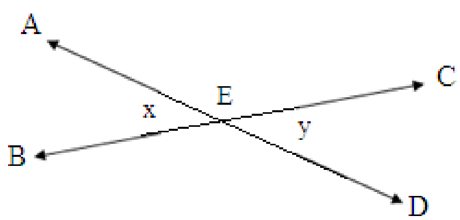
Geometry

The notes are same as before. Please review them, then and do the Questions in class and homework.

1. Vertical Angles

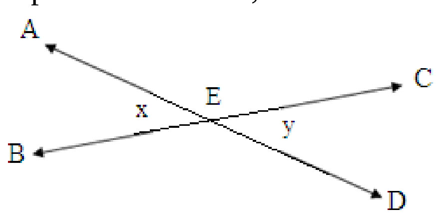
In this diagram, 2 straight lines, AB and CD, intersect at E



As you can see, when this occurs, 4 angles are created. Angles x and y are opposite one another. The name given to a pair of angles such as these is *vertical angles*.

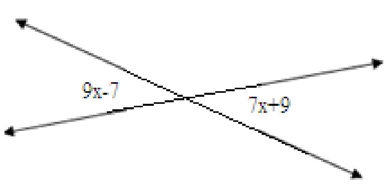
If we were to measure each of these angles, we would find the measure of both angle x and y to be the same. Therefore we will state that when 2 lines intersect, the *vertical angles formed are equal* in measure.

Example 1: Let's assume that the measure of angle y is 42^0. If angle "x" is represented by the expression "a + 16", can we solve for "a"?



Solution: If angle x = a + 16, and angle y = 42, we can set up the following equation: a + 16 = 42. This is a simple one-step equation, a + 16 = 42, a = 26.

Example 2: Find the value of x.



Solution: We can clearly see from the diagram that the angles represented by "9x - 7" and "7x + 9" are vertical angles. We also know that vertical angles are equal. Therefore we can set up the following equation:

$9x - 7 = 7x + 9, \quad x = 8.$

2. Supplementary Angles

If the sum of the measures of *two* angles is 180^0, then the angles are called supplementary angles. Each angle is called the supplement of the other angle.

For example, if one angle measures 700, then it's supplement will measure 1100 = (180-70)0. Supplementary angles are a pair of angles, if more than 2 angles add up to 1800, they are not supplementary!

Example 1: Two angles are supplementary. One of the angles is three times as large as the other. Find the measure of both angles.

Solution:
First, assign variables to represent the two angles.
Let x = the smaller angle
Let 3x = the larger angle

Second, write the equation.
3x + x = 180 (the sum of the angles is 180^0)

Third, solve the equation.
3x + x = 180 (combine like terms), x = 45

Therefore, the measure of the smaller angle (x) is 45^0 and the measure of the larger angle (3x) is 135^0.

Example 2: Find the number of degrees in an angle which is 20^0 less than four times it's supplement.

Solution:
Let 4x - 20 = one of the angles
Let x =the supplement of the angle
(4x-20) + (x) = 180, x = 40

Therefore, one of the angles (4x - 20) is 140^0, and the supplement of that angle is (180 - 140) = 40^0.

Example 3: Find the supplement of the angle which is represented by the expression $(4x - 60)^0$.

Solution:
For this problem let's use the fact that the easiest way to find the supplement of an angle is to subtract the given angle from 180^0.

So, if we let one of the angles be "x", and it's supplement be $(4x - 60)^0$, then we should be able to find the other angle by solving the following equation:
x = 180 - (4x - 60) (distribute the "-" sign), x = 48^0

Therefore the supplement is 48^0.

3. Complementary Angles

If the sum of the measures of two angles is 90^0, then the angles are called *complementary angles*. Each of the angles is called the *complement* of the other angle.

For example, if one angle measures 30^0, then the measure of it's *complement* is (90-30), or 60^0.

Remember: *Complementary angles are a pair of angles.*
If the sum of the measure of 3 or more angles is 90^0, that does not make them complementary angles!

Example 1: Two angles are complementary. The measure of one of the angles is 24^0 greater than the measure of the other angle. Find the measure of each of the angles.

Let x = the smaller angle
Let x + 24 = the larger angle
(x) + (x+24) = 90, x = 33

Therefore x, the smaller angle, measures 33^0 and x+24, the larger angle measures 57^0.

