Chapter 3 Polynomials 1

1. Highest Common Factor

The **highest common factor (HCF)** of two numbers (or expressions) is the largest number (or expression) which is a factor of both.

Consider the highest common factor of 24 and 40.

$24 = 2 \times 12 = 4 \times 6 = 8 \times 3 = 2 \times 2 \times 2 \times 3$

$40 = 2 \times 20 = 4 \times 10 = 8 \times 5 = 2 \times 2 \times 2 \times 5$

The common factors of 24 and 40 are 2, 4, and 8. So, the highest common factor is 8.

Note: The highest common factor is the product of the common prime factors.

$\therefore \text{HCF} = 2 \times 2 \times 2 = 8$

The **highest common factor of algebraic expressions** is useful in factorisation; and it is the product of the common prime factors, which includes both common numerical and algebraic factors.

Example 1

Find the highest common factor of $4a^2c^2$, $6ab^3cp$ and $2a^3bc^3p^2$.

Solution:

$$4a^2c^2 = 2 \times 2 \times a \times a \times c \times c$$
$$6ab^3cp = 2 \times 3 \times a \times b \times b \times b \times c \times p$$
$$2a^3bc^3p^2 = 2 \times a \times a \times a \times b \times c \times c \times c \times p \times p$$
$$\therefore \text{HCF} = 2 \times a \times c = 2ac \qquad \{\text{Product of common factors}\}$$

Example 2

Find the highest common factor of:

a. $10a^2bc$, $15ab^2c$, $25abc^2$

b. $14x^2y^3z^3$, $21x^3y^3z$, $28x^5y^2z^2$

Solution:

a. $10a^2bc = 2 \times 5 \times a^2 \times b \times c$

$15ab^2c = 3 \times 5 \times a \times b^2 \times c$

$25abc^2 = 5 \times 5 \times a \times b \times c^2$

$\therefore \text{HCF} = 5 \times a \times b \times c = 5abc \qquad \{\text{Product of common factors}\}$

b. $14x^2y^3z^3 = 2\times 7\times x^2\times y^3\times z^3$

$21x^3y^3z = 3\times 7\times x^3\times y^3\times z$

$28x^5y^2z^2 = 2\times 2\times 7\times x^5\times y^2\times z^2$

$\therefore$ HCF $= 7\times x^2\times y^2\times z \ = 7x^2y^2z$ {Product of common factors}

2. Factorization using the Common Factor

We know that:

$$a(b+c) = ab+ac$$

The reverse process, $ab+ac = a(b+c)$, is called **taking out the common factor**.

Consider the factorisation of the expression $7x+35$.

Clearly, $7x = 7\times x$ $\quad$ $35 = 5\times 7$

$\therefore$ HCF $= 7$

Thus $7x+35 = 7\times x+5\times 7 = 7(x+5)$

Note that the common factor of 7 has been taken out and placed in front of the brackets. The expression inside the brackets is obtained by dividing each term by 7.

In general:
To factorize an algebraic expression, take out the highest common factor and place it in front of the brackets. Then the expression inside the brackets is obtained by dividing each term by the highest common factor.

Example 1: Factor the following.

a. $16x+40$

b. $ux+vx$

c. x^2-x

d. $12x^2y-8xy^2$

e. $-6x-12y+15z$

Solution:

a. $16x+40$ $\quad$ {HCF $=8$}

$= 8(2x+5)$

b. $ux+vx$ $\quad$ {HCF $= x$}

$= x(u+v)$

c. $x^2 - x$ $\{\text{HCF} = x\}$

$= x(x-1)$

d. $12x^2y - 8xy^2$ $\{\text{HCF} = 4xy\}$

$= 4xy(3x - 2y)$

e. $-6x - 12y + 15z$ $\{\text{HCF} = -3\}$

$= -3(2x + 4y - 5z)$

Note: The answer is neater if the first term in the bracket is positive. So, we use –3 as the common factor rather than 3.

