

cálculo 2, stewart vol.1, ed. 8, cap 7.3

$$1 \int \frac{1}{x^2 \sqrt{4-x^2}} dx \quad \left[\begin{array}{l} x = 2 \sin(\theta) \quad \text{onde} \quad -\pi/2 \leq \theta \leq \pi/2 \\ dx = 2 \cos(\theta) d\theta \quad \text{e } \theta \neq 0 \end{array} \right.$$

$$\Rightarrow \int \frac{1 \cdot 2 \cos(\theta) d\theta}{(2 \sin(\theta))^2 \sqrt{4 - (2 \sin(\theta))^2}} = \int \frac{2 \cos(\theta) d\theta}{4 \sin^2(\theta) \cdot \sqrt{4 - 4 \sin^2(\theta)}}$$

$$= \int \frac{\cos(\theta) d\theta}{2 \sin^2(\theta) \cdot \sqrt{4(1 - \sin^2(\theta))}} = \int \frac{\cos(\theta) d\theta}{\sin^2(\theta) \cdot 2 \cdot \sqrt{(1 - \sin^2(\theta))} \cdot \sqrt{4}}$$

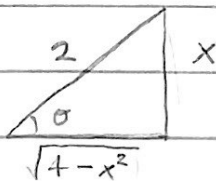
$$= \frac{1}{4} \int \frac{\cos(\theta) d\theta}{\sin^2(\theta) \cdot \sqrt{\cos^2(\theta)}} \quad \left[\begin{array}{l} \sqrt{\cos^2(\theta)} = |\cos(\theta)|, \text{ analisando o de-} \\ \text{mínio de } \theta, |\cos(\theta)| = \cos(\theta) \end{array} \right.$$

$$= \frac{1}{4} \int \frac{\cos(\theta) d\theta}{\sin^2(\theta) \cdot \cos(\theta)} = \frac{1}{4} \int \frac{1}{\sin^2(\theta)} d\theta = \frac{1}{4} \int \csc^2(\theta) d\theta$$

$$= \frac{1}{4} \cdot (-\cotg(\theta) + c) = -\frac{\cotg(\theta)}{4} + c.$$

$$x = 2 \cdot \sin(\theta)$$

$$\sin(\theta) = \frac{x}{2}$$



$$4 = x^2 + \text{adj}^2$$

$$\text{adj}^2 = 4 - x^2$$

$$\cdot \cotg(\theta) = \frac{x}{\sqrt{4-x^2}} \quad \Rightarrow \quad -\frac{\cotg(\theta)}{4} + c = -\frac{\frac{1}{\cotg(\theta)}}{4} + c = -\frac{1}{4 \cotg(\theta)} + c$$

$$= -\frac{1}{4 \left(\frac{x}{\sqrt{4-x^2}} \right)} + c = -\frac{\sqrt{4-x^2}}{4x} + c$$

$$3 \int \frac{\sqrt{x^2-4}}{x} dx \quad \left[\begin{array}{l} x = 2 \cdot \sec(\theta) \quad \underbrace{0 \leq \theta \leq \pi/2 \text{ ou } \pi \leq \theta \leq 3\pi/2}_{\text{↪ escolhida}} \\ dx = 2 \cdot \sec(\theta) \tan(\theta) d\theta \end{array} \right.$$

$$= \int \frac{\sqrt{4 \sec^2(\theta) - 4} \cdot 2 \sec(\theta) \tan(\theta) d\theta}{2 \sec(\theta)} = \int \frac{\sqrt{4 \cdot (\sec^2 - 1)} \cdot 2 \tan(\theta) d\theta}{2}$$

