Percents and Ratio

1. Percents

1) What is a Percent?
A percent is a ratio of a number to 100. A percent can be expressed using the percent symbol %.

Example: 10 percent or 10% are both the same, and stand for the ratio 10:100.

2) Percent as a fraction
A percent is equivalent to a fraction with denominator 100.

Example: 5% of something = 5/100 of that thing.

Example: 2 1/2% is equal to what fraction?
Answer:
2 1/2% = (2 1/2)/100 = 5/200 = 1/40

Example: 52% most nearly equals which one of 1/2, 1/4, 2, 8, or 1/5?
Answer: 52% = 52/100. This is very close to 50/100, or 1/2.

Example: 13/25 is what %?
We want to convert 13/25 to a fraction with 100 in the denominator:
13/25 = $(13 \times 4)/(25 \times 4)$ = 52/100, so 13/25 = 52%.

Alternatively, we could say: Let 13/25 be n%, and let us find n. Then 13/25 = n/100, so cross multiplying, $13 \times 100 = 25 \times n$, so $25n = 13 \times 100 = 1300$. Then $25n \div 25 = 1300 \div 25$, so $n = 1300 \div 25 = 52$. So 13/25 = n% = 52%.

Example: 8/200 is what %?
Method 1: 8/200 = $(4 \times 2)/(100 \times 2)$, so 8/200 = 4/100 = 4%.
Method 2: Let 8/200 be n%. Then 8/200 = n/100, so $200 \times n = 800$, and $200n \div 200 = 800 \div 200 = 4$, so n% = 4%.

Example: Write 80% as a fraction in lowest terms.
80% = 80/100, which is equal to 4/5 in lowest terms.

3) Percent as a decimal
Percent and hundredths are basically equivalent. This makes conversion between percent and decimals very easy.

To convert from a decimal to a percent, just move the decimal 2 places to the right. For example, 0.15 = 15 hundredths = 15%.

Example:
0.0006 = 0.06%

Converting from percent to decimal form is similar, only you move the decimal point 2 places to the left. You must also be sure, before doing this, that the percentage itself is expressed in decimal form, without fractions.

Example:
Express 3% in decimal form. Moving the decimal 2 to the left (and adding in 0's to the left of the 3 as place holders,) we get 0.03.

Example:
Express 97 1/4% in decimal form. First we write 97 1/4 in decimal form: 97.25.
Then we move the decimal 2 places to the left to get 0.9725, so 97 1/4% = 0.9725.
This makes sense, since 97 1/4% is nearly 100%, and 0.9725 is nearly 1.

2. Ratio

1) What is a ratio?
A "ratio" is just a comparison between two different things.

For instance, someone can look at a group of people, count noses, and refer to the "ratio of men to women" in the group. Suppose there are thirty-five people, fifteen of whom are men. Then the ratio of men to women is 15 to 20.

Notice that, in the expression "the ratio of men to women", "men" came first. This order is very important, and must be respected: whichever word came first, its number must come first. If the expression had been "the ratio of women to men", then the numbers would have been "20 to 15".

Expressing the ratio of men to women as "15 to 20" is expressing the ratio in words. There are two other notations for this "15 to 20" ratio:
odds notation: 15 **:** 20
fractional notation: $^{15}/_{20}$

Given a pair of numbers, you should be able to write down the ratios.

For example:
There are 16 ducks and 9 geese in a certain park. Express the ratio of ducks to geese in all three formats.

$$16 : 9,\ \frac{16}{9},\ 16\ \textit{to}\ 9$$

Consider the above park. Express the ratio of geese to ducks in all three formats.

$$9 : 16,\ \frac{9}{16},\ 9\ \textit{to}\ 16$$

Let's return to the 15 men and 20 women in our original group. I had expressed the ratio as a fraction, namely, $^{15}/_{20}$. This fraction reduces to $^{3}/_{4}$. This means that you can also express the ratio of men to women as $^{3}/_{4}$, 3 **:** 4, or "3 to 4".

The ratio "15 to 20" refers to the *absolute* numbers of men and women, respectively, in the group of thirty-five people.

The simplified or reduced ratio "3 to 4" tells you only that, for every three men, there are four women. The simplified ratio also tells you that, in any representative set of seven people (3 + 4 = 7) from this group, three will be men. In other words, the men comprise $^{3}/_{7}$ of the people in the group. These relationships and reasoning are what you use to solve many word problems:

- In a certain class, the ratio of passing grades to failing grades is 7 to 5. How many of the 36 students failed the course?

