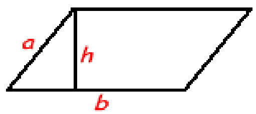
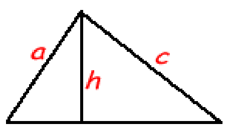
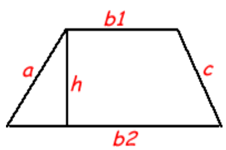
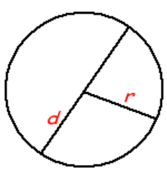
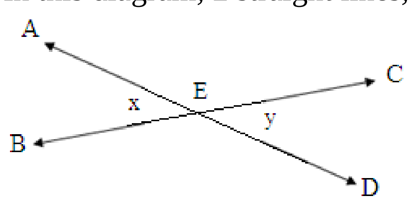
# Geometry 1

## 1. Geometric formula

| Shapes | Formula |
|---|---|
|   | Rectangle:<br>Area = Length X Width<br>A = lw<br>Perimeter = 2 X Lengths + 2 X Widths<br>P = 2l + 2w |
|   | Parallelogram<br>Area = Base X Height<br>a = bh |
|   | Triangle<br>Area = 1/2 of the base X the height<br>a = 1/2 bh<br>Perimeter = a + b + c<br>(add the length of the three sides) |
|   | Trapezoid<br>area $A = \left(\frac{b1 + b2}{2}\right)h$<br>Perimeter = a + b1 + b2 + c |
|   | Circle Try the Online tool.<br>The distance around the circle is a circumference.<br>The distance across the circle is the diameter (d).<br>The radius (r) is the distance from the center to a point on the circle. ($\pi = 3.14$) More about circles.<br>$d = 2r$, $c = \pi d = 2\pi r$, $A = \pi r^2$, ($\pi = 3.14$) |

## 2. Vertical Angles

In this diagram, 2 straight lines, AB and CD, intersect at E

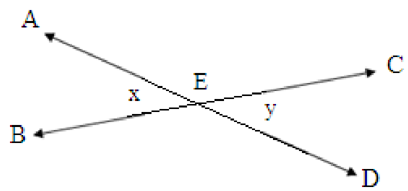



As you can see, when this occurs, 4 angles are created. Angles x and y are opposite one another.

The name given to a pair of angles such as these is *vertical angles*.

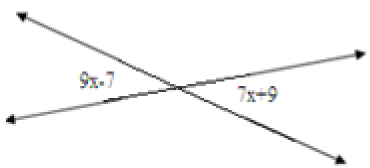
If we were to measure each of these angles, we would find the measure of both angle x and y to be the same. Therefore we will state that when 2 lines intersect, the *vertical angles formed are equal* in measure.

**Example 1:** Let's assume that the measure of angle y is $42^0$. If angle "x" is represented by the expression "a + 16", can we solve for "a"?




Solution: If angle x = a + 16, and angle y = 42, we can set up the following equation:
a + 16 = 42.
This is a simple one-step equation, a + 16 = 42, a = 26.

**Example 2:** Find the value of x.




Solution: We can clearly see from the diagram that the angles represented by "9x - 7" and "7x + 9" are vertical angles. We also know that vertical angles are equal. Therefore we can set up the following equation:
9x - 7 = 7x + 9,     x = 8.

**3. Supplementary Angles**

If the sum of the measures of *two* angles is $180^0$, then the angles are called supplementary angles. Each angle is called the supplement of the other angle.

For example, if one angle measures $70^0$, then its supplement will measure $110^0 = (180\text{-}70)^0$.
Supplementary angles are a pair of angles, if more than 2 angles add up to $180^0$, they are not supplementary!

**Example 1:** Two angles are supplementary. One of the angles is three times as large as the other. Find the measure of both angles.

Solution: First, assign variables to represent the two angles.
Let x be the smaller angle
Let 3x be the larger angle

Second, write the equation.
3x + x = 180 (the sum of the angles is $180^0$)

Third, solve the equation.
$3x + x = 180$ (combine like terms), $x = 45$

Therefore, the measure of the smaller angle (x) is $45^0$ and the measure of the larger angle (3x) is $135^0$.

