Ratio, Proportion and Percent

1. Ratio

A ratio is a pair of numbers that compares two quantities or describes a rate. A ratio can be written in three ways:

Using the word *to* 3 to 4
Using a colon 3 : 4 These are all read *3 to 4*.
Writing a fraction 3/4

When a ratio is used to compare two different kinds of quantities, such as miles to gallons, it is called a rate.

Equal ratios make the same comparison. To find equal ratios, multiply or divide each term of the given ratio by the same number.

2. Proportion

A proportion is a statement that two ratios are equal. A proportion shows that the numbers in two different ratios compare to each other in the same way.

The proportion $\frac{2}{3} = \frac{10}{15}$ is read *2 is to 3 as 10 is to 15.*

In a proportion, the **cross products** are equal.

3 and 10 are the **means,** or the terms in the middle of the proportion.
2 and 15 are the **extremes**, or the terms at the beginning and end of the proportion.

$$2 \times 15 = 3 \times 10$$
$$30 = 30$$

If the product of the means equals the product of the extremes, then you have a proportion.

To solve a proportion with an unknown term represented by a variable, set the cross products equal to each other. Then solve the resulting equation using division.

Formulae

$$\frac{a}{b} \pm \frac{c}{b} = \frac{a \pm c}{b} \qquad \frac{a}{b} + \frac{c}{d} = \frac{ad \pm bc}{bd}$$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \qquad \frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$$

If $\frac{a}{b}=\frac{c}{d}$ （or $a:b=c:d$）， a, b, c, d cannot be 0，

then $ad=bc$ $\quad \frac{b}{a}=\frac{d}{c} \quad \frac{a}{c}=\frac{b}{d} \quad \frac{a+b}{b}=\frac{c+d}{d}$

3. Unit Pricing

A useful application of proportions is unit pricing. The unit price of an item is the price per unit of measure. The unit could be an ounce, quart, pound, or some other unit.

To find the unit price of an item, set up and solve the proportion:

$$\frac{price\ paid}{quantity\ bought\ in\ unit}=\frac{unit\ price}{1\ unit}$$

Since the denominator of the second ratio is 1, the proportion becomes:

$$\frac{price\ paid}{quantity}=unit\ price$$

4. Scale Drawing

A **scale drawing** can be a reduction (such as a map or floor plan) or an enlargement (such as a drawing of a blood cell) of an actual object. The scale of the drawing is a ratio of the size of the drawing to the size of actual object, or:

To find an unknown length for a scale drawing, write and solve a proportion.

$$scale=\frac{size\ of\ drawing}{size\ of\ actual\ object}$$

5. Percent

The word *percent* means *for each hundred*. A percent is a ratio whose second term is 100. The symbol % is used for percent.

$$100\%=\frac{100}{100}=1$$

Percents less than 100% are numbers less than 1. Percents greater than 100% are numbers greater than 1.

$$75\%=\frac{75}{100}=\frac{3}{4} \qquad 160\%=\frac{160}{100}=1\frac{3}{5}$$

Decimals, fractions, and percents can be converted from one form to another.

• To write a decimal as a percent, multiply the decimal by 100 and write a percent sign after it.

• To write a percent as a decimal, divide the percent by 100 and omit the percent sign.

• To write a fraction as a percent, first change the fraction to a decimal. Then use the method above. (Another way is to find an equal ratio with a denominator of 100.)

• To write a percent as a fraction, first write the percent as the numerator without the percent sign. Use 100 as the denominator. Then write the fraction in lowest terms.

6. Percent of Increase and Decrease

To find the percent of increase or decrease, first subtract to find the amount of change. Then compare the amount of change to the original amount.

$$\textit{Percent of change (increase or decrease)} = \frac{(\textit{amount of change }(\textit{increase or decrease}))}{(\textit{original amount})}$$

7. Applications of Percent

There are many everyday situations that involve the use of percent. Determining the selling price of an item is a common problem for retailers and consumers. For the buyer, the amount a regular or list price is reduced is called a discount.

The rate of discount is the percent that the regular price is reduced.

• discount = regular price × rate of discount
• sale price = regular price – discount

For the retailer, the **cost** is the original amount paid for an item.

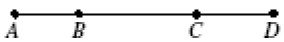
The **markup** is the percent of increase. The **amount of markup** is the increase.

• amount of markup = cost × markup
• selling price = cost + amount of markup

Questions in class

1. Four points are on a line segment, as shown.

If *A B*: *BC* = 1 : 2 and *BC* : *CD* = 8: 5, then *AB* : *BD* equals what?



2. If x and y are positive numbers and the average of 4, 20 and x is equal to the average of y and 16, then what is the ratio $x : y$?

3. In a pack of construction paper, the numbers of blue and red sheets are originally in the ratio 2:7. Each day, Laura uses 1 blue sheet and 3 red sheets. One day, she uses 3 red sheets and the last blue sheet, leaving her with 15 red sheets. How many sheets of construction paper were in the pack originally?

4. Angela and Barry share a piece of land. The ratio of the area of Angela's portion to the area of Barry's portion is 3 : 2. They each grow corn and peas on their piece of land. The entire piece of land is covered by corn and peas in the ratio 7 : 3. On Angela's portion of the land, the ratio of corn to peas is 4 : 1. What is the ratio of corn to peas for Barry's portion?

5. At Matilda's birthday party, the ratio of people who ate ice cream to people who ate cake was 3 : 2. People who ate both ice cream and cake were included in both categories. If 120 people were at the party, what is the maximum number of people who could have eaten both ice cream and cake?

6. Given the following information: Of the 135,000 voters 56% were women. 52% of the women and 47.5% of the men voted for the Democrat. How many votes did the Democrat get?

7. Arnold started getting an allowance at his tenth birthday. Each month he got three times as many dollars as he was years old. If he saved 25% of his allowance, how much money would he have saved by the day before his 18th birthday?

8. The cost of living in each quarter (3 months) increased by 2% over the previous quarter. To the nearest tenth of a percent, to what annual percentage rate of increase does this correspond?

9. Given a right triangle with sides of length a, b, and c and area, $a^2 + b^2 - c^2$. Find $\frac{c}{b}$, the ratio of the legs of the right triangle.

10. The capacity of a car's radiator is nine liters. The mixture of antifreeze and water is 30% antifreeze. The temperature is predicted to drop rapidly requiring the mixture to be 65% antifreeze. How much of the mixture in the radiator must be drawn off and replaced with pure antifreeze?