calculo 2, stewart vol. 1, ed. 8, cap 7.2

1 $\int sen^3(x) \cdot cos^2(x)\, dx = \int sen^2(x) \cdot sen(x) \cdot cos^2(x)\, dx$

$= \int (1-cos^2(x)) \cdot sen(x) \cdot cos^2(x)\, dx$

$\hookrightarrow u = cos(x) \qquad du = -sen(x)\, dx$

$= \int (1-u^2) \cdot u^2 \cdot sen(x)\, dx = -\int (1-u^2) \cdot u^2 \cdot du$

$= -\int u^2 - u^4\, du = -\left(\int u^2\, du - \int u^4\, du\right) = -\left(\frac{u^3}{3} - \frac{u^5}{5}\right) + c$

$= \frac{u^5}{5} - \frac{u^3}{3} + c = \frac{cos^5(x)}{5} - \frac{cos^3(x)}{3} + c$ //

3 $\int_0^{\pi/2} sen^7(x) \cdot cos^5(x)\, dx \Rightarrow \int sen^7(x) \cdot cos^5(x)\, dx$

$= \int (sen^2(x))^3 \cdot sen(x) \cdot cos^5(x)\, dx = \int (1-cos^2(x))^3 \cdot cos^5(x) \cdot sen(x)\, dx$

$\hookrightarrow u = cos(x) \qquad du = -sen(x)$

$-\int (1-u^2)^3 \cdot u^5\, du = -\int (1-2u^2+u^4) \cdot (1-u^2) \cdot u^5\, du$

$= -\int (1-2u^2+u^4-u^2+2u^4-u^6) \cdot u^5\, du$

$= -\int (1-3u^2+3u^4-u^6) \cdot u^5\, du = -\int u^5 - 3u^7 + 3u^9 - u^{11}\, du$

$= -\left(\int u^5\, du - 3\int u^7\, du + 3\int u^9\, du - \int u^{11}\, du\right)$

$= -\frac{u^6}{6} + 3 \cdot \frac{u^8}{8} - 3 \cdot \frac{u^{10}}{10} + \frac{u^{12}}{12} + c$

$= \left(-\frac{cos^6(x)}{6} + \frac{3}{8} \cdot cos^8(x) - \frac{3}{10} \cdot cos^{10}(x) + \frac{cos^{12}(x)}{12} + c\right)\Bigg|_0^{\pi/2}$

$\Rightarrow \cancel{\left(-\frac{0}{6} + \frac{3}{8} \cdot 0 - \frac{3}{10} \cdot 0 + \frac{0}{12} + c\right)} - \left(-\frac{1}{6} + \frac{3}{8} \cdot 1 - \frac{3}{10} \cdot 1 + \frac{1}{12} \cancel{+c}\right)$

$= +\frac{1}{6} - \frac{3}{8} + \frac{3}{10} - \frac{1}{12} = \frac{1}{120}$

5 $\int sen^5(2x) \cdot cos^2(2x)\, dx = \frac{1}{2} \int sen^5(u) \cdot cos^2(u) \cdot du$

$u = 2x \qquad du = 2\, dx$

$= \frac{1}{2} \int \underbrace{(sen^2(u))^2}_{1-cos^2(u)} \cdot sen(u) \cdot cos^2(u)\, du \qquad K = cos(u) \qquad dK = -sen(u)\, du$

$$= \frac{1}{2}\int (1-\cos^2(u))^2 \cdot sen(u) \cdot \cos^2(u)\, du \qquad k = \cos(2x)$$

$$= -\tfrac{1}{2}\int (1-k^2)^2 \cdot k^2 \cdot dk = -\tfrac{1}{2}\int (1-2k^2+k^4)\cdot k^2\, dk$$

$$= -\tfrac{1}{2}\int k^2 - 2k^4 + k^6\, dk = -\tfrac{1}{2}\left(\int k^2\, dk - 2\int k^4\, dk + \int k^6 dk\right)$$

$$= -\frac{1}{2}\cdot\left(\frac{k^3}{3} - 2\cdot\frac{k^5}{5} + \frac{k^7}{7}\right) + c = -\frac{1}{6}\cdot k^3 + \frac{1}{5}k^5 - \frac{1}{14}k^7 + c$$

$$= -\frac{1}{6}\cos^3(2x) + \frac{1}{5}\cos^5(2x) - \frac{1}{14}\cos^7(2x) + c$$

