cálculo 2, stewart vol.1, ed. 8, cap 7.3

1 $\int \frac{1}{x^2\sqrt{4-x^2}}\,dx$ $\left[\begin{array}{l} x = 2\,\text{sen}(\theta) \quad \text{onde } -\pi/2 \leq \theta \leq \pi/2 \\ dx = 2\cos(\theta)\,d\theta \quad\quad \text{e } \theta \neq 0 \end{array}\right.$

$$\Rightarrow \int \frac{1 \cdot 2\cos(\theta)\,d\theta}{(2\,\text{sen}(\theta))^2\sqrt{4-(2\,\text{sen}(\theta))^2}} = \int \frac{\cancel{2}\cos(\theta)\,d\theta}{\cancel{4}^{2}\,\text{sen}^2(\theta)\cdot\sqrt{4-4\,\text{sen}^2(\theta)}}$$

$$= \int \frac{\cos(\theta)\,d\theta}{2\cdot\text{sen}^2(\theta)\cdot\sqrt{4(1-\text{sen}^2(\theta))}} = \int \frac{\cos(\theta)\,d\theta}{\text{sen}^2(\theta)\cdot 2\cdot\sqrt{(1-\text{sen}^2(\theta))}\cdot\sqrt{4}}$$

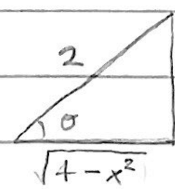
$$= \frac{1}{4}\int \frac{\cos(\theta)\,d\theta}{\text{sen}^2(\theta)\cdot\sqrt{\cos^2(\theta)}}$$ $\left[\sqrt{\cos^2(\theta)} = |\cos(\theta)|\right.$, analisando o domínio de θ, $|\cos(\theta)| = \cos(\theta)$

$$= \frac{1}{4}\int \frac{\cancel{\cos(\theta)}\,d\theta}{\text{sen}^2(\theta)\cdot\cancel{\cos(\theta)}} = \frac{1}{4}\int \frac{1\,d\theta}{\text{sen}^2(\theta)} = \frac{1}{4}\cdot\int \text{cossec}^2(\theta)\,d\theta$$

$$= 1/4\cdot(-\text{cotg}(\theta)+c) = -\frac{\text{cotg}(\theta)}{4} + c$$

$x = 2\cdot\text{sen}(\theta)$

$\text{sen}(\theta) = \frac{x}{2}$



$4 = x^2 + \text{adj}^2$

$\text{adj}^2 = 4 - x^2$

$\text{tg}(\theta) = \frac{x}{\sqrt{4-x^2}}$ ➜ $-\frac{\text{cotg}(\theta)}{4} + c = -\frac{\frac{1}{\text{tg}(\theta)}}{4} + c = -\frac{1}{4\,\text{tg}(\theta)} + c$

$$= -\frac{1}{4\left(\frac{x}{\sqrt{4-x^2}}\right)} + c = \frac{-\sqrt{4-x^2}}{4x} + c \quad //$$

3 $\int \frac{\sqrt{x^2-4}}{x}\,dx$ $\left[\begin{array}{l} x = 2\cdot\sec(\theta) \\ dx = 2\cdot\sec(\theta)\,\text{tg}(\theta) \end{array}\right.$ $\underbrace{0 \leq \theta \leq \pi/2}_{\text{escolhida}}$ ou $\pi \leq \theta \leq 3\pi/2$

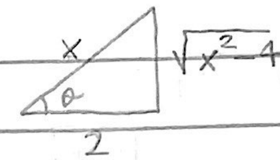
$$= \int \frac{\sqrt{4\sec^2(\theta)-4}\cdot 2\cancel{\sec(\theta)}\,\text{tg}(\theta)\,d\theta}{2\cancel{\sec(\theta)}} = \int \frac{\sqrt{4}\cdot\sqrt{(\sec^2-1)}\cdot\cancel{2}\,\text{tg}(\theta)\,d\theta}{\cancel{2}}$$

$$= 2\int\sqrt{\text{tg}^2\theta}\cdot\text{tg}\,\theta\,d\theta = 2\int\text{tg}\,\theta\cdot\text{tg}\,\theta\,d\theta = 2\cdot\int\text{tg}^2\theta\,d\theta$$

$$= 2\cdot\int\sec^2\theta - 1\,d\theta = 2\left(\int\sec^2\theta\,d\theta - \int 1\,d\theta\right)$$

$$= 2\cdot(\text{tg}\,\theta - \theta + c) = 2\text{tg}\,\theta - 2\theta + c = [\ldots]$$

