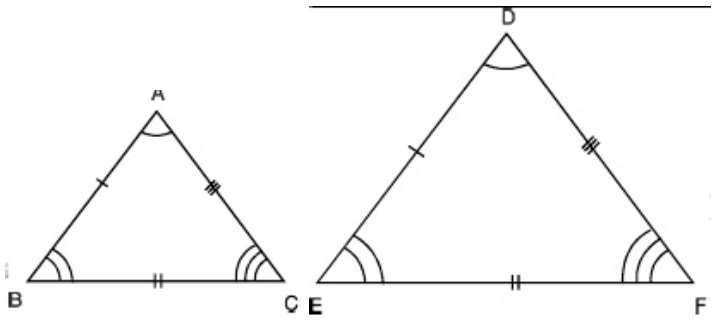
Chapter 9 Similar Triangles (1)

Two triangles are **similar** if

(i) their corresponding angles are equal, and

(ii) their corresponding sides are proportional

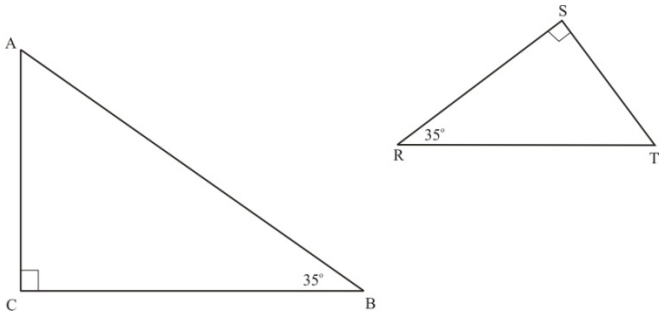


We say that ΔABC is similar to ΔDEF and denote it by writing ΔABC ~ΔDEF

The symbol '~' stands for the phrase "is similar to"

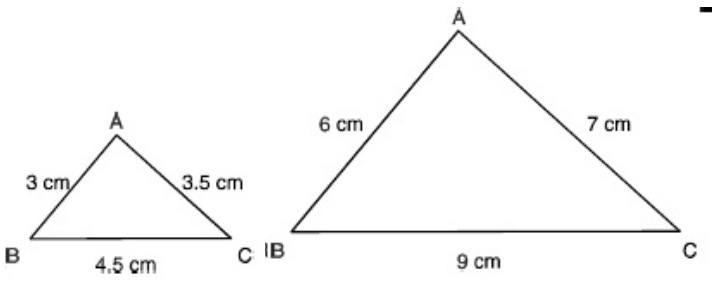
If ΔABC ~ΔDEF, then by definition ∠A = ∠D, ∠B = ∠E, ∠D = ∠F and AB/DE = BC/EF = CA/FD

1) AA~ criterion for similarity



If in two triangles, the corresponding angles are equal the triangles are similar.

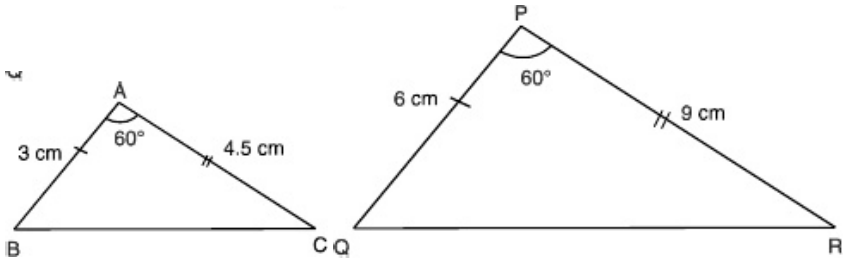
2) SSS~ criterion for similarity



$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

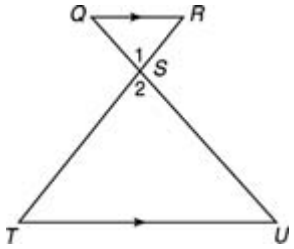
If the corresponding sides of two triangles are proportional the triangles are similar.

3) SAS~ criterion for similarity



If one angle of a triangle is equal to one angle of the other triangle and the sides containing these angles are proportional, the triangles are similar.

Example 1: Use Figure to show that $\Delta QRS \sim \Delta UTS$.

