Unit 2: Momentum, Impulse and Energy

Grade 12 Physics

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Olympiads School

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Files for You to Download

Please download from the school website if you have not done so:

- The print version of the handouts for this unit.
  - `Phys12-2a-workEnergy-print.pdf`
  - `Phys12-2b-momentumImpulse-print.pdf`

  As usual, if you wish to print the slides out, we recommend printing 4 slides per page.
- `Phys12-C04-HW.pdf`–The homework assignment for this week.

Reminder: Always download/print the PDF file for the unit before each class. There is no point copying notes that are already printed out for you. Instead, take notes on things I say that aren't necessarily on the slides.

Where Are We In the Course

1. Fundamentals of Dynamics
2. Momentum, Impulse and Energy
3. Gravitational, Electric and Magnetic Fields
4. Wave Nature of Light
5. Modern Physics:
    5.1 Special Relativity
    5.2 Introduction to Quantum Mechanics

Go Back to Work!

The central theme in Grades 11 and 12 Physics: **work** and **energy**

Work:

- Work is done when a force $\mathbf{F}$ is applied to displace an object by $\Delta\mathbf{d}$
- Mechanism in which energy is transformed from one form to another, but
- Work itself is not energy

Energy:

- The ability to do work
- Two broad categories: *potential* and *kinetic*
  - Kinetic energy is contained in objects that are in motion
  - Potential energy has the *potential* to do work, but is not doing work

Definition of Work

The work done by a constant force $\mathbf{F}$ to displace an object by $\Delta\mathbf{d}$ is defined as:

$$\boxed{W = F\Delta d\cos\theta}$$

Quantity	**Symbol**	**Unit**
Work	W	J
Magnitude of force	F	N
Magnitude of displacement	Δd	m
Angle between force and displacement vectors	θ	(no SI unit)

- Force and displacement are both vectors, but work is a *scalar* quantity
- Calculus is needed if force is not constant
- Area under the force-displacement graph in 1-D

Definition of Work

$$\boxed{W = F\Delta d \cos\theta}$$

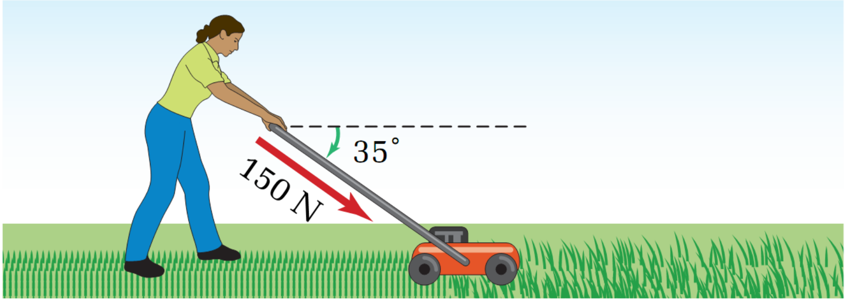
- No work is done if $F = 0$ (no force applied)
- No work is done if $\Delta d = 0$ (no displacement)
- No work is done if $\cos\theta = 0 \;\rightarrow\; \theta = 90^\circ$ (force and displacement vectors are perpendicular to each other)
- The value of $\cos\theta$ is from -1 to $+1$, which means that work done by a force can be *positive* or *negative*

Definition of Work

- **Work done by a force**
  - We can quantify work by calculating the work done by a specific force
  - Example: A boy pushes a cart forward. The “work done by the boy” is the work done by the applied force.
- **Work done on an object**
  - There may be more than one force acting on an object
  - The *sum* of all the work done on the object by each force
  - The work done by the net force
  - Also called the **net work** W_{net}

Example Problem

Example 1: A woman pushes a lawnmower with a force of $150\,\mathrm{N}$ at an angle of 35° down from the horizontal. The lawn is $10.0\,\mathrm{m}$ wide and required 15 complete trips across the back. How much work does she do?



Example Problem

Example 2: You drive a nail horizontally into a wall, using a $0.45\,\mathrm{kg}$ hammerhead. If the hammerhead is moving horizontally at $5.5\,\mathrm{m/s}$ and in one blow drives the nail into the wall a distance of $3.4\,\mathrm{cm}$, determine the average force acting on

- The hammerhead
- The nail

Defining Kinetic Energy

When a constant net force accelerates an object, work is being done. We can combine one of the kinematic equations and Newton's second law of motion to find out what "quantity of motion" changes:

$$v_2^2 = v_1^2 + 2a\Delta d = v_1^2 + 2\frac{F_{\text{net}}}{m}\Delta d$$

$$2\frac{F_{\text{net}}}{m}\Delta d = v_2^2 - v_1^2$$

$$F_{\text{net}}\Delta d = \frac{1}{2}m(v_2^2 - v_1^2) = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

We can use calculus to get the same result even if F is not constant.

