Equations (2)

1. More Solving Equations with Brackets

Example 1: Solve the equation $2(x+5)-7=3(x-2)$.

$$2(x+5)-7=3(x-2)$$
$$2x+10-7=3x-6$$
$$2x+3=3x-6$$
$$2x+3-3x=3x-6-3x$$
$$-x+3=-6$$
$$-x+3-3=-6-3$$
$$-x=-9$$
$$\frac{-x}{-1}=\frac{-9}{-1}$$
$$x=9$$

If you put 9 back in for x in the original problem you will see that 9 is the solution.

Example 2: Solve the equation $-2(x+5)+4(x-1)=x+1$

Using the distributive property and then combining like terms to simplify the left side of the equation we get:

$$-2(x+5)+4(x-1)=x+1$$
$$-2x-10+4x-4=x+1$$
$$2x-14=x+1$$

Solving for x we get:

$$2x-14=x+1$$
$$2x-14-x=x+1-x$$
$$x-14=1$$
$$x-14+14=1+14$$
$$x=15$$

If you put 15 back in for *x* in the original problem you will see that 15 is the solution to our problem.

2. Solving Equations with Fractions and Decimals

Example 3: Solve the equation $\frac{5}{4}x + \frac{1}{2} = 2x - \frac{1}{2}$.

$$\frac{5}{4}x + \frac{1}{2} = 2x - \frac{1}{2}$$
$$(4)\left(\frac{5}{4}x + \frac{1}{2}\right) = (4)\left(2x - \frac{1}{2}\right)$$
$$5x + 2 = 8x - 2$$
$$5x + 2 - 8x = 8x - 2 - 8x$$
$$-3x + 2 = -2$$
$$-3x + 2 - 2 = -2 - 2$$
$$-3x = -4$$
$$\frac{-3x}{-3} = \frac{-4}{-3}$$
$$x = \frac{4}{3}$$

If you put 4/3 back in for x in the original problem you will see that 4/3 is the solution we are looking for.

Example 4: Solve $\frac{3}{4}t - 2 = \frac{1}{2}(t + 2)$
$$\left[\frac{3}{4}t - 2 = \frac{1}{2}(t + 2)\right] \times 4$$
$$3t - 8 = 2(t + 2)$$
$$3t - 8 = 2t + 4$$
$$t = 12$$

Contradiction - A contradiction is an equation with one variable that has no solution

Example 5: Solve the equation $4x - 1 = 4(x + 3)$.

$$4x - 1 = 4(x + 3)$$
$$4x - 1 = 4x + 12$$
$$4x - 1 - 4x = 4x + 12 - 4x$$
$$-1 = 12$$

Where did our variable, *x*, go??? It disappeared on us. Also note how we ended up with a FALSE statement, -1 is not equal to 12. This does not mean that $x = 12$ nor $x = -1$.

Whenever your variable drops out AND you end up with a false statement, then after all of your hard work, there is **NO SOLUTION**.

So, the answer is no solution.

Identity - An identity is an equation with one variable that has all numbers as a solution.

Example 6: Solve the equation $5x + 10 = 5(x + 2)$.

$$\begin{aligned} &5x + 10 = 5(x + 2) \\ &5x + 10 = 5x + 10 \\ &5x + 10 - 5x = 5x + 10 - 5x \\ &10 = 10 \end{aligned}$$

This time when our variable dropped out, we ended up with a TRUE statement. Whenever that happens, your answer is **ALL REAL NUMBERS**.

3. Application in Proportion

There are some terminologies related to proportions that you need to know. In the proportion: $\frac{a}{b} = \frac{c}{d}$ the values in the "*b*" and "*c*" positions are called the **"means" of the proportion**, while the values in the "*a*" and "*d*" positions are called the **"extremes" of the proportion**. A basic defining property of a proportion is that the product of the means is equal to the product of the extremes. In other words, given the proportional statement: $\frac{a}{b} = \frac{c}{d}$, you can conclude that $ad = bc$. (This is the cross-multiplication.) This relationship is occasionally turned into a homework problem, such as:

Example 1: Is $^{24}/_{140}$ proportional to $^{30}/_{176}$?

