Number Theory

Concepts

1. Integers: Whole numbers and the opposites of whole numbers.

2. Rational Numbers: Numbers that can be written as a fraction (or ratio) of two integers.

3. Irrational Numbers: Numbers that cannot be written as a fraction of two integers and are on the number line.

The numbers defined above are in some ways similar as well as different.

The numbers are similar in that they are all components in the **real number system.**

The numbers are different in their definitions.

For examples

Integers	***Rational***	***Irrational***
5, -9, 0, 2, -16	2/5, -7/9, 3/8, $2.\overline{3}$	π , $\sqrt{2}$, $\sqrt{5}$

Since integers can be written in fractional form they can also be considered rational numbers. However, not all rational numbers can be integers!

4. Least Common Multiple

The smallest (non-zero) number that is a multiple of two or more numbers.

Least Common Multiple is made up of the words *Least*, *Common* and *Multiple*:
What is a "Multiple"?

The multiples of a number are what you get when you **multiply it by other numbers** (such as if you multiply it by 1, 2, 3, 4, 5, etc). Just like the multiplication table.

Here are some examples:

The multiples of **3** are **3, 6, 9, 12, 15, 18, 21, etc** ...
The multiples of **12** are **12, 24, 36, 48, 60, 72, etc...**

What is a "Common Multiple"?

When you list the multiples of two (or more) numbers, and find the **same value in both lists**, then that is a ***common multiple*** of those numbers.

For example, when you write down the multiples of **4** and **5**, the *common* multiples are those that are found in both lists:

The multiples of 4 are 4,8,12,16,**20**,24,28,32,36,**40**,44,...

The multiples of 5 are 5,10,15,**20**,25,30,35,**40**,45,50,...

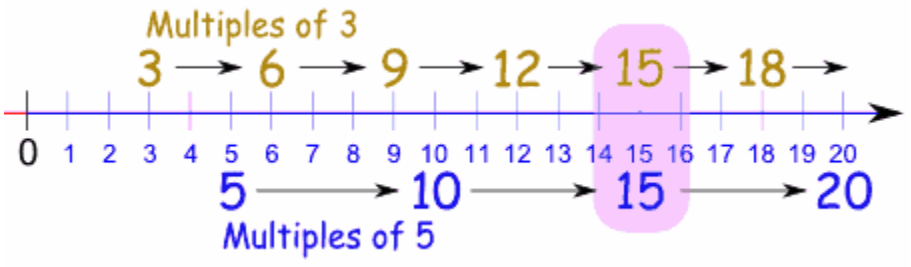
Notice that **20** and **40** appear in both lists?
So, the common multiples of 4 and 5 are: **20, 40,** (and 60, 80, etc ..., too)

What is the "Least Common Multiple"?
It is simply the **smallest** of the common multiples.

In our previous example, the smallest of the common multiples is 20, so the *Least* Common Multiple of 4 and 5 is **20**.

Finding the Least Common Multiple
It is a really easy thing to do. Just start listing the multiples of the numbers until you get a match.

Example: Find the least common multiple for 3 and 5:
The multiples of 3 are ***3, 6, 9, 15, ...***,
and the multiples of 5 are ***5, 10, 15, 20, ...***, like this:



As you can see on this number line, the first time the multiples matchup is 15.
Answer: 15

More than 2 Numbers
You can also find the least common multiple of 3 (or more) numbers.

Example: Find the least common multiple for 4, 6, and 8

Multiples of 4 are: 4, 8, 12, 16, 20, **24**, 28, 32, 36, ...
Multiples of 6 are: 6, 12, 18, **24**, 30, 36, ...
Multiples of 8 are: 8, 16, **24**, 32, 40, ...
So, **24** is the least common multiple (I can't find a smaller one !)

Hint: You can have smaller lists for the bigger numbers.

5. Greatest Common Factor

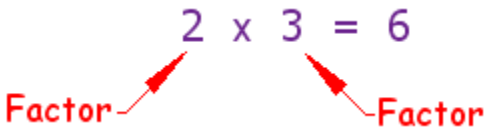
The highest number that divides exactly into two or more numbers. It is the "greatest" thing for simplifying fractions!

1) Greatest Common Factor is made up of three words

- *Greatest*,
- *Common* and
- *Factor*

2) Factor

Factors are the numbers you multiply together to get another number:



Sometimes we want to find ALL the factors of a number:

The factors of 12 are **1, 2, 3, 4, 6** and **12** ...

... because **2** × **6** = 12, or **4** × **3** = 12, or **1** × **12** = 12.

3) What is a "Common Factor"?

