Dynamics

Unit 1: Fundamentals of Dynamics

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Grade 12 Physics
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Where Are We In the Course

1. Fundamentals of Dynamics
2. Momentum, Impulse and Energy
3. Gravitational, Electric and Magnetic Fields
4. Wave Nature of Light
5. Modern Physics
   5.1 Theory of Special Relativity
   5.2 Introduction to Quantum Mechanics

Dynamics

Now that we can mathematically describe the motion of any object, we have to be able describe *what* causes motion.

- Newton’s three laws of motion

Newton's First Law

An object at rest or in uniform motion will remain at rest or in uniform motion unless acted on by a net external force.

- Uniform motion means constant velocity
- An object "at rest" is also in uniform motion because $\mathbf{v} = \mathbf{0}$
- The mass of the object must be constant
- As long as an object moves in uniform motion, it must be that $\mathbf{F}_{\text{net}} = \mathbf{0}$
- e.g.: a spacecraft in "deep space" has no forces acting on it
- e.g.: a hockey puck sliding on very smooth ice has gravity and normal force, but the net force is zero
- e.g.: a car travelling on a highway at $100\,\mathrm{km/h}$ has many forces acting on it, but the net force is zero

Newton's Second Law

The sum of the external forces acting on an object is proportional to its mass and its acceleration.

$$\boxed{\mathbf{F}_{\text{net}} = \Sigma\mathbf{F} = m\mathbf{a}}$$

Quantity	Symbol	SI Unit
Net force	$\mathbf{F}_{\text{net}}$	N
Mass	m	kg
Acceleration	$\mathbf{a}$	$\mathrm{m/s^2}$

- This equation is a "special case" that assumes that mass is constant
- This isn't exactly what Newton said. But it requires us to learn another concept first, which we will do in the next unit!

Newton's Third Law

For every action there is an equal and opposite reaction.

$$\boxed{\mathbf{F}_{\text{A on B}} = -\mathbf{F}_{\text{B on A}}}$$

For every action force on an object (B) due to another object (A), there is a reaction force which is equal in magnitude but opposite in direction, on object (A), due to object (B). **The reaction forces act on different objects!**

A Simple Example

Example 1: Old-style television picture tubes and computer monitors use cathode ray tubes, where light is produced when fast-moving electrons collide with phosphor molecules on the surface of the screen. The electrons (mass $m = 9.1 \times 10^{-31}$ kg) are accelerated from rest in the electron "gun" at the back of the vacuum tube. Find the velocity of an electron when it exits the gun after experiencing an electric force of 5.8×10^{-15} N over a distance of 3.5 mm.

Forces

Force is the interaction between the objects

- When there is interaction, then forces are created
- A “push” or a “pull”

There are two types of forces:

- **Contact forces** act between two objects that are in contact with one another
- **Non-contact forces** act between two objects without them touching each other.
  - Also called “action-at-a-distance” force

Forces

Newton considered all forces acting at a single point of an object called the centre of mass ("CM")

- The centre of gravity is also called the centre of gravity ("CG")
- If the density of an object is constant, then the CG is also the geometric centre (centroid) of the object

Common Forces

Common forces that we encounter in Grade 12 Physics include:

- Gravitational force
- Normal force
- Static and kinetic friction
- Spring force (discussed in unit 2)
- Tension
- Applied force
- Electrostatic force (discussed in Unit 3)
- Magnetic force (discussed in Unit 3)

Gravity

The force of attraction between all objects with mass:

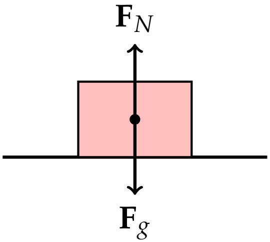
$$\boxed{\mathbf{F}_g = m\mathbf{g}}$$

Quantity	Symbol	SI Unit
Gravitational force (weight)	$\mathbf{F}_g$	N
Mass	m	kg
Acceleration due to gravity	$\mathbf{g}$	$\mathrm{m/s^2}$

