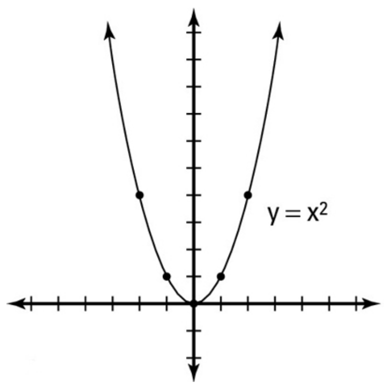
Chapter 4 Quadratic Functions (2)

1. Graphing Quadratic Functions by table of values

Let us start with the basic graph $y = x^2$. As with other functions, you can graph a quadratic function by plotting points with coordinates that make the equation true. Let us graph it using table of values.

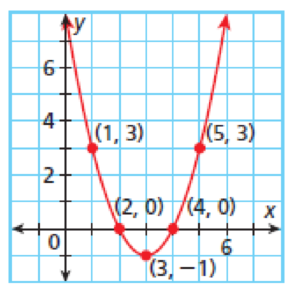
x	y
-2	4
-1	1
0	0
1	1
2	4



Let us try another one.

Example: Graph $y = x^2 - 6x + 8$

x	y
1	$1^2 - 6(1) + 8 = 3$
2	0
3	-1
4	0
5	3



2. Graphing Quadratic Functions by transformation

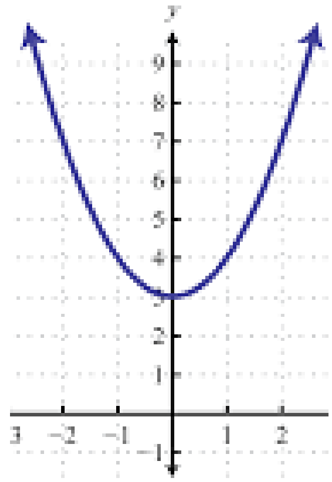
You can also graph quadratic functions by applying transformations to the parent function $y = x^2$.

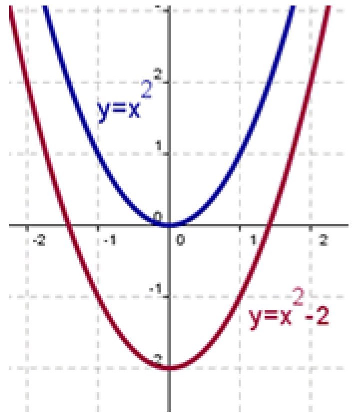
How do we do that?

Investigation - Sketch the graphs of these three quadratic relations on the same set of axes.

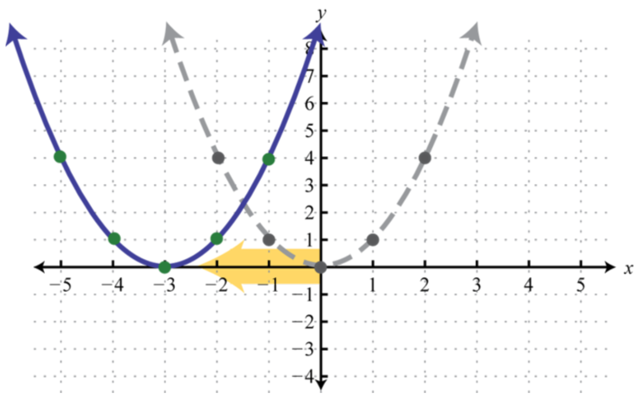
a) $y = x^2$ b) $y = x^2 + 3$ c) $y = x^2 - 2$ d) $y = (x + 3)^2$ e) $y = (x - 3)^2$

What did you notice?

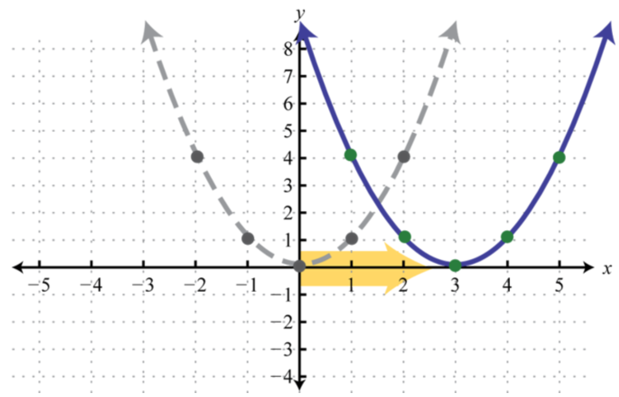


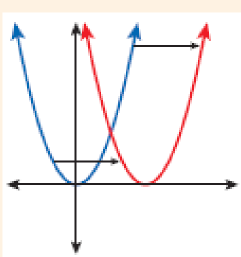
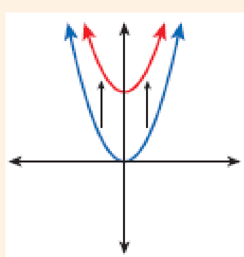


