Geometry 1

1. Parallel Lines and Pairs of Angles

1) Parallel Lines

Lines are parallel if they are always the same distance apart (called "equidistant"), and will never meet. Just remember:

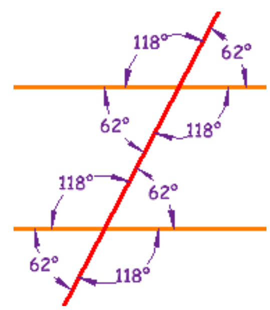
Always the same distance apart and never touching.
The red line is parallel to the blue line in both these cases:

Example 1 Example 2

Parallel lines also point in the same direction.

2) Pairs of Angles

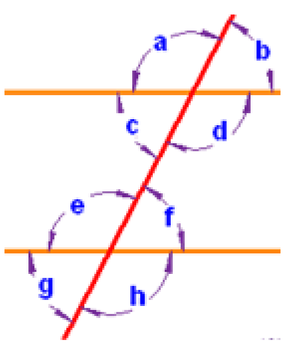
When parallel lines get crossed by another line (which is called a Transversal), you can see that many angles are the same, as in this example:



These angles can be made into pairs of angles which have special names.

3) Testing for Parallel Lines

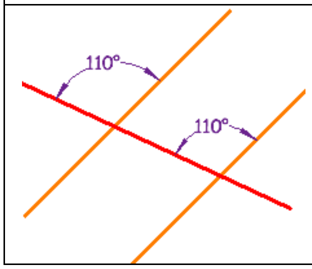
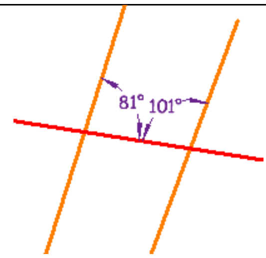
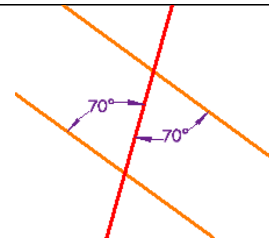
Some of those special pairs of angles can be used to test if lines really are parallel:



If Any Pair Of ...	*Example:*
Corresponding Angles are equal, or	$a = e$
Alternate Interior Angles are equal, or	$c = f$
Alternate Exterior Angles are equal, or	$b = g$
Consecutive Interior Angles add up to 180°	$d + f = 180°$

... then the lines are Parallel

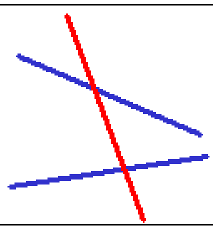
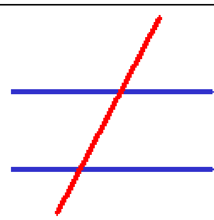
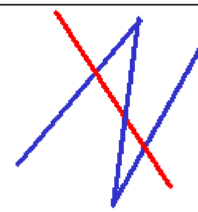
Examples

These lines are parallel, because a pair of Corresponding Angles are equal.	These lines are not parallel, because a pair of Consecutive Interior Angles do not add up to 180° (81° + 101° =182°)	These lines are parallel, because a pair of Alternate Interior Angles are equal
		

4) Transversals

A Transversal is a line that crosses at least two other lines.

The red line is the transversal in each example:

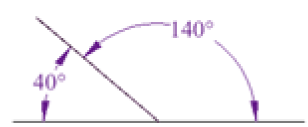
Transversal crossing two lines	this Transversal crosses two parallel lines	... and this one cuts across three lines
		

5) Supplementary Angles

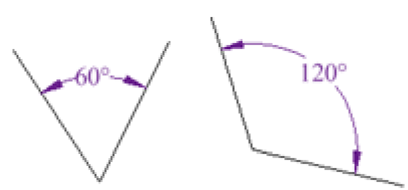
Two Angles are Supplementary if they add up to 180 degrees.

These two angles (140° and 40°) are Supplementary Angles, because they add up to 180°.

Notice that together they make a straight angle.



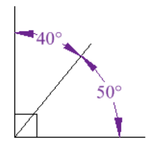
But the angles don't have to be together.
These two are supplementary because 60° + 120° = 180°



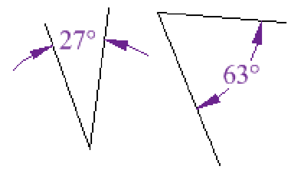
6) Complementary Angles

Two Angles are Complementary if they **add up to 90 degrees** (a Right Angle).

