cálculo 2, stewart vol. 1, ed. 8, cap 7.1

1) $\int x \cdot e^{2x}\, dx$ $\quad f(x) = x \quad f'(x) = 1$

$g'(x) = e^{2x} \quad g(x) = \int e^{2x}\, dx = \frac{1}{2} \cdot \int e^{u}\, du = \frac{1}{2} \cdot e^{u}$

$u = 2x \quad du = 2\, dx \quad = \frac{e^{2x}}{2} + c$

$\int x \cdot e^{2x}\, dx = x \cdot \frac{e^{2x}}{2} - \int 1 \cdot \frac{e^{2x}}{2}$

$= \frac{x \cdot e^{2x}}{2} - \frac{1}{2} \int e^{2x} = \frac{x \cdot e^{2x}}{2} - \frac{1}{2}\left(\frac{e^{2x}}{2}\right) = \frac{x e^{2x}}{2} - \frac{1}{4} \cdot e^{2x} + c$

3) $\int x \cdot \cos(5x)\, dx \leftarrow \begin{cases} f(x) = x & f'(x) = 1 \\ g'(x) = \cos(5x) & g(x) = \int \cos(5x) = \frac{1}{5} \int \cos(u)\, du \end{cases}$

$= x \cdot \frac{1}{5} \operatorname{sen}(5x) - \frac{1}{5} \int \operatorname{sen}(\underbrace{5x}_{u})\, dx \qquad = \frac{1}{5} \operatorname{sen}(5x) + c$

$= \frac{x \cdot \operatorname{sen}(5x)}{5} - \frac{1}{25} \int \operatorname{sen}(u)\, du = \frac{x \cdot \operatorname{sen}(5x)}{5} - \frac{1}{25} \cdot (-\cos(5x)) + c$

5) $\int x \cdot e^{\frac{x}{2}}\, dx$

$f(x) = x \quad f'(x) = 1 \quad \nearrow u = x/2 \quad du = 1/2\, dx$

$g'(x) = e^{\frac{x}{2}} \quad g(x) = \int e^{\frac{x}{2}}\, dx = 2 \cdot \int e^{u} \cdot \frac{1}{2}\, dx = 2 \int e^{u}\, du = 2 \cdot e^{u}$

$= 2 \cdot e^{\frac{x}{2}} + c$

$\int x \cdot e^{\frac{x}{2}}\, dx = x \cdot 2e^{x/2} - \int 1 \cdot 2 \cdot e^{\frac{x}{2}}\, dx$

$= 2x e^{x/2} - 2 \int e^{x/2}\, dx = 2x e^{x/2} - 4 e^{x/2} + c$

7) $\int (x^2 + 2x) \cdot \cos(x)$

$f(x) = x^2 + 2x \quad f'(x) = 2x + 2$

$g'(x) = \cos(x) \quad g(x) = \int \cos(x)\, dx = \operatorname{sen}(x) + c$

$= (x^2 + 2x) \cdot \operatorname{sen}(x) - \underline{\int (2x+2) \cdot \operatorname{sen}(x)\, dx} = (x^2+2x) \cdot \operatorname{sen}(x) - ((2x+2) \cdot -\cos(x)$

$f(x) = 2x + 2 \quad f'(x) = 2 \qquad - \int 2 \cdot -\cos(x)$

$g'(x) = \operatorname{sen}(x) \quad g(x) = \int \operatorname{sen}(x)\, dx = -\cos(x)$

$= (x^2 + 2x) \cdot \operatorname{sen}(x) + \cos(x) \cdot (2x+2) + 2 \operatorname{sen}(x) + c$

9 $\int \cos^{-1}(x)\, dx = \int \frac{1}{\cos(x)}\, dx = \int \sec(x)\, dx$

$= \int \sec(x) \left(\frac{\sec(x) + tg(x)}{\sec(x) + tg(x)} \right) dx = \int \frac{\sec^2(x) + \sec(x)\, tg(x)\, dx}{\underbrace{\sec(x) + tg(x)}_{u}}$

$du = \sec(x) . tg(x) + \sec^2(x)\, dx$

$= \int \frac{1}{u}\, du = \ln|u| + c = \ln|\sec(x) + tg(x)| + c$

11 $\int x^4 . \ln x\, dx$

$f(x) = \ln(x) \qquad f'(x) = \frac{1}{x}$

$g'(x) = x^4 \qquad g(x) = \frac{x^5}{5}$

$\int x^4 \ln(x)\, dx = \frac{\ln(x) . x^5}{5} - \int \frac{1}{x} . \frac{x^5}{5}\, dx = \frac{\ln(x) . x^5}{5} - \frac{1}{5} \int x^4\, dx$

$= \frac{\ln(x) . x^5}{5} - \frac{1}{5} . \frac{x^5}{5} + c$

13 $\int x . \text{cossec}^2 x\, dx$

$f(x) = x \qquad f'(x) = 1$

$g'(x) = \text{cossec}^2(x) \qquad g(x) = \int \text{cossec}^2(x)\, dx = - \text{cotg}(x) + c$

