

cálculo 2, Stewart vol. 1, ed. 8, cap. 7.1

1)  $\int x \cdot e^{2x} dx$

$f(x) = x$

$f'(x) = 1$

$g'(x) = e^{2x}$

$g(x) = \int e^{2x} dx = \frac{1}{2} \cdot \int e^u du = \frac{1}{2} \cdot e^u$   
 $u = 2x \quad du = 2 dx \quad = \frac{e^{2x}}{2} + c$

$\int x \cdot e^{2x} dx = x \cdot \frac{e^{2x}}{2} - \int 1 \cdot \frac{e^{2x}}{2}$

$= x \cdot \frac{e^{2x}}{2} - \frac{1}{2} \int e^{2x} = x \cdot \frac{e^{2x}}{2} - \frac{1}{2} \left( \frac{e^{2x}}{2} \right) = \frac{x e^{2x}}{2} - \frac{1}{4} e^{2x} + c$

3)  $\int x \cdot \cos(5x) dx$

$f(x) = x \quad f'(x) = 1$

$g'(x) = \cos(5x) \quad g(x) = \int \cos(5x) = \frac{1}{5} \int \cos(u) du$

$= x \cdot \frac{1}{5} \sin(5x) - \frac{1}{5} \int \sin(5x) dx$

$= \frac{1}{5} \sin(5x) + c$

$= x \cdot \sin(5x) - \frac{1}{25} \int \sin(u) du = x \cdot \sin(5x) - \frac{1}{25} (-\cos(5x)) + c$

5)  $\int x \cdot e^{\frac{x}{2}} dx$

$f(x) = x$

$f'(x) = 1$

$u = \frac{x}{2} \quad du = \frac{1}{2} dx$

$g'(x) = e^{\frac{x}{2}}$

$g(x) = \int e^{\frac{x}{2}} dx = 2 \cdot \int e^u \cdot \frac{1}{2} dx = 2 \int e^u du = 2 \cdot e^u$   
 $= 2 \cdot e^{\frac{x}{2}} + c$

$\int x \cdot e^{\frac{x}{2}} dx = x \cdot 2e^{\frac{x}{2}} - \int 1 \cdot 2 \cdot e^{\frac{x}{2}} dx$

$= 2x e^{\frac{x}{2}} - 2 \int e^{\frac{x}{2}} dx = 2x e^{\frac{x}{2}} - 4 e^{\frac{x}{2}} + c$

7)  $\int (x^2 + 2x) \cdot \cos(x) dx$

$f(x) = x^2 + 2x$

$f'(x) = 2x + 2$

$g'(x) = \cos(x)$

$g(x) = \int \cos(x) dx = \sin(x) + c$

$= (x^2 + 2x) \cdot \sin(x) - \int (2x + 2) \cdot \sin(x) dx = (x^2 + 2x) \cdot \sin(x) - ((2x + 2) \cdot -\cos(x))$

$f(x) = 2x + 2$

$f'(x) = 2$

$- \int 2 \cdot -\cos(x)$

$g'(x) = \sin(x)$

$g(x) = \int \sin(x) dx = -\cos(x)$

$= (x^2 + 2x) \cdot \sin(x) + \cos(x) \cdot (2x + 2) + 2 \sin(x) + c$

$$9 \int \cos^{-1}(x) dx = \int \frac{1}{\cos(x)} dx = \int \sec(x) dx$$

$$= \int \sec(x) \left( \frac{\sec(x) + \tan(x)}{\sec(x) + \tan(x)} \right) dx = \int \frac{\sec^2(x) + \sec(x) \tan(x)}{\sec(x) + \tan(x)} dx$$

$$du = \sec(x) \cdot \tan(x) + \sec^2(x) dx$$

$$= \int \frac{1}{u} du = \ln|u| + c = \ln|\sec(x) + \tan(x)| + c$$

$$11 \int x^4 \cdot \ln x dx$$

$$f(x) = \ln(x)$$

$$f'(x) = \frac{1}{x}$$

$$g'(x) = x^4$$

$$g(x) = \frac{x^5}{5}$$

$$\int x^4 \ln(x) dx = \frac{\ln(x) \cdot x^5}{5} - \int \frac{1}{x} \cdot \frac{x^5}{5} dx = \frac{\ln(x) \cdot x^5}{5} - \frac{1}{5} \int x^4 dx$$

$$= \frac{\ln(x) \cdot x^5}{5} - \frac{1}{5} \cdot \frac{x^5}{5} + c$$

$$13 \int x \cdot \cos \sec^2 x dx$$

$$f(x) = x$$

$$f'(x) = 1$$

$$g'(x) = \cos \sec^2(x)$$

$$g(x) = \int \cos \sec^2(x) dx = -\cot(x) + c$$

$$= -x \cdot \cot(x) - \int (1 - \cot(x)) dx = -x \cdot \cot(x) + \int \cot(x) dx$$

