Magnetism and Applications

Unit 3: Force Fields

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Olympiads School

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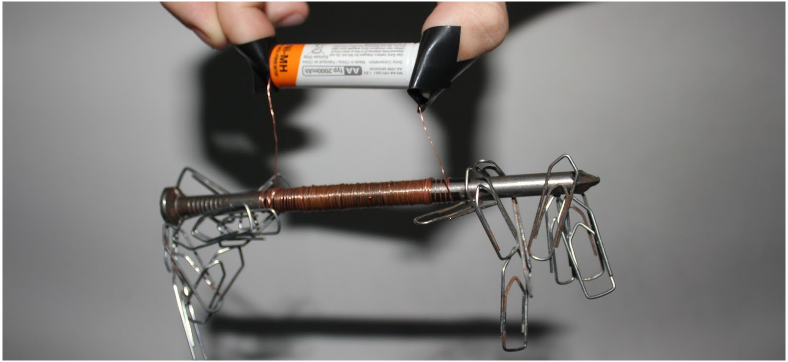
Where Are We In the Course

1. Fundamentals of Dynamics
2. Momentum, Impulse and Energy
3. Gravitational, Electric and Magnetic Fields
4. Wave Nature of Light
5. Theory of Special Relativity
6. Introduction to Quantum Mechanics

Magnetic Field

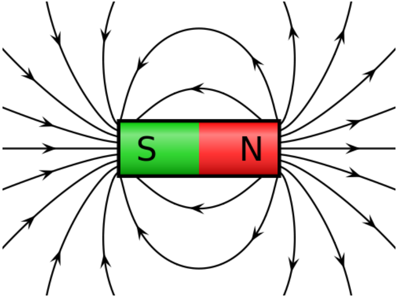
Let's Review Some Basics

- A magnetic field is created by charges that are moving
    - a single charge, or
    - a current along a wire
- Example: building electromagnets using a battery, copper wires and a nail



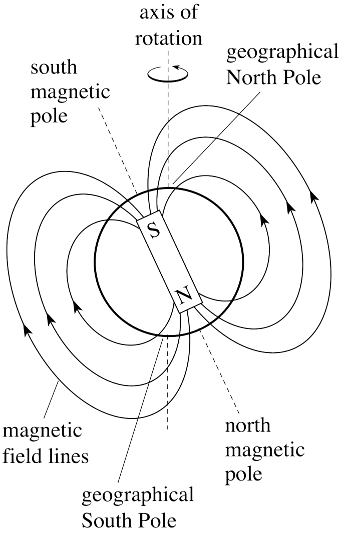
We’re Also Familiar with Permanent Magnets

- Generally made of iron, nickel, cobalt, some alloys of rare-earth materials, some minerals (e.g. lodestone)
- Atoms in these materials can be organized such that the electrons are always creating a small current inside
- We’re told that the magnetic field runs from “north” to “south” pole, like the diagram shown below. If this is the case, someone lied to you. . .



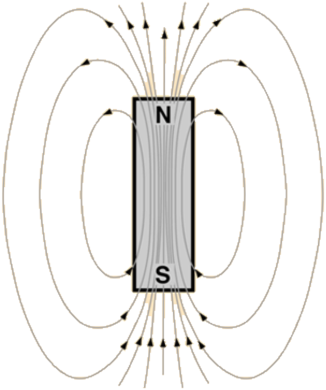
A Gigantic “Permanent” Magnet

- Earth is also a “permanent” magnet
  - Magnetic field generated by electric currents in the conductive material in its core
  - Current created by convection currents due to heat escaping from the core
- Magnetic field lines run from magnetic north to magnetic south
- By our convention, our South Pole is actually the “magnetic north pole”, and our North Pole is the “magnetic south pole”

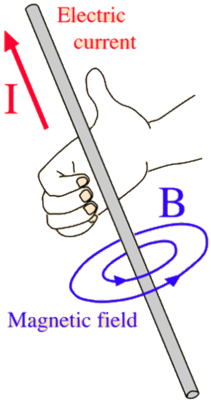


What About Permanent Magnet?