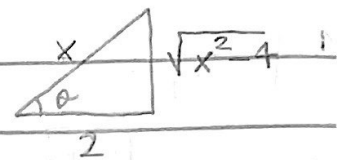
$$= 2 \int \sqrt{\tan^2 \theta} \cdot \tan \theta d\theta = 2 \int \tan \theta \cdot \tan \theta d\theta = 2 \cdot \int \tan^2 \theta d\theta$$

$$= 2 \cdot \int \sec^2 \theta - 1 d\theta = 2 \left(\int \sec^2 \theta d\theta - \int 1 d\theta \right)$$

$$= 2 \cdot (\tan \theta - \theta + c) = 2 \tan \theta - 2\theta + c = [\dots]$$

$$x = 2 \sec(\theta) = 2 \cdot \frac{1}{\cos \theta} = \frac{2}{\cos \theta}$$

$$\cos \theta = \frac{2}{x} = \frac{\text{adj}}{\text{hip}}$$



$$\tan \theta = \frac{\text{op}}{\text{adj}} = \frac{\sqrt{x^2 - 4}}{2}$$

$$\theta = \arccos\left(\frac{2}{x}\right)$$

$$[\dots] = \sqrt{x^2 - 4} - 2 \arccos\left(\frac{2}{x}\right) + c = \sqrt{x^2 - 4} - 2 \arccos\left(\frac{2}{x}\right) + c$$

$$5 \int \frac{\sqrt{x^2 - 1}}{x^4} dx$$

$$x = 1 \cdot \sec \theta$$

$$0 \leq \theta \leq \pi/2 \text{ or } \pi \leq \theta \leq 3\pi/2$$

$$dx = \sec \theta \cdot \tan \theta$$

↪ inválido

$$= \int \frac{\sqrt{\sec^2 \theta - 1} \cdot \sec \theta \cdot \tan \theta}{\sec^4 \theta} d\theta = \int \frac{|\tan \theta| \cdot \sec \theta \cdot \tan \theta}{\sec^3 \theta \cdot \sec \theta} d\theta$$

$$= \int \frac{\tan^2 \theta}{\sec^3 \theta} d\theta = \int \frac{(\sec^2 \theta - 1)}{\sec^3 \theta} d\theta = \int \frac{\sec^2 \theta}{\sec^3 \theta} d\theta - \int \frac{1}{\sec^3 \theta} d\theta$$

$$= \int \frac{1}{\sec \theta} d\theta - \int \frac{1}{\sec^3 \theta} d\theta = \int \cos \theta d\theta - \int \cos^3 \theta d\theta$$

$$= \sin \theta - \int \cos^3 \theta d\theta = [\dots]$$

$$\int \cos^3 \theta d\theta = \int \cos^2 \theta \cdot \cos \theta d\theta = \int (1 - \sin^2 \theta) \cdot \cos \theta d\theta$$

$$= \int \cos \theta d\theta - \int \sin^2 \theta \cos \theta d\theta = \sin \theta - \int u^2 du$$

$$\hookrightarrow u = \sin \theta \quad du = \cos \theta d\theta$$

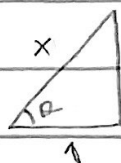
$$= \sin \theta - \frac{u^3}{3} + c = \sin \theta - \frac{\sin^3 \theta}{3} + c$$

$$[\dots] = \cancel{\sin \theta} - \left(\cancel{\sin \theta} - \frac{\sin^3 \theta}{3} \right) + c = \frac{\sin^3 \theta}{3} + c = [\dots]$$

$$x = \sec \theta$$

$$x = \frac{1}{\cos \theta}$$

$$\cos \theta = \frac{1}{x} = \frac{\text{ad}}{\text{hip}}$$



$$\sqrt{x^2 - 1}$$

$$\sin \theta = \frac{\text{op}}{\text{hip}} = \frac{\sqrt{x^2 - 1}}{x}$$

$$[\dots] = \frac{\left(\frac{\sqrt{x^2-1}}{x}\right)^3}{3} + C = \frac{\left(\sqrt{x^2-1}\right)^3 \cdot 1}{x^3 \cdot 3} + C = \frac{(x^2-1)^{3/2}}{x^3} \cdot \frac{1}{3} + C$$

