cálculo 1, stewart vol.1, ed 8, cap 7.8

1 a) a função $\frac{x}{x-1}$ tem assíntota vertical em $x=1$

b) intervalo é infinito

c) intervalo é infinito

d) a função $\frac{1}{tg(x)} = cotg(x)$ Tem assíntota vertical em $x=0$.

2 a) não é imprópria

b) é imprópria, $x \to \pi/2$, $\tan(x) \to \pm\infty$

c) é imprópria, assíntota vertical em $x=-1$

d) é imprópria, intervalo infinito

5 $$\int_3^\infty \frac{1}{(x-2)^{3/2}}\,dx = \lim_{t\to\infty}\int_3^t \frac{1}{(x-2)^{3/2}}\,dx$$

$$\Rightarrow \int \frac{1}{(x-2)^{3/2}}\,dx = \int \frac{du}{u^{3/2}} = \int u^{-3/2}\,du = \frac{u^{-1/2}}{-1/2} = -\frac{2}{\sqrt{u}} = -\frac{2}{\sqrt{x-2}} + c$$

$u = x-2 \qquad du = 1\,dx$

$$\Rightarrow \left(\frac{-2}{\sqrt{x-2}} + c\right)\Bigg|_3^t = \frac{-2}{\sqrt{t-2}} + \frac{2}{\sqrt{3-2}} = \frac{-2}{\sqrt{t-2}} + 2$$

$$\Rightarrow \lim_{t\to\infty} \frac{-2}{\sqrt{t-2}} + 2$$, quando $t \to \infty$, $\sqrt{t-2} \to \infty$

$$= 0 + \lim_{t\to\infty} 2 = 2$$

7 $$\int_{-\infty}^0 \frac{1}{3-4x}\,dx = \lim_{s\to-\infty}\int_s^0 \frac{1}{3-4x}\,dx$$

$$\Rightarrow \int \frac{1}{3-4x}\,dx = \int \frac{1}{u}\cdot\frac{-4}{-4}\,dx = -\frac{1}{4}\int \frac{1}{u}\,du = \frac{-1}{4}\ln|3-4x| + c$$

$u = 3-4x \qquad du = -4\,dx$

aplicando $\Big|_s^0$

$$\Rightarrow \left(-\tfrac{1}{4}\cdot \ln|3-4x|+c\right)\Big|_s^0$$

$$= -\frac{\ln|3-0|}{4} + \frac{\ln|3-4s|}{4} = \frac{\ln\left|\frac{3-4s}{3}\right|}{4}$$

aplicando o limite

$$\Rightarrow \lim_{s\to-\infty} \frac{\ln\left|\frac{3-4s}{3}\right|}{4} = +\infty \text{, diverge}$$

9 $\int_2^\infty e^{-5x}\,dx = \lim_{t\to+\infty}\int_2^t e^{-5x}\,dx$

$$\Rightarrow \int e^{-5x}\,dx = -\frac{1}{5}\int e^u\,du = -\tfrac{1}{5}\,e^{-5x}$$

$u=-5x \quad du=-5$

$$\Rightarrow \left(\frac{-e^{-5x}}{5}\right)\Big|_2^t = \left(\frac{-e^{-5t}}{5}\right)-\left(\frac{-e^{-10}}{5}\right) = \frac{-e^{-5t}+e^{-10}}{5}$$

$$\Rightarrow \lim_{t\to\infty}\frac{-e^{-5t}+e^{-10}}{5} = \lim_{t\to\infty} -\frac{e^{-5t}}{1}\cdot\frac{1}{5} + \lim_{t\to\infty}\frac{e^{-10}}{5} = \left(\lim_{t\to\infty} -\frac{1}{e^{5t}\cdot 5}\right)^{\nearrow 0} + \frac{1}{5e^{10}} = \frac{1}{5e^{10}}$$

11 $\int_0^\infty \frac{x^2}{\sqrt{1+x^3}}\,dx = \lim_{t\to\infty}\int_0^t \frac{x^2}{\sqrt{1+x^3}}\,dx$

$$\Rightarrow \int \frac{x^2}{\sqrt{1+x^3}}\,dx = \frac{1}{3}\int\frac{du}{\sqrt{u}} = \frac{1}{3}\int u^{-1/2}\,du = \frac{1}{3}\cdot\frac{\sqrt{u}\cdot 2}{1}+c$$

$u=1+x^3 \quad du=3x^2\,dx$

$$\Rightarrow \left(\frac{2}{3}\sqrt{1+x^3}+c\right)\Big|_0^t = \frac{2\sqrt{1+t^3}}{3} - \frac{2\sqrt{1+0}}{3} = \frac{2\sqrt{1+t^3}}{3}-\frac{2}{3}$$

$$\Rightarrow \left(\lim_{t\to\infty}\frac{2\sqrt{1+t^3}}{3}\right)-\frac{2}{3} = +\infty \text{, diverge}$$