**Example 2:** Find the number of degrees in an angle which is $20^0$ less than four times its supplement.

Solution: Let $4x - 20$ be one of the angles.
Let x be the supplement of the angle
$(4x-20) + (x) = 180$, $x = 40$
Therefore, one of the angles (4x-20) is $140^0$, and the supplement of that angle is $(180 - 140) = 40^0$.

**Example 3:** Find the supplement of the angle which is represented by the expression $(4x - 60)^0$.

For this problem let's use the fact that the easiest way to find the supplement of an angle is to subtract the given angle from $180^0$.

So, if we let one of the angles be "x", and it's supplement be $(4x-60)^0$, then we should be able to find the other angle by solving the following equation:
$x = 180 - (4x - 60)$ (distribute the "-" sign), $x = 48^0$
Therefore the supplement is $48^0$

**4. Complementary Angles**

If the sum of the measures of two angles is $90^0$, then the angles are called *complementary angles*. Each of the angles is called the *complement* of the other angle.

For example, if one angle measures $30^0$, then the measure of it's *complement* is (90-30), or $60^0$.

Remember: *Complementary angles are a pair of angles.*
If the sum of the measure of 3 or more angles is $90^0$, that does not make them complementary angles!

**Example 1:** Two angles are complementary. The measure of one of the angles is $24^0$ greater than the measure of the other angle. Find the measure of each of the angles.

Let x = the smaller angle
Let x + 24 = the larger angle
$(x) + (x+24) = 90$, $x = 33$

Therefore x, the smaller angle, measures $33^0$ and x+24, the larger angle measures $57^0$.

**Example 2:** Find the number of degrees in angle which exceeds three times its complement by $22^0$.

Let x be the smaller complementary angle
Let 3x+22 be the larger complementary angle.

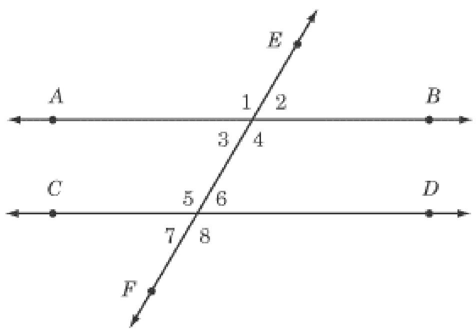
(x) + (3x+22) = 90, x = 17

Therefore the smaller angle, x, is $17^0$, and the larger angle, 3x+ 22, is $73^0$.

**5. Angles Formed by Parallel Lines Cut by a Transversal**

In the diagram below you can see that lines AB and CD are parallel and that the third line, EF is a transversal.

When this occurs you can also see that 8 different angles have been created which have been numbered 1 – 8.




It turns out that each of these angles can be paired with another, and that each pair of angles has a special name, as well as a special property.

**1) Corresponding Angles**

In the diagram above, the following pairs of angles are called *corresponding angles*:
<1 and <5;  <2 and <6;  <3 and <7;  <4 and <8

If you look at where each of the angles in a pair are located, you will notice that they are in the same relative position where the transversal intersects one of the parallel lines and the same point of intersection on the other parallel line.

In other words, the position of one of the angles *corresponds* to the position of the other angle in the pair.
As for their special property: Corresponding angles are equal.
< 1 = < 5;  < 2 = < 6;  < 3 = < 7;  < 4 = < 8

**2) Alternate Interior Angles**

In the diagram above, angles 3,4,5 and 6 are called *interior* angles because they are between the two parallel lines. If the angles lie on opposite sides of the transversal, but not on the same parallel line, they are called *alternate interior* angles.

The pairs of alternate interior angles in this diagram are:
< 3 and < 6;  < 4 and < 5

As for their special property: Alternate Interior angles are equal.
< 3 = < 6;  < 4 = < 5

**3) Alternate Exterior Angles**

In the diagram above, angles 1, 2, 7 and 8 are called *exterior* angles because they do not lie between the parallel lines.