$$7 \int_0^a \frac{dx}{(a^2+x^2)^{3/2}} \quad \left| \begin{array}{l} x = a \tan \theta \\ dx = a \sec^2 \theta d\theta \end{array} \right. \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2} \text{ e } \theta \neq 0$$

$$\Rightarrow \int \frac{a \cdot \sec^2 \theta d\theta}{(a^2 + a^2 \tan^2 \theta)^{3/2}} = \int \frac{a \sec^2 \theta d\theta}{(a^2 \cdot (1 + \tan^2 \theta))^{3/2}} = \int \frac{a \sec^2 \theta d\theta}{(a^2 \cdot \sec^2 \theta)^{3/2}}$$

$$= \int \frac{a \sec^2 \theta d\theta}{\sqrt{a^6 \cdot \sec^6 \theta}} = \int \frac{a \cdot \sec^2 \theta d\theta}{|a|^3 \cdot |\sec \theta|^3}, \text{ como } a > 0 \text{ e } \sec \theta > 0$$

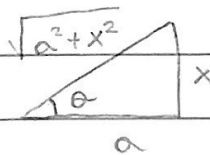
$$= \int \frac{a \sec^2 \theta d\theta}{a^3 \sec^3 \theta} = \int \frac{1}{a^2 \cdot \sec \theta} d\theta \quad \text{como } a \text{ é uma constante,}$$

$$= \frac{1}{a^2} \int \frac{1}{\sec \theta} d\theta = \frac{1}{a^2} \int \cos^2 \theta d\theta = \frac{1}{a^2} \left(\int \frac{1}{2} d\theta + \frac{1}{2} \int \cos(2\theta) d\theta \right)$$

$$= \frac{1}{a^2} \left(\frac{1}{2} \theta + \frac{1}{4} \sin(2\theta) \right) = \frac{\theta}{2a^2} + \frac{\sin(2\theta)}{4a^2} = \frac{\theta}{2a^2} + \frac{2 \sin \theta \cos \theta}{4a^2}$$

volvendo θ para x

$$x = a \tan \theta \quad \tan \theta = \frac{\text{op}}{\text{adj}} = \frac{x}{a}$$



$$\sin \theta = \frac{\text{op}}{\text{hip}} = \frac{x}{\sqrt{a^2+x^2}} \quad \cos \theta = \frac{\text{adj}}{\text{hip}} = \frac{a}{\sqrt{a^2+x^2}} \quad \theta = \arctan\left(\frac{x}{a}\right)$$

$$= \frac{\arctan\left(\frac{x}{a}\right)}{2a^2} + \frac{\left(\frac{x}{\sqrt{a^2+x^2}}\right) \cdot \left(\frac{a}{\sqrt{a^2+x^2}}\right)}{2a^2} = \frac{\arctan\left(\frac{x}{a}\right)}{2a^2} + \frac{x \cdot a}{\left(\sqrt{a^2+x^2}\right)^2} \cdot \frac{1}{2a^2}$$

$$= \frac{\arctan\left(\frac{x}{a}\right)}{2a^2} + \frac{x \cdot a}{2a^2 \cdot (a^2+x^2)} \Big|_0^a$$

$$\Rightarrow \left(\frac{\arctan(1)}{2a^2} + \frac{a \cdot a}{2a^2 \cdot (a^2+a^2)} \right) - \left(\frac{\cancel{\arctan(0)}}{2a^2} + \frac{\cancel{0 \cdot a}}{2a^2 \cdot (a^2)} \right)$$

$$= \frac{\arctan(1)}{2a^2} + \frac{1}{4a^2}$$

(3)

$$11 \int_0^{\frac{1}{2}} x \cdot \sqrt{1-4x^2} dx \Rightarrow \int x \cdot \sqrt{1-4x^2} dx = \int x \cdot \sqrt{u} dx$$

