revisão – cálculo

① a) $(f+g)(x) = f(x) + g(x) = x + x^2 - 1$

$\hookrightarrow D(f+g)(x) = \{x \in \mathbb{R}\} = \mathbb{R}$

$\frac{g}{f} = \frac{x^2 - 1}{x} \rightarrow D(g/f)(x) = \{x \in \mathbb{R} \mid x \neq 0\}$

b) $f+g = x + \frac{1}{\sqrt{x}} = \frac{\sqrt{x} \cdot x + 1}{\sqrt{x}} \rightarrow D(f+g)(x) = \{x \in \mathbb{R} \mid x > 0\}$

$\frac{g}{f} = \frac{1/\sqrt{x}}{x/1} = \frac{1}{\sqrt{x}} \cdot \frac{1}{x} = \frac{1}{\sqrt{x} \cdot x} \rightarrow D(f/g)(x) = \{x \in \mathbb{R} \mid x > 0\}$

c) $f+g = 1 + \sqrt{x-1} \rightarrow D(f+g)(x) = \{x \in \mathbb{R} \mid x \geq 1\}$

$g/f = \frac{\sqrt{x-1}}{1} \rightarrow D(f/g)(x) = \{x \in \mathbb{R} \mid x \geq 1\}$

d) $f+g = 1 + \frac{1}{(x-2)^2} = \frac{(x-2)^2 \cdot 1 + 1}{(x-2)^2} \rightarrow (x-2)^2 \neq 0$

$x^2 - 2 \cdot x \cdot 2 + 4 \neq 0$

$x^2 - 4x + 4 \neq 0$

$x' \neq 2 \quad x'' \neq 2$

$\rightarrow D(f+g)(x) = \{x \in \mathbb{R} \mid x \neq 2\}$

$g/f = \frac{1}{(x-2)^2} \cdot \frac{1}{1} = \frac{1}{(x-2)^2} \rightarrow D(f/g)(x) =$

e) $|x| + |x-3| \rightarrow D(f+g)(x) = \{x \in \mathbb{R}\}$

$g/f = \frac{|x-3|}{|x|} \rightarrow D(f/g)(x) = \{x \in \mathbb{R} \mid x \neq 0\}$

$Q(t) = Q_0 \, e^{k \cdot t}$ → valor inicial

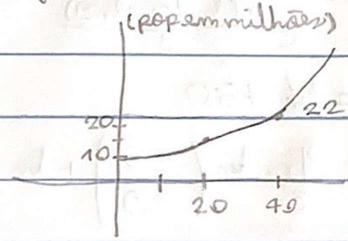
2 a)

– A população atual é de 10 milhões de habitantes

– Em 20 anos será:

$P(20) = 10 \cdot e^{0{,}02 \cdot 20} = 10 \cdot e^{0{,}4} \cong 14{,}91 \cong 15$ milhões de habitantes

– gráfico de $P(t) = 10 \cdot e^{0{,}02 \cdot t}$:



b)

$Q(t) = Q_0 \cdot e^{k \cdot t}$ → 3000 $\qquad \boxed{9000 = 3000 \cdot e^{k \cdot 30}}$

$\frac{9000}{3000} = 3 = e^{k \cdot 30} \qquad \log_a b = x \quad a^x = b$

→ $\log_e 3 = k \cdot 30 \quad \Rightarrow \boxed{\log_e x = \ln x}$

$\ln 3 = k \cdot 30 \qquad k = \frac{\ln 3}{30}$

$Q(t) = 3000 \cdot e^{\left(\frac{\ln 3}{30} \cdot 60\right)} = 3000 \cdot e^{(\ln 3 \cdot 2)} = 3000 \cdot e^{\ln 3^2}$

$= 3000 \cdot e^{\ln 9} = 3000 \cdot 9$

c) $20000 = Q_0 \cdot e^{k \cdot 10} \qquad 25000 = Q_0 \cdot e^{k \cdot 15}$

$Q_0 = \frac{20000}{e^{k \cdot 10}} = Q_0 = \frac{25000}{e^{k \cdot 15}}$

$\frac{4}{5} = \frac{e^{k \cdot 10}}{e^{k \cdot 15}} = e^{k \cdot 10 - k15} = e^{k \cdot 5} \qquad \ln e^{k \cdot 5} = \ln \frac{5}{4} \qquad k \cdot 5 \cdot 1 = \ln 5/4$

$k = \ln \frac{5}{4} / 5$

$20000 = Q_0 \cdot e^{\frac{\ln(5/4)}{5} \cdot 10} = Q_0 \cdot e^{\ln(5/4)^2}$

$20000 = Q_0 \cdot \left(\frac{5}{4}\right)^2 = Q_0 \cdot \frac{25}{16} \qquad Q_0 = 12800$ habitantes em 1980

