

Equations (3)

1. Rearranging Equations

Sometimes there are more than one variable in an algebraic equation. In order to solve for a particular variable, we can rearrange the equation by:

- 1) Imagining all other variables are numbers
- 2) Solving the equation just like usual (i.e. expand brackets, collect like terms, inverse operations, etc.)
- 3) So that the target variable is isolated.

Example 1. Solve for y.

$$2x + 5y = -10$$

$$5y = -2x - 10$$

$$y = -\frac{2}{5}x - 2$$

Example 2. Solve for x.

$$\frac{3}{5} = \frac{1}{3}x + \frac{1}{4}y$$

$$36 = 20x + 15y$$

$$20x = 36 - 15y$$

$$x = \frac{9}{5} - \frac{3}{4}y$$

Example 3: The volume of a cone can be written as $V = \frac{1}{3}\pi r^2 h$, where V is the volume, r is radius of the base circle and h is height of the cone.

- a. Express height in terms of other variables.

$$V = \frac{1}{3}\pi r^2 h$$

$$3V = \pi r^2 h$$

$$h = \frac{3V}{\pi r^2}$$

- b. How much is the height when the radius is 2 cm and the volume is 20 cm³?

$$h = \frac{3(20)}{\pi 2^2} = 4.77 \text{ cm}$$

- c. Express radius in terms of other variables.

$$V = \frac{1}{3}\pi r^2 h$$

$$3V = \pi r^2 h$$

$$r^2 = \frac{3V}{\pi h}$$

$$r = \sqrt{\frac{3V}{\pi h}}$$

- d. How much is the radius when the height is 8 cm and the volume is 54 cm³?

$$r = \sqrt{\frac{3(54)}{8\pi}} = 2.5 \text{ cm}$$

2. Modelling with Algebra

Algebra is an efficient way to express mathematical ideas. Algebraic modelling is one of many ways to solve a problem.

Example 1: The difference between two numbers is 55. Twice the smaller plus the larger is 76. Find the two numbers.

Step 1: Let statement.

Let x be the smaller number.

So the bigger number is $x + 55$.

Step 2: Form an equation and solve.

$$2x + x + 55 = 76$$

$$3x = 21$$

$$x = 7$$

Step 3: Find all answers and finish with a conclusion sentence.

$$7 + 55 = 62$$

Therefore, the numbers are 7 and 62.

Example 2: Mr. Skyvington operates a variety store with his two sons, Jerry and Koko. Jerry makes twice as much as Koko, who only works part-time. Mr. Skyvington makes \$200 per week more than Jerry. The total weekly payroll is \$1450. How much does each family member earn per week?

Solution: Let x be Koko's earning.

Jerry's earning is $2x$, and Mr. Skyvington's earning is $2x + 200$.

$$x + 2x + 2x + 200 = 1450$$

$$5x = 1250$$

$$x = 250$$

$$2x = \$500 \text{ and } 2x + 200 = \$700$$

Therefore Koko's earning is \$250, Jerry's earning is \$500, and Mr. Skyvington's earning is \$700.

Example 3: John can wax his car in 3 hours. Jim can do the same job in 5 hours. How long will it take them if they work together?

Step 1: Let statement.

Let x be the time.

Step 2: Form an equation and solve.

People	John	Jim
$\frac{\text{Time spent}}{\text{Total time needed to do job alone}}$	$\frac{x}{3}$	$\frac{x}{5}$

$$\frac{x}{3} + \frac{x}{5} = 1$$

Multiply by 15 to eliminate fractions:

$$5x + 3x = 15,$$

$$8x = 15,$$

$$x = 1\frac{7}{8} \text{ hours.}$$

Step 3: Find all answers and finish with a conclusion sentence.

Therefore, it will take 1 and 7/8 hours if they work together.

Example 4: Ava purchased a hamburger with 20 dimes and nickels. The hamburger costs \$1.55. How many dimes and nickels did she have?

Let x be number of nickels.

There are $(20 - x)$ number of dimes.

$$5x + 10(20 - x) = 155$$

$$-5x + 200 = 155$$

$$5x = 45$$

$$x = 9$$

Therefore, there are 9 nickels and 11 dimes.

Example 5: Rick is three times older than his son. Three years ago, the sum of their ages was 50. How old is each now?

Let x be Rick's son's age.

	Now	3 years ago
Rick's Son	x	$x - 3$
Rick	$3x$	$3x - 3$

$$x - 3 + 3x - 3 = 50$$

$$4x = 56$$

$$x = 14$$

$$14 \times 3 = 42$$

Therefore, Rick's son's age is 14 years old and Rick's age is 42 years.

Practice in class

1. John is 3 years older than Jim. Jim is 4 years less than twice David's age. How old are the three boys if their ages add up to 35?
2. A cashier has 3 more dimes than nickels and twice as many nickels as quarters. Find the number of each kind of coin if the total value of the coins is \$3.05.
3. You are driving along at 55 mph when you are passed by a car doing 85 mph. How long will it take for the car that passed you to be one mile ahead of you?
4. If the perimeter of a rectangle is 18 inches, and one side is one inch longer than the other, how long are the sides?
5. Suppose \$10,000 is invested at 9% interest. How much money must be invested at 12% to produce a return of 11% on the entire amount invested?
6. Find a number such that 5 more than one-half the number is three times the number.
7. The price of gasoline increased by 25% between January and March. If the price per gallon in March was \$1.15, what was the price per gallon in January?
8. David can paint the living room in 4 hours and Anna can paint it in 6 hours. How long will it take the two of them to paint the living room if they work together?