Algebra

1. Translating 2-Step Verbal Expressions

In the language of algebra we use letters to represent the unknowns (variables) and symbols to represent the operations.

For examples:

Ex 1: Five times a number *y* increased by 9, is 67.
$5y + 9 = 67$

Ex 2: 97 diminished by twice a number *x* is 10.
$97 - 2x = 10$

Ex 3: The quotient of sixteen and *x* is 4.
$16/x = 4$

Ex 4: If six is increased by the product of three and *x*, the result is 106.
$56+ 3x = 106$

Ex 5: If a number *y* is added to three times *y* the result is fifty-two.
$3y + y = 52$

2. Adding and Subtracting Monomials

The same rules apply to adding and subtracting monomials that apply to integers. We also call this "combining" monomials.

We can only combine terms that are exactly alike!!!!
(In other words, the variables, if any, must be exactly the same. If one term's variable has an exponent and the other does not, they are <u>not</u> like terms.)

Examples of like terms are:
$5x$ and $-7x$; $-4p$ and $9p$; $-3y^2$ and $-y^2$; 10 and -14

These are <u>not</u> like terms:
$6x$ and $-4y$; $2ab$ and $3cd$; $8x$ and $-9x^2$

To combine monomials, we must remember two important rules:

- If the signs are the same, add and keep the sign.
- If the signs are different, subtract and keep the sign of the larger number.

Now let's look at some different problems...

-5x + 3x - 8x - x (Remember there is a "1" in front of the "x" making it a "-1x".)	Following order of operations, we must solve left to right. -5+3 is -2 -2 -8 is -10 -10 -1 is -11	Now that we have solved the integer part of the solution we now attach the x! Answer: -11x
$7x^2 - 8x^2 + 11x^2 - 6x^2$	Again, solve left to right. 7 -8 is -1 -1 +11 is 10 10 -6 is 4	Answer: $4x^2$

What if the problem has various terms in it?

3ab - 10x - 7ab - 2x	We can only combine the terms that are exactly alike! So we can combine the "ab" terms and the "x" terms.	3ab and -7ab is -4ab -10x and -2x is -12x Put the two terms together for our answer: -4ab -12x
-11y -c + 4y -7c	We can combine the "y" terms and the "c" terms.	-11y and 4y is -7y -c and -7c is -8c Put the two terms together for our answer: -7y -8c

3. Multiplying Monomials

1) Monomial
Definition: A term can be a variable, a number (called a constant), or a variable with a number attached (called a coefficient). Single terms are called "monomials". Monomial is a single term.

Examples of Monomials:
x, y, a, 12, -7, 2x, 9z, 5m, x^2

2) Multiply monomials
If there are coefficients on the terms then we must first multiply coefficients by coefficients.
Then we multiply the variable of one term by the variable in the other term.

For example: (3x)(4y)
First : (3)(4) = 12
Then: (x)(y) = xy
So the answer is: 12xy

If the variable in one term is the same as the variable in the other term then we must follow the rule of exponents for multiplication.

For example: $(5x)(7x)$
First: $(5)(7) = 35$
Then $(x)(x) = x^2$
So the answer is $35x^2$

Finally we need to remember that when multiplying terms we must also pay attention to the signs on the terms.

When multiplying remember:
$(+)(+) = +$
$(+)(-) = -$
$(-)(+) = -$
$(-)(-) = +$

For example:
$(-4x^2)(7y^5z^3)$
First: $(-4)(7) = -28$
Then: $(x^2)(y^5z^3) = x^2y^5z^3$
So the answer is: $-28x^2y^5z^3$

4. Dividing Monomials

To understand the concept and procedure for *dividing monomials* it is suggested that you first review the lesson on *multiplying monomials*.

In division we follow the same two-step process.

1) Because multiplication and division are inverse (opposite) operations, instead of multiplying the coefficients of the monomials we must divide them.

2) And secondly we must follow the rules for dividing variables, which means that when dividing *like* variables we must *subtract the exponents* instead of adding them.

For Example: $(14x^4) \div (2x^2) = ?$
First...divide the coefficients: $14 \div 2 = 7$
Second...divide the variables: $x^4 \div x^2 = x^{4-2} = x^2$
So the answer is: $(14x^4) \div (2x^2) = 7x^2$

5. Adding and Subtracting Polynomials

We are going to expand our discussion on adding monomials to include larger algebraic expressions called *polynomials*.

The prefix "poly" means "many". So a polynomial is an expression made up of *many* terms.

Remember that a term can be a *variable*, a *number* (called a constant), or a *variable with a coefficient* (a number attached to the front of a variable).

For example, x, 13, 5y are all terms.
When you connect two, or more terms, with either a "+" or a "-" sign you create different types of *polynomials.*

An expression made up of 2 terms is called a binomial.
3x + 5, 2y -15, x + 6f are examples of *binomials*.

An expression made up of 3 terms is called a trinomial.
4a + 7y - 2z, $3x^2$ - 7x + 4 are examples of *trinomials*.

When 4 or more terms are connected we simply call them polynomials.

Now let's see how adding and subtracting polynomials is done.

Example 1: Add: 3x+6y+7z and 5x-4y+9z

Solution: That means that in our example above we will combine only those terms in the first polynomial which are like other terms in the second polynomial.
3x + 6y + 7z + 5x - 4y + 9z = 8x + 2y + 16z

Notice that the variables did not change in any way...we only added or subtracted the coefficients!