  The ratio, "7 to 5" (or 7 : 5 or $^{7}/_{5}$), tells me that, of every 7 + 5 = 12 students, five failed. That is, $^{5}/_{12}$ of the class flunked. Then ($^{5}/_{12}$) (36) = 15 students failed.

- In the park mentioned above, the ratio of ducks to geese is 16 to 9. How many of the 300 birds are geese?

  The ratio tells me that, of every 16 + 9 = 25 birds, 9 are geese. That is, $^{9}/_{25}$ of the birds are geese. Then there are ($^{9}/_{25}$)(300) = 108 geese.

Generally, ratio problems will just be a matter of stating ratios or simplifying them.

For instance:

a) Express the ratio in simplest form: $10 to $45
This exercise wants me to write the ratio as a *reduced* fraction:
.$^{10}/_{45} = ^{2}/_{9}$.

This reduced fraction is the ratio's expression in simplest fractional form. Note that the units (the "dollar" signs) "canceled" on the fraction, since the units, "$", were the same on both values. When both values in a ratio have the same unit, there should generally be no unit on the reduced form.

b) Express the ratio in simplest form: 240 miles to 8 gallons
When I simplify, I get (240 miles) / (8 gallons) = (30 miles) / (1 gallon), or, in more common language, 30 miles per gallon.

In contrast to the answer to the previous exercise, this exercise's answer did need to have units on it, since the units on the two parts of the ratio, the "miles" and the "gallons", do not "cancel" with each other.

2) If $\frac{a}{b} = \frac{c}{d}$ **then** $\frac{b}{a} = \frac{d}{c}$**.**

3) If $\frac{a}{b} = \frac{c}{d}$ **or** $a:b = c:b$**, then** $a \cdot d = b \cdot c$

4) Comparing Ratios

To compare ratios, write them as fractions. The ratios are equal if they are equal when written as fractions.

Example:
Are the ratios 3 to 4 and 6:8 equal?
The ratios are equal if 3/4 = 6/8.
These are equal if their cross products are equal; that is, if $3 \times 8 = 4 \times 6$. Since both of these products equal 24, the answer is yes, the ratios are equal.

Remember to be careful! Order matters! A ratio of 1:7 is not the same as a ratio of 7:1.

Examples:
Are the ratios 7:1 and 4:81 equal? No!
7/1 > 1, but 4/81 < 1, so the ratios can't be equal. Are 7:14 and 36:72 equal?

Notice that 7/14 and 36/72 are both equal to 1/2, so the two ratios are equal.
A percent is a ratio of a number to 100. A percent can be expressed using the percent symbol %.

Example:
10 percent or 10% are both the same, and stand for the ratio 10:100.

3. Percent as a ratio

The word **percent** comes from the Latin *per centum* meaning "out of one hundred," so we can think of 22% as "22 out of 100." Thus a percent is a **symbol** representing a ratio of the part -- in this case 22 -- to the whole -- 100.

A percent must be changed to a number (fraction or decimal) before we can compute with it. Hence 22% is the ratio 22 : 100, which gives:

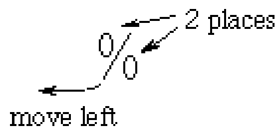
$$\frac{22}{100} = .22$$

Other examples:

percent notation	**ratio notation**	**number notation**
30%	30 : 100	30 / 100 = **0.3**
8%	8 : 100	8 / 100 = **0.08**
63.7%	63.7 : 100	63.7 / 100 = **0.637**
100%	100 : 100	100 / 100 = **1**
212%	212 : 100	212 / 100 = **2.12**
5 3/4%	5 3/4 : 100 or 5.75 : 100	5.75 / 100 = **0.0575**

We will most often use the decimal form (in bold above) because it is easy to find on a calculator by dividing by 100. Let's make some notes from these examples.

- The mechanics of changing a percent to a decimal number involves moving the decimal point two places to the left. The percent symbol can help us remember this:



- When we change percents smaller than 10 percent to numbers, we need to place holding 0s -- as with 8% and 5 3/4%.

- 100% is 1, reinforcing the idea that x% describes x's part of "the whole."

- Percents representing more than 1 are possible (212%) -- they are used to talk about something growing really large rather than as "part out of the whole."

