Chapter 10 Congruent Triangles

1. Congruent Polygons

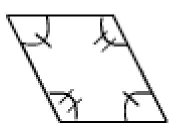
Congruent Polygons - Two polygons are congruent if their corresponding sides and angles are congruent.

Note: Two sides are congruent if they have the same length and angles are congruent if they have the same measure.

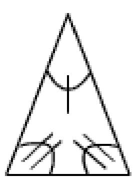
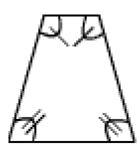
We indicate that angles are congruent by putting the same number of slash marks through each angle. Congruent polygons necessarily have to have the same shape and the same size. They also have to have the same number of sides.

We can think of congruent polygons as coming from the same stamp. The location where they reside may differ and rotating or flipping congruent polygons does not change the fact that they are congruent.

Example 1: Consider the polygon shown below.



Determine which could be congruent to the above parallelogram.

A.  B.  C. 

Solution

First notice that polygon A has three sides while the given polygon has 4 sides. These cannot be congruent. Polygon B has the correct number of sides, but the congruent angles in polygon B are adjacent (next to each other) while the congruent angles in the given polygon are opposite angles. In fact Polygon B is a trapezoid, while the given Polygon is a parallelogram. Polygon B cannot be congruent to the given polygon.

Polygon C has four sides, and the opposite angles are congruent, just like the given polygon. Polygon C could be congruent to the given polygon.

Notice that Polygon C has a vertex pointing downward while the given polygon does not. This is irrelevant, since rotating a polygon does not change which polygons it is congruent to.

Once we have determined that two polygons are congruent, we can then look at corresponding sides and corresponding angles and state that their measures are equal. You can tell which side is equal to which by following the order that the vertices are stated.

Example 2: Polygons ABCDEF and GHIJKL are congruent. Which sided is congruent to the side CD?

Solution
First note that other than the congruency statement, no other information is given. You cannot just look at the figures and try to estimate the length is side CD and then try to find a side that looks as though it has the same length.

Instead, pay attention to the order at which the letters are presented. In particular C is the third vertex named in ABCDEF and D is the fourth vertex.

We can now count from G until we get to the third and fourth named vertex in GHIJKL. These are I and J. We can conclude that IJ is congruent to CD.

2. Conditions for Congruent Triangles

Congruent triangles are triangles that have the same **size** and **shape**. This means that the corresponding sides are equal and the corresponding angles are equal.

In the above diagrams, the corresponding sides are *a* and *d*; *b* and *e* ; *c* and *f*.
The corresponding angles are *x* and *s*; *y* and *t*; *z* and *u*.

We can tell whether two triangles are congruent without testing all the sides and all the angles of the two triangles. There are four rules to check for congruent triangles. They are called the SSS rule, SAS rule, ASA rule and AAS rule.

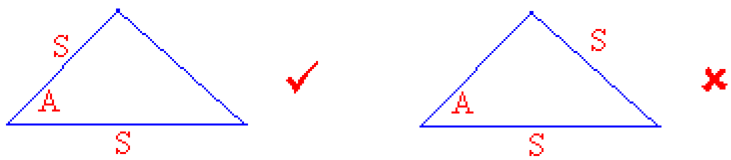
There is also another rule for right triangles called the Hypotenuse Leg (HL) rule. As long as one of the rules is true, it is sufficient to prove that the two triangles are congruent.

SSS Rule

If three sides of one triangle are equal to three sides of another triangle, then the triangles are congruent. (Hence the name of this rule: Side-Side-Side, SSS)

SAS Rule

If two sides and the included angle (Side-Angle-Side, SAS) of one triangle are equal to two sides and included angle of another triangle, then the triangles are congruent. An included angle is the angle formed by the two given sides.



