Chapter 4 Linear and Non–Linear Relations (2)

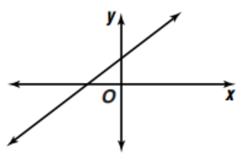
1. Linear Relations

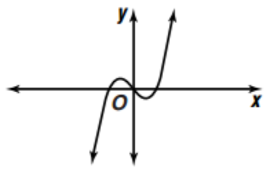
Graphically, a LINEAR RELATION is a relation between two variables which creates a straight line when graphed on a Cartesian plane. Otherwise, it is **NON-LINEAR**.

Algebraically, the equation of a linear line is always in the form of "y = mx + b", and has a degree of 1. The equation of a non-linear line has a degree of any number, other than one.

Example: Determine the follow as linear or non-linear.

1. $y = 5$ **2.** $2x + 3y = 10$ **3.** $y = 7x^2$

4. 

5. 

Answer: 1. Linear 2. Linear 3. Non-linear 4. Linear 5. Non-linear

Consider the following sentence: The cost (in dollars) of buying pens is equal to ten times the number of pens bought.

If *c* represents the cost in dollars and *p* represents the number of pens bought, then this sentence can be expressed mathematically as

$c = 10p$ where $p \in N$

When $p = 1$, $c = 10 \times 1 = 10$ When $p = 2$, $c = 10 \times 2 = 20$

When $p = 3$, $c = 10 \times 3 = 30$ When $p = 4$, $c = 10 \times 4 = 40$

Thus the mathematical sentence $c = 10p$ relates the values of c to the values of p.
It defines a binary relation on the natural numbers.
The ordered (p, c) pairs $(1, 10)$, $(2, 20)$, $(3, 30)$, $(4, 40)$ etc. belong to the relation defined by $c = 10p$.

This suggests the following definition:

A **relation** is a set of ordered pairs, and is usually defined by a rule.
In the above example, $\{(1,10), (2, 20), (3, 30), (4, 40), \ldots\}$ is a relation and it can be described by the rule $c = 10p$, where $p \in N$.

Domain - The domain of a relation is the set of all first elements (usually x values) of its ordered pairs. In the example discussed, the domain = $\{p \mid p = 1, 2, 3, 4, \ldots\}$ or N – natural numbers.

Range - The range of a relation is the set of all second elements (usually *y* values) of its ordered pairs. In the example discussed, the range $=\{c \mid c = 10, 20, 30, 40, ...\}$.

Note: The graph of *c* against *p* is **discrete** because *p* is an element of the set of natural numbers. The values of *c* depend upon *p*. So, we say that *p* is an **independent variable** and *c* is a **dependent variable.**

Example 1: State the domain and range of the following relations:

a. $\{(1,1), (2,4), (3,9), (4,16), (5,25)\}$
b. $\{(0,8), (2,12), (4,16), (6,20), (8,24)\}$

Solution:

a. Domain $= \{1,2,3,4,5\}$
Range $= \{1,4,9,16,25\}$

b. Domain $= \{0,2,4,6,8\}$
Range $= \{8,12,16,20,24\}$

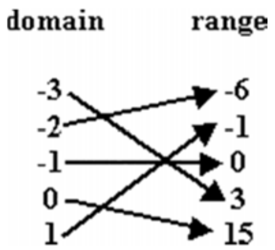
Functions - A relation is said to be a function if each element of the domain determines exactly one element of the range. i.e. there can only be ONE y-value for each x-value.

For example, the relation $c = 10p$, where $p \in N$, is a function since each element of the domain determines exactly one element of the range.

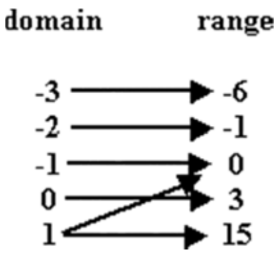
Algebraic Test: If a relation is given as an equation, and the substitution of any value for x results in one and only one value of y, we have a function.

Geometric Test: is by using the **"Vertical Line Test".** Given the graph of a relation, if you can draw a vertical line that crosses the graph in more than one place, then the relation is not a function.

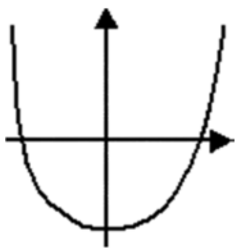
Example 2: Determine if the follow relation is a function.

a) 

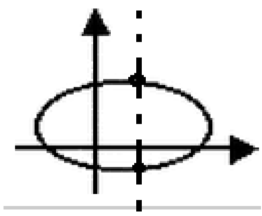
a) This is a function. You can tell by tracing from each *x* to each *y*. There is only one *y* for each *x*; there is only one arrow coming from each *x*.

b) 

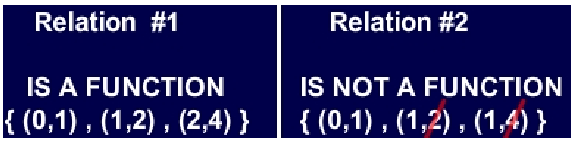
b) This one is not a function: there are *two* arrows coming from the number 1; the number 1 is associated with two *different* range elements. So this is a relation, but it is not a function.

c) 

c) This graph shows a function, because there is no vertical line that will cross this graph twice.

