Statistics and Probability

1. Collect and display Data

Double Bar and Double Line Graphs

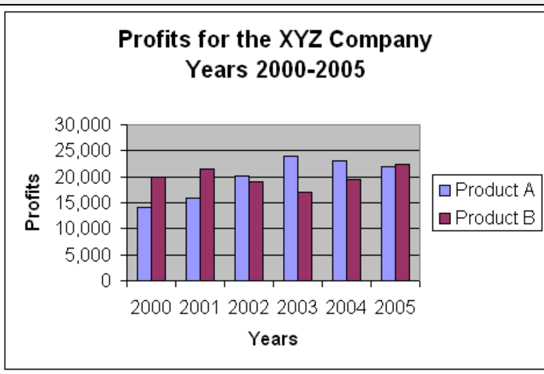
When collecting, organizing and making sense of raw data, often it is best to make use of bar and line graphs. If comparing two sets of data we use double bar and double line graphs.

Example

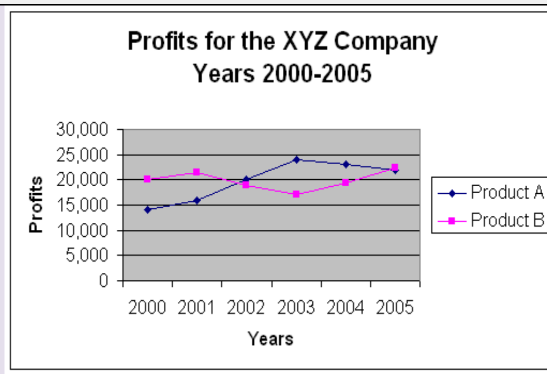
Using the following data, construct a double bar graph and a double line graph.
The profits for the XYZ Company from

2000 to 2005 were as follows:
2000: Product A - $14,000, Product B - $20,000
2001: Product A - $16,000, Product B - $21,500
2002: Product A - $20,100, Product B - $19,000
2003: Product A - $24,000, Product B - $17,000
2004: Product A - $23,000, Product B - $19,500
2005: Product A - $22,000, Product B - $22,500

Double bar graph



Double line graph



2. Analysis of Data

- Mean: (same as average) Add the numbers and divide by the number of numbers.
- Mode: The number that occurs the most.
- Median: The number is the middle of the data set (after the numbers are ordered least to greatest or vice versa).
- Range: The difference between the highest and lowest numbers (subtract).

Example

A car salesman sold cars at the following prices: $31,000, $35,000, $37,000, $31,000, $31,000, $4,000 Find the mean, mode, median and range of these prices.	
Mean:	**Mode:**
The sum of the prices is $169,000. Divide by 6 (the number of prices) and we round to $28, 167.	The price that occurs the most is $31,000.
Median:	**Range:**
When we order the prices, our list is: $4,000, $31,000, $31,000, $31,000, $35,000, $37,000 This list actually has two numbers in the middle: $31,000 and $31,000. When this happens we take the average of the two and get $31,000.	The difference of the lowest and highest prices is $37,000 - $4,000 = $33,000.

3. Probability

Probability is defined as the measure of the *"likelihood"* of an event, or *outcome*.
Another way of saying that is, "What is the *chance* of something happening?"

Probability is all about the *certainty*, the *uncertainty*, and the *prediction* of something happening. Some events are *impossible*, other events are *certain to occur*, while many are *possible, but not certain* to occur.

For examples

1) Impossible Events:
Moving Mt. Everest to New York State
The sun turning to ice
Oscar the Grouch becoming President of the U.S.
Rolling a 7 on a single die
Finding the color purple on the American flag.

2) Certain Events:
Rolling a 1, 2, 3, 4, 5, or 6 on a single die
Today will be 24 hours long
The number 10 is less than the number 11
George Washington was the first U.S. President
Flipping a coin and getting either a head or a tail

3) Possible, But Not Certain Events:
Picking the King of Hearts from a deck of cards
Landing on "Boardwalk" in *Monopoly*
Rolling a 3 on a single die
Getting a "100" on a Math test
Flipping a coin and getting a "head"

Now let's look at how we determine the *Probability* of something happening

4. Probability of an event

The probability of an event is the chance that it will occur, expressed as a ratio of a specific event to all possible events.

$$Probability = \frac{number\ of\ \ actual\ \ events}{number\ of\ \ possible\ \ events}$$

Or....

$$Probability = \frac{number\ of\ \ favorable\ outcomes}{number\ of\ \ possible\ outcomes}$$

- The probability of an event that is *certain* to occur is 1.
- The probability of an event that is *impossible* to occur is 0.
- The probability of an event that is *possible* is >0 and <1

Example
For this experiment we have a bag containing 10 cards. Each card is numbered differently from 1 - 10. In each trial we will reach into the bag and pull out exactly 1 card, look at it, and then return it to the bag.

1) What is the probability of choosing a "7"?

Let's see, there is only 1 *favorable outcome*, because there is only 1 - "7" in the bag. There are a total of *10 possible outcomes* because we could have selected any of the 10 cards.
Therefore the probability of choosing the "7" is:

$$P(7) = \frac{1}{10}$$

2) What is the probability of choosing an *<u>even</u>* numbered card?