③ a) $Q(t) = Q_0 \cdot e^{-Kt}$
$\quad \hookrightarrow 750$

$290 = 750 \cdot e^{-K \cdot 3} = 750 \cdot e^{-3K}$

$290 = 750 \cdot \frac{1}{e^{3K}}$ $\quad e^{3K} = \frac{75}{29}$

$3K \cdot \cancel{\ln e}^{1} = \ln 75 - \ln 29$

$3K = \ln 75 - \ln 29$

$K = \frac{\ln 75 - \ln 29}{3}$

$\underbrace{Q(5)}_{y} = 750 \cdot e^{-\left(\frac{\ln 75 - \ln 29}{3}\right) \cdot 5}$

$y = 750 \cdot \frac{1}{e^{\frac{5(\ln 75 - \ln 29)}{3}}}$

$\ln\left(e^{\frac{5(\ln 75 - \ln 29)}{3}} \cdot y\right) = \ln 750$

$\frac{5(\ln 75 - \ln 29)}{3} \cdot \cancel{\ln e}^{1} + \ln y = \ln 750 \quad \Rightarrow \quad \frac{1}{3} \cdot \ln\left(\frac{75^5}{29^5}\right) + \ln y = \ln(750)$

$\Rightarrow \cancel{\ln}\left(\frac{75^{5/3}}{29^{5/3}} \cdot y\right) = \cancel{\ln}(750) \quad \Rightarrow \quad y \cong 153,91$ toneladas

b) $e^{-0,2582 \cdot t} = \frac{1}{2} \quad \Rightarrow \quad \frac{1}{e^{-0,2582 \cdot t}} = \frac{1}{2}$

$e^{+0,2582 \cdot t} = 2 \quad \Rightarrow \quad +0,2582 \cdot t = \ln 2$

$\cong 2,68 \cong 2,7$ anos

④ a) PAR $\to f(x) = f(-x)$

ÍMPAR $\to f(-x) = -f(x)$

$f(1) = 2 - 3 + 1 = 0$
$f(-1) = 2 - 3 + 1 = 0$
função par

b) $f(x) = 5x^3 - 7x$

$f(-x) = 5(-x)^3 - 7\cdot -x = -5x^3 + 7x$

função ímpar

c) $f(x) = x^6 - 1$

$f(-x) = (-x)^6 - 1 = x^6 - 1$

função par

d) $f(y) = \frac{y^3 - y}{y^2 + 1}$ $\quad f(-y) = \frac{-y^3 + y}{(-y)^2 + 1} = \frac{-y^3 + y}{y^2 + 1}$

nenhuma

e) $f(x) = \frac{x - 1}{x + 1}$ $\quad f(-x) = \frac{-x - 1}{-x + 1}$

nenhuma

⑤ a) $f \circ g = 2(3x+2)^2 - 3x + 2$

$= 2(9x^2 + 12x + 4) - 3x + 2 = 18x^2 + 24x + 8 - 3x + 2$

$= 18x^2 + 21x + 10 \rightarrow D(f \circ g)(x) = \mathbb{R}$

$g \circ f = 3(2x^2 - x) + 2 = 6x^2 - 3x + 2 \rightarrow D(g \circ f)(x) = \mathbb{R}$

$f \circ f = 2(2x^2 - x)^2 - 2x^2 - x = 2 \cdot (4x^4 - 4x^3 + x^2) - 2x^2 - x$

$= 8x^4 - 8x^3 + x^2 - 2x^2 - x$

$= 8x^4 - 8x^3 - x^2 - x \rightarrow D(f \circ f)(x) = \mathbb{R}$

$g \circ g = 3(3x+2) + 2 = 9x + 8 \rightarrow D(g \circ g)(x) = \mathbb{R}$

b) $f \circ g = 1 - \left(\frac{1}{x}\right)^2 = 1 - \frac{1}{x^2} = \frac{x^2 - 1}{x^2} \rightarrow D = \{x \in \mathbb{R} \mid x \neq 0\}$

$g \circ f = \frac{1}{1 - x^2} \rightarrow D = \{x \in \mathbb{R} \mid x \neq 1\} = \mathbb{R} - 1$

$f \circ f = 1 - (1 - x^2)^2 = 1 - (1 - 2x^2 - x^4) \rightarrow D = \mathbb{R}$

$g \circ g = \frac{1}{1/x} = x \rightarrow D = \mathbb{R}$

c) $f \circ g = \left(\frac{x+1}{x+3}\right) + \frac{1}{\left(\frac{x+1}{x+3}\right)} = \frac{x+1}{x+3} + \frac{x+3}{x+1} \rightarrow D = \{x \in \mathbb{R} \mid x \neq -3 \text{ e } -1\}$