$x = 2\sec(\theta) = 2\cdot\frac{1}{\cos\theta} = \frac{2}{\cos\theta}$ $\qquad \cos\theta = \frac{2}{x} = \frac{adj}{hip}$



$tg\,\theta = \frac{op}{adj} = \frac{\sqrt{x^2-4}}{2}$ $\qquad \theta = \arccos\left(\frac{2}{x}\right)$

$[\dots] = \frac{\cancel{2}\sqrt{x^2-4}}{\cancel{2}} - 2\arccos\left(\frac{2}{x}\right) + c = \sqrt{x^2-4} - 2\arccos\left(\frac{2}{x}\right) + c$

5 $\int \frac{\sqrt{x^2-1}}{x^4}\,dx$ $\qquad x = 1\cdot\sec\theta \qquad \underbrace{0 \leq \theta \leq \pi/2}_{\text{escolhido}}$ ou $\pi \leq \theta \leq 3\pi/2$

$dx = \sec\theta\cdot tg\,\theta$

$= \int \frac{\sqrt{\sec^2\theta - 1}\cdot\sec\theta\cdot tg\,\theta\,d\theta}{\sec^4\theta} = \int \frac{|tg\,\theta|\cdot\cancel{\sec\theta}\cdot tg\,\theta\,d\theta}{\sec^3\theta\cdot\cancel{\sec\theta}}$

$= \int \frac{tg^2\theta}{\sec^3\theta}\,d\theta = \int \frac{(\sec^2\theta - 1)\,d\theta}{\sec^3\theta} = \int \frac{\cancel{\sec^2\theta}}{\sec^{\cancel{3}}\theta}\,d\theta - \int \frac{1}{\sec^3\theta}\,d\theta$

$= \int \frac{1}{\sec\theta}\,d\theta - \int \frac{1}{\sec^3\theta}\,d\theta = \int \cos\theta\,d\theta - \int \cos^3\theta\,d\theta$

$= sen\,\theta - \int \cos^3\theta\,d\theta = [\dots]$

$\int \cos^3\theta\,d\theta = \int \cos^2\theta\cdot\cos\theta\,d\theta = \int (1 - sen^2\theta)\cdot\cos\theta\,d\theta$

$= \int \cos\theta\,d\theta - \int \underbrace{sen^2\theta}\cos\theta\,d\theta = sen\,\theta - \int u^2\,du$

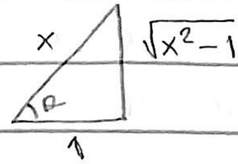
$u = sen\,\theta \qquad du = \cos\theta\,d\theta$

$= sen\,\theta - \frac{u^3}{3} + c = sen\,\theta - \frac{sen^3\theta}{3} + c$

$[\dots] = \cancel{sen\,\theta} - \left(\cancel{sen\,\theta} - \frac{sen^3\theta}{3}\right) + c = \frac{sen^3\theta}{3} + c = [\dots]$

$x = \sec\theta$

$x = \frac{1}{\cos\theta} \qquad \cos\theta = \frac{1}{x} = \frac{ad}{hip}$



$sen\,\theta = \frac{op}{hip} = \frac{\sqrt{x^2-1}}{x}$

$$[\ldots] = \frac{\left(\frac{\sqrt{x^2-1}}{x}\right)^3}{3} + c = \frac{\left(\sqrt{x^2-1}\right)^3}{x^3} \cdot \frac{1}{3} + c = \frac{(x^2-1)^{3/2}}{x^3} \cdot \frac{1}{3} + c$$

7 $\displaystyle\int_0^a \frac{dx}{(a^2+x^2)^{3/2}}$

$x = a \operatorname{tg}\theta \qquad -\pi/2 < \theta < \pi/2 \text{ e } \theta \neq 0$

