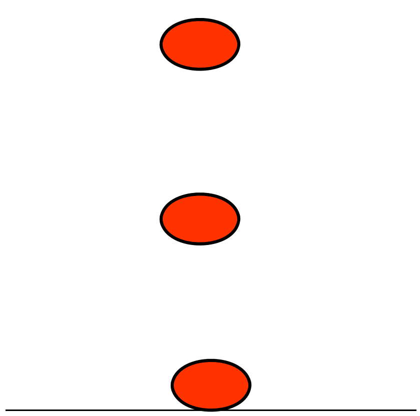
Gravitational Potential Energy

Gravitational potential energy is proportional to an object's mass *m*, acceleration due to gravity *g*, and height *h*, measured above a *reference level*

$$E_g = mgh$$

Quantity	Symbol	SI Unit
Gravitational potential energy	E_g	J (joules)
Mass	m	kg (kilograms)
Acceleration due to gravity	g	m/s^2 (metres per second squared)
Height above reference level	h	m (metres)

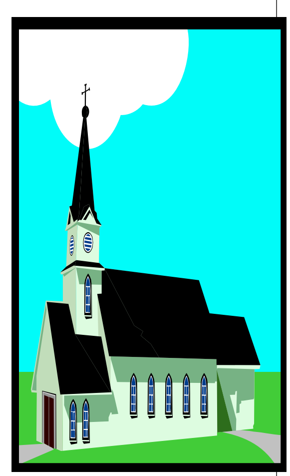
Potential Gravitational Energy - Reference Level



$E_{p2} > E_{p1}$

$E_{p1} > 0$

R.L. $E_p = 0$

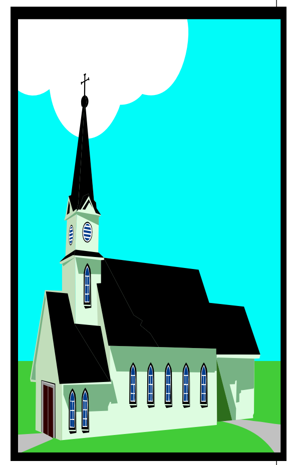


Potential Gravitational Energy - Reference Level

$E_p > 0$

R.L. $E_p = 0$

$E_p < 0$



Potential Gravitational Energy

Hint: Choose the reference level at the lowest point in which an object can be located

Sample Problem #9

You are about to drop a 3.0 kg rock onto a tent peg. Calculate the gravitational potential energy of the rock after you lift it to a height of 0.68 m above the tent peg.

Solution:

$E_g = mgh = (3.0\text{kg})(9.81\text{m/s}^2)(0.68\text{ m})$

$= 2.0 \times 10^1 \text{ J}$

Sample Problem #10

A 65.0 kg rock climber did 1.60×10^4 J of work against gravity to reach a ledge. How high did the rock climber ascend?

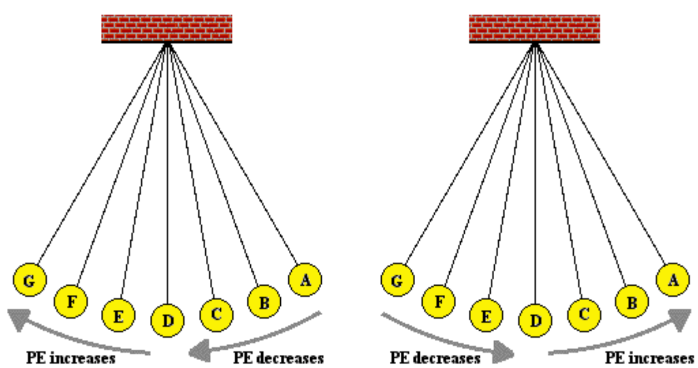
Solution:

$E_g = mgh \Rightarrow h = \frac{E_g}{mg}$

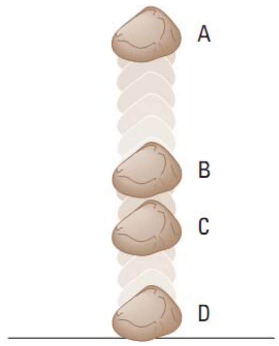
$h = \frac{1.6\times10^4 J}{65\ kg\times9.81\ m/s^2}$

h = 25 m

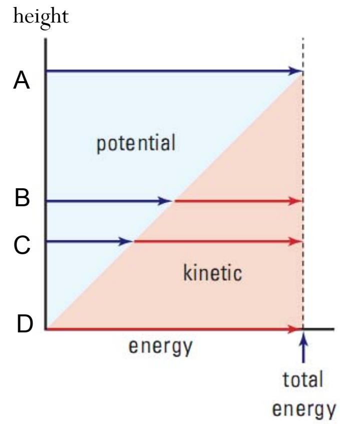
Motion of a Pendulum – Energy is Conserved



Conservation of Energy



Gravitational potential energy is being transformed into kinetic energy.



The graph shows the amount of potential and kinetic energy of the rock at different positions, up to the moment before it hits the ground.

Conservation of Energy

The total mechanical energy of a system always remains constant

$$E_T = E_k + E_g = \text{constant}$$

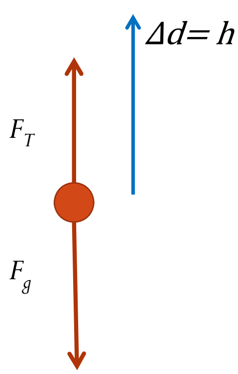
This equation applies when we can neglect friction (generation of heat), and other forms of energy (light, sound, etc.).

Sample Problem #11

A crane lifts a car, with a mass of 1.5×10^3 kg, at a constant velocity, to a height of 14 m from the ground. It turns and drops the car, which then falls freely back to the ground. Neglecting air friction, find

- The work done by the crane in lifting the car
- The gravitational energy of the car at its highest point, in relation to the ground
- The velocity of the car just before it strikes the ground after falling freely from 14 m

Solution:



$F_T = F_g = mg$ (N1LM)

$W = F_T\Delta d = mgh = (1.5\times10^3\ \text{kg})(9.81\ \text{m/s}^2)(14\ \text{m})$
$= 2.1\times10^5$ J

$E_p = mgh = W = 2.1\times10^5$ J

Solution (cont.)

Position 1 $E_{1p} = 2.1\text{x}10^5$ J, $E_{1k} = 0$, $E_{1T} = E_{1p} + 0 = 2.1\text{x}10^5$ J

$E_{k2} = mv_2^2/2$ $\qquad v_2 = \sqrt{\frac{2E_{k2}}{m}} = \sqrt{\frac{2\times2.1\times10^5}{1.5\times10^3}}$ J = 16 m/s

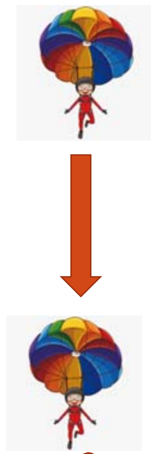
Position 2 $E_{2T} = E_{1T}$; $E_{2p} = 0$; E_{2k} =?; $E_{2T} = E_{2k} + 0 = 2.1\text{x}10^5$ J

Sample Problem #12

A 65.0-kg skydiver steps out from a hot air balloon that is $5.00\text{x}10^2$ m above the ground. After free-falling a short distance, he deploys his parachute, finally reaching the ground with a velocity of 8.0 m/s.

- Find the gravitational potential energy of the skydiver, relative to the ground, before he jumps
- Find the kinetic energy of the skydiver just before he lands
- How much work did the non-conservative frictional force do?

