Unit 2: Momentum, Impulse and Energy

Grade 12 Physics

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Where Are We In the Course

1. Fundamentals of Dynamics
2. Momentum, Impulse and Energy
3. Gravitational, Electric and Magnetic Fields
4. Wave Nature of Light
5. Modern Physics
   5.1 Special Relativity
   5.2 Introduction to Quantum Mechanics

A New Concept: Momentum

You can’t stop a bullet train by yourself. Why not?

- A train is *fast*
- A train is *massive*

A New Concept: Momentum

You can't stop a bullet either (unless you wear a bullet-proof vest). Why not?

- A bullet isn't massive
- A bullet sure is *fast*!

A New Concept: Momentum

What makes it so difficult to stop a train, or a speeding bullet, or a car?

- Both the train, the speeding bullet and the car have a lot of *momentum*
- Momentum is related to both the *mass* and *velocity* of an object
- “Mass in motion”: the tendency for the object to remain in the same state of motion
- Newton referred to momentum as the “quantity of motion”

Then What is Momentum?

Momentum is a vector quantity that is directly proportional to both an object's mass and its velocity:

$$\boxed{\mathbf{p} = m\mathbf{v}}$$

Quantity	Symbol	SI Unit
Momentum	$\mathbf{p}$	kg m/s
Mass	m	kg
Velocity	$\mathbf{v}$	m/s

A Very Simple Example: Hockey Puck

Example 1: Determine the momentum of a $0.300\,\mathrm{kg}$ hockey puck travelling across the ice at a velocity of $5.55\,\mathrm{m/s}$ [N].

Impulse

Impulse is the change in momentum (also a vector!):

$$\boxed{\mathbf{J} = \Delta\mathbf{p} = \mathbf{p}_2 - \mathbf{p}_1}$$

Assuming that masss m is constant, we can find out how $\Delta\mathbf{p}$ relates to $\mathbf{F}$:

$$\mathbf{F}_{\text{net}} = m\mathbf{a} = m\frac{\Delta\mathbf{v}}{\Delta t} = \frac{\Delta(m\mathbf{v})}{\Delta t} = \frac{\Delta\mathbf{p}}{\Delta t}$$

In fact, this is how Newton's second law actually is defined: the net force is the *rate of change of momentum*. Rearranging the equation, we have a new

$$\boxed{\Delta\mathbf{p} = \mathbf{F}_{\text{net}}\Delta t = \mathbf{J}}$$

Impulse

Impulse is generally defined based on the applied force and the time interval:

$$\boxed{\mathbf{J} = \Delta\mathbf{p} = \mathbf{F}_{\text{net}}\Delta t}$$

Quantity	Symbol	SI Unit
Impulse	$\mathbf{J}$	N s
Average net force	$\mathbf{F}_{\text{net}}$	N
Time interval	Δt	s

- Every force acting on an object generates an impulse, regardless of whether it changes the motion of the object
- The change in momentum $\mathbf{p}$ is based on the *net* force $\mathbf{F}_{\text{net}}$
- The net force $\mathbf{F}_{\text{net}}$ is averaged over the entire interval Δt

For the over-achievers amongst you...

NOTE: You don't have to know this for this class. If you know calculus, or if you're studying it right now, you can get a more precise definition of impulse:

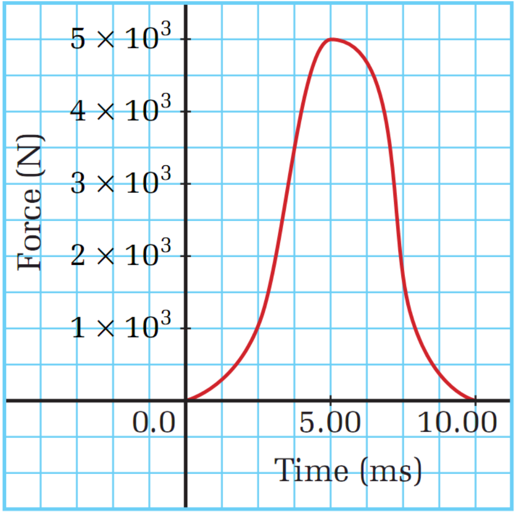
$$\mathbf{J} = \int_{t_1}^{t_2} \mathbf{F}_{\text{net}}(t)\,dt$$

This can get very complicated if $\mathbf{F}_{\text{net}}$ is changing in both magnitude and direction with time

Example Problem: Golf Club

Example 2: If a golf club exerts an average force of 5.25×10^3 N [W] on a golf ball over a time interval of 5.45×10^{-4} s, what is the impulse of the interaction?