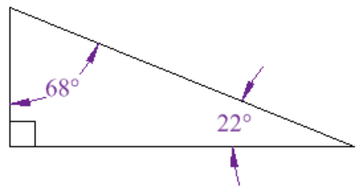
These two angles (40° and 50°) are Complementary Angles, because they add up to 90°.
Notice that together they make a right angle.



But the angles don't have to be together.
These two are complementary because
27° + 63° = 90°



Right Angled Triangle
In a right angled triangle, the two acute angles are complementary, because in a triangle the three angles add to 180°, and 90° have been taken by the right angle.



7) Complementary vs Supplementary

A related idea is Complementary Angles, they add up to 90° .

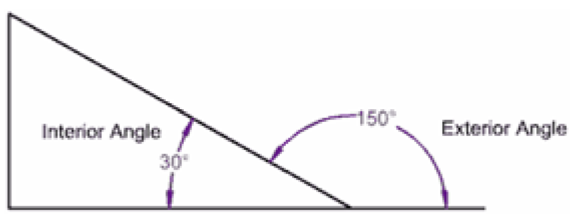
How can you remember which is which? Easy! Think:

- "C" of Complementary stands for "Corner" ∟ (a Right Angle), and
- "S" of Supplementary stands for "Straight" (180 degrees is a straight line)

2. Interior and Exterior Angle

An Interior Angle is an angle inside a shape.

The Exterior Angle is the angle between any side of a shape, and a line extended from the next side.



Note: If you add up the Interior Angle and Exterior Angle you get a straight line, 180°.

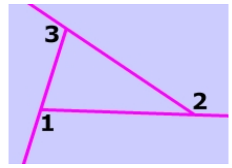
1) Exterior Angles of Polygons

The Exterior Angle is the angle between any side of a shape, and a line extended from the next side.

The sum of the measures of the exterior angles of a polygon, one at each vertex, is 360°.

For examples:
A Polygon is any flat shape with straight sides.

The Exterior Angles of a Polygon add up to 360°.
$\angle 1 + \angle 2 + \angle 3 = 360^0$



2) Interior Angles sum of Polygons

An interior angle of a regular polygon with n sides is $\frac{(n-2)\times 180}{n}$.

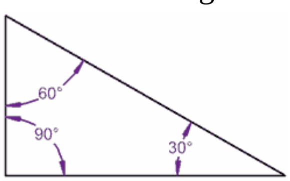
Example:

To find the measure of an interior angle of a regular octagon, which has 8 sides, apply the formula above as follows:
((8-2) × 180) /8 = 135°

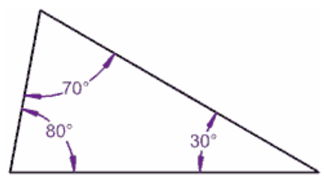
The sum of the measures of the interior angles of a polygon with *n* sides is (n-2)180.

3) Triangles

The Interior Angles of a Triangle add up to 180°



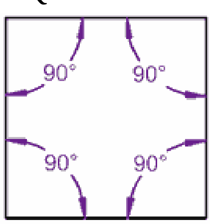
90° + 60° + 30° = 180°



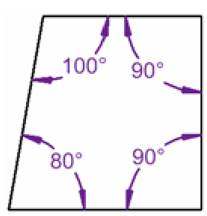
80° + 70° + 30° = 180°

4) Quadrilaterals (Squares, etc)

(A Quadrilateral is any shape with 4 sides)



90° + 90° + 90° + 90° = 360°
A Square adds up to 360°



80° + 100° + 90° + 90° = 360°
Let's tilt a line by 10° ... still adds up to 360°!

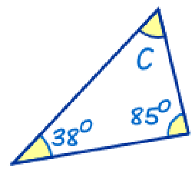
The Interior Angles of a Quadrilateral add up to 360°

2. Triangles

In a triangle, the three angles always add to 180°: **A + B + C = 180°.**

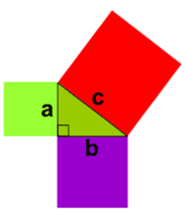
Example: Find the Missing Angle "C"

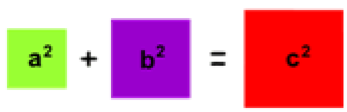
Start With: A + B + C = 180°
Fill in what we know: 38° + 85° + C = 180°
Rearrange: C = 180° - 38° - 85°
Calculate: C = 57°



3. Pythagorean Triples

A "Pythagorean Triple" is a set of positive integers, **a**, **b** and **c** that fits the rule:
$a^2 + b^2 = c^2$
And when you make a triangle with sides **a**, **b** and **c** it will be a right angled triangle:





Note:
c is the **longest side** of the triangle, called the "hypotenuse"
a and **b** are the other two sides

Example: 3, 4, 5 is an example of such triples.

