Fractions 1

1. Adding and Subtracting Fractions

If the fractions have the same denominator, their sum is the sum of the numerators over the denominator. If the fractions have the same denominator, their difference is the difference of the numerators over the denominator. We do not add or subtract the denominators! Reduce if necessary.

Examples: 3/8 + 2/8 = 5/8
9/2 - 5/2 = 4/2 = 2

If the fractions have different denominators:
1) First, find the least common denominator.
2) Then write equivalent fractions using this denominator.
3) Add or subtract the fractions. Reduce if necessary.

Example: 3/4 + 1/6 = ?
The least common denominator is 12.
3/4 + 1/6 = 9/12 + 2/12 = 11/12.

Example: 9/10 - 1/2 = ?
The least common denominator is 10.
9/10 - 1/2 = 9/10 - 5/10 = 4/10 = 2/5.

Example: 2/3 + 2/7 = ?
The least common denominator is 21
2/3 + 2/7 = 14/21 + 6/21 = 20/21.

2. Adding and Subtracting Mixed Numbers

To add or subtract mixed numbers, simply convert the mixed numbers into improper fractions, then add or subtract them as fractions.

Example: 9 1/2 + 5 3/4 = ?

Converting each number to an improper fraction, we have 9 1/2 = 19/2 and 5 3/4 = 23/4.
We want to calculate 19/2 + 23/4. The LCM of 2 and 4 is 4, so
19/2 + 23/4 = 38/4 + 23/4 = (38 + 23)/4 = 61/4.

Converting back to a mixed number, we have 61/4 = 15 1/4.

The strategy of converting numbers into fractions when adding or subtracting is often useful, even in situations where one of the numbers is whole or a fraction.

Example: 13 - 1 1/3 = ?

In this situation, we may regard 13 as a mixed number without a fractional part. To convert it into a fraction, we look at the denominator of the fraction 4/3, which is 1 1/3 expressed as an improper fraction. The denominator is 3, and 13 = 39/3. So 13 - 1 1/3 = 39/3 - 4/3 = (39-4)/3 = 35/3, and 35/3 = 11 2/3.

Example: 5 1/8 - 2/3 = ?

This time, we may regard 2/3 as a mixed number with 0 as its whole part. Converting the first mixed number to an improper fraction, we have 5 1/8 = 41/8. The problem becomes
5 1/8 - 2/3 = 41/8 - 2/3 = 123/24 - 16/24 = (123 - 16)/24 = 107/24.
Converting back to a mixed number, we have 107/24 = 4 11/24.

Example: 92 + 4/5 =?
This is easy. To express this as a mixed number, just put the whole number and the fraction side by side. The answer is 92 4/5.

3. Multiplying Fractions and Whole Numbers

To multiply a fraction by a whole number, write the whole number as an improper fraction with a denominator of 1, then multiply as fractions.

Example: 8 × 5/21 = ?
We can write the number 8 as 8/1. Now we multiply the fractions.
8 × 5/21 = 8/1 × 5/21 = (8 × 5)/(1 × 21) = 40/21

Example: 2/15 × 10 = ?
We can write the number 10 as 10/1. Now we multiply the fractions.
2/15 × 10 = 2/15 × 10/1 = (2 × 10)/(15 × 1) = 20/15 = 4/3

4. Multiplying Fractions and Fractions

When two fractions are multiplied, the result is a fraction with a numerator that is the product of the fractions' numerators and a denominator that is the product of the fractions' denominators.

Example: 4/7 × 5/11 = ?
The numerator will be the product of the numerators: 4 × 5, and the denominator will be the product of the denominators: 7 × 11.

The answer is (4 × 5)/(7 × 11) = 20/77.
Remember that like numbers in the numerator and denominator cancel out.

Example: 14/15 × 15/17 = ?
Since the 15's in the numerator and denominator cancel, the answer is
14/15 × 15/17 = 14/1 × 1/17 = (14 × 1)/(1 × 17) = 14/17

Example: 4/11 × 22/36 = ?
In the solution below, first we cancel the common factor of 11 in the top and bottom of the product, then we cancel the common factor of 4 in the top and bottom of the product.
4/11 × 22/36 = 4/1 × 2/36 = 1/1 × 2/9 = 2/9

5. Multiplying Mixed Numbers

To multiply mixed numbers, convert them to improper fractions and multiply.

Example: 4 1/5 × 2 2/3 = ?.
Converting to improper fractions, we get 4 1/5 = 21/5 and 2 2/3 = 8/3. So the answer is
4 1/5 × 2 2/3 = 21/5 × 8/3 = (21 × 8)/(5 × 3) = 168/15 = 11 3/15.

Examples:
3/4 × 1 1/8 = 3/4 × 9/8 = 27/32.
3 × 7 3/4 = 3 × 31/4 = (3 × 31)/4 = 93/4 = 23 1/4.

6. Reciprocal

The reciprocal of a fraction is obtained by switching its numerator and denominator.

To find the reciprocal of a mixed number, first convert the mixed number to an improper fraction, then switch the numerator and denominator of the improper fraction. Notice that when you multiply a fraction and its reciprocal, the product is always 1.
Example: Find the reciprocal of 31/75.
We switch the numerator and denominator to find the reciprocal: 75/31.