- Near the surface of Earth, $\mathbf{g} = 9.81\,\mathrm{m/s^2}$ [down]
- $\mathbf{g}$ is also called the “gravitational field”; will be discussed in Unit 3
- The direction of $\mathbf{F}_g$ is how “down” is *defined*.
- Motion under *only* gravitational force is called **free fall**

Normal Force

Normal force is a force a surface exerts on another object that it is in contact with

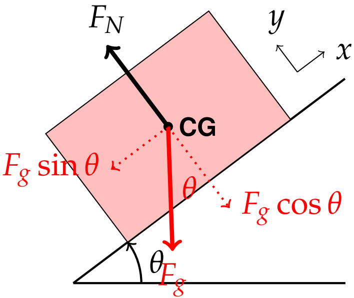


$$\mathbf{F}_g = m\mathbf{g} = -\mathbf{F}_N$$

- Always *perpendicular* to the contact surface (this is where the term *normal* came from)
- **Special case:** When an object is on a horizontal surface with no additional applied force, the magnitude of the normal force is equal to the magnitude of the weight of the object, i.e. $F_N = F_g$

Normal Forces on an Incline

When an object sits on an incline, normal force decreases.

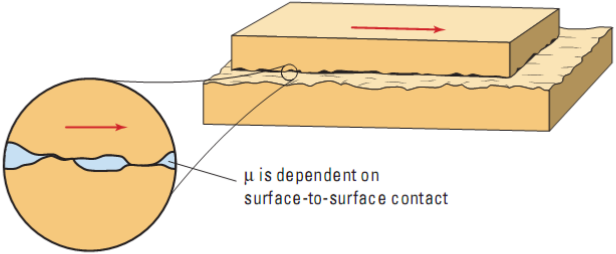


- $F_N = F_g \cos\theta$
- F_g has a component along the ramp $F_g \sin\theta$ that wants to slide the block down.
- There may also be a friction force F_f that opposes the motion

Friction

Friction is force that opposes the sliding of two surface across one another

- Always act in the direction opposite to motion or attempted motion
- Two types: *static friction* and *kinetic friction*



Static Friction

- When there is no relative motion between the two surfaces
- Increases with increasing applied force F_a. It is at maximum when the object is just about to move.

$$\boxed{\max F_s = \mu_s F_N}$$

Quantity	Symbol	SI Unit
Maximum static friction	$\max F_s$	N
Static friction coefficient	μ_s	(no unit)
Normal force	F_N	N

The above equation only deals with the *magnitude* of the friction force. A proper free-body diagram is required to express the direction of $\mathbf{F}_s$

Kinetic Friction

- The friction between two surfaces that are moving relative to each other
- (Nearly) constant along the path of movement as long as the normal force stays constant:

$$\boxed{F_k = \mu_k F_N}$$

Quantity	Symbol	SI Unit
Kinetic friction	F_k	N
Kinetic friction coefficient	μ_k	(no unit)
Normal force	F_N	N

μ_k is always lower than μ_s, otherwise nothing will ever move:

$$\mu_k \leq \mu_s$$

Tension in a Cable

Tension is the force exerted on and by a cable, rope, or string.

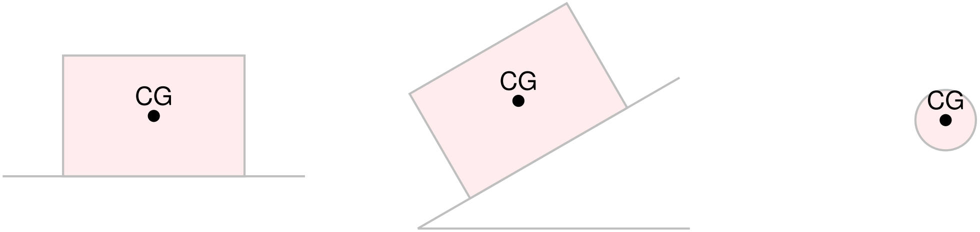
- You can't push on a rope
- Assume the cable/rope/string to be mass less
- Force can change direction when used with pulleys

How do engineers determine the amount of tension needed for a specific object (bridges, floors or light fixtures)?

