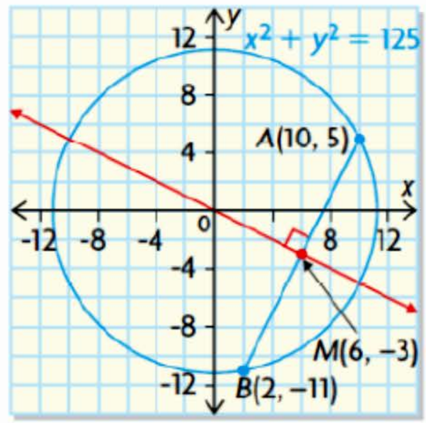
Chapter 2 Analytic Geometry (2)

Continue from last class …

Example 3

Show that points $A(10, 5)$ and $B(2, -11)$ lie on the circle with equation $x^2 + y^2 = 125$. Also show that the perpendicular bisector of **chord** AB passes through the centre of the circle.

(**chord:** A line segment that joins two points on a curve)



sub $A(10,5)$ *and* $B(2,-11)$ *into* $x^2 + y^2 = 125$:

$10^2 + 5^2 = 125$ $\quad$ $2^2 + (-11)^2 = 125$

$100 + 25 = 125$ $\quad$ $4 + 121 = 125$

$125 = 125$ $\quad$ $125 = 125$

Since left side is equal to the right side, therefore, point A & B are on the circle.

$$M\left(\frac{10+2}{2}, \frac{5+(-11)}{2}\right) = (6, -3)$$

$$m_{AB} = \frac{-11-5}{2-10} = \frac{-16}{-8} = 2$$

$$m_M = -\frac{1}{2}$$

sub $m_M = -0.5$ & $M(6,-3)$ *into* $y = mx + b$:

$-3 = -0.5(6) + b$

$-3 + 3 = b$

$b = 0$

$\therefore y = -0.5x$

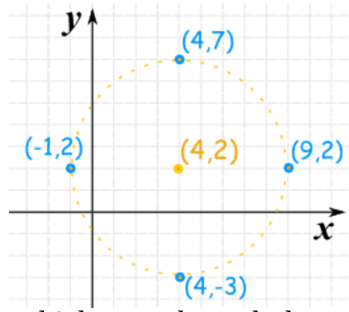
sub the center $(0,0)$ *into the equation*:

$0 = -0.5(0)$

$0 = 0$

Since left side is equal to the right side, therefore, the perpendicular bisector passes through the centre.

Example 4: Plot $(x-4)^2 + (y-2)^2 = 25$



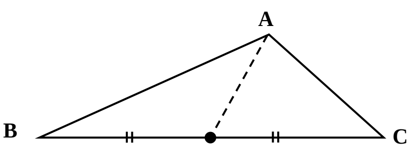
Centre: (4, 2)
Radius: 5

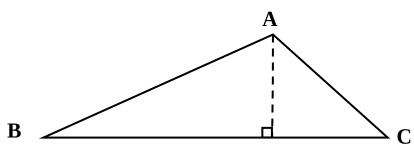
Right Bisector: A line perpendicular to a line segment, which goes through the midpoint of the line segment.

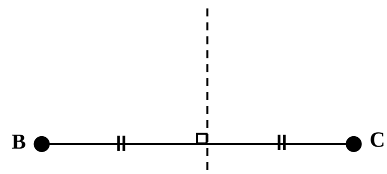
Median: A line segment that joins a vertex of a triangle to the midpoint of the opposite side.

Altitude: In a triangle, the altitude is the perpendicular distance from a vertex to the opposite side.

Label the dotted line in each diagram with the appropriate term.







How do I find….?

A. **Equation of Median**

Given ΔABC, the equation of the median from vertex A can be found by:

i) Find the midpoint of BC.

ii) Calculate the slope of the line segment joining A to the midpoint of BC.

iii) Use the slope and the vertex A to write the equation of the median.

B. **Equation of Right Bisector**

Given two points A and B, the equation of the right bisector can be found by:

i) Find the midpoint of AB.

ii) Find the slope of AB.

iii) Find the perpendicular slope to AB (negative reciprocal).

iv) Use the midpoint and perpendicular slope to find the equation of the right bisector.

