cálculo 2, stewart vol. 1, ed. 8, cap 7.4

1 a) $\frac{A}{(1+2x)} + \frac{B}{(3-x)}$ b) $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{1+x}$

3 a) $x^2 + x^4 = x^2(1+x^2)$ b) $x^3 - 3x^2 + 2x = x(x^2 - 3x + 2)$

$\frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{(1+x^2)}$ $= x(x-1)(x-2)$

$\frac{A}{x} + \frac{B}{x-1} + \frac{C}{x-2}$

5 a) $x^2 - 4 = (x-2)(x+2)$ b) quadrados irredutíveis

grau do numerador é maior $\frac{Ax+B}{x^2-x+1} + \frac{Cx+D}{x^2+2} + \frac{Ex+F}{(x^2+2)^2}$

9 $\int \frac{5x+1}{(2x+1)(x-1)} dx$ $\frac{A}{2x+1} + \frac{B}{x-1} = \frac{5x+1}{(2x+1)(x-1)}$

$\frac{A(x-1) + B(2x+1)}{(2x+1)(x-1)} = \frac{5x+1}{(2x+1)(x-1)} \Rightarrow Ax - A + 2Bx + B = 5x+1$

$Ax + 2Bx - A + B = 5x + 1$

$x(A+2B) + (-A+B)$

$\begin{cases} A+2B = 5 \\ -A+B = 1 \end{cases}$ $B = 1 + A$ $A + 2 + 2A = 5$ $3A = 3$ $A = 1$ $B = 2$

resolvendo a integral equivalente

$\int \frac{1}{2x+1} dx + \int \frac{2}{x-1} dx = 1 \cdot \frac{1}{2} \cdot \int \frac{1}{u} du + 2 \cdot \int \frac{1}{v} dv$

$u = 2x+1$ $du = 2$ $v = x-1$ $dv = 1$

$= \frac{1}{2} \cdot \ln|2x+1| + 2 \cdot \ln|x-1| + c$

11 $\int_0^1 \frac{2}{2x^2+3x+1} dx$ $2x^2 + 3x + 1 = 0$

$-3 \pm \sqrt{9-8} / 4 = 0$ $x' = -1$

$-3 \pm 1 / 4 = 0$ $x'' = -1/2$

$2 \int \frac{1}{2(x+1)(x+\frac{1}{2})} dx = 2 \int \frac{1}{(2x+1)(x+1)} dx \Rightarrow \frac{A}{2x+1} + \frac{B}{x+1} = \frac{1}{(2x+1)(x+1)}$

$A(x+1) + B(2x+1) = Ax + A + 2Bx + B = x(A+2B) + A + B$

$$\begin{cases} A + B = 1 & A = 1 - B & B = -1 \\ A + 2B = 0 & 1 - B + 2B = 0 & A = 2 \end{cases}$$

$$\int \frac{2}{(2x+1)}\,du - \int \frac{1}{(x+1)}\,du = \ln|2x+1| - \ln|x+1| + c$$

aplicando $\Big|_0^1$

$$[(\ln|3| - \ln|2| + c) - (\cancel{\ln|1|} - \cancel{\ln|1|} + \cancel{c})] \cdot 2 = \ln\left|\tfrac{3}{2}\right| \cdot 2$$

13 $\displaystyle\int \frac{ax}{x^2 - bx}\,du$ $\qquad x^2 - bx = x(x-b)$

$$\frac{A}{x} + \frac{B}{x-b} = \frac{ax}{x(x-b)} = \frac{A(x-b) + Bx}{x(x-b)}$$

$$\begin{cases} A + B = a \\ -A = 0 \end{cases} \qquad \begin{aligned} &Ax - Ab + Bx = ax \\ &x(A+B) + b(-A) = ax \end{aligned}$$

$B = a$

$$= \int \frac{0\,du}{x} + \int \frac{a}{x-b}\,du = \int \frac{a}{x-b}\,du = a\int \frac{du}{x-b} = a\int \frac{1}{u}\,du$$

$$u = x - b \qquad du = 1\,dx$$

$$= a \cdot \ln|u| = a \cdot \ln|x-b| + c$$

17 $\displaystyle\int_1^2 \frac{4x^2 - 7x - 12}{x(x+2)(x-3)}$ $\qquad \dfrac{A}{x} + \dfrac{B}{(x+2)} + \dfrac{C}{(x-3)} = \dfrac{4x^2 - 7x - 12}{x(x+2)(x-3)}$

$A(x+2)(x-3) + Bx(x-3) + Cx(x+2)$

$= A(x^2 - 3x + 2x - 6) + Bx^2 - 3Bx + Cx^2 + 2Cx$

$= Ax^2 - Ax - 6A + Bx^2 - 3Bx + Cx^2 + 2Cx$

$= Ax^2 + Bx^2 + Cx^2 - Ax - 3Bx + 2Cx - 6A$

$= x^2(A + B + C) + x(-A - 3B + 2C) - 6A$

$$\begin{cases} A + B + C = 4 \\ -A - 3B + 2C = -7 \\ -6A = -12 \end{cases} \qquad \begin{cases} B + C = 2 \quad (\cdot 3) \\ -3B + 2C = -5 \end{cases}$$

$$5C = 1$$

$A = 2 \qquad B = 9/5 \qquad C = 1/5$

resolvendo a integral equivalente

$$\int \frac{2}{x}\,dx + \frac{9}{5}\int \frac{1}{x+2}\,dx + \frac{1}{5}\int \frac{1}{x-3}\,dx$$

$$= 2\ln|x| + \frac{9}{5}\ln|x+2| + \frac{1}{5}\ln|x-3| + c$$

aplicando $\int_1^2$

$$\left(2\ln|2| + \frac{9}{5}\ln|4| + \cancelto{0}{\frac{1}{5}\ln|-1|}\right) - \left(\cancelto{0}{2\ln|1|} + \frac{9}{5}\ln|3| + \frac{1}{5}\ln|-2|\right)$$

$$= 2\ln(2) + \frac{9}{5}\ln(4) - \frac{9}{5}\ln(3) - \frac{1}{5}\ln(2) = \frac{9}{5}\ln(2) + \frac{9\ln(4) - \ln(3)}{5}$$

$$= \frac{\ln(8/3)}{5}\cdot 9$$