4. Similar Triangles

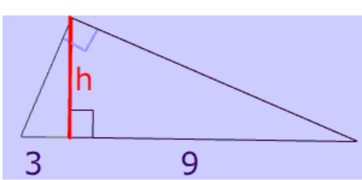
Mean Proportion - any value that can be expressed just the way that 'x' is in the proportion.

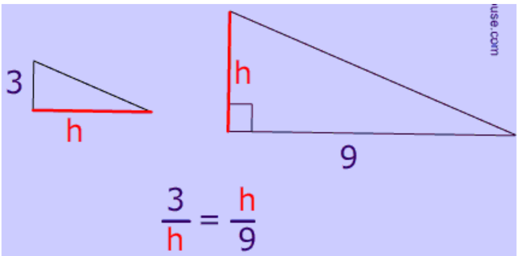
$$\frac{15}{x} = \frac{x}{10}$$

We call x the **Geometric mean.**

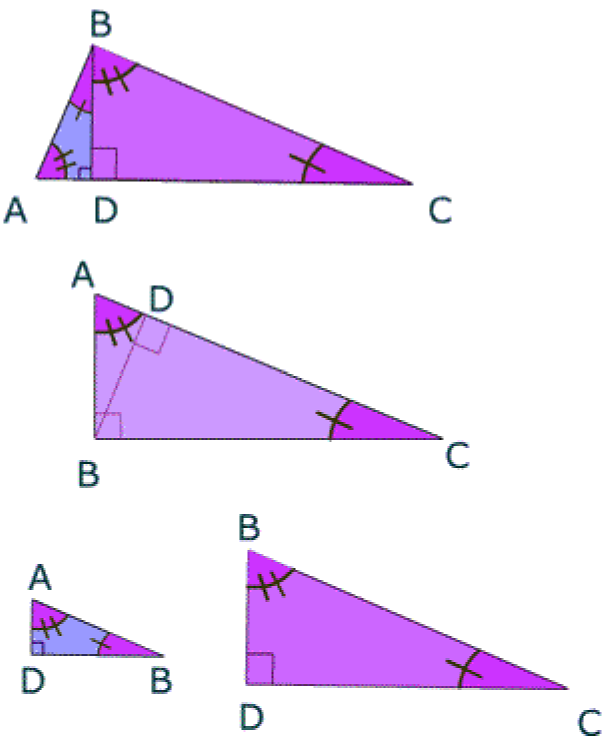
So what does this have to do with right similar triangles?

It turns out the when you drop an altitude (h in the picture below) from the right angle of a right triangle, the length of the altitude becomes a geometric mean.
This occurs because you end up with similar triangles which have proportional sides and the altitude is the long leg of 1 triangle (left side) and the short leg of the other similar triangle – see below.



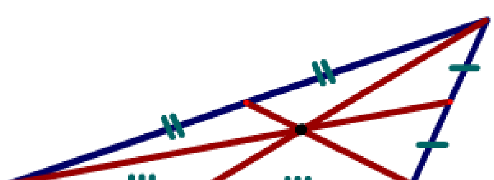


Below is a picture of the similar triangles created when you drop the altitude from a right angle of a right triangle. All three right angles are similar.



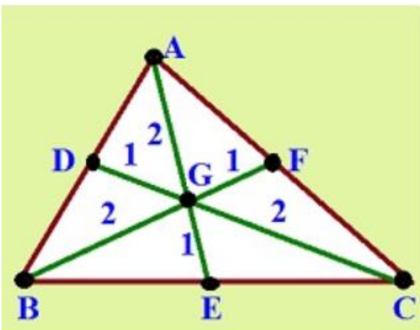
5. Centroid

The **Centroid** is a point of concurrency of the triangle. It is the point where all 3 medians intersect and is often described as the triangle's center of gravity.