13 $\int_{-\infty}^\infty x e^{-x^2}\,dx = \lim_{t\to\infty}\int_0^t xe^{-x^2}\,dx + \lim_{s\to-\infty}\int_{-\infty}^0 xe^{-x^2}\,dx$

resolvendo a integral indefinida

$$\int x e^{-x^2} dx = -\frac{1}{2} \int e^{u} du = -\frac{1}{2} \cdot e^{-x^2} + c = \frac{-e^{-x^2}}{2} + c$$

$$u = -x^2 \qquad du = -2x$$

resolvendo os limites

$$\lim_{t \to \infty} \frac{-e^{-t^2}}{2} + \frac{e^{0}}{2} = \left(\lim_{t \to +\infty} - \frac{1}{e^{t^2} \cdot 2} \right) + \frac{1}{2} = \frac{1}{2}$$

$$\lim_{s \to -\infty} \frac{-e^{0}}{2} + \lim_{s \to -\infty} \frac{e^{-s^2}}{2} = \frac{-1}{2} + \lim_{s \to -\infty} \frac{1}{e^{s^2} \cdot 2} = \frac{-1}{2}$$

resultado final

$$\frac{1}{2} - \frac{1}{2} = 0$$

17 $\displaystyle\int_1^{\infty} \frac{1}{x^2+x}\,dx = \lim_{t\to\infty} \int_1^{t} \frac{1}{x^2+x}\,dx$

$\Rightarrow \displaystyle\int \frac{1}{x^2+x}\,dx = \int \frac{1}{x(x+1)}\,dx \quad , \quad \frac{A}{x} + \frac{B}{x+1} = \frac{1}{x(x+1)}$

$A(x+1) + Bx = 1 \qquad A = 1$

$Ax + A + Bx = 1 \qquad B = -1$

$= \displaystyle\int \frac{1}{x}\,dx - \int \frac{1}{x+1}\,dx = \ln|x| - \ln|x+1| + c$

$\Rightarrow$ aplicando $\Big|_1^t$

$\ln|t| - \ln|t+1| - \cancel{\ln|1|}^{\,0} + \ln|2|$

$\Rightarrow \displaystyle\lim_{t\to\infty} \ln\left|\frac{t}{t+1}\right| + \lim_{t\to\infty} \ln|2| = \ln\left(\lim_{t\to\infty} \frac{t}{t+1}\right) + \ln|2|$

$\overset{L.H.}{=} \ln\left(\displaystyle\lim_{t\to\infty} \frac{1}{1}\right) + \ln|2| = \cancel{\ln|1|}^{\,0} + \ln|2| = \ln(2)$

21 $\displaystyle\int_1^{\infty} \frac{\ln x}{x}\,dx = \lim_{t\to\infty} \int_1^{t} \frac{\ln x}{x}$

$\Rightarrow \displaystyle\int \frac{\ln x}{x}\,dx = \int u\,du = \frac{u^2}{2} = \left(\frac{\ln(x)^2}{2}\right)\Bigg|_1^t$

$u = \ln x \qquad du = 1/x\,dx$

$\Rightarrow \dfrac{\ln(t)^2}{2} - \cancel{\dfrac{\ln(1)^2}{2}}^{\,0} = \dfrac{\ln(t)^2}{2}$

$\Rightarrow \displaystyle\lim_{t\to\infty} \frac{\ln(t)^2}{2} = \frac{\lim_{t\to\infty} \ln(t)^2}{\lim_{t\to\infty} 2} = \frac{\ln\left(\lim_{t\to\infty} t\right)^2}{2} = \frac{\ln(\infty)^2}{2} = +\infty$

ou seja, diverge

49 $\int_0^{\infty} \frac{x}{x^3+1}\,dx = \left.\underbrace{\int_0^{1} \frac{x}{x^3+1}\,dx}_{\text{com certeza é convergente (intervalo fechado)}} + \int_1^{\infty} \frac{x}{x^3+1}\right]$ testar

para $x \geq 1$, $0 \leq \frac{x}{x^3+1} \leq \frac{x}{x^3+1}$

$\int_1^{\infty} \frac{x}{x^3}\,dx = \int_1^{\infty} \frac{1}{x^2}\,dx = \lim_{t\to\infty} \left(-\frac{1}{x}\right)\Big|_1^t = \lim_{t\to\infty} -\cancel{\frac{1}{t}}^{0} + \frac{1}{1} = 1$

logo, pelo teste da comparação, $\int_1^{\infty} \frac{x}{x^3+1}$, também converge

e como as duas partes da soma convergem, a integral inicial também irá convergir

51 $\int_1^{\infty} \frac{x+1}{\sqrt{x^4-x}}\,dx$

testando se a integral é divergente

$0 \leq \frac{x+1}{\sqrt{x^4}} \leq \frac{x+1}{\sqrt{x^4-x}} \longrightarrow \int_1^{\infty} \frac{x+1}{\sqrt{x^4}}\,dx = \int_1^{\infty} \frac{x+1}{x^2}\,dx$

