# Chapter 5 Analytic Geometry (1)

## 1. Intercepts

***x*–intercept -** is where the graph crosses the *x* axis. Algebraically, it is the x value when y = 0.

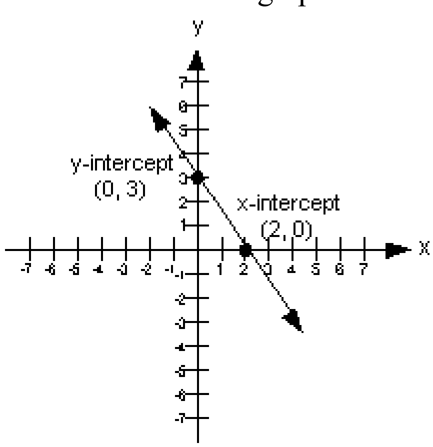
The word 'intercept' looks like the word 'intersect'. Think of it as where the graph intersects the x–axis.

With that in mind, what value is *y* always going to be on the *x*–intercept? No matter where you are on the *x*–axis, *y* value is always 0. We will use that bit of information to help us find the *x*–intercept when given an equation.

***y*–intercept -** is where the graph crosses the *y* axis. Algebraically, it is the y value when x = 0.

This time it is *x* value that is 0. Anywhere you would cross the *y*–axis, *x* value is always 0. We will use this tidbit to help us find the *y*–intercept when given an equation.

Below is an illustration of a graph of a linear function which highlights the *x* and *y* intercepts:




In the above illustration, the *x*–intercept is the point (2, 0) and the *y*–intercept is the point (0, 3).

## 2. Sketching a Graph Using Intercepts

**Step 1:** Find the *x*– and *y*– intercepts.
You find the *x*–intercept by plugging in 0 for *y* and solving for *x*.
You find the *y*–intercept by plugging in 0 for *x* and solving for *y*.

**Step 2:** Plot the intercepts and point(s) found in steps 1.
Remember that intercepts are points on the graph too. They are plotted just like any other point.

**Step 3:** Draw the graph by connecting the two points.
The graph of a linear function is a straight line, therefore, **two points determines a line**.

**Example 1:** Graph each linear function by finding the *x*– and *y*– intercepts. $y = 5 - 3x$

**Step 1:** Find the *x*– and *y*– intercepts.

Let's first find the *x*–intercept.

What value are we going to use for *y*? You are correct if you said $y = 0$.

$$y = 5 - 3x \qquad -5 = -3x$$
$$0 = 5 - 3x \qquad \frac{-5}{-3} = x$$

$x = 5/3$
The *x*–intercept is (5/3, 0).

Next, we will find the y– intercept.

What value are we going to plug in for *x*? If you said $x = 0$ you are right.

$$y = 5 - 3x \qquad y = 5 - 0$$
$$y = 5 - 3(0) \qquad y = 5$$

The *y*–intercept is (0, 5)

If you want to be more accurate in graphing, you can find one more point.
We can plug in any *x* value we want as long as we get the right corresponding *y* value and the function exists there.
Let's put in an easy number $x = 1$:

$$y = 5 - 3x \qquad y = 5 - 3$$
$$y = 5 - 3(1) \qquad y = 2$$

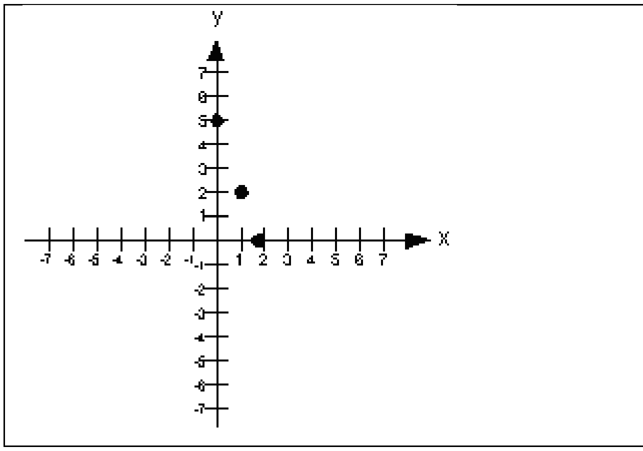
So the ordered pair (1, 2) is another solution to our function.