Properties of the Centroid

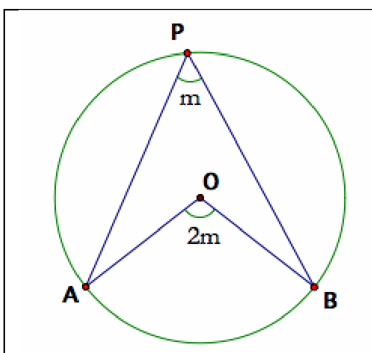
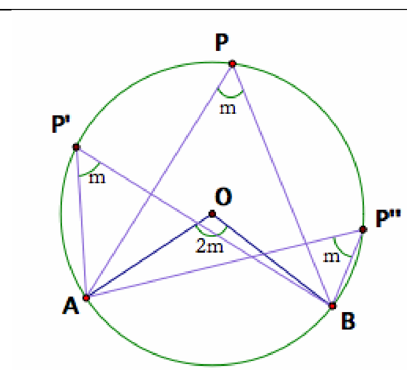
- It is formed by the intersection of the medians.
- It is one of the points of concurrency of a triangle.
- It is always located inside the triangle
- The centroid divides each median in a ratio of 2:1. In other words, the centroid will always be 2/3 of the way along any given median.



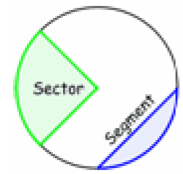
6. Central Angle Theorem

The Central Angle Theorem states that the central angle from two chosen points A and B on the circle is always twice the inscribed angle from those two points. The inscribed angle can be defined by any point along the outer arc AB and the two points A and B.

Note: The Central Angle Theorem says the inscribed angle APB can have the point P anywhere along the outer arc AB, and the angle relationship will still hold.

 *The Central Angle Theorem*	 *All inscribed angles of same arc AB are equal in size*

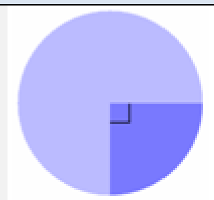
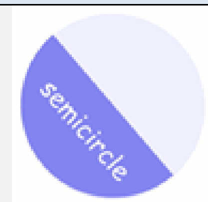
7. Circle Sector and Segment



Slices
There are two main "slices" of a circle:
The "pizza" slice is called a **Sector**.
And the slice made by a chord is called a **Segment**.

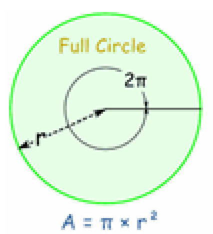
1) Common Sectors

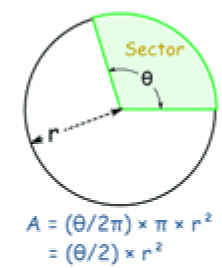
The Quadrant and Semicircle are two special types of Sector:

Quarter of a circle is called a **Quadrant**	Half a circle is called a **Semicircle.**
	

2) Area of a Sector

You can work out the Area of a Sector by comparing its angle to the angle of a full circle.
Note: I am using radians for the angles.





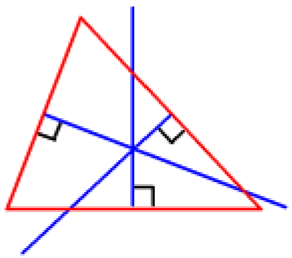
This is the reasoning:

- A circle has an angle of 2π and an Area of: πr^2
- So a Sector with an angle of θ (instead of 2π) must have an area of: $(\theta/2\pi) \times \pi r^2$
- Which can be simplified to: $(\theta/2) \times r^2$

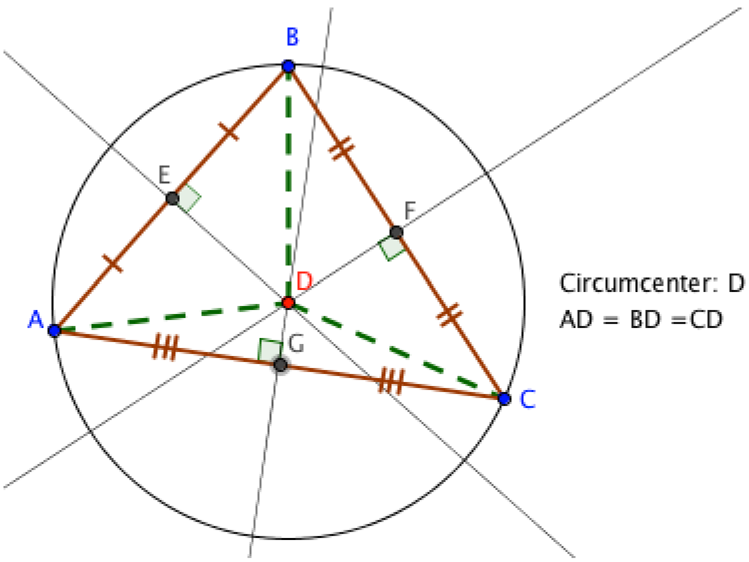
Area of Sector = $\frac{1}{2} \times \theta \times r^2$ *(when θ is in radians)*
Area of Sector = $\frac{1}{2} \times (\theta \times \pi/180) \times r^2$ *(when θ is in degrees)*