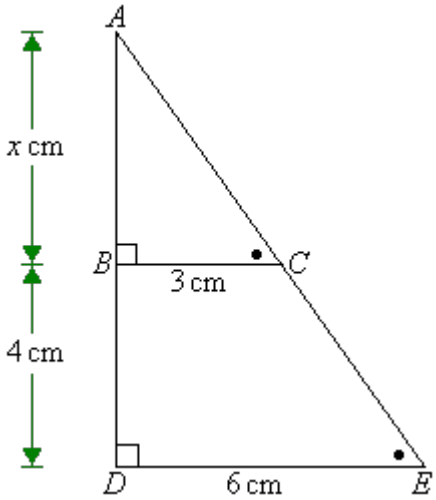


$\because$ QR // TU (Given)

$\therefore$ $m\angle R = m\angle T$ and $m\angle Q = m\angle U$ (alternate interior angles)

$\therefore$ $\Delta QRS \sim \Delta UTS$ (*AA~*)

Example 2: Find the value of the unknown in the following diagram.



$\because \angle ABC = \angle D = 90^o$ (given)
$\angle A = \angle A$ (shared)
$\therefore$ $\Delta ADE \sim \Delta ABC$ (*AA~*)

$$\therefore \frac{AD}{AB} = \frac{DE}{BC}$$

$$\frac{x+4}{x} = \frac{6}{3}$$

$$\frac{x+4}{x} = 2 \qquad \text{(Multiply both sides by } x\text{)}$$

$$x\left(\frac{x+4}{x}\right) = x \times 2$$

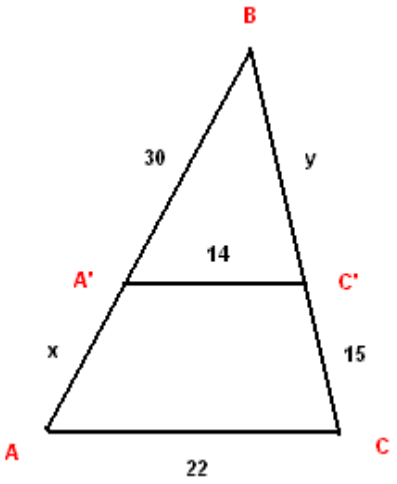
$$x+4 = 2x \qquad \text{(Subtract } x \text{ from both sides)}$$

$$x+4-x = 2x-x$$

$$4 = x$$

$$x = 4$$

Example 3: In the triangle ABC shown below, A'C' is parallel to AC. Find the length y of BC' and the length x of A'A.



$\because$ A'C' // AC (Given)

$\therefore$ $m \angle BA'C' = m \angle A$ and $m \angle BC'A' = m \angle C$ (corresponding angles)

$\therefore$ $\Delta BA'C' \sim \Delta BAC$ ($AA\sim$)

$(30 + x) / 30 = 22 / 14 = (y + 15) / y$

$(30 + x) / 30 = 22 / 14$

$420 + 14\,x = 660$
$x = 17.1$ (rounded to one decimal place).

$22 / 14 = (y + 15) / y$
$y = 26.25$

Class Practice:

1. The ratio of the corresponding sides of two similar triangles is 3 : 2.

The ratio of their corresponding heights is _____

The ratio of their perimeters is _____

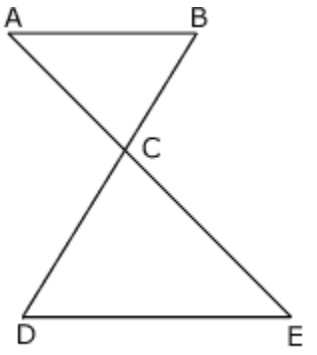
The ratio of their areas is _____

2. Which of the following are always similar?

I. All equilateral triangles. II. All circles III. All regular hexagons.

IV. All isosceles triangles. V. All parallelograms.

3. In the figure, AB || DE. AC = 4 cm, CE = 6 cm and DB = 15 cm. Find BC.



Picture not drawn to scale

4. ΔAPQ ~ ΔACB. If AQ = 2 cm, PC = 5 cm, QB = 13 cm and BC = 15 cm, then find the length of AP rounded to the nearest decimal.