$g(x)=(x+3)^2$ *shift left*



$h(x)=(x-3)^2$ *shift right*

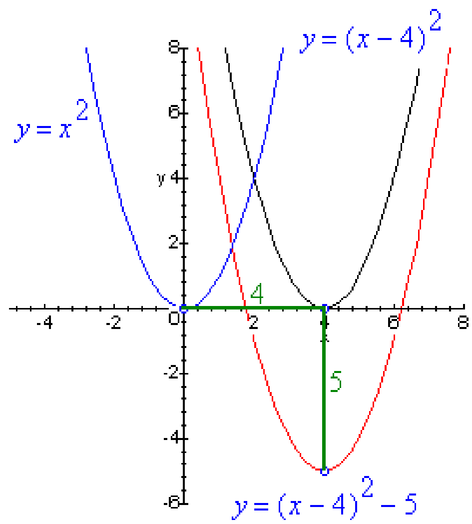


<table>
<tr><th colspan="2">Translations of Quadratic Functions</th></tr>
<tr><th>Horizontal Translations</th><th>Vertical Translations</th></tr>
<tr><td>Horizontal Shift of $|h|$ Units</td><td>Vertical Shift of $|k|$ Units</td></tr>
<tr><td>
$f(x) = x^2$
$f(x - h) = (x - h)^2$
Moves left for $h < 0$
Moves right for $h > 0$</td><td>
$f(x) = x^2$
$f(x) + k = x^2 + k$
Moves down for $k < 0$
Moves up for $k > 0$</td></tr>
</table>

Example 1: Sketch the graph of $y = (x - 4)^2 - 5$ with the description of the transformations.

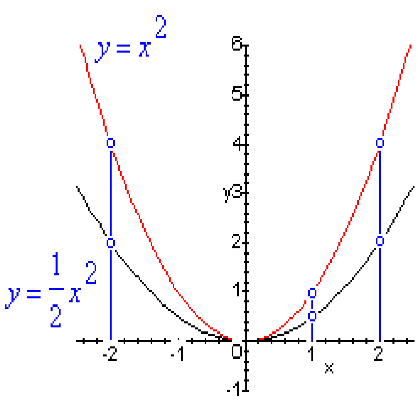
Descriptions:

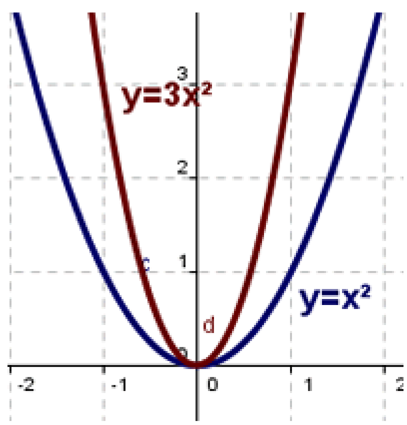
1) We start with the graph of $y = x^2$.
2) $h = 4$ → Horizontal shift 4 units right → $y = (x - 4)^2$.
3) $k = -5$ → Vertical shift 5 units down. → $y = (x - 4)^2 - 5$.

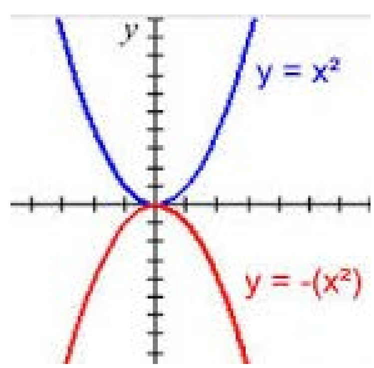


Investigation - Sketch the graphs of these three quadratic relations on the same set of axes.

a) $y = x^2$ b) $y = \frac{1}{2}x^2$ c) $y = 3x^2$ d) $y = - x^2$

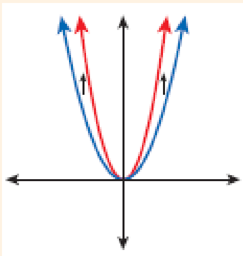






What did you notice?

Vertical Stretch/Compression by a Factor of |a|



Output values change.

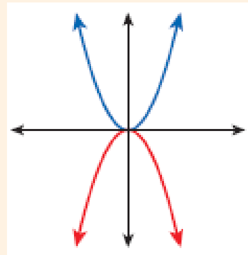
$f(x) = x^2$

$a \cdot f(x) = ax^2$

$|a| > 1$ stretches away from the x-axis.