Example 2: Factor the following.

a. $(x+3)^2 - x - 3$

b. $(x-5)^2 - 2x + 10$

c. $25a^2(a-6) - 10a(a-6)^2$

Solution:

a. $(x+3)^2 - x - 3$

It seems to have no common factor. However, if -1 is taken out of the last two terms we can factorise the expression.

$$
\begin{aligned}
(x+3)^2 - x - 3 &= (x+3)^2 - (x+3) \\
&= (x+3)(x+3-1) \\
&= (x+3)(x+2)
\end{aligned}
$$

b.
$$
\begin{aligned}
(x-5)^2 - 2x + 10 &= (x-5)^2 - 2(x-5) \\
&= (x-5)(x-5-2) \\
&= (x-5)(x-7)
\end{aligned}
$$

c. $25a^2(a-6) - 10a(a-6)^2$ $\{\text{HCF} = 5a(a-6)\}$

$= 5a(a-6)[5a - 2(a-6)]$

$= 5a(a-6)[5a - 2a + 12]$

$= 5a(a-6)(3a + 12)$

$= 15a(a-6)(a+4)$

3. Factor by Grouping

Here are the steps required for factoring by grouping:

Step 1: Decide if the four terms have anything in common, called the greatest common factor or GCF. If so, factor out the GCF. Do not forget to include the GCF as part of your final answer.

Step 2: Create smaller groups within the problem, usually done by grouping the first two terms together and the last two terms together.

Step 3: Factor out the GCF from each of the two groups. In the second group, you have a choice of factoring out a positive or negative number. To determine whether you should factor out a positive or negative number, you need to look at the signs before the second and fourth terms. If the two signs are the same (both positive or both negative) you need to factor out a positive number. If the two signs are different, you must factor out a negative number.

Step 4: If the factors inside of the parenthesis are exactly the same, it is time for the 2 for 1 special. The one thing that the two groups have in common should be what is in parenthesis, so you can factor out what is inside the parenthesis, but only write what is inside the parenthesis once. If what is inside the parenthesis does not match, you need to rearrange the four terms and try again until you get a perfect match. If you have rearranged the problems a couple of times and still have not found a perfect match, then the problem does not factor.

Step 5: Determine if the remaining factors can be factored any further.

Example 1: Factor: $x^3 - 5x^2 + 3x - 15$

Step 1: Decide if the four terms have anything in common, called the greatest common factor or GCF. If so, factor out the GCF. Do not forget to include the GCF as part of your final answer. In this case, the four terms only have a 1 in common which is of no help.	$x^3 - 5x^2 + 3x - 15$
Step 2: Create smaller groups within the problem, usually done by grouping the first two terms together and the last two terms together.	$\underline{x^3 - 5x^2} + \underline{3x - 15}$
Step 3: Factor out the GCF from each of the two groups. In this problem, the signs in front of the $5x^2$ and the 15 are different, so you need to factor out a positive 3.	$\underline{x^2}(x-5) + \underline{3}(x-5)$
Step 4: Notice that what is inside the parenthesis is a perfect match, so it is time for the 2 for 1 special. The one thing that the two groups have in common is (x – 5), so you can factor out (x – 5) leaving the following:	$(x-5)(x^2+3)$ or $(x^2+3)(x-5)$

Step 5: Determine if any of the remaining factors can be factored further. In this case they cannot so the final answer is:	$(x-5)(x^2+3)$ or $(x^2+3)(x-5)$

Example 2: Factor: $4x^2+20x-3xy-15y$

Step 1: Decide if the four terms have anything in common, called the greatest common factor or GCF. If so, factor out the GCF. Do not forget to include the GCF as part of your final answer. In this case, the four terms only have a 1 in common which is of no help.	$4x^2+20x-3xy-15y$
Step 2: Create smaller groups within the problem, usually done by grouping the first two terms together and the last two terms together.	$\underline{4x^2+20x}-\underline{3xy-15y}$
Step 3: Factor out the GCF from each of the two groups. In this problem, the signs in front of the 20x and the 15y are different, so you need to factor out a negative 3y.	$\underline{4x}(x+5)\underline{-3y}(x+5)$
Step 4: Notice that what is inside the parenthesis is a perfect match, so it is time for the 2 for 1 special. The one thing that the two groups have in common is (x + 5), so you can factor out (x + 5) leaving the following:	$(x+5)(4x-3y)$ or $(4x-3y)(x+5)$
Step 5: Determine if any of the remaining factors can be factored further. In this case they cannot so the final answer is:	$(x+5)(4x-3y)$ or $(4x-3y)(x+5)$

Example 3: Solve: $3x^3-6x^2+15x-30$

Step 1: Decide if the four terms have anything in common, called the greatest common factor or GCF. If so, factor out the GCF. Do not forget to include the GCF as part of your final answer. In this case, the four terms have a 3 in common, which leaves:	$3(x^3-2x^2+5x-10)$
Step 2: Create smaller groups within the problem, usually done by grouping the first two terms together and the last two terms together.	$3(\underline{x^3-2x^2}+\underline{5x-10})$

