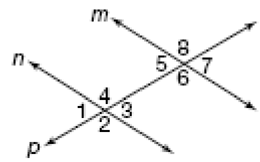
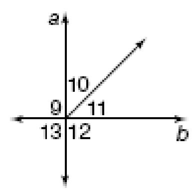
Chapter 9 Angles

1. Angle Relationships

When two lines intersect, the two pairs of opposite angles formed are called vertical angles. Vertical angles are always congruent, meaning they have the same measure. In Figure 1, $\angle 1$ and $\angle 3$ are vertical angles, so $\angle 1$ is congruent to $\angle 3$.

In Figure 1, two parallel lines, *n* and *m*, are intersected by a third line, *p*, called a **transversal.** Since $m \parallel n$, the following statements are true.

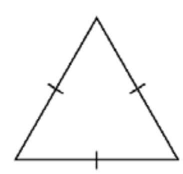
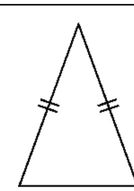
$\angle 5$ and $\angle 3$ are a pair of congruent **alternate interior angles.** $\angle 2$ and $\angle 8$ are a pair of congruent **alternate exterior angles.** $\angle 1$ and $\angle 5$ are a pair of congruent **corresponding angles.**	 Figure 1
In Figure 2, lines *a* and *b* are perpendicular. $\angle 10$ and $\angle 11$ are complementary angles since the measure of $\angle 10$ plus the measure of $\angle 11$, $\angle 10 + \angle 11 = 90^0$. $\angle 9$ and $\angle 13$ are supplementary angles since $\angle 9 + \angle 13 = 180^0$.	 Figure 2

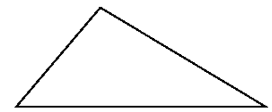
2. Triangles and Angles

Classifying Triangles by Sides or Angles

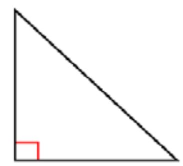
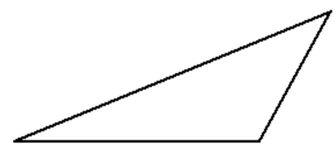
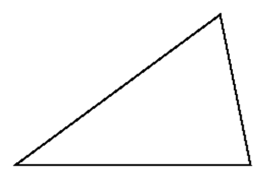
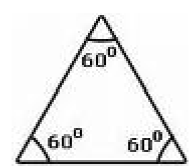
Triangles can be classified either according to their sides or according to their angles. All of each may be of different or the same sizes; any two sides or angles may be of the same size; there may be one distinctive angle.

The types of triangles classified by their *sides* are the following:

Equilateral triangle: A triangle with all three sides equal in measure. He slash marks indicate equal measure.	
Isosceles triangle: A triangle in which at least two sides have equal measure	

Scalene triangle: A triangle with all three sides of different measures	

The types of triangles classified by their *angles* include the following:

Right triangle: A triangle that has a right angle in its interior	
Obtuse triangle: A triangle having an obtuse angle (greater than 90° but less than 180°) in its interior.	
Acute triangle: A triangle having all acute angles (less than 90°) in its interior	
Equiangular triangle: A triangle having all angles of equal measure	

Because the sum of all the angles of a triangle is 180°, the following theorem is easily shown.

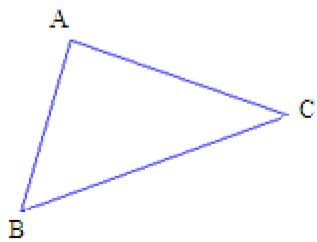
Theorem: Each angle of an equiangular triangle has a measure of 60°.

3. Interior angles of a Polygons

The sum of the measures of the interior angles of a polygon with n sides is $(n-2) \times 180$.