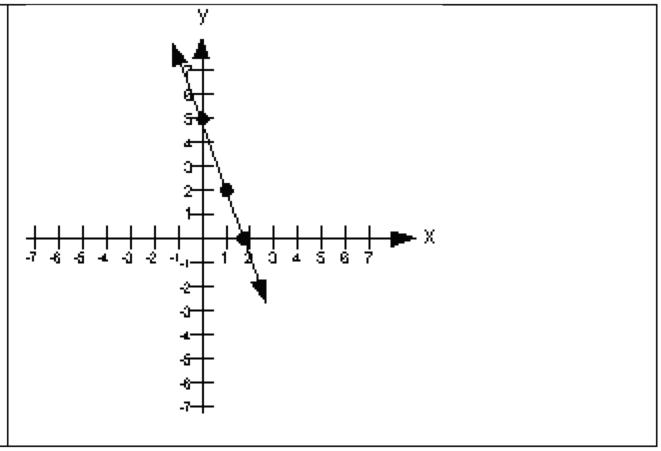
Note that we could have plugged in any value for x: 5, 10, –25..., but it is best to keep it as simple as possible.

The solutions that we found are:

| x | y | (x, y) |
|---|---|---|
| 5/3 | 0 | (5/3, 0) |
| 0 | 5 | (0, 5) |
| 1 | 2 | (1, 2) |

| | |
|---|---|
| **Step 2:** Plot the intercepts and point(s) found in steps 1. | **Step 3:** Draw the graph. |




**Example 2:** Graph each linear function by finding the *x*– and *y*– intercepts. $-3x = y$

**Step 1:** Find the *x*– and *y*– intercepts.

Let's first find the *x*–intercept.

What value are we going to use for *y*? You are correct if you said $y = 0$.

$-3x = y$

$-3x = 0$ $\quad \dfrac{-3x}{-3} = \dfrac{0}{3}$ $\quad x = 0$

The *x*–intercept is (0, 0).

Next, we will find the *y*– intercept.

What value are we going to plug in for *x*? If you said $x = 0$, you are right.

$-3x = y$ $\quad -3(0) = y$ $\quad y = 0$

The *y*–intercept is (0, 0)

Hey, look at that, we ended up with the exact same point for both our *x*– and *y*–intercepts. As mentioned above, there is only one point that can be both an *x*– and *y*– intercept at the same time, the origin (0, 0).

**Step 2:** Since there are not enough points, we need to find at least one more point.
To be more accurate, we can find two additional solutions so we have a total of three points.

We can plug in any *x* value we want as long as we get the right corresponding *y* value and the function exists there.

Let's put in an easy number $x = 1$:

$-3x = y$ $\quad -3(1) = y$

$y = -3$

So the ordered pair (1, –3) is another solution to our function.

Let's put in another easy number x = –1:

$-3x = y \qquad -3(-1) = y \qquad 3 = y$

So the ordered pair (–1, 3) is another solution to our function.

Note that we could have plugged in any value for x: 5, 10, –25..., but it is best to keep it as simple as possible.

The solutions that we found are:

| *x* | *y* | *(x, y)* |
|---|---|---|
| 0 | 0 | (0, 0) |
| 1 | –3 | (1, –3) |
| –1 | 3 | (–1, 3) |

| **Step 3:** Plot the intercepts and point(s) found in steps 1 and 2. | **Step 4:** Draw the graph. |
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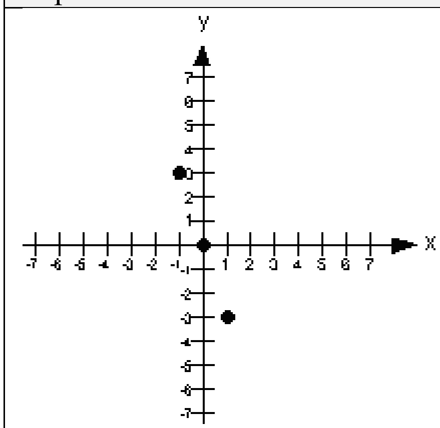
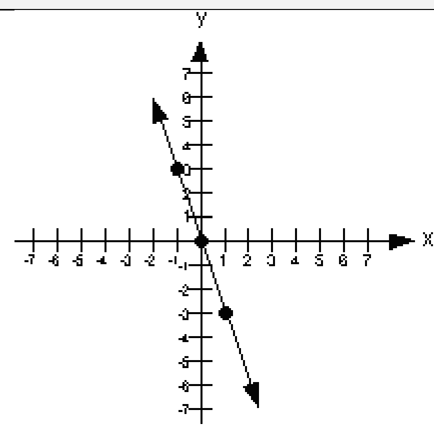
|  |  |

