Chapter 1 Rational Numbers and Exponents

1. Rational Numbers

Number Systems: Natural numbers, N = {1, 2, 3, 4, …}
Whole numbers, W = {0, 1, 2, 3, …}
Integers, I = {…-3, -2, -1, 0, 1, 2, 3, …}

NOW… Rational numbers, Q = $\{ \frac{a}{b} \mid a, b \in I, b \neq 0 \}$ Q is for quotient
This is read as "the set of numbers that can be written as fractions where a and b are integers, a as the numerator and b as the denominator, but b cannot equal zero (since zero can never be a divisor).

A rational number is any number that can be written as a ratio of two integers. All integers are rational numbers, for instance, 3 can be written as 3/1. All terminating or repeated decimals are also rational numbers, for instance, 0.25 can be written as ¼ and 0.3… can be written as 1/3.

1) Types of Fractions

a) Proper $\frac{1}{2}$ b) Improper $\frac{4}{3}$ c) Mixed $2\frac{3}{8}$

2) Equivalent Fractions – can multiply or divide the top and bottom by the same number.

Ex. 1: Reduce the following fractions.

a) $\frac{4}{6}$ b) $\frac{6}{3}$ c) $\frac{-24}{9}$ d) $\frac{-52}{-16}$ e) $-\frac{-4}{24}$

Ex. 2: Fill in the blank.

a) $\frac{5}{\underline{\quad}} = \frac{25}{40}$ b) $\frac{4}{7} = \frac{\underline{\quad}}{28}$ c) $\frac{-6}{\underline{\quad}} = \frac{-1}{3}$ d) $\frac{75}{\underline{\quad}} = \frac{5}{1}$

3) Rational Number Operation

Recall: BEDMAS – Brackets, exponents, division/multiplication, addition/subtraction

Ex. 1: Evaluate without using a calculator. Show your work.

a)
$$\frac{1}{2} + \frac{3}{4} \times \left(-\frac{5}{6}\right)$$
$$= \frac{1}{2} + \left(-\frac{15}{24}\right)$$
$$= \frac{1(12)}{2(12)} - \frac{15}{24}$$
$$= \frac{12}{24} - \frac{15}{24}$$
$$= \frac{-3}{24}$$
$$= \frac{-1}{8}$$

b)
$$-3 + \left(\frac{3}{4} - \frac{1}{2}\right)^2$$
$$= -3 + \left(\frac{3}{4} - \frac{2}{4}\right)^2$$
$$= -3 + \left(\frac{1}{4}\right)^2$$
$$= -3 + \frac{1}{16}$$
$$= \frac{-48}{16} + \frac{1}{16}$$
$$= \frac{-47}{16}$$

c)
$$1\frac{2}{3} + \left(2\frac{1}{5} \div \frac{7}{10}\right)$$
$$= 1\frac{2}{3} + \left(\frac{11}{5} \div \frac{7}{10}\right)$$
$$= 1\frac{2}{3} + \left(\frac{11}{\cancel{5}_1} \times \frac{\cancel{10}^2}{7}\right)$$
$$= 1\frac{2}{3} + \left(\frac{11}{1} \times \frac{2}{7}\right)$$
$$= \frac{5^{\times 7}}{3_{\times 7}} + \frac{22^{\times 3}}{7_{\times 3}}$$
$$= \frac{35}{21} + \frac{66}{21}$$
$$= \frac{101}{21}$$

Practice:

Evaluate each expression. Show your work.

a) $-4\frac{2}{5}+\left(\frac{-2}{3}\right)\times\left(-3\frac{5}{6}\right)$ b) $-\frac{1}{5}\times\left(\frac{1}{2}-2\frac{3}{5}\right)\div\frac{-3}{8}$

c) $-2\frac{1}{4}\div\left(\frac{5}{3}-1\frac{1}{2}\right)+3$ d) $-\frac{1}{2}+1\frac{2}{3}\times\frac{6}{-7}$

2. Exponents, Powers

Definition of Exponents

$x^n = x\cdot x\cdot x\cdots x$ (There are n x's in the product) x = ***base,*** n = ***exponent***

Exponents are another way to write multiplication. The exponent tells you how many times a base appears in a PRODUCT.

Example 1: Evaluate $\left(\frac{1}{4}\right)^3$.

$$\left(\frac{1}{4}\right)^3 = \left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right) = \frac{1}{64}$$

Example 2: Evaluate $(-6)^2$.

$$(-6)^2 = (-6)(-6) = 36$$

Note how I included the - when I expanded this problem out. If the - is inside the ( ) of an exponent, then it is included as part of the base.

Example 3: Evaluate -6^2.

$$-6^2 = -6\cdot 6 = -36$$

It may look alike, but they ARE NOT the same. Can you see the difference between the two? Hopefully, you noticed that in example 2, there was a ( ) around the - and the 6.

In this problem, there is no -. This means the - is NOT part of the base, so it will not get expanded like it did in example 2. It is interpreted as finding the negative or opposite of 6 squared.

3. Scientific Notation-Large Numbers

Do you know this number, 300,000,000 m/sec.? It's the Speed of light!

Do you recognize this number, 0.000 000 000 753 kg.? This is the mass of a dust particle!

Scientists have developed a shorter method to express very large numbers. This method is called **scientific notation**. Scientific Notation is based on powers of the base number 10.
The number 123,000,000,000 in scientific notation is written as: 1.23×10^{11}

The first number 1.23 is called the ***coefficient***. It must be greater than or equal to 1 and less than 10.

The second number is called the ***base***. It must always be 10 in scientific notation. The base number 10 is always written in exponent form. In the number $1.23 \text{ x } 10^{11}$ the number 11 is referred to as the exponent or power of ten.

To write a number in scientific notation:
Put the decimal after the first digit and drop the zeroes.

$\boxed{1.23}000000000$

In the number 123,000,000,000, the coefficient will be 1.23

To find the exponent, count the number of places from the decimal to the end of the number.

In 123,000,000,000 there are 11 places. Therefore we write 123,000,000,000 as: 1.23×10^{11}

Exponents are often expressed using other notations. The number 123,000,000,000 can also be written as:

1.23E+11 or as $1.23 \text{ X } 10^{11}$

For small numbers we use a similar approach. Numbers less than 1 will have a negative exponent. A millionth of a second is:

0.000001 sec. or 1.0E-6 or 1.0^-6 or 1.0×10^{-6}

4. Square Roots

HERE ARE THE FIRST TEN square numbers and their roots:

Square numbers	1	4	9	16	25	36	49	64	81	100
Square roots	1	2	3	4	5	6	7	8	9	10

We write, for example, $\sqrt{36} = 6$.
"The square root of 36 is 6."

This mark $\sqrt{\ }$ is called the **radical sign** (after the Latin *radix* = root). The number under the radical sign is called the **radicand**. In the example, 36 is the radicand.

Example 1: Evaluate $\sqrt{13 \cdot 13}$.

Solution. $\sqrt{13 \cdot 13} = 13$.

For, $13 \cdot 13$ is a square number. And the square root of $13 \cdot 13$ is 13!

If a is any positive number, then $a \cdot a$ is obviously a square number, and
$\sqrt{a \cdot a} = a$

Example 2: Evaluate the following.

a) $\sqrt{28 \cdot 28} = 28$. b) $\sqrt{135 \cdot 135} = 135$.

c) $\sqrt{2 \cdot 3 \cdot 5 \cdot 2 \cdot 3 \cdot 5} = 2 \cdot 3 \cdot 5 = 30$.

d) $\sqrt{a^6} = \sqrt{a \cdot a \cdot a \cdot a \cdot a \cdot a} = a^3$