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Applications of Vectors (1)

1. Calculate the dot product, $\vec{u} \cdot \vec{v}$, to one decimal place accuracy, given that

- a. $|\vec{u}|=10, |\vec{v}|=2$, and the angle between \vec{u} and \vec{v} is 40°
- b. $\vec{u} = 3\hat{i} - \hat{j} + 4\hat{k}$ and $\vec{v} = -\hat{i} + 2\hat{j} + 5\hat{k}$

2. If the vectors $2\vec{a} + \vec{b}$ and $\frac{1}{2}\vec{a} - \vec{b}$ are perpendicular to each other and $2|\vec{b}| = 3|\vec{a}|$ find the angle $\theta = \angle(\vec{a}, \vec{b})$.

3. Find the angle between each pair of vectors:

a. $\vec{u} = 3\hat{i} - \hat{j}$ and $\vec{v} = -\hat{i} + 2\hat{j}$

b. $(2, 1, -3)$ and $(1, 0, 4)$

4. For each of the following pairs of vectors, find the value of a which makes u orthogonal to v :

- a. $u = (3, -4)$ and $v = (a, 6)$
- b. $u = 2\hat{i} + \hat{j} + 3\hat{k}$ and $v = a\hat{i} + 2\hat{j} - \hat{k}$
- c. $u = (3, a, -2)$ and $v = (1 - a, -3, 4)$

5. Use the dot product to determine if $\triangle ABC$ is right-angled, given the coordinates of its vertices. If it is, state which angle measured 90° .

a. $A(3, -1), B(0, -2), C(2, 0)$

b. $A(1, -1, 4), B(-2, 5, 3), C(3, 0, 4)$

6. The parallelogram $PQRS$ has vertices $P(7, 12), R(20, 5)$, and $S(4, 3)$.

- a. Find the coordinates of Q .
- b. Find the measure of $\angle PSR$
- c. Calculate the area of the parallelogram.

7. If \vec{u} has magnitude 11, \vec{v} has magnitude 5, and the angle between \vec{u} and \vec{v} is 140° , what is the magnitude of $\vec{u} \times \vec{v}$ to one decimal place accuracy?

8. Find the cross product $\vec{u} \times \vec{v}$ given that

a. $\vec{u} = 3\hat{i} - \hat{j} + 4\hat{k}$ and $\vec{v} = -\hat{i} + 2\hat{j} + 5\hat{k}$

b. $\vec{u} = (1, 2, 3)$ and $\vec{v} = (4, -1, 5)$

9. Given the vectors $\vec{u} = (-2, 1, -1)$ and $\vec{v} = (-1, 2, -1)$

- Find a unit vector perpendicular to both \vec{u} and \vec{v} .
- Find two vectors of magnitude 11 which are perpendicular to both \vec{u} and \vec{v} .

10. For each pair of vectors $u \rightarrow$ and $v \rightarrow$, find the vector projection of $u \rightarrow$ on $v \rightarrow$.

- a. $u \rightarrow = (-2, 1, -1)$ and $v \rightarrow = (2, 1, 3)$
- b. $u \rightarrow = (-2, 1, -1)$ and $v \rightarrow = (4, -2, 2)$

11. For each pair of vectors $u \rightarrow$ and $v \rightarrow$ in Question 11, find the scalar projection of $u \rightarrow$ on $v \rightarrow$.

12. Determine if the vectors $(1, 3, 2)$, $(5, 0, -1)$, and $(-4, 3, 3)$ are coplanar.

13. Find the volume of the parallelepiped defined by the vectors $\vec{a} = (0, 1, -3)$, $\vec{b} = (1, 2, 3)$ and $\vec{c} = (-1, 0, 1)$.

14. Find all unit vectors perpendicular to $(1, 2, 3)$ that make equal angles with the unit vectors \hat{i} and \hat{j} .