In the bag there are 5 *even* numbered cards: 2,4,6,8,and 10.
That means there are *5 favorable outcomes* out of the *10 possible outcomes.*
Therefore the probability of choosing an even card is:

$$P(Even) = \frac{5}{10} = \frac{1}{2}$$

3) What is the probability of choosing a card numbered 1 - 10?

After all, there are 10 cards in the bag, and each card is numbered differently from 1 - 10
Therefore the probability of choosing a card numbered 1-10:

$$P(1-10)=\frac{10}{10}=1$$

4) What is the probability of choosing a "12"?

In the bag there are *no* cards with the number "12" on it. Therefore the number of favorable outcomes is 0.
So, the probability of choosing a "12" is"

$$P(12)=\frac{0}{10}=0$$

Remember:
The sum of the probability of an event happening and the probability of an event not happening is ALWAYS 1!

Example
In this experiment we will simply roll a single die one time, record the result, and roll the die again.

What is the probability of rolling a "3"?
Since a die has 6 faces, but only one of them a "3"

$$P(3)=\frac{1}{6}$$

Now, what is the probability of NOT rolling a "3"?
Since there are 5 other possible outcomes

$$P(not3)=\frac{5}{6}$$

Do you see that if you add the probability of rolling the "3" to the probability of NOT rolling the "3" that the sum is 1?

$$\frac{1}{6}+\frac{5}{6}=1!$$

That means that if you know the probability of something happening, to determine the probability of it NOT happening, simply subtract the favorable probability from 1.

If the probability of it raining today is 2/5, the probability of it NOT raining is 1 - 2/5 = 3/5!

5. Probability of two or more Independent Events

1) Compound event
A compound event consists of two or more events.

There are a couple of things to note about this experiment. Choosing two pairs of socks from the same drawer is a compound event. Since the first pair was replaced, choosing a red pair on the first try has no effect on the probability of choosing a red pair on the second try. Therefore, these events are independent.

2) Independent event

Two events, A and B, are **independent** if the fact that A occurs does not affect the probability of B occurring.

Some other examples of independent events are:

- Landing on heads after tossing a coin AND rolling a 5 on a single 6-sided die.
- Choosing a marble from a jar AND landing on heads after tossing a coin.
- Choosing a 3 from a deck of cards, replacing it, AND then choosing an ace as the second card.
- Rolling a 4 on a single 6-sided die, AND then rolling a 1 on a second roll of the die

To find the probability of two independent events that occur in sequence, find the probability of each event occurring separately, and then multiply the probabilities. This multiplication rule is defined symbolically below. Note that multiplication is represented by AND.

Multiplication Rule:

When two events, A and B, are independent, the probability of both occurring is:
P(A and B) = P(A) · P(B)

(Note: Another multiplication rule will be introduced in the next lesson.) Now we can apply this rule to find the probability for Experiment 1.

Example

A coin is tossed and a single 6-sided die is rolled. Find the probability of landing on the head side of the coin and rolling a 3 on the die.

Probabilities:

$$P(head) = \frac{1}{2}; \quad P(3) = \frac{1}{6}.$$

$$P(head\ \ and\ \ 3) = P(head) \cdot P(3) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

Questions in class

1. One stamp is randomly selected from a 10 x 10 sheet of stamps. What is the probability that the stamp is not on the border?

2. What is the coefficient of correlation for the data set below?
Coefficient of correlation (r): measure of the strength and direction of the linear relationship between two variables. Its value varies between -1 and 1: 1 means perfect correlation, 0 means no correlation, positive values means the relationship is positive (when one goes up so does the other), negative values mean the relationship is negative (when one goes up the other goes down).

x	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	3.83	3.67	3.51	3.35	3.19	3.03	2.87	2.71	2.55	2.39	2.23	2.07	1.91

3. Two numbers are chosen randomly without replacement from the set given here:
{-2, -4/3, -1/2, 0, 1/2, 3/4, 3}
What is the probability that the numbers will be the slopes of two perpendicular lines?

4. The five tiles shown below are in a bag. $\boxed{1}$ $\boxed{2}$ $\boxed{3}$ $\boxed{4}$ $\boxed{5}$
Three of the tiles are drawn at random (without replacement). What is the probability that the 3 numbers could be lengths (in decimeters) of the sides of a triangle?

5. Numbers that are palindromes read the same forward and backward. For example, 30203 is a five-digit palindrome. If a single number is chosen randomly from the set of all three-digit numbers, find the probability that the number will be a palindrome.

6. Two people are to be selected from a group of 4 men and 2 women. What is the probability that the two women will be selected?

7. Tennessee license plates consist of 3 letters followed by 3 digits (0 – 9), for example ABA121. How many such plates have no repeated digit and do not use the letter Z?

8. Two prime numbers are randomly selected without replacement from among the first eight prime numbers. What is the probability that their sum will be 24?

9. The following table describes a group of musicians. If three musicians are selected at random without replacement, find the probability that at least one is playing piano.

	flute	piano
male	6	3
female	2	4

10. If the probability that Julie makes her next shot in the basketball game is .36, then what is the probability that Julie does not make the shot?

11. From a standard deck of 52 playing cards a single card is drawn, recorded and then put back in the deck. Find the probability of each of the following events.
(a) P(a "face" card); (b) P(a "4"); (c) P(a red "15")