Defining Kinetic Energy

The “quantity of motion” that changes when work is done is called **kinetic energy**:

$$\boxed{K = \frac{1}{2}mv^2}$$

Every force that does work on an object changes its kinetic energy

- Positive work increase its K (e.g. applied force)
- Negative work decrease its K (e.g. friction)

Work-energy theorem: When multiple forces act on an object, the net work W_{net} is equal to the change in K, i.e.

$$\boxed{W_{\mathrm{net}} = \Delta K}$$

Kinetic Energy

Translational kinetic energy is the energy of objects that are in translational (not spinning) motion:

$$\boxed{K = \frac{1}{2}mv^2}$$

Quantity	Symbol	Unit
Kinetic energy	K	J
Mass	m	kg
Speed	v	m/s

Rotational kinetic energy is a more difficult topic that is discussed in AP Physics.

Defining Potential Energy

The other category of energy is **potential energy**. When a **conservative force** acts on an object, it changes the amount of potential energy that the object has. Conservative forces can include:

- Gravitational force
- Spring force
- Electrostatic force (aka *electric force*, or *coulomb force*)

These forces are called *conservative* because they lead to the concept of **conservation of mechanical energy**.

Gravitational Potential Energy

Any object subjected to gravity has a weight of:

$$F = mg$$

When an object is free falling, the work done by gravitational force is:

$$W = -F_g \Delta h = -mg(h_2 - h_1) = -(\underbrace{mgh_2}_{U_{g2}} - \underbrace{mgh_1}_{U_{g1}})$$

Work done by gravity is *positive*, while the change in height Δh is *negative*. From this we obtain an expression for **gravitational potential energy**:

$$\boxed{U_g = mgh}$$

At a “reference level” where $h = 0$, $U_g = 0$.

Gravitational Potential Energy

$$\boxed{U_g = mgh}$$

- Positive work done by gravity always *decreases* gravitational potential energy, i.e.

$$W = -\Delta U_g = -(U_{g2} - U_{g1})$$

- When gravitational force is doing work (i.e. an object is falling), the change in height Δh is assumed to be small enough that g is treated as a constant
    - This is not the case if you are launching a satellite into space
- The reference level can be chosen anywhere. The choice of where to place the reference level should make problem solving easier.

Work Done by Conservative Forces

The fact that positive work done by a **conservative force** always *decreases* a related potential energy is the basis for conservation of mechanical energy:

$$\boxed{W = -\Delta U}$$

The change in potential energy is *path independent*, which means that it only depends on the initial and final position, but not the path taken.

Work Done by Non-Conservative Forces

Forces that are non-conservative do not correspond to a potential energy:

- Kinetic friction
- Aerodynamic forces (lift and drag)

The work done by non-conservative forces are path dependent (e.g. work done by friction)

Elastic Potential Energy

Elastic potential energy is stored while the material *deforms*. It is released when the material returns to its original shape (state). Examples:

- Diving board
- Rubber band
- Exercise ball
- Spring

A deformation that cannot be restored is called a "plastic deformation". Every material will deform elastically, then plastically, then fracture, but the mechanics of *how* this happens is complex. In Physics 12, we will only look at elastic deformation of springs.

Hooke's Law

In an **ideal spring**, the force applied to compress or extend the spring is proportional to the displacement called **Hooke's law**:

$$\boxed{\mathbf{F}_a = k\mathbf{x}}$$

Quantity	**Symbol**	**SI Unit**
Applied force	$\mathbf{F}_a$	N
Spring constant	k	N/m
Amount of extension/compression	$\mathbf{x}$	m

- The **spring constant** k (or **force constant**) is the stiffness of the spring
- To *stretch* the spring by x requires applied force of $F_a = kx$
- To *compress* the spring by $-x$ requires a force of $F_a = -kx$

Spring Force

When an applied force compresses or extends a spring, Newton's third law of motion tells us that there will also be reaction force *by* the spring. This is called the spring force:

$$\boxed{\mathbf{F}_s = -k\mathbf{x}}$$

Quantity	**Symbol**	**SI Unit**
Spring force	$\mathbf{F}_s$	N
Spring constant	k	N/m
Amount of extension/compression	$\mathbf{x}$	m

We want to look at the work done by the spring force.

