First Name:_____ Last Name: _____ Student ID: _____

Applications of Vectors (1)

1. Calculate the dot product, $\vec{u} \cdot \vec{v}$, to one decimal place accuracy, given that

a. $|\vec{u}|=10, |\vec{v}|=2$, and the angle between $\vec{u}$ and $\vec{v}$ is 40∘
b. $\vec{u} = 3\hat{\imath} - \hat{\jmath} + 4\hat{k}$ and $\vec{v} = -\hat{\imath} + 2\hat{\jmath} + 5\hat{k}$

2. If the vectors $\overrightarrow{2a} + \vec{b}$ and $\frac{1}{2}\vec{a} - \vec{b}$ are perpendicular to each other and $2\,|\vec{b}| = 3\,|\vec{a}|$ find the angle $\theta = \angle(\vec{a}, \vec{b})$.

3. Find the angle between each pair of vectors:

a. $\vec{u} = 3\hat{\imath} - \hat{\jmath}$ and $\vec{v} = -\hat{\imath} + 2\hat{\jmath}$

b. (2,1,−3) and (1,0,4)

4. For each of the following pairs of vectors, find the value of a which makes $\vec{u}$ orthogonal to $\vec{v}$:

a. $\vec{u} = (3,-4)$ and $\vec{v} = (a,6)$
b. $\vec{u} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$ and $\vec{v} = a\hat{\imath} + 2\hat{\jmath} - \hat{k}$
c. $\vec{u} = (3,a,-2)$ and $\vec{v} = (1-a,-3,4)$

5. Use the dot product to determine if $\triangle ABC$ is right-angled, given the coordinates of its vertices. If it is, state which angle measured 90∘.

a. A (3,−1), *B* (0,−2), *C* (2,0)

b. *A* (1,−1,4), *B* (−2,5,3), *C* (3,0,4)

6. The parallelogram *PQRS* has vertices *P* (7,12),*R* (20,5), and *S* (4,3).

a. Find the coordinates of *Q*.

b. Find the measure of $\angle PSR$

c. Calculate the area of the parallelogram.

7. If $\vec{u}$ has magnitude 11, $\vec{v}$ has magnitude 5, and the angle between $\vec{u}$ and $\vec{v}$ is 140∘, what is the magnitude of $\vec{u} \times \vec{v}$ to one decimal place accuracy?

8. Find the cross product $\vec{u} \times \vec{v}$ given that

a. $\vec{u} = 3\hat{\imath} - \hat{\jmath} + 4\hat{k}$ and $\vec{v} = -\hat{\imath} + 2\hat{\jmath} + 5\hat{k}$

b. $\vec{u} = (1,2,3)$ and $\vec{v} = (4,-1,5)$

9. Given the vectors $\vec{u} = (-2,1,-1)$ and $\vec{v} = (-1,2,-1)$

a. Find a unit vector perpendicular to both $\vec{u}$ and $\vec{v}$.
b. Find two vectors of magnitude 11 which are perpendicular to both $\vec{u}$ and $\vec{v}$.

10. For each pair of vectors $\vec{u}$ and $\vec{v}$, find the vector projection of $\vec{u}$ on $\vec{v}$.

a. $\vec{u} = (-2,1,-1)$ and $\vec{v} = (2,1,3)$
b. $\vec{u} = (-2,1,-1)$ and $\vec{v} = (4,-2,2)$

11. For each pair of vectors $\vec{u}$ and $\vec{v}$ in Question 11, find the scalar projection of $\vec{u}$ on $\vec{v}$.

12. Determine if the vectors (1, 3, 2), (5,0,−1), and (−4,3,3) are coplanar.

13. Find the volume of the parallelepiped defined by the vectors $\vec{a}$= (0, 1, −3), $\vec{b}$= (1, 2, 3) and $\vec{c}$ = (−1,0,1).

14. Find all unit vectors perpendicular to (1, 2, 3) that make equal angles with the unit vectors $\hat{\imath}$ and $\hat{\jmath}$.