$= -x . \text{cotg}(x) - \int 1 . - \text{cotg}(x)\, dx = - x . \text{cotg}(x) + \int \text{cotg}(x)\, dx$

$= - x . \text{cotg}(x) + \ln|\text{sen}(x)| + c$

$\Rightarrow \int \text{cotg}(x)\, dx = \int \frac{\cos(x)}{\text{sen}(x)}\, dx \qquad \begin{array}{l} u = \text{sen}(x) \\ du = \cos(x)\, dx \end{array}$

$= \int \frac{1}{u}\, du = \ln|u| + c = \ln|\text{sen}(x)| + c$

15 $\int \ln(x) . x^{1/3}\, dx$

$f(x) = \ln(x) \qquad f'(x) = \frac{1}{x}$

$g'(x) = x^{1/3} \qquad g(x) = \int x^{1/3}\, dx = \frac{x^{4/3}}{4/3} = \frac{3x^{4/3}}{4}$

$= \ln(x) . \frac{3x^{4/3}}{4} - \int \frac{1}{x} . \frac{3 . x^{4/3}}{4} = \frac{\ln(x) . 3x^{4/3}}{4} - \frac{3}{4} \int x^{1/3} = \frac{\ln(x) . 3x^{3/4}}{4} - \frac{9x^{4/3}}{16} + c$

17 $\int \operatorname{arctg}(4x)\,dx$

$u = 4x \qquad \frac{du}{dx}(4x) = 4 \qquad du = 4\,dx$

$= \int \operatorname{arctg}(u)\,\frac{4}{4}\,dx = \frac{1}{4}\cdot\int \operatorname{arctg}(u)\,du$

$f(u) = \operatorname{arctg}(u) \qquad f'(u) = \frac{1}{u^2+1}$

$g'(u) = 1\,du \qquad g(u) = u$

$= \left(\operatorname{arctg}(u)\cdot u - \underbrace{\int \frac{1}{u^2+1}\cdot u\,du}\right)\cdot\frac{1}{4}$

$K = u^2+1 \qquad dK = 2u\,du$

$= \left(\operatorname{arctg}(u)\cdot u - \frac{1}{2}\int \frac{1}{K}\,dK\right)\cdot\frac{1}{4} = \left(\operatorname{arctg}(u)\cdot u - \frac{\ln|K|}{2} + c\right)\cdot\frac{1}{4}$

$= \left(\operatorname{arctg}(4x)\cdot 4x - \frac{1}{2}\cdot\ln|(4x)^2+1| + c\right)\cdot\frac{1}{4}$

$= \operatorname{arctg}(4x)\cdot x - \frac{1}{8}\cdot\ln|16x^2+1| + c$

19 $\int x^3\cdot e^2\,dx$

$f(x) = x^3 \qquad f'(x) = 3x^2$

$g'(x) = e^x \qquad g(x) = \int e^x\,dx = e^x$

$= x^3\cdot e^x - \int 3x^2\cdot e^2\,dx = x^3e^x - 3\cdot\left(x^2\cdot e^x - \int 2x\cdot e^x\,dx\right)$

$= x^3e^x - 3\cdot\left[x^2\cdot e^x - 2\cdot\left(x\cdot e^x - e^x\right)\right] + c$

$= x^3e^x - 3\cdot\left[x^2e^x - 2xe^x - 2e^x\right] + c$

$= x^3e^x - 3x^2e^x - 6xe^x - 6e^x + c$

aplicando os limites de integração

$$\left(\frac{1}{2}\cdot\frac{sen(\pi/2)}{\pi}+\frac{cos(\pi/2)}{\pi^2}\right)-\left(0+\frac{cos(0)}{\pi^2}\right)$$

$$=\frac{1}{2\pi}+0-\frac{1}{\pi^2}=\frac{1}{2\pi}-\frac{1}{\pi^2}$$

25 $\int_0^2 x\, sen(x)\, dx$

$f(x)=x \qquad f'(x)=1$

$g'(x)=sen(x) \qquad g(x)=-cos(x)$

$$=-x\,cos(x)-\int -cos(x)\,dx=\left(-x\,cos(x)+sen(x)+c\right)\Big|_0^2$$

$$\left(-2\,cos(2)+sen(2)\right)-\left(-0\,cos(0)+sen(0)\right)$$

$$=-2\,cos(2)+sen(2)$$

27 $\int_1^5 \frac{\ln x}{x^2}\,dx$

$f(x)=\ln x \qquad f'(x)=1/x$

$g'(x)=1/x^2 \qquad g(x)=\frac{-1}{x}$

$$\Rightarrow -\frac{\ln x}{x}-\int\frac{1}{x}\cdot\frac{-1}{x}\,dx=-\frac{\ln x}{x}+\int x^{-2}\,dx=\left(-\frac{\ln x}{x}+\frac{-1}{x}+c\right)\Big|_1^5$$

$$=\left(-\frac{\ln 5}{5}-\frac{1}{5}\right)-\left(-\frac{\cancel{\ln 1}^{0}}{1}-\frac{1}{1}\right)=-\frac{\ln 5}{5}-\frac{1}{5}+1=-\frac{\ln 5}{5}+\frac{4}{5}$$

