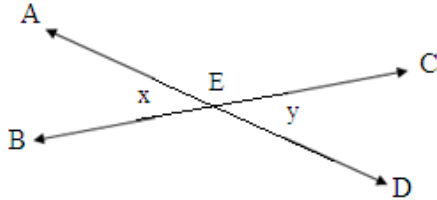


Geometry

1. Vertical Angles

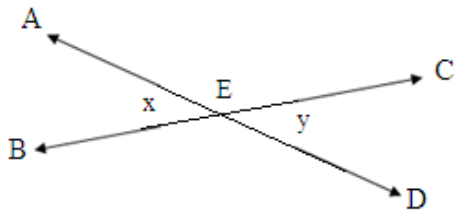
In this diagram, 2 straight lines, AB and CD, intersect at E



As you can see, when this occurs, 4 angles are created. Angles x and y are opposite one another. The name given to a pair of angles such as these is *vertical angles*.

If we were to measure each of these angles, we would find the measure of both angle x and y to be the same. Therefore we will state that when 2 lines intersect, the *vertical angles formed are equal* in measure.

Example 1: Let's assume that the measure of angle y is 42° . If angle "x" is represented by the expression " $a + 16$ ", can we solve for "a"?

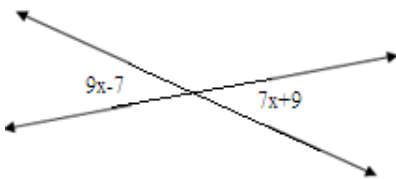


Solution: If angle $x = a + 16$, and angle $y = 42$, we can set up the following equation:

$$a + 16 = 42.$$

This is a simple one-step equation, $a + 16 = 42$, $a = 26$.

Example 2: Find the value of x.



Solution: We can clearly see from the diagram that the angles represented by " $9x - 7$ " and " $7x + 9$ " are vertical angles. We also know that vertical angles are equal. Therefore we can set up the following equation:

$$9x - 7 = 7x + 9, \quad x = 8.$$

2. Supplementary Angles

If the sum of the measures of two angles is 180° , then the angles are called **supplementary angles**. Each angle is called the **supplement** of the other angle.

For example, if one angle measures 70° , then its supplement will measure $110^\circ = (180-70)^\circ$.
Supplementary angles are a pair of angles, if more than 2 angles add up to 180° , they are not supplementary!

Example 1: Two angles are supplementary. One of the angles is three times as large as the other. Find the measure of both angles.

Solution: First, assign variables to represent the two angles.

Let x = the smaller angle

Let $3x$ = the larger angle

Second, write the equation.

$3x + x = 180$ (the sum of the angles is 180°)

Third, solve the equation.

$3x + x = 180$ (combine like terms)

$x = 45$

Therefore, the measure of the smaller angle (x) is 45° and the measure of the larger angle ($3x$) is 135° .

Example 2: Find the number of degrees in an angle which is 20° less than four times its supplement.

Solution: Let $4x - 20$ be one of the angles.

Let x be the supplement of the angle

$(4x-20) + (x) = 180$

$x = 40$

Therefore, one of the angles ($4x-20$) is 140° , and the supplement of that angle is $(180 - 140) = 40^\circ$.

Example 3: Find the supplement of the angle which is represented by the expression $(4x - 60)^\circ$.

For this problem let's use the fact that the easiest way to find the supplement of an angle is to subtract the given angle from 180° .

So, if we let one of the angles be " x ", and its supplement be $(4x-60)^\circ$, then we should be able to find the other angle by solving the following equation:

$x = 180 - (4x - 60)$ (distribute the "-" sign)

$x = 48^\circ$

Therefore the supplement is 48° .

3. Complementary Angles

If the sum of the measures of two angles is 90° , then the angles are called **complementary angles**. Each of the angles is called the **complement** of the other angle.

For example, if one angle measures 30° , then the measure of its *complement* is $(90-30)$, or 60° .

Remember: *Complementary angles are a pair of angles.*

If the sum of the measure of 3 or more angles is 90° , that does not make them complementary angles!

Example 1: Two angles are complementary. The measure of one of the angles is 24° greater than the measure of the other angle. Find the measure of each of the angles.

Let x be the smaller angle

Let $x + 24$ be the larger angle

$$(x) + (x+24) = 90$$

$$x = 33$$

Therefore x , the smaller angle, measures 33° and $x+24$, the larger angle measures 57° .

Example 2: Find the number of degrees in angle which exceeds three times it's complement by 22° .

