Equations

►Basic knowledge

1. Linear Equation:
A mathematical expression is that has an equal sign and linear expressions.

2. Variable:
A number that you don't know, often represented by "x" or "y" but any letter will do!

3. Linear Expression:
A mathematical statement is that performs functions of addition, subtraction, multiplication, and division.

Note: *Variable(s)* in linear expressions

- Cannot have exponents (or powers)
  For example, *x* squared or x^2
- Cannot multiply or divide each other
  For example: "x" times "y" or xy; "x" divided by "y" or x/y
- Cannot be found under a root sign or square root sign (sqrt)
  For example: $\sqrt{x}$ or the "square root of x"; sqrt (x)

4. Linear equation form
A **linear equation** is any equation that can be written in the form: $ax+b=c$, where *a* and *b* are real numbers and *x* is a variable.

This form is sometimes called the **standard form** of a linear equation. Note that most linear equations will not start off in this form. Also, the variable may or may not be an *x* so don't get too locked into always seeing an *x* there.

To solve linear equations we will make heavy use of the following facts.

1) If $a=b$, then $a+c=b+c$, for any *c*. All this is saying is that we can add a number, *c*, to both sides of the equation and not change the equation.

2) If $a=b$, then $a-c=b-c$, for any *c*. As with the last property we can subtract a number, *c*, from both sides of an equation.

3) If $a=b$, then $a \cdot c=b \cdot c$, for any *c*. Like addition and subtraction we can multiply both sides of an equation by a number, *c*, without changing the equation.

4) If $a = b$, then $\dfrac{a}{c} = \dfrac{b}{c}$, for any non-zero c. We can divide both sides of an equation by a non-zero number, c, without changing the equation.

5) Solving an equation means find root(s) for the equation.

If r is a root of the equation, replacing x with r will satisfy the equation.

For example, $ax + b = c$, then $ax = b - c$ and $x = \dfrac{b - c}{a}$

Example 1

Driving between two towns at 110 km/h instead of 100 km/h saves 9 minutes. What is the distance in kilometres between the two towns?

Solution

Let x represent the distance, in kilometres, between the two towns. Driving at 100 km/h, it takes $x/100$ hours to travel between the towns. Driving at 110 km/h, it takes $x/110$ hours. We know that these two times differ by 9 minutes, or 9/60 hours. Thus, $\dfrac{x}{110} + \dfrac{9}{60} = \dfrac{x}{100}$

$x = 165$. The two towns are 165 km apart.

Example 2

The average (mean) of a list of n numbers is 7. When the number -11 is added to the list, the new average is 6. What is the value of n?

Solution

The average of a list of numbers is their sum divided by how many numbers are in the list. Thus if n numbers have an average of 7 their sum is $7n$.

When -11 is added to the list of numbers, there are then $n + 1$ numbers whose sum is

$7n$ -11. Using this, we obtain $\dfrac{7n - 11}{n + 1} = 6$, $n = 17$.

Example 3

In a school, 500 students voted on each of two issues. Of these students, 375 voted in favour of the first issue, 275 voted in favour of the second, and 40 students voted against both issues. How many students voted in favour of both issues?

Solution

Let x be the number of students who voted in favour of both issues. We construct a Venn diagram of the results of the vote:

Since the total number of students is 500, then

$37 - x + x + 275 - x + 40 = 500$

$x = 190$

So 190 students voted in favour of both.

Questions in class

1. Find the value of *N* that will make the equation below an identity. (An identity is an equation for which all real numbers are solutions.)
$9\,x - 3(x + 2) = 2(3x + 8) - 2N$

2. The average (mean) of a list of *n* numbers is 7. When the number −11 is added to the list, the new average is 6. What is the value of *n*?

3. Solve for x.
$$\frac{8(\dfrac{5(3x+12)}{22}-7)-1}{3}+4=25$$

4. Together Akira and Jamie weigh 101 kg. Together Akira and Rabia weigh 91 kg. Together Rabia and Jamie weigh 88 kg. How many kilograms does Akira weigh?

5. If a, b and c are positive integers with $a \times b = 13$ (1),
$b \times c = 52$ (2), and $c \times a = 4$ (3),
determine the value of $a \times b \times c$.

6. In a school, 500 students voted on each of two issues. Of these students, 375 voted in favour of the first issue, 275 voted in favour of the second, and 40 students voted against both issues. How many students voted in favour of both issues?

7. The sum of three consecutive multiples of 120 is the same as the sum of five consecutive multiples of 96. If the least of the three multiples of 120 is 840, then what is the value of the greatest of the multiples of 96?

8. An airplane, flying with a tail wind, travels 1200 miles in 5 hours; the return trip, against the wind, takes 6 hours. Find the cruising speed of the plane and the speed of the wind (assume that both are constant).

9. Driving between two towns at 110 km/h instead of 100 km/h saves 9 minutes. What is the distance in kilometers between the two towns?