cálculo 1, stewart, vol 1, ed 5, cap 4.4

1 a) $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0}$ = indeterminado

b) $\lim_{x \to a} \frac{f(x)}{p(x)} = \frac{0}{\infty} = 0$

c) $\lim_{x \to a} \frac{h(x)}{p(x)} = \frac{1}{\infty} = 0$

d) $\lim_{x \to a} \frac{p(x)}{f(x)} = \frac{\infty}{0} = \infty, -\infty$ ou não existe

5 $\lim_{x \to -1} \frac{x^2-1}{x+1} \overset{L.H.}{=} \lim_{x \to -1} \frac{2x}{1} = -2$

7 $\lim_{x \to 1} \frac{x^9-1}{x^5-1} \overset{L.H.}{=} \lim_{x \to 1} \frac{9x^8}{5x^4} = \frac{9}{5}$

9 $\lim_{x \to \frac{\pi}{2}^+} \frac{\cos x}{1-\operatorname{sen} x} \overset{L.H.}{=} \lim_{x \to \frac{\pi}{2}^+} \frac{-\operatorname{sen} x}{-\cos x} \overset{L.H.}{=} \lim_{x \to \frac{\pi}{2}^+} \frac{\overset{1}{\operatorname{sen} x}}{\cos x \to 0^+} = +\infty$

11 $\lim_{x \to 0} \frac{e^x-1}{x^3} \overset{L.H.}{=} \lim_{x \to 0} \frac{e^x}{3x^2}$ = indeterminado

13 $\lim_{x \to 0} \frac{tg(px)}{tg(qx)} \overset{L.H.}{=} \lim_{x \to 0} \frac{\sec^2(px) \cdot p}{\sec^2(qx) \cdot q} = \frac{1^2 \cdot p}{1^2 \cdot q} = \frac{p}{q}$

$F = tg\, x \qquad G = Kx$

$F' = \sec^2 x \qquad G' = K$

15 $\lim_{x \to \infty} \frac{\ln x}{x}$, por substituição $\frac{+\infty}{\infty}$

$\overset{L.H.}{=} \lim_{x \to \infty} \frac{\frac{1}{x}}{1} = \lim_{x \to \infty} \frac{1}{x} = 0$

17 $\lim_{x \to 0^+} \frac{\ln x}{x}$, por substituição $\frac{-\infty}{0}$

19 $\lim_{x \to 0} \frac{5^x - 3^x}{x} \overset{L.H.}{=} \lim_{x \to 0} \frac{5^x \cdot \ln 5 - 3^x \cdot \ln 3}{1} = \ln 5 - \ln 3$

21 $\lim_{x\to 0} \frac{e^x - 1 - x}{x^2} \overset{L.H.}{=} \lim_{x\to 0} \frac{e^x - 1}{2x} \overset{L.H.}{=} \lim_{x\to 0} \frac{e^x}{2} = \frac{1}{2}$

23 $\lim_{x\to\infty} \frac{e^x}{x^3} \overset{L.H}{=} \lim_{x\to\infty} \frac{e^x}{3x^2} \overset{L.H.}{=} \lim_{x\to\infty} \frac{e^x}{6x} \overset{L.H}{=} \lim_{x\to\infty} \frac{e^x}{6} = \infty$

25 $\lim_{x\to 0} \frac{\operatorname{sen}^{-1} x}{x} \overset{L.H.}{=} \lim_{x\to 0} \frac{\frac{1}{\sqrt{1-x^2}}}{1} = 1$

27 $\lim_{x\to 0} \frac{1-\cos x}{x^2} \overset{L.H}{=} \lim_{x\to 0} \frac{+\operatorname{sen} x}{2x} \overset{L.H}{=} \lim_{x\to 0} \frac{\cos x}{2} = \frac{1}{2}$

29 $\lim_{x\to 0} \frac{x + \operatorname{sen} x}{x + \cos x} = 0$

31 $\lim_{x\to\infty} \frac{x}{\ln(1+2e^x)} \overset{L.H}{=} \lim_{x\to\infty} \frac{1}{\frac{1}{1+2e^x} \cdot 2e^x} = \lim_{x\to\infty} \frac{1}{\frac{2e^x}{1+2e^x}}$

$= \lim_{x\to\infty} \frac{1+2e^x}{2e^x} \overset{L.H}{=} \lim_{x\to\infty} \frac{2e^x}{2e^x} = \lim_{x\to\infty} 1 = 1$

33 $\lim_{x\to 1} \frac{1 - x + \ln x}{1 + \cos \pi x} \overset{L.H}{=} \lim_{x\to 1} \frac{-1 + \frac{1}{x}}{-\operatorname{sen}(\pi x)\cdot\pi} = \lim_{x\to 1} \frac{\frac{-x+1}{x}}{-\operatorname{sen}(\pi x)\cdot\pi}$