Why Average Force?



Why did the previous example use **average force**?

A Slightly Longer Example

Example 3: A student practices her tennis volleys by hitting a tennis ball against a wall.

- If the $0.060\,\mathrm{kg}$ ball travels $48\,\mathrm{m/s}$ before hitting the wall and then bounces directly backward at $35\,\mathrm{m/s}$, what is the impulse of the interaction?
- If the duration of the interaction if $25\,\mathrm{ms}$, what is the average force exerted on the ball by the wall?

Conservation of Momentum

Conservation of momentum is derived through Newton's third law of motion. When objects interact, the total momentum *before* the interaction is the same as *after* the interaction:

$$\boxed{\sum_i \mathbf{p}_i = \sum_i \mathbf{p}_i'}$$

Examples:

- Collision of two or more objects
- A rocket expelling gas from the engine nozzle
- Skaters pushing on each other on a friction-less ice surface
- An exploding bomb

Conservation of Momentum

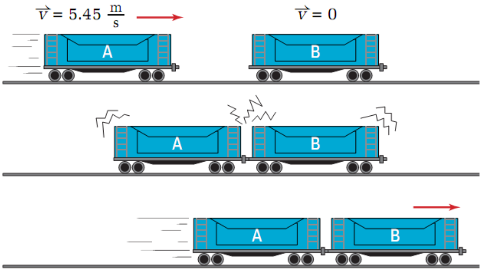
For a collision between two objects A and B:

$$\boxed{m_A\mathbf{v}_A + m_B\mathbf{v}_B = m_A\mathbf{v}'_A + m_B\mathbf{v}'_B}$$

Quantity	**Symbol**	**SI Unit**
Mass of objects A and B	m_A, m_B	kg (kilograms)
Velocities of A and B before	$\mathbf{v}_A, \mathbf{v}_B$	m/s (metres per second)
Velocities of A and B after	$\mathbf{v}'_A, \mathbf{v}'_B$	m/s (metres per second)

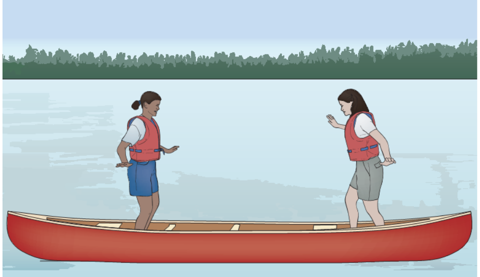
Example Problem: Boxcars

Example 4: A 1.75×10^4 kg boxcar is rolling down a track towards a stationary boxcar that has a mass of 2.00×10^4 kg. Just before the collision, the first boxcar is moving east at $5.45\,\mathrm{m/s}$. when the boxcars collide, they lock together and continue to down the track. What is the velocity of the two boxcars immediately after the collision?



Example Problem: Canoe

Example 5: Two people A and B stand in a canoe on top of the water. Find the velocity of the canoe and person B at the instant that person A start to take a step, if her velocity is $0.75\,\mathrm{m/s}$ [forward]. Assume person A has a mass of $65\,\mathrm{kg}$ and the combined mass of the canoe, A and B, is $115\,\mathrm{kg}$.



Conservation of Momentum

What happens if I let go of this balloon?

We will talk about this more in Unit 3 when we talk about propulsion in space

A More Difficult Example: Glancing Collision

A glancing collision involves motion in 2D. Since momentum is a vector, the calculation will involve some vector arithmetic.

Example 6: A billiard ball of mass $0.155\,\text{kg}$ ("cue ball") moves with a velocity of $1.25\,\text{m/s}$ towards a stationary billiard ball ("eight ball") of identical mass and strikes it with a glancing blow. The cue ball moves off at an angle of 29.7° clockwise from its original direction, with a speed of $0.956\,\text{m/s}$.

