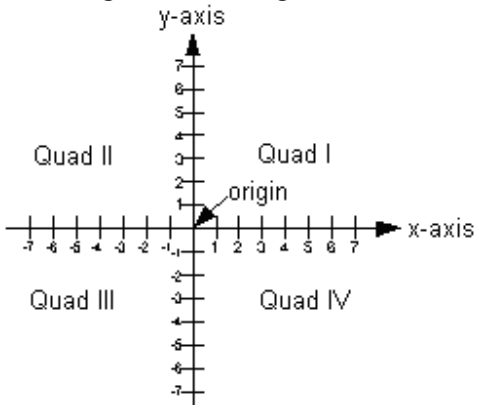
Chapter 4 Linear and Non–Linear Relations (1)

1. Rectangular Coordinate System

The following is the rectangular coordinate system:



It is made up of two number lines:

- The horizontal number line is the *x*– axis.
- The vertical number line is the *y*– axis.
- The origin is where the two intersect. This is where both number lines are 0.

It is split into four quadrants which are marked on this graph with Roman numerals.
Each point on the graph is associated with an ordered pair.

When dealing with an *x, y* graph, the *x* coordinate is always first and the *y* coordinate is always second in the ordered pair (*x, y*). It is a solution to an equation in two variables. Even though there are two values in the ordered pair, be careful that it associates to ONLY ONE point on the graph, the point lines up with both the *x* value of the ordered pair (*x*–axis) and the *y* value of the ordered pair (*y*–axis).

Example 1: Plot the ordered pairs and name the quadrant or axis in which the point lies. A(2, 3), B(–1, 2), C(–3, –4), D(2, 0), and E(0, 5).

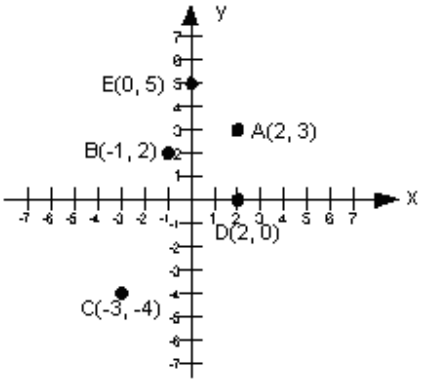
A(2, 3) lies in quadrant I.

B(–1, 2) lies in quadrant II.

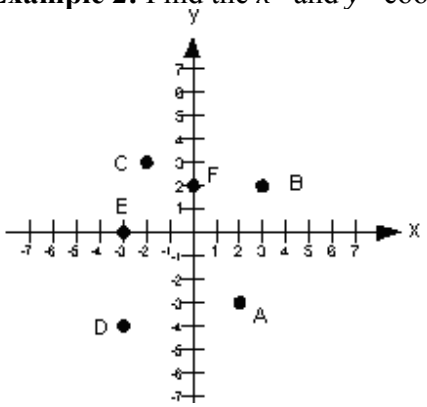
C(–3, –4) lies in quadrant III.

D(2, 0) lies on the *x*–axis.

E(0, 5) lies on the *y*–axis.



Example 2: Find the *x*– and *y*– coordinates of the following labeled points



Remember that each ordered pair associates with only one point on the graph. Just line up the *x* value and then the *y* value to get your ordered pair.

Since point A corresponds to 2 on the *x*–axis and –3 on the *y*–axis, then A's ordered pair is (2, –3).
Since point B corresponds to 3 on the *x*–axis and 2 on the *y*–axis, then B's ordered pair is (3, 2).
Since point C corresponds to –2 on the *x*–axis and 3 on the *y*–axis, then C's ordered pair is (–2, 3).
Since point D corresponds to –3 on the *x*–axis and – 4 on the *y*–axis, then D's ordered pair is (–3, – 4).
Since point E corresponds to –3 on the *x*–axis and 0 on the *y*–axis, then E's ordered pair is (–3, 0).
Since point F corresponds to 0 on the *x*–axis and 2 on the *y*–axis, then F's ordered pair is (0, 2).

2. Solutions of Equations in Two Variables

The **solutions** to equations in two variables consist of two values that when substituted into their corresponding variables in the equation, make a true statement.

In other words, if your equation has two variables *x* and *y*, and you plug in a value for *x* and its corresponding value for *y* and the mathematical statement comes out to be true, then the *x* and *y* value that you plugged in would together be a solution to the equation.

Equations in two variables can have more than one solution.

We usually write the solutions to equations in two variables in ordered pairs.