$= \lim_{x\to 1} \frac{(-x+1)}{x \cdot -\operatorname{sen}(\pi x)\cdot\pi} = \lim_{x\to 1} \frac{x-1}{x\pi\cdot\operatorname{sen}(x\pi)} \overset{L.H}{=} \lim_{x\to 1} \frac{1}{\pi \operatorname{sen}(x\pi) + x\pi\cdot\cos(\pi x)\cdot\pi}$

$= \frac{1}{\pi\cdot 0 + \pi\cdot -1\cdot\pi} = \frac{1}{-\pi^2}$

37 $\lim_{x\to 0^+} \sqrt{x}\cdot\ln x$, por substituição $0\cdot -\infty$

$= \lim_{x\to 0^+} \frac{\sqrt{x}}{\frac{1}{\ln x}}$, por substituição $= \frac{0}{0}$

$\overset{L.H.}{=} \lim_{x\to 0^+} \frac{\frac{1}{2}\cdot x^{-\frac{1}{2}}}{[\ln x^{-1}]'} = \lim_{x\to 0^+} \frac{\frac{1}{2\sqrt{x}}}{-\ln x^{-2}\cdot\frac{1}{x}} = \lim_{x\to 0^+} \frac{\frac{1}{2\sqrt{x}}}{\frac{1}{-(\ln x)^2\cdot x}} = \lim_{x\to 0^+} \frac{1}{2\sqrt{x}} \cdot \frac{-\ln^2 x\cdot x}{1}$

$= \lim_{x\to 0^+} \frac{-\ln^2 x\cdot \cancel{x}\cdot\sqrt{x}}{2\cancel{x}} = \frac{-\infty\cdot 0}{2} = 0$

$$49 \quad \lim_{x\to\infty} (x - \ln x) \quad , \quad +\infty - \infty$$

$$= \lim_{x\to\infty} \ln e^x - \ln x = \lim_{x\to\infty} \ln \frac{e^x}{x} = \ln\left(\lim_{x\to\infty} \frac{e^x}{x}\right) \overset{L.H.}{=} \ln\left(\lim_{x\to\infty} \frac{e^x}{1}\right)$$

$\frac{\infty}{\infty}$

$$= \ln\left(\frac{\infty}{1}\right) = \ln \infty = \infty$$

$$51 \quad \lim_{x\to 0^+} x^{x^2} \quad , \quad 0^{0^2} \qquad y = x^{x^2} \to \ln(y) = \ln(x^{x^2})$$

$$\ln(y) = x^2 \cdot \ln x$$

$$\lim_{x\to 0^+} \ln(y) = \lim_{x\to 0^+} (x^2 \cdot \ln x) = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x^2}} \overset{L.H.}{=} \lim_{x\to 0^+} \frac{\frac{1}{x}}{-2\cdot x^{-3}}$$

$$= \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-2}{x^3}} = \lim_{x\to 0^+} \frac{1}{x} \cdot \frac{x^3}{-2} = \lim_{x\to 0^+} \frac{x^3}{-2x} = \lim_{x\to 0^+} \frac{x^2}{-2} = 0 = y$$

$$\lim_{x\to 0^+} y = \lim_{x\to 0^+} e^{\ln(y)} = e^{(\lim_{x\to 0^+} \ln(y))} = e^0 = 1$$

$$53 \quad \lim_{x\to 0} (1-2x)^{\frac{1}{x}} \quad , \quad 1^\infty \qquad y = (1-2x)^{\frac{1}{x}} \to \ln(y) = \frac{1}{x} \cdot \ln(1-2x)$$

$\frac{0}{0}$

$$\lim_{x\to 0} \ln(y) = \lim_{x\to 0} \frac{1}{x} \cdot \ln(1-2x) = \lim_{x\to 0} \frac{\ln(1-2x)}{x} \overset{L.H.}{=} \lim_{x\to 0} \frac{\frac{1}{1-2x}\cdot -2}{1}$$

$$= \lim_{x\to 0} \frac{-2}{1-2x} = \frac{-2}{1} = -2$$

$$\lim_{x\to 0} y = \lim_{x\to 0} e^{\ln(y)} = e^{(\lim_{x\to 0} \ln(y))} = e^{-2} = \frac{1}{e^2}$$

$$57 \quad \lim_{x\to\infty} x^{\frac{1}{x}} \quad , \quad \infty^0 \qquad y = x^{\frac{1}{x}} \to \ln(y) = \frac{1}{x} \ln x$$

$$\lim_{x\to\infty} \ln(y) = \lim_{x\to\infty} \frac{\ln(x)}{x} \overset{L.H.}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = \lim_{x\to\infty} \frac{1}{x} = 0$$

$$\lim_{x\to\infty} y = \lim_{x\to\infty} e^{(\ln(y))} = e^{(\lim_{x\to\infty} \ln(y))} = e^0 = 1$$