cálculo 2, stewart vol. 2, ed 8, cap 14.3

15 $f(x,y) = x^4 + 5xy^3$

$f_x = 4x^3 + 5y^3$ $\qquad$ $f_y = 0 + 15xy^2 = 15xy^2$

17 $f(x,t) = t^2 \cdot e^{-x}$

$f_x = \cancel{(t^2)' \cdot e^{-x}} + t^2 \cdot (e^{-x})'$ $\qquad$ $f_t = (t^2)' \cdot e^{-x} + \cancel{t^2 \cdot (e^{-x})'}$

$= t^2 \cdot (e^{-x})' = t^2 \cdot (-e^{-x})$ $\qquad$ $= 2t \cdot e^{-x}$

18 $f(x,t) = \sqrt{3x+4t}$ $\qquad$ $f = \sqrt{x}$ $\qquad$ $f' = \frac{1}{2} \cdot x^{-1/2} = \frac{1}{2\sqrt{x}}$

$f_x = \frac{1}{2\sqrt{3x+4t}} \cdot 3$ $\qquad$ $g = 3x+4t$ $\qquad$ $g' = \ldots$

$f_t = \frac{1}{2\sqrt{3x+4t}} \cdot 4 = \frac{2}{\sqrt{3x+4t}}$

19 $z = f(x,t) = \ln(x+t^2)$ $\qquad$ $(\ln x)' = \frac{1}{x}$

$f_x = \frac{1}{x+t^2} \cdot 1 + 0 = \frac{1}{x+t^2}$ $\qquad$ $f_t = \frac{1}{x+t^2} \cdot 2t = \frac{2t}{x+t^2}$

20 $z = f(x,y) = x \operatorname{sen}(xy)$

$f_x = (x)' \cdot \operatorname{sen}(xy) + x \cdot (\operatorname{sen}(xy))'$ $\qquad$ $\frac{d}{dx}(\operatorname{sen}(xy)) = \cos(xy) \cdot y$

$= \operatorname{sen}(xy) + \cos(xy) \cdot xy$

$f_y = \cancel{(x)' \cdot \operatorname{sen}(xy)} + x \cdot (\operatorname{sen}(xy))'$ $\qquad$ $\frac{d}{dy}(\operatorname{sen}(xy)) = \cos(xy) \cdot x$

$= x^2 \cos(xy)$

21 $f(x,y) = \frac{x}{y}$ $\quad \to \quad$ $(x)' \cdot y - x \cdot (y)' / y^2$

$f_x = \frac{y}{y^2} = \frac{1}{y}$ $\qquad$ $f_y = \frac{-x}{y^2}$

23 $f(x,y) = \frac{ax+by}{cx+dy}$ $\quad \to \quad$ $\frac{(ax+by)' \cdot (cx+dy) - (ax+by) \cdot (cx+dy)'}{(cx+dy)^2}$

$f_x = \frac{a \cdot (cx+dy) - (ax+by) \cdot c}{(cx+dy)^2} = \frac{\cancel{acx} + ady - \cancel{acx} - cby}{(cx+dy)^2} = \frac{y(ad-cb)}{(cx+dy)^2}$

$f_y = \frac{b(cx+dy) - (ax+by) \cdot d}{(cx+dy)^2} = \frac{bcx + \cancel{bdy} - adx - \cancel{bdy}}{(cx+dy)^2} = \frac{x(bc-ad)}{(cx+dy)^2}$

25 $g(u,v) = (u^2 v - v^3)^5$ $\quad f = x^5 \quad f' = 5x^4$

$g = u^2 v - v^3 \quad g' = \dots$

$g_u = 5(u^2 v - v^3)^4 \cdot 2uv$

$g_v = 5(u^2 v - v^3)^4 \cdot (u^2 - 3v^2)$

27 $R(p,q) = \tan^{-1}(pq^2)$ $\quad (tg^{-1})' = \dfrac{1}{1+x^2}$

$R_p = \dfrac{1}{1+(pq)^2} \cdot q^2$

$R_q = \dfrac{1}{1+(pq)^2} \cdot 2pq$

29 $F(x,y) = \int_y^x \cos(e^t)\,dt$

$F_x = \dfrac{d}{dx}\left(\int_y^x \cos(e^t)\,dt\right) = \cos(e^x)$

$F_y = \dfrac{d}{dx}\left(\int_y^x \cos(e^t)\,dt\right) \longrightarrow \int_y^x \cos(e^t)\,dt = -\int_x^y \cos(e^t)\,dt$

$= -\cos(e^y)$

31 $f(x,y,z) = x^3 y z^2 + 2yz$

$f_x = 3x^2 y z^2$

$f_y = x^3 z^2 + 2z$

$f_z = 2x^3 y z + 2y$

33 $w = f(x,y,z) = \ln(x+2y+3z)$

$f_x = \dfrac{1}{(x+2y+3z)} \cdot 1 \qquad f_y = \dfrac{1}{(x+2y+3z)} \cdot 2 \qquad f_z = \dfrac{1}{(x+2y+3z)} \cdot 3$

35 $p = f(x,y,z) = \sqrt{x^4 + y^2 \cos(z)}$ $\qquad f = \sqrt{x} \quad f' = \dfrac{1}{2\sqrt{x}}$