## 3. Special lines

**1) Vertical Lines** $\quad$ ***x = c***

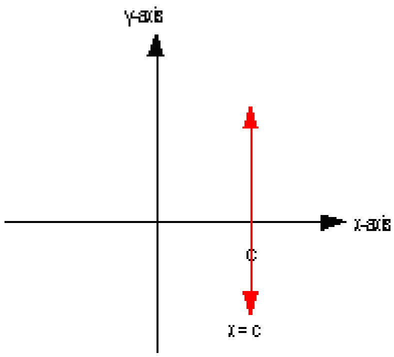
If you have an equation *x* = *c*, where *c* is a constant, and you want to graph it on a two dimensional graph, this would be a vertical line with *x*–intercept of (*c*, 0).

Even though you do not see a *y* in the equation, you can still graph it on a two dimensional graph. Remember that the graph is the set of all solutions for a given equation. In this case, the solution doesn't depend on the y value.

If all the points are solutions then any ordered pair that has an *x* value of *c* would be a solution.

As long as *x* never changes value, it is always *c*, then you have a solution. In that case, you will end up with a vertical line.

Below is an illustration of a vertical line $x = c$:

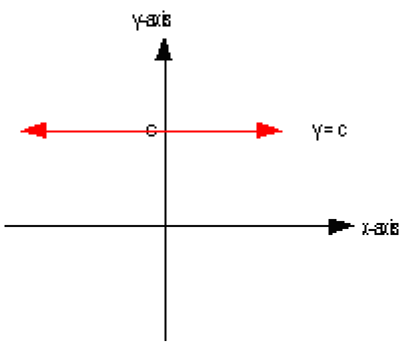



**2) Horizontal Lines** $\boldsymbol{y = c}$

If you have an equation $y = c$, where $c$ is a constant, and you want to graph it on a two dimensional graph, this would be a horizontal line with $y$– intercept of $(0, c)$. Even though you do not see an $x$ in the equation, you can still graph it on a two dimensional graph.

Remember that the graph is the set of all solutions for a given equation. If all the points are solutions then any ordered pair that has a $y$ value of $c$ would be a solution. As long as y never changes value, it is always c, then you have a solution. The solution doesn't depend on the x values. In that case, you will end up with a horizontal line.

Below is an illustration of a horizontal line $y = c$:




**Example 1:** Graph the linear relation $y = 4$.
It looks like it fits the form $y = c$. With that in mind, what kind of line are we going to end up with? Horizontal!

Note how the directions did not specify that we had to use intercepts to do our graph. Any time you take a math test or do homework, make sure that you follow directions carefully. If it specifies a certain way to do a problem, then you need to follow that plan. If it does not specify, like in this example, then you can use whatever "legitimate" way works to get the job done.

**Step 1:** Find the $x$– and $y$– intercepts

Since this is a special type of line, the $y$–intercept (where $x = 0$) is at $(0, 4)$.

Do we have any x–intercept? The answer is no. Since *y* has to be 4, then it can never equal 0, which is the criterion of an *x*–intercept. Also, think about it, if we have a horizontal line that crosses the *y*–axis at 4, it will never ever cross the *x*–axis.

It doesn't matter what *x* is, *y* is always 4. So for our solutions we just need two to three ordered pairs such that $y = 4$. Some points that we can use are (0, 4), (1, 4) and (2, 4). These are all ordered pairs that fit the criteria of *y* having to be 4.