Step 3: Factor out the GCF from each of the two groups. In this problem, the signs in front of the $2x^2$ and the 10 are the same, so you need to factor out a positive 5.	$3(\underline{x^2}(x-2)\underline{+5}(x-2))$
Step 4: Notice that what is inside the parenthesis is a perfect match, so it is time for the 2 for 1 special. The one thing that the two groups have in common is (x – 2), so you can factor out (x – 2) leaving the following:	$3(x-2)(x^2+5)$ or $3(x^2+5)(x-2)$
Step 5: Determine if any of the remaining factors can be factored further. In this case they cannot so the final answer is:	$3(x-2)(x^2+5)$ or $3(x^2+5)(x-2)$

Example 4: Solve: $x^2+ab-ax-bx$

Step 1: Decide if the four terms have anything in common, called the greatest common factor or GCF. If so, factor out the GCF. Do not forget to include the GCF as part of your final answer. In this case, the four terms only have a 1 in common which is of no help.	$x^2+ab-ax-bx$
Step 2: Create smaller groups within the problem, usually done by grouping the first two terms together and the last two terms together.	$\underline{x^2+ab}-\underline{ax-bx}$
Step 3: Factor out the GCF from each of the two groups. In this problem, the signs in front of the ab and the bx are different, so you need to factor out a negative x.	$1(x^2+ab)-x(a+b)$
Step 4: Notice that what is inside the parenthesis is not a perfect match, so you need to rearrange the four terms and try again. When you rearrange the terms, try to follow a pattern when you write them down. In this case notice that the first terms has an x, the second an a, the third an x, and the fourth an a.	$\underline{x^2-ax}-\underline{bx+ab}$
Step 5: Factor out the GCF from each of the two groups. In this problem, the signs in front of the ax and the ab are different, so you need to factor out a negative b.	$\underline{x}(x-a)\underline{-b}(x-a)$

Step 6: Notice that what is inside the parenthesis is a perfect match, so it is time for the 2 for 1 special. The one thing that the two groups have in common is (x – a), so you can factor out (x – a) leaving the following:	$(x-a)(x-b)$ or $(x-b)(x-a)$
Step 7: Determine if any of the remaining factors can be factored further. In this case they cannot so the final answer is:	$(x-a)(x-b)$ or $(x-b)(x-a)$

Example 5 – Solve: $x^3+2x^2-9x-18$

Step 1: Decide if the four terms have anything in common, called the greatest common factor or GCF. If so, factor out the GCF. Do not forget to include the GCF as part of your final answer. In this case, the four terms only have a 1 in common which is of no help.	$x^3+2x^2-9x-18$
Step 2: Create smaller groups within the problem, usually done by grouping the first two terms together and the last two terms together.	$\underline{x^3+2x^2}-\underline{9x-18}$
Step 3: Factor out the GCF from each of the two groups. In this problem, the signs in front of the $2x^2$ and the 18 are different, so you need to factor out a negative 9.	$\underline{x^2}(x+2)\underline{-9}(x+2)$
Step 4: Notice that what is inside the parenthesis is a perfect match, so it is time for the 2 for 1 special. The one thing that the two groups have in common is (x + 2), so you can factor out (x + 2) leaving the following:	$(x+2)(x^2-9)$ or $(x^2-9)(x+2)$
Step 5: Determine if any of the remaining factors can be factored further. In this case one of the factors is a difference of square, which can be factored into:	$(x+2)(x+3)(x-3)$ or $(x+3)(x-3)(x+2)$

4. Quadratic Trinomials

An expression ax^2+bx+c, where a, b and c are constants and $a \neq 0$, is called a **quadratic trinomial** in x. It has three terms which are an x^2 term, an x term and an independent (i.e. no x) term.

E.g. x^2+5x+6, x^2-5x+6, x^2-x-6, x^2+x-12, $2x^2+7x+3$ etc. are quadratic trinomials.

Factors of Quadratic Trinomials of the Type x^2+bx+c – *Simple Trinomials*

The Distributive Law is used in reverse to factorize a quadratic trinomial, as illustrated below.

Consider the expansion of $(x+2)(x+3)$.

$$\begin{aligned}(x+2)(x+3) &= x(x+3)+2(x+3)\\ &= x^2+3x+2x+6\\ &= x^2+5x+6\end{aligned}$$

We notice that:
5, the coefficient of *x*, is the sum of 2 and 3.
6, the independent term, is the product of 2 and 3.

