cálculo 1, stewart, vol 1, ed 5, cap 2.3

11 $\lim_{x \to 2} \frac{x^2+x-6}{x-2}$ $S=-1$ $x'=2$ $x''=-3$

$P=-6$

$\frac{\cancel{(x-2)}(x+3)}{\cancel{(x-2)}} \rightarrow \lim_{x \to 2} x+3 = 5$

12 $\lim_{x \to -4} \frac{x^2+5x+4}{x^2+3x-4} \rightarrow \frac{0}{0}$ por substituição

x^2+3x-4 $S=3$

$(x'+4)(x''-1)$ $P=-4$

$x'=-4$ $x''=1$

x^2+5x+4 $S=-5$

$(x+1)(x+4)$ $P=4$

$x'=-4$ $x''=-1$

$\lim_{x \to -4} \frac{(x+1)\cancel{(x+4)}}{\cancel{(x+4)}(x-1)} = \lim_{x \to -4} \frac{(x+1)}{(x-1)} = \frac{3}{-5}$

13 $\lim_{x \to 2} \frac{x^2-x+6}{x-2} = \frac{2^2-2+6}{2-6} = \frac{4-2+6}{0} = \frac{8}{0} = \nexists$

14 $\lim_{x \to 4} \frac{x^2-4x}{x^2-3x-4} = \frac{x\cancel{(x-4)}}{(x+1)\cancel{(x-4)}} = \frac{x}{x+1} = \frac{4}{5}$

x^2-3x-4 $S=+3$

$(x+1)(x-4)$ $P=-4$

$x'=4$ $x''=-1$

15 $\lim_{t \to -3} \frac{t^2-9}{2t^2+7t+3}$ por substituição direta temos $0/0$

fatorando $\Rightarrow \frac{(t-3)\cdot(t+3)}{2t^2+7t+3} = \frac{(t-3)\cancel{(t+3)}}{2(t+1/2)\cancel{(t+3)}} = \frac{(t-3)}{2t+1}$

$t'=-1/2$, $t''=-3$

$$\lim_{t\to-3} \frac{t-3}{2t+1} = \frac{\lim_{t\to-3} t-3}{\lim_{t\to-3} 2t+1} = \frac{\lim_{t\to-3} t - \lim_{t\to-3} 3}{\lim_{t\to-3} 2t + \lim_{t\to-3} 1} = \frac{-3-3}{\lim_{t\to-3} 2 \cdot \lim_{t\to-3} t + 1} = \frac{-6}{2\cdot-3+1}$$

$= -6/-5$

16 $\lim_{x\to1} \frac{2x^2+3x+1}{x^2-2x-3} = \frac{2\cdot1^2+3\cdot1+1}{1+2-3} = \frac{0}{0}$ por substituição direta

$2x^2+3x+1=0$ $\quad$ $x^2-2x-3=0$

$\Delta = 3^2-4\cdot2\cdot1 = 9-8=1$ $\quad$ $\Delta = 4-4\cdot1\cdot-3 = 4+12=16$

$\frac{-3\pm\sqrt{1}}{2\cdot2} = \frac{-3\pm1}{4} \begin{cases}-1\\ -1/2\end{cases}$ $\quad$ $\frac{+2\pm\sqrt{16}}{2} = \frac{2\pm4}{2} \begin{cases}-1\\ 3\end{cases}$

$= 2(x+\frac{1}{2})(x+1)$ $\quad$ $(x+3)(x+1)$

$\lim_{x\to-1} \frac{2(x+\frac{1}{2})\cancel{(x+1)}}{(x+3)\cancel{(x+1)}} = \lim_{x\to-1} \frac{2x+1}{x+3} = \frac{-1}{2}$

17 $\lim_{h\to0} \frac{(-5+h)^2-25}{h}$ fatorando $\frac{\cancel{25}+2\cdot-5\cdot h+h^2-\cancel{25}}{h}$

$= \frac{-10h+h^2}{h} = \frac{\cancel{h}(-10+h)}{\cancel{h}}$

$\lim_{h\to0} -10+h = -10$

$= -10+h$

18 $\lim_{h\to0} \frac{(2+h)^3-8}{h}$ $\quad$ $8+3\cdot2^2\cdot h+3\cdot2\cdot h^2+h^3-8/h$

$= \cancel{8}+12h+6h^2+h^3-\cancel{8}/h$

$\lim_{h\to0} 12+6h+h^2 = 12$ $\quad$ $= \frac{\cancel{h}(12+6h+h^2)}{\cancel{h}} = 12+6h+h^2$

19 $\lim_{x\to-2} \frac{x+2}{x^3+8} = \lim_{x\to-2} \frac{\cancel{x+2}}{\cancel{(x+2)}(x^2-2x+4)} = \lim_{x\to-2} \frac{1}{x^2-2x+4} = \frac{1}{12}$

20 $\lim_{t\to1} \frac{t^4-1}{t^3-1} = \lim_{t\to1} \frac{t^3+t^2+t+1}{t^2+t+1} = \frac{4}{3}$