(a) What is the final velocity of the eight ball?

(b) Is the collision elastic?

We can't answer the second part of the question yet (be patient, we'll come back to this example later), but we can definitely solve the first part.

A Full View of Collision

There are two types of collisions, either an **elastic collision** or an **inelastic collision**

Elastic collisions

- Momentum is conserved
- Kinetic energy is conserved
  - During the collision, the kinetic energy is first transformed into an elastic potential energy (e.g. compressing a spring)
  - All of the energy is then released back as kinetic energy
- Usually in these collisions the two objects don't actually make contact with each other
- **You must NOT assume that a collision is elastic unless you are told!**

Collisions

Inelastic collisions (the majority of all collisions)

- Momentum is conserved
- Kinetic energy is lost due to heat, friction or sound
- Energy is conserved to motion just before and/or just after the collision (depends on the situation)

A special case of inelastic collision is called a **completely inelastic collision** (or **perfectly inelastic collision**), the objects stick together

- Momentum is conserved
- Most of the kinetic energy is lost

Example Problem

Example 7: A $0.0520\,\mathrm{kg}$ golf ball is moving velocity of $2.10\,\mathrm{m/s}$ when it collides, head on, with a stationary $0.155\,\mathrm{kg}$ billiard ball. If the golf ball rolls directly backwards with a velocity of $-1.04\,\mathrm{m/s}$, is the collision elastic?

To solve this type of problem, using the conservation of momentum to find the velocities of the golf ball and the billiard ball. Then, sum the total kinetic energies of both balls, and compare that to the total kinetic energy of the balls before the collision.

Example Problem

Example 8: A car ($1000\,\mathrm{kg}$) travels at a speed of $20\,\mathrm{m/s}$ towards a stationary truck ($3000\,\mathrm{kg}$). The car rear ends the truck elastically. What is the velocity of the truck after the collision?

For this example we are not concerned with the *implausibility* of such a collision; we just want to figure out what happens *if* it actually happens.

Solving the Example Problem

In order to solve this question we need *both* the conservation of kinetic energy and the conservation of momentum. Here, A is the car, B is the truck.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B$$
$$\frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2} m_A v_A'^2 + \frac{1}{2} m_B v_B'^2$$

Solving the Example Problem

Momentum

$$m_A v_A + \cancel{m_B v_B} = m_A v'_A + m_B v'_B$$

We can eliminate v_B, since the truck wasn't moving:

$$m_A v_A = m_A v'_A + m_B v'_B$$

Move terms with m_A to the left:

$$\boxed{m_A(v_A - v'_A) = m_B v'_B} \quad (1)$$

Kinetic Energy

$$\frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2} m_A v_A'^2 + \frac{1}{2} m_B v_B'^2$$

We can multiply every term by 2, then remove the v_B term, since it's zero:

$$m_A v_A^2 = m_A v_A'^2 + m_B v_B'^2$$

Again, moving m_A terms to the left:

$$\boxed{m_A(v_A^2 - v_A'^2) = m_B v_B'^2} \quad (2)$$

Solving the Example Problem

Dividing (2) by (1), we get:

$$\frac{(2)}{(1)} = \frac{m_A(v_A^2 - v_A'^2)}{m_A(v_A - v_A')} = \frac{m_B v_B'^2}{m_B v_B'} = v_B'$$

We can cancel out the m_A terms on the left, then expand on the squared terms on top:

$$\frac{(v_A + v_A')(v_A - v_A')}{(v_A - v_A')} = v_B'$$

Now we get:

$$v_B' = v_A + v_A'$$
$$v_A' = v_A - v_B'$$

We substitute back to the momentum equation (1)

Solving the Example Problem

$$\boxed{v'_A = \frac{m_A - m_B}{m_A + m_B} v_A} \quad \boxed{v'_B = \frac{2m_A}{m_A + m_B} v_A}$$