This suggests that to factorise the quadratic trinomial x^2+5x+6, we need to find two numbers whose sum is 5 (the coefficient of x); but whose product is 6 (the independent term).

Two such numbers are 2 and 3 because $2+3=5$ and $2\times 3=6$.
$\therefore\ x^2+5x+6=(x+2)(x+3)$

Note: The product of two linear factors yields a quadratic trinomial; and the factors of a quadratic trinomial are linear factors.

Now consider the expansion of $(x+a)(x+b)$.

$$\begin{aligned}(x+a)(x+b) &= x(x+b)+a(x+b)\\ &= x^2+xb+ax+ab\\ &= x^2+x(a+b)+ab\end{aligned}$$

Factors of last term

$a\times b$

$$\therefore\ x^2+(a+b)x+ab=(x+a)(x+b)$$

$x\times x$

Factors of first term

Coefficient of $x=a+b=$ Sum of a and b.
Independent term $=ab=$ Product of a and b.

In general:

To factorise a quadratic trinomial, find two numbers whose sum is equal to the coefficient of x; and whose product is equal to the independent term.

E.g. To factorise $x^2+9x+18$, find two numbers whose sum is 9 (the coefficient of x), and whose product is 18 (the independent term). Two such numbers are 3 and 6 because $3+6=9$ and $3\times 6=18$.

$\therefore\ x^2+9x+18=(x+3)(x+6)$

Example 1: Factor the following

a. y^2-5y+4

b. p^2-p-6

Solution:

a. y^2-5y+4

Coefficient of $y=-5$

Independent term $=4$

Find two numbers whose sum is -5 (coefficient of y), and whose product is 4 (independent term).

Two such numbers are -1 and -4 because $-1-4=-5$ and $-1\times -4=4$.

$\therefore\ y^2-5y+4=(y-1)(y-4)$

b. p^2-p-6

Coefficient of $p=-1$

Independent term $=-6$

Find two numbers whose sum is -1 (coefficient of p), and whose product is -6 (independent term).

Two such numbers are -3 and $+2$ because $-3+2=-1$ and $-3\times 2=-6$.

$\therefore\ p^2-p-6=(p-3)(p+2)$

Check:

$$
\begin{aligned}
(p-3)(p+2) &= p(p+2)-3(p+2)\\
&= p^2+2p-3p-6\\
&= p^2-p-6
\end{aligned}
$$

Example 2: Factor $x^2+3x-10$.

Solution. The binomial factors will have this form: $(x\quad a)(x\quad b)$

What are the factors of 10?
They are (1, 10), (2, 5).
Since the product is -10, one of the factor should be negative.
Since the sum is 3, we will choose +5 and -2.

$x^2 + 3x - 10 = (x - 2)(x + 5)$.

Note: When 1 is the coefficient of x^2, the order of the factors does not matter.
$(x - 2)(x + 5) = (x + 5)\ (x - 2)$.

Example 3: Factor $x^2 - x - 12$.

Solution. We must find factors of -12 whose algebraic sum will be the coefficient of x : -1.
Choose -4 and $+ 3$. (-4) x 3 = -12 and (-4) + 3 = -1
$x^2 - x - 12 = (x - 4)(x + 3)$.

Practice in class: Factor. Again, the order of the factors does not matter.

a) $x^2 + 5x + 6 = (x + 2)(x + 3)$
b) $x^2 - x - 6 = (x - 3)(x + 2)$
c) $x^2 + x - 6 = (x + 3)(x - 2)$
d) $x^2 - 10x + 9 = (x - 1)(x - 9)$
e) $x^2 + x - 12 = (x + 4)(x - 3)$
f) $x^2 - 6x - 16 = (x - 8)(x + 2)$

Example 4: Factor completely $6x^8 + 30x^7 + 36x^6$.

Solution. To factor completely means to first remove any common factors

$6x^8 + 30x^7 + 36x^6 = 6x^6(x^2 + 5x + 6)$.

Now continue by factoring the trinomial:

$= 6x^6(x + 2)(x + 3)$.

Practice in class: Factor completely. First remove any common factors.

a) $x^3 + 6x^2 + 5x$ **=** $\boldsymbol{x(x^2 + 6x + 5) = x(x + 5)(x + 1)}$
b) $x^5 + 4x^4 + 3x^3$ **=** $\boldsymbol{x^3(x^2 + 4x + 3) = x^3(x + 1)(x + 3)}$

Example 5: Factor by making the leading term positive.
$-x^2 + 5x - 6 = -(x^2 - 5x + 6) = -(x - 2)(x - 3)$.