$dx = a \sec^2\theta \, d\theta$

$$\Rightarrow \int \frac{a \cdot \sec^2\theta \, d\theta}{(a^2 + a^2 \operatorname{tg}^2\theta)^{3/2}} = \int \frac{a \sec^2\theta \, d\theta}{(a^2 \cdot (1+\operatorname{tg}^2\theta))^{3/2}} = \int \frac{a \sec^2\theta \, d\theta}{(a^2 \cdot \sec^2\theta)^{3/2}}$$

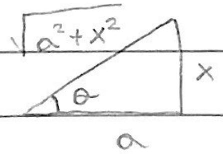
$$= \int \frac{a \sec^2\theta \, d\theta}{\sqrt{a^6 \cdot \sec^6\theta}} = \int \frac{a \cdot \sec^2\theta \, d\theta}{|a|^3 \cdot |\sec\theta|^3}, \text{ como } a > 0 \text{ e } \sec\theta > 0$$

$$= \int \frac{\cancel{a \sec^2\theta} \, d\theta}{a^3 \sec^{3}\theta} = \int \frac{1}{a^2 \cdot \sec^2\theta} \, d\theta \quad \text{como } a \text{ é uma constante,}$$

$$= \frac{1}{a^2} \int \frac{1}{\sec^2\theta} \, d\theta = \frac{1}{a^2} \int \cos^2\theta \, d\theta = \frac{1}{a^2}\left(\int \frac{1}{2} \, d\theta + \frac{1}{2} \int \cos(\underbrace{2\theta}_{u=2\theta}) \, d\theta\right)$$

$$= \frac{1}{a^2}\left(\frac{1}{2}\theta + \frac{1}{4}\operatorname{sen}(2\theta)\right) = \frac{\theta}{2a^2} + \frac{\operatorname{sen}(2\theta)}{4a^2} = \frac{\theta}{2a^2} + \frac{\cancel{2} \operatorname{sen}\theta \cos\theta}{\cancel{4} 2a^2}$$

voltando θ para x



$x = a \operatorname{tg}\theta \qquad \operatorname{tg}\theta = \frac{op}{adj} = \frac{x}{a}$

$\operatorname{sen}\theta = \frac{op}{hip} = \frac{x}{\sqrt{a^2+x^2}} \qquad \cos\theta = \frac{adj}{hip} = \frac{a}{\sqrt{a^2+x^2}} \qquad \theta = \operatorname{arctg}\left(\frac{x}{a}\right)$

$$= \frac{\operatorname{arctg}\left(\frac{x}{a}\right)}{2a^2} + \frac{\left(\frac{x}{\sqrt{a^2+x^2}}\right) \cdot \left(\frac{a}{\sqrt{a^2+x^2}}\right)}{2a^2} = \frac{\operatorname{arctg}\left(\frac{x}{a}\right)}{2a^2} + \frac{x \cdot a}{\left(\sqrt{a^2+x^2}\right)^2} \cdot \frac{1}{2a^2}$$

$$= \frac{\operatorname{arctg}\left(\frac{x}{a}\right)}{2a^2} + \frac{x \cdot a}{2a^2 \cdot (a^2+x^2)} \Bigg|_0^a$$

$$\Rightarrow \left(\frac{\operatorname{arctg}(1)}{2a^2} + \frac{a \cdot a}{2a^2 \cdot (a^2+a^2)}\right) - \left(\cancel{\frac{\operatorname{arctg}(0)}{2a^2}}^{\,0} + \cancel{\frac{0 \cdot a}{2a^2 \cdot (a^2)}}^{\,0}\right)$$

$$= \frac{\operatorname{arctg}(1)}{2a^2} + \frac{1}{4a^2}$$

?

