First Name:_____ Last Name: _____ Student ID: _____

Relationships between Points, Lines and Planes (1)

1. Without solving, determine if the given line intersects the given plane at a single point or not:

a. **π**: 3x - y + 4z - 8 = 0

l: (x, y, z) = (3, 0, 5) + t(7, -11, 18)

b. **π**: $\vec{r}$ = (0, 0, 1) + t(1, 1, -1) + u(13, 3, 2)

l: $\vec{r}$ = (5, -1, 4) + t(1, -2, 3)

c. **π**: 6x -2y + 2z +6 = 0

l: $\vec{r}$ = (4, 12, -19) + t(2, -3, -9)

2. For each part of Question 1, find the intersection of the line with the plane.

3. Find the intersection of the line and the plane.

a. $\frac{x+4}{3}$ = z, y = 0 and $x-2y-3z+4=0$

b. $\begin{cases} x = 2t \\ y = 1 - t \\ z = -4 + t \end{cases}$ and $\begin{cases} x = 2 + 2m + 2n \\ y = 3 - m \\ z = -5 + m - n \end{cases}$

4. Which of the following lines lie in the plane $3x-y-3z-12=0$?

a. $\vec{r}=(2,-9,1)+t(1,0,1)$

b. $\vec{r}=(4,0,0)+t(2,3,-1)$

c. $\vec{r}=(1,5,-2)+t(3,2,1)$

5. Find the point on the line $\vec{r}=(6,7,-8)+t(-2,4,1)$ that is also

a. on the *xy* plane.
b. on the *yz* plane.

6. Find the point on the plane $x-2y+z-8=0$ that is closest to (10,12,4) (Hint: it is on a line that passes through the given point, perpendicular to the plane.)

7. Find the value(s) of a and b such that the given plane and line
$4x-2y-z+12=0$, $\vec{r}=(3,4,a)+t(7,b,6)$

a. Intersect at every point on the line.
b. Intersect at a single point.
c. Do not intersect.

8. For each pair of planes, determine whether they intersect in a line, are coincident, or are parallel and distinct:

a. $2x-y+3z-8=0$ and $6x-3y+9z+3=0$
b. $4x+3y+z-8=0$ and $3x-y-2z-2=0$
c. $12x-6y+15z-9=0$ and $20x-10y+25z-15=0$

9. Find the intersection of the two planes given.

a. $2x-2y+5z+10=0$ and $2x+y-4z+7=0$
b. $3x-2y+5z+3=0$ and $6x-4y+10z+7=0$

10. Find the Cartesian equation of the plane that is perpendicular to the plane $\vec{r}=(4,-5,2)+s(2,1,3)+t(-1,4,0)$ and intersects it at the line $\vec{r}=(4,-5,2)+t(1,-1,1)$.

11. Given the two planes below, find the value(s) of k that make the two planes intersect in the desired way, if possible. Explain your reasoning.

$6x-9y+15z=21$ and $10x-15y+kz=35$

a. Along a whole plane.

b. Along a single line.

c. No intersection.

d. At a single point.

Extra Questions

1. Find the scalar equations of two planes which intersect along the line $\vec{r} = (3,2,-1)+t(1,-2,1)$.

2. Give the equations of three planes which all intersect at the y-axis.

3. Find the intersection of the two planes given.

a. $\vec{r} = (1,2,3)+t(1,1,1)+u(2,1,0)$ and $\vec{r} = (0,0,1)+t(0,-1,1)+u(1,2,-1)$

b. $6x-10y+4z+30= 0$ and $15x-25y+10z+75= 0$