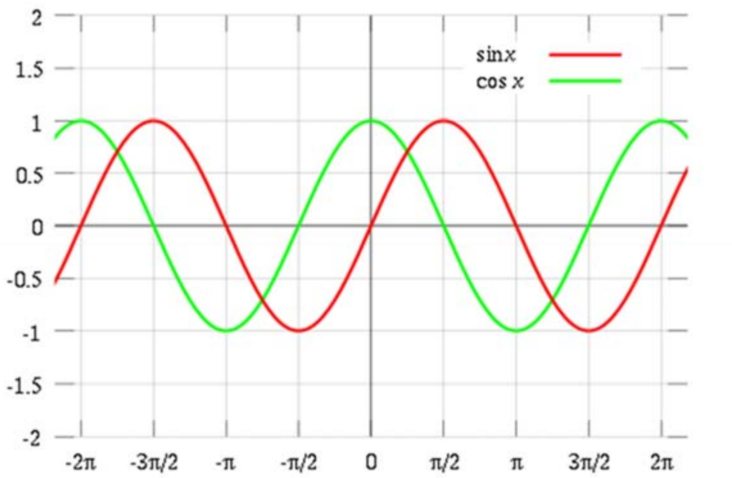
Trigonometric Functions (3)

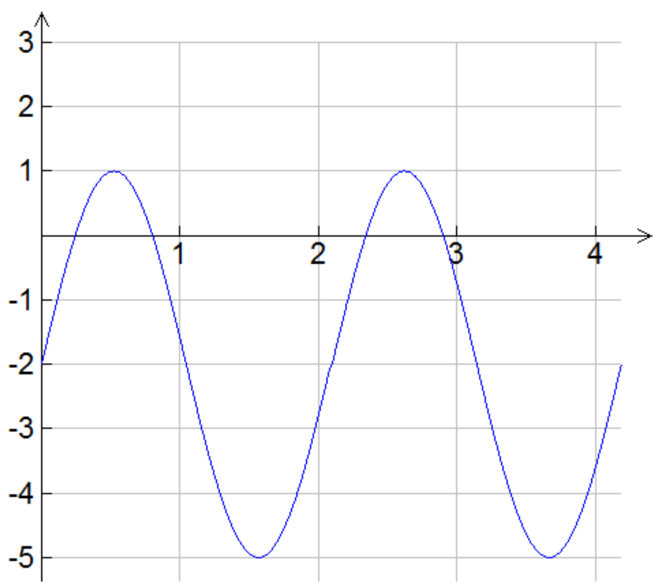
Recall the sin and cos graph:



***The sine function and cosine function are <u>congruent</u> sinusoidal curves; the cosine curve is the sine curve translated 90^0 to the left.**

- To convert y = sin θ to a cosine function, ∴ $\sin\theta = \cos(\theta - 90^0)$
- To convert y = cos θ to a sine function, ∴ $\cos\theta = \sin(\theta + 90^0)$

Example 1: Sketch the sinusoidal graph that satisfies the properties below.
a) Base function y = sin x Amplitude = 3 Period = 120^0 EOA: y = -2; # of cycles = 2



b) What is the equation of this function?
a = 3; k = 360 / 120 = 3; c = -2; d = 0

➔ **f(x) = 3 sin (3x) – 2**

c) What is the equation of this function in terms of cos x?

➔ **f(x) = 3 cos (3x - 90^0) – 2**

Example 2: Given the following, determine the function in two ways.

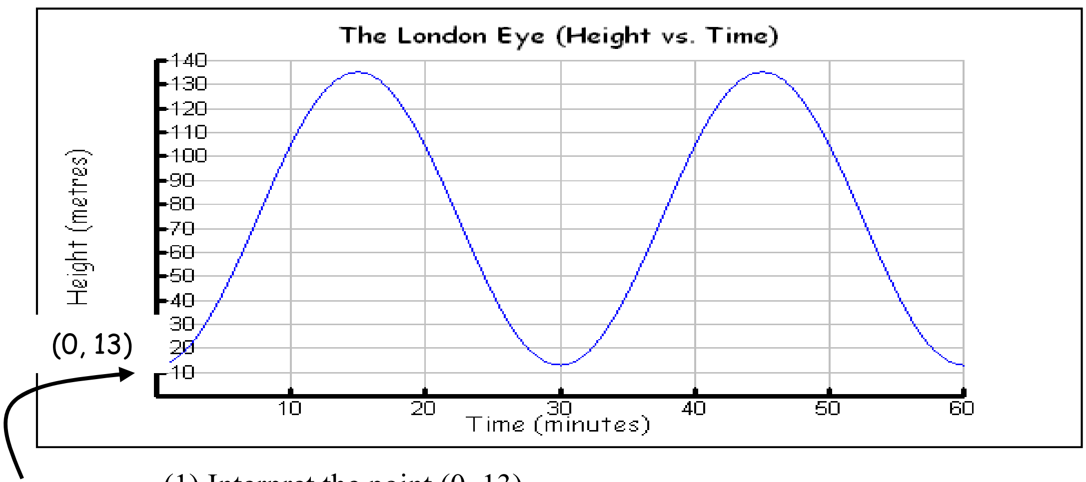
Amplitude = 0.5 → a = 0.5
Period = 60^0 → k = 360 / 60 = 6
EOA: y = -3 → c = -3
Phase shift: 5^0 right → d = 5
Graph starts at min → - cos x

