calculo 1, stewart, vol 1, ed 5, cap 3.6

1 $xy + 2x + 3x^2 = 4$

a) $\frac{d}{dx}(x \cdot f(x) + 2x + 3x^2) = \frac{d}{dx}(4)$

$(x \cdot f(x))' + (2x)' + (3x^2)' = 0 \Rightarrow 1 \cdot f(x) + x \cdot f'(x) + 2 + 6x = 0$

$\hookrightarrow x' \cdot f(x) + x \cdot f'(x), \quad x' = 1$

$x \cdot f'(x) = -2 - 6x - f(x)$

$x \cdot y' = -2 - 6x - y \qquad y' = \frac{-(2 + 6x + y)}{x}$

b) $xy + 2x + 3x^2 = 4 \qquad y = \frac{4 - 2x - 3x^2}{x} \qquad \left[\begin{array}{l} (4 - 2x - 3x^2)' \\ = -6x - 2 \end{array} \right.$

$y' = \frac{(-6x-2) \cdot x + (4 - 2x - 3x^2)}{x^2} = \frac{-6x^2 - \cancel{2x} - 4 + \cancel{2x} + 3x^2}{x^2}$

$= \frac{-3x^2 - 4}{x^2} = \frac{-(3x^2 + 4)}{x^2}$

3 $\frac{1}{x} + \frac{1}{y} = 1$

a) $\frac{d}{dx}\left(\frac{1}{x} + \frac{1}{y}\right) = \frac{d}{dx}(1) \Rightarrow \left(\frac{1}{x}\right)' + \left(\frac{1}{y}\right)' = 0 \Rightarrow (x^{-1})' + \left(\frac{1}{f(x)}\right)' = 0$

$\left(\frac{1}{f(x)}\right)' = \frac{(1)' \cdot f(x) - f'(x) \cdot 1}{f(x)^2} = \frac{-y'}{y^2}$

$\Rightarrow -x^{-2} + \frac{-y'}{y^2} = 0 \qquad \Rightarrow \frac{y'}{y^2} = \frac{-1}{x^2} \qquad \Rightarrow y' = \frac{-y^2}{x^2}$

b) $\frac{1}{y} = 1 - \frac{1}{x} \Rightarrow \frac{1}{y} = \frac{x-1}{x} \Rightarrow y = \frac{x}{x-1}$

$y' = \frac{(x)' \cdot (x-1) - (x \cdot (x-1)')}{(x-1)^2} = \frac{(x-1) - (x \cdot 1)}{(x-1)^2} = \frac{(\cancel{x} - 1) - \cancel{x}}{(x-1)^2} = -\frac{1}{(x-1)^2}$

5 $x^2 + y^2 = 1 \qquad 2x + 2(f(x)) \cdot f'(x) = 0$

$\frac{d}{dx}(x^2 + (f(x))^2) = \frac{d}{dx}(1) \qquad y' = \frac{-2x}{2y} = \frac{-x}{y}$

7 $x^3 + x^2y + 4y^2 = 6$ d. $3x^2 + (x^2f(x))' + (4f(x)^2)' = 0$

$\frac{d}{dx}(x^3 + x^2f(x) + 4f(x)^2) = \frac{d}{dx}(6)$ $(x^2f(x))' = 2x \cdot f(x) + x^2 \cdot f'(x)$

$(4f(x)^2)' \Rightarrow F(x) = 4x^2$ $G(x) = f(x)$

$F'(x) = 8x$ $G'(x) = f'(x)$

$3x^2 + 2x \cdot f(x) + x^2 \cdot f'(x) + 8(f(x)) \cdot f'(x) = 0$

$x^2 \cdot f'(x) + 8(f(x)) \cdot f'(x) = -3x^2 - 2xy$

$y'(x^2 + 8y) = -(3x^2 + 2xy)$ $y' = \frac{-(3x^2 + 2xy)}{(x^2 + 8y)} = \frac{-x(3x + 2y)}{x^2 + 8y}$

