Chapter 8 Geometric Relationships (1)

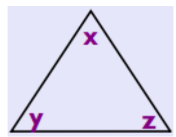
1. Angle Relationships in Triangles and Other Polygons

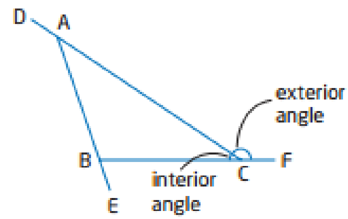
Vertex - point where two or more sides meet. The vertex is always labelled with a capital letter. Ex. A, B, C.

Interior Angle - Angle formed on the inside of a polygon by two sides meeting at the vertex.

Exterior Angle - Angle formed on the outside of a geometric shape by extending one of the sides past the vertex.

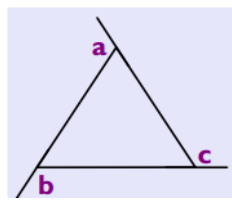
1) The three interior angles of a triangle add up to 180^o
$x + y + z = 180^o$

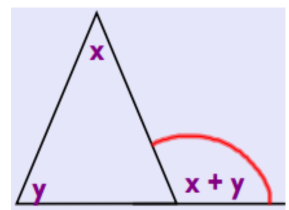




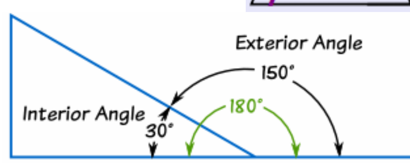
2) The exterior angles of a triangle add up to 360^o
$a + b + c = 360^o$

3) The exterior angle at each vertex of a triangle is equal to the sum of the interior angles at the other two vertices. (see diagram to the right)

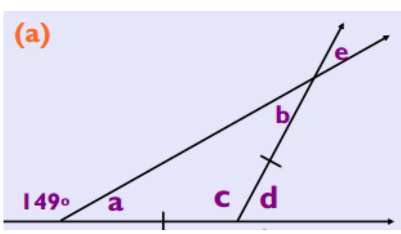


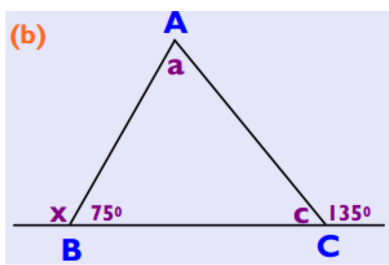


4) The interior angle is always supplementary to an exterior angle at that vertex.



Example: Find the unknown angles.

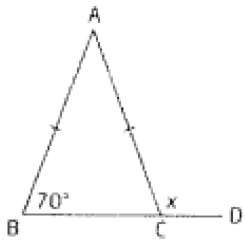


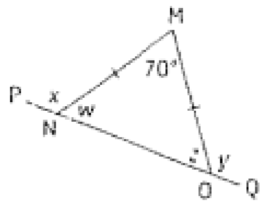


a) $\angle a = \angle b = \angle e = 180 - 149 = 21^o$
$\angle$ c = $180 - 21 \times 2 = 138^o$
$\angle$ d = $21 \times 2 = 42^o$

b) $\angle x = 180 - 75 = 105^o$
$\angle$ c = $180 - 135 = 45^o$
$\angle$ a = $180 - 45 - 75 = 60^o$

Class Practice: Find the unknown angles.

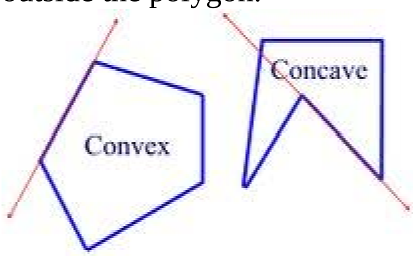
1. 

2. 

Polygon - a plane figure that is bounded by a finite chain of straight line segments closing in a loop to form a closed chain.

Convex Polygon – a polygon with no part of any line segment joining two points on the polygon outside the polygon.

Concave Polygon – a polygon with parts of some line segments joining two points on the polygon outside the polygon.



For n-sided polygon:

1) The sum of the interior angles of an n-sided polygon is $180^{\circ}(n-2)$.
2) The sum of the exterior angles of an n-sided polygon is 360°.
3) Number of distinct diagonals in an n-sided polygon is $\frac{n(n-3)}{2}$

Example:

1. Find the measure of each interior angle of a regular polygon with 8 sides.
   $180^{\circ}(8-2)/8 = 135^{\circ}$

2. How many sides does a polygon have if the number of distinct diagonals is 5?
   $$\frac{n(n-3)}{2} = 5$$
   $$n(n-3) = 10$$
   Guess and check: n = 5 sides

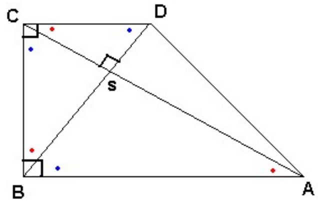
2. Midpoints and Medians in Triangles

A **conjecture** is a guess that can be either supported, proved, or disproved. In order to prove a conjecture is true, we need to conduct proper geometric proving procedures. Using measurement by hand or Geometer's Sketchpad (GSP) cannot prove anything, but just support or reject the conjecture.

A **counterexample** is an example that can be used to reject a conjecture. We only need one counterexample to disprove a conjecture.

Example: Disprove the conjecture: The quadrilateral that has perpendicular diagonals is either a kite or a rhombus.

