cálculo 1, stewart, vol 1, ed 5, cap 5.3

7 $\int_0^x \sqrt{1+2t}\, dt = g(x)$

$g'(x) = \sqrt{1+2x}$

9 $g(y) = \int_2^y t^2 \cdot \operatorname{sen} t\, dt$ $\quad g'(y) = y^2 \cdot \operatorname{sen} y$

11 $F(x) = \int_x^2 \cos(t^2)\, dt = -\int_2^x \cos(t^2)\, dt$

$F'(x) = -(\cos(x^2)) = -\cos x^2$

13 $h(x) = \int_2^{\frac{1}{x}} \operatorname{arctg} t\, dt$ $\quad \frac{d}{dx}(h(x)) = \frac{d}{dx}\left(\int_2^{\frac{1}{x}} \operatorname{arctg} t\, dt\right)$

$F = \int_2^x \operatorname{arctg} t\, dt$ $\quad F' = \operatorname{arctg} x$ $\quad G = \frac{1}{x}$ $\quad G' = -x^{-2} = -\frac{1}{x^2}$

$h'(x) = \operatorname{arctg}\left(\frac{1}{x}\right) \cdot \frac{-1}{x^2} = \frac{-\operatorname{arctg}\left(\frac{1}{x}\right)}{x^2}$

15 $\int_3^{\sqrt{x}} \frac{\cos t}{t}\, dt$ $\quad F = \int_3^x \frac{\cos t}{t}\, dt$ $\quad F' = \frac{\cos x}{x}$

$G = x^{\frac{1}{2}}$ $\quad G' = \frac{1}{2} \cdot x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$f'(x) = \frac{\cos\sqrt{x}}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \frac{\cos\sqrt{x}}{2x}$

17 $\int_{1-3x}^1 \frac{u^3}{1+u^2}\, du = -\int_1^{1-3x} \frac{u^3}{1+u^2}\, du$

$F = \int_1^x \frac{u^3}{1+u^2}$ $\quad F' = \frac{x^3}{1+x^2}$ $\quad G = 1-3x$ $\quad G' = -3$

$f'(x) = -\left(\frac{(1-3x)^3}{1+(1-3x)^2} \cdot -3\right) = \frac{3(1-3x)^3}{1+(1-3x)^2}$

19 $\int_{-1}^{3} x^5\,dx = F(3) - F(-1)$

$= \left(\frac{x^6}{6}+c\right)\Big|_{-1}^{3} = \left(\frac{3^6}{6}+\cancel{c} - \left(\frac{-1^6}{6}+\cancel{c}\right)\right) = \frac{3^6}{6} - \frac{(-1)^6}{6} = \frac{364}{3}$

21 $\int_2^8 (4x+3)\,dx = \left(4\frac{x^2}{2}+3x+c\right)\Big|_2^8$

$= \left(2\cdot 8^2 + 3\cdot 8 + \cancel{c} - \left(2\cdot 2^2 + 3\cdot 2 + \cancel{c}\right)\right) = 128+24-8-6 = 138$

23 $\int_0^1 x^{4/3}\,dx = \left(\frac{x^{7/3}}{\frac{7}{3}}\right)\Big|_0^1 = \left(\frac{3\sqrt[3]{x^7}}{7}\right)\Big|_0^1 = \left(\frac{3\cdot 1}{7}+\cancel{c}-\left(\frac{3\cdot 0}{7}+\cancel{c}\right)\right) = \frac{3}{7}$

25 $\int_1^2 \frac{3}{t^4}\,dx = \int_1^2 3t^{-4}\,dx = \left(\frac{\cancel{3}\cdot t^{-3}}{-\cancel{3}}+c\right)\Big|_1^2 = \left(\frac{-1}{t^3}+c\right)\Big|_1^2$

$= \frac{-1}{2^3}+\cancel{c} - \left(\frac{-1}{1}+\cancel{c}\right) = -\frac{1}{8}+1 = \frac{7}{8}$

27 $\int_{-5}^{5} 2x^{-3}\,dx = \left(\cancel{2}\frac{x^{-2}}{-\cancel{2}}+c\right)\Big|_{-5}^{5} = \left(-\frac{1}{x^2}+c\right)\Big|_{5}^{5}$

$= -\frac{1}{25}+\cancel{c} - \left(\frac{-1}{25}+\cancel{c}\right) = -\frac{1}{25}+\frac{1}{25} = 0$ não existe

29 $\int_0^2 x(2+x^5)\,dx = \int_0^2 2x + x^6\,dx = \left(\cancel{2}\cdot\frac{x^2}{\cancel{2}} + \frac{x^7}{7}+c\right)\Big|_0^2 = \left(x^2+\frac{x^7}{7}+c\right)\Big|_0^2$

