Interference of Light

Unit 5: The Wave Nature of Light

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Grade 12 Physics
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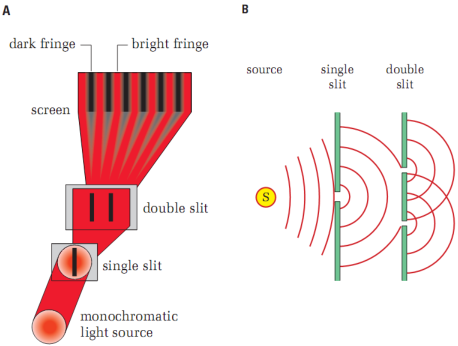
Thomas Young's Double-Slit Experiment

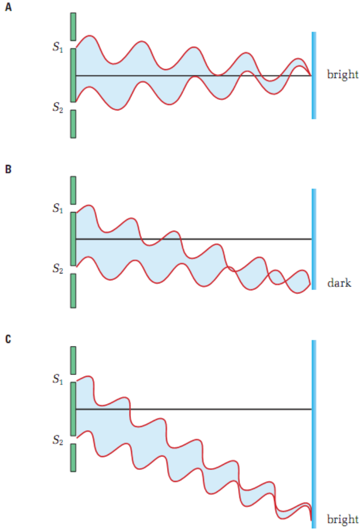
First definitive evidence that light is a wave



- **Monochromatic light** light with a single colour (frequency); the light source can be a laser, LED , or gas lamp (most likely what Young used)
- **Slit:** an opening; also called an **aperture**
- The **screen** far away from the slits is also called the **projection**

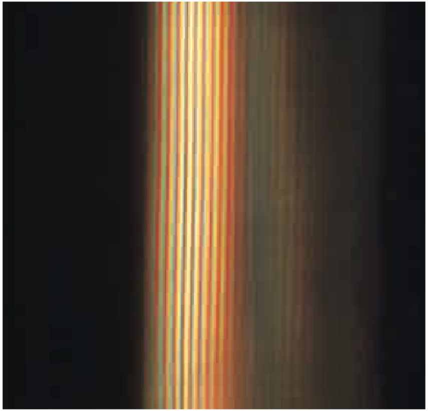
Double-slit experiment showed that light causes interference, just like any other wave

Thomas Young's Double-Slit Experiment



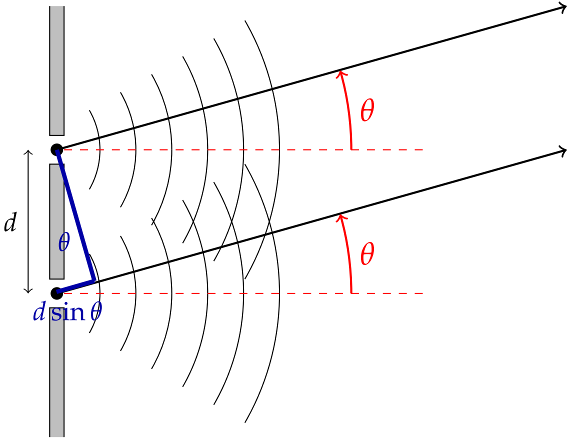
- At **A**, the path from slits S_1 and S_2 are the same, therefore we have **constructive interference** and the projection is bright
- At **B**, the path from S_1 and S_2 are diffed by half a wavelength, and therefore there is **destructive interference** and the projection is dark
- At **C**, the path from S_1 and S_2 are diffed by one wavelength, and therefore there is **constructive interference** again, and again, the projection is bright

Interference Pattern: Bright and Dark Fringes



The “bright fringes” are from constructive interference; the “dark fringes” are from destructive interference.

Let's Work This Out!



- We have two slits at distance d apart, emitting *coherent* light
- Huygens' principle: light passing through the slits become point sources
- Assume that the screen is far enough from the slits that we can treat the two beams of light from the slits as being parallel
- Using basic geometry, we can see that the path difference from the two slit to the projection is $d\sin\theta$

Double-Slit Interference

Constructive Interference

A **bright fringe** (i.e. constructive interference, anti-nodal point) will happen if the path length difference ($d \sin \theta$) is an integer (n) multiple of wavelength (λ):

$$\boxed{\pm n\lambda = d \sin \theta_n} \qquad n = 0, 1, 2, 3, \ldots$$

Quantity	Symbol	SI Unit
Integer number of full wavelengths	n	(none)
Wavelength of light	λ	m
Distance between slits	d	m
Angle between slit separation and line perpendicular to light rays	θ	(none)