9 $x^2y + xy^2 = 3x$ $\frac{d}{dx}(x^2y + xy^2) = \frac{d}{dx}(3x)$

$(x^2y)' + (xy^2)' = 3$ $(x^2y)' = 2x \cdot y + x^2 \cdot y'$

$(x\,f(x)^2)' = 1 \cdot y^2 + x \cdot ((f(x))^2)'$

$((f(x))^2)' = 2y \cdot y'$

$2xy + x^2y' + y^2 + 2yx \cdot xy' = 3$

$x^2y' + 2yx^2y' = 3 - y^2 - 2xy$

$y'(x^2 + 2yx^2) = 3 - y^2 - 2xy$ $y' = \frac{3 - y^2 - 2xy}{x^2 + 2yx^2}$

11 $x^2y^2 + x\,\text{sen}\,y = 4$ $(x^2f(x)^2)' + (x\,\text{sen}(f(x)))' = 0$

$(x^2f(x)^2)' = 2x \cdot y^2 + x^2 \cdot 2y \cdot y' = 2xy^2 + 2yy'x^2$

$(x\,\text{sen}(f(x)))' = 1 \cdot \text{sen}(y) + x \cdot \cos y \cdot y'$

$(\text{sen}(f(x)))' \Rightarrow F(x) = \text{sen}\,x$ $G(x) = y$

$F'(x) = \cos x$ $G'(x) = y'$ $y' = \frac{-(2xy^2 + \text{sen}(y))}{2yx^2 + \cos(y)x}$

$2xy^2 + 2yx^2y' + \text{sen}(y) + \cos(y)xy' = 0$

$2yx^2y' + \cos(y)xy' = -2xy^2 - \text{sen}(y)$

$y'(2yx^2 + \cos(y)x) = -(2xy^2 + \text{sen}(y))$

13 $(4\cos(x))\,\text{sen}(y) = 1$ $(4\cos(x))' \cdot \text{sen}(f(x)) + 4\cos(x) \cdot (\text{sen}(f(x)))' = 0$

$(\text{sen}(f(x)))' =$ $F(x) = \text{sen}\,x$ $F'(x) = \cos x$ $G(x) = f(x)$ $G'(x) = f'(x)$

$-4 \cdot \text{sen}\,x \cdot \text{sen}(y) + 4 \cdot \cos(x) \cdot \cos y \cdot y' = 0$

$y' = \frac{\cancel{4}\,\text{sen}(x) \cdot \text{sen}(y)}{\cancel{4}\cos(x) \cdot \cos(y)} = \text{tg}(x)\,\text{tg}(y)$

15 $e^{x^2y} = x+y$ $\qquad \frac{d}{dx}(e^{x^2y}) = \frac{d}{dx}(x+f(x))$

$e^{g(x)}$ $\qquad g(x) = x^2 f(x)$ $\qquad \Rightarrow e^{g(x)} \cdot g'(x) = e^{x^2y} \cdot (2xy + x^2y')$

$$\begin{cases} F'(x) = e^x & G'(x) = g'(x) \\ & = (x^2)' \cdot y + x^2 \cdot y' \\ & = 2xy + x^2 y' \end{cases}$$

$\Rightarrow e^{x^2y} \cdot (2xy + x^2y') = 2e^{(x^2y)}xy + e^{(x^2y)} \cdot x^2y' = 1 + y'$

$\Rightarrow 2e^{(x^2y)}xy - 1 = y' - e^{(x^2y)}x^2y' = y'(1 - e^{x^2y}x^2)$

$$y' = \frac{2e^{(x^2y)}xy - 1}{1 - e^{x^2y}\,x^2}$$

17 $\sqrt{xy} = 1 + x^2y$ $\qquad d(\sqrt{xy}) = d(\sqrt{g(x)})$ $\qquad F(x) = \sqrt{x}$ $\qquad G(x) = xy$

$F'(x) = \frac{1}{2}x^{-\frac{1}{2}}$

$G'(x) = (x)' \cdot f(x) + x \cdot f'(x)$

$= f(x) + x f'(x) = y + xy'$

$d(\sqrt{g(x)}) = \frac{1}{2}(xy)^{-\frac{1}{2}} \cdot (y + xy') = \frac{1}{2} \cdot \frac{1}{\sqrt{xy}} \cdot (y + xy') = \underbrace{\frac{y+xy'}{2\sqrt{xy}}}$