Free Body Diagrams

A free-body diagram is used to visualize the forces acting on an object. It is very useful tool that should be used all the time. *We generally draw the forces from an inertial frame of reference.*

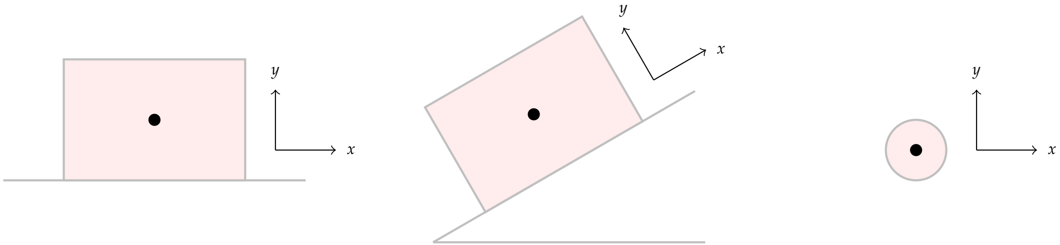
Step 1: Draw a “big dot” to represent the centre of mass of the object.



Free Body Diagrams

Step 2: Define a coordinate system (x and y axes)

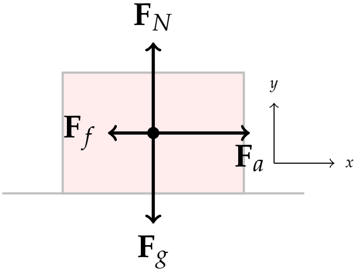
- Axes are defined to simplify the problem (not to make it more complicated!)
- Define them such that the sum of forces along one axis (usually y) is always zero

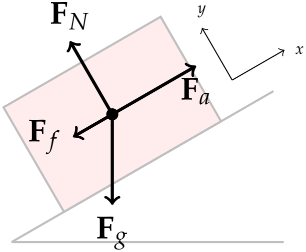


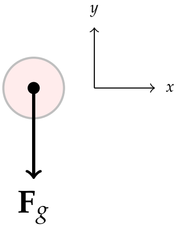
Free Body Diagrams

Step 3: Add all forces acting on the object

- If it has mass, weight $\mathbf{F}_g$ acts downward
- If it is on a surface, there is also a normal force $\mathbf{F}_N$
- If there is friction, first think about which direction the object will move without it, then draw $\mathbf{F}_s$ or $\mathbf{F}_k$ in the opposite direction
- If it is being pushed/pulled, there may be an applied force $\mathbf{F}_a$ or tension force $\mathbf{F}_T$





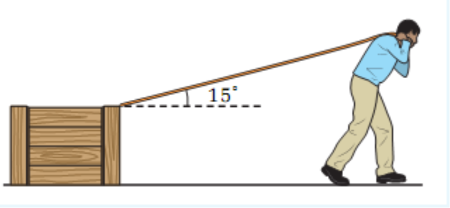


Solving Force Problem

- Break down the forces into the x and y components
- Sum the forces in the direction that doesn't have a net force (usually y axis)
- Sum the forces in the other axis, and find out what the acceleration is
- Use kinematic equations to solve the motion of the object

Example Problem

Example 1: To move a $45\,\mathrm{kg}$ wooden crate across a wooden floor ($\mu = 0.20$), you tie a rope onto the crate and pull on the rope. While you are pulling the rope with a force of $115\,\mathrm{N}$, it makes an angle of 15° with the horizontal. How much time elapses between the time at which the crate just starts to move and the time at which you are pulling it with a velocity of $1.4\,\mathrm{m/s}$?



Example Problem

Example 2: You are holding an $85\,\mathrm{kg}$ trunk at the top of a ramp that slopes from a moving van to the ground, making an angle of 35° with the ground. You lose your grip and the trunk begins to slide.

- If the coefficient of friction between the trunk and the ramp is 0.42, what is the acceleration of the trunk?
- If the trunk slides $1.3\,\mathrm{m}$ before reaching the bottom of the ramp, for what time interval did it slide?

Example: Vertical Motion

Example 3: A $55\,\mathrm{kg}$ person is standing on a scale in an elevator. If the scale is calibrated in *newtons*, what is the reading on the scale when the elevator is not moving? If the elevator begins to accelerate upward at $0.75\,\mathrm{m/s^2}$, what will be the reading on the scale?

