cálculo 1, stewart, vol 1, ed 5, cap 2.9

4 I – c II – a III – d IV – b

21 $f(x) = 37$ $D(f) = \mathbb{R}$

$f'(x) = 0$ $D(f'(x)) = \mathbb{R}$

23 $f(x) = 1 - 3x^2$ $D(f) = \mathbb{R}$

$$f'(x) = \lim_{h \to 0} \frac{1 - 3(x+h)^2 - (1 - 3x^2)}{h} = \lim_{h \to 0} \frac{\cancel{+1} - 3(x^2 + 2xh + h^2) \cancel{-1} + 3x^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{-3x^2} - 6xh - 3h^2 \cancel{+3x^2}}{h} = \lim_{h \to 0} \frac{\cancel{h}(-6x - 3h)}{\cancel{h}} = -6x \quad D(f'(x)) = \mathbb{R}$$

25 $f(x) = x^2 - 3x + 5$ $D(f) = \mathbb{R}$

$$f'(x) = \lim_{h \to 0} \frac{((x+h)^2 - 3(x+h) + 5) - (x^2 - 3x + 5)}{h} = \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h \cancel{-3x} - 3h \cancel{+5} \cancel{-x^2} \cancel{+3x} \cancel{-5}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x - 2)}{\cancel{h}} = 2x - 2 \quad D(f'(x)) = \mathbb{R}$$

27 $g(x) = \sqrt{1+2x}$ $\qquad D(g) = \{x \in \mathbb{R} \mid x \geq -\frac{1}{2}\}$

$$\frac{\sqrt{1+2(x+h)} - \sqrt{1+2x}}{h} = \frac{\sqrt{1+2x+2h} - \sqrt{1+2x}}{h} \cdot \frac{\sqrt{1+2x+2h} + \sqrt{1+2x}}{\sqrt{1+2x+2h} + \sqrt{1+2x}}$$

$$= \frac{\cancel{1} + \cancel{2x} + 2\cancel{h} - \cancel{1} - \cancel{2x}}{\cancel{h}\left(\sqrt{1+2x+2h} + \sqrt{1+2x}\right)} = = \frac{2}{\sqrt{1+2x+2h} + \sqrt{1+2x}} \qquad \lim_{h \to 0} \frac{2}{\sqrt{1+2x+2h} + \sqrt{1+2x}}$$

$$= \frac{\cancel{2}}{\cancel{2}\sqrt{1+2x}} = \frac{1}{\sqrt{1+2x}} = g'(x) \qquad D(g'(x)) = \{x \in \mathbb{R} \mid x > -\tfrac{1}{2}\}$$

29 $G(t) = \dfrac{4t}{t+1}$ $\qquad D(G(t)) = \mathbb{R} - 1$

$$\Rightarrow \frac{\frac{4(x+h)}{x+h+1} - \frac{4x}{x+1}}{h} = \frac{\frac{(x+1)(4x+4h) - (x+h+1)(4x)}{(x+h+1)(x+1)}}{h} = \frac{(x+1)(4x+4h) - (x+h+1)(4x)}{h(x+h+1)(x+1)}$$

$$= \frac{\cancel{4x^2} + \cancel{4xh} + \cancel{4x} + 4h - (\cancel{4x^2} + \cancel{4xh} + \cancel{4x})}{h(x+h+1)(x+1)} = \frac{4}{(x+h+1)(x+1)}$$

$$\lim_{h \to 0} \frac{4}{(x+h+1)(x+1)} = \frac{4}{(x+1)^2} \qquad D = \mathbb{R} - (-1)$$

31 $f(x) = x^4$ $\qquad D(f) = \mathbb{R}$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^4 - x^4}{h} = \lim_{h \to 0} \frac{(x^2+2xh+h^2)(x^2+2xh+h^2) - x^4}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^4} + 2x^3h + x^2h^2 + 2x^3h + 4x^2h^2 + 2xh^3 + x^2h^2 + 2xh^3 + h^4 - \cancel{x^4}}{h}$$

$$= \lim_{h \to 0} \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h} = \lim_{h \to 0} \frac{h(4x^3 + 6x^2h + 4xh^2 + h^3)}{h} = 4x^3$$

$D = \mathbb{R}$