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Derivatives (2)**1. Differentiate:**

a. $f(x) = \frac{3x+1}{x-1}$	b. $f(x) = \frac{x^2-1}{x^2+1}$
c. $f(x) = \sqrt{3x^2 - 1}$	d. $f(x) = \sqrt{\frac{x}{x+2}}$
e. $f(x) = (5x - 1)^3(x^2 + 1)^5$	f. $f(x) = \left(\frac{x-1}{x+1}\right)^4$

2.

- Suppose that $h(x)$ is an odd, differentiable function. Show that $h'(x)$ is an even function.
- Suppose that $g(x)$ is an even, differentiable function. Show that $g'(x)$ is an odd function.

3. Cotangent Curves

Two curves which both pass through the point P are said to be cotangent at P if they have the same tangent line at P . For what values of the constants a and b are the curves $C_1: y = \frac{8}{x} - 2$, and $C_2: y = x^2 + ax + b$ cotangent at the point P at which $x=2$?

4. Suppose that f is a function satisfying the equation $f(x + y) = f(x) + f(y)$ for all real x and y , and that $f'(0) = c$, a constant.

i. Show that $f(0) = 0$, and hence that $f'(0) = \lim_{h \rightarrow 0} \frac{f(h)}{h}$

ii. Use the definition of the derivative, $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$, and the given equation for

$f(x + y)$, to show that $f'(x) = c$ for all real x , i.e., that the graph of $y = f(x)$ has slope c for all real x .

5. Find the equation of the normal line to the graph of the given function at the given point:

$$f(x) = -2x + 2x^2 ; P(1, 0)$$

6. Find $f^{(n)}(x)$ for the following functions

a. $f(x) = x^3 + x^2 + x + 1$

b. $f(x) = \frac{1}{x-a}$

7.

- a. Let $f(x) = x|x|$. Find $f'(0)$, or show that it does not exist.
- b. Let $f(x) = |x|$, and show that $f'(x) = \frac{x}{|x|}$.
- c. Consider the function $f(x) = ||x|-1|$.
 - i. Sketch the graph of $y = |x|-1$.
 - ii. Use your graph in part i) to sketch the graph of $y = f(x)$.
 - iii. Find an explicit piecewise definition of $f(x)$.
 - iv. Indicate any points where $f'(x)$ does not exist, giving reasons. Find an explicit expression for $f'(x)$ wherever it does exist. (It will be a piecewise-defined derivative function.)

8. Find the points on the curve $y = x^3 - x^2 - x + 1$ where the tangent lines are horizontal.

9. Given $y = x^3 - 8x + 7$ where $x = f(t)$. If $f(0) = 3$ and $f'(0) = 2$, what is the value of $\frac{dy}{dt}$

when $t = 0$?

10. Determine the slope of the tangent line to the curve $x^2 + 2xy - y^3 = 0$ at $(1, -1)$.