11 $\displaystyle\int_0^{1/2} x\cdot\sqrt{1-4x^2}\,dx \Rightarrow \int x\cdot\sqrt{1-4x^2}\,dx = \int x\cdot\sqrt{u}\,dx$

$u = 1-4x^2 \qquad du = -4\cdot 2x = -8x$

$\displaystyle = -\frac{1}{8}\int\sqrt{u}\,du = -\frac{1}{8}\cdot\frac{2\cdot(1-4x^2)^{3/2}}{3} = -\frac{1}{12}\cdot(1-4x^2)^{3/2}$

$\displaystyle \Rightarrow \left(-\frac{1}{12}\cdot(\underbrace{1-4\cdot\tfrac{1}{4}}_{0})^{3/2}\right) - \left(-\frac{1}{12}\cdot(1-0)^{3/2}\right) = -0 + \frac{1}{12} = \frac{1}{12}$

13 $\displaystyle\int \frac{\sqrt{x^2-9}}{x^3}\,dx$

$x = 3\sec\theta \qquad 0\leq\theta\leq \pi/2$

$dx = 3\sec\theta\, tg\,\theta\, d\theta$

$\displaystyle \Rightarrow \int\frac{\sqrt{9\sec^2\theta-9}\cdot 3\cancel{\sec\theta}\,tg\,\theta\,d\theta}{27\sec^3\theta} = \int\frac{\sqrt{9}\cdot\sqrt{tg^2\theta}\;3\,tg\,\theta\,d\theta}{27\sec^2\theta}$

$\displaystyle = \frac{1}{3}\int\frac{tg\,\theta\cdot tg\,\theta}{\sec^2\theta}\,d\theta = \frac{1}{3}\int\frac{tg^2\theta}{\sec^2\theta}\,d\theta = \frac{1}{3}\left(\int\cancelto{1}{\frac{\sec^2\theta}{\sec^2\theta}}\,d\theta - \int\frac{1}{\sec^2\theta}\,d\theta\right)$

$\displaystyle = \frac{1}{3}\left(\theta - \int\cos^2\theta\,d\theta\right) = [\cdots]$

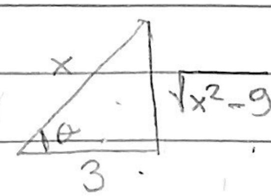
$\displaystyle \int\cos^2\theta\,d\theta = \int\frac{1}{2}\,d\theta + \frac{1}{4}\int\cos(u)\,du = \frac{\theta}{2} + \frac{sen(2\theta)}{4} + c$

$u = 2\theta$

$\displaystyle [\cdots] = \frac{1}{3}\left(\theta - \left(\frac{\theta}{2} + \frac{\cancel{2}\,sen\,\theta\cos\theta}{\cancel{4}\,2}\right)\right) + c = \frac{1}{3}\left(\theta - \frac{\theta}{2} - \frac{sen\,\theta\cos\theta}{2}\right)$

$\displaystyle = \frac{\theta}{6} - \frac{sen\,\theta\cos\theta}{6} = [\cdots]$

$\displaystyle x = 3\sec\theta = 3\cdot\frac{1}{\cos\theta} \qquad \cos\theta = \frac{3}{x}$



$\displaystyle sen\,\theta = \frac{\sqrt{x^2-9}}{x}$

$\displaystyle \theta = \arccos\left(\frac{3}{x}\right)$

$\displaystyle [\cdots] = \frac{\arccos\left(\frac{3}{x}\right)}{6} - \frac{\left(\frac{\sqrt{x^2-9}}{x}\right)\cdot\left(\frac{3}{x}\right)}{6} = \frac{\arccos\left(\frac{3}{x}\right)}{6} - \frac{1}{\cancel{6}\,2}\cdot\frac{\cancel{3}\sqrt{x^2-9}}{x^2}$

$\displaystyle = \frac{1}{6}\arccos\left(\frac{3}{x}\right) - \frac{\sqrt{x^2-9}}{2x^2} + c$

17 $\displaystyle\int \frac{x}{\sqrt{x^2-7}}\,dx = \int \frac{x}{\sqrt{x^2-7}}\,dx$ | $x = \sqrt{7}\,\sec\theta \quad 0 < \theta \le \pi/2$; $dx = \sqrt{7}\,\sec\theta\, tg\,\theta\, d\theta$

