cálculo 2, stewart vol 2, ed 8, cap 10.4

1 $r = \theta^2$ $| 0 \leq \theta \leq \pi/4$

$A = \int_0^{\pi/4} 1/2 \cdot (\theta^2)^2 \, d\theta = 1/2 \cdot \int_0^{\pi/4} \theta^4 \, d\theta = 1/2 \left(\frac{\theta^5}{5} \right) \Big|_0^{\pi/4}$

$= \frac{1}{2} \cdot \left(\frac{(\pi/4)^5}{5} \right) = \frac{1}{2} \cdot \frac{\left(\frac{\pi^5}{1024}\right)}{5} = \frac{1}{2} \cdot \frac{\pi^5}{1024} \cdot \frac{1}{5} = \frac{\pi^5}{10240}$

3 $r = \operatorname{sen} \theta + \cos \theta$ $0 \leq \theta \leq \pi$

$A = \int_0^{\pi} 1/2 \cdot (\operatorname{sen}\theta + \cos\theta)^2 \, d\theta$

$= 1/2 \int_0^{\pi} (\operatorname{sen}\theta)^2 + 2 \operatorname{sen}\theta \cos\theta + (\cos\theta)^2 \, d\theta$

I $\int (\operatorname{sen}\theta)^2 \, d\theta = \int 1/2 \, d\theta - \int 1/2 \cdot \cos 2\theta \, d\theta$ $u = 2\theta$ $du = 2 \, d\theta$

$= \frac{\theta}{2} - \frac{\operatorname{sen} 2\theta}{4} + c$

II $2 \int \operatorname{sen}\theta \cos\theta \, d\theta$ $u = \operatorname{sen}\theta$ $du = \cos\theta \, d\theta$

$= 2 \int u \, du = \frac{\cancel{2} u^2}{\cancel{2}} = \operatorname{sen}^2\theta + c$

III $\int \cos^2\theta \, d\theta = \int 1/2 \, d\theta + \int 1/2 \cos 2\theta \, d\theta$

$= \frac{\theta}{2} + \frac{\operatorname{sen} 2\theta}{4} + c$

$= 1/2 \left(\frac{\theta}{\cancel{2}} - \cancel{\frac{\operatorname{sen} 2\theta}{4}} + \operatorname{sen}^2\theta + \frac{\theta}{\cancel{2}} + \cancel{\frac{\operatorname{sen} 2\theta}{4}} \right) \Big|_0^{\pi} = 1/2 \cdot \left[\theta + \operatorname{sen}^2\theta \right] \Big|_0^{\pi}$

$= 1/2 \, (\pi + 0^2) = \pi/2$

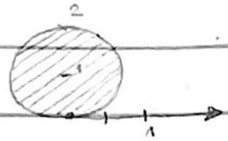
5 $r^2 = \operatorname{sen} 2\theta$ $r = \sqrt{\operatorname{sen} 2\theta}$

$V = \int_0^{\pi/2} 1/2 \cdot (\sqrt{\operatorname{sen} 2\theta})^2 \, d\theta = 1/2 \int_0^{\pi/2} \operatorname{sen} 2\theta \, d\theta = \frac{1}{2} \cdot \left(- \frac{\cos(2x)}{2} \right) \Big|_0^{\pi/2}$

$= \left(- \frac{\cos(2x)}{4} \right) \Big|_0^{\pi/2} = \left(\frac{1}{4} + \frac{1}{4} \right) = \frac{1}{2}$

7 $r = \sqrt{\theta}$

$V = \int_0^{2\pi} 1/2 \, (\sqrt{\theta})^2 \, d\theta = 1/2 \int_0^{2\pi} \theta \, d\theta = \frac{1}{2} \cdot \left(\frac{\theta^2}{2} \right) \Big|_0^{2\pi} = \pi^2$

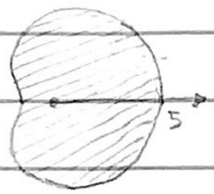
9 

$r = 2 \operatorname{sen} \theta$

$A = \frac{1}{2} \int_0^{\pi} (2 \operatorname{sen} \theta)^2 \, d\theta$

$= 2 \int_0^{\pi} \operatorname{sen}^2 \theta \, d\theta = 2 \cdot \left(\frac{\theta}{2} - \frac{\operatorname{sen} 2\theta}{4} \right) \Big|_0^{\pi}$

$= 2 \cdot \left(\frac{\pi}{2} - \frac{0}{4} - \frac{0}{2} + \frac{0}{4} \right) = \pi$

11 

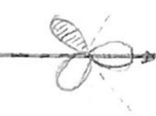
$r = 3 + 2 \cos \theta$

$A = \frac{1}{2} \int_0^{2\pi} (3 + 2 \cos \theta)^2 \, d\theta$

$= \frac{1}{2} \int_0^{2\pi} 9 + 12 \cos \theta + 4 \cos^2 \theta \, d\theta$

$= \frac{1}{2} \left(\operatorname{sen}(2x) + 12 \operatorname{sen}(x) + 11x \right) \Big|_0^{2\pi}$