These equations work for *all* elastic impact where object B (in this example, the truck) is stationary when impact occurs. Substituting values for m_A, m_B and v_A, we get:

$$v'_A = \frac{m_A - m_B}{m_A + m_B} v_A = \frac{(1000 - 3000)}{(1000 + 3000)} \times 20 = \boxed{-10\,\mathrm{m/s}}$$

$$v'_B = \frac{2m_A}{m_A + m_B} v_A = \frac{(2 \times 1000)}{(1000 + 3000)} \times 20 = \boxed{10\,\mathrm{m/s}}$$

This Example tells us much more!

$$\boxed{v'_A = \frac{m_A - m_B}{m_A + m_B} v_A} \quad \boxed{v'_B = \frac{2m_A}{m_A + m_B} v_A}$$

Let's say $m_A = m_B = m$, then:

$$v'_A = \frac{m_A - m_B}{m_A + m_B} v_A \qquad v'_B = \frac{2m_A}{m_A + m_B} v_A$$

$$v'_A = \frac{m - m}{m + m} v_A \qquad v'_B = \frac{2m}{m + m} v_A$$

$$v'_A = 0 \qquad v'_B = v_A$$

If both masses are the same, then *all* of the momentum and energy are transferred from A to B!

This Example tells us much more!

$$\boxed{v'_A = \frac{m_A - m_B}{m_A + m_B} v_A} \qquad \boxed{v'_B = \frac{2m_A}{m_A + m_B} v_A}$$

Let's say $m_A \gg m_B$, then we can “ignore” m_B:

$$v'_A = \frac{m_A - m_B}{m_A + m_B} v_A \qquad v'_B = \frac{2m_A}{m_A + m_B} v_A$$

$$v'_A \approx \frac{m_A}{m_A} v_A \qquad v'_B \approx \frac{2m_A}{m_A} v_A$$

$$v'_A \approx v_A \qquad v'_B \approx 2v_A$$

Object A continues to move like nothing happened, but object B is pushed to move at an even higher speed.

This Example tells us much more!

$$\boxed{v'_A = \frac{m_A - m_B}{m_A + m_B} v_A} \qquad \boxed{v'_B = \frac{2m_A}{m_A + m_B} v_A}$$

Similarly, if $m_A \ll m_B$ then we can “ignore” m_A:

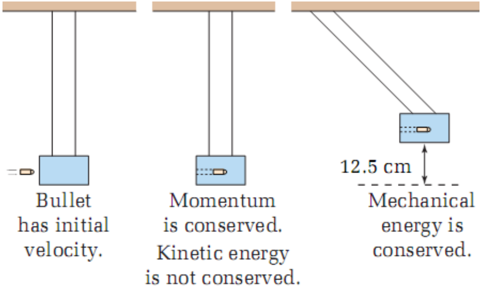
$$v'_A = \frac{m_A - m_B}{m_A + m_B} v_A$$
$$v'_A \approx \frac{-m_B}{m_B} v_A$$
$$v'_A \approx -v_A$$

$$v'_B = \frac{2m_A}{m_A + m_B} v_A$$
$$v'_B \approx \frac{0}{m_B} v_A$$
$$v'_B \approx 0$$

Object A bounces off B, and travels in the opposite direction with the same velocity magnitude.

Example Problem

Example 9: A forensic expert needed to find the velocity of a bullet fired from a gun in order to predict the trajectory of a bullet. He fired a $5.50\,\mathrm{g}$ bullet into a ballistic pendulum with a bob that had a mass of $1.75\,\mathrm{kg}$. The pendulum swung to a height of $12.5\,\mathrm{cm}$ above its rest position before dropping back down. What was the velocity of the bullet just before it hit and became embedded in the pendulum bob?



Example Problem

Example 10: A block of wood with a mass of $0.500\,\mathrm{kg}$ slides across the floor towards a $3.50\,\mathrm{kg}$ block of wood. Just before the collision, the small block is travelling at $3.15\,\mathrm{m/s}$. Because some nails are sticking out of the blocks, the blocks stick together when they collide. Scratch marks on the floor show that they slid $2.63\,\mathrm{cm}$ before coming to a stop. What is the coefficient of friction between the wooden blocks and the floor?

We don't have to use any kinematic or dynamic equations to solve this problem. We only need the conservation momentum equation, and the definition of kinetic energy.