Let x be the smaller complementary angle

Let $3x+22$ be the larger complementary angle.

$$(x) + (3x+22) = 90$$

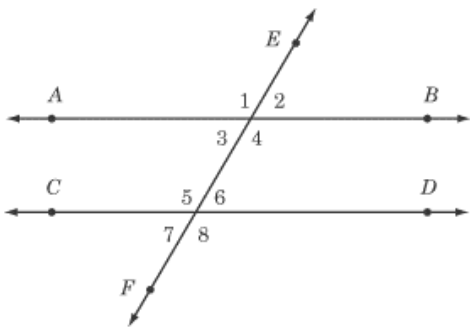
$$x = 17$$

Therefore the smaller angle, x , is 17° , and the larger angle, $3x+ 22$, is 73° .

4. Angles Formed by Parallel Lines Cut by a Transversal

In the diagram below you can see that lines AB and CD are parallel and that the third line, EF is a transversal.

When this occurs you can also see that 8 different angles have been created which have been numbered 1 – 8.



It turns out that each of these angles can be paired with another, and that each pair of angles has a special name, as well as a special property.

1) Corresponding Angles

In the diagram above, the following pairs of angles are called *corresponding angles*:

$\angle 1$ and $\angle 5$; $\angle 2$ and $\angle 6$; $\angle 3$ and $\angle 7$; $\angle 4$ and $\angle 8$

If you look at where each of the angles in a pair are located, you will notice that they are in the same relative position where the transversal intersects one of the parallel lines and the same point of intersection on the other parallel line.

In other words, the position of one of the angles *corresponds* to the position of the other angle in the pair.

As for their special property: Corresponding angles are equal.

$$\angle 1 = \angle 5; \angle 2 = \angle 6; \angle 3 = \angle 7; \angle 4 = \angle 8$$

2) Alternate Interior Angles

In the diagram above, angles 3, 4, 5 and 6 are called ***interior angles*** because they are between the two parallel lines. If the angles lie on opposite sides of the transversal, but not on the same parallel line, they are called ***alternate interior angles***.

The pairs of alternate interior angles in this diagram are:

$$\angle 3 \text{ and } \angle 6; \angle 4 \text{ and } \angle 5$$

As for their special property: Alternate Interior angles are equal.

$$\angle 3 = \angle 6; \angle 4 = \angle 5$$

3) Alternate Exterior Angles

In the diagram above, angles 1, 2, 7 and 8 are called ***exterior angles*** because they do not lie between the parallel lines.

Just like *alternate interior angles*, if the exterior angles lie on opposite sides of the transversal, but not at the same parallel line, they are called ***alternate exterior angles***.

The pairs of alternate exterior angles in the diagram are:

$$\angle 1 \text{ and } \angle 8; \angle 2 \text{ and } \angle 7$$

As for their special property: Alternate Exterior angles are equal.

$$\angle 1 = \angle 8; \angle 2 = \angle 7$$

4) Interior Angles on the Same Side of the Transversal

As the name clearly implies, the diagram above shows that :

$$\angle 3 \text{ and } \angle 5; \angle 4 \text{ and } \angle 6$$

are pairs of angles which are not only interior angles, but also lie on the same side of the transversal.

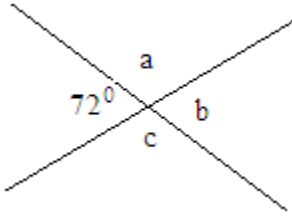
The special property of these angle pairs: Interior Angles on the Same Side of the Transversal are SUPPLEMENTARY (THEIR SUM IS ALWAYS 180°)

$$\angle 3 + \angle 5 = 180^\circ$$

$$\angle 4 + \angle 6 = 180^\circ$$

5. Calculate the Value of Missing Angles

When two lines intersect four angles are formed. Look at the diagram below:



As you can see the two intersecting lines have created 4 angles. The measure of one of the angles has been given as 72° . The other 3 angles are labeled a, b and c.

Our task is to determine the measure of each of those other 3 angles.

Determining the measure of angle "b" is as easy as remembering that the 72° angle and angle "b" are known as vertical angles, and vertical angles are equal.

That means that the measure of angle "b" is also 72° .

We can also see that angles "a" and "c" are also vertical angles. So if we can determine the measure of one of the angles, the other will have the same measure.

To do this we need to see that the original 72° angle and angle "a" are *supplementary angles*. Supplementary angles are two angles whose sum is 180° .

We know they are supplementary because the angles combine to create a straight line. And as you know *a straight line is a straight angle*, and *the measure of a straight angle is 180°* .

Now that we have reviewed the important vocabulary all we need to do is subtract 72 from 180, and that will give us the measure of angle "a".



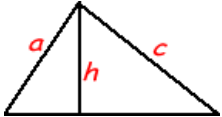
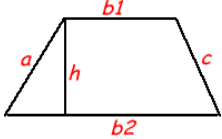
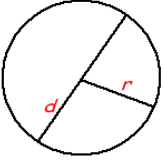
$$180 - 72 = 108$$

The measure of angle "a" is 108° .

And, because angle "a" and angle "c" are vertical angles, the measure of angle "c" is also 108° .

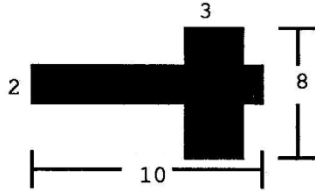
6. Geometric formula

Shapes	Formula
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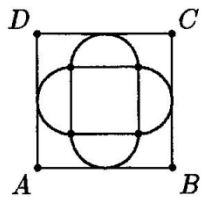
	<p>Rectangle:</p> <p>Area = Length X Width</p> $A = lw$ <p>Perimeter = 2 X Lengths + 2 X Widths</p> $P = 2l + 2w$
	<p>Parallelogram</p> <p>Area = Base X Height</p> $A = bh$
	<p>Triangle</p> <p>Area = 1/2 of the base X the height</p> $A = \frac{1}{2}bh$ <p>Perimeter = $a + b + c$</p> <p>(add the length of the three sides)</p>
	<p>Trapezoid</p> <p>area $A = \left(\frac{b_1 + b_2}{2}\right)h$</p> <p>Perimeter = $a + b_1 + b_2 + c$</p>
	<p>Circle</p> <p>The distance around the circle is a circumference.</p> <p>The distance across the circle is the diameter (d).</p> <p>The radius (r) is the distance from the center to a point on the circle. (Pi = 3.14...)</p> <p>$d = 2r, \quad c = \pi d = 2\pi r, \quad A = \pi r^2$</p>

Questions in class

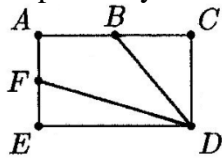
1. Find the shaded area formed by the two intersecting perpendicular rectangles, in square units.



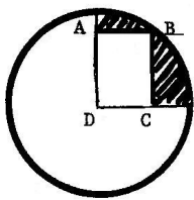
2. Around the outside of a 4 by 4 square, construct four semicircles (as shown in the figure) with the four sides of the square as their diameters. Another square, ABCD, has its sides parallel to the corresponding sides of the original square, and each side of ABCD is tangent to one of the semicircles. What is the area of the square ABCD?



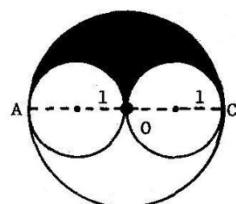
3. The rectangle shown has length $AC = 32$, $AE = 20$, and B and F are midpoints of AC and AE, respectively. What is the area of the quadrilateral ABDF?



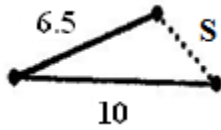
4. What is the measure of the acute angle formed by the hands of a clock at 4:20 a.m.?
5. ABCD is a rectangle, D is the center of the circle, and B is on the circle. If $AD = 4$ and $CD = 3$, then what is the area of the shaded region?



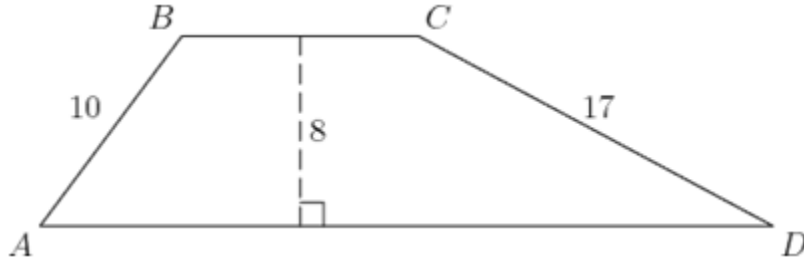
6. The large circle has diameter AC. The two small circles have their centers on AC and just touch at O, the center of the large circle. If each small circle has radius 1, what is the value of the ratio of the area of the shaded region to the area of one of the small circles?



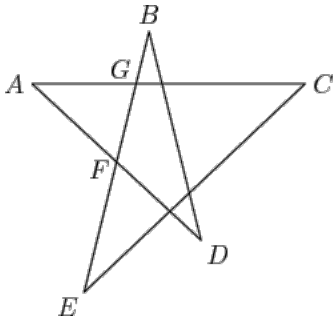
7. The sides of a triangle have lengths 6.5, 10, and S , where s is a whole number. What is the smallest possible value of S ?



8. The area of trapezoid $ABCD$ is 164 cm^2 . The altitude is 8 cm, AB is 10 cm, and CD is 17 cm. What is BC , in centimeters?



9. If $\angle A = 20^\circ$ and $\angle AFG = \angle AGF$. Then $\angle B + \angle D = ?$



10. The area of rectangle $ABCD$ is 72. If point A and the midpoints of BC and CD are joined to form a triangle, what is the area of that triangle?

