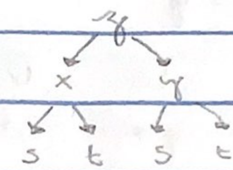
cálculo 2, stewart vol. 2, ed 8, cap 14.5

7 $z = (x-y)^5$, $x = s^2 t$, $y = st^2$



$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial t}$$

a) $\frac{\partial z}{\partial x} = 5(x-y)^4 \cdot 1$ b) $\frac{\partial z}{\partial y} = 5(x-y)^4 \cdot (-1)$

c) $\frac{\partial x}{\partial t} = s^2$ d) $\frac{\partial y}{\partial t} = 2ts$

$$\Rightarrow \frac{\partial z}{\partial t} = 5(x-y)^4 \cdot s^2 - 5(x-y)^4 \cdot 2st = 5(x-y)^4 \cdot (s^2 - 2st)$$

$$\Rightarrow \frac{\partial z}{\partial s} = 5(x-y)^4 \cdot 2st - 5(x-y)^4 \cdot t^2 = 5(x-y)^4 \cdot (2st - t^2)$$

9 $z = \ln(3x+2y)$, $x = s \cdot \text{sen}(t)$, $y = t \cdot \cos(s)$

a) $\frac{\partial z}{\partial x} = \frac{1}{3x+2y} \cdot 3$ b) $\frac{\partial z}{\partial y} = \frac{1}{3x+2y} \cdot 2$

$$\frac{\partial z}{\partial t} = \frac{3}{3x+2y} \cdot s \cdot \cos(t) + \frac{2}{3x+2y} \cdot \cos(s)$$

$$\frac{\partial z}{\partial s} = \frac{3}{3x+2y} \cdot \text{sen}(t) + \frac{2}{3x+2y} \cdot t \cdot (-\text{sen}(s))$$

11 $z = e^r \cdot \cos\theta$, $r = st$, $\theta = \sqrt{s^2+t^2}$

a) $\frac{\partial z}{\partial r} = e^r \cdot \cos\theta$ b) $\frac{\partial z}{\partial \theta} = -e^r \cdot \text{sen}\,\theta$

$$\frac{\partial z}{\partial s} = e^r \cdot \cos\theta \cdot t - e^r \cdot \text{sen}\,\theta \cdot \frac{1}{2\sqrt{s^2+t^2}} \cdot 2s$$

$$\frac{\partial z}{\partial t} = e^r \cdot \cos\theta \cdot s - e^r \cdot \text{sen}\,\theta \cdot \frac{1}{2\sqrt{s^2+t^2}} \cdot 2t$$

13 $p(t) = f(g(t), h(t))$, $x = g(t)$, $y = h(t)$

$p(t) = f(x,y)$

$$\frac{\partial p}{\partial t} = \underbrace{\frac{\partial f}{\partial x}}_{f_x(x,y)} \cdot \frac{\partial x}{\partial t} + \underbrace{\frac{\partial f}{\partial y}}_{f_y(x,y)} \cdot \frac{\partial y}{\partial t}$$

$g(2) = 4$ $h(2) = 5$

$g'(2) = -3$ $h'(2) = 6$

$f_x(4,5) = 2$ $f_y(4,5) = 8$

$\rightarrow \frac{\partial x}{\partial t} = \frac{\partial (g(t))}{\partial t} = g'(t)$ $\rightarrow h'(t)$

$p'(t)$

$$\frac{\partial p}{\partial t} = f_x(x,y) \cdot g'(t) + f_y(x,y) \cdot h'(t) = f_1(g(t), h(t)) \cdot g'(t) + f_2(g(t), h(t)) \cdot h'(t)$$

$$p'(2) = f_1(g(2), h(2)) \cdot g'(2) + f_2(g(2), h(2)) \cdot h(2) = f_1(4,5) \cdot (-3) + f_2(4,5) \cdot 6$$
$$= 2 \cdot (-3) + 8 \cdot 6 = 42$$

15 $x(u,v) = e^u + \operatorname{sen} v$ $y(u,v) = e^u + \cos v$

$$\frac{\partial g}{\partial u} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial u}$$

$$g_u(u,v) = f_x(x(u,v), y(u,v)) \cdot x_u(u,v) + f_y(x(u,v), y(u,v)) \cdot y_u(u,v)$$

$$g_u(0,0) = f_x(x(0,0), y(0,0)) \cdot x_u(0,0) + f_y(x(0,0), y(0,0)) \cdot y_u(0,0)$$

$\rightarrow x_u = e^u$ $\rightarrow y_u = e^u$

$$g_u(0,0) = f_x(1,2) \cdot 1 + f_y(1,2) \cdot 1$$
$$= 2 \cdot 1 + 5 \cdot 1 = 7$$

$$g_v(0,0) = f_x(1,2) \cdot x_v(0,0) + f_y(1,2) \cdot x_v(0,0)$$

$\rightarrow x_v = \cos v$ $\rightarrow y_v = -\operatorname{sen} v$

$$g_v(0,0) = 2 \cdot 1 + 5 \cdot 0 = 2$$

17 $u = f(x,y)$ $x = x(r,s,t)$ $y = y(r,s,t)$

$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial u}{\partial y} \cdot \frac{\partial y}{\partial r}$$

de forma analoga para s e t, apenas substituindo r por s e por t