Included Angle Non-included angle

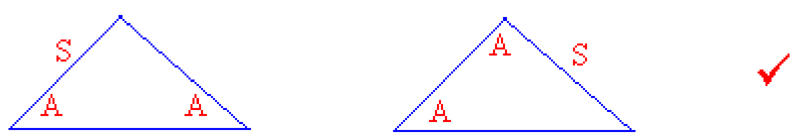
ASA Rule

If two angles and the included side of one triangle (Angle-Side-Angle, ASA) are equal to two angles and **included** side of another triangle, then the triangles are congruent. An included side is the side between the two given angles.

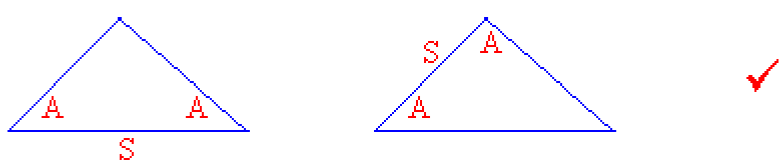
AAS Rule

If two angles and a non-included side of one triangle (Angle-Angle-Side, AAS) are equal to two angles and a non-included side of another triangle, then the triangles are congruent. (This rule may sometimes be referred to as SAA).

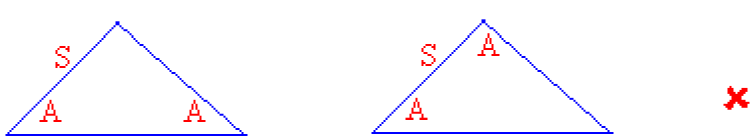
For the ASA rule the given side must be included and for AAS rule the side given must not be included. The trick is we must use the same rule for both the triangles that we are comparing.



Compare AAS with AAS



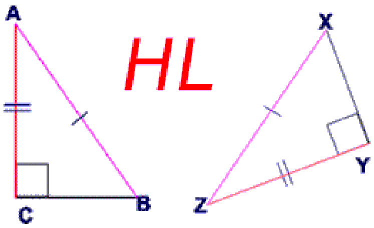
Compare ASA with ASA



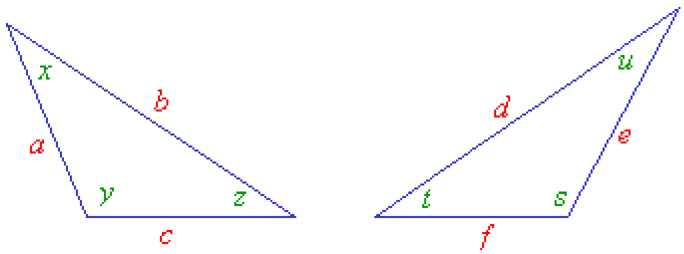
Compare AAS with ASA

HL Rule

It is used only for right angled triangles. If you are given the hypotenuses and one pair of legs are same, then the triangles are congruent.



Example 1:



Which of the following conditions would be sufficient for the above triangles to be congruent?

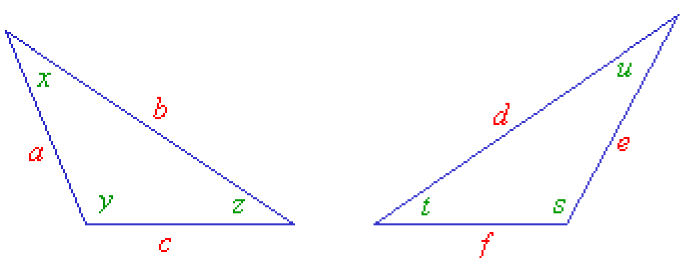
a) $a = e, x = u, c = f$,
b) $a = e, y = s, z = t,$
c) $x = u, y = t, z = s,$
d) $a = f, y = t, z = s$

Solution for a):

Step 1: $a = e$ gives the S; $x = u$ gives the A; $c = f$ gives the S

Step 2: Beware! *x* and *u* are not the included angles. This is not SAS but ASS which is **not** one of the rules. Note that you cannot compare donkeys with triangles!

Answer: $a = e, x = u, c = f$ is **not** sufficient for the above triangles to be congruent.