$g \circ f = \frac{(x + \frac{1}{x}) + 1}{(x + \frac{1}{x}) + 3} = \frac{\frac{x^2 + 1 + x}{x}}{\frac{x^2 + 1 + 3x}{x}} = \frac{x^2 + 1 + x}{x^2 + 1 + 3x} \rightarrow x = \frac{-3 \pm \sqrt{5}}{2}$

$D(g \circ f)(x) = \{x \in \mathbb{R} \mid x \neq \frac{-3 \pm \sqrt{5}}{2}\}$

$f \circ f = \left(x + \frac{1}{x}\right) + \frac{1}{x + \frac{1}{\cancel{x}}} = \frac{x^2 + 1}{x} + \left(\frac{x}{x^2 + 1}\right) \rightarrow D = \{x \in \mathbb{R} \mid x \neq 0\}$

$g \circ g = \frac{\frac{x+1}{x+3} + 1}{\frac{x+1}{x+3} + 3} = \frac{\frac{x+1+x+3}{x+3}}{\frac{x+1+3x+9}{x+3}} = \frac{2x+4}{4x+10 \neq 0} \rightarrow D = \{x \in \mathbb{R} \mid x \neq \frac{-5}{2}\}$

$x \neq \frac{-10}{4} \rightarrow \frac{-5}{2}$

⑥ a) $2((1-x)^2)-1 = 2\cdot(1-2x+x^2)-1 = 2-4x+2x^2-1$

$= 2x^2-4x+1$

b) $\sqrt{((x+3)^2+2)-1} = \sqrt{((x^2+6x+9)+2)-1} = \sqrt{x^2+6x+10}$

c) $\dfrac{2}{\cos(\sqrt{x+3})+1}$

⑦ a) $g = x^2+1$ $\quad f = (x)^{10}$

b) $g = \sqrt{x}$ $\quad f = \operatorname{sen}(x)$

c) $g = x^2$ $\quad f = \dfrac{x}{x+4}$

d) $g = x+3$ $\quad f = 1/x$

e) $g = \cos(t)$ $\quad f = \sqrt{x}$

f) $g = \tan(t)$ $\quad f = \dfrac{x}{1+x}$

⑧ a) $g(x) = \dfrac{1}{x}$ $\quad g(f(x)) = \dfrac{1}{f(x)} = x$ $\quad f(x) = \dfrac{1}{x}$

b) $g(x) = \dfrac{x+2}{x+1}$ $\quad g(f(x)) = \dfrac{f(x)+2}{f(x)+1} = x$

$x\cdot f(x) + x\cdot 1 = f(x)+2$

$x\cdot f(x) - f(x) = 2-x$ $\quad f(x) = \dfrac{2-x}{x-1}$

$f(x)(x-1) = 2-x$

c) $g(x) = x^2$ $\quad g(f(x)) = f(x)^2 = x$

$f(x) = \sqrt{x}$

d) $g(f(x)) = f(x)^2 - 2f(x) = x$ $\quad y = x^2-2x$

$x^2-2x-y=0$

$x_{1,2} = \dfrac{+2 \pm \sqrt{(-2)^2-4\cdot 1\cdot(-y)}}{2} = \dfrac{2\pm\sqrt{4+4y}}{2} = \dfrac{2\pm\sqrt{4\cdot(1+y)}}{2}$

$= \dfrac{2\pm 2\sqrt{1+y}}{2} = \dfrac{2(1\pm\sqrt{1+y})}{2} = 1\pm\sqrt{1+y}$ $\quad f(x) = 1+\sqrt{1+x}$

tilibra

e) $g(f(x)) = 2 + \frac{3}{f(x)+1} = x \qquad \frac{3}{f(x)+1} = x-2$

$3 = (x-2)(f(x)+1) = x \cdot f(x) + x - 2f(x) - 2$

$3 - x + 2 = x \cdot f(x) - 2f(x)$

$3 - x + 2 = (x-2) \cdot f(x)$

$f(x) = \frac{3-x+2}{(x-2)} = \frac{5-x}{x-2}$

f) $g(f(x)) = f^2 - 4f + 3 = x \quad \longrightarrow \quad x^2 - 4x + 3 - y = 0$

$f \to x \qquad x \to y$

$x_{1,2} = \frac{+4 \pm \sqrt{16 - 4 \cdot 1 \cdot (3-y)}}{2} = \frac{+4 \pm \sqrt{16 - (12 - 4y)}}{2}$

$= \frac{+4 \pm \sqrt{16 - 12 + 4y}}{2} = \frac{4 \pm \sqrt{4 + 4y}}{2} = \frac{4 \pm \sqrt{4(1+y)}}{2}$