Double-Slit Interference

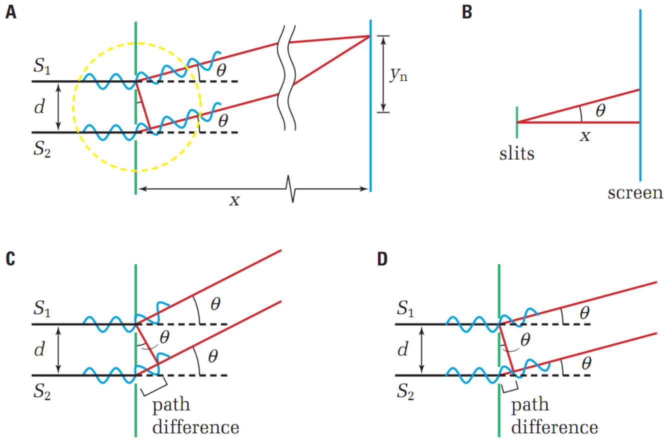
Destructive Interference

Conversely, a **dark fringe** (i.e. destructive interference, nodal point) occurs when the path length difference ($d\sin\theta$) is an half-number ($n+\frac{1}{2}$) multiple of wavelength (λ):

$$\boxed{\pm\left(n+\frac{1}{2}\right)\lambda = d\sin\theta_n} \qquad n = 0, 1, 2, 3, \ldots$$

- As $\sin\theta$ has a maximum value of 1, there is an actual maximum number $n_{\max}$ of bright (or dark) fringes.
- The total number of bright (or dark) fringes is $2n_{\max}+1$

Double-Slit Interference



Approximation of The Wavelength of Light

We can estimate the wavelength of light based on the distances between bright fringes using the **small-angle approximation**, where:

$$\theta \approx \tan\theta \approx \sin\theta$$

The angle θ must be measured in **radians**, not degrees. (As an exercise, compute $\tan\theta$ and $\sin\theta$ for $\theta = 0.001$)

We can already relate the distance from slits to the screen (x), and the distance of the n-th bright fringe from the centre (y_n) to θ_n. Applying the approximation, we have:

$$\tan\theta_n = \frac{y_n}{x} \approx \sin\theta_n$$

Approximation of The Wavelength of Light

Now the Fun Begins

We can substitute our approximation into the constructive interference equation:

$$n\lambda \approx \frac{y_n d}{x}$$

For $n = 0$, $y_0 = 0$ as well, therefore for $n = 1$, $y_1 = y_1 - y_0 = \Delta y$, the distance between bright fringes. The wavelength can be approximated:

$$\boxed{\lambda \approx \frac{\Delta y d}{x}}$$

Approximation of The Wavelength of Light

This equation applies equally to dark fringes as well as bright fringes.

$$\boxed{\lambda \approx \frac{\Delta y d}{x}}$$

Quantity	Symbol	SI Unit
Wavelength	λ	m (metre)
Distance between fringes	Δy	m (metre)
Distance between slits	d	m (metre)
Distance from slits to screen	x	m (metre)

Since the approximation is based on small angles, we generally apply this to Δy close to the centre, where light from both slits are deflected by a small angle.

Important Notes

- We have applied the double-slit problem specifically to light, but it can be applied to any wave (e.g. ocean waves) as well
- The “slits” don’t actually need to be slits; any point source will do
- The projection/screen doesn’t need to be a real screen either; it just has to be a line where wave intensity can be measured

Important Question

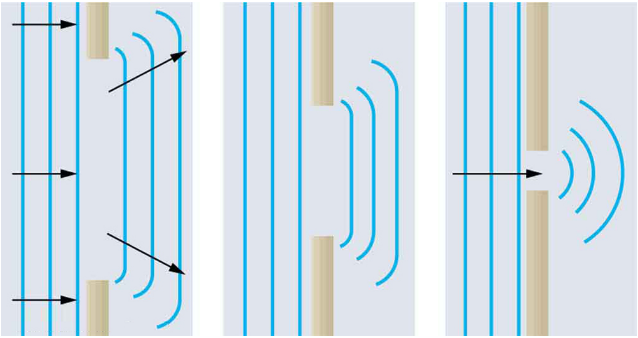
We know that when light passes through the double slit, it spreads out. The single slit also does the same thing, but **why**?

Diffraction of Waves

When a wave goes through an small opening, it **diffracts**. This happens with sound waves, ocean waves. . . and light.

(The photo is from the Port of Alexandria in Egypt. The shape of the entire harbour is created because of diffraction of ocean wave.)

Diffraction of Waves



The smaller the opening (compared to the wavelength of the incoming wave) the greater the diffraction effects.