7 $\int_0^{\pi/2} \cos^2(x)\, dx \Rightarrow \int \cos^2(x)\, dx$

$$= \int \frac{1}{2} + \frac{1}{2}\cos(2x)\, dx$$

$$= \int \frac{1}{2}\, dx + \frac{1}{2}\int \cos(\underbrace{2x}_{u})\, dx = \frac{1}{2}\cdot x + \frac{1}{2}\cdot\frac{1}{2}\int \cos(u)\, du$$

$$= \frac{x}{2} + \frac{1}{4}\cdot - sen(u) = \left(\frac{x}{2} - \frac{1}{4}\cdot sen(2x) + c\right)\Bigg|_0^{\pi/2}$$

$$\Rightarrow \left(\frac{\pi/2}{2} - \frac{sen(2\cdot\pi/\cancel{2})}{4} \cancel{+c}\right) - \left(\frac{0}{2} - \frac{sen(2.0)}{4} \cancel{+c}\right)$$

$$= \left(\frac{\pi}{4} - \frac{0}{4}\right) - \left(\cancel{-\frac{0}{4}}\right) = \frac{\pi}{4}$$

9 $\int_0^{\pi} \cos^4(\underbrace{2x}_{u})\, dx \Rightarrow \frac{1}{2}\int \cos^4(u)\, du$

$u = 2x \quad du = 2\, dx$

$$= \frac{1}{2}\int \left(\frac{1}{2} + \frac{1}{2}\cos(2u)\right)^2 du = \frac{1}{2}\cdot\int \frac{1}{4} + \cancel{2}\cdot\frac{1}{2}\cdot\frac{1}{\cancel{2}}\cos(2u) + \frac{\cos^2(2u)}{4}\, du$$

$$= \frac{1}{2}\left(\int \frac{1}{4}\, du + \frac{1}{2}\int \cos(2u)\, du + \frac{1}{4}\int \cos^2(2u)\, du\right)$$

$$\blacktriangleright \int \cos(\underbrace{2u}_{k})\, du = \frac{1}{2}\cdot - sen(k) = -\frac{sen(2u)}{2}$$

$$\blacktriangleright \int \cos^2(\underbrace{2u}_{k=2u})\,du = \frac{1}{2}\int \cos^2(k)\,dk = \frac{1}{2}\int \frac{1}{2} + \frac{1}{2}\cos(2k)\,dk$$

$$= \frac{1}{2}\left(\frac{1}{2}k + \frac{1}{2}\int \cos(2k)\,dk\right) = \frac{1}{4}\cdot k + \frac{1}{4}\cdot\frac{1}{2}\cdot -\mathrm{sen}(2k) = \frac{2u}{4} - \frac{\mathrm{sen}(4u)}{8}$$

$$\Rightarrow \frac{1}{2}\left(\frac{u}{4} + \frac{1}{2}\cdot\left(\frac{-\mathrm{sen}(2u)}{2}\right) + \frac{1}{4}\cdot\left(\frac{u}{2} - \frac{\mathrm{sen}(4u)}{8}\right)\right)$$

$$= \frac{u}{8} + \frac{-\mathrm{sen}(2u)}{8} + \frac{1}{8}\cdot\left(\frac{u}{2} - \frac{\mathrm{sen}(4u)}{8}\right) = \frac{x}{4} - \frac{\mathrm{sen}(4x)}{8} + \frac{x}{8} - \frac{\mathrm{sen}(8x)}{8}$$

aplicando os limites de definição

$$= \left(\frac{\pi}{4} - \cancel{\frac{0}{8}} + \frac{\pi}{8} - \cancel{\frac{0}{8}} + \cancel{c}\right) - \left(\frac{0}{4} - \frac{0}{8} + \frac{0}{8} - \frac{0}{8} + \cancel{c}\right)$$

$$= \frac{\pi}{4} + \frac{\pi}{8} = \frac{3\pi}{8}$$

11 $\displaystyle\int_0^{\pi/2} \mathrm{sen}^2(x)\cdot\cos^2(x)\,dx \Rightarrow \int \left(\frac{1}{2} - \frac{1}{2}\cos(2x)\right)\cdot\left(\frac{1}{2} + \frac{1}{2}\cos(2x)\right)\,dx$