29 $\int_0^\pi x\, sen(x)\, cos(x)$

$f(x)=x \qquad f'(x)=1$

$g'(x)=sen(x)\,cos(x) \qquad g(x)=\int sen(x)\,cos(x)\,dx=\int u\,du=\frac{sen^2(x)}{2}$

$u=sen(x) \quad du=cos(x)$

$$\Rightarrow \frac{x\,sen^2(x)}{2}-\int\frac{sen^2(x)}{2}\,dx=\frac{x\,sen^2(x)}{2}-\frac{1}{2}\left(\int\frac{1}{2}\,dx-\frac{1}{2}\int cos(2x)\,dx\right)$$

$$=\left(\frac{x\,sen^2(x)}{2}-\frac{x}{4}+\frac{sen(2x)}{8}\right)\Big|_0^\pi=\left(\frac{\pi\cdot\cancel{0}}{\cancel{2}}-\frac{\pi}{4}+\frac{\cancel{0}}{\cancel{8}}\right)-\left(0-0+0\right)$$

$$=\frac{-\pi}{4}$$

31 $\int_1^5 \frac{x}{e^x}\,dx$ $\quad f(x) = x \quad f'(x) = 1$

$g'(x) = e^{-x} \quad g(x) = -e^{-x}$

$\Rightarrow -xe^{-x} - \int 1 \cdot -e^{-x}\,dx$

$= -xe^{-x} + \int e^{-x}\,dx = -xe^{-x} - e^{-x} + c$

$\Rightarrow (-5e^{-5} - e^{-5}) - (-1e^{-1} - e^{-1})$

$= \frac{-5}{e^5} - \frac{1}{e^5} + \frac{1}{e} + \frac{1}{e} = \frac{-5}{e^5} - \frac{1}{e^5} + \frac{2}{e} = -\frac{6}{e^5} + \frac{2}{e}$

33 $\int_0^{\pi/3} sen(x) \cdot \ln(\cos x)\,dx$

$f(x) = \ln(\cos x) \quad f'(x) = \frac{1}{\cos(x)} \cdot -sen(x) = -\frac{sen(x)}{\cos(x)} = -tg(x)$

$g'(x) = sen(x) \quad g(x) = -\cos(x)$

$\Rightarrow \ln(\cos(x)) \cdot -\cos(x) - \int -tg(x) \cdot -\cos(x)\,dx$

$= -\ln(\cos(x)) \cdot \cos(x) - \int \frac{sen(x) \cdot \cancel{\cos(x)}}{\cancel{\cos(x)}}\,dx = -\ln(\cos(x)) \cdot \cos(x) + \cos(x)$

$\Rightarrow (-\ln(\cos(\pi/3)) \cdot \cos(\pi/3) + \cos(\pi/3)) - (-\ln(\cos(0)) \cdot \cos(0) + \cos(0))$

$= -\ln(1/2) \cdot \frac{1}{2} + \frac{1}{2} + \cancel{\ln(1) \cdot 1}^{\,0} - 1 = -(\cancel{\ln(1)}^{\,0} - \ln(2)) \cdot \frac{1}{2} + \frac{1}{2} - 1$

$= \frac{\ln(2)}{2} - \frac{1}{2}$

35 $\int_1^2 x^4 (\ln(x))^2\,dx$

$f(x) = (\ln(x))^2 \quad f'(x) = 2\ln(x)/x$ $\quad \int \; f(x) = \ln(x) \quad f'(x) = 1/x$

$g'(x) = x^4 \quad g(x) = x^5/5$ $\quad g'(x) = x^4 \quad g(x) = x^5/5$

$\Rightarrow \frac{\ln(x)^2 \cdot x^5}{5} - \int \frac{x^{\cancel{5}\,4}}{5} \cdot \frac{2\ln(x)}{\cancel{x}} = \frac{\ln(x)^2 \cdot x^5}{5} - \frac{2}{5}\int x^4 \ln(x)\,dx$

$= \frac{\ln(x)^2 \cdot x^5}{5} - \frac{2}{5}\left(\frac{\ln(x)\,x^5}{5} - \frac{1}{5}\int \frac{1}{\cancel{x}} \cdot x^{\cancel{5}\,4}\,dx\right) = \frac{\ln(x)^2 x^5}{5} - \frac{2\ln(x)\,x^5}{25} + \frac{2}{125} x^5 + c$

aplicando $\Big|_1^2$

$= \frac{32\ln^2(2)}{5} - \frac{64\ln(2)}{25} + \frac{62}{125}$