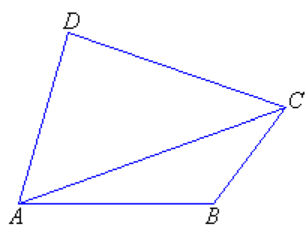
1) Triangle or ('3-gon')

Sum of interior angles: $(3-2) \times 180 = 180°$



2) Quadrilateral which has four sides ('4-gon')

Sum of interior angles: (4-2) ×180 = 360°

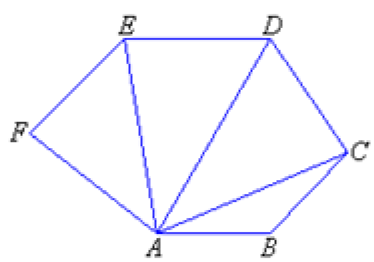


Clearly, the diagonal *AC* divides the quadrilateral into 2 triangles.

$$\therefore \text{ Angle sum of a quadrilateral} = 2\times 180^\circ \qquad \{\because \text{ Angle sum of a triangle} = 180^\circ\}$$
$$= 360^\circ$$

3) Hexagon which has six sides (‘6-gon')

Sum of interior angles: (6-2) ×180 = 720°



Clearly, the diagonals *AC*, *AD*, and *AE* divides the hexagon into 4 triangles.

$$\therefore \text{ Angle sum of a hexagon} = 4\times 180^\circ \qquad \{\because \text{ Angle sum of a triangle} = 180^\circ\}$$
$$= 720^\circ$$

In order to find the measure of a single interior angle of a regular polygon (a polygon with sides of equal length and angles of equal measure) with n sides, we just divide the sum of the interior angles or (n - 2) × 180 by the number of sides or n.

An interior angle of a regular polygon with n sides is $\frac{(n-2)\times 180^0}{n}$

Example: To find the measure of an interior angle of a regular octagon, which has 8 sides, apply the formula above as follows:

((8-2) × 180)/8 = 135°

Example:
1) What is the total number of degrees of all interior angles of a triangle?
2) What is the total number of degrees of all interior angles of the polygon on the left?
3) What is the sum measure of the interior angles of the polygon (a pentagon) on the left?

Example 1: Find the number of degrees in the sum of the interior angles of an octagon.

An octagon has 8 sides. So n = 8. Using our formula from above, that gives us:
$180(8-2) = 180(6) = 1080^0$.

Example 2: How many sides does a polygon have if the sum of its interior angles is 720^0?

Since, this time, we know the number of degrees, we set the formula above equal to 720^0, and solve for n.

180(n-2) = 720	Set the formula = 720^0
n-2 = 4	Divide both sides by 180
n= 6	Add 2 to both sides

From the above discussion, we observed that:

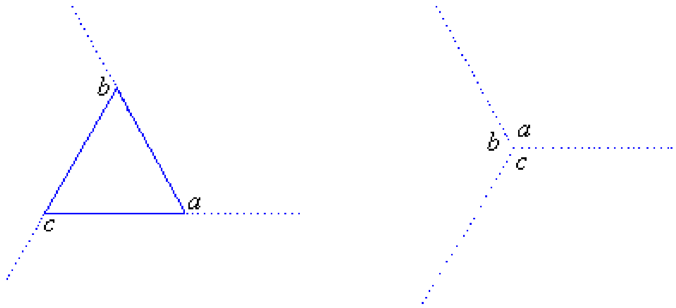
$$\text{Angle sum of a polygon of 4 sides} = 2\times 180^\circ = (4-2)\times 180^\circ$$
$$\text{Angle sum of a polygon of 5 sides} = 3\times 180^\circ = (5-2)\times 180^\circ$$
$$\text{Angle sum of a polygon of 6 sides} = 4\times 180^\circ = (6-2)\times 180^\circ$$

In general:

$$\begin{aligned}\text{Angle sum of a polygon of } n \text{ sides} &= (n-2)\times 180^\circ \\ &= 2(n-2) \text{ right angles} \\ &= (2n-4) \text{ right angles}\end{aligned}$$

Note that a polygon of *n* sides is called an ***n*-gon.**

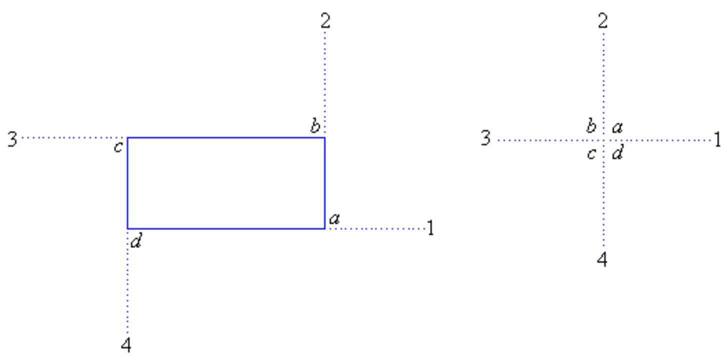
4) External Angle Sum of Polygons



Let the exterior angles of a triangle be rearranged so that they have the same vertex as shown above.