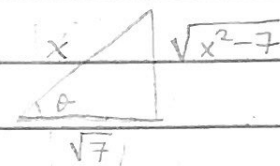
$$= \int \frac{\sqrt{7}\sec\theta \cdot \sqrt{7}\sec\theta \cdot tg\,\theta\, d\theta}{\sqrt{7\sec^2\theta - 7}}$$

$$= \int \frac{\sqrt{7}\sec^2\theta\, tg\,\theta\, d\theta}{\cancel{\sqrt{7}}\cdot \underbrace{\sqrt{\sec^2\theta - 1}}_{tg^2\theta}} = \int \frac{\sqrt{7}\sec^2\theta\,\cancel{tg\,\theta}\, d\theta}{\cancel{tg\,\theta}} = \int \sqrt{7}\sec^2\theta\, d\theta$$

$tg^2\theta$, $\sqrt{tg^2\theta} = |tg\,\theta| = tg\,\theta$

$= \sqrt{7}\; tg\,\theta + c = [\cdots]$

$x = \sqrt{7}\sec\theta = \dfrac{\sqrt{7}}{\cos\theta}$ $\quad \cos\theta = \dfrac{\sqrt{7}}{x} = \dfrac{adj}{hip}$



$tg\,\theta = \dfrac{op}{adj} = \dfrac{\sqrt{x^2-7}}{\sqrt{7}}$

$[\cdots] = \cancel{\sqrt{7}} \cdot \dfrac{\sqrt{x^2-7}}{\cancel{\sqrt{7}}} + c = \sqrt{x^2-7} + c$ //

19 $\displaystyle\int \frac{\sqrt{1+x^2}}{x}\,dx$ | $x = 1\cdot tg\,\theta \quad -\pi/2 < \theta < \pi/2$; $dx = \sec^2\theta\, d\theta \quad \theta \neq 0$

$$= \int \frac{\sqrt{1+tg^2\theta}\cdot \sec^2\theta\, d\theta}{tg\,\theta} = \int \frac{\sec\theta\cdot\sec^2\theta\, d\theta}{tg\,\theta} = \int \frac{\sec^3\theta\, d\theta}{tg\,\theta}$$

$$= \int \frac{(1+tg^2\theta)\cdot\sec\theta\, d\theta}{tg\,\theta} = \int \frac{\sec\theta + tg^2\theta\,\sec\theta\, d\theta}{tg\,\theta} = \int \frac{\sec\theta\, d\theta}{tg\,\theta} + \int \frac{tg^{\cancel{2}}\theta\,\sec\theta\, d\theta}{\cancel{tg\,\theta}}$$

$$= \int \frac{\sec\theta\, d\theta}{tg\,\theta} + \int \sec\theta\cdot tg\,\theta = [\cdots]$$

resolvendo a primeira integral

$$\int \frac{\sec\theta\, d\theta}{tg\,\theta} = \int \frac{1}{\cancel{\cos\theta}}\cdot\frac{\cancel{\cos\theta}}{sen\,\theta}\, d\theta = \int \frac{1}{sen\,\theta}\, d\theta = \int cossec\,\theta\, d\theta$$

$u = cossec\,\theta + cotg\,\theta \quad du = -cossec\,\theta\cdot cotg\,\theta - cossec^2\theta$

$$= \int cossec\,\theta \left(\frac{cossec\,\theta + cotg\,\theta}{cossec\,\theta + cotg\,\theta}\right) d\theta = \int \frac{cossec^2\theta + cossec\,\theta\cdot cotg\,\theta}{cossec\,\theta + cotg\,\theta}\, d\theta$$

$= -\int \frac{1\,du}{u} = -\ln|u| + c = -\ln|\text{cossec}\,\theta + \text{cotg}\,\theta| + c$

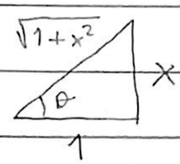
resolvendo a segunda integral

$\int \sec\theta \cdot tg\,\theta \; d\theta = \sec\theta + c$

resultado em theta

$[\cdots] = -\ln|\text{cossec}\,\theta + \text{cotg}\,\theta| + \sec\theta + c$

$x = tg\,\theta = \frac{op}{adj}$ $\qquad$ $\text{cossec}\,\theta = \frac{1}{\text{sen}\,\theta} = \frac{\sqrt{1+x^2}}{x}$ $\qquad$ $\sec\theta = \sqrt{1+x^2}$