- Magnetic fields don't actually run from a "North" pole to a "South" pole
- In fact, they run in a loop (see right)
- The magnetic field lines continues inside the bar magnet



Magnetic Field Generated By a Wire



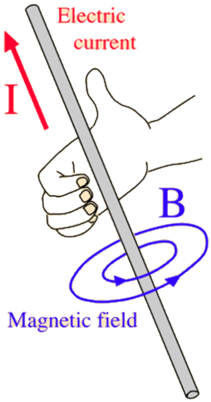
In fact, the magnetic field generated by a current also runs in a loop, given by the equation:

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

Quantity	Symbol	SI Unit
Magnetic field intensity	B	T (teslas)
Current	I	A (amperes)
Radial distance from the wire	r	m (metres)

The constant $\mu_0 = 4\pi \times 10^{-7}$ T · m / A is the "permeability of free space"

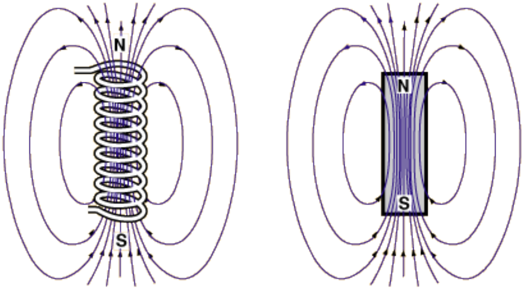
Magnetic Field Generated By a Wire



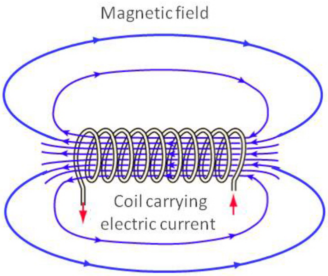
- Both electric current ($\mathbf{I}$) and magnetic field ($\mathbf{B}$) are vectors
- Direction of $\mathbf{I}$ based on movement of positive charges
  - In an *actual* wire, negative charges (electrons) are moving in the opposite direction
- Direction of $\mathbf{B}$ is determined using **right hand rule**

Wounding Wires Into a Coil

- A **solenoid** is when you wound a wire into a coil
- You create a magnet very similar to a bar magnet, with an effective north pole and a south pole
- Magnetic field inside the solenoid is uniform
- Magnetic field strength can be increased by the addition of an iron core



Magnetic Field Inside a Solenoid



- The magnetic field **inside** the solenoid given by:

$$\boxed{B = \mu n I}$$

- Direction of $\mathbf{B}$ determined by **right hand rule**

Quantity	**Symbol**	**SI Unit**
Magnetic field intensity	B	T (teslas)
Number of coils	n	integer, no units
Current	I	A (amperes)
Effective permeability	μ	$\mathrm{T \cdot m / A}$ (tesla metres per ampere)

Tesla

The unit of magnetic field, not the car!

- The strength of a magnetic field
- A charge of one coulomb, travelling with a speed of $1\,\mathrm{m/s}$ perpendicular to the magnetic field ($\theta = 90^\circ$ and $\sin\theta = 1$) experiences a force of $1\,\mathrm{N}$

$$\text{tesla} = \frac{\text{newton}}{\text{coulomb}\dfrac{\text{metre}}{\text{second}}}$$

So What Does the Magnetic Field Do?

Gravitational Field $\mathbf{g}$

- Generated by any object that has a mass
- Affects everything that has mass

Electric Field $\mathbf{E}$

- Generated by all charged particles
- Affects all charged particles
- The charged particle can be at rest or moving

Magnetic Field $\mathbf{B}$

- Generated by **electric currents** and **moving charged particles**
- Affects electric currents and moving charged particles

Force on a Moving Charge in a Magnetic Field

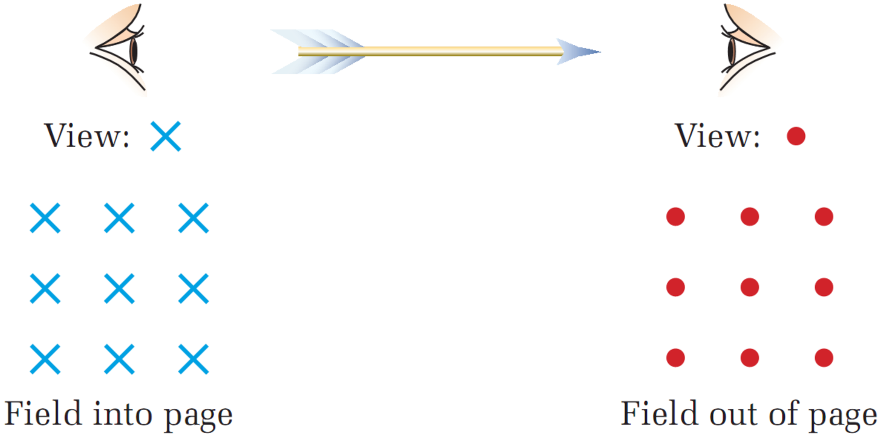
When a moving charge (q) enters a magnetic field ($\mathbf{B}$) with a velocity $\mathbf{v}$, the magnetic field exerts a force ($\mathbf{F}_M$) on the charge:

$$\boxed{F_M = qvB\sin\theta}$$

Quantity	Symbol	SI Unit
Magnetic force on the moving charge	F_M	N (newtons)
Electric charge of the particle	q	C (coulombs)
Speed of the charged particle	v	m/s (metres per second)
Magnetic field strength	B	T (teslas)
Angle between particle and magnetic field	θ	

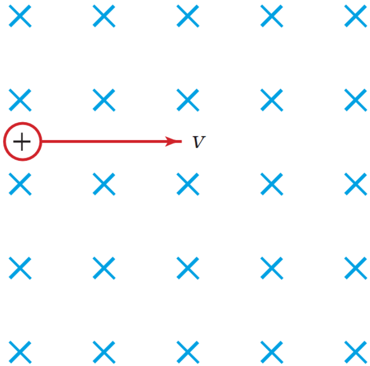
The direction of $\mathbf{F}_M$ is perpendicular to both $\mathbf{v}$ and $\mathbf{B}$

Convention for Diagrams



Example Problem

Example 8: A particle carrying a charge of $2.50\,\mu\mathrm{C}$ enters a magnetic field travelling at $3.40 \times 10^5\,\mathrm{m/s}$ to the right of the page. If a uniform magnetic field is pointing directly into the page and has a strength of $0.500\,\mathrm{T}$, what is the magnitude and direction of the force acting on the charge as it just enters the magnetic field?



Force on a Current-Carrying Conductor in a Magnetic Field

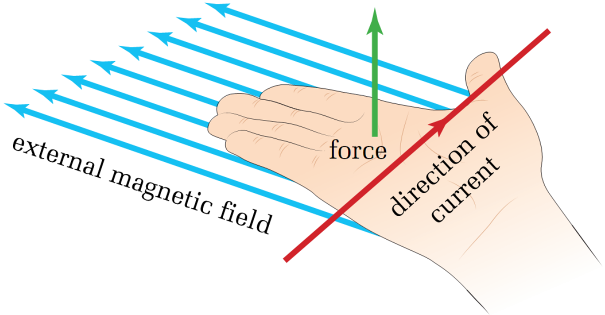
The magnetic field also exerts a force on a conductor carrying a current.

$$\boxed{F_M = IlB\sin\theta}$$

Quantity	Symbol	SI Unit
Magnetic force on the conductor	F_M	N (newtons)
Electric current in the conductor	I	A (amperes)
Length of the conductor inside magnetic field	l	m (metres)
Magnetic field strength	B	T (teslas)
Angle between conductor and magnetic field	θ	

This should not come as a surprise as a current is just a stream of charged particles

Right Hand Rule for Induced Magnetic Force



- The direction of current is the *conventional current*, which assumes the flow of positive charges. In fact, in a conductor, negatively charged electrons flow in the opposite direction.
- Single charged particle follows the same convention.

Example Problem

Example 9: A wire segment of length $40\,\mathrm{cm}$, carrying a current of $12\,$A, crosses a magnetic field of $0.75\,$T [up] at an angle of [up 40° right]. What magnetic force is exerted on the wire?

Circular Motion Caused by a Magnetic Field

When a charged particle enters a magnetic field at right angle...