Example 2: Find the number of degrees in angle which exceeds three times it's complement by 22^0.

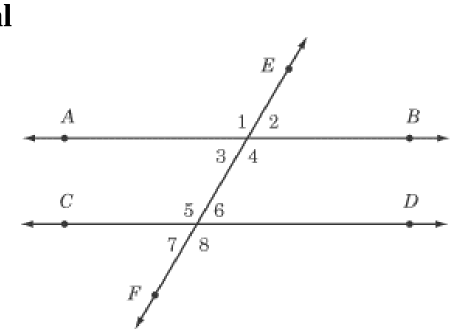
Let x = the smaller complementary angle
Let 3x+22 = the larger complementary angle.
(x) + (3x+22) = 90,
x = 17

Therefore the smaller angle, x, is 17^0, and the larger angle, 3x+ 22, is 73^0.

4. Angles Formed by Parallel Lines Cut by a Transversal

In the diagram below you can see that lines AB and CD are parallel and that the third line, EF is a transversal.

When this occurs you can also see that 8 different angles have been created which have been numbered 1 – 8.



It turns out that each of these angles can be paired with another, and that each pair of angles has a special name, as well as a special property.

1) Corresponding Angles

In the diagram above, the following pairs of angles are called *corresponding angles*:
<1 and <5; <2 and <6; <3 and <7; <4 and <8

If you look at where each of the angles in a pair are located, you will notice that they are in the same relative position where the transversal intersects one of the parallel lines and the same point of intersection on the other parallel line.

In other words, the position of one of the angles *corresponds* to the position of the other angle in the pair. As for their special property: Corresponding angles are equal.
< 1 = < 5; < 2 = < 6; < 3 = < 7; < 4 = < 8

2) Alternate Interior Angles

In the diagram above, angles 3, 4, 5 and 6 are called *interior* angles because they are between the two parallel lines. If the angles lie on opposite sides of the transversal, but not on the same parallel line, they are called *alternate interior* angles.

The pairs of alternate interior angles in this diagram are:
< 3 and < 6; < 4 and < 5
As for their special property: Alternate Interior angles are equal.
< 3 = < 6; < 4 = < 5

3) Alternate Exterior Angles

In the diagram above, angles 1, 2, 7 and 8 are called *exterior* angles because they do not lie between the parallel lines. Just like *alternate interior* angles, if the exterior angles lie on opposite sides of the transversal, but not at the same parallel line, they are called *alternate exterior angles*.

The pairs of alternate exterior angles in the diagram are:
< 1 and < 8; < 2 and < 7
As for their special property: Alternate Exterior angles are equal.
< 1 = < 8; < 2 = < 7

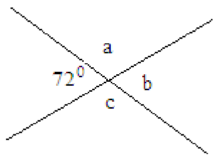
4) Interior Angles on the Same Side of the Transversal

As the name clearly implies, the diagram above shows that :
< 3 and < 5; < 4 and < 6
are pairs of angles which are not only interior angles, but also lie on the same side of the transversal.

The special property of these angle pairs: Interior Angles on the Same Side of the Transversal are SUPPLEMENTARY (THEIR SUM IS ALWAYS 180^0)
<3 + <5 = 180^0
<4 + <6 = 180^0

5. Calculate the Value of Missing Angles

When two lines intersect four angles are formed. Look at the diagram below:



As you can see the two intersecting lines have created 4 angles. The measure of one of the angles has been given as 72^0. The other 3 angles are labeled a, b and c.

Our task is to determine the measure of each of those other 3 angles.

Determining the measure of angle "b" is as easy as remembering that the 72^0 angle and angle "b" are known as *vertical angles*, and *vertical angles are equal*.

That means that the measure of angle "b" is also 72^0.
We can also see that angles "a" and "c" are also vertical angles. So if we can determine the measure of one of the angles, the other will have the same measure.

To do this we need to see that the original 72^0 angle and angle "a" are *supplementary angles*. Supplementary angles are two angles whose sum is 180^0.

We know they are supplementary because the angles combine to create a straight line. And as you know *a straight line is a straight angle*, and *the measure of a straight angle is 180^0*.

