Lesson 8 (on Calculus)

Unit 8 – Curves Sketching

Algorithm for Curve Sketching

1. Domain

a. denominator ≠ 0 (rational functions)
b. radicand ≥ 0 (even roots)
c. logarithmic argument > 0 (logarithmic functions)

2. Intercepts

a. f (x) = 0 (x-int or zeroes)
b. numerator = 0 (for rational functions) Ö
c. y − int = f (0) (if exists)

3. Symmetry

a.f (−x) = f (x) (even functions are symmetric about the y-axis)
b. f (−x) = − f (x) (odd functions are symmetric about the origin)
c. f (x + T) = f (x) (periodic functions have cycles)

4. Asymptotes

a. compute $\lim\limits_{x \to \pm\infty} f(x)$ (horizontal asymptote)

b. compute $\lim\limits_{x \to a^-} f(x)$, $\lim\limits_{x \to a^+} f(x)$ (vertical asymptote where a is a zero of the denominator but not of the numerator)

c. compute long division (to find the oblique asymptotes for rational functions)

5. First Derivative

a. compute f '(x)

b. find critical points (f '(x) = 0 or f '(x) DNE)

c. create the sign chart for f '(x)

d. find intervals of increase/decrease

e. find the local extrema (using first derivative test) and global extrema (if function is defined on a closed interval)

6. Second Derivative

a. compute f ' '(x) Ö find points where f ' '(x) = 0 or f ' '(x) DNE

b. create the sign chart for f ''(x)

c. find points of inflection

d. find intervals of concavity upward/downward

e. check the local extrema using the second derivative test (if necessary)

7. Curve Sketching

a. use broken lines to draw the asymptotes

b. plot x- and y- intercepts, extrema, and inflection points

c. draw the curve near the asymptotes

d. sketch the curve

Ex. Sketch the graph for y = $\frac{x^2}{x-1}$.

Domain: x∈ R \{1} .

Intercepts: f (x) = 0 ⇨ x=0,   f(0) = 0.

Symmetry: f(-x) = $\frac{x^2}{-x-1}$ ⇨ The function y = f (x) is neither odd nor even.

Asymptotes: f(x) = x + 1 + $\frac{1}{x-1}$ ⇨ y = x +1 is the equation of the oblique asymptote.

First Derivative: *f′(x)* = $\frac{x(x-2)}{(x-1)^2}$, *f′(x)* = 0 ⇨ x = 0 or x = 2

f′(x) DNE at x=1

f(0) = 0, f(2) = 4, f(1) DNE

x		0		1		2	
$f(x)$	↗	0	↘	DNE	↘	4	↗
$f'(x)$	+	0	-	DNE	-	0	+

(0,0) is a local maximum point.

(1,4) is a local minimum point.

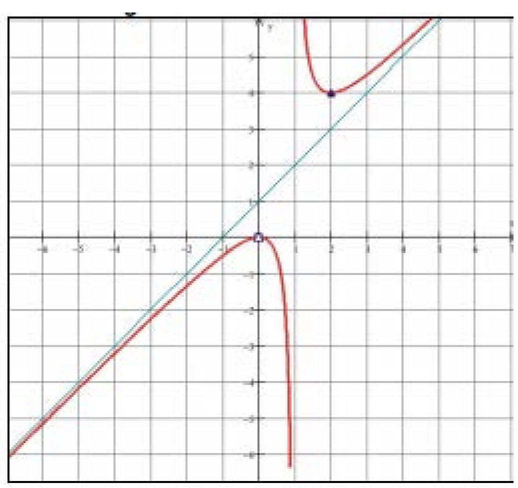
Second Derivative: $f''(x) = \frac{2}{(x-1)^3}$

$f''(1)$ DNE

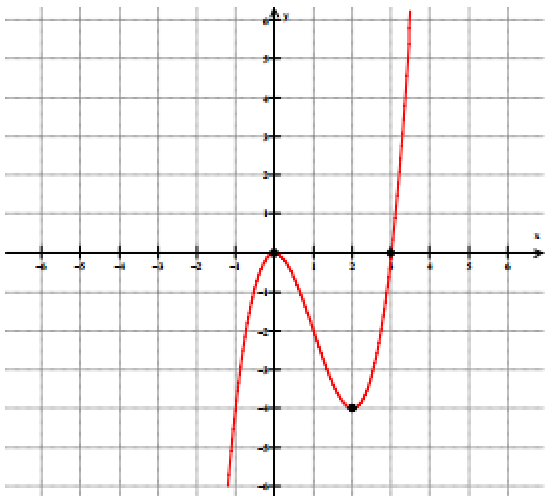
x		1	
$f(x)$	∩	DNE	∪
$f''(x)$	-	DNE	+

There are no inflection points.

Curve Sketching:



Ex. In the next figure is given the graph of the derivative f '(x) of a function f (x).



a) Find intervals where the function f (x) is increasing or decreasing.

The function f (x) is increasing where f '(x) > 0: (3, ∞) .

The function f (x) is decreasing where f '(x) < 0: (−∞, 0) or (0, 3).

b) Find intervals where the graph of f (x) is concave upward or downward.

The graph of f (x) is concave upward where f '(x) is increasing: (−∞, 0) or (2, ∞) .

The graph of f (x) is concave downward where f '(x) is decreasing: (0, 2) .