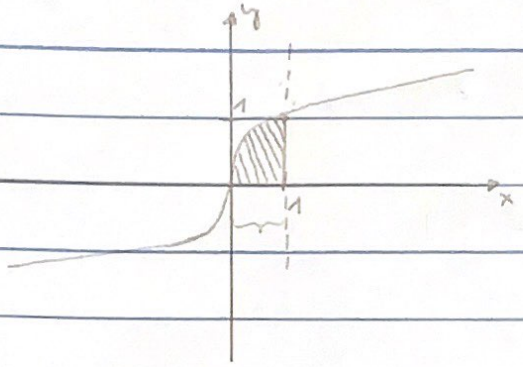
cálculo 2, stewart vol 1, ed 8, cap 6.3

3 $y = \sqrt[3]{x}$ $\quad y = 0$ $\quad x = 1$ em torno do eixo y

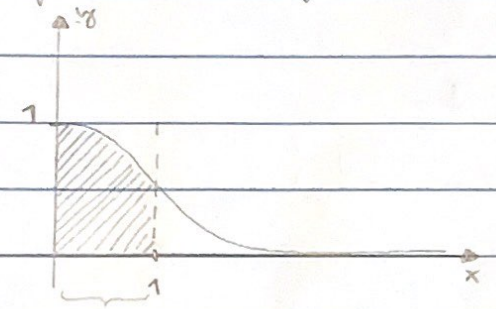


$$V = \int_0^1 2\pi \cdot x \cdot \sqrt[3]{x}\, dx$$

$$= 2\pi \int_0^1 x^{4/3}\, dx$$

$$= 2\pi \cdot \left(\frac{3x^{7/3}}{7}\right)\Big|_0^1 = \frac{2\pi \cdot 3}{7} = \frac{6\pi}{7}$$

5 $y = e^{-x^2}$ $\quad y = 0$ $\quad x = 0$ $\quad x = 1$



$$V = \int_0^1 2\pi \cdot x \cdot e^{-x^2}\, dx$$

$$= 2\pi \int_0^1 x \cdot e^{-x^2}\, dx$$

$$\Rightarrow \int x \cdot e^{-x^2}\, dx \quad ; \quad u = -x^2 \quad du = -2x\, dx$$

$$= \frac{\int -2x \cdot e^{-x^2}\, dx}{-2}$$

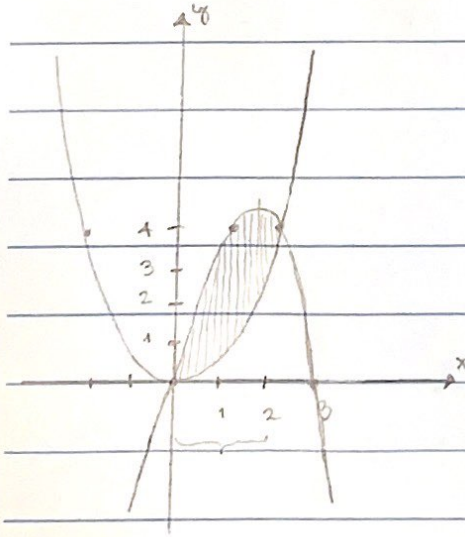
$$= \frac{-1}{2}\int e^u\, du = \frac{-1}{2} e^u + c = \frac{-1}{2} e^{-x^2} + c$$

$$\Rightarrow 2\pi \left(-\tfrac{1}{2} e^{-x^2}\right)\Big|_0^1 = 2\pi \cdot \left(\frac{-e^{-1}}{2} + \frac{e^0}{2}\right) = \cancel{2}\pi \cdot \left(\frac{-1}{\cancel{2}e} + \frac{1}{\cancel{2}}\right)$$

7 $y = x^2$ $\quad y = 6x - 2x^2$

$$x^2 = 6x - 2x^2$$

$$3x^2 - 6x = 0 \quad \begin{cases} x'' = 2 \\ x' = 0 \end{cases}$$



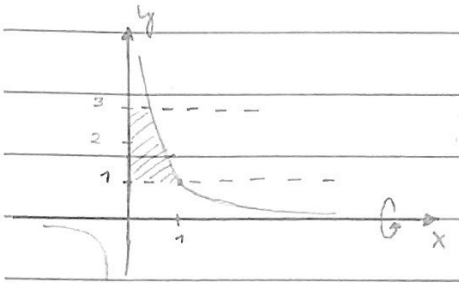
$$V = \int_0^2 2\pi \cdot x \cdot \left((6x - 2x^2) - (x^2)\right) dx$$

$$= 2\pi \int_0^2 x \cdot (6x - 3x^2)\, dx = 2\pi \int_0^2 6x^2 - 3x^3\, dx$$

$$= 2\pi \left(\frac{6x^3}{3} - \frac{3x^4}{4}\right)\Big|_0^2 = 2\pi \left(16 - 12\right) = 8\pi$$

