

First Name: _____ Last Name: _____ Student ID: _____

An introduction to calculus (1)

1. Determine the slope of the secant to the given curve between the specified values of x .

a. $y = 3^x - 4$, $x = 0$, $x = 2$

b. $y = \tan(x)$, $x = \frac{\pi}{3}$, $x = \frac{\pi}{6}$

c. $y = \log_8 x$, $x = \frac{1}{2}$, $x = 4$

d. $y = \sin(x) - \cos(x)$, $x = -\frac{\pi}{4}$, $x = \frac{3\pi}{4}$

2. Consider the function $y = f(x) = \frac{3}{2}x^2 - 2x$.

a. Find the slope of the tangent line at the generic point $P(a, f(a))$

b. Find the point where the tangent line is horizontal.

c. Find the point P such that $m_P = 1$.

d. Find the point P such that the tangent line at P is perpendicular to the line $L: 11x - y = 3$.

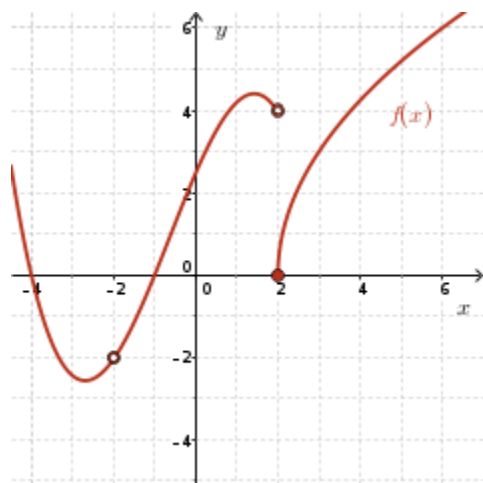
3. The altitude of a rock climber t hours after she begins her ascent up a mountain is modelled by the equation $a(t) = -5t^2 + 30t$, where the altitude, $a(t)$, is measured in metres.

- a. Determine the altitude of the rock climber 2 hours after she begins her climb.
- b. Determine the altitude of the rock climber 3 hours after she begins her climb.
- c. Determine the average rate of change of the altitude of the rock climber between 2 and 3 hours after she begins her climb.
- d. Determine the instantaneous rate of change of the altitude of the rock climber 3 hours after she begins her climb.
- e. What is the significance of the instantaneous rate of change value found in part d)? Explain what this value tells us about the rock climber's travel at this point.

4. The path of a robot along a track is modelled by the curve $y = 2x^2 + 1$. As the robot moves, it passes through the point $P(-1, 3)$. At this point, it attempts to shoot a ball at a target located at the point $(1, -5)$. If the ball travels along the tangent line to the curve at point P , will the ball hit the target?

5. On an evening walk, a man passes under a streetlight and notices that the length of his shadow increases as he walks away from the base of the streetlight. If the man is 2 m tall and the streetlight is 6 m tall, determine how the length of the man's shadow is changing, in terms of the rate at which he is walking away from the streetlight. Support your answer with a diagram.

6. Given the graph of $f(x)$, evaluate the following expressions involving $f(x)$.



a. $\lim_{x \rightarrow -1} f(x)$

b. $\lim_{x \rightarrow -2} f(x)$

c. $\lim_{x \rightarrow 2^+} f(x)$

d. $\lim_{x \rightarrow -2^-} f(x)$

e. $f(-2)$

f. $\lim_{x \rightarrow 2} f(x)$

g. $\lim_{x \rightarrow 2^-} f(x)$

h. $f(2)$

i. $\lim_{x \rightarrow 3} f(x)$

7. Sketch the graph of a function that has the following characteristics:

$$\lim_{x \rightarrow -1^-} f(x) = 3$$

$$\lim_{x \rightarrow -1^+} f(x) = 1$$

$$\lim_{x \rightarrow 3} f(x) = 2$$

$f(3)$ does not exist

8. Sketch the graph of a function that has the following characteristics:

i. $\lim_{x \rightarrow 3^-} f(x) \rightarrow +\infty$

ii. $\lim_{x \rightarrow -1^+} f(x) \rightarrow -\infty$

iii. $\lim_{x \rightarrow 1} f(x) = 1$

iv. $f(0) = 0$

9. Given that $\lim_{x \rightarrow a} f(x) = 4$ and $\lim_{x \rightarrow a} g(x) = -2$, find the following limits:

a. $\lim_{x \rightarrow a} (f(x) + g(x))$

b. $\lim_{x \rightarrow a} f(x) g(x)$

c. $\lim_{x \rightarrow a} \frac{f(x)+2}{2-2g(x)}$

d. $\lim_{x \rightarrow a} \frac{\sqrt{f(x)}}{g(x)}$