$$= \int \frac{1}{4} - \frac{1}{4}\cos^2(2x)\,dx$$

$$= \tfrac{1}{4}\cdot x - \tfrac{1}{4}\int \cos^2(2x)\,dx = x/4 - \tfrac{1}{4}\cdot\left(x/2 - \mathrm{sen}(4x)/8\right)$$

$$= \left(\frac{x}{4} - \frac{x}{8} + \frac{\mathrm{sen}(4x)}{32}\right)\Bigg|_0^{\pi/2}$$

$$= \left(\frac{\pi}{8} - \frac{\pi}{16} + \frac{0}{32} + \cancel{c}\right) - \left(\frac{0}{8} - \frac{0}{16} + \frac{0}{32} + \cancel{c}\right)$$

$$= \frac{\pi}{8} - \frac{\pi}{16} = \frac{\pi}{16}$$

13 $\int \sqrt{\cos\theta}\ \sin^3\theta\ d\theta = \int \cos^{\frac{1}{2}}\theta \cdot (1-\cos^2\theta) \cdot \operatorname{sen}\theta\ d\theta$

$= \int \cos^{\frac{1}{2}}\theta \cdot \operatorname{sen}\theta\ d\theta - \int \cos^2\theta \cdot \cos^{\frac{1}{2}}\theta \cdot \operatorname{sen}\theta\ d\theta$

$u = \cos\theta \qquad du = -\operatorname{sen}\theta\ d\theta$

$= -\int u^{\frac{1}{2}}\ du + \int u^2 \cdot u^{\frac{1}{2}}\ du = -\frac{u^{3/2}}{3/2} + \frac{u^{7/2}}{7/2} + c =$

$= -\frac{2}{3} \cdot \cos^{\frac{3}{2}}\theta + \frac{2}{7}\cos^{\frac{7}{2}}\theta + c$

15 $\int \operatorname{cotg}(x) \cdot \cos^2(x)\ dx \qquad \operatorname{cotg}(x) = \frac{\cos(x)}{\operatorname{sen}(x)}$

$= \int \frac{\cos^3(x)\ dx}{\operatorname{sen}(x)} = \int \frac{(1-\operatorname{sen}^2(x)) \cdot \cos(x)}{\operatorname{sen}(x)} = \int \frac{\cos(x) - \operatorname{sen}^2(x)\cos(x)}{\operatorname{sen}(x)}\ dx$

$= \int \frac{\cos(x)\ dx}{\operatorname{sen}(x)} - \int \frac{\operatorname{sen}^{\cancel{2}}(x)\cos(x)\ dx}{\cancel{\operatorname{sen}(x)}} = \int \frac{1}{u}\ du - \int u\ du$

$u = \operatorname{sen}(x) \qquad du = \cos(x)\ dx$

$= \ln|u| - \frac{u^2}{2} = \ln|\operatorname{sen}(x)| - \frac{\operatorname{sen}^2(x)}{2} + c$

17 $\int \operatorname{sen}^2(x)\ \operatorname{sen}(2x)\ dx = \int \operatorname{sen}^2(x) \cdot 2\operatorname{sen}(x)\cos(x)\ dx$

$= 2\int \operatorname{sen}^3(x)\cos(x)\ dx = 2\int u^3\ du = \frac{2u^4}{4} + c = \frac{\operatorname{sen}^4(x)}{2} + c$

$u = \operatorname{sen}(x) \qquad du = \cos(x)\ dx$

19 $\int x \cdot \operatorname{sen}^2(x)\ dx = [\ldots]$

$f(x) = x \qquad f'(x) = 1$

$g'(x) = \operatorname{sen}^2(x) \qquad g(x) = \underbrace{\int \operatorname{sen}^2(x)\ dx}_{①} = \frac{x}{2} - \frac{\operatorname{sen}(x)\cos(x)}{2}$

① $\int \operatorname{sen}^2(x)\ dx = \int \frac{1}{2}\ dx - \frac{1}{2}\int \cos(2x)\ dx = \frac{x}{2} - \frac{\operatorname{sen}(2x)}{4}$