f(x) = -0.5 cos (6(x - 5)) – 3 OR f(x) = -0.5 sin (6(x – 5) + 90^0) – 3 = -0.5 sin (6x + 60) – 3

Key Points

- The sine and cosine functions can be used as models to solve problems that represent many types of repetitive motions and trends
- A graph that models a sinusoidal function will always form a **series of symmetrical waves** that repeat at regular intervals. The amplitude of the sine or cosine function depends on the situation being modelled
- One cycle of motion corresponds to one period of the function
- The <u>distance</u> of a circular path is calculated from the **circumference** of the path. The <u>speed</u> of an object following a circular path can be calculated by dividing the **distance** by the **period**, the time to complete one revolution.
- The **diameter** of a circular path is equal to the total distance from the maximum point to the minimum point of the function. The **radius** is, therefore, equal to the amplitude of the function.

Example: The London Eye in London, England, is one of the largest Ferris wheels in the world! The sinusoidal curve below shows the height of a particular capsule on the wheel over time. Let's see what information we can gather about this popular attraction from its graph.



(1) Interpret the point (0, 13).

The height of the Ferris wheel is 13 m.

(2) Predict the measurement of the **diameter** of the London Eye. ___135 – 13 = 122 m___.

(3) How long does it take for the wheel to make **one complete revolution**? __30 min__.

(4) What is the **equation of the axis** for the wheel? __y = 122/2 = 61__.

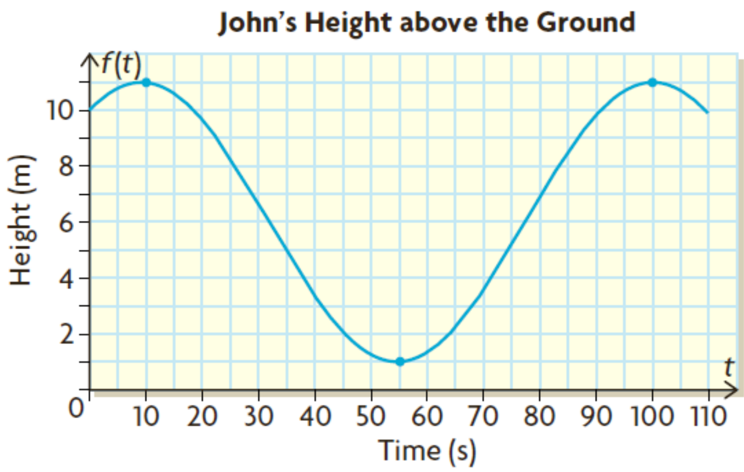
(5) Calculating the speed of the London Eye:
a) What is the formula for calculating speed? ⇨ Speed = 2πr / period

b) What pieces of information do we need to calculate the speed of the London Eye?

Radius and Period.

c) Determine the speed at which a capsule moves around the wheel.
$\pi(61)^2 / 30 = 390$ m / min

Example: A group of students is tracking a friend, John, who is riding a Ferris wheel when he is at a height of 10 m. They know that John reaches the maximum height of 11 m at 10 s and then reaches the minimum height of 1 m at 55 s. How can you develop the equation of a sinusoidal function that models John's height above the ground to determine his height at 78 s? Sketch a graph first.



First plot the two points we know: (10, 11) and (55, 1). Since it takes John 45 s to go from the highest point to the lowest, then it would take him 90 s to do one complete revolution and be back to a height of 11 m at 100 s.

Equation of the axis: y = (11+1)/2 = 6
Vertical stretch / amplitude: *a* = 11 – 6 = 5
Horizontal compression: k = 360/90 = 4

If I use the cosine function, the first maximum is at x = 0. The first maximum of the new function is at x = 10. So there was a horizontal translation of 10. **d = 10.**

➔ **y = 5 cos (4(x - 10)) + 6**
y = 5 cos (4(78 - 10)) + 6 = 6.17 m