$\text{sen}\,\theta = \frac{x}{\sqrt{1+x^2}}$ $\qquad$ $\cos\theta = \frac{1}{\sqrt{1+x^2}}$

$\text{cotg}\,\theta = \frac{1}{x}$

resultado em x

$= \sqrt{1+x^2} - \ln\left|\frac{\sqrt{1+x^2}}{x} + \frac{1}{x}\right| + c$

21 $\int_0^{0,6} \frac{x^2}{\sqrt{9-25x^2}}$ $\longrightarrow$ $\sqrt{9-25x^2} = \sqrt{9-u^2} = \sqrt{9-9\,\text{sen}^2\theta}$

$u = 5x$ $\qquad$ $u = 3\,\text{sen}\,\theta$ $\qquad$ $u^2 = 25x^2$

$5x = 3\,\text{sen}\,\theta$ $\qquad$ $x = 3/5\,\text{sen}\,\theta$

$\int \frac{9/25\,\text{sen}^2\theta}{\sqrt{9-9\,\text{sen}^2\theta}}\,dx$ $\qquad$ $dx = 3/5\cos\theta\,d\theta$

domínio de θ : $-\pi/2 < \theta < \pi/2$

$= \int \frac{9/25\,\text{sen}^2\theta \cdot 3/5\cos\theta\,d\theta}{\sqrt{9}\cdot\sqrt{1-\text{sen}^2\theta}}$ $\qquad$ $\sqrt{1-\text{sen}^2\theta} = \sqrt{\cos^2\theta} = |\cos\theta| = \cos\theta$

$= \int \frac{27\,\text{sen}^2\theta\,\cancel{\cos\theta}}{125\cdot 3\cdot\cancel{\cos\theta}}\,d\theta = \frac{9}{125}\int \text{sen}^2\theta\,d\theta$

resolvendo a integral

$\int \text{sen}^2\theta\,d\theta = \int \frac{1}{2}\,d\theta - \int \frac{1}{2}\cdot\cos(\underbrace{2\theta}_{u\,;\;du=2})\,d\theta = \frac{1}{2}\theta - \frac{1}{4}\,\text{sen}(2\theta)$

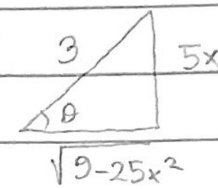
$$= \frac{\theta}{2} - \frac{2 \operatorname{sen}\theta \cos\theta}{4} = \frac{\theta}{2} - \frac{\operatorname{sen}\theta \cos\theta}{2}$$

substituindo a integral resolvida

$$= \frac{9}{125} \cdot \left(\frac{\theta}{2} - \frac{\operatorname{sen}\theta \cos\theta}{2} \right) = [\cdots]$$

voltando para x

$$\left[\begin{array}{l} x = \frac{3}{5} \operatorname{sen}\theta \qquad \operatorname{sen}\theta = \frac{5x}{3} = \frac{op}{hip} \\ \cos\theta = \frac{adj}{hip} = \frac{\sqrt{9-25x^2}}{3} \qquad \theta = \operatorname{arcsen}\left(\frac{5x}{3}\right) \end{array} \right.$$



$$[\cdots] = \frac{9}{125} \left(\frac{\operatorname{arcsen}\left(\frac{5x}{3}\right)}{2} - \frac{\frac{5x}{3} \cdot \frac{\sqrt{9-25x^2}}{3}}{2} \right)$$

$$= \frac{9}{125} \left(\frac{\operatorname{arcsen}\left(\frac{5x}{3}\right)}{2} - \frac{5x \cdot \sqrt{9-25x^2}}{9} \cdot \frac{1}{2} \right)$$

$$= \frac{9 \operatorname{arcsen}\left(\frac{5x}{3}\right)}{250} - \frac{45x\sqrt{9-25x^2}}{2250} = \frac{9 \operatorname{arcsen}\left(\frac{5x}{3}\right)}{250} - \frac{x\sqrt{9-25x^2}}{50} + c$$

aplicando os limites de integração $\Big|_0^{0.6}$, onde $0.6 = 3/5$

$$\frac{9 \cdot \operatorname{arcsen}(1)}{250} - \frac{3}{5} \cdot \frac{\sqrt{9 - 25 \cdot \frac{9}{25}}}{5} - \cancel{\frac{9 \cdot \operatorname{arcsen}(0)}{250}} + \cancel{\frac{0 \cdot \sqrt{9-0^2}}{50}}$$

$$= \frac{9 \cdot \pi/2}{250} - \cancel{\frac{3\sqrt{0}}{25}} = \frac{9\pi}{500}$$

