cálculo 1, stewart, vol 1, ed 5, cap 3.4

1 $f(x) = x - 3\operatorname{sen} x$

$f'(x) = (x)' - (3\operatorname{sen} x)' = 1 - 3\cos x$

2 $f(x) = x \cdot \operatorname{sen} x$

$f'(x) = (x)' \cdot \operatorname{sen} x + x \cdot \cos x = \operatorname{sen} x + x \cdot \cos x$

3 $f(x) = \operatorname{sen} x + 10\operatorname{tg} x$

$f'(x) = \cos x + 10\,(\operatorname{tg} x)' = \cos x + 10 \cdot \frac{1}{\cos^2 x} = \cos x + \frac{10}{\cos^2 x}$

4 $f(x) = 2\operatorname{cossec} x + 5\cos x$

$f'(x) = (2 \cdot \operatorname{cossec} x)' + 5 \cdot -\operatorname{sen} x = 2 \cdot -\operatorname{cossec} \cdot \operatorname{cotg} - 5\operatorname{sen} x$

5 $g(t) = t^3 \cos t$

$g'(t) = (t^3)' \cdot \cos t + t^3 \cdot (\cos t)' = 3t^2 \cdot \cos t + t^3 \cdot -\operatorname{sen} t$

6 $g(t) = 4\operatorname{sec} t + \operatorname{tg} t$

$g'(t) = (4\operatorname{sen} t)' + (\operatorname{tg} t)' = 4\cos t + \operatorname{sec}^2 t$

7 $h(x) = \operatorname{cossec} x + e^x \cdot \operatorname{cotg} x$

$h'(x) = -\operatorname{cossec} x \cdot \operatorname{cotg} x + ((e^x)' \cdot \operatorname{cotg} x + e^x \cdot (\operatorname{cotg} x)')$

$= -\operatorname{cossec} x \cdot \operatorname{cotg} x + e^x \cdot \operatorname{cotg} x - \operatorname{cossec}^2 x \cdot e^x$

8 $f(x) = e^x (\cos x + c \cdot x) =$

$f'(x) = (e^x)' \cdot [\cos x + cx] + [e^x](\cos x + cx)' \Rightarrow$

$(\cos x + cx)' = -\operatorname{sen} x + c \cdot 1 = -\operatorname{sen} x + c$

$\Rightarrow e^x \cdot \cos x + e^x cx + e^x (-\operatorname{sen} x + c)$

9 $f(x) = \frac{x}{\cos x}$

$f'(x) = \frac{1 \cdot \cos x + x \cdot -\operatorname{sen} x}{\cos^2 x} = \frac{\cos x - \operatorname{sen} x \cdot x}{\cos^2 x}$

10 $y = \frac{1 + \operatorname{sen} x}{x + \cos x}$ $\qquad (1 + \operatorname{sen} x)' = \cos x$

$(x + \cos x)' = 1 + -\operatorname{sen} x = 1 - \operatorname{sen} x$

$y' = \frac{\cos x \cdot (x + \cos x) - (1 + \operatorname{sen} x)(1 - \operatorname{sen} x)}{(x + \cos x)^2}$

11 $f(x) = \frac{\sec x}{1+\sec x}$ $\quad (\sec)' = \sec \cdot \operatorname{tg}$

$(1+\sec) = 0 + \sec \cdot \operatorname{tg}$

$f'(x) = \frac{(\sec x \cdot \operatorname{tg} x)\cdot(1+\sec x) - \sec x\,(\sec x \cdot \operatorname{tg} x)}{(1+\sec x)^2}$

$= \frac{\sec x \cdot \operatorname{tg} x + \sec^2 x \cdot \operatorname{tg} x - \sec^2 x \cdot \operatorname{tg} x}{(1+\sec x)^2} = \frac{\sec x \cdot \operatorname{tg} x}{(1+\sec x)^2}$

12 $f(x) = \frac{\operatorname{tg} x - 1}{\sec x}$ $\quad (\operatorname{tg} x - 1)' = \sec^2 x$

