cálculo 1, stewart, vol 1, ed 5, cap 5.5

1 $\int \cos 3x \, dx$, $u = 3x$

$= \int \cos(3x) \cdot \frac{3}{3} \, dx = \frac{1}{3} \int \cos \underbrace{(3x)}_{u} \cdot \underbrace{3 \, dx}_{du} = \operatorname{sen}(3x) + c \cdot \frac{1}{3}$

$\int \cos(u) \cdot du = \operatorname{sen}(u) + c$

3 $\int x^2 \sqrt{x^3+1} \, dx$, $u = x^3 + 1$ $\quad \frac{du}{dx}(x^3+1) = 3x^2$

$du = 3x^2 \, dx$

$\int \sqrt{x^3+1} \cdot \frac{3}{3} x^2 \, dx = \frac{1}{3} \int \underbrace{\sqrt{x^3+1}}_{u} \cdot \underbrace{3x^2 \, dx}_{du} = \frac{1}{3} \, 2 \cdot \frac{(x^3+1)^{\frac{3}{2}}}{3} + c$

$\int \sqrt{u} \, du = \frac{u^{\frac{3}{2}}}{\frac{3}{2}} = 2 \frac{u^{\frac{3}{2}}}{3} + c$

5 $\int \frac{4}{(1+2x)^2} \, dx$, $u = 1 + 2x$ $\quad \frac{du}{dx}(1+2x) = 2$ $\quad du = 2 \, dx$

$2 \cdot \int \frac{1}{\underbrace{(1+2x)^2}_{u^2}} \cdot \underbrace{2 \, dx}_{du} \Rightarrow \int \frac{1}{u^2} \cdot du = -u^{-1} + c = -\frac{1}{u} + c$

$\hookrightarrow 2 \cdot \left(\frac{-1}{1+2x} + c \right) = \frac{-2}{1+2x} + c$

7 $\int 2x (x^2+3)^4 \, dx$, $u = x^2 + 3$ $\quad \frac{du}{dx}(x^2+3) = 2x$ $\quad du = 2x \, dx$

$\Rightarrow \int (u)^4 \cdot du = \frac{u^5}{5} + c \rightarrow \frac{(x^2+3)^5}{5} + c$

9 $\int (3x-2)^{20} \, dx$, $u = 3x - 2$ $\quad \frac{du}{dx}(3x-2) = 3$

$du = 3 \, dx$

$\frac{1}{3} \int (3x-2)^{20} \cdot 3 \, dx \quad \frac{1}{3} \int u^{20} \cdot du = \frac{1}{3} \frac{u^{21}}{21} + c$

$= \frac{1}{3} \frac{(3x-2)^{21}}{21} + c$

11 $\int \frac{1+4x}{\sqrt{1+x+2x^2}}\,dx$, $u = 1+x+2x^2$ $\quad \frac{du}{dx}(1+x+2x^2) = 1+4x$

$du = 1+4x\,dx$

$$\int \frac{1}{\sqrt{u}}\,du = \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + c = 2\sqrt{u} + c = 2\sqrt{1+x+2x^2} + c$$

13 $\int \frac{dx}{5-3x} = \int \frac{1}{5-3x}\,dx = -\frac{1}{3}\int \frac{1}{\underbrace{5-3x}_{u}} \cdot \underbrace{-3\,dx}_{du}$

$u = 5-3x \quad \frac{du}{dx}(5-3x) = -3$

$du = -3\,dx$

$$-\frac{1}{3}\int \frac{1}{u} \cdot du = -\frac{1}{3} \cdot |\ln(|u|) + c| = -\frac{1}{3} \cdot \ln(|5-3x|) + c$$