$= 2^2 + \frac{2^7}{7} + \cancel{c} - (0+0+\cancel{c}) = 4 + \frac{128}{7} = \frac{156}{7}$

31 $\int_0^{\pi/4} \sec^2 t\,dt = (\text{tg}\, t + c)\Big|_0^{\pi/4} = \text{tg}\,\frac{\pi}{4} + \cancel{c} - \left(\cancel{\text{tg}\, 0}^{\,0}+\cancel{c}\right) = 1$

33 $\int_{\pi}^{2\pi} \text{cossec}^2 x\,dx = (\text{cotg}\, x + c)\Big|_{\pi}^{2\pi} =$? não existe

35 $\int_1^9 \frac{1}{2x}\,dx = \int_1^9 \frac{1}{x}\cdot\frac{1}{2}\,dx = \frac{1}{2}\int_1^9 \frac{1}{x}\,dx = \frac{1}{2}\left|\ln(|x|)+c\right|\Big|_1^9$

$= \frac{1}{2}\left(\ln(9)+\cancel{c}-(\ln(1)+\cancel{c})\right) = \frac{1}{2}\ln(9) - 0 = \ln(9^{\frac{1}{2}}) = \ln(\sqrt{9}) =$
$= \ln(3)$

39 $\int_{-1}^{1} e^{u+1}\,du \Rightarrow \int_{-1}^{1} e^{t}\,dt = e^t$

$t = u+1$

$\hookrightarrow \left|e^{u+1}+c\right|\Big|_{-1}^{1} = e^2+\cancel{c}-(e^0+\cancel{c}) = e^2-1$

41 $\int_0^2 f(x)\,dx \qquad f(x) = \begin{cases} x^4 \text{ se } & 0 \leq x < 1 \\ x^5 \text{ se } & 1 \leq x \leq 2 \end{cases}$

$\lim_{x\to 1^+}\left(\int_0^x x^4\,dx\right) + \int_1^2 x^5\,dx = \lim_{x\to 1^+}\left(\left|\frac{x^5}{5}+c_1\right|\Big|_0^x\right) + \left(\frac{x^6}{6}+c_2\right)\Big|_1^2$

$= \frac{1^5}{5}+\cancel{c_1}-\left(\frac{0^5}{5}+\cancel{c_1}\right)+\frac{2^6}{6}+\cancel{c_2}-\left(\frac{1^6}{6}+\cancel{c_2}\right) = \frac{1}{5}-0+\frac{64}{6}-\frac{1}{6}$

$= \frac{1}{5}+\frac{63}{6} = \frac{107}{10} = 10{,}7$

49 $g(x) = \int_{2x}^{3x} \frac{u^2-1}{u^2+1}\,du = \int_{2x}^{0} \frac{u^2-1}{u^2+1}\,du + \int_0^{3x} \frac{u^2-1}{u^2+1}\,du$

$= -\int_0^{2x} \frac{u^2-1}{u^2+1}\,du + \int_0^{3x} \frac{u^2-1}{u^2+1}\,du$

$\hookrightarrow \frac{d}{dx}\left(\int_0^{2x} \frac{u^2-1}{u^2+1}\,du\right) \Rightarrow \quad F = \int_0^x f(u)\,du \qquad G = 2x$

$F' = \frac{x^2-1}{x^2+1} \qquad G' = 2$

$\Rightarrow \frac{(2x)^2-1}{(2x)^2+1}\cdot 2 = \frac{(4x^2-1)\cdot 2}{4x^2+1} = \frac{8x^2-2}{4x^2+1}$

$\hookrightarrow \frac{d}{dx}\left(\int_0^{3x} \frac{u^2-1}{u^2+1}\,du\right) \Rightarrow \quad F = \int_0^x f(u)\,du \qquad G = 3x$

$F' = \frac{x^2-1}{x^2+1} \qquad G' = 3$

$\Rightarrow \frac{(3x)^2-1}{(3x)^2+1}\cdot 3 = \frac{(9x^2-1)\cdot 3}{9x^2+1} = \frac{27x^2-1}{9x^2+1}$

$g'(x) = \left(\frac{8x^2 - 2}{4x^2 + 1}\right) + \frac{27x^3 - 3}{9x^3 + 1}$

55 $f(1) = 12$ $\int_1^4 f'(x)\,dx = 17$ $\int f'(x) = f(x)$

$\int_1^4 f'(x) = \left(f(x)\right)\Big|_1^4 = f(4) - f(1) = f(4) - 12 = 17$

$f(4) = 5$