Algebra 1

1. "Number" Word Problems

"Number" word problems are fairly contrived, but they're also fairly standard, so you should learn how to handle them. After all, the point of these problems isn't their relation to "real life", but your ability to extract the math from the English.

Example 1: The sum of two consecutive integers is 15. Find the numbers.

They've given me two pieces of information here.

First, I know that I am adding two numbers, and their sum is fifteen.

Second, I know that the numbers are nice neat round numbers (like –3 or 6), not messy ones (like –4.628 or $^{17}/_{32}$), and that the second number is one more than the first.

This last piece of information comes from the fact that "consecutive integers" (or "consecutive whole numbers", if they're restricting the possibilities to only positive numbers) are one unit apart.

Examples of "consecutive integers" would be –12 and –11, 1 and 2, and 99 and 100. Using these facts, I can set up the translation.

I will represent the first number by "n". Then the second number has to be "$n + 1$". Their sum is then:
$n + (n + 1) = 15$
$2n + 1 = 15$
$2n = 14$
$n = 7$

The exercise did not ask me for the value of the variable n; it asked for the identity of two numbers. So my answer is not "$n = 7$"; the actual answer is:
"The numbers are 7 and 8."

Example 2: The product of two consecutive negative even integers is 24. Find the numbers.

They have told me quite a bit about these two numbers: the numbers are even and they are negative. (The fact that they are negative may help if I come up with two solutions — a positive and a negative — so I'll know which one to pick.) Since even numbers are two apart (for example, –4 and –2 or 10 and 12), then I also know that the second number is two greater than the first.

I also know that, when I multiply the two numbers, I will get 24. In other words, letting the first number be "n" and the second number be "$n + 2$", I have:

$(n)(n + 2) = 24$
By guess and check, we have the numbers are -6 and -4.

Example 3: Twice the larger of two numbers is three more than five times the smaller, and the sum of four times the larger and three times the smaller is 71. What are the numbers?

The point of exercises like this is to give you practice in unwrapping and unwinding these words, and turning the words into algebraic equations. The point is in the solving, not in the relative "reality" of the problem. That said, how do you solve this? The best first step is to start labelling:

the larger number: x
the smaller number: y
twice the larger: $2x$
three more than five times the smaller: $5y + 3$
relationship between ("is"): $2x = 5y + 3$
four times the larger: $4x$
three times the smaller: $3y$
relationship between ("sum of"): $4x + 3y = 71$

Now I have two equations in two variables:
$2x = 5y + 3$
$4x + 3y = 71$

I will solve, say, the first equation for x:
$x = (5/2)y + (3/2)$

Then I'll plug the right-hand side of this into the second equation in place of the "x":
$4[(5/2)y + (3/2)] + 3y = 71$
$10y + 6 + 3y = 71$
$13y + 6 = 71$
$13y = 65$
$y = 65/13 = 5$

Now that I have the value for y, I can solve for x:
$x = (5/2)y + (3/2)$
$x = (5/2)(5) + (3/2)$
$x = (25/2) + (3/2)$
$x = 28/2 = 14$

As always, I need to remember to answer the question that was actually asked. The solution here is not "$x = 14$", but is the following sentence:
The larger number is 14, and the smaller number is 5.

2. "Age" Word Problems

Age problems are algebra word problems that deal with the ages of people currently, in the past or in the future.

If the problem involves a single person, then it is similar to an Integer Problem. Read the problem carefully to determine the relationship between the numbers. This is shown in the example involving a single person.

Example 1: Five years ago, John's age was half of the age he will be in 8 years. How old is he now?

Solution:
Step 1: Let *x* be John's age now. Look at the question and put the relevant expressions above it.

$x-5$ $\frac{1}{2}$ $x+8$

Five years ago, John's age was half of the age he will be in 8 years.

Step 2: Write out the equation.

$$x-5=\frac{1}{2}(x+8)$$

$$x-5=\frac{1}{2}x+4$$

Isolate variable *x*

$$x-\frac{1}{2}x=4+5$$

$$\frac{1}{2}x=9$$

$$x=18$$

John is now 18 years old.

Example 2: One-half of Heather's age two years from now plus one-third of her age three years ago is twenty years. How old is she now?

Solution: This problem refers to Heather's age two years in the future and three years in the past. So I'll pick a variable and label everything clearly:
Let Heather's age now be H
age two years from now: $H + 2$
age three years ago: $H - 3$

Now I need certain fractions of these ages:
one-half of age two years from now: $(\frac{1}{2})(H + 2) = \frac{H}{2} + 1$
one-third of age three years ago: $(\frac{1}{3})(H - 3) = \frac{H}{3} - 1$

The sum of these two numbers is twenty, so I'll add them and set this equal to 20:
$H/2 + 1 + H/3 - 1 = 20$
$H/2 + H/3 = 20$
$3H + 2H = 120$
$5H = 120$
$H = 24$
Heather is 24 years old.

Note: Remember that you can always check your answer to any "solving" exercise by plugging that answer back into the original problem. If Heather is 24 now, then she will be 26 in two years, half of which is 13, and she was 21 three years ago, a third of which is 7. Adding, I get 13 + 7 = 20, so the solution works.

Here's an easy word problem:
Suzy is ten years older than Billy, and next year she will be twice as old as Billy. How old are they now?