The solutions that we found are:

| *x* | *y* | *(x, y)* |
|---|---|---|
| 0 | 4 | (0, 4) |
| 1 | 4 | (1, 4) |
| –1 | 4 | (-1, 4) |

| **Step 2:** Plot the intercepts and point(s) found in steps 1 and 2. | **Step 3:** Draw the graph. |
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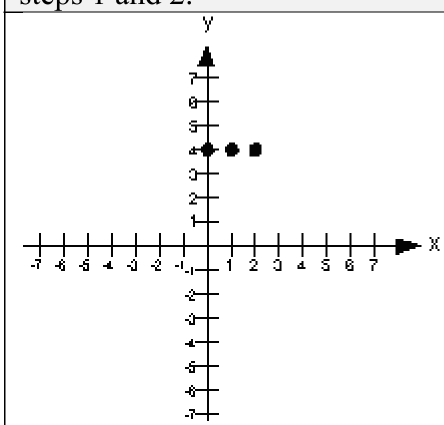
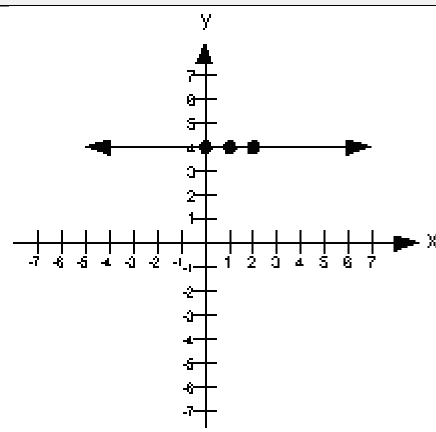
|  |  |

**Example 2:** Graph the linear equation $x + 3 = 0$.

Note how if we subtract 3 from both sides, we can write this as $x = –3$, which means it can be written in the form $x = c$. So, what type of line are we going to end up with? Vertical!

**Step 1:** Find the *x*– and *y*– intercepts.
Since this is a special type of line, it does not matter what *y* is, as long as *x* is –3. Note that the *x*–intercept is at (–3, 0).

Do we have a *y*–intercept? The answer is no. Since *x* can never equal 0, then there will be no y–intercept for this equation.

Some points that would be solutions are (–3, 0), (–3, 1), and (–3, 2).

Again, I could have picked an infinite number of solutions.

The solutions that we found are:

| *x* | *y* | *(x, y)* |
|---|---|---|
| –3 | 0 | (–3, 0) |
| –3 | 1 | (–3, 1) |
| –3 | 2 | (–3, 2 ) |

| **Step 2:** Plot the intercepts and point(s) found in steps 1 and 2. | **Step 3:** Draw the graph. |
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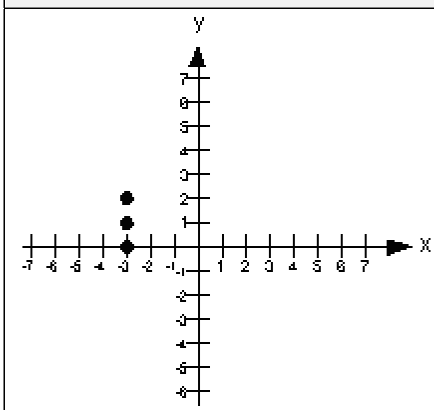
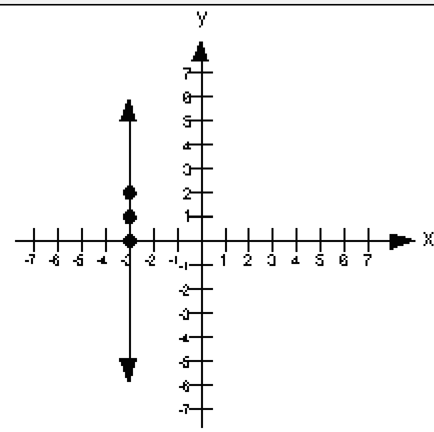
|  |  |