$= \frac{4 \pm 2\sqrt{1+y}}{2} = \frac{\cancel{2}(2 \pm \sqrt{1+y})}{\cancel{2}} = 2 \pm \sqrt{1+y}$

$f(x) = 2 + \sqrt{1+x}$

9 a) Domínio $(f(x)) = \{x \in \mathbb{R} \mid x \leq 10/3\}$

$f^{-1}(x) =$

$y = \sqrt{10 - 3x}$

$y^2 = 10 - 3x$

$y^2 - 10 = -3x$

$y^2 - 10/3 = -x$

$x = -(y^2 - 10)/3$

$x = \frac{10 - y^2}{3}$

$f^{-1}(x) = \frac{10 - x^2}{3}$

b) $D(f(x)) = \mathbb{R}$

$x = e^{y^3}$

$\ln x = \ln e^{y^3}$

$\ln x = y^3 \cdot \ln e^{\to 1}$

$f^{-1}(x) =$

$\ln x = y^3$

$y = \sqrt[3]{\ln x}$

$f^{-1}(x) = \sqrt[3]{\ln(x)}$

$\hookrightarrow D = \{x \in \mathbb{R} \mid x > 0\}$

c) $D(f(x)) = \{x \in \mathbb{R} \mid x > -3\}$ $\boxed{f^{-1}(x) = e^x - 3}$

$x + 3 > 0$

$x = \ln(y+3)$

$e^x = e^{\ln(y+3)}$ $y = e^x - 3$

$D(f^{-1}(x)) = \mathbb{R}$ $e^x = y + 3$

d) $D(f(x)) = \{x \in \mathbb{R} \mid x \neq -3/2\}$ $f^{-1}(x) =$

$x = \dfrac{4y - 1}{2y + 3}$

$2y \cdot x + 3x = 4y - 1$ $y = \dfrac{-3x - 1}{2x - 4}$

$2y \cdot x - 4y = -1 - 3x$

$y(2x - 4) = -3x - 1$ $D(y) = \{x \in \mathbb{R} \mid x \neq 2$

e) $D(y) = \{x \in \mathbb{R} \mid x \neq 0\}$

$f^{-1}(x) \Rightarrow$ $x = \dfrac{1 + e^y}{1 - e^y}$ $x(1 - e^y) = 1 + e^y$?

10 a) $2\ln(x) = 1$ $x^2 = e$

$\ln(x^2) = 1$ $x = \sqrt{e}$

$e^{\ln(x^2)} = e^1$

b) $e^{2x+3} = 7$ $2x + 3 = \ln 7$

$\ln e^{2x+3} = \ln 7$ $2x = \ln 7 - 3$

$2x + 3 \cdot \cancel{\ln e}^1 = \ln 7$ $x = \ln 7 - 3/2$

c) $\ln(5 - 2x) = -3$ $-2x = e^{-3} - 5$

$e^{\ln(5-2x)} = e^{-3}$ $+x = -\dfrac{(e^{-3} - 5)}{2} = \dfrac{-e^{-3} + 5}{2} = \dfrac{-\frac{1}{e^3} + 5}{2}$

$5 - 2x = e^{-3}$

d) $\ln(x) + \ln(x - 1) = 1$ $\Delta = 1 - 4 \cdot 1 \cdot -e = 1 + 4e$

$\ln(x \cdot (x - 1)) = 1$ $x_{1,2} = \dfrac{+1 \pm \sqrt{1 + 4e}}{2}$

$e^{\ln(x \cdot (x-1))} = e^1$

$x \cdot (x - 1) = e$ $x = \dfrac{1 + \sqrt{1 + 4e}}{2}$

$x^2 - x = e$ $x^2 - x - e = 0$

e) $2^{x-5} = 3$

$\log_2 2^{x-5} = \log_2 3$ $\qquad x = \log_2 3 + 5$

$x-5 \cdot \cancel{\log_2 2}^{1} = \log_2 3$

$x - 5 = \log_2 3$

f) $\ln(\ln(x)) = 1$ $\qquad e^{\ln x} = e^e$

$e^{\ln(\ln(x))} = e^1$ $\qquad x = e^e$

$\ln(x) = e$

11 a) $3 - e^{2x} \geqslant 0$ $\qquad \ln 3 \geqslant 2x$ $\qquad x \leq \frac{\ln 3}{2}$

$3 \geqslant e^{2x}$ $\qquad \frac{\ln 3}{2} \geqslant x$

$\ln 3 \geqslant \ln e^{2x}$ $\qquad D(f(x)) = \{x \in \mathbb{R} \mid x \leq \frac{\ln 3}{2}\}$

b) $2x - 1 > 0$

$2x > 1$ $\qquad D(f(x)) = \{x \in \mathbb{R} \mid x > 1/2\}$

$x > 1/2$