Solution



Position 1

E_{p1} = mgh =(65.0)(9.81)(5.00x10^2) J = 3.19 x10^5 J

E_{k1} = 0

E_{T1} = 3.19 x10^5 J + 0 = 3.19 x10^5 J

Position 2

E_{p2} = 0

E_{k2} =$mv_2^2/2$ = (0.5)(65.0)$(8.0)^2$ J = 2.1 x10^3 J

E_{T2} = 0 + E_{k2} = 2.1 x10^3 J

$E_{T1} = E_{T2} + W_{N\text{-}C}$

$W_{N\text{-}C} = E_{T1} - E_{T2}$ = 3.19 x10^5 J - 2.1 x10^3 J = 3.17 x10^5 J

Sample Problem #13

- Two objects (m_1 = 5.00 kg and m_2 = 3.00 kg) are connected by a light string passing over a light, frictionless pulley. The 5.00-kg object is released at a point h = 4.00 m above the table. (a) Determine the speed of each objects when the two pass each other. (b) Determine the speed of each objects at the moment the 5.00-kg object hits the table. (c) How much higher does the 3.00-kg object travel after the 5.00-kg object hits the table.

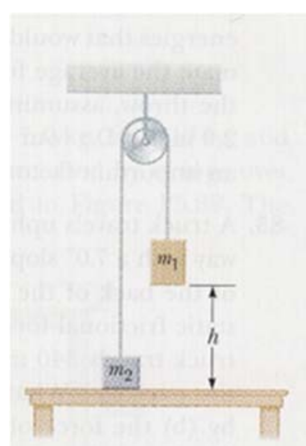
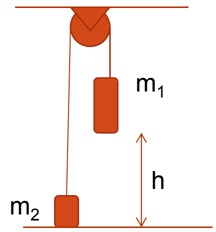


Figure P5.71

Solution

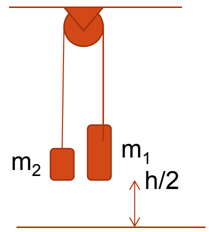
m_1 = 5.00 kg
m_2 = 3.00 kg
h = 4.00 m

Position 1.



$E_{T1} = E_{g1} = m_1gh$

Position 2.



$E_{T2} = E_{g1} + E_{g2} + E_{k1} + E_{k2}$

$E_{T2} = (m_1+m_2)gh/2 + (m_1+m_2)v^2/2$

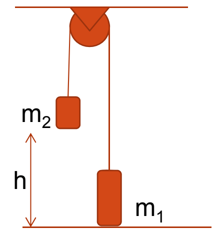
$E_{T2} = E_{T1}$

$(m_1+m_2)gh/2 + (m_1+m_2)v^2/2 = m_1gh$

$v = \sqrt{gh}\left(\frac{m_1-m_2}{m_1+m_2}\right)$

v = 3.13 m/s

Position 3.



$E_{T3} = E_{g2} + E_{k1} + E_{k2}$

$E_{T3} = m_2gh + (m_1+m_2)v^2/2$

$E_{T3} = E_{T1}$

$m_2gh + (m_1+m_2)v^2/2 = m_1gh$

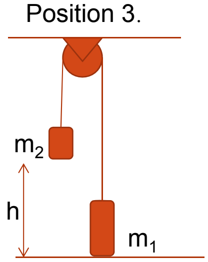
$v = \sqrt{2gh}\left(\frac{m_1-m_2}{m_1+m_2}\right)$

v = 4.43 m/s

Solution

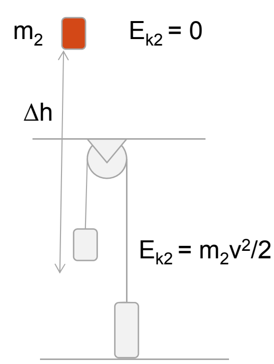
m_1 = 5.00 kg
m_2 = 3.00 kg
h = 4.00 m

Position 3.



v_2 = 4.43 m/s

Position 4.



LCE:

$m_2g\Delta h = m_2v_2^2/2$

$\Delta h = v_2^2/2g$

Δh = 1.00 m

Sample Problem #14

- An ball of mass m is suspended from the top of the cart by a string of length L. The cart and the object are initially moving to the right at a constant speed v_0. The cart comes to a rest after colliding and sticking to a bumper and the suspended object swings through an angle Θ. Show that the initial speed is $v_0 = \sqrt{2gL(1 - cos\theta)}$.