Example 1: Determine whether each ordered pair is a solution of the given equation. $y = 5x - 7$: (2, 3), (1, 5), (–1, –12)

Let's start with the ordered pair (2, 3).
Which number is the *x* value and which one is the *y* value? If you said $x = 2$ and $y = 3$, you are correct!

Let's plug (2, 3) into the equation and see what we get:

LS = y	RS = 5x – 7
= 3	= 5(2) – 7
	= 3

LS = RS
Therefore, (2, 3) is a solution to the equation y = 5x – 7.

Now let's take a look at (1, 5).
Which number is the *x* value and which one is the *y* value? If you said $x = 1$ and $y = 5$, you are right!

Let's plug (1, 5) into the equation and see what we get:

LS = y	RS = 5x – 7
= 5	= 5(1) – 7
	= -2

LS ≠ RS
Whoops, it looks like we have ourselves a FALSE statement. This means that (1, 5) is NOT a solution to the equation $5x – 7$.

Now let's look at (–1, –12).
Which number is the *x* value and which one is the *y* value? If you said $x = –1$ and $y = –12$, you are right!

Let's plug (–1, –12) into the equation and see what we get:

LS = y	RS = 5x – 7
= -12	= 5(-1) – 7
	= -12

LS = RS
Therefore, (–1, –12) is another solution to the equation $y = 5x – 7$.

Note that you were only given three ordered pairs to check, however, there are an infinite number of solutions to this equation. It would very cumbersome to find them all.

Example 2: Determine whether each ordered pair is a solution of the given equation. $x = 3$;
a) (3, 5) b) (2, 3) c) (3, 4)

This equation looks a little different than the one on example 3. In this equation, we only have an *x* value to plug in. So as long as the *x* value is 3, then we have a solution to the equation. It doesn't matter what *y*'s value is.

a) Let's start with the ordered pair (3, 5).
Which number is the *x* value and which one is the *y* value? If you said $x = 3$ and $y = 5$, you are correct!

This is a TRUE statement since x = 3, so (3, 5) is a solution to the equation $x = 3$.

b) Now let's take a look at (2, 3).
Since x is not 3, (2, 3) is NOT a solution to the equation $x = 3$.

c) Now let's look at (3, 4).
We have another TRUE statement since x is 3. This means (3, 4) is another solution to the equation $x = 3$.

Note that you were only given three ordered pairs to check, however, there are an infinite number of solutions to this equation. It would be very cumbersome to find them all.

3. Finding the Corresponding Value in an Ordered Pair Given One Variable's Value

Again, the solutions to equations in two variables consist of two values that when substituted into their corresponding variables in the equation, make a true statement.
Sometimes you are given a value of one of the variables and you need to find the corresponding value of the other variable.

The steps involved in doing that are:
Step 1: Plug given value for variable into equation.
Step 2: Solve the equation for the remaining variable.

Example 1: Complete each ordered pair so that it is a solution of the equation
2x + 3y = 5: (1,) and (, –1).

In the ordered pair (1,), is 1 that is given the *x* or the *y* value? If you said *x*, you are correct.
Plugging in 1 for *x* into the given equation and solving for *y* we get:

$$\begin{aligned} 2x + 3y &= 5 \\ 2 + 3y &= 5 \\ 3y &= 3 \\ y &= 1 \end{aligned} \qquad \begin{aligned} 2(1) + 3y &= 5 \\ 2 + 3y - 2 &= 5 - 2 \\ \frac{3y}{3} &= \frac{3}{3} \end{aligned}$$

So, the ordered pair (1, 1) would be a solution to the given equation.

In the ordered pair (, –1), is the –1 that is given the *x* or the *y* value?
If you said *y*, you are correct.
Plugging in –1 for *y* into the given equation and solving for *x* we get:

$$\begin{aligned} 2x + 3y &= 5 \\ 2x - 3 &= 5 \\ 2x &= 8 \\ x &= 4 \end{aligned} \qquad \begin{aligned} 2x + 3(-1) &= 5 \\ 2x - 3 + 3 &= 5 + 3 \\ \frac{2x}{2} &= \frac{8}{2} \end{aligned}$$

So, the ordered pair (4, –1) would be another solution to the given equation.

Example 2: Complete the table of values for the equation $x = -\frac{1}{2}$.

x	y
	0
	–1
-1/2	

We need to find the corresponding values of our variables that are solutions to the given equation.

Plugging in 0 for y into the given equation and solving for x we get: $x = -\frac{1}{2}$

So, the ordered pair (–1/2, 0) would be a solution to the given equation.