23 $\displaystyle\int \frac{1}{\sqrt{x^2+2x+5}} dx \qquad x^2+2x+5 = x^2+2x+1+4 = (x+1)^2+4$

$$= \int \frac{1}{\sqrt{(x+1)^2+4}} dx \qquad \left[\begin{array}{l} x+1 = 2 \operatorname{tg}\theta \qquad \pi/2 < \theta < \pi/2 \\ dx = 2\sec^2\theta \, d\theta \end{array} \right.$$

$$= \int \frac{2\sec^2\theta \, d\theta}{\sqrt{4\operatorname{tg}^2\theta + 4}} = \int \frac{2}{2} \frac{\sec^2\theta}{\sqrt{\operatorname{tg}^2+1}} d\theta = \int \frac{\sec^2\theta}{\sqrt{\sec^2\theta}} d\theta = \int \sec\theta \, d\theta$$

$$= \ln|\sec\theta + \operatorname{tg}\theta| + c = \ln\left| \frac{\sqrt{(x+1)^2+4} + (x+1)}{2} \right| + c$$

27 $\int \sqrt{x^2+2x}\, dx = \int \sqrt{x^2+2x+1-1}\, dx$ $\quad \left[\begin{array}{l} x+1 = \sec\theta \qquad 0 \leq \theta \leq \pi/2 \\ dx = \sec\theta \, tg\,\theta \, d\theta \end{array} \right.$

$= \int \sqrt{(x+1)^2-1}\, dx$

$= \int \sqrt{\sec^2\theta - 1} \cdot \sec\theta \, tg\,\theta \, d\theta = \int tg\,\theta \cdot \sec\theta \, tg\,\theta \, d\theta$

$= \int \sec\theta \cdot tg^2\theta \, d\theta = \int \sec\theta \cdot (\sec^2\theta - 1)\, d\theta = \int \sec^3\theta \, d\theta - \int \sec\theta \, d\theta$

$= \underbrace{\int \sec^2\theta \cdot \sec\theta \, d\theta}_{①} - \ln|\sec\theta + tg\,\theta| = [\ldots]$

① $\int \sec^2\theta \cdot \sec\theta \, d\theta$

$f(x) = \sec\theta \qquad f'(x) = \sec\theta \cdot tg\,\theta$

$g'(x) = \sec^2\theta \qquad g(x) = tg\,\theta$

$= \sec\theta \cdot tg\,\theta - \int \sec\theta \cdot tg\,\theta \cdot tg\,\theta \, d\theta$

$= \sec\theta \cdot tg\,\theta - \int \sec\theta \cdot tg^2\theta \, d\theta = \sec\theta \, tg\,\theta - \int (\sec^2\theta - 1)\cdot \sec\theta \, d\theta$

$= \sec\theta \, tg\,\theta - \left(\int \sec^3\theta \, d\theta - \int \sec\theta \, d\theta\right)$

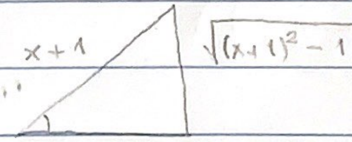
$\Rightarrow 2\int \sec^3\theta \, d\theta = \sec\theta \, tg\,\theta + \int \sec\theta \, d\theta \cancel{- \int \sec^3\theta} + \cancel{\int \sec^3\theta}$

$\Rightarrow \int \sec^3\theta = \dfrac{\sec\theta \, tg\,\theta}{2} + \dfrac{\ln|\sec\theta + tg\,\theta|}{2} + c$

resultado em θ

$[\ldots] = \dfrac{\sec\theta \, tg\,\theta}{2} + \dfrac{\ln|\sec\theta + tg\,\theta|}{2} - \ln|\sec\theta + tg\,\theta|$