$f_x = \dfrac{1}{\cancel{2}\sqrt{x^4+y^2\cdot\cos(z)}} \cdot \cancel{2}\,4x^3 \qquad f_y = \dfrac{1}{2\sqrt{x^4+y^2\cos(z)}} \cdot 2y\cos(z)$

$f_z = \dfrac{1}{2\sqrt{x^4+y^2\cdot\cos(z)}} \cdot (-y^2 \cdot \mathrm{sen}(z))$

37 $h(x,y,z,t) = x^2 y \cos(z/t)$

$h_x = 2xy\cos(z/t)$ $\qquad f = \cos(u) \qquad f' = -\mathrm{sen}(u)$

$h_y = x^2 \cos(z/t)$

$h_z = x^2 y \cdot (-\mathrm{sen}(z/t) \cdot 1/t)$

$h_t = x^2 y \cdot (-\mathrm{sen}(z/t) \cdot z/t^2)$

41 $R(s,t) = t e^{s/t}$

$R_t = (t)' \cdot e^{s/t} + t \cdot (e^{s/t})'$ $\qquad \frac{d}{dt}(e^{s/t}) = e^{s/t} \cdot (s e^{-1})' = e^{s/t} \cdot \left(\frac{-s}{e^2}\right)$

$= e^{s/t} + \left(\frac{\cancel{t} \cdot e^{s/t} \cdot s}{t^{\cancel{2}}}\right)$

$= e^{s/t} - \frac{e^{s/t} \cdot s}{t}$, apliando $(s,t) = (0,1)$

$= e^{\cancel{0/1}} ① - \cancel{\frac{e^{0/1} ① \cdot 0}{1}} = 1$

44 $f(x,y,z) = x^{yz}$ $\qquad (a^u)' = a^u \ln(a) \qquad u = xy$

$f_z = (x^u \cdot \ln(x)) \cdot (u)'$

$= (x^{yz} \ln(x)) \cdot (yz)' = (x^{yz} \cdot \ln(x)) \cdot (y)$, apliando $(e,1,0)$

$= (\cancel{e^0} ① \cdot \cancel{\ln(e)} ①) \cdot (1) = 1$

47 $\frac{\partial}{\partial x}(x^2 + 2y^2 + 3z^2) = \cancel{\frac{\partial}{\partial x}(1)}^{0} = 0$

$\frac{\partial}{\partial x}(x^2) + \cancel{\frac{\partial}{\partial x}(2y^2)}^{0} + \frac{\partial}{\partial x}(3z^2) \longrightarrow \frac{\partial}{\partial x}(3z^2) = \frac{\partial}{\partial x}(3 \cdot (z(x))^2)$

$\Rightarrow 2x + 0 + 6z \cdot \frac{\partial z}{\partial x} = 0$ $\qquad = 6(z) \cdot \frac{\partial z}{\partial x}$

$\boxed{\frac{\partial z}{\partial x} = \frac{-2x}{6z} = \frac{-x}{3z}}$, agora para $\frac{\partial z}{\partial y}$:

$\cancel{\frac{\partial}{\partial y}(x^2)}^{0}\ 0 + \frac{\partial}{\partial y}(2y^2) + \frac{\partial}{\partial y}(3z^2) = \cancel{\frac{\partial}{\partial y}(1)}^{0}$

$\Rightarrow 4y + 6z \cdot \frac{\partial z}{\partial y} = 0 \Rightarrow \boxed{\frac{\partial z}{\partial y} = \frac{-4y}{6z} = \frac{-2y}{3z}}$

49 $\frac{\partial}{\partial x}(e^z) = \frac{\partial}{\partial x}(xyz) \rightarrow e^z \cdot \frac{\partial z}{\partial x} = y \cdot \left(\frac{\partial}{\partial x}(xz)\right)$

$\left[\frac{\partial}{\partial x}(xz) = \frac{\partial}{\partial x}(x \cdot z(x)) = (x)' \cdot z + x \cdot \frac{\partial z}{\partial x} = z + x\frac{\partial z}{\partial x}\right.$

$\rightarrow e^z \cdot \frac{\partial z}{\partial x} = y \cdot \left(z + x\frac{\partial z}{\partial x}\right) \rightarrow e^z \cdot \frac{\partial z}{\partial x} - xy \cdot \frac{\partial z}{\partial x} = yz$

$\rightarrow \frac{\partial z}{\partial x}(e^z - xy) = yz \qquad \frac{\partial z}{\partial x} = \frac{yz}{(e^z - xy)}$

51 $\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}(f(x) + g(y)) = \frac{\partial}{\partial x}f(x) + \cancel{\frac{\partial}{\partial x}g(y)} = f'(x)$

$\frac{\partial z}{\partial y} = \cancel{\frac{\partial}{\partial y}f(x)} + \frac{\partial}{\partial y}g(y) = g'(y)$

53 $f(x,y) = x^4y - 2x^3y^2$

$f_x = 4x^3y - 6x^2y^2 \qquad f_y = x^4 - 4x^3y$

$f_{xx} = 12x^2y - 12xy^2 \qquad f_{yy} = -4x^3$

$f_{xy} = 4x^3 - 12x^2y \qquad f_{yx} = 4x^3 - 12x^2y$

59 $u = f(xy) = x^4y^3 - y^4$

$f_x = 4x^3y^3 \qquad f_y = 3x^4y^2 - 4y^3$

$f_{xy} = 12x^3y^2 \qquad f_{yx} = 12x^3y^2$