Since there is no "y" in the equation, no matter what y is, it is always $x = -\frac{1}{2}$. Similarly, if given that x = - 1/2, y can be any number because it doesn't depend on y.

Filling in the table we get:

x	y
–1/2	0
–1/2	–1
–1/2	Any #

4. Graphing a Linear Equation by Table of Values (TOV)

If the equation is linear:

Step 1: Find at least three ordered pair solutions. Organize them into a table.
You do this by plugging in at least ANY three values for x and find their corresponding y values.

Yes, it can be ANY three values you want, 1, –3, or even 10,000. However, we usually take the easier ones which should include a positive number, a negative number, and zero. Remember there are an infinite number of solutions. As long as you find the corresponding y value that goes with each x, you have a solution.

Step 2: Plot the points found in step 1.
Remember that each ordered pair corresponds to only one point on the graph.

The point lines up with both the x value of the ordered pair (x–axis) and the y value of the ordered pair (y–axis).

Step 3: Draw the graph.
A linear equation will graph as a straight line.
If you know it is a linear equation and your points don't line up, then you either need to check your math in step 1 and/or that you plotted all the points found correctly.

Example 1: Graph the linear equation $y = 5x - 3$.

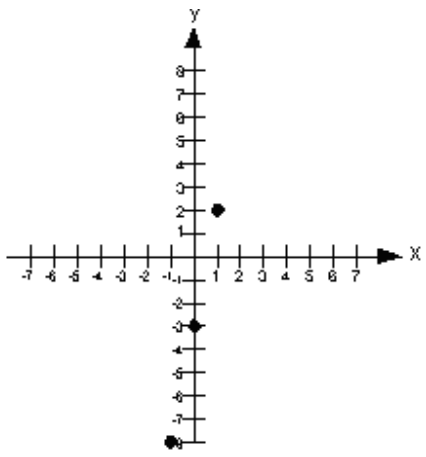
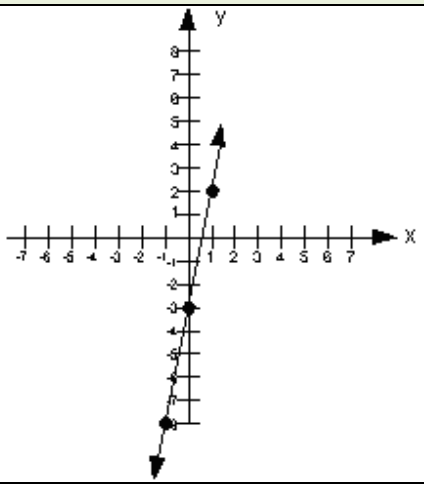
Step 1: Find three ordered pair solutions.
We are going to use a chart to organize my information. A chart keeps track of the x values that you are using and the corresponding y value found when you used a particular x value.

If you do this step the same each time, then it will make it easier for you to remember how to do it.

We usually pick out three points when I know we are dealing with a line. The three x values we are going to use are –1, 0, and 1. We want to keep it as simple as possible.

The following is the table of values we ended up with after plugging in the values we mentioned for x.

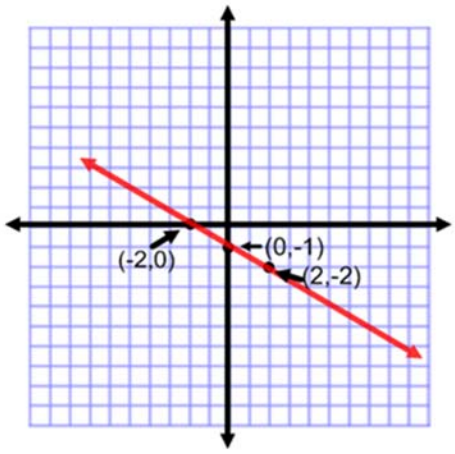
x	$y = 5x - 3$
–1	y = 5(–1) – 3 = –8
0	y = 5(0) – 3 = –3
1	y = 5(1) – 3 = 2

Step 2: Plot the points found in step 1.	**Step 3:** Draw the graph.
	

Example 2: Graph the line for y = $-\frac{1}{2}x - 1$

If the coefficient is a fraction, choose 0 or a multiple of the denominator as your x coordinates to produce an integer y coordinate!

x	-½x -1	y	Ordered Pairs
-2	-½(-2) - 1	0	(-2,0)
0	-½(0)-1	-1	(0,-1)
2	-½(2)-1	-2	(2,-2)



Practice in class

1. Use table of values to graph the following lines.

a) y = 2x + 5

b) y = -4x – 3