Before looking at how percents come up in science, we need to be able to answer questions like those below.

1) What is 20% of 35? It's 7.

2) 11 is what percent of 20? It's 55%.

3) 6.5% of what number is 80.02? It's 1231.076923 (or 1200 to two sig figs).

4) What is 10 after being increased by 15% ? It's 11.5.

5) What is 201.1 after being decreased by 25% ? It's 150.825.

4. Interest

Interest is a fee paid to borrow money. It is usually charged as a percent of the total amount borrowed. The percent charged is called the interest rate.

The amount of money borrowed is called the principal. There are two types of interest, simple interest and compound interest.

Example: A bank charges 7% interest on a $1000 loan. It will cost the borrower 7% of $1000, which is $70, for each year the money is borrowed. Note that when the loan is up, the borrower must pay back the original $1000.

5. Simple Interest

Simple interest is interest figured on the principal only, for the duration of the loan. Figure the interest on the loan for one year, and multiply this amount by the number of years the money is borrowed for.

Example: A bank charges 8% simple interest on a $600 loan, which is to be paid back in two years. It will cost the borrower 8% of $600, which is $48, for each year the money is borrowed.

Since it is borrowed for two years, the total charge for borrowing the money will be $96. After the two years the borrower will still have to pay back the original $600.

Challenge Examples:

Example 1:
The table to the right displays the grade distribution of the 30 students in a mathematics class on the last two tests. For example, exactly one student received a 'D' on Test 1 and a 'C' on Test 2 (see circled entry). What percent of the students received the same grade on both tests?

TEST 1 \ TEST 2	A	B	C	D	F
A	2	2	1	0	0
B	1	4	3	0	0
C	1	3	5	2	0
D	0	0	1	1	1
F	0	0	2	1	0

A) 12% B) 25% C) $33\frac{1}{3}$% D) 40% E) 50%

Solution
(D) A student received the same grade on both tests if s/he is counted on the main diagonal (from the top left to the bottom right) of the table. Thus the number of students receiving the same grade on both tests is 2 + 4 + 5 + 1 + 0 = 12. Consequently 12/30 = 4/10 = 40% of the students received the same grade on both tests.

Example 2:
Sale prices at the Ajax Outlet Store are 50% below original prices. On Saturdays an additional discount of 20% off the sale price is given. What is the Saturday price of a coat whose original price is $180?
A) $54 B) $72 C) $90 D) $108 E) $110

Solution: The sale price of the coat is 50% of $180 or $90. The additional discount is 20% of $90 or $18, so the Saturday price is S90 - $18 = $72
OR

The Saturday price is 80% of the sale price and the sale price is 50% of the original price, so the Saturday price is 0.8(0.5($180)) = 0.8($90) = $72. Answer is B

Questions in class

Ratio problems:

1. The ratio of the number of games won to the number of games lost (no ties) by the Middle School Middies is 11/4. To the nearest whole percent, what percent of its games did the team lose?

2. In a far-off land three fish can be traded for two loaves of bread and a loaf of bread can be traded for four bags of rice. How many bags of rice is one fish worth?

3. Diameter ACE is divided at C in the ratio 2:3. The two semicircles, ABC and CDE, divide the circular region into an upper (shaded) region and a lower region. Find the ratio of the area of the upper region to that of the lower region.

4. An American traveling in Italy wishes to exchange American dollars for Italian lire. If 3000 lire = $ 1.60, how many lire will the traveler receive in exchange for $ 1.00?

Percent problems:

1. A burger at Ricky C's weighs 120 grams, of which 30 grams are filler. What percent of the burger is not filler?

2. A merchant offers a large group of items at 30% off. Later. the merchant takes 20% off these sale prices and claims that the final price of these items is 50% off the original price. What is the total discount?

3. A collector offers to buy state quarters for 2000% of their face value. At that rate how much will Bryden get for his four state quarters?

4. Ara and Shea were once the same height. Since then Shea has grown 20% while Ara has grown half as many inches as Shea. Shea is now 60 inches tall. How tall, in inches, is Ara now?

5. Walter has exactly one penny, one nickel, one dime and one quarter in his pocket. What percent of one dollar is in his pocket?

6. A team won 40 of its first 50 games. How many of the remaining 40 games must this team win so it will have won exactly 70% of its games for the season?