17 $\int \sqrt{4-x}\,dx$, $u = 4-x \quad \frac{du}{dx}(4-x) = -1 \quad du = -1\,dx$

$$-1\int \underbrace{\sqrt{4-x}}_{} \cdot \underbrace{-1\,dx}_{} = -1\int \sqrt{u} \cdot du = -1 \cdot \left(\frac{2u^{\frac{3}{2}}}{3} + c\right)$$

$$= -\frac{2 \cdot (4-x)^{\frac{3}{2}}}{3} - c$$

19 $\int sen(\pi \cdot t)\,dt$, $u = \pi \cdot t \quad \frac{du}{dt}(\pi \cdot t) = \pi \quad du = \pi\,dt$

$$\frac{1}{\pi}\int sen(\underbrace{\pi \cdot t}_{})\,\underbrace{\pi\,dt}_{} = \frac{1}{\pi}\int sen(u) \cdot du = \frac{1}{\pi}\left(-\cos(u) + c\right) = \frac{-\cos(\pi \cdot t)}{\pi} + c$$

21 $\int \frac{(\ln x)^2}{x}\,dx$, $u = \ln x \quad \frac{du}{dx}(\ln x) = \frac{1}{x}, \quad du = \frac{1}{x}\,dx$

$$\int \underbrace{(\ln x)^2}_{u}\,\underbrace{\frac{1}{x}\,dx}_{du} = \int u^2\,du = \frac{u^3}{3} + c = \frac{(\ln x)^3}{3} + c = \frac{(\ln x)^3}{3} + c$$

23 $\int \frac{\cos\sqrt{t}}{\sqrt{t}}\,dt$, $u = \sqrt{t}$ $\quad \frac{du}{dx}(\sqrt{t}) = \frac{1}{2}t^{-\frac{1}{2}} = \frac{1}{2\sqrt{t}}$, $du = \frac{1}{2\sqrt{t}}\,dx$

$\int \cos\sqrt{t} \cdot \frac{1}{\sqrt{t}}\,dt = 2\int \cos\underbrace{(\sqrt{t})}_{u} \cdot \frac{1}{2\sqrt{t}}\,dt$ (with $\frac{1}{2\sqrt{t}}\,dt \to du$)

$= 2\int \cos u \cdot du = 2\cdot(\operatorname{sen}(u) + c) = 2\operatorname{sen}(\sqrt{t}) + c$

27 $\int e^x \cdot \sqrt{1+e^x}\,dx$, $u = 1+e^x$ $\quad \frac{du}{dx}(1+e^x) = e^x$ $\quad du = e^x\,dx$

$\int \sqrt{u}\,du = \frac{2\,u^{3/2}}{3} + c = \frac{2\sqrt[2]{(1+e^x)^3}}{3}$

31 $\int \frac{dx}{x\ln x}$, $u = \ln x$ $\quad \frac{du}{dx}(\ln x) = \frac{1}{x}$ $\quad du = \frac{1}{x}\,dx$

$= \int \frac{1}{\ln x}\cdot\frac{1}{x}\,dx = \int \frac{1}{u}\,du = \ln(|u|) + c = \ln(|\ln(x)|) + c$

33 $\int \sqrt{\operatorname{cotg} x}\cdot \operatorname{cossec}^2 x\,dx$, $u = \operatorname{cotg} x$, $\frac{du}{dx}(\operatorname{cotg} x) = -\operatorname{cossec}^2 x$

$du = -\operatorname{cossec}^2 x\,dx$

$\int \sqrt{u}\,du = \frac{-2\,u^{\frac{3}{2}}}{3} + c = \frac{-2\cdot\sqrt[2]{(\operatorname{cotg} x)^3}}{3} + c$

35 $\int \operatorname{cotg} x\,dx = \int \frac{\cos x}{\operatorname{sen} x}\,dx = \int \frac{1}{\operatorname{sen} x}\cos x\,dx$

$u = \operatorname{sen} x$ $\quad \frac{du}{dx}(\operatorname{sen} x) = \cos x$ $\quad du = \cos x\,dx$

$\int \frac{1}{u}\,du = \ln(|u|) + c = \ln(|\operatorname{sen} x|) + c$