$\Rightarrow \int \frac{x+1}{x^2}\,dx = \int \frac{x}{x^2}\,dx + \int \frac{1}{x^2}\,dx = \ln|x| - \frac{1}{x} + c$

$\Rightarrow \lim_{t\to\infty} \left(\ln|x| - \frac{1}{x}\right)\Big|_1^t = \lim_{t\to\infty} \ln|t| - \cancel{\frac{1}{t}}^{0} - \cancel{\ln|1|}^{0} + \frac{1}{1} = \left(\lim_{t\to\infty} \ln|t|\right) + 1$

$= +\infty + 1$, divergente e, pelo teste da comparação, a integral será também divergente

53 $\int_0^{1} \frac{\sec^2(x)}{x\sqrt{x}}\,dx$, $0 \leq \frac{1}{x^{3/2}} \leq \frac{\sec^2(x)}{x^{3/2}}$ para $0 \leq x \leq 1$

$\int_0^1 x^{-3/2} = \lim_{t\to 0^+} -\frac{2}{\sqrt{1}} + \frac{2}{\sqrt{t}} = +\infty$, divergente

logo, pelo teorema da comparação $\int_0^1 \frac{\sec^2(x)}{x^{3/2}}$ também é divergente

59 $\int_0^1 x^p \ln(x)\, dx$

para $p=1$, $\int_0^1 x \cdot \ln(x)\, dx = \lim_{t \to 0^+} \int_t^1 x \cdot \ln(x)\, dx$

$\Rightarrow \int x \ln(x)\, dx = \ln(x) \cdot \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2}\, dx$

$f(x) = \ln(x) \qquad f'(x) = 1/x$

$g'(x) = x \qquad g(x) = x^2/2$

$= \frac{\ln(x) \cdot x^2}{2} - \frac{1}{2} \int x\, dx = \frac{\ln(x) \cdot x^2}{2} - \frac{1}{2} \cdot \frac{x^2}{2}$

$\lim_{t \to 0^+} \left(\cancel{\frac{\ln(1) \cdot 1^2}{2}} - \frac{1^2}{4} - \frac{\ln(t)\, t^2}{2} + \frac{t^2}{4} \right) = -\frac{1}{4}$

$\hookrightarrow \lim_{t \to 0^+} \ln(t) \cdot t^2 = \lim_{t \to 0^+} \frac{\ln(t)}{\frac{1}{t^2}}, \frac{\infty}{\infty}, \overset{LH}{=} \lim_{t \to 0^+} \frac{\frac{1}{t}}{-2 \cdot t^{-3}} = \lim_{t \to 0^+} \frac{\frac{1}{t}}{\frac{-2}{t^3}}$

$\overset{LH}{=} \lim_{t \to 0^+} \frac{1}{\cancel{t}} \cdot \frac{t^{\cancel{3}\,2}}{-2} = \lim_{t \to 0^+} \frac{t^2}{-2} = 0$

para $p \neq 1$

$\Rightarrow \int x^1 \ln(x)\, dx$

$f(x) = \ln(x) \qquad f'(x) = \frac{1}{x}$

$g'(x) = x^p \qquad g(x) = x^{p+1}/p+1$

$= \frac{\ln(x) \cdot x^{p+1}}{p+1} - \int \frac{1}{x} \cdot \frac{x^{p+1}}{p+1}\, dx = \frac{\ln(x) \cdot x^{p+1}}{p+1} - \frac{1}{p+1} \cdot \frac{x^{p+1}}{p+1}$

$\lim_{t \to 0^+} \left(\cancel{\frac{\ln(1) \cdot 1^{p+1}}{p+1}} - \frac{1^{p+1}}{p+1} - \frac{\ln(t) \cdot t^{p+1}}{p+1} + \cancel{\frac{t^{p+1}}{p+1}} \right) =$

$\hookrightarrow \lim_{t \to 0^+} \ln(t) \cdot t^{p+1} = \lim_{t \to 0^+} \frac{\ln(t)}{\frac{1}{t^{p+1}}} \overset{LH}{=} \lim_{t \to 0^+} \frac{\frac{1}{t}}{\frac{-(p+1)}{t^{-(p+2)}}} = \lim_{t \to 0^+} \frac{1}{t} \cdot \frac{t^{-p-2}}{-p-1}$

$= \lim_{t \to 0^+} \frac{t^{-p-3}}{-(p+1)}$ será $= 0$ se $p \neq -1$

ou seja, a função é convergente para $p \neq 1$