Now that we have reviewed the important vocabulary all we need to do is subtract 72 from 180, and that will give us the measure of angle "a".

180 - 72 = 108

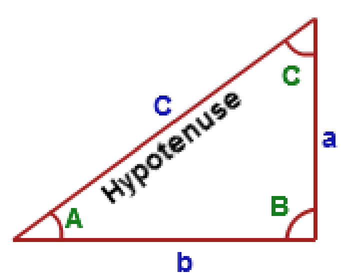
The measure of angle "a" is 108^0.

And, because angle "a" and angle "c" are vertical angles, the measure of angle "c" is also 108^0.

6. Right Triangle Formula

Right triangle is a triangle whose one of the angle is right angle. Also the sum of other two angles is equal to 90 degrees.

Right Triangle Formula is used to calculate the area, perimeter, unknown sides and unknown angles of the right triangle.



1) Right Triangle Formula is given below as,

$$\text{Area of right triangle} = \frac{1}{2}ab$$

$$\text{Perimeter of right triangle} = a + b + c$$

2) The unknown sides in a right triangle can be calculated using the Pythagoras formula.

$$c^2 = a^2 + b^2$$

3) Special Right Triangle 45º- 45º- 90º

The 45º- 45º- 90º triangle is one of two special right triangles we will be investigating.

The "special" nature of these triangles is their ability to yield exact answers instead of decimal approximations when dealing with trigonometric functions.

45º-45º-90º (Isosceles Right Triangle) Pattern Formulas

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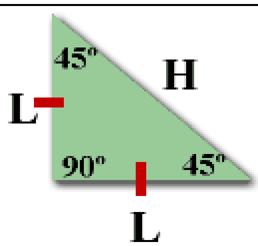
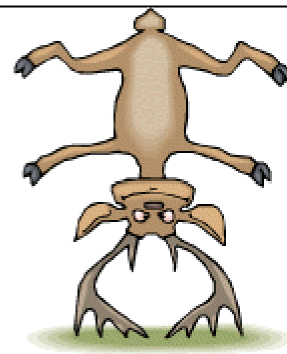
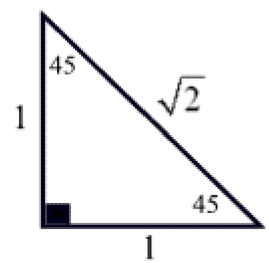
<td rowspan="2">

H = hypotenuse
L = leg</td>
<td>$H = L\sqrt{2}$</td>
<td rowspan="2"></td>
<td rowspan="2">
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<td>$L = \frac{1}{2}H\sqrt{2}$</td>
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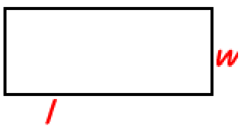
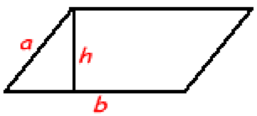
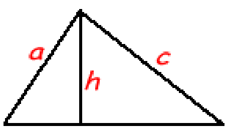
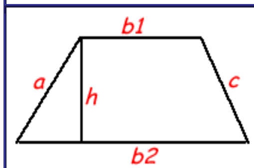
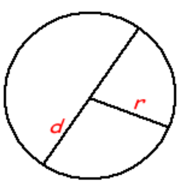
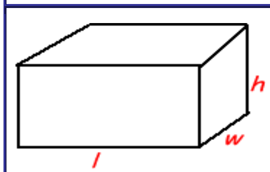
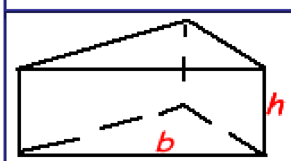
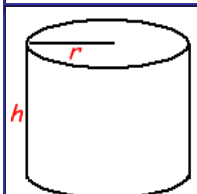
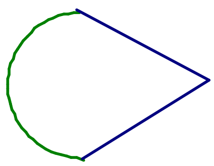
►Geometric formula