Example: Find the reciprocal of 12 1/2.
First, convert the mixed number to an improper fraction: 12 1/2 = 25/2. Next, we switch the numerator and denominator to find the reciprocal: 2/25.

7. Dividing Fractions

To divide a number by a fraction, multiply the number by the reciprocal of the fraction.

Examples:
7 ÷ 1/5 = 7 × 5/1 = 7 × 5 = 35
1/5 ÷ 16 = 1/5 ÷ 16/1 = 1/5 × 1/16 = (1 × 1)/(5 × 16) = 1/80
3/5 ÷ 7/12 = 3/5 × 12/7 = (3 × 12)/(5 × 7) = 36/35 or 1 1/35

8. Dividing Mixed Numbers

To divide mixed numbers, you should always convert to improper fractions, then multiply the first number by the reciprocal of the second.

Examples:
1 1/2 ÷ 3 1/8 = 3/2 ÷ 25/8 = 3/2 × 8/25 = (3 × 8)/(2 × 25) = 24/50
1 ÷ 3 3/5 = 1/1 ÷ 18/5 = 1/1 × 5/18 = (1 × 5)/(1 × 18) = 5/18
3 1/8 ÷ 2 = 25/8 ÷ 2/1 = 25/8 × 1/2 = (25 × 1)/(8 × 2) = 25/16 or 1 9/16.

9. Simplifying Complex Fractions

A complex fraction is a fraction whose numerator or denominator is also a fraction or mixed number.

Example of complex fractions:

$$\frac{\frac{1}{4}}{\frac{2}{3}},\ \frac{\frac{3}{7}}{100},\ \frac{11}{\frac{2}{3}},\ \frac{23\frac{1}{5}}{\frac{2}{3}}$$

otherwise written as (1/4)/(2/3), (3/7)/100, 11/(2/3), and (23 1/5)/(2/3).

To simplify complex fractions, change the complex fraction into a division problem: divide the numerator by the denominator.

The first of these examples becomes
(1/4)/(2/3) = 1/4 ÷ 2/3 = 1/4 × 3/2 = 3/8.
The second of these becomes
(3/7)/100 = 3/7 ÷ 100 = 3/7 × 1/100 = 3/700.

The third of these becomes
11/(2/3) = 11 ÷ 2/3 = 11 × 3/2 = 33/2 = 16 1/2.

The fourth of these becomes
(23 1/5)/(2/3) = 23 1/5 ÷ 2/3 = 116/5 ÷ 2/3 = 116/5 × 3/2 = 174/5 = 34 4/5.

10. Repeating Decimals

Every fraction can be written as a decimal.

For example, 1/3 is 1 divided by 3.
If you use a calculator to find 1 ÷ 3, the calculator returns 0.333333... This is called a repeating decimal.

To represent the idea that the 3's repeat forever, one uses a horizontal bar (overstrike) as shown below:

$0.\overline{3} = 0.333333..$

$1.412412412412... = 1.\overline{412}$

$104.6278787878... = 104.62\overline{78}$

Example: What is the repeating decimal for 1/7? Dividing 7 into 1, we get 0.142857142..., and we see the pattern begin to repeat with the second 1, so $\frac{1}{7} = 0.\overline{142857}$.

11. Rounding a Fraction to the Nearest Hundredth

Divide to the thousandths place. If the last digit is less than 5, drop it. This is particularly useful for converting a fraction to a percent, if we want to convert to the nearest percent.
1/3 = 1 ÷ 3 = 0.333… which rounds to 0.33

If the last digit is 5 or greater, drop it and round up.
2/7 = 2 ÷ 7 = 0.285 which rounds to 0.29

►Questions

1. On an exam with *q* questions, Marie correctly answered 15 of the first 20 but just 1/3 of the rest. If her total score was 50%, what was *q*?

2. Find the value of this product of 98 numbers: $(1-\frac{2}{3})(1-\frac{2}{4})(1-\frac{2}{5})\cdots(1-\frac{2}{98})(1-\frac{2}{99})(1-\frac{2}{100})$.

3. A recipe for brownies requires $2\frac{2}{3}$ cups of flour. There are 45 cups of flour in the bag. After as many whole recipes of brownies as possible have been made for the Spring Fling, how many cups of flour will be left over?

4. Simplify the fraction $\dfrac{1}{1+\dfrac{1}{2+\dfrac{1}{3}}}$

5. What is the value of $\dfrac{\frac{1}{2}-\frac{1}{3}}{\frac{1}{6}-\frac{1}{18}}$?

6. Evaluate $\frac{1}{98}+\frac{99\times 97}{98}-98$.

7. Evaluate the following expression. $\dfrac{2^{310}-2^{301}}{3^4\cdot 2^{300}}$

8. What is the value of $2+\dfrac{2}{2+\dfrac{2}{2+\frac{2}{3}}}$?

9. Evaluate the expression $\dfrac{1}{\frac{1}{2}+\frac{1}{3}+\frac{1}{5}}$.

10. What is the value of $\dfrac{3}{1+\dfrac{2}{2+1}}$?