$$= -x \cdot \cot(x) + \ln|\sin(x)| + c$$

$$\Rightarrow \int \cot(x) dx = \int \frac{\cos(x)}{\sin(x)} dx \quad \begin{matrix} u = \sin(x) \\ du = \cos(x) dx \end{matrix}$$

$$= \int \frac{1}{u} du = \ln|u| + c = \ln|\sin(x)| + c$$

$$15 \int \ln(x) \cdot x^{\frac{1}{3}} dx$$

$$f(x) = \ln(x)$$

$$f'(x) = \frac{1}{x}$$

$$g'(x) = x^{\frac{1}{3}}$$

$$g(x) = \int x^{\frac{1}{3}} dx = \frac{x^{\frac{4}{3}}}{\frac{4}{3}} = \frac{3}{4} x^{\frac{4}{3}}$$

$$= \ln(x) \cdot \frac{3}{4} x^{\frac{4}{3}} - \int \frac{1}{x} \cdot \frac{3}{4} x^{\frac{4}{3}} = \ln(x) \cdot \frac{3}{4} x^{\frac{4}{3}} - \frac{3}{4} \int x^{\frac{1}{3}} = \ln(x) \cdot \frac{3}{4} x^{\frac{4}{3}} - \frac{9}{16} x^{\frac{4}{3}} + c$$

$$17 \int \arctg(4x) dx$$

$$u = 4x \quad \frac{du}{dx}(4x) = 4 \quad du = 4 dx$$

$$= \int \arctg(u) \frac{4}{4} dx = \frac{1}{4} \cdot \int \arctg(u) du$$

$$\left[ \begin{array}{l} f(u) = \arctg(u) \quad f'(u) = \frac{1}{u^2+1} \\ g'(u) = 1 du \quad g(u) = u \end{array} \right.$$

$$= \left( \arctg(u) \cdot u - \int \frac{1}{u^2+1} \cdot u du \right) \cdot \frac{1}{4}$$

$$\left[ \begin{array}{l} k = u^2+1 \quad dk = 2u du \end{array} \right.$$

$$= \left( \arctg(u) \cdot u - \frac{1}{2} \int \frac{1}{k} dk \right) \cdot \frac{1}{4} = \left( \arctg(u) \cdot u - \frac{\ln|k|}{2} + c \right) \cdot \frac{1}{4}$$

$$= \left( \arctg(4x) \cdot 4x - \frac{1}{2} \cdot \ln|(4x)^2+1| + c \right) \cdot \frac{1}{4}$$

$$= \arctg(4x) \cdot x - \frac{1}{8} \cdot \ln|16x^2+1| + c$$

$$19 \int x^3 \cdot e^x dx$$

$$f(x) = x^3 \quad f'(x) = 3x^2$$

$$g'(x) = e^x \quad g(x) = \int e^x dx = e^x$$

$$= x^3 \cdot e^x - \int 3x^2 \cdot e^x dx = x^3 e^x - 3 \cdot (x^2 \cdot e^x - \int 2x \cdot e^x dx)$$

$$= x^3 e^x - 3 \cdot [x^2 \cdot e^x - 2 \cdot (x \cdot e^x - e^x)] + c$$

$$= x^3 e^x - 3 \cdot [x^2 e^x - 2x e^x - 2e^x] + c$$

$$= x^3 e^x - 3x^2 e^x - 6x e^x - 6e^x + c$$

aplicando os limites de integração

$$\left( \frac{1}{2} \cdot \frac{\sin(\frac{\pi}{2})}{\pi} + \frac{\cos(\frac{\pi}{2})}{\pi^2} \right) - \left( 0 + \frac{\cos(0)}{\pi^2} \right)$$

$$= \frac{1}{2\pi} + 0 - \frac{1}{\pi^2} = \frac{1}{2\pi} - \frac{1}{\pi^2}$$

25  $\int_0^2 x \sin(x) dx$

$$f(x) = x$$

$$f'(x) = 1$$

$$g'(x) = \sin(x)$$

$$g(x) = -\cos(x)$$

$$= -x \cos(x) - \int -\cos(x) dx = (-x \cos(x) + \sin(x) + c) \Big|_0^2$$

$$(-2 \cos(2) + \sin(2)) - (-0 \cos(0) + \sin(0))$$

$$= -2 \cos(2) + \sin(2)$$

27  $\int_1^5 \frac{\ln x}{x^2} dx$

$$f(x) = \ln x$$

$$f'(x) = \frac{1}{x}$$

$$g'(x) = \frac{1}{x^2}$$

$$g(x) = -\frac{1}{x}$$

$$\Rightarrow -\frac{\ln x}{x} - \int \frac{1}{x} \cdot \frac{-1}{x} dx = -\frac{\ln x}{x} + \int x^{-2} dx = \left( -\frac{\ln x}{x} + \frac{-1}{x} + c \right) \Big|_1^5$$

$$= \left( -\frac{\ln 5}{5} - \frac{1}{5} \right) - \left( -\frac{\ln 1}{1} - \frac{1}{1} \right) = -\frac{\ln 5}{5} - \frac{1}{5} + 1 = -\frac{\ln 5}{5} + \frac{4}{5}$$