$t^4-(1^4) = \cancel{(t-1)}(t^3+t^2+t+1)$

$t^3-(1^3) = \cancel{(t-1)}(t^2+t+1)$

21 $\dfrac{\sqrt{9+h}-3}{h} \cdot \dfrac{\sqrt{9+h}+3}{\sqrt{9+h}+3} = \dfrac{\cancel{9}+\cancel{h}-\cancel{9}}{\cancel{h}(\sqrt{9+h}+3)} = \dfrac{1}{\sqrt{9+h}+3}$

$\lim\limits_{h\to 0} \dfrac{1}{\sqrt{9+h}+3} = \dfrac{1}{6}$

22 $\dfrac{\sqrt{4u+1}-3}{u-2} \cdot \dfrac{\sqrt{4u+1}+3}{\sqrt{4u+1}+3} = \dfrac{4u+1-9}{u-2\,(\sqrt{4u+1}+3)} = \dfrac{4\cancel{(u-2)}}{\cancel{(u-2)}(\sqrt{4u+1}+3)}$

$\lim\limits_{u\to 2} \dfrac{\sqrt{4u+1}-3}{u-2} = \dfrac{4}{\sqrt{4u+1}+3} = \dfrac{4}{\sqrt{\lim\limits_{u\to 2} 4u+1}+3} = \dfrac{4}{3+3} = \dfrac{4}{6} = \dfrac{2}{3}$

23 $\lim\limits_{x\to -4} \left(\dfrac{1/4 + 1/x}{4+x}\right) = \lim\limits_{x\to -4} \dfrac{\frac{x+4}{4x}}{x+4} = \lim\limits_{x\to -4} \dfrac{\cancel{x+4}}{4x} \cdot \dfrac{1}{\cancel{x+4}} = \lim\limits_{x\to -4} \dfrac{1}{4x}$

$= \dfrac{1}{\lim\limits_{x\to -4} 4 \cdot \lim\limits_{x\to -4} x} = \dfrac{1}{-16}$

24 $\dfrac{x^2+2x+1}{x^4-1}$ — $x'=x''=-1$ $= \dfrac{(x+1)(x+1)}{(x^2-1)(x^2+1)} = \dfrac{\cancel{(x+1)}(x+1)}{\cancel{(x+1)}(x-1)(x^2+1)}$

$\lim\limits_{x\to 1} \dfrac{(x+1)}{(x-1)(x^2+1)} = \lim\limits_{x\to 1} \dfrac{x+1}{x^3+x-x^2-1} = \dfrac{2}{0} = \nexists$

$\lim\limits_{x\to 1^+} = +\infty \qquad \lim\limits_{x\to 1^-} = -\infty$

25 $\dfrac{\sqrt{1+t}-\sqrt{1-t}}{t} \cdot \dfrac{\sqrt{1+t}+\sqrt{1-t}}{\sqrt{1+t}+\sqrt{1-t}} = \dfrac{(1+t)-(1-t)}{t(\sqrt{1+t}+\sqrt{1-t})} = \dfrac{2\cancel{t}}{\cancel{t}(\sqrt{1+t}+\sqrt{1-t})}$

$= \dfrac{2}{\sqrt{1+t}+\sqrt{1-t}} \qquad \lim\limits_{t\to 0} \dfrac{2}{2} = 1$

26 $\left(\dfrac{1}{t} - \dfrac{1}{t^2+t}\right) = \dfrac{t^2+\cancel{t}-\cancel{t}}{t(t^2+t)} = \dfrac{t^2}{t^3+t^2} = \dfrac{\cancel{t^2}}{\cancel{t^2}\cdot(t+1)} = \dfrac{1}{t+1}$

$\lim\limits_{t\to 0} \dfrac{1}{t+1} = \dfrac{1}{1} = 1$

27 $\dfrac{4-\sqrt{x}}{16x-x^2} = \dfrac{4-\sqrt{x}}{x(16-x)} = \dfrac{\cancel{4-\sqrt{x}}}{x\cancel{(4-\sqrt{x})}(4+\sqrt{x})} = \dfrac{1}{4x+\sqrt{x}}$

$\lim\limits_{x\to 16} \dfrac{1}{4\cdot x+\sqrt{x}} = \dfrac{1}{4\cdot 16+4} = \dfrac{1}{128}$

tilibra

28 $$\frac{\frac{1}{3+h}-\frac{1}{3}}{h} = \frac{\frac{\cancel{3-3}+h}{9+3h}}{h} = \frac{\cancel{h}}{9+3h}\cdot\frac{1}{\cancel{h}} = \frac{1}{9+3h} \qquad \lim_{h\to 0}\frac{1}{9+3h} = \frac{1}{9}$$