$= 22\pi \cdot \frac{1}{2} = 11\pi$

17 $\frac{2\pi}{3}$ é um laço 

$r = 4 \cos 3\theta$

$A = \int_{-\pi/6}^{\pi/6} \frac{1}{2} 4 \cos 3\theta \, d\theta$

$= 2 \int_{-\pi/6}^{\pi/6} \cos 3\theta \, d\theta$

$= 2 \left(\frac{\operatorname{sen} 3\theta}{3} \right) \Big|_{-\pi/6}^{\pi/6} = 2 \cdot \left(\frac{1}{3} + \frac{1}{3} \right.$

para achar os pontos que fecham o laço basta saber

$r = 0$

$4 \cos 3\theta = 0$ em $\theta = \frac{\pi}{6}, -\frac{\pi}{6}$

19 $r = \operatorname{sen} 4\theta$

$\operatorname{sen} 4\theta = 0$ em $\frac{\pi}{4} \cdot k$

$[0, \pi/4]$

$A = \int_0^{\pi/4} \frac{1}{2} (\operatorname{sen} 4\theta)^2 \, d\theta$

$= \frac{1}{2} \int_0^{\pi/4} \operatorname{sen}^2 4\theta \, d\theta$

$= \frac{1}{2} \cdot \left(\frac{\operatorname{sen}(8x) - 8x}{16} \right) \Big|_0^{\pi/4} = \frac{\pi}{16}$

21 $r = 1 + 2\operatorname{sen}(t)$

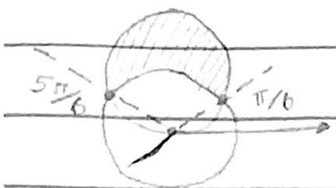
$A = \int_{7\pi/6}^{11\pi/6} \frac{1}{2} \cdot (1 + 2 \operatorname{sen} \theta)^2 \, d\theta$

$1 + 2\operatorname{sen}(t) = 0$

$\operatorname{sen}(t) = -\frac{1}{2}$

$t = \frac{7\pi}{6}$ e $\frac{11\pi}{6}$

23 $r = 4 \operatorname{sen} \theta$ e $r = 2$ se encontram em



$4 \operatorname{sen} \theta = 2 \qquad \operatorname{sen} \theta = 1/2$

$\theta = \pi/6$ e $\theta = -5\pi/6$

$$A = \int_{\pi/6}^{5\pi/6} 1/2 \cdot \left((4 \operatorname{sen} \theta)^2 - (2)^2\right) d\theta$$

$$= \left(-2\left(\operatorname{sen}(2\theta) - x\right)\right)\Big|_{\pi/6}^{5\pi/6}$$

$$= 4/3\,\pi + 2\sqrt{3}$$

27 $r = 3 \cos \theta \qquad r = 1 + \cos \theta$

$3 \cos \theta = 1 + \cos \theta \rightarrow 2 \cos \theta = 1 \rightarrow \cos \theta = 1/2$

pontos de encontro : $\pi/3$ e $-\pi/3$

qual e maior dentro do intervalo $[\pi/3, 2\pi/3]$?

$3 \cos 0 = 3 \qquad 1 + \cos 0 = 2$

$$A = \int_{\pi/3}^{\pi/3} 1/2 \cdot \left((3\cos\theta)^2 - (1+\cos\theta)^2\right) d\theta$$

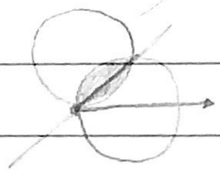
$$= 1/2 \int_{\pi/3}^{\pi/3} -1 - 2\cos\theta + 8\cos^2\theta$$

$$= 1/2 \cdot \left(2 \operatorname{sen}(2\theta) - 2 \operatorname{sen}(\theta) + 3\theta\right)\Big|_{-\pi/3}^{\pi/3}$$

$$= 1/2 \cdot 2\pi = \pi$$

29 $r = 3 \operatorname{sen} \theta \qquad r = 3 \cos \theta$

$\cancel{3} \operatorname{sen} \theta = \cancel{3} \cos \theta \rightarrow \operatorname{sen} \theta = \cos \theta$ em $\pi/4$ e $5\pi/4$



$$A = \int_0^{\pi/4} 1/2\,(3 \operatorname{sen} \theta)^2\, d\theta + \int_{\pi/4}^{\pi/2} 1/2\,(3\cos\theta)^2\, d\theta$$

$$= \frac{9(\pi-2)}{16} + \frac{9(\pi-2)}{16} = \frac{9\pi}{8} - \frac{9}{4}$$