29  $\int_0^\pi x \sin(x) \cos(x) dx$

$$f(x) = x$$

$$f'(x) = 1$$

$$g'(x) = \sin(x) \cos(x)$$

$$g(x) = \int \sin(x) \cos(x) dx = \frac{1}{2} \sin^2(x)$$

$$u = \sin(x) \quad du = \cos(x)$$

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$$\Rightarrow \frac{x \sin^2(x)}{2} - \int \frac{\sin^2(x)}{2} dx = \frac{x \sin^2(x)}{2} - \frac{1}{2} \left( \int \frac{1}{2} dx - \frac{1}{2} \int \cos(2x) dx \right)$$

$$= \left( \frac{x \sin^2(x)}{2} - \frac{x}{4} + \frac{\sin(2x)}{8} \right) \Big|_0^\pi = \left( \frac{\pi \cdot 0}{2} - \frac{\pi}{4} + \frac{0}{8} \right) - \left( 0 - 0 + 0 \right)$$

$$= -\frac{\pi}{4}$$



$$31 \int_1^5 \frac{x}{e^x} dx \quad f(x) = x \quad f'(x) = 1$$

$$g'(x) = e^{-x} \quad g(x) = -e^{-x}$$

$$\Rightarrow -xe^{-x} - \int 1 \cdot -e^{-x} dx$$

$$= -xe^{-x} + \int e^{-x} dx = -xe^{-x} - e^{-x} + c$$

$$\Rightarrow (-5e^{-5} - e^{-5}) - (-1e^{-1} - e^{-1})$$

$$= \frac{-5}{e^5} - \frac{1}{e^5} + \frac{1}{e} + \frac{1}{e} = \frac{-5}{e^5} - \frac{1}{e^5} + \frac{2}{e} = \frac{-6}{e^5} + \frac{2}{e}$$

$$33 \int_0^{\pi/3} \sec(x) \cdot \ln(\cos(x)) dx$$

$$f(x) = \ln(\cos(x)) \quad f'(x) = \frac{1}{\cos(x)} \cdot -\sin(x) = -\frac{\sin(x)}{\cos(x)} = -\tan(x)$$

$$g'(x) = \sec(x) \quad g(x) = -\cos(x)$$

$$\Rightarrow \ln(\cos(x)) \cdot -\cos(x) - \int -\tan(x) \cdot -\cos(x) dx$$

$$= -\ln(\cos(x)) \cdot \cos(x) - \int \frac{\sin(x) \cdot \cos(x)}{\cos(x)} dx = -\ln(\cos(x)) \cdot \cos(x) + \cos(x)$$

$$\Rightarrow (-\ln(\cos(\pi/3)) \cdot \cos(\pi/3) + \cos(\pi/3)) - (-\ln(\cos(0)) \cdot \cos(0) + \cos(0))$$

$$= -\ln(1/2) \cdot \frac{1}{2} + \frac{1}{2} + \ln(1) \cdot 1 - 1 = -(\ln(1) - \ln(2)) \cdot \frac{1}{2} + \frac{1}{2} - 1$$

$$= \frac{\ln(2)}{2} - \frac{1}{2}$$

$$35 \int_1^2 x^4 (\ln(x))^2 dx$$

$$f(x) = (\ln(x))^2 \quad f'(x) = \frac{2 \ln(x)}{x}$$

$$g'(x) = x^4$$

$$g(x) = \frac{x^5}{5}$$

$$f(x) = \ln(x) \quad f'(x) = \frac{1}{x}$$

$$g'(x) = x^4$$

$$g(x) = \frac{x^5}{5}$$

$$\Rightarrow \frac{\ln(x)^2 \cdot x^5}{5} - \int \frac{x^5 \cdot 2 \ln(x)}{5} = \frac{\ln(x)^2 \cdot x^5}{5} - \frac{2}{5} \int x^4 \ln(x) dx$$

$$= \frac{\ln(x)^2 \cdot x^5}{5} - \frac{2}{5} \left( \frac{\ln(x) x^5}{5} - \frac{1}{5} \int 1 \cdot x^4 dx \right) = \frac{\ln(x)^2 x^5}{5} - \frac{2 \ln(x) x^5}{25} + \frac{2 x^5}{125} + c$$

aplicando  $|_1^2$

$$= \frac{32 \ln^2(2)}{5} - \frac{64 \ln(2)}{25} + \frac{62}{125}$$