8. Circumcenter

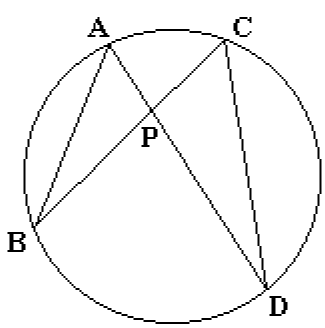
The three <u>perpendicular bisectors</u> of a triangle meet in a single point, called the *circumcenter*.



Circumcenter Theorem: The vertices of a triangle are equidistant from the circumcenter.



9. Intersecting Chords Theorem

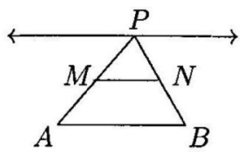


Given a point P in the interior of a circle, pass two lines through P that intersect the circle in points A and D and, respectively, B and C. Then AP·DP = BP·CP.

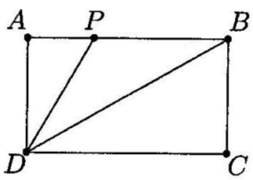
► Questions in class

1. Points M and N are the midpoints of sides PA and PB of ΔPAB. As P moves along a line that is parallel to side AB, how many of the four quantities listed below change?

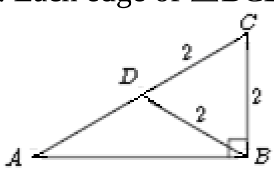
(a) the length of the segment MN, (b) the perimeter of ΔPAB, (c) the area of ΔPAB,
(d) the area of trapezoid ABNM



2. In rectangle ABCD, AD = 1, P is on AB, and DB and DP trisect $\angle$ADC. What is the perimeter of ΔBDP?



3. Each edge of $\triangle$BCD has length 2, D lies on $\overline{AC}$, and m$\angle$ABC = 90°. What is the length of AB?



4. Each of the small circles in the figure has radius one. The innermost circle is tangent to the six circles that surround it, and each of those circles is tangent to the large circle and to its small-circle neighbours. Find the area of the shaded region.

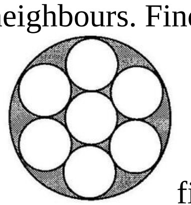


fig.4

5. Circles of radius 2 and 3 are externally tangent and are circumscribed by a third circle, as shown in the figure. Find the area of the shaded region.

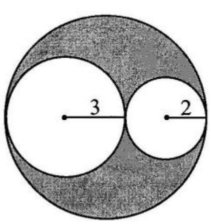


fig.5

6. The sides of a triangle have lengths of 15, 20, and 25. Find the length of the shortest altitude.

7. What is the volume of a sphere of radius 6?

8. In the Figure, *angle EAB* and *angle ABC* are right angles, *AB* = 4, *BC* = 6, *AE* = 8, and *AC* and *BE* intersect at *D.* What is the difference between the areas of *ΔADE* and *ΔBDC?*

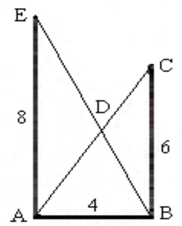


fig.8

9. A square has sides of length 10 and a circle centered at one of its vertices has radius 10. What is the area of the union of the regions enclosed by the square and the circle?

10. An annulus is the region between two concentric circles. The concentric circles in the figure have radii b and c, with b > c. Let OX be a radius of the larger circle, let XZ be tangent to the smaller circle at Z. and let OY be the radius of the larger circle that contains Z Let a = XZ, d = YZ, and e = XY. What is the area of the annulus?