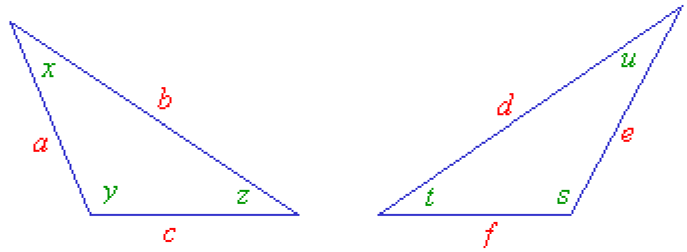


Solution for b):

Step 1: $a = e$ gives the S; $y = s$ gives the A; $z = t$ gives the A

Step 2: a and e are non-included sides. Follows the **AAS rule**.

Answer: $a = e, y = s, z = t$ is sufficient show that the above are congruent triangles.

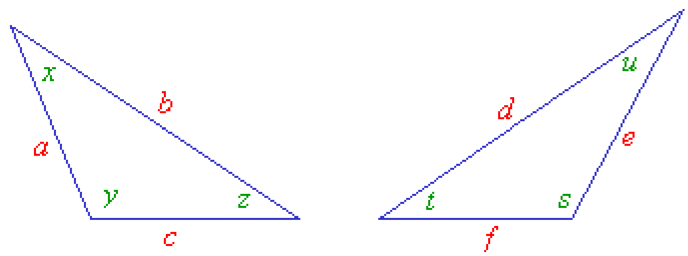


Solution for c):

Step 1: $x = u$ gives the A; $y = t$ gives the A; $z = s$ gives the A

Step 2: AAA is **not** one of the rules.

Answer: $x = u, y = t, z = s$ is **not** sufficient for the above triangles to be congruent.



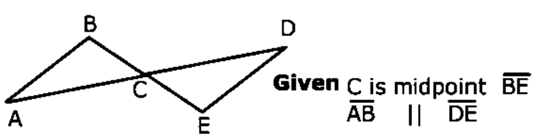
Solution for d):

Step 1: a, y, z follows AAS (non-included side); f, t, s follows the ASA (included side)

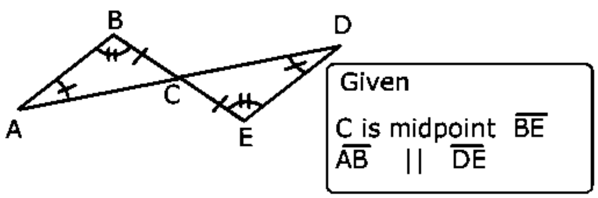
Step 2: Comparing AAS with ASA is not allowed.

Answer: $a = f, y = t, z = s$ is **not** sufficient to show that the above are congruent triangles.

Example 2: Given that C is the midpoint of BE and AB // DE. Prove that $\triangle ABC \cong \triangle DEC$.

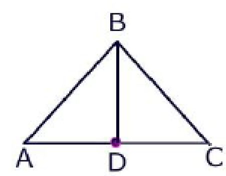


Solution

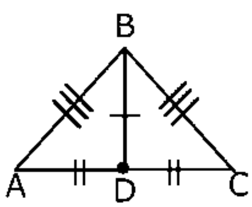


Statement	Reason
$\angle BAC = \angle CDE$	alternate interior angles of parallel lines are congruent
$\overline{BC} = \overline{CE}$	definition of midpoint
$\angle ABC = \angle DEC$	alternate interior angles of parallel lines are congruent
$\Delta ABC \cong \Delta DEC$	**AAS Postulate**

Example 3: Given *AB*≅*BC*, *BD* is a <u>median</u> of side AC**.** Prove: $\triangle ABD \cong \triangle CBD$.
Note: Median cuts AC in halves.



Solution



Statement	Reason
$\overline{AB} = \overline{BC}$	Given
$\overline{BD} = \overline{BD}$	Reflexive Property
$\overline{AD} = \overline{DC}$	$\overline{BD}$ is a median bisects a segment into 2 equal parts
Δ**ABD** $\cong \Delta$ **CBD**	**SSS Postulate**

Note: Reflexive Property of Equality - *The property that a = a.*