Just like *alternate interior* angles, if the exterior angles lie on opposite sides of the transversal, but not at the same parallel line, they are called *alternate exterior angles*.

The pairs of alternate exterior angles in the diagram are:
< 1 and < 8;  < 2 and < 7

As for their special property: Alternate Exterior angles are equal.
< 1 = < 8;  < 2 = < 7

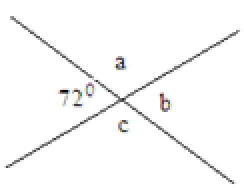
**4) Co-interior Angles - Interior Angles on the Same Side of the Transversal**

As the name clearly implies, the diagram above shows that :
< 3 and < 5;  < 4 and < 6
are pairs of angles which are not only interior angles, but also lie on the same side of the transversal.

The special property of these angle pairs: Interior Angles on the Same Side of the Transversal are SUPPLEMENTARY (THEIR SUM IS ALWAYS $180^0$)
<3 + <5 = $180^0$
<4 + <6 = $180^0$

**6. Calculate the Value of Missing Angles**

When two lines intersect four angles are formed. Look at the diagram below:




As you can see the two intersecting lines have created 4 angles.  The measure of one of the angles has been given as $72^0$. The other 3 angles are labeled a, b and c.

Our task is to determine the measure of each of those other 3 angles.

Determining the measure of angle "b" is as easy as remembering that the $72^0$ angle and angle "b" are known as *vertical angles*, and *vertical angles are equal*.

That means that the measure of angle "b" is also $72^0$.

We can also see that angles "a" and "c" are also vertical angles. So if we can determine the measure of one of the angles, the other will have the same measure.

To do this we need to see that the original $72^0$ angle and angle "a" are *supplementary angles*. Supplementary angles are two angles whose sum is $180^0$.

We know they are supplementary because the angles combine to create a straight line. And as you know *a straight line is a straight angle*, and *the measure of a straight angle is* $180^0$.

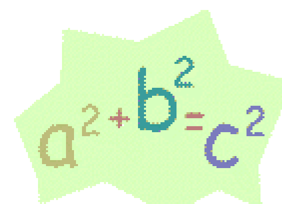
Now that we have reviewed the important vocabulary all we need to do is subtract 72 from 180, and that will give us the measure of angle "a".

180 - 72 = 108
The measure of angle "a" is $108^0$.

And, because angle "a" and angle "c" are vertical angles, the measure of angle "c" is also $108^0$.

**7. Pythagorean Theorem**

Over 2,500 years ago, a Greek mathematician named Pythagoras popularized the concept that a relationship exists between the hypotenuse and the legs of right triangles and that this relationship is true for ***all*** right triangles. The Egyptians knew of this concept, as it related to 3, 4, 5 right triangles, long before the time of Pythagoras. It was Pythagoras, however, who generalized the concept and who is attributed with its first geometrical demonstration. Thus, it has become known as the **Pythagorean Theorem.**

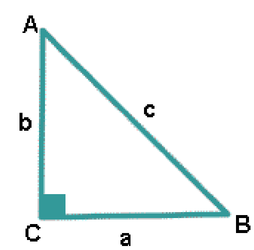


for right triangles

**"In any right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs."**

This relationship can be stated as: $c^2 = a^2 + b^2$ for any right triangle and is known as the Pythagorean Theorem.

*where a and b are legs, c is the hypotenuse (c is across from the right angle).*



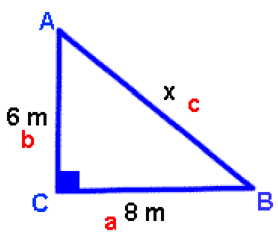

There are certain sets of numbers that have a very special property in relation to the Pythagorean Theorem. Not only do these numbers satisfy the Pythagorean Theorem, but **any** multiples of these numbers also satisfy the Pythagorean Theorem.