$0 < |a| < 1$ compresses toward the x-axis.

Reflection Across x-axis



Output values change.

$f(x) = x^2$

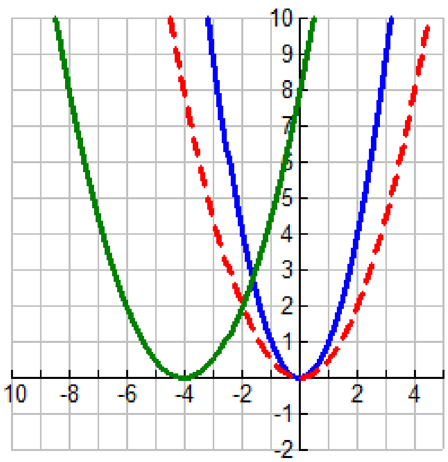
$-f(x) = -(x^2)$
$= -x^2$

The function is flipped across the x-axis.

Example 2: Graph the following quadratics <u>using transformations</u> from the parent $y = x^2$:

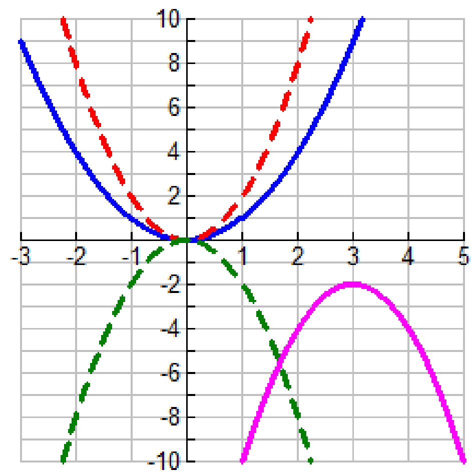
$y = \frac{1}{2}(x+4)^2$

1. Blue: Start with the parent function y = x^2
2. Red: Vertical compression by a factor of ½ (multiply all y-values by ½)
3. Green: Horizontal shift 4 units left.



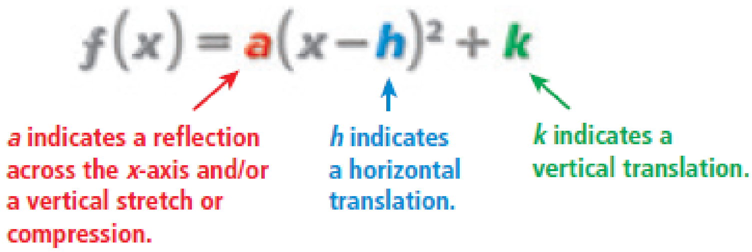
$y = -2(x-3)^2 - 2$

1. Blue: Start with the parent function y = x^2
2. Red: Vertical stretch by a factor of 2.
3. Green: Vertical reflection over x-axis. (Step 2 and 3 can be done together by multiply all y-values by -2)
4. Pink: Horizontal shift 3 units right and vertical shift 2 units down.



3. Vertex Form of a Quadratic Relation

The **Vertex Form** of a Quadratic Relation: $\mathbf{y = a(x - h)^2 + k}$ with vertex (h, k).



Example 3: Find the vertex of the quadratic relation $y = \frac{2}{3}(x+3)^2 - 7$. Then describe the transformations compared to $y = x^2$

h = -3, k = -7
Vertex (h, k) = (-3, -7)

Transformations:
1) vertical compressed by a factor of 2/3.
2) no reflection
3) vertical shift down by 7 units and horizontal shift left by 3 units.

Example 4: Use the description to write the quadratic function in vertex form.
The parent function $y = x^2$ is reflected across the x-axis, vertically stretched by a factor of 6, and translated 3 units left.

Step 1 Identify how each transformation affects the constants in vertex form.
Reflection across x-axis: a is negative
Vertical stretch by 6: $|a|$ = **6**
➔ a = **-6**

Translation left 3 units: h = -3

Step 2 Write the transformed function.

$y = a(x - h)^2 + k$ *Vertex form of a quadratic function*

$= -6(x - (-3))^2 + 0$ *Substitute -6 for a, -3 for h, and 0 for k.*

$= -6(x + 3)^2$ *Simplify.*

Example 5: Write an equation for the quadratic relation based on the descriptions:

a) Horizontal translated 7 units to the right. → $y = (x - 7)^2$

b) Compressed vertically by a factor of 1/8. → $y = 1/8\ x^2$

c) Horizontal shift 1 units to the left. → $y = (x + 1)^2$

d) Vertical shift 10 units downward. → $y = x^2 – 10$

e) Vertical reflected, vertical translated 3 units up. → $y = -x^2 + 3$

f) A stretch by a factor of 4, shift left 5, and up 2. → $y = 4(x + 5)^2 + 2$