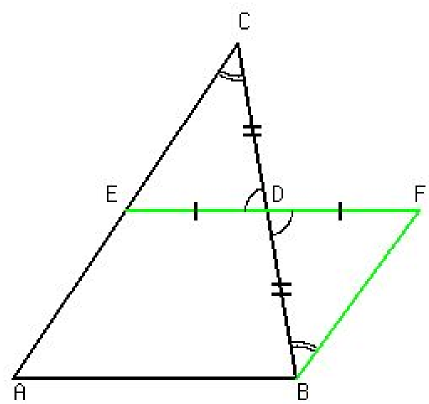
Solution: Any quadrilateral could have perpendicular diagonals.



Midpoint: A point that divides a line segment into two equal segments.

Theorem 1: A line segment joining the midpoints of two sides of a triangle is parallel to the third side.

Proof:
Continue the straight line segment ED to its own length to the point F and connect the points B and F by the straight line segment BF.



∵ Point E, D are the midpoints
∴ CD = BD, and CE = EA
∵ ∠ EDC = ∠ FDB (vertical ∠)
CD = BD
ED = FD
∴ Triangles EDC and FDB are congruent (SAS)
∴ ∠ ECD = ∠ DBF and CE = EA = BF
∴ AC // BF (Alternate ∠)
∴ AE // BF and AE = BF
∴ EF // AB and EF = AB

Theorem 2: The straight segment connecting midpoints of the two sides of a triangle (midsegment) is of half of the length of the third side of the triangle.

Proof:
Continue with the above proof:

∵ AB = EF and EF = DF
∴ ED = ½ EF
∴ ED = ½ AB

Theorem 3: The height of a triangle formed by joining the midpoints of two sides of a triangle is half the height of the original triangle.

Proof:

∵ Point E, D are the midpoints

∴ CD = BD, and CE = EA

∴ AE = ½ AB

∵ EF // BG

∴ ∠ AEF = ∠ ABG (corresponding ∠)

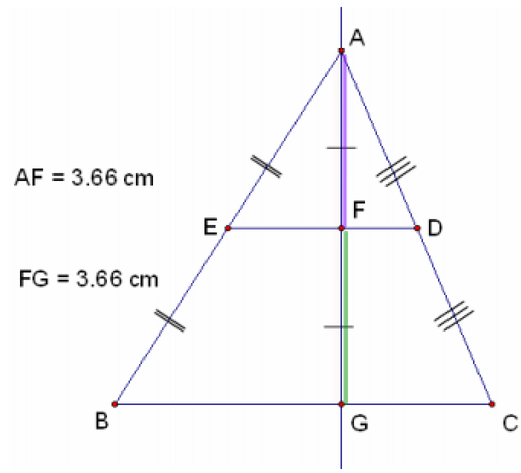
∵ AG is the height

∴ ∠ AFE = ∠ AGB = 90º

∵ ∠ FAE = ∠ GAB

∴ Triangles AEF and ABG are similar (AAA)

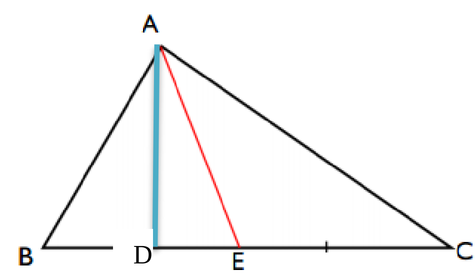
∴ AF = ½ AG



Median: the line segment joining a vertex of a triangle to the midpoint of the opposite side.

Bisect: Divide into 2 equal parts

Theorem 1: The medians of a triangle bisect its area.



E is the midpoint of BC.

Proof:

Construct a height AD from A to BC.

∵ AD ⊥ BE and AD ⊥ EC and AD ⊥ BC

∴ Area of ABE = ½ (BE)(AD), Area of AEC = ½ (EC)(AD)

∵ BE = EC

∴ Area of ABE = Area of AEC

Practice in class:

1. Calculate the total sum of the interior angles for the following convex polygons.

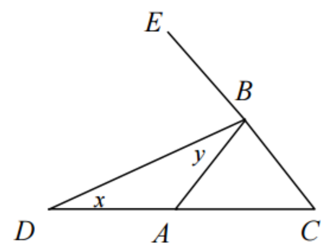
a. 4 sides   b. 6 sides   c. 9 sides

2. Determine the number of sides of each convex polygon with the following interior angle sums.

a. 540˚   b. 900˚   c. 1080˚

3. In ΔABC, CA is extended to D and CB is extended to E.

a. If ∠ABC is 47º and ∠ACB is 49º, what is the measure of ∠DAB?



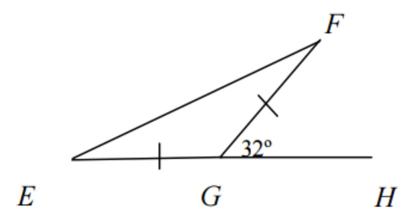
b. Determine ∠ABE.

c. If ∠ABC is b and ∠ACB is c, write a formula for ∠DAB in terms of b and c.

d. If we join D and B, we form a new triangle, ΔBCD. If ∠ABD is y and ∠ADB is x, what is the measure of ∠BAC in terms of x and y?

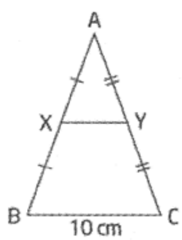
e. Can ∠ABD ever be greater than ∠BAC? Why or why not?

4. In isosceles ΔEFG, GE = GF. EG is extended to point H. If ∠HGF is 32º, what is the measurement angle of ∠GEF?



5. Find the length of XY in each of the triangles below.

a)



b)