There are Two Types of Diffraction

- **Fresnel diffraction**
  - “Near-field” diffraction
  - The distance between aperture and the projection is small
  - The short distance to the projection causes the diffraction pattern observed to differ in size and shape
- **Fraunhofer diffraction**
  - “Far-field diffraction”
  - The distance between the aperture and the projection is large
  - Will only focus on this form of diffraction in Physics 12 because the pattern is easier to understand

Fresnel Number

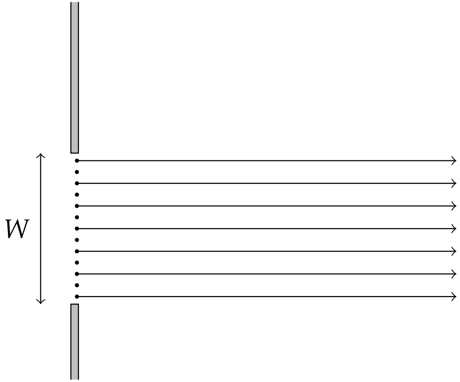
The Fresnel Number tell us when to use Fresnel diffraction (difficult!), and when to use Fraunhofer diffraction (easier):

$$F = \frac{W^2}{\lambda L}$$

Quantity	Symbol	SI Unit
Fresnel Number	F	(no units)
Characteristic length of the aperture	W	m (metres)
Wavelength of light	λ	m (metres)
Distance from aperture to projection	L	m (metres)

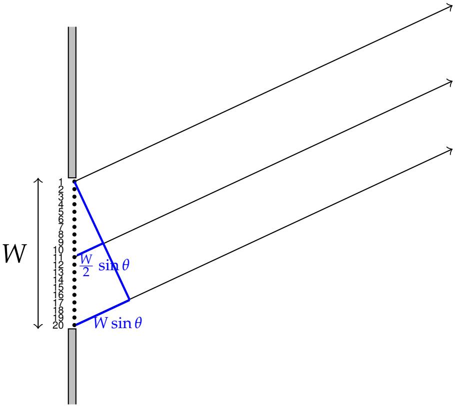
- Fresnel diffraction if $F \gg 1$; Fraunhofer diffraction if $F \ll 1$
- In Physics 12, we will only deal with Fraunhofer diffraction

Let's Work This Out Again!



- Similar to the double-slit problem, we apply Huygens' principle again
- This time we treat the slit as wide enough that there is a series (an infinite series, actually) of point waves at the slit
- We can easily see that the light from the wavelet that travel perpendicular to the slit (aperture) will not interfere with one another
- i.e. a bright fringe at the middle. **This is called the central maximum.**

At Some Other Angle θ



- Like what we did with double-slit, we can find the path difference between the wavelet on the top (1) and bottom (20): $W\sin\theta$
- At some θ, the path difference between 1 and 20 will be an integer multiple of the wavelength $(m\lambda)$
- In this case, the path difference between 1 and 11 is a half-number multiple of the wavelength (i.e. destructive interference) and they cancel each other
- Similarly, 2 cancels 12, 3 cancels 13...

RESULT: COMPLETE DESTRUCTIVE INTERFERENCE

Dark Fringes: Destructive Interference

Dark fringes exists on the screen at regular, whole-numbered intervals ($m = 1, 2, 3\ldots$):

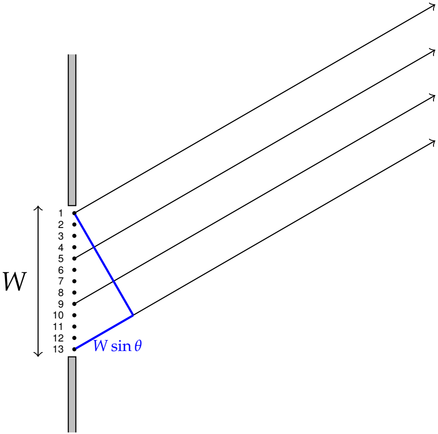
$$\boxed{\pm m\lambda = W \sin\theta_m}$$

Again, we can apply our small-angle approximation equation, and end up with:

$$\boxed{y_m = \frac{m\lambda L}{W}}$$

Pro-tip: This equation looks very similar to the double-slit equation for *bright* fringes, so be *very* careful when you use them!

At Some Other Angle θ



- Again, we follow what we did with the the previous case, and we find that at some angle θ, the path difference between the top and bottom is $W \sin\theta = \frac{3}{2}\lambda$
- Beam from (1) and (5) differ by $\frac{\lambda}{2}$, so they have destructive interference; similarly 2 and 6, 3 and 7, 4 and 8, 9 and 13 will all interfere destructively
- But some of the beams will not, so we have a bright fringe at the projection
- This bright fringe is not as bright as the central one because of the destructive interference

Bright Fringes: Constructive Interference

Bright fringes exist on the screen at regular, half-numbered intervals ($m = 1, 2, 3 \ldots$):