Shapes	Formula
	**Rectangle:** Area = Length X Width A = lw Perimeter = 2 X Lengths + 2 X Widths P = 2l + 2w
	**Parallelogram** Area = Base X Height a = bh
	**Triangle** Area = 1/2 of the base X the height a = 1/2 bh Perimeter = a + b + c (add the length of the three sides)
	**Trapezoid** area $A = \left(\frac{b1 + b2}{2}\right)h$ Perimeter = a + b1 + b2 + c
	**Circle** Try the Online tool. The distance around the circle is a circumference. The distance across the circle is the diameter (d). The radius (r) is the distance from the center to a point on the circle. (Pi = 3.14) More about circles. d = 2r c = πd = 2 πr A = πr^2 (π=3.14)
	**Rectangular Solid** Volume = Length X Width X Height V = lwh Surface = 2lw + 2lh + 2wh
	**Prisms** Volume = Base X Height v=bh Surface = 2b + Ph *(b is the area of the base P is the perimeter of the base)*
	**Cylinder** Volume = $\pi r^{2\ x}$ height V = πr^2 h Surface = 2π radius x height S = 2πrh + 2πr^2

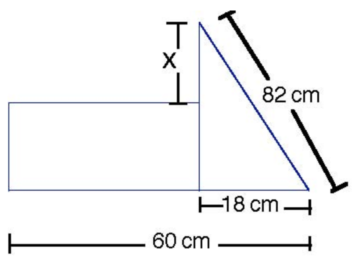
These Geometric formulae come from TDSB website

Questions in class

1. The arc in the figure shown here is a semi-circle. The segments are congruent to each other and form a 60˚ angle. The perimeter of the entire figure is $5\pi + 20$ units. What is the diameter of the circle?

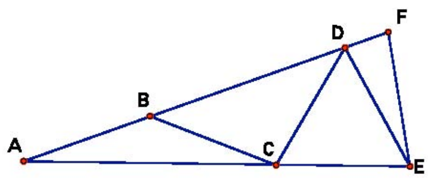


2. If the rectangle has area 2268 cm^2, find the value of x.

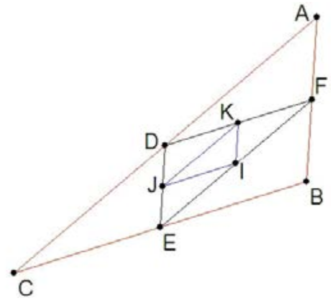


3. Triangle ABC is similar to triangle DEF. AB = 8 cm, BC =12 cm, CA = 16 cm, DE = 12cm, EF =18 cm, FD =24 cm. If the measure of angle A is 47˚, angle B is 104˚, what is the measure of angle E?

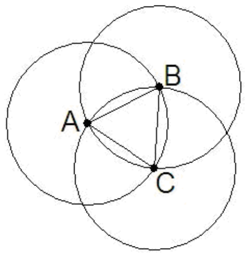
4. In the triangle shown here, AB = BC = CD = DE = EF and AE = AF. If the measure of $\angle BAC$ is 20˚, what is the measure of $\angle DEF$?



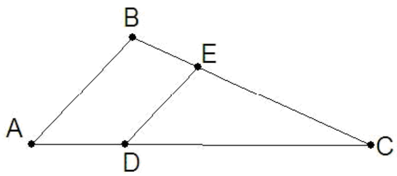
5. In the picture below, points D, E, F, I, J and K are midpoints. If the area of ΔABC is 144 square units, what is the area ΔKIJ?



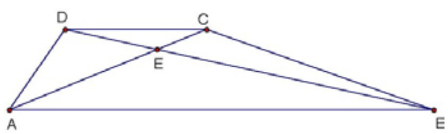
6. Each of the three circles pictured below has a radius of 2 units. Points A, B and C are each the center of one of the circles as pictured. What is the area of triangle ABC?



7. Triangles ABC and DEC are similar triangles. BE = 8 units, AD = 10 units, EC = 20 units.
Find AC.



8. In trapezoid ABCD shown below, AB is parallel to DC and AB =3CD. E is the intersection of the two diagonals of the trapezoid. What is the ratio between the area of triangle ABE and the area of triangle CDE?



9. In trapezoid ABDE, the distance between E and F is 3 units, the distance between F and H is 1.5 units and the distance between H and D is 1 unit. $\overline{ED}$ is parallel to $\overline{AB}$. $\overline{FA}$ and $\overline{HB}$ are both perpendicular to $\overline{ED}$. What is the ratio of the area of triangle AFE to trapezoid ABDE?