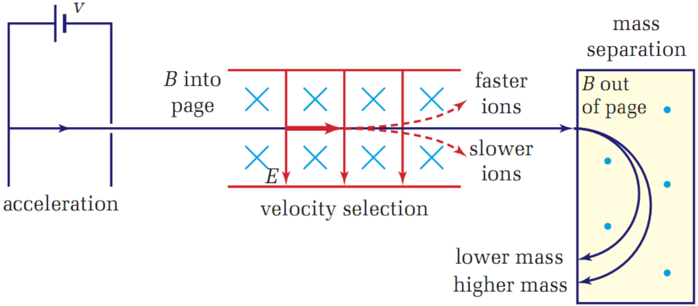
- Magnetic force $\mathbf{F}_M$ perpendicular to both velocity $\mathbf{v}$ and magnetic field $\mathbf{B}$.
- Results in circular motion

Centripetal force $\mathbf{F}_c$ given by the magnetic force $\mathbf{F}_M$. We can solve for the radius r of the motion:

$$\frac{mv^2}{r} = qvB$$
$$r = \frac{mv}{qB}$$

Mass Spectrometer

- Separates particles of different mass
- Measures the mass of the particle by measuring displacement
- Three major components:
    - Particle accelerator
    - Velocity selector
    - Mass separator



Simple Particle Accelerator

- Made of:
  - A pair of parallel plates
  - Accelerating potential difference
  - Particle source
- Ionized particle source passes through the plates and get accelerated by the potential difference then get shot out of the other end of the plate.
- e.g.: particle gun or electron gun

$$\frac{1}{2}mv^2 = qV$$

Velocity Selector

- Often associates with the parallel plate particle accelerator.
- Filters the beam of particles to let particles with the same velocity to pass by only.
- Consists of a crossed (perpendicular) electric and magnetic field.
- When electric and magnetic forces are balanced, particle travels straight through.

Velocity Selector

The particle will travel straight through when magnetic force $\mathbf{F}_M$ and electric force $\mathbf{F}_q$ are balanced:

$$F_M = F_q$$

Substitute the expressions for F_M and F_q, then solve for v:

$$qvB = qE$$

$$v = \frac{E}{B}$$

We can adjust which particle velocity can go straight through by adjusting the relative strength of the electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$.

Mass Separation

Particles of different masses are separated by allowing to go into circular motion inside a magnetic field. Centripetal force $\mathbf{F}_c$ given by magnetic force $\mathbf{F}_M$

$$F_c = F_M$$

Substitute the expressions for F_c and F_M, then solve for m:

$$\frac{mv^2}{r} = qvB$$

$$m = \frac{rqB}{v}$$

Can tell the mass of the particle by knowing its charge q, speed v, magnetic field strength B and the radius of the circular motion r.

Example Problem

Example 9: A positive ion, having a charge of 3.2×10^{-19} C, enters at the extreme left of the parallel plate assembly associated with the velocity selector and mass spectrometer shown previously.

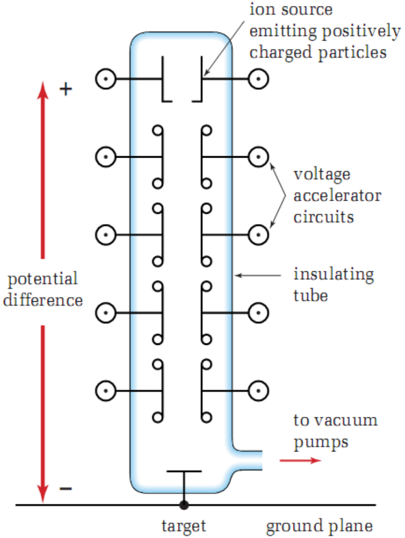
- If the potential difference across the simple accelerator is 1.2×10^{3} V, what is the kinetic energy of the particle as it leaves through the hole in the right plate?

Example Problem (cont.)

- The parallel plates of the velocity selector are separated by $12\,\mathrm{mm}$ and have an electric potential difference across them of $360\,\mathrm{V}$. If a magnetic field of strength $0.10\,\mathrm{T}$ is applied at right angles to the electric field, what is the speed of the particles that will be selected to pass on the mass spectrometer?
- When these particles then enter the mass spectrometer, which shares a magnetic field with the velocity selector, the radius of the resulting circular path followed by the particles is $6.3\,\mathrm{cm}$. What is the mass of the charged particles?

The Cockcroft-Walton Proton Accelerator

- Capable of 1 MeV (mega electron volt)
- $1\ \mathrm{eV} = 1.6021 \times 10^{-19}$ J



The Cyclotron

- Large number of small increases in potential.
- 30 MeV is very common