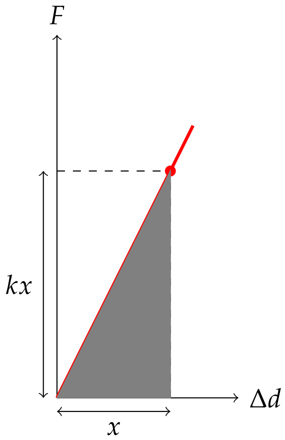
Mass-Spring Simulation

Click for external link: Hooke's Law

Example Problem

Example 3: A typical compound archery bow requires a force of $133\,\mathrm{N}$ to hold an arrow at "full draw" (pulled back) of $71\,\mathrm{cm}$. Assuming that the bow obeys Hooke's law, what is its spring constant?

Elastic Potential Energy



- Work done to extend/compress a spring is the area under the force-displacement graph
- If we use some calculus, we can see that work done is the potential stored in the spring

$$\boxed{W = U_e = \frac{1}{2}kx^2}$$

Example Problem

Example 4: A spring with spring constant of $75\,\mathrm{N/m}$ is resting on a table.

- If the spring is compressed by $28\,\mathrm{cm}$, what is the increase in its potential energy?
- What force must be applied to hold the spring in this position?

Conservation of Mechanical Energy

Positive work done by conservative forces (e.g. gravity, electrostatics, spring) on an object does two things:

1. Decrease its potential energy, while
2. Increase its kinetic energy by the same amount

Mathematically,

$$W = -\Delta U = \Delta K$$

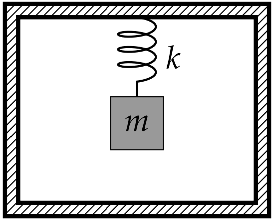
More importantly, this shows that mechanical energy is always conserved:

$$\boxed{\Delta K + \Delta U = 0}$$

Conservation of Energy

We can also look at the conservation of energy this way. If only conservative forces are acting on objects, the objects form an **isolated system**.

- A system of objects that does not interact with its surroundings
- “Interaction” can be through
  - Friction
  - Exchange of heat
  - Sound emission
- Think of a bunch of objects inside an insulated box



Isolated Systems and Conservation of Energy

The system is isolated from the surrounding environment, therefore

- The environment can't do any work on it
- Energy inside the system cannot escape either

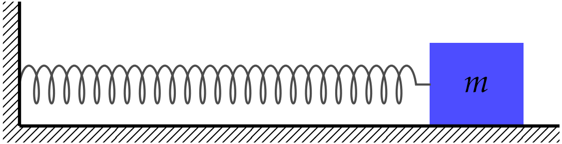
Therefore energy is conserved, i.e.

$$\boxed{U_1 + K_1 = U_2 + K_2}$$

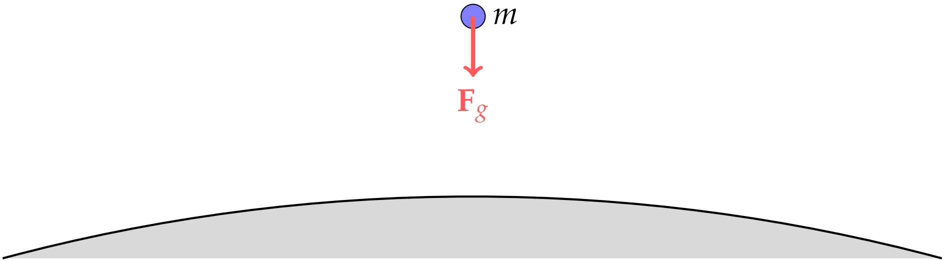
Forces are now *internal* to the system, and that work only converts kinetic energy into potential energies inside the system, and vice versa.

Example: Mass Sliding on a Horizontal Spring

- Assume no friction in any part of the system
- The isolated system consists of the mass and the spring
- Energies:
    - Kinetic energy of the mass
    - Elastic potential energy stored in the spring

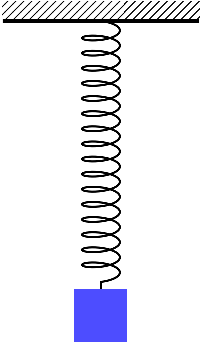


Example: Gravity



- The isolated system consists only of the mass and the Earth
- Assuming no friction
- Energies:
  - Kinetic energy of the mass
  - Gravitational potential energy of the mass

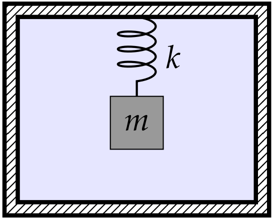
Example: Vertical Spring-Mass System



- The system consists of a mass, a spring and Earth
- Energies:
  - Kinetic energy of the mass
  - Gravitational potential energy of the mass
  - Elastic potential energy stored in the spring
- The total energy of the system is conserved if there is no friction

What if there is friction?