Example Problem

Example 4: An elevator filled with people has a total mass of $2245\,\mathrm{kg}$. As the elevator begins to rise, the acceleration is $0.55\,\mathrm{m/s^2}$. What is the tension in the cable that is lifting the elevator?

Connected Bodies

- Usually the objects are connected by a cable or a solid linkage with negligible mass
- All objects have the same acceleration
- Require multiple free-body diagrams

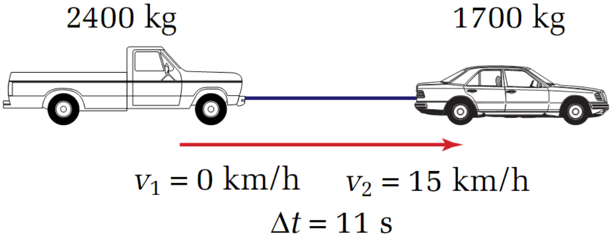
Solving Connected-Bodies Problems

To solve a connected-bodies problem, you can follow these procedures:

1. Draw a free-body diagram (FBD) on each of the objects
2. Sum all the forces on all the objects along the direction of motion
    - Direction of motion are usually very obvious
    - All action/reaction and tension forces should cancel, because they are *internal* forces and not *external* forces
3. Compute the acceleration of the entire system using Newton's second law
    - Remember that every object has the same acceleration!
4. Go back to the FBD of each of the objects and compute any unknown forces

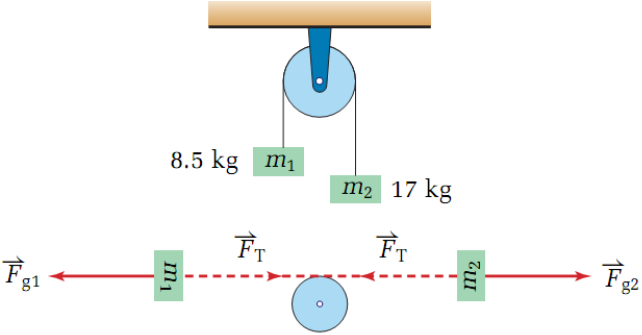
Example Problem: Towing

Example 5: A $1700\,\mathrm{kg}$ car is towing a larger vehicle with mass of $2400\,\mathrm{kg}$. The two vehicles accelerate uniformly from a stoplight, reaching a speed of $15\,\mathrm{km/h}$ in $11\,\mathrm{s}$. Find the force needed to accelerate the connected vehicles, as well as the minimum strength of the rope between them.



Example Problem: Atwood Machine

An **Atwood machine** is made of two objects connected by a rope that runs over a pulley. The pulley allows the direction of force and direction of motion to change between two objects.



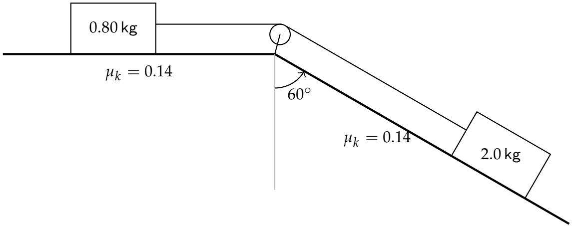
Example 6: The object on the left (m_1) has a mass of $8.5\,\mathrm{kg}$ and the object on the right (m_2) has a mass of $17\,\mathrm{kg}$.

(a) What is the acceleration of the masses?

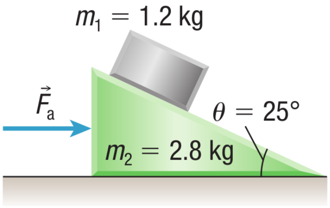
(b) What is the tension in the rope?

A More Typical Problem

Example 7: Two blocks are connected by a mass-less string over a friction-less pulley as shown in the diagram. What is the acceleration of the blocks? And what is the tension in the string?



Another Connected Body Problem



Example 8: In the figure above, m_1 does not slide with respect to the surface with m_2 when the horizontal force shown is applied. Determine the magnitude of the horizontal. Assume there is no friction.