$d(1 + x^2y) = (1)' + (x^2 f(x))' = \underbrace{2x \cdot f(x) + x^2 \cdot f'(x)}$

$\frac{y+xy'}{2\sqrt{xy}} = 2xy + x^2y' \Rightarrow y + xy' = (2xy + x^2y') \cdot 2\sqrt{xy}$

$\Rightarrow y + xy' = 4xy\sqrt{xy} + 2\sqrt{xy}\;x^2y'$

$\Rightarrow xy' - 2\sqrt{xy} \cdot x^2y' = 4xy\sqrt{xy} - y$

$y'(x - 2\sqrt{xy}\,x^2) = 4xy\sqrt{xy} - y$

$$y' = \frac{4xy\sqrt{xy} - y}{x - 2\sqrt{xy}\,x^2}$$

25 $x^2+xy+y^2=3$ $\qquad \frac{d}{dx}(x^2+xy+y^2)=0$

$= 2x+(xf(x))' + (f(x)^2)' = 2x+f(x)+xf'(x)+2f(x)\cdot f'(x)$

$= 2x+y+xy'+2yy'$

$xy'+2yy' = -2x-y = -(2x+y)$

$y'(x+2y) = -(2x+y)$ $\qquad y' = \frac{-(2x+y)}{x+2y} = m$

m em $(1,1) \rightarrow y' = m = \frac{-(2+1)}{1+2} = \frac{-3}{3} = -1$

$y-y_0 = m(x-x_0)$ $\qquad y-1 = -1(x-1)$

$y = -x+1+1 = -x+2$

27 $\frac{d}{dx}(x^2+y^2) = \frac{d}{dx}(x^2+(f(x))^2) = 2x+2f(x)\cdot f'(x) = 2x+2yy'$

$\frac{d}{dx}((2x^2+2y^2-x)^2) = \frac{d}{dx}(g(x)^2) = 2g(x)g'(x)$

$g'(x) = (2x^2+2y^2-x)' = 4x+(2(f(x))^2)'-1$

$(2(f(x))^2)' \rightarrow F(x) = 2x^2 \qquad G'(x) = f(x) \rightarrow 4f(x)\cdot f'(x)$

$F'(x) = 4x \qquad G'(x) = f'(x)$

$= 4x+\cancel{4}^{2}yy'-1 = 2x\cancel{+2yy'}$

$2yy' = 2x+1-4x = -2x+1$ $\qquad y' = \frac{2x+1}{2y}$

$m = \frac{2 \cdot 0 + 1}{2 \cdot \frac{1}{2}}$ para $(0, 1/2)$ $= \frac{1}{1} = 1$

$y - \frac{1}{2} = 1 \cdot (x - 0)$ $\quad y = x + \frac{1}{2}$

41 $y = tg^{-1} \sqrt{x} = arctg(\sqrt{x})$

$y' = [arctg(g(x))]'$ onde $F(x) = arctg(x)$ $\quad G(x) = \sqrt{x}$

$F'(x) = \frac{1}{x^2+1}$ $\quad G'(x) = \frac{1}{2} x^{-1/2}$

$y' = \frac{1}{(\sqrt{x})^2 + 1} \cdot \frac{1}{2} \cdot x^{-\frac{1}{2}} = \frac{1}{x+1} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{x}} = \frac{1}{2\sqrt{x}(x+1)}$

42 $y = \sqrt{tg^{-1} x} = \sqrt{arctg\, x}$

$y' = \frac{1}{2} (arctg\, x)^{-1/2} \cdot \frac{1}{x^2+1} = \frac{1}{2} \cdot \frac{1}{\sqrt{arctg\, x}} \cdot \frac{1}{x^2+1}$

43 $y = sen^{-1}(2x+1) = arcsen(2x+1)$

$y' = \frac{1}{\sqrt{1-(2x+1)^2}} \cdot 2 = \frac{2}{\sqrt{1-(4x^2+4x+1)}} = \frac{2}{\sqrt{-4x^2+4x}}$