$$u = 1-4x^2 \quad du = -4 \cdot 2x = -8x$$

$$= \frac{-1}{8} \int \sqrt{u} du = \frac{-1}{8} \cdot \frac{2}{3} \cdot (1-4x^2)^{\frac{3}{2}} = \frac{-1}{12} \cdot (1-4x^2)^{\frac{3}{2}}$$

$$\Rightarrow \left(\frac{-1}{12} \cdot (1-4 \cdot \frac{1}{4})^{\frac{3}{2}} \right) - \left(\frac{-1}{12} \cdot (1-0)^{\frac{3}{2}} \right) = -0 + \frac{1}{12} = \frac{1}{12}$$

$$13 \int \frac{\sqrt{x^2-9}}{x^3} dx \quad x = 3 \sec \theta \quad 0 \leq \theta \leq \frac{\pi}{2}$$

$$dx = 3 \sec \theta \tan \theta d\theta$$

$$\Rightarrow \frac{\int \sqrt{9 \sec^2 \theta - 9} \cdot 3 \sec \theta \tan \theta d\theta}{27 \sec^3 \theta} = \frac{\int \sqrt{9} \cdot \sqrt{\tan^2 \theta} 3 \tan \theta d\theta}{27 \sec^2 \theta}$$

$$= \frac{1}{3} \int \frac{\tan \theta \cdot \tan \theta d\theta}{\sec^2 \theta} = \frac{1}{3} \int \frac{\tan^2 \theta d\theta}{\sec^2 \theta} = \frac{1}{3} \left(\int \frac{\sec^2 \theta}{\sec^2 \theta} d\theta - \int \frac{1}{\sec^2 \theta} d\theta \right)$$

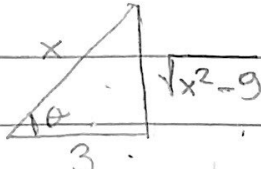
$$= \frac{1}{3} \left(\theta - \int \cos^2 \theta d\theta \right) = [\dots]$$

$$\int \cos^2 \theta d\theta = \int \frac{1}{2} d\theta + \frac{1}{4} \int \cos(2u) du = \frac{\theta}{2} + \frac{\sin(2\theta)}{4} + c$$

$$[\dots] = \frac{1}{3} \left(\theta - \left(\frac{\theta}{2} + \frac{\sin 2\theta}{4} \right) \right) + c = \frac{1}{3} \left(\theta - \frac{\theta}{2} - \frac{\sin 2\theta}{4} \right)$$

$$= \frac{\theta}{6} - \frac{\sin 2\theta}{12} = [\dots]$$

$$x = 3 \sec \theta = 3 \cdot \frac{1}{\cos \theta} \quad \cos \theta = \frac{3}{x}$$

$$\theta = \arccos \left(\frac{3}{x} \right)$$


$$\sin \theta = \frac{\sqrt{x^2-9}}{x}$$

$$[\dots] = \frac{\arccos \left(\frac{3}{x} \right)}{6} - \left(\frac{\sqrt{x^2-9}}{x} \right) \cdot \left(\frac{3}{x} \right) = \frac{\arccos \left(\frac{3}{x} \right)}{6} - \frac{1}{x^2} \cdot 3 \sqrt{x^2-9}$$

$$= \frac{1}{6} \arccos \left(\frac{3}{x} \right) - \frac{\sqrt{x^2-9}}{2x^2} + c$$

$$17 \int \frac{x}{\sqrt{x^2-7}} dx = \int \frac{x}{\sqrt{x^2-7}} dx \quad \begin{cases} x = \sqrt{7} \sec \theta & 0 < \theta \leq \frac{\pi}{2} \\ dx = \sqrt{7} \sec \theta \tan \theta d\theta \end{cases}$$

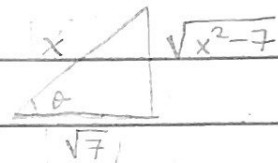
$$= \int \frac{\sqrt{7} \sec \theta \cdot \sqrt{7} \sec \theta \cdot \tan \theta d