**4. Slope**

**1) Definition of slope**

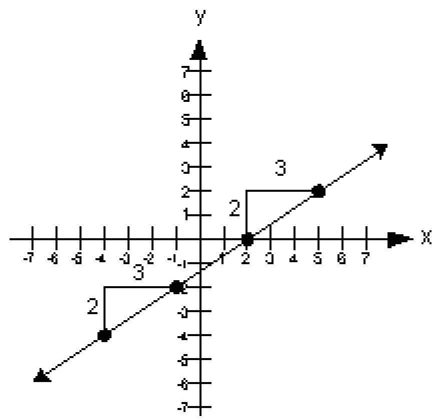
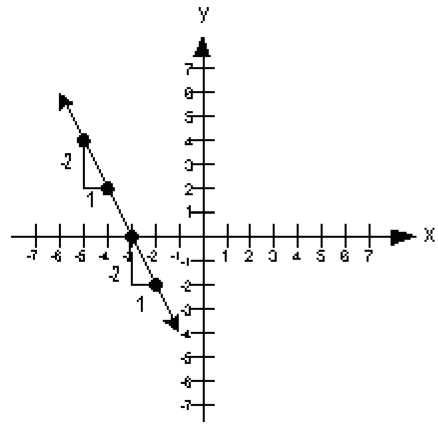
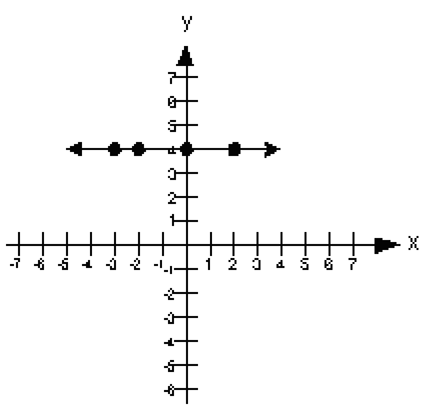
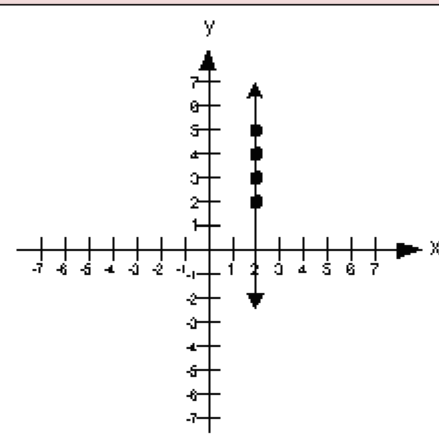
The slope of a line measures the **steepness of the line**. Most of you are probably familiar with associating slope with "**rise over run**".

**Rise** means how many units you move up or down from point to point. On the graph that would be a **change in the *y* values**.

**Run** means how far left or right you move from point to point. On the graph, that would mean a **change of *x* values**.

Here are some visuals to help you with this definition:

| **Positive slope:** | **Negative slope:** |
|---|---|

| | |
|---|---|
|  |  |
| $slope = \frac{rise}{run} = \frac{2}{3}$ | $slope = \frac{rise}{run} = \frac{-2}{1} = -2$ |
| Note that when a line has a positive slope it goes up left to right. | Note that when a line has a negative slope it goes down left to right. |
| **Zero slope:** | **Undefined slope:** |
|  |  |
| $slope = 0$ | $slope =$ undefined |
| Note that when a line is horizontal the slope is 0. | Note that when the line is vertical the slope is undefined. |

**2) Slope Formula Given Two Points**

Given two points $(x_1, y_1)$ and $(x_2, y_2)$

$$m = \frac{rise}{run} = \frac{chsnge\ in\ y}{change\ in\ x} = \frac{y_2 - y_1}{x_2 - x_1}$$

The subscripts just indicate that these are two different points. It doesn't matter which one you call point 1 and which one you call point 2 as long as you are consistent throughout that problem.

Note that we use the letter *m* to represent slope.

**Example 1:** Find the slope of the straight line that passes through (–5, 2) and (4, –7).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{-7-2}{4-(-5)} = \frac{-7-2}{4+5} = \frac{-9}{9} = -1$$

Make sure that you are careful when one of your values is negative and you have to subtract it as we did in line 2. 4 – (–5) is not the same as 4 – 5.
The slope of the line is –1. This would be a slopping downward line.