29 $$\frac{1}{t\sqrt{1+t}} - \frac{1}{t} = \frac{t - t\sqrt{1+t}}{t\,(t\sqrt{1+t})} = \frac{\cancel{t}\,(1-\sqrt{1+t})}{\cancel{t}\,(t\sqrt{1+t})} = \frac{1-\sqrt{1+t}}{t\sqrt{1+t}}\cdot\frac{1+\sqrt{1+t}}{1+\sqrt{1+t}}$$

$$= \frac{\cancel{1-1}-t}{t\sqrt{1+t}+t\sqrt{1+t}^{\,2}} = \frac{-\cancel{t}}{\cancel{t}\,(\sqrt{1+t}+1+t)} = \frac{-1}{\sqrt{1+t}+1+t}$$

$$\lim_{t\to 0}\frac{-1}{\sqrt{1+t}+1+t} = \frac{-1}{2}$$

36 $$\lim \sqrt{x^3+x^2}\cdot \operatorname{sen}\left(\frac{\pi}{x}\right) \Rightarrow -1 \leq \operatorname{sen}\left(\frac{\pi}{x}\right) \leq 1$$

multiplicando por $\sqrt{x^3+x^2}$

$$\underbrace{-\sqrt{x^3+x^2}}_{f(x)} \leq \underbrace{\sqrt{x^3+x^2}\cdot \operatorname{sen}\left(\frac{\pi}{x}\right)}_{g(x)} \leq \underbrace{\sqrt{x^3+x^2}}_{h(x)}$$

$$\lim_{x\to 0}\sqrt{x^3+x^2} = \sqrt{\lim_{x\to 0} x^3+x^2} = \sqrt{0} = 0$$

logo, pelo teorema do confronto, $\lim_{x\to 0} g(x) = 0$.

$$\lim_{x\to 0} -\sqrt{x^3+x^2} = -\sqrt{\lim_{x\to 0} x^3+x^2} = \sqrt{0} = 0$$

37 $$\lim_{x\to 4} 4x-9 = 7 \qquad \lim_{x\to 4} x^2-4x+7 = 7$$

pelo teorema do confronto $4x-9 \leq f(x) \leq x^2-4x+7$

$\hookrightarrow \lim_{x\to 4} f(x) = 5$

38 $$\underbrace{2x}_{f(x)} \leq g(x) \leq \underbrace{x^4-x^2+2}_{h(x)} \qquad \lim_{x\to 1} f(x) = 2 \qquad \lim_{x\to 1} h(x) = 2$$

$\lim_{x\to 1} g(x) = 2$

39 $\lim_{x\to 0} x^4\cdot\cos\frac{2}{x}$, como $x^4 \geq 0$ e $-1 \leq \cos\frac{2}{x} \leq 1$

$$-x^4 \leq x^4\cos\frac{2}{x} \leq x^4 \qquad \lim_{x\to 0} -x^4 = -0 = 0 \qquad \lim_{x\to 0} x^4 = 0$$

$\hookrightarrow \lim_{x\to 0} x^4\cos\left(\frac{2}{x}\right) = 0$

40 $\lim_{x\to 0^+} \sqrt{x} \cdot e^{\text{sen}\left(\frac{\pi}{x}\right)}$ $\sqrt{x} \geq 0$ $-1 \leq \text{sen}\left(\frac{\pi}{x}\right) \leq 1$

logo $\frac{1}{e} \leq e^{\text{sen}\left(\frac{\pi}{x}\right)} \leq e$

$\sqrt{x} \cdot \frac{1}{e} \leq \sqrt{x} \cdot e^{\text{sen}\left(\frac{\pi}{x}\right)} \leq \sqrt{x} \cdot e$

$\lim_{x\to 0^+} \sqrt{x} \cdot \frac{1}{e} = 0 \cdot \frac{1}{e} = 0$ $\lim_{x\to 0^+} \sqrt{x} \cdot e = 0 \cdot e = 0$

pelo Teorema do confronto $\lim_{x\to 0^+} \sqrt{x} \cdot e^{\text{sen}\left(\frac{\pi}{x}\right)} = 0$

42 $\lim_{x\to -6} \frac{2x+12}{|x+6|}$ se $x \to -6^+$, $|x+6| = x+6$

se $x \to -6^-$, $|x+6| = -(x+6)$

$\lim_{x\to -6^+} \frac{2x+12}{|x+6|} = \frac{2\cancel{(x+6)}}{\cancel{x+6}} = 2$

$\neq$ logo, o lim $\nexists$

$\lim_{x\to -6^-} \frac{2x+12}{|x+6|} = \frac{2\cancel{(x+6)}}{\cancel{-(x+6)}} = -2$

45 $\lim_{x\to 0^-} \left(\frac{1}{x} - \frac{1}{|x|}\right)$ se $x \to 0^-$, $|x| = -x$

$\lim_{x\to 0^-} \left(\frac{1}{x} - \frac{1}{-x}\right) = \lim_{x\to 0^-} \frac{-x-x}{-x^2} = \lim_{x\to 0^-} \frac{-2\cancel{x}}{-x^{\cancel{2}}} = \frac{-2}{-x} = \frac{+2}{x}$

como $\lim_{x\to 0^-} f(x) = \frac{2}{x} = \nexists = -\infty$