$[\ldots] = \frac{x^2}{2} - \frac{x\operatorname{sen}(2x)}{4} - \int \frac{x}{2} - \frac{\operatorname{sen}(2x)}{4}\ dx = \frac{x^2}{2} - \frac{x\operatorname{sen}(2x)}{2} - \frac{1}{2} \cdot \frac{x^2}{2} - \int \operatorname{sen}(2x)\ dx$

$= \frac{x^2}{2} - \frac{x\operatorname{sen}(2x)}{4} - \frac{x^2}{4} - 2\int \underbrace{\operatorname{sen}(x)}_{u}\ \underbrace{\cos(x)\ dx}_{du} = \frac{x^2}{4} - \frac{x\operatorname{sen}(2x)}{4} - \operatorname{sen}^2(x) + c$

21 $\int tg(x)\ sec^3(x)\ dx = \int tg(x)\ sec^2(x)\ sec(x)\ dx$

$u = sec(x) \qquad du = sec(x)\ tg(x)\ dx$

$= \int u^2\ sec(x)\ tg(x)\ dx = \int u^2\ du = \frac{u^3}{3} + c = \frac{sec^3(x)}{3} + c$

23 $\int tg^2(x)\ dx = \int sec^2(x) - 1\ dx = \int sec^2(x) dx - \int 1\ dx = tg(x) - x + c$

25 $\int tg^4(x)\ sec^6(x)\ dx$

$= \int tg^4(x) \cdot (sec^2(x))^2 \cdot sec^2(x)\ dx = \int tg^4(x) \cdot (tg^2(x) + 1)^2 \cdot sec^2(x)\ dx$

$= \int tg^4(x) \cdot (tg^4(x) + 2\ tg^2(x) + 1) \cdot sec^2(x)\ dx$

$= \int tg^8(x)\ sec^2(x)\ dx + 2 \int tg^6(x)\ sec^2(x)\ dx + \int tg^4(x)\ sec^2(x)\ dx$

$u = tg(x) \qquad du = sec^2(x)\ dx$

$= \int u^8\ du + 2 \int u^6\ du + \int u^4\ du$

$= \frac{u^9}{9} + 2\frac{u^7}{7} + \frac{u^5}{5} = \frac{tg^9(x)}{9} + 2\frac{tg^7(x)}{7} + \frac{tg^5(x)}{5} + c$

27 $\int tg^3(x)\ sec(x)\ dx = \int tg(x)\ sec(x) \cdot tg^2(x)\ dx$

$= \int tg(x)\ sec(x) \cdot (sec^2(x) - 1)\ dx = \int u^2 - 1\ du = \int u^2\ du - \int 1\ du$

$u = sec(x) \qquad du = sec(x)\ tg(x)\ dx$

$= \frac{u^3}{3} - u + c = \frac{sec^3(x)}{3} - sec(x) + c$

29 $\int tg^3(x)\ sec^6(x)\ dx = \int tg^3(x) \cdot sec^2(x) \cdot (tg^2(x) + 1)^2\ dx$

$= \int tg^3(x)\ sec^2(x) \cdot (tg^4(x) + 2\ tg^2(x) + 1)\ dx$

$= \int tg^7(x) \cdot sec^2(x)\ dx + 2 \int tg^5(x) \cdot sec^2(x)\ dx + \int tg^3(x)\ sec^2(x)\ dx$

$u = tg(x) \qquad du = sec^2(x)\ dx$

$= \int u^7\ du + 2 \int u^5\ du + \int u^3\ du = \frac{u^8}{8} + 2 \cdot \frac{u^6}{6} + \frac{u^4}{4} + c$

$= \frac{tg^8(x)}{8} + \frac{tg^6(x)}{3} + \frac{tg^4(x)}{4} + c$

31 $\int tg^5(x)\,dx = \int tg^3(x)\,(sec^2(x)-1)\,dx$

$= \int tg^3(x)\,sec^2(x)\,dx - \int tg^3(x)\,dx = \int u^3\,du - \int tg(x)\,(sec^2(x)-1)\,dx$

$u = tg(x) \quad du = sec^2(x)\,dx$

$= \dfrac{u^4}{4} - \int tg(x)\,sec^2(x)\,dx - \int tg(x)\,dx = \dfrac{u^4}{4} - \int u\,du - \int \dfrac{sen(x)}{cos(x)}\,dx$