**Example 2:** Find the slope of the straight line that passes through (1, 1) and (5, 1).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{1-1}{5-1} = \frac{0}{4} = 0$$

It is ok to have a 0 in the numerator. Remember that 0 divided by any non–zero number is 0. The slope of the line is 0. This would be a horizontal line.

**Example 3:** Find the slope of the straight line that passes through (3, 4) and (3, 6).

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \frac{6-4}{3-3} = \frac{2}{0}$$

Since we did not have a change in the *x* values, the denominator of our slope became 0. This means that we have an undefined slope. If you were to graph the line, it would be a vertical line, as shown above. The slope of the line is undefined.

**5. Distance Between Two Points**

Finding the distance between (-2,-3) and (-2, 4) is easy, since the x-coordinates are the same, they make a vertical line. The distance is just the difference of the y-coordinates.

d = |4 – (-3)| = 7

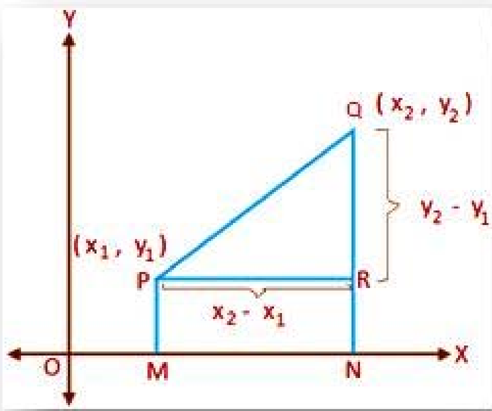
Similarly for (-2, -3) and (-4, -3), the y-coordinates are the same, they make a horizontal line. The distance is just the difference of the x-coordinates.

d = |-4 – (-2)| = 2

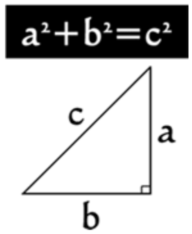
Note: remember the distance is always positive → need the absolute value sign.

How about (-2, -3) and (-4, 4)? We need the distance formula.

The Distance Formula is a variant of the Pythagorean Theorem that you used back in geometry.




Because of the Pythagorean Theorem, we have the following Distance Formula to find PQ.




**Distance Formula:** Given the two points ($x_1$, $y_1$) and ($x_2$, $y_2$), the distance between these points is given by the formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Don't let the subscripts scare you. They only indicate that there is a "first" point and a "second" point; that is, that you have two points. Whichever one you call "first" or "second" is up to you. The distance will be the same, regardless.

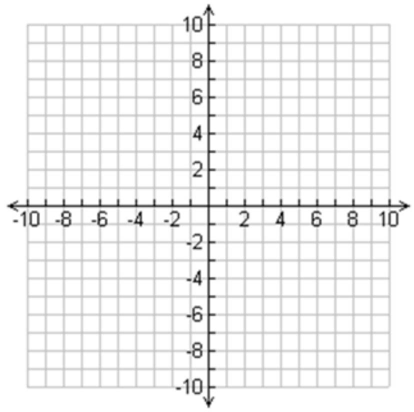
**Example:** Find the distance between the points (–2, –3) and (–4, 4).

I just plug the coordinates into the Distance Formula:

$$\begin{aligned} d &= \sqrt{(-4-(-2))^2 + (4-(-3))^2} \\ &= \sqrt{(-4+2)^2 + (4+3)^2} \\ &= \sqrt{(-2)^2 + (7)^2} \\ &= \sqrt{4+49} = \sqrt{53} \approx 7.28 \end{aligned}$$

**Class practice**

1. Graph $7x + 11 = y$ using x- and y-intercept.




2. Find the slope of the line passes through and describe it as rising, falling, horizontal, and vertical.

a) (-5, -8), (2, -5) b) (2, 1), (4, 5) c) (-1, 2), (-1, 5) d) (2, 1), (-3, 1)

3. Find the distance between the two points.

a) (1, 4), (1, 9) b) (-4, 11), (-4, 3) c) (13, 6), (15, 8)