For these ratios to be proportional (that is, for them to be a true proportion when they are set equal to each other), I have to be able to show that the product of the means is equal to the product of the extremes. In other words, they want me to find the product of 140 and 30 and the product of 24 and 176, and then see if these products are equal. So I'll check:

$140 \times 30 = 4200 \qquad 24 \times 176 = 4224$

While these values are close, they are not equal, so I know the original fractions cannot be proportional to each other. So the answer is that **they are not proportional.**

Solving proportions is simply a matter of stating the ratios as fractions, setting the two fractions equal to each other, cross-multiplying, and solving the resulting equation. You'll probably start out by just solving proportions, like this:

Example 2: Find the unknown value in the proportion: $(2x + 1) : 2 = (x + 2) : 5$

First, I convert the colon-based odds-notation ratios to fractional form:

$$\frac{2x+1}{2} = \frac{x+2}{5}$$

Then I solve the proportion:

$$\frac{2x+1}{2} = \frac{x+2}{5}$$

$5(2x + 1) = 2(x + 2)$
$10x + 5 = 2x + 4$
$8x = -1$
$\mathbf{x = {}^{-1}/_{8}}$

Example 3: Solve $$\frac{6x-7}{4} + \frac{3x-5}{7} = \frac{5x+78}{28}$$

This is NOT a proportion. Therefore, you cannot cross multiply. Get rid of the denominators by multiplying both sides by LCM 28, the smallest number that 4, 7, and 28 will divide into evenly. Recall the line that separates the numerator and the denominator also functions as a parenthesis. It instructs the reader to treat the numerator as one number and the denominator as one number.

$$28\left(\frac{6x-7}{4} + \frac{3x-5}{7}\right) = 28\left(\frac{5x+78}{28}\right)$$

Simplify:

$$28\left(\frac{6x-7}{4}\right) + 28\left(\frac{3x-5}{7}\right) = 28\left(\frac{5x+78}{28}\right)$$

$$7(6x-7) + 4(3x-5) = 1(5x+78)$$

$$42x - 49 + 12x - 20 = 5x + 78$$

Add the *x* terms and the constants on the left side of the equation.

$$54x - 69 = 5x + 78$$

Subtract $5x$ from both sides of the equation: $49x - 69 = 78$

Add 69 to both sides of the equation: $49x = 147$

Divide both sides by 49: $x = 3$

Class Practice:

1. Solve for x.

a) $\frac{10}{128} = \frac{(7/3)}{w}$

b) (2x - 3) : 4 = (x + 8) : 5

c) $\frac{(\frac{3}{80})}{x} = \frac{x}{(\frac{3}{5})}$

d) $\frac{4}{3}x + 5 = \frac{1}{2}x - 7$

Practice in class

1. Solve

(a) $\frac{x}{2} = 4$ (b) $\frac{3x}{5} = -9$ (c) $6 = \frac{m}{4}$ (d) $2 \times \frac{x}{-7} = 6$

(e) $\frac{1}{4}x - 3 = 4$ (f) $\frac{4}{5}x - 3 = 5$ (g) $7 = 1 + \frac{2}{3}x$ (h) $-5 + \frac{1}{4}x = -7$

(i) $2y + \frac{1}{2} = \frac{2}{3}$ (j) $\frac{7}{6}x - 2 = \frac{1}{3}$ (k) $\frac{n}{4} - 1 = \frac{n}{5}$ (l) $3 - \frac{m}{2} = 5 - \frac{m}{3}$

(m) $\frac{2}{3}y - 3 = \frac{4}{5}y - 5$ (n) $\frac{1}{2}x + \frac{1}{3}x = 10$ (o) $\frac{3}{4}x - \frac{1}{8}x = 5$

(p) $\frac{1}{3}(2x - 1) = 1$ (q) $\frac{1}{6}(2y + 6) = 2$ (r) $\frac{1}{2}(x - 5) = \frac{x}{4}$

(s) $\frac{3}{5}(2x + 15) = 3$ (t) $\frac{1}{2}(x - 1) = \frac{1}{4}(x + 1)$ (u) $\frac{4x - 1}{5} = \frac{2x + 3}{2}$

(v) $\frac{y - 7}{3} = \frac{y - 2}{4} - 1$