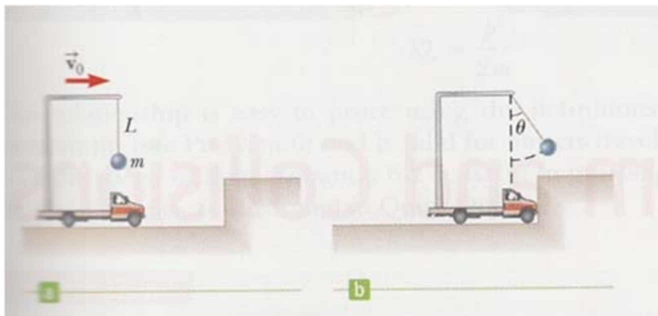
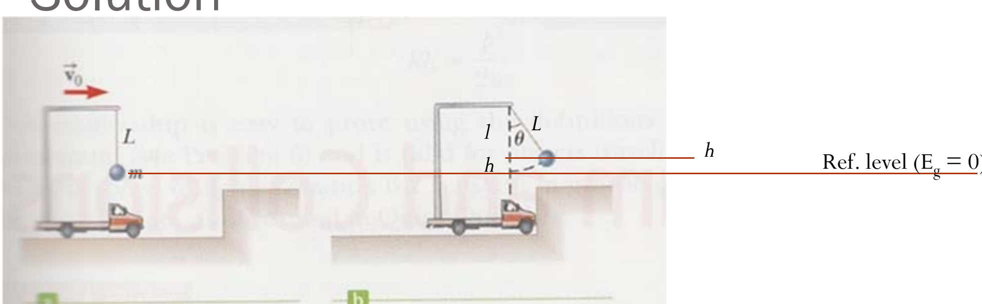


Figure P5.88

Solution



(a)

Energy of the ball:

$E_T = E_K + E_g$

$E_g = 0$ (ref. level)

$E_T = mv_0^2/2$

(b)

Energy of the ball:

$E_T = E_K + E_g$

$E_k = 0 \quad (v = 0)$

$E_T = mgh$

$h = L - l$

$h = L - L \cos \Theta = L(1 - \cos \Theta)$

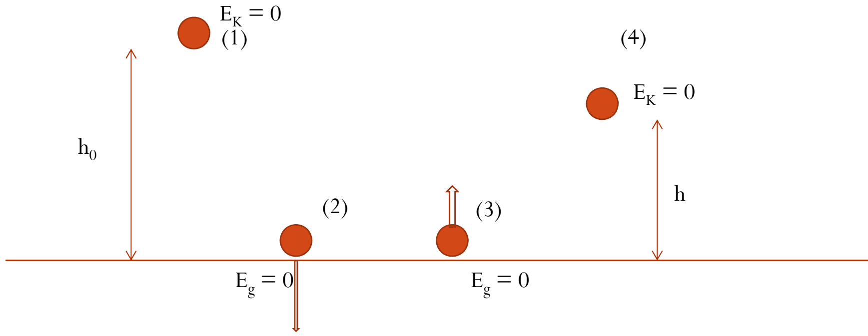
LCE:

$\cancel{m}v_0^2/2 = \cancel{m}gh$

$$v_0 = \sqrt{2gL(1 - cos\theta)}.$$

Sample Problem #15

A basketball rolls off the rim and falls to the floor from a height of 3.05 m. Then it bounces up and loses 15% of its kinetic energy. To what height will it rise this time?



$E_{K3} = 0.85E_{K2}$

LCE:

$E_{T1} = E_{T2}$ and $E_{T3} = E_{T4}$

$E_{T1} = mgh_0$

$E_{T2} = E_{K2} = mgh_0$

$E_{T3} = E_{K3} = 0.85E_{K2} = 0.85mgh_0$

$E_{T4} = mgh = 0.85mgh_0$

$h = 0.85h_0$

h = 0.85 x 3.05 m = 2.59 m

May energy be with you