Practice in class. Factor by making the leading term positive.
$-x^2 - 2x + 3 = -(x^2 + 2x - 3) = -(x + 3)(x - 1)$

5. Quadratics in different arguments

Here is the *form* of a quadratic trinomial with **argument *x***: $ax^2 + bx + c$.

The **argument** is whatever is being squared. x is being squared. x is called the **argument.** The argument appears in the middle term.

Now here is a quadratic whose argument is x^3: $3x^6 + 2x^3 - 1$.
x^6 is the *square* of x^3.

But that quadratic has the same constants as $3x^2 + 2x - 1$. In a sense, it is the same quadratic only with a different argument. For it is the constants that distinguish a quadratic.

Now, since the quadratic with argument x can be factored in this way:
$3x^2 + 2x - 1 = (3x - 1)(x + 1)$, then the quadratic with argument x^3 is factored in the same way:
$3x^6 + 2x^3 - 1 = (3x^3 - 1)(x^3 + 1)$.

Whenever a quadratic has constants 3, 2, −1, then for *any* argument, the factoring will be (3 times the argument − 1)(argument + 1).

Example 1

$$z^2 - 3z - 10 = (z + 2)(z - 5).$$
$$x^8 - 3x^4 - 10 = (x^4 + 2)(x^4 - 5).$$

The trinomials on the left have the same constants 1, −3, −10 but different arguments. That is the only difference between them. In the first, the argument is z. In the second, the argument is x^4.

(The square of x^4 is x^8.) Each quadratic is *factored* as (argument + 2)(argument − 5).

Every quadratic with constants 1, −3, −10 will be factored that way.

Practice in class:

a) Write the form of a quadratic trinomial with argument z. Answer: $\boldsymbol{az^2 + bz + c}$

b) Write the form of a quadratic trinomial with argument x^4. Answer: $\boldsymbol{ax^8 + bx^4 + c}$

c) Write the form of a quadratic trinomial with argument x^n. Answer: $\boldsymbol{ax^{2n} + bx^n + c}$

Practice in class: Multiply out each of the following, which have the same constants, but different argument.

a) $(z + 3)(z - 1) = \boldsymbol{z^2 + 2z - 3}$ b) $(y + 3)(y - 1) = \boldsymbol{y^2 + 2y - 3}$

Practice in class: Factor each quadratic trinomials.

a) $x^4 - x^2 - 2 = \boldsymbol{(x^2 - 2)(x^2 + 1)}$

b) $y^6 + 2y^3 - 8 = \mathbf{(y^3 + 4)(y^3 - 2)}$

6. The Difference of Two Squares

When the sum of two numbers multiplies their difference -- $(a + b)(a - b)$
-- then the product is the difference of their squares: $\mathbf{(a + b)(a - b) = a^2 - b^2}$

Symmetrically, the difference of two squares can be *factored*:
$x^2 - 25 = (x + 5)(x - 5)$
x^2 is the square of x. 25 is the square of 5.

Example 1: Multiply $(x^3 + 2)(x^3 - 2)$.

Recognize the form: $(a + b)(a - b)$. The product will be the difference of two squares:
$(x^3 + 2)(x^3 - 2) = x^6 - 4$.

x^6 is the square of x^3. 4 is the square of 2.

When confronted with the form $(a + b)(a - b)$, the student should *not* do the FOIL method. The student should recognize immediately that the product will be $a^2 - b^2$.

Also, the order of factors never matters:
$(a + b)(a - b) = (a - b)(a + b) = a^2 - b^2$.

Example 2: Factor the following.

a. $p^2 - 64$ c. $x^2 - 36y^2$
b. $121 - y^2$ d. $(x-2)^2 - 49$

Solution:

a. $p^2 - 64$
$= p^2 - 8^2$
$= (p+8)(p-8)$

b. $121 - y^2$
$= 11^2 - y^2$
$= (11+y)(11-y)$

c. $x^2 - 36y^2$
$= x^2 - (6y)^2$
$= (x+6y)(x-6y)$

d. $(x-2)^2 - 49$
$= (x-2)^2 - 7^2$
$= (x-2+7)(x-2-7)$
$= (x+5)(x-9)$

Practice in class. Factor.