$(\sec x)' = \sec x \cdot \operatorname{tg} x$

$f'(x) = \frac{\sec^3 x - (\operatorname{tg} x - 1)\cdot \sec x \cdot \operatorname{tg} x)}{\sec^2 x}$

$= \frac{\sec^3 x - \operatorname{tg}^2 x \cdot \sec x + \sec x \operatorname{tg} x}{\sec^2 x}$

$= \frac{\sec x(\sec^2 x - \operatorname{tg}^2 x + \operatorname{tg} x)}{\sec^2 x} = \frac{\sec^2 x - \operatorname{tg}^2 x + \operatorname{tg} x}{\sec x}$

13 $f'(x) = \frac{\operatorname{sen} x}{x^2}$ $\quad (\operatorname{sen} x)' = \cos x$

$f'(x) = \frac{\cos x \cdot x^2 - \operatorname{sen} x \cdot 2x}{x^4}$

17 $\frac{d}{dx}(\operatorname{cossec} x) = -\operatorname{cossec} x \cdot \operatorname{cotg} x$

$\operatorname{cossec} x = \frac{1}{\operatorname{sen} x}$

$(\operatorname{cossec} x)' = -\frac{(1 \cdot \cos x)}{\operatorname{sen}^2 x} = \frac{-\cos x}{\operatorname{sen}^2 x} = \overbrace{\frac{-1}{\operatorname{sen} x}}^{-\operatorname{cossec} x} \cdot \overbrace{\frac{\cos x}{\operatorname{sen} x}}^{\operatorname{cotg} x}$

18 $\frac{d}{dx}(\sec x) = \sec x \cdot \operatorname{tg} x$ $\quad \sec x = \frac{1}{\cos x}$

$(\sec x)' = \frac{-(1 \cdot -\operatorname{sen} x)}{\cos^2 x} = \frac{\operatorname{sen} x}{\cos^2 x} = \frac{1}{\cos x} \cdot \frac{\operatorname{sen} x}{\cos x} = \sec x \cdot \operatorname{tg} x$

19 $\frac{d}{dx}(\operatorname{cotg} x) = -\operatorname{cossec}^2 x$ $\quad \operatorname{cotg} x = \frac{1}{\operatorname{tg} x}$

$(\operatorname{cotg} x)' = \frac{-(1 \cdot \sec^2 x)}{\operatorname{tg}^2 x} = \frac{-\sec^2 x}{\operatorname{tg}^2 x} = -\left(\frac{1}{\frac{\cos x \cos x}{\operatorname{sen} x \operatorname{sen} x}}\right)$

$= -\frac{1}{\cos x \cos x} \cdot \frac{\cos x \cos x}{\operatorname{sen} x \operatorname{sen} x} = \frac{-1}{\operatorname{sen}^2 x} = -\operatorname{cossec} x \cdot \operatorname{cossec} x$

21 $y = \operatorname{tg} x$

$y' = \sec^2 x = \frac{1}{\cos^2 x}$ $\quad f'(\pi/4) = \frac{1}{(\cos \frac{\pi}{4})^2} = \frac{1}{(\sqrt{2}/2)^2} = \frac{1}{2/4} = 2$

$m = 2$

$y - y_0 = 2(x - x_0)$ $\quad y - 1 = 2(x - \pi/4)$

$y = 2x - \frac{\pi}{2} + 1$

23 $y = x + \cos x$ $\quad y' = 1 + (-\operatorname{sen} x) = 1 - \operatorname{sen} x$

$f'(0) = 1 - \operatorname{sen}(0) = 1 - 0 = 1$

$m = 1$

$y - 1 = 1 \cdot (x - 0)$ $\quad y = x + 1$

25 $y = x \cos x$ $\quad y' = \cos x + x \cdot -\operatorname{sen} x$

$f(\pi) = -1 + \pi \cdot -0 = -1$ $\quad m = -1$

$y + \pi = -1(x - \pi) = -x + \pi$

$y = -x + \pi - \pi = -x$ $\quad \boxed{y = -x}$