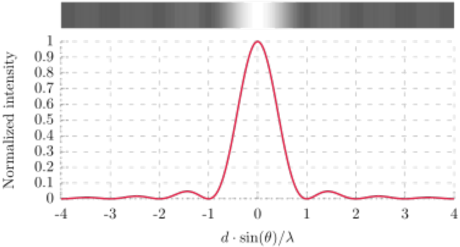
$$\boxed{\pm\left(m + \frac{1}{2}\right)\lambda = W\sin\theta_m}$$

Again, similar to the dark fringes, we apply our small-angle approximation equation:

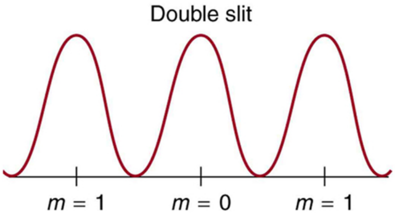
$$\boxed{y_m = \pm\left(m + \frac{1}{2}\right)\frac{\lambda L}{W}}$$

Single-Slit Diffraction, A Summary

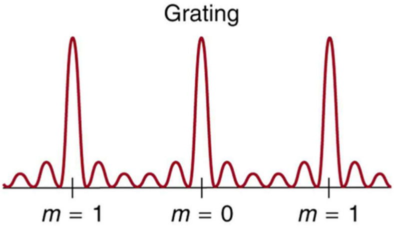
- Similar to the double-slit interference, single-slit diffraction projects a series of alternating bright fringes (“maxima”) and dark fringes (“minima”) in the far field
- The bright fringe in the middle (“central maximum”) is twice as wide and very bright
- Subsequent bright fringes on either side (“higher-order maxima”) are much dimmer because of the partial destructive interference



Diffraction Grating: What if there are more than 2 slits?



(a)



(b)

- We can apply the same analysis from double-slit to a diffraction grating
- Use equation for double-slit interference to locate bright fringes

$$n\lambda = d \sin \theta_n$$

- Interference pattern is sharper
- Bright fringes are narrower

Example Problem

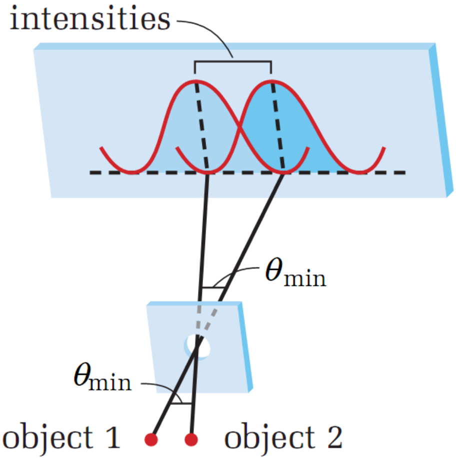
Example 1: Viewing a $645\,\mathrm{nm}$ red light through a narrow slit cut into a piece of paper yields a series of bright and dark fringes. You estimate that five dark fringes appear in a space of $1.0\,\mathrm{mm}$. If the paper is $32\,\mathrm{cm}$ from your eye, calculate the width of the slit.

Resolving Power

The ability of an optical instrument (e.g. the human eye, microscope, camera) to distinguish two distinct objects.

WHY? When light from any object passes through an "optical instrument", it **diffracts**, therefore "blurring" the object.

Resolving Power



In the **Rayleigh criterion**, two objects are resolved if the angle $\theta > \theta_{\text{min}}$, where θ_{min} is when the first minimum (dark fringe) from object 1 overlaps with the central maximum (bright fringe in the middle) from object 2.

Resolving Power

In order to resolve two objects, the minimum angle between rays from the two objects passing through a rectangular aperture is given by:

Rectangular aperture:

$$\boxed{\theta_{\min} = \frac{\lambda}{W}}$$

Circular aperture:

$$\boxed{\theta_{\min} = \frac{1.22\lambda}{D}}$$

Quantity	Symbol	SI Unit
Minimum angle of separation	$\theta_{\min}$	(radian)
Wavelength	λ	m (metre)
Characteristic length	W	m (metre)
Diameter of circular aperture	D	m (metre)

Note: The angle $\theta_{\min}$ is measured in **radians** not degree.

Example Problem

Example 2: A skydiver is falling toward the ground. How close to the ground will she have to be before she is able to distinguish two yellow baseballs lying $25.0\,\mathrm{cm}$ apart, reflecting $625\,\mathrm{nm}$ light in air? Her pupil diameter is $3.35\,\mathrm{mm}$. Assume that the speed of light inside the human eye is $2.21 \times 10^{8}\,\mathrm{m/s}$.

Dispersion of Light Through Diffraction

The examples for single- and double-slit patterns that have all been based on a single wavelength of light, but we know that the equations depends on wavelength. So what happens to our diffraction pattern when the light source is a white light?