If you don't use algebra, you probably have to solve it by trial and error. This sometimes works fine. Here it is a little slow. And, in other problems, it is next to impossible.

1) Translate into equations

With algebra, the solution is easy. The only problem is to convert the above sentence into equations, because equations are what we need to use algebra. How is this?
S = 10 + B
S + 1 = 2(B + 1)

That is a direct translation of the word problem. I used S to represent Suzy's age (this year), and B for Billy's age. I could use other letters, but these are easier to remember.

Do you see how this translation is done? "Suzy is ten years older than Billy" is an equation (S = 10 + B), but is in words instead of symbols. "Next year she will be twice as old as Billy" is a little more complicated, but is just another equation (S + 1 = 2(B + 1)).

The translation process seldom gets much more difficult than the above. But, you may have to weed out extra information. If I had started the word problem with, "Suzy is six inches taller than Billy, and...," you are getting extra info which has nothing to do with the problem.

You would have an extra equation (Z = 6 + L, where Z is Suzy's height, and L is Billy's height). You would find that this equation does not affect the other two equations, at all. You could write down this equation, but you would end up solving the other two equations.

2) Solve the equations

This is just regular algebra (two equations and two unknowns). There are several ways to continue:

Solve for one variable (in one equation) and substitute in the other equation.
Subtract one equation from another (after changing their form, perhaps), to solve for one variable.

If we solved for one variable, then we go back and solve for the other variable. Let's use the first method. S = 10 + B. Substitute that into the second equation:
10 + B + 1 = 2(B + 1)
B = 9
Going back into the first equation, we find that S = 19. Billy is 9, Suzy is 19.

3) Check your work

Checking your work may seem like a waste of time, especially when you get good at algebra. But, remember that you went through a translation process above. That is not an exact science. Check your work; see that you didn't get something backward.

We can put the numbers (the answers) back into the word problem to see if they are correct. Above, we notice that next year Billy will be 10 and Suzy will be 20. And she will indeed be twice as old as Billy.

Addendum #1: Key words

The mystery seems to be in step #1, "Translate into equations". It is sometimes easier said than done. And you really should practice doing word problems. But there are (fairly obvious) clues (key words) in the statement of the problem:

"and" often means "plus"
"difference" often means "larger-smaller"
"five more than" means "+5"
"next year" means "(this year)+1"
"twice" means "times 2"
ages are normally in whole numbers

We see statements like "Billy has two and a half dollars less than Sally." So, B = S - 2.50. How about this:

In seven years, Billy's age will be one and a half times Sally's age.

What will Billy's age be in seven years? B + 7. What will Sally's age be? S + 7. What do we get from the above sentence? B + 7 = (1.5)(S + 7). Right?

Addendum #2: Famous formulas

In geometry, we have formulas for areas and volumes; see Areas and Perimeters and Some Triangle Formulas. Also see The Pythagorean Theorem. In science, we have d = vt or distance is velocity (speed) times time, and other well-known formulas. Velocity is a "rate;" we also have rates of interest involving money, which work the same way.

If we don't know these formulas (or if we cannot figure them out from simpler ideas), then we cannot solve certain algebra problems.

And we can use these formulas backward. In other words, we can solve for any of the variables, if we know the value of the other variables.

Puzzle: Sally's age is 12 years plus half her age. How old is she?

You should be able to get it by trial and error, even though there are easier ways. By the way, 18 doesn't work. 12 + 6 is 18. But 6 is not half of 18.

Answer: Algebra is the easy way:

A = 12 + A/2
A/2 = 12 [Subtracting A/2 from both sides]
A = 24

You can also reason it out, without much algebra. Her age is some number plus half her age.
Well it must be half her age plus half her age, right?
That is the only way to add up to her age.
So half her age is 12. And she is 24.

Questions in class (After the class, please do the following problems again)

1. When will a 53 year old man be 10 times the age of his son who is now 8 years of age?

2. A set of 10 numbers has sum 100. Each number of the set is increased by 20, then multiplied by 20 then decreased by 20. What is the sum of the numbers in the new set?

3. Adam lived a quarter of his life as a boy, a fifth as a young man, a third in middle-age and 13 years in retirement. How old was he when he died?

4. How many two digit numbers are equal to seven times the sum of their digits?

5. The sum of the squares of two integers is 34 and the difference of the squares of those same integers is 16. Find the cube of the smallest of those integers.

6. x and 2x are both 3 digit integers. If the sum of the digits of x is 19, what is the sum of the digits of 2x?

7. The sum of three consecutive integers is equal to four times the smallest of them. What is the value of the product of the three integers?

8. I have a number such that if I multiply the number by 4 and subtract 12, I get twice as much as when I first subtract 12 and then multiply by 4. What is the sum of the digits of my number?

9. The average mark on the first 3 tests Jean took was 76. What average mark does Jean need on the next two tests to have an average of 80 for all of the tests?

10. In 10 years, Samuel will be 1/2 as old as Roman. Five years ago, Samuel was only 1/3 as old as Roman. How old is Samuel now?

11. You are given a set of three numbers. If the numbers are added together two at a time, the sums are 23, 32 and 39. What is the sum of the three numbers?

12. The sum of 9 consecutive integers is 369. What is the middle integer?