a) $x^2 - 100y^2 = (x + 10y)(x - 10y)$

b) $4y^2 - 1 = (2y + 1)(2y - 1)$

c) $x^4 - y^4 = (x^2 + y^2)(x^2 - y^2)$
$= (x^2 + y^2)(x + y)(x - y)$

d) $xy^2 - xz^2 = x(y^2 - z^2)$
$= x(y + z)(y - z)$

Practice in class:
(1) $x^2 - 144 = (x + 12)(x - 12)$ (2) $9 - y^2 = (3 - y)(3 + y)$
(3) $1 - 4x^2 = (1 + 2x)(1 - 2x)$
(4) $x^{12} - y^{12} = (x^6 + y^6)(x^6 - y^6) = (x^6 + y^6)(x^3 + y^3)(x^3 - y^3)$

7. The Square of a Binomial

The square of a binomial come up so often that the student should be able to write the final product immediately. It will turn out to be a very specific trinomial.
To see that, let us square the binomial $(x + 5)$:
$(x + 5)^2 = (x + 5)(x + 5) = x^2 + 10x + 25$.

$x^2 + 10x + 25$ is called a perfect square trinomial. It is the square of a binomial. To see what happens when we square any binomial, let us square $(a + b)$:
$(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$

The square of any binomial produces the following three terms:
1. The square of the first term of the binomial: a^2
2. Twice the product of the two terms: $2ab$
3. The square of the second term: b^2

The square of every binomial -- every perfect square trinomial -- has that form: $a^2 + 2ab + b^2$. To recognize that is to know an important product in the "multiplication table" of algebra.

Example 1: Square the binomial $(x + 6)$.

Solution. $(x + 6)^2 = x^2 + 12x + 36$
1. x^2 is the square of x.
2. $12x$ is twice the product of $x \cdot 6$. ($x \cdot 6 = 6x$. Twice that is $12x$.)
3. 36 is the square of 6.

Example 2: Square the binomial $(3x - 4)$.

Solution. $(3x - 4)^2 = 9x^2 - 24x + 16$
1. $9x^2$ is the square of $3x$.
2. $-24x$ is twice the product of $3x \cdot -4$. ($3x \cdot -4 = -12x$. Twice that is $-24x$.)
3. 16 is the square of -4.

Note: If the binomial has a minus sign, then the minus sign appears only in the middle term of the trinomial.

Therefore, using the double sign ± ("plus or minus"), we can state the rule as follows:

$(a \pm b)^2 = a^2 \pm 2ab + b^2$

This means: If the binomial is $a + b$, then the middle term will be $+2ab$; but if the binomial is $a - b$, then the middle term will be $-2ab$

Example 3: $(5x^3 - 1)^2 = 25x^6 - 10x^3 + 1$

1. $25x^6$ is the square of $5x^3$.
2. $-10x^3$ is twice the product of $5x^3 \cdot -1$. ($5x^3 \cdot -1 = -5x^3$. Twice that is $-10x^3$.)
3. 1 is the square of -1.

Example 4: Is this a perfect square trinomial: $x^2 + 14x + 49$?

Answer. Yes. It is the square of $(x + 7)$.
x^2 is the square of x. 49 is the square of 7. And $14x$ is twice the product of $x \cdot 7$.
In other words, $x^2 + 14x + 49$ could be *factored* as $x^2 + 14x + 49 = (x + 7)^2$

Note: If the coefficient of x had been any number but 14, this would not have been a perfect square trinomial.

Example 5: Is this a perfect square trinomial: $x^2 + 50x + 100$?

Answer. No, it is not. Although x^2 is the square of x, and 100 is the square of 10, $50x$ is not twice the product of $x \cdot 10$. (Twice their product is $20x$.)

Example 6: Is this a perfect square trinomial: $x^8 - 16x^4 + 64$?

Answer. Yes. It is the perfect square of $x^4 - 8$.

Practice in class

1. Expand

a) $(x + 8)^2 = x^2 + 16x + 64$
b) $(r + s)^2 = r^2 + 2rs + s^2$
c) $(x + 1)^2 = x^2 + 2x + 1$
d) $(x - 1)^2 = x^2 - 2x + 1$
e) $(2x + 1)^2 = 4x^2 + 4x + 1$
f) $(3x - 2)^2 = 9x^2 - 12x + 4$

2. Factor if possible.

a) $p^2 + 2pq + q^2 = (p + q)^2$
b) $x^2 - 4x + 4 = (x - 2)^2$
c) $x^2 + 6x + 9 = (x + 3)^2$
d) $25x^2 + 30x + 9 = (5x + 3)^2$
e) $4x^2 - 28x + 49 = (2x - 7)^2$
f) $4r^2 + 28r + 49 = (2r + 7)^2$