Example 2: Subtract: 8a+5b-6c from 10a+8b+7c

Solution: Remember the polynomial after the word "from" is placed first in the subtraction problem.
(10a+8b+7c) - (8a+5b-6c)

Clear the parentheses by distributing the signs...
10a+8b+7c-8a-5b+6c
Then combine the like terms...
10a+8b+7c-8a-5b+6c = 2a+3b+13c

6. Multiply polynomial

Distributive Property: a(b + c) = ab + ac

This property is used whenever we need to multiply a polynomial by a monomial.

For example: 4(3x + 5y - 6z)

Do you see that this is an example of multiplying a monomial (the 4) by a polynomial? (3x + 5y -6z).

The distributive property works like this: Multiply every term inside the parentheses by the monomial in front of the parentheses.

$4(3x + 5y - 6z) = 4(3x) + 4(5y) - 4(6z) = 12x + 20y - 24z$

7. Dividing a Polynomial

Example: Divide $(4x^4 + 6x^3 - 10x^2) \div (2x)$

All we need to do is to divide (2x) into each of the terms, one at a time....
$(4x^4 + 6x^3 - 10x^2) \div (2x)$

Step 1: divide 4x4 by 2x
$4x^4 \div 2x = 2x^3$

Step 2: divide 6x3 by 2x
$6x^3 \div 2x = 3x^2$

Step 3: divide $-10x^2$ by 2x
$-10x^2 \div 2x = -5x$

After doing each of the separate divisions we put each of the individual answers together....make sure to connect them with the correct sign.

So...the final answer is: $2x^3 + 3x^2 - 5x$

8. Solving and Graphing Linear Inequalities

In this lesson we will practice solving multi-step inequalities and then graph the solution on a number line.

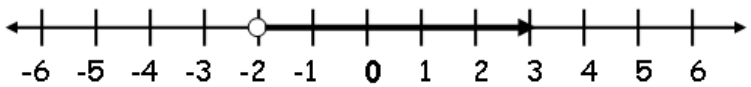
For example: $8x + 13 > 4x + 5$
$8x > 4x - 8$
(add -13 to both sides)
$4x > - 8$
(add -4x to both sides)
$x > - 2$
(divide both sides by -4)

The rules for graphing on a number line are as follows:
The arrow points in the same direction as the inequality sign use an "open" circle if the inequality is < or >.

Use a "closed" circle if the inequality is $\leq$ or $\geq$.

For example: $x > -2$

The graph will be an "open" circle at -2 and the arrow will point to the right.

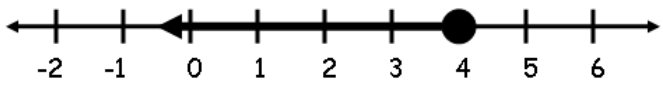


For example: $9a - 10 \leq 3a + 14$

$9a \leq 3a + 24$
(add +10 to both sides)
$6a \leq 24$
(add -3a to both sides)
$a \leq 4$
(divide both sides by 6)

Now let's graph the solution.

The graph should have a "closed" circle on 4 and the arrow should point to the left.



Questions in class

1. The temperature of a chemical is dropping *d* degrees per minute. At 8:00 pm, the temperature was 0^0 C. What was the temperature in degrees centigrade *m* minutes before 8:00 pm?

2. Bread A has 1 g of fiber and 4 g of protein per slice, while Bread B has 2 g of fiber and 3 g of protein per slice. After Tom ate some slices of each type of bread, he calculated that there were 16 g of fiber and 39 g of protein in the bread he ate. What is the total number of slices of bread Tom ate?

3. A pencil and a magazine cost $4.25. A magazine and a notebook cost $6.50. A notebook and a pencil cost $5.75. How much does the pencil cost?

4. Adult tickets for a game cost $7. Student tickets for the same game cost $4. The ratio of adult tickets to student tickets sold was 1:3. If the value of all the tickets sold was $1,368, how many tickets were sold altogether?

5. Beth can paint a house in 5 hours. Tim can paint half of the same house in 75 minutes. How long would it take both of them working together to paint the house?

6. If $xy = 48$ and $x^2 + y^2 = 1026.25$, then what is the value of $(x + y)^2$?

7. If $x + y = 27$ and $x^2 + y^2 = 365$, what is the value of $x \cdot y$?

8. Solve for x: $\sqrt{x-5} + 3 = 8$

9. Which of the following pairs of algebraic expressions are equivalent?

a. $(x+2)^2 - x^2$ and $2(x+2) + 2x$ b. $(x+3)^2$ and $x^2 + 9$

c. $2(xy)$ and $2x \cdot 2y$ d. $\sqrt{x^2+9}$ and $x + 3$ e. $0x^0$ and 0^0

10. What is the solution to the equation: $9t - 6(t + 1) = 3t + 6$.

11. Which of the following is a solution to the inequality: $7 - 2(a - 5) < 8$

a. a = 4 b. a = 10 c. a = 0

d. Any real number will satisfy the equation. e. The equation has no solution.

12. An identity is an equation that is true for all real numbers. Which of the following equations is an identity?

a. $a^2 + b^2 = a + b$ b. $\frac{a}{3} + \frac{b}{4} = \frac{a+b}{7}$ c. $a - b = b - a$

d. $(ab)c = ac \cdot bc$ e. $-(a-b) = b - a$