$\left[\sec\theta = x+1 = \dfrac{1}{\cos\theta} \right.$



$tg\,\theta = \sqrt{x^2+2x}$

$= \dfrac{(x+1)\cdot\sqrt{x^2+2x}}{2} - \dfrac{\ln|(x+1) + \sqrt{x^2+2x}|}{2} + c$ //

25 $\int x^2 \sqrt{3+2x-x^2}\, dx$, $x^2-2x+1 = (x-1)^2$

$= \int x^2 \sqrt{-(x^2-2x-3)}\, dx = \int x^2 \sqrt{-(x^2-2x+1-4)}\, dx = \int x^2 \sqrt{-([x-1]^2-4)}\, dx$

$= \int x^2 \sqrt{4-(x-1)^2}\, dx$

$\left[\begin{array}{l} x-1 = 2 \cdot sen\,\theta \qquad x = 2\,sen\,\theta + 1 \qquad -\pi/2 \le \theta \le \pi/2 \\ dx = 2\cos\theta\, d\theta \end{array}\right.$

$= \int (4\,sen^2\theta + 4\,sen\,\theta + 1) \cdot \sqrt{4 - 4\,sen^2\theta}\, dx = \int (4\,sen^2\theta + 4\,sen\,\theta + 1) \cdot 2\sqrt{1 - sen^2\theta}\, dx$

$[\sqrt{1-sen^2\theta} = \sqrt{\cos^2\theta} = |\cos\theta| = \cos\theta$ no domínio do θ

$= \int (4\,sen^2\theta + 4\,sen\,\theta + 1) \cdot 2\cos\theta\ \cancel{dx} \to 2\cos\theta\, d\theta$

$= \underbrace{16\int sen^2\theta \cos^2\theta\, d\theta}_{③} + \underbrace{16\int sen\,\theta \cos^2\theta\, d\theta}_{②} + \underbrace{4\int \cos^2\theta\, d\theta}_{①}$

① $\int \cos^2\theta\, d\theta = \int \frac{1}{2}\, d\theta + \frac{1}{2}\int \cos(2\theta)\, d\theta = \theta/2 + \frac{1}{2}\cdot\left(\frac{sen(2\theta)}{2}\right)$

$= \theta/2 + sen(2\theta)/4$

② $\int sen\,\theta \cos^2\theta\, d\theta = -\int u^2\, du = -\frac{u^3}{3} = -\frac{\cos^3\theta}{3}$

$u = \cos\theta \qquad du = -sen\,\theta$

③ $\int sen^2\theta \cos^2\theta\, d\theta = \int (1-\cos^2\theta)\cos^2\theta\, d\theta = \int \cos^2\theta\, d\theta - \int \cos^4\theta\, d\theta = [\ldots]$

$\Rightarrow \int \cos^4\theta\, d\theta = \int (\cos^2\theta)^2\, d\theta = \int \left(\frac{1}{2} + \frac{1}{2}\cdot\cos(2\theta)\right)^2 d\theta = \int \frac{1}{4}\, d\theta + \int \frac{1}{2}\cos(2\theta)\, d\theta$

$+ \int \frac{1}{4}\cos^2(2\theta)\, d\theta = \frac{\theta}{4} + \frac{1}{4}\, sen(2\theta) + \frac{1}{4}\int \cos^2(\underbrace{2\theta}_{t=2\theta})\, d\theta = \frac{\theta}{4} + \frac{sen(2\theta)}{4} + \frac{\theta}{8} + \frac{sen(4\theta)}{32}$

$[\ldots] = \frac{4\theta}{8} + \cancel{\frac{sen(2\theta)}{4}} - \left(\frac{3\theta}{8} + \cancel{\frac{sen(2\theta)}{4}} + \frac{sen(4\theta)}{32}\right) = \frac{\theta}{8} - \frac{sen(4\theta)}{32}$

resultado

$= 2\theta - \frac{sen(4\theta)}{2} - \frac{16\cos^3\theta}{3} + 2\theta + sen(2\theta)$

voltar para x...