9 $xy = 1$ $\quad x = 0$ $\quad y = 1$ $\quad y = 3$

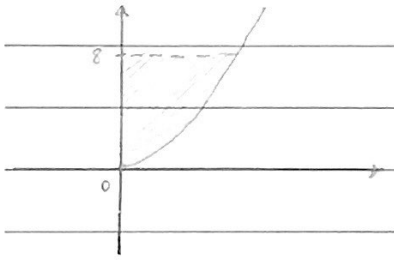
$x = 1/y$



$$V = \int_1^3 2\pi \cdot y \cdot \frac{1}{y}\, dy$$
$$= 2\pi \int_1^3 1\, dy$$
$$= 2\pi \cdot (y)\Big|_1^3 = 4\pi$$

11 $y = x^{3/2}$ $\quad y = 8$ $\quad x = 0$

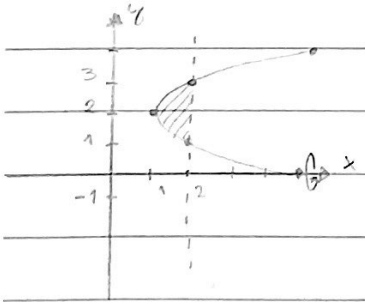
$y^{2/3} = x$ para $y \geq 0$



$$V = 2\pi \int_0^8 y \cdot y^{2/3}\, dy$$
$$= 2\pi \int_0^8 y^{5/3}\, dy = 2\pi \left(\frac{3\, y^{8/3}}{8}\right)\Big|_0^8$$
$$= 2\pi \frac{3}{8} \cdot 8^{8/3} = \pi \frac{3}{4} (2^3)^{8/3} = 192\pi$$

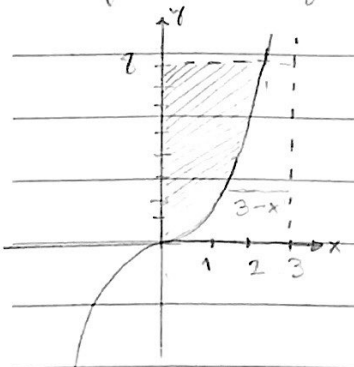
13 $x = 1 + (y-2)^2$ $\quad x = 2$

$x = y^2 - 4y + 5$



$$V = \int_1^3 2\pi \cdot y \cdot (2 - (y^2 - 4y + 5))\, dy$$
$$= 2\pi \int_1^3 -y^3 + 4y^2 - 3y\, dy$$
$$= 2\pi \left(-\frac{y^4}{4} + \frac{4y^3}{3} - \frac{3y^2}{2}\right)\Big|_1^3 = \frac{16\pi}{3}$$

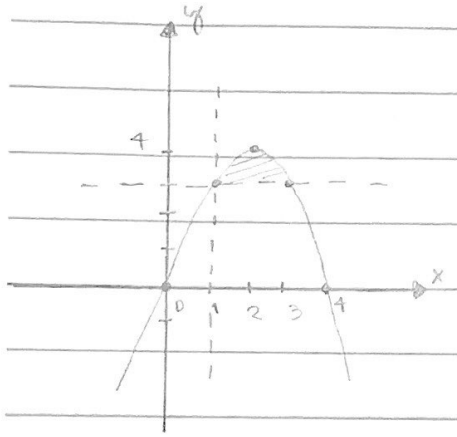
15 $y = x^3$ $\quad y = 8$ $\quad x = 0$ em volta de $x = 3$



raio $= 3 - x$ $\quad$ altura $= 8 - x^3$

$$V = \int_0^2 2\pi \cdot (3-x) \cdot (8 - x^3)\, dx$$
$$= 2\pi \int_0^2 (x^4 - 3x^3 - 8x + 24)\, dx = 2\pi \cdot \left(\frac{x^5}{5} - \frac{3x^4}{4} - 4x^2 + 24x\right)\Big|_0^2$$
$$= \frac{264\pi}{5}$$

17 $y = 4x - x^2$ $\quad y = 3$ $\quad$ em torno de $x = 1$



$$V = \int_a^b 2\pi (r \cdot h)\, dx$$

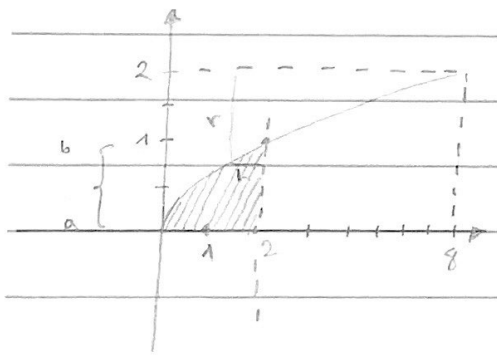
$a = 1 \qquad b = 3 \qquad r = x - 1 \qquad h = f(x) - 3$

$$V = \int_1^3 2\pi (x-1) \cdot ((4x - x^2) - 3)\, dx$$

$$= 2\pi \int_1^3 -x^3 + 5x^2 - 7x + 3\, dx$$

$$= \frac{8\pi}{3}$$

19 $x = 2y^2$ $\quad$ para $y \geq 0$ $\quad x = 2$ $\quad$ em torno de $\quad y = 2$



$$V = \int_a^b 2\pi\, (r \cdot h)\, dy$$

$a = 0 \qquad b = 1$

$r = 2 - y \qquad h = 2 - 2y^2$

$$V = \int_0^1 2\pi \cdot (2-y)\,(2-2y^2)$$

...

$$= 13\pi/3$$