Let us say you have worked out the factors of two or more numbers:

Example: The factors of 12 are **1, 2, 3, 4, 6** and **12**

The factors of 30 are **1, 2, 3, 5, 6, 10, 15** and **30**

Then the ***common* factors** are those that are found in both numbers:

- Notice that **1, 2, 3** and **6** appear in both lists?
- So, the **common factors** of 12 and 30 are: **1, 2, 3** and **6**

It is a *common* factor when it is a factor of two or more numbers.
(It is then *"common to"* those numbers.)

Here is another example:
Example: What are the common factors of 15, 30 and 105?

The factors of 15 are **1, 3, 5,** and **15**
The factors of 30 are **1, 2, 3, 5, 6, 10, 15** and **30**

The factors of 105 are **1, 3, 5, 7, 15, 21, 35** and **105**

The factors that are common to all three numbers are **1, 3, 5** and **15**
In other words, the **common factors** of 15, 30 and 105 are **1, 3, 5** and **15**

What is the "Greatest Common Factor"?
It is simply the **largest** of the common factors. In our previous example, the largest of the common factors is 15, so the **Greatest Common Factor** of 15, 30 and 105 **is 15**

4) Finding the Greatest Common Factor

Here are three ways:

a) you can

- find all **factors** of both numbers
- then select the ones that are **common** to both, and
- then choose the **greatest**.

Example:

Two Numbers	**All Factors**	**Common Factors**	**Greatest Common Factor**	***Example Simplified Fraction***
9 and 12	**9**: 1,3,9 **12**: 1,2,3,4,6,12	1,3	**3**	$^{9}/_{12}$ » $^{3}/_{4}$

And another example:

Two Numbers	**All Factors**	**Common Factors**	**Greatest Common Factor**	***Example Simplified Fraction***
6 and 18	**6**: 1,2,3,6 **18**: 1,2,3,6,9,18	1,2,3,6	**6**	$^{6}/_{18}$ » $^{1}/_{3}$

b) Find the prime factors and combine the common ones together:

Two Numbers	**Thinking ...**	**Greatest Common Factor**	***Example Simplified Fraction***
24 and 108	**2** × **2** × 2 × **3** = 24, and **2** × **2** × **3** × 3 × 3 = 108	2 × 2 × 3 = **12**	$^{24}/_{108}$ » $^{2}/_{9}$

c) And sometimes you can just **play around** with the factors until you discover it:

Two Numbers	**Thinking ...**	**Greatest Common Factor**	***Example Simplified Fraction***
9 and 12	**3** × 3 = 9 and **3** × 4 = 12	**3**	$^{9}/_{12}$ » $^{3}/_{4}$

But in that case you had better be careful you have found the **greatest** common factor.

6. Prime Number

A Prime Number can be divided evenly only by 1, or itself. (In other words, its factors are only 1 or itself)

Example: 13 can only be divided evenly by 1 or 13, so it is a prime number.

If it is not a Prime Number it is called a Composite Number

Example: 14 can be divided evenly by 1, 2, 7 and 14 so it is a composite number.

7. Composite Number

A **Composite Number** can be divided evenly by numbers other than 1 or itself.

Example: is 6 a Prime Number or Composite Number?
6 can be divided evenly by 2, or by 3, as well as by 1 or 6:
6 = 1 × 6; 6 = 2 × 3
So 6 is a **Composite Number**

Sometimes a number can be divided evenly **many ways**:

Example: 12 can be divided evenly by 1, 2, 3, 4, 6 and 12:
1 × 12 = 12; 2 × 6 = 12; 3 × 4 = 12
So 12 is also a **Composite Number**

Note this: any whole number greater than 1 is either **Prime** or **Composite**

Questions in class

1. The number 315 can be written as the product of two odd integers each greater than 1. In how many ways can this be done?

2. The expression $n!$ means the product of the positive integers from 1 to n. For example, $5!=1\times2\times3\times4\times5$. What is the value of $6!–4!$?

3. The sum of seven consecutive positive integers is always
(**A**) odd (**B**) a multiple of 7 (**C**) even (**D**) a multiple of 4 (**E**) a multiple of 3

4. Kalyn writes down all of the integers from 1 to 1000 that have 4 as the sum of their digits. If a/b (in lowest terms) is the fraction of these numbers that are prime, then $a+b$ is

5. The prime numbers between 10 and 20 are added together to form the number Q. What is the largest prime divisor of Q?

6. Sally picks four consecutive positive integers. She divides each integer by four, and then adds the remainders together. What is the sum of the remainders?

7. The product of four different positive integers is 360. What is the maximum possible sum of these four integers?

8. What is the number halfway between 1/ 8 and 1/10?