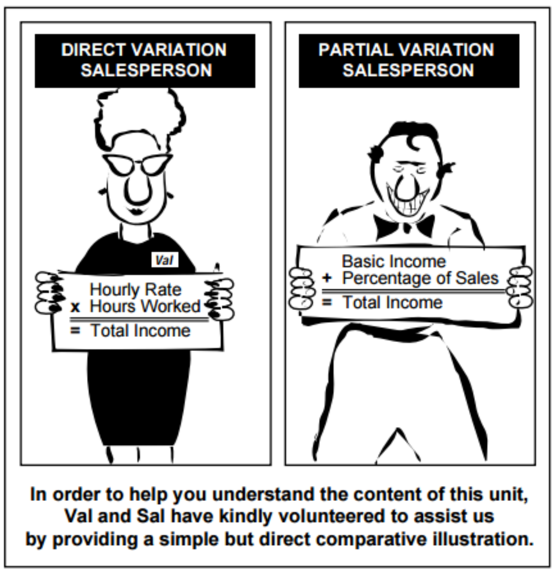
d) 

d) This graph does not show a function, because any number of vertical lines will intersect this oval twice. For instance, the*y*-axis intersects (crosses) the line twice.

e) 

Since relation #1 has ONLY ONE y value for each x value, this relation is a function.

On the other hand, relation #2 has TWO distinct y values '2' and '4' for the same x value of '1'. Therefore, relation #2 does not satisfy the definition of a function. It is a relation.

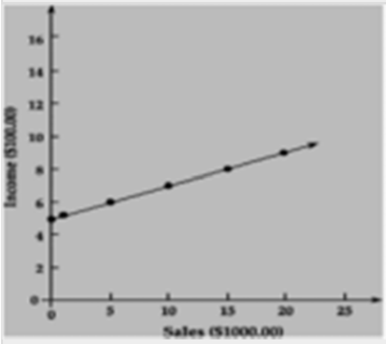
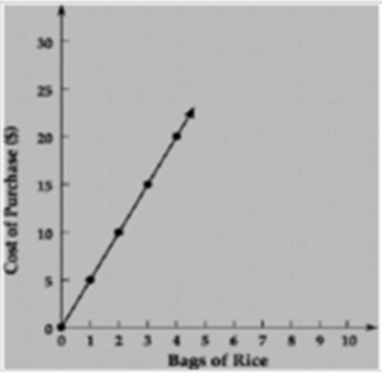


2. Direct vs. Partial Variation

Direct Variation - A relationship between two variables in which one variable is a constant multiple of the other. When graphing the line DOES pass through the origin.
Represented by $y = mx$ form
X and Y values vary directly with each other

Partial Variation - A relationship between two variables in which one variable is a constant multiple of the other plus a constant value. Graph DOES NOT pass through the origin.
Represented by y = mx + b form
X and Y values don't vary directly with each other

Characteristics

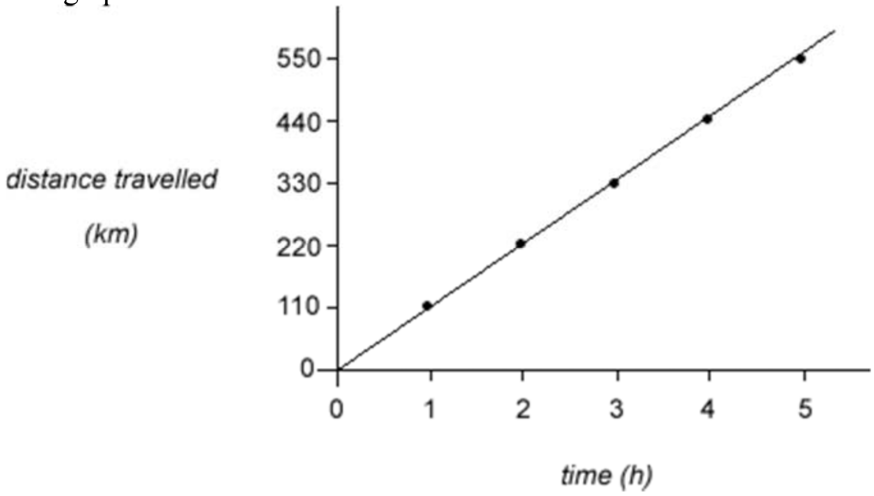
Direct Variation	**Partial Variation**
straight line	straight line
constant of variation	constant of variation
no fixed cost	fixed cost
x and y values	x and y values
starts at origin (0,0)	starts anywhere but origin
y=mx	y=mx+b
	

Example 1: While driving down a highway at a constant speed, the following times and distances were recorded:

Time (h)	*Distance Travelled (km)*
1	110
2	220
3	330
4	440
5	550

a. The relationship between the time spent driving and the distance traveled can also be expressed as the following function: $D = 110t$

b. Sketch the graph of this function.



Example 2: A medium pizza costs \$7 plus \$1.50 per topping.

a) Identify the fixed cost and the variable cost.

The fixed cost is \$7 and the variable cost is \$1.50

b) Determine the equation relating cost, C, in dollars and the number of toppings, n.

C = 7 + 1.5n

c) Use the equation to determine the cost of a medium pizza with 6 toppings.

n = 6
C = 7 + 1.5(6) = \$16

d) Graph this partial variation relation.