$v = cos(x) \qquad dv = -sen(x)\,dx$

$= \dfrac{u^4}{4} - \dfrac{u^2}{2} + \int \dfrac{1}{v}\,dv = \dfrac{tg^4(x)}{4} - \dfrac{tg^2(x)}{2} + \ln|cos(x)| + c$

33 $\int x\,sec(x)\cdot tg(x)\,dx$

$f(x) = x \qquad f'(x) = 1$

$g'(x) = sec(x)\,tg(x) \qquad g(x) = sec(x)$

$= x\,sec(x) - \int sec(x)\,dx = x\,sec(x) - \ln|sec(x) + tg(x)| + c$

35 $\int_{\pi/6}^{\pi/2} cotg^2(x)\,dx \qquad cossec^2(x) = 1 + cotg^2(x)$

$= \int cotg^2(x)\,dx = \int (cossec^2(x) - 1)\,dx = \int cossec^2(x)\,dx - \int 1\,dx$

$= -cotg\,x - x + c$

aplicando os limites de integração

$(-cotg(\pi/2) - \pi/2) - (-cotg(\pi/6) - \pi/6)$

$cotg(x) = \dfrac{1}{tg(x)} \qquad \lim_{x\to\pi/2} tg(x) = \pm\infty \qquad \lim_{x\to\infty} \dfrac{1}{tg(x)} = 0$

$cotg(\pi/6) = \dfrac{1}{tg(\pi/6)} = \dfrac{1}{tg(30°)} = \dfrac{1}{\sqrt{3}/3} = \dfrac{3}{\sqrt{3}}$

$= -\dfrac{\pi}{2} + \dfrac{\pi}{6} + \dfrac{3}{\sqrt{3}} = -\dfrac{2\pi}{6} \cdot \sqrt{3} = \sqrt{3} - \dfrac{\pi}{3}$

37 $\int_{\pi/4}^{\pi/2} cotg^5(x)\,cossec^3(x)\,dx$

$\int cotg^5(x)\,cossec^3(x)\,dx = \int (cotg^2(x))^2\cdot cossec^2(x)\cdot cotg(x)\,cossec(x)\,dx$

$= \int (cossec^2 - 1)^2 \cdot cossec^2 \cdot cotg\,cossec\,dx$

$= \int (cossec^4 - 2\,cossec^2 + 1)\cdot cossec^2 \cdot cotg\,cossec\,dx$

$= \int cossec^6 \cdot cotg\,cossec\,dx - 2\int cossec^4 \cdot cotg\,cossec\,dx + \int cossec^2 \cdot cotg\,cossec\,dx$

$u = \text{cossec}(x) \qquad du = -\text{cossec}(x)\,\text{cotg}(x)\,dx$

$= -\int u^6\,du + 2\int u^4\,du - \int u^2\,du$

$= -\frac{u^7}{7} + \frac{2u^5}{5} - \frac{u^3}{3} + c = -\frac{\text{cossec}^7(x)}{7} + \frac{2\,\text{cossec}^5(x)}{5} - \frac{\text{cossec}^3(x)}{3} + c$

aplicando os limites de integração

$\left(-\frac{\text{cossec}^7(\pi/2)}{7} + \frac{2\,\text{cossec}^5(\pi/2)}{5} - \frac{\text{cossec}^3(\pi/2)}{3}\right) - \left(-\frac{\text{cossec}^7(\pi/4)}{7} + \frac{2\,\text{cossec}^5(\pi/4)}{5}\right.$

$\left.- \frac{\text{cossec}^3(\pi/4)}{3}\right) =$

$\text{cossec}\left(\frac{\pi}{2}\right) = \frac{1}{\text{sen}(\pi/2)} = \frac{1}{1} = 1$

$\text{cossec}\left(\frac{\pi}{4}\right) = \frac{1}{\text{sen}(\pi/4)} = \frac{1}{\sqrt{2}/2} = \frac{2}{\sqrt{2}} = \sqrt{2}$

$= -\frac{1}{7} + \frac{2\cdot 1}{5} - \frac{1}{3} + \frac{\sqrt{2}^7}{7} - \frac{2\sqrt{2}^5}{5} + \frac{\sqrt{2}^3}{3} = \frac{-8+22\sqrt{2}}{105}$