C. Equation of Altitude

Given ΔABC, the equation of the altitude from vertex A can be found by:

i) Find the slope of BC.

ii) Find the perpendicular slope for BC (negative reciprocal).

iii) Use the perpendicular slope and vertex A to find the equation of the altitude through vertex A.

Example:

Triangle ABC has vertices A(3, 4), B(-5, 2), and C(1, -4).

Determine

a) an equation for CD, the median from C to AB

1. Find the midpoint of AB: $\left(\frac{3+(-5)}{2}, \frac{4+2}{2}\right) = (-1, 3)$

2. Find the slope of CD: $m = \frac{3-(-4)}{-1-1} = -\frac{7}{2}$

3. Write an equation for CD: $y - (-4) = -\frac{7}{2}(x - 1)$

Simplify: **7x + 2y + 1 = 0**

b) an equation for GH, the right bisector of AB

1. Find the slope of AB: $m = \frac{2-4}{-5-3} = \frac{1}{4}$

2. So, slope of the right bisector is -4.

3. Write the equation: $y - 3 = -4(x - (-1))$

Simplify: **4x + y + 1 = 0**

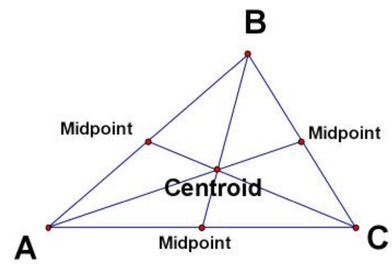
c) an equation for CE, the altitude from C to AB

1. We have the slope = -4 and C(1, -4).

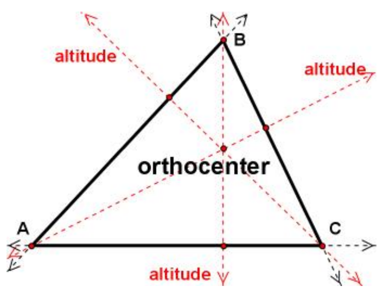
2. Write the equation: $y - (-4) = -4(x - 1)$

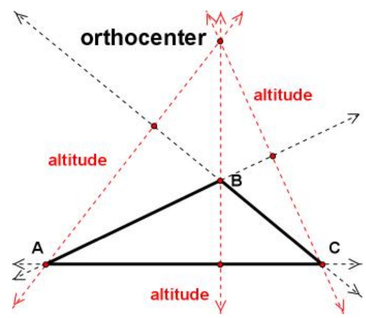
Simplify: **4x + y = 0**

Centroid - the point at which the three medians intersect.

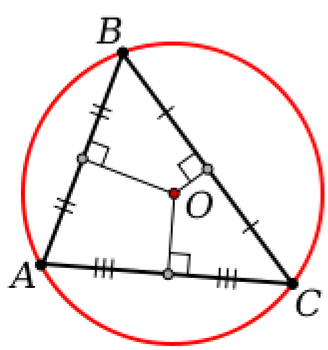


Orthocenter - The point at which the three (possibly extended) altitudes of a triangle intersect.





Circumcentre – A point at which the three perpendicular bisector of a triangle intersect.



To find the above special points, we just need 2 lines of median, altitude, and perpendicular bisector, and solve the linear system and find (x, y).

4. Distance from a point to a line

Given a point P and a line k, the shortest distance from P to k, call that PR, can be found:

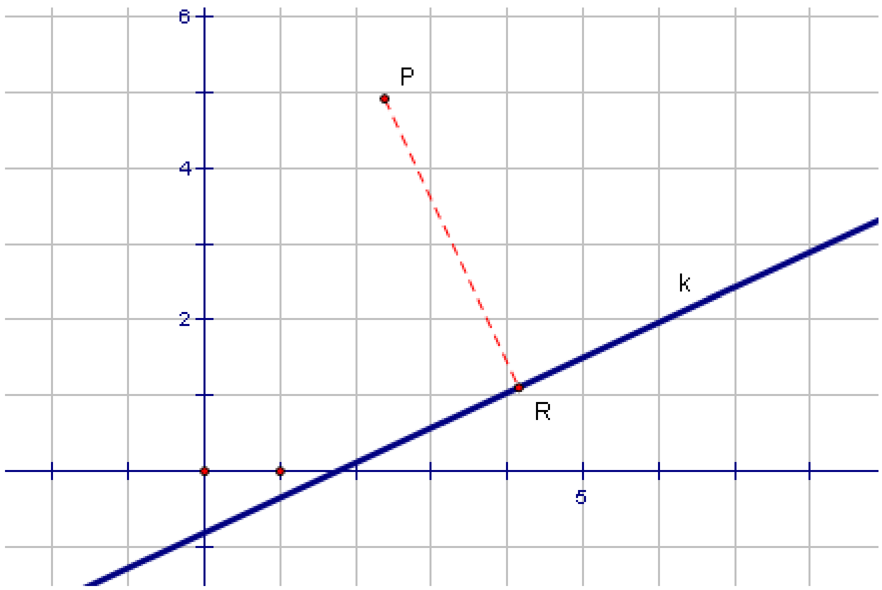
1. Find the slope of k

2. Find the perpendicular slope for k (negative reciprocal).

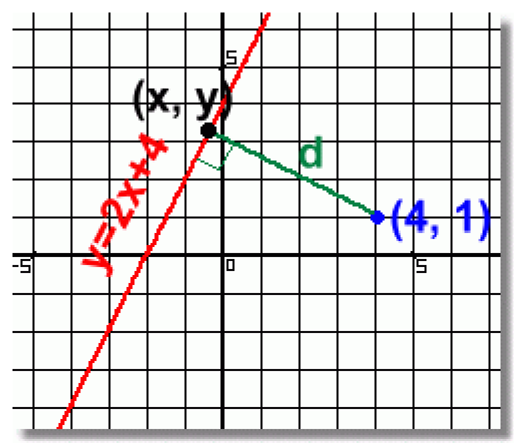
3. Use the perpendicular slope and point P to find the equation of the line l through P and k

4. Use equations of lines k and l and either the elimination or substitution to find the coordinates of point R of intersection

5. Use coordinates of R and P and the length of a line segment formula to find this distance.



Example:
Find the distance from point (4, 1) to the line y=2x+4.



Step 1: Find the equation of the line represented by distance **d**

Since the slope of **y=2x+4** is **2**, the slope of the line representing distance **d** must be **-1/2**, since it's perpendicular.

The line representing distance **d** has slope **-1/2** and passes through the point **(4, 1)**.
Using the point-slope formula for the equation of a line:

$y - y_1 = m(x - x_1)$
$y - 1 = (-1/2)(x - 4)$
$y - 1 = -1/2x + 2$
$y = -1/2x + 3$

Step 2: Find the intersection point of the two lines.

$y = -1/2x + 3$
$y = 2x + 4$

Solve:
$-1/2x + 3 = 2x + 4$
$-x + 6 = 4x + 8$
$-5x = 2$
$x = -0.4$
$y = 2(-0.4) + 4$
$y = 3.2$

The point (x, y) where the lines intersect is **(-0.4, 3.2)**

<u>Step 3</u>: Find the length of the line representing distance **d**

Find the distance between **(-0.4, 3.2)** and **(4, 1)**:

$$d = \sqrt{(4 - (-0.4))^2 + (1 - 3.2)^2}$$
$$= \mathbf{4.92}$$

In short, we have a formula to find the distance from a point (x_1, y_1) to a line
$Ax + By + C = 0$

$$d = \left| \frac{x_1 A + y_1 B + C}{\sqrt{A^2 + B^2}} \right|$$

We use the absolute value, since d is a distance, and thus avoid any confusion arising from the ± radical.

Note that the absolute value,|| of a number is defined as follows:
$|b| = b$ for $b \geqq 0$ And $|b| = -b$ *for* $b < 0$

That is, for the positive number 2,
$|2|=2$

For the negative number -2,
$|-2|= -(-2)=2$

The absolute value of $\frac{6-12}{3}$ is $\left|\frac{6-12}{3}\right| = \left|\frac{-6}{3}\right| = |-2| = -(-2) = 2$

Example

Find the distance from the point (2, 1) to the line $4x+2y+7=0$.

Solution:

$$d = \left| \frac{4\times 2 + 2\times 1 + 7}{\sqrt{4^2+2^2}} \right| = \frac{8+2+7}{\sqrt{20}} = \frac{17\sqrt{5}}{10}$$