Then $\quad a+b+c=360°$ $\qquad$ {Angle sum at a point}

Let us now consider the external angle sum of a rectangle.



Let the exterior angles of a rectangle be rearranged so that they have the same vertex as shown above.

Then $\quad a+b+c+d=360°$ $\qquad$ {Angle sum at a point}

In general:
The external angle sum of a polygon is 360°.

Example 1: Calculate the exterior and interior angle of a regular pentagon.

Solution: A regular pentagon has five equal angles.

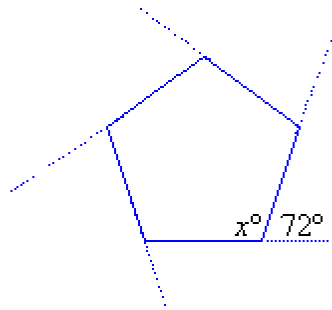
$$\text{Sum of 5 exterior angles} = 360°$$

$$\therefore \text{Each exterior angle} = \frac{360°}{5} = 72°$$

Let the interior angle be x^o.

$$\therefore\ x+72=180 \qquad \{\text{Supplementary adjacent angles}\}$$
$$x+72-72=180-72$$
$$x=108$$

So, each interior angle is 108^o.

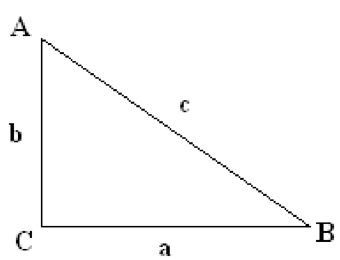


5. The Pythagorean Theorem

Over 2,500 years ago, a Greek mathematician named Pythagoras developed a proof that the relationship between the hypotenuse and the legs is true for ***all*** right triangles.

"In any right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs."

This relationship can be stated as: $c^2 = a^2 + b^2$ and is known as the Pythagorean Theorem.



a, b are legs.

c is the hypotenuse

(c is across from the right angle).

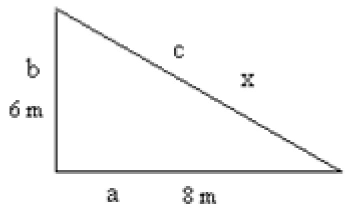
There are certain sets of numbers that have a very special property. Not only do these numbers satisfy the Pythagorean Theorem, but any multiples of these numbers also satisfy the Pythagorean Theorem.

For example: the numbers 3, 4, and 5 satisfy the Pythagorean Theorem. If you multiply all three numbers by 2 (6, 8, and 10), these new numbers ALSO satisfy the Pythagorean Theorem.

The special sets of numbers that possess this property are called Pythagorean Triples.

The most common Pythagorean Triples are:
3, 4, 5; 5, 12, 13; 8, 15, 17.

Example 1: In the figure, find x.



$$c^2 = a^2 + b^2$$

$$x^2 = 8^2 + 6^2 = 64+32 = 100$$

x = 10

This problem could also be solved using the Pythagorean Triple 3, 4, 5. Since 6 is 2 times 3, and 8 is 2 times 4, then x must be 2 times 5.

Example 2: A triangle has sides 6, 7 and 10. Is it a right triangle?

Let a = 6, b = 7 and c = 10. The longest side MUST be the hypotenuse, so c = 10. Now, check to see if the Pythagorean Theorem is true.

$$c^2 = a^2 + b^2$$

$c^2 = 10^2 = 100$ $a^2 + b^2 = 6^2 + 7^2 = 36 + 49 = 85$
So, $c^2 \neq a^2 + b^2$

Since the Pythagorean Theorem is NOT true, this triangle is NOT a right triangle.