Energy is always conserved as long as your system is defined properly



- The system consists of a mass, a spring, Earth and all the air molecules inside the box
- As the mass vibrates, friction with air slows it down
- While the mass loses energy, the temperature of the air rises due to friction
- Energies:
    - Kinetic and gravitational potential energies of the mass
    - Elastic potential energy stored in the spring
    - Internal kinetic energy of the air molecules (translational and vibrational)
- Total energy is conserved even as the mass stops moving

The Generalized Work-Energy Theorem

When non-conservative forces (such as friction) are also present, we have to modify our equation to account for the work done by those forces:

$$\boxed{W_{\text{nc}} = \Delta U + \Delta K}$$

And the conservation of energy equation now becomes

$$\boxed{\Delta U + \Delta K + \Delta E_{\text{int}} = 0}$$

where E_{int} is the internal (thermal) energy of the system.

When We Solve Problems

When solving conservation of energy problems

- If only conservative forces (gravity, electrostatic, elastic) are present:

$$K_1 + U_1 = K_2 + U_2$$

  where K is the total kinetic energy, and U is the total potential energy.

- If non-conservative forces (e.g. friction) are also involved, we modify the equation to:

$$K_1 + U_1 + W_{\text{nc}} = K_2 + U_2$$

  where W_{nc} is the work done by non-conservative forces.

Example Problem

Example 5: A skier is gliding along with a speed of $2.00\,\mathrm{m/s}$ at the top of a ski hill, $40.0\,\mathrm{m}$ high. The skier then begins to slide down the icy (friction-less) hill.

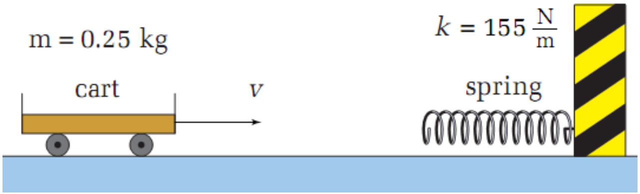
(a) What will be the skier's speed at a height of $25.0\,\mathrm{m}$?

(b) At what height will the skier have a speed of $10.0\,\mathrm{m/s}$?

In real life, there will always be *some* friction. In that case, we will need to know the non-conservative work done by friction.

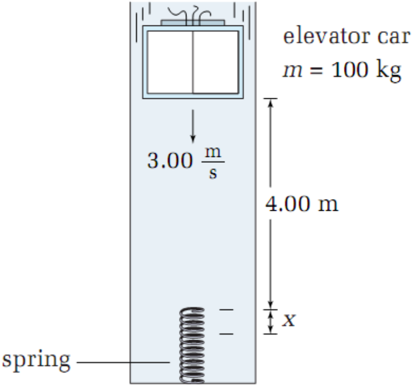
Example Problem

Example 6: A toy cart with a mass of $0.25\,\mathrm{kg}$ travels along a frictionless horizontal track and collides head on with a spring that has a spring constant of $155\,\mathrm{N/m}$. If the spring was compressed by $6.0\,\mathrm{cm}$, how fast was the cart initially travelling?



Example Problem

Example 7: A freight elevator car with a total mass of $100\,\text{kg}$ is moving downward at $3.00\,\text{m/s}$, when the cable snaps. The car falls $4.00\,\text{m}$ onto a huge spring with a spring constant of $8.00 \times 10^3\,\text{N/m}$. By how much will the spring be compressed when the elevator car reaches zero velocity?



Power

Power is the rate at which work is done, i.e. the rate at which energy is being transformed:

$$\boxed{P = \frac{W}{\Delta t}} \qquad \boxed{P = \frac{\Delta E}{\Delta t}}$$

Quantity	Symbol	SI Unit
Power	P	W
Energy transformed	ΔE	J
Work done	W	J
Time interval	Δt	s

In engineering, power is often more critical than the actual amount of work done.

Power

If a constant force is used to push an object at a constant velocity, the power produced by the force is:

$$P = \frac{W}{\Delta t} = \frac{F\Delta d}{\Delta t} \quad \longrightarrow \quad \boxed{P = Fv}$$

Application: aerodynamics

- When an object moves through air, the applied force must overcome air resistance (drag force), which is proportional with v^2
- Therefore “aerodynamic power” must scale with v^3 (i.e. doubling your speed requires $2^3 = 8$ times more power)
- Important when aerodynamic forces dominate

Efficiency

The ratio of useful energy or work output to the total energy or work input

$$\boxed{\eta = \frac{E_o}{E_i} \times 100\%} \qquad \boxed{\eta = \frac{W_o}{W_i} \times 100\%}$$

Quantity	Symbol	SI Unit
Useful output energy	E_o	J
Input energy	E_i	J
Useful output work	W_o	J
Input work	W_i	J
Efficiency	η	no units

Efficiency is always $0 \leq \eta \leq 100\%$