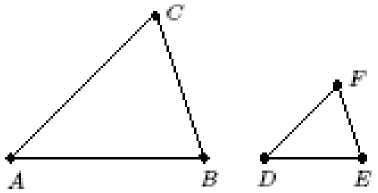
Geometry 2

►Similarity of triangles

1. Definition

Two triangles are said to be similar if there is a correspondence between their vertices such that corresponding angles are congruent. The notation ΔABC ∽ ΔDEF means that ΔABC is similar to ΔDEF under the correspondence A ↔ D, B ↔ E, and C ↔ F, or more specifically that

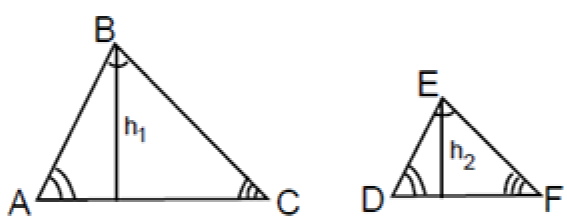
$\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$.



The first thing to notice is that in Euclidean geometry, it is only necessary to check that two of the corresponding angles are congruent.

Each of the congruence rules has a corresponding similarity rule, by replacing side-length equality by proportionality. Thus, triangles may be determined to be similar by any of the following rules.

Similar triangles have proportional sides, that is, $\frac{ED}{AB} = \frac{EF}{BC} = \frac{DF}{AC} = \frac{h_1}{h_2}$



• SAS Rule
If two sides of one triangle are in the same proportion as the two sides of another, and the included angles of the sides that correspond are equal then the triangles are similar.

• SSS Rule
If three sides of one triangle are in the same proportion as the three sides of another, then the triangles are similar.

• AA Rule (or AAA Rule)
If two angles of one triangle are equal to two angles of another, then the triangles are similar. (The equality of the two remaining corresponding angles are then necessarily equal.)

• RHS Rule
If the hypotenuse and one other side of a right-angled triangle are in the same proportion as the hypotenuse and one side of another right-angled triangle, then the triangles are similar.

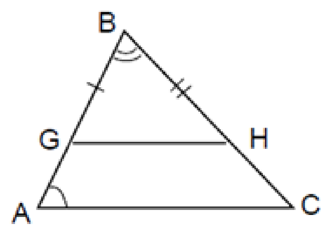
As with congruence, when we say two triangles ABC and XY Z are similar we mean that the correspondence of vertex A to X, B to Y and C to Z determines the similarity. We denote that two triangles ABC and XY Z are similar by writing $\Delta ABC \backsim \Delta XYZ$.

Theorem 1.
If a line joins the midpoints of two sides of a triangle then that line is parallel to the third side and its length is equal to one half of the length of the third side.

Theorem 2.
A line parallel to one side of a triangle divides the other two sides in the same proportion. That is

If $GH // AC$ then

$$\frac{BG}{BH} = \frac{BA}{BC}$$



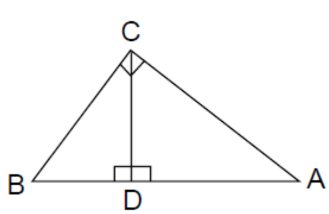
Theorem 3.
The bisector of one side of a triangle divides the opposite side in the same ratio as the other two sides.

2. Right triangles

Theorem 4. Right Angle Similarity Theorem: In the right ΔABC. drawing the perpendicular from the right angle at C to the side AB gives the three similar right triangles; ΔABC ΔACD *and* ΔBDC, and

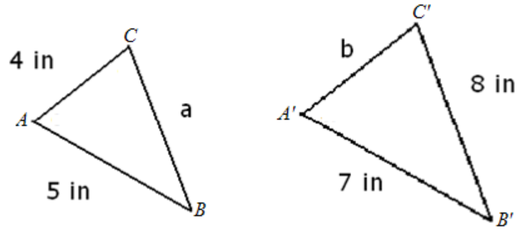
$$\frac{BD}{BC} = \frac{BC}{BA}$$

This implies that the BC is the ***geometric mean*** of BD and BA

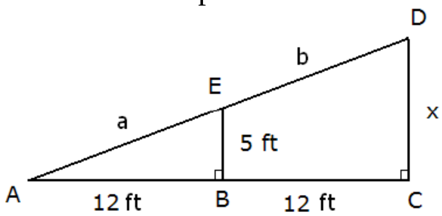


►Questions in class

1. The triangles shown below are similar. Find the exact values of a and b shown on the picture below.

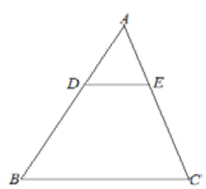


2. Consider the picture shown below.

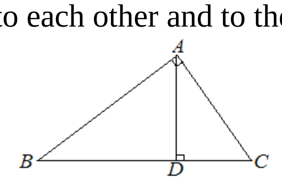


(a) Use the Pythagorean Theorem to find the value of a.
(b) Prove that the triangles ABE and ACD are similar.
(c) Use similar triangles to find the value of x.
(d) Find the value of b.

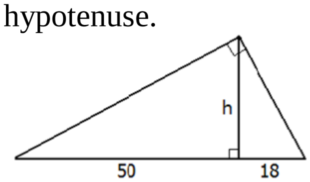
3. In triangle ABC, if DE//BC and AB=14, BC=15, DE=6, find the length of AD.



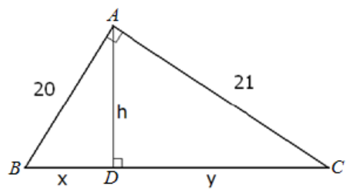
4. Prove the following statement. Let ABC be any right triangle, the right angle at point A. The altitude drawn from C to the hypotenuse splits the triangle into two right triangles that are similar to each other and to the original triangle.



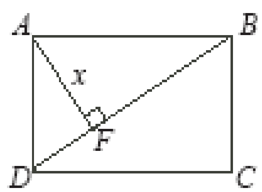
5. The picture below shows a right triangle. Find the length of h, the height drawn to the hypotenuse.



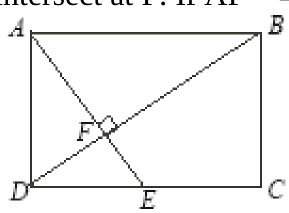
6. Find x, y, and h based on the picture below.



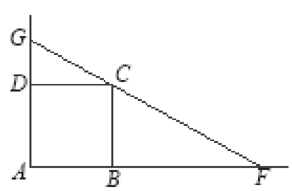
7. In rectangle ABCD, F is on diagonal BD so that AF is perpendicular to BD. Also, BC = 30, CD = 40 and AF = x. Determine the value of x.



8. In rectangle ABCD, point E is on side DC. Line segments AE and BD are perpendicular and intersect at F. If AF = 4 and DF = 2, determine the area of quadrilateral BCEF.



9. In the diagram, line segment FCG passes through vertex C of square ABCD, with F lying on AB extended and G lying on AD extended. Prove that $\frac{1}{AB} = \frac{1}{AF} + \frac{1}{AG}$.



10. In triangle *ABC*, $\angle ABC = 90°$. Rectangle *DEFG* is inscribed in ΔABC, as shown. Squares *JKGH* and *MLFN* are inscribed in ΔAGD and ΔCFE, respectively. If the side length of *JHGK* is *v*, the side length of *MLFN* is *w*, and *DG* = *u*, prove that $u = v + w$.