**For example:** the numbers 3, 4, and 5 satisfy the Pythagorean Theorem. If you multiply all three numbers by 2 (6, 8, and 10), these new numbers ALSO satisfy the Pythagorean Theorem.
The special sets of numbers that possess this property are called **Pythagorean Triples.**

The most common Pythagorean Triples are: (3, 4, 5); (5, 12, 13); (8, 15, 17)

**REMEMBER: The Pythagorean Theorem ONLY works in Right Triangles!**

**Example 1:** Find *x*.




$$c^2 = a^2 + b^2$$
$$x^2 = 64 + 36$$
$$x^2 = 100$$
$$\sqrt{x^2} = \sqrt{100}$$
$$x = 10$$

This problem could also be solved using the Pythagorean Triple 3, 4, 5. Since 6 is 2 times 3, and 8 is 2 times 4, then *x* must be 2 times 5.

**Example 2:** A triangle has sides 6, 7 and 10. Is it a right triangle?

Let *a* = 6, *b* = 7 and *c* = 10. The longest side MUST be the hypotenuse, so *c* = 10. Now, check to see if the Pythagorean Theorem is true.

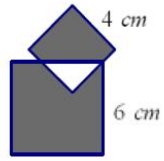
$$c^2 = a^2 + b^2$$
$$10^2 \; ? \; 6^2 + 7^2$$
$$100 \; ? \; 36 + 49$$
$$100 \neq 85$$

Since the Pythagorean Theorem is NOT true, this triangle is NOT a right triangle.

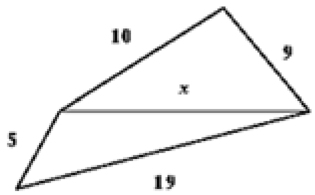
**►Questions in class**

1. A ladder 10m long runs from the top of a wall to a point 6m out from the foot of the wall. How high is the wall?

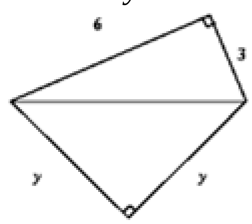
2. What is the difference between the areas of the shaded portions of the two squares?



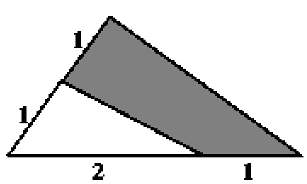

3. The distance *x* is known to be one of the following answers. Which is it?
   (A) 9   (B) 10   (C) 14   (D) 15   (E) 20



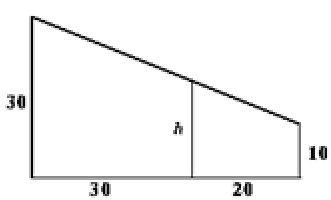

4. Determine *y* from the following figure (in which two 90 degree angles are indicated).



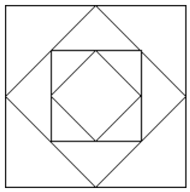

5. What fraction of the area of the large triangle is shaded?




6. What is the value of the length *h*?

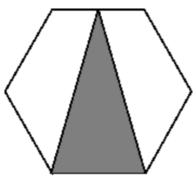


7. Calculate the total length of all of the line segments in the figure below if the sides of the small square in the center each measure 1 cm.

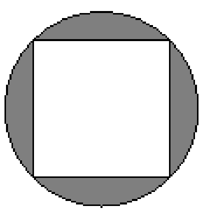


8. The area of a triangle with a height of 12 cm is 24 $cm^2$. How long is the base of the triangle?

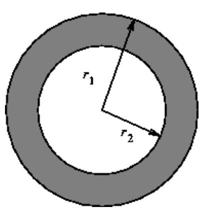
9. What fraction of the area of the regular hexagon is the shaded triangle?



10. What fraction of the area of the circle lies outside the square?



11. The shaded and unshaded areas in these concentric circles are equal. What is the ratio $r_1 / r_2$ of the larger to the smaller radius?




12. In the diagram, *ABCD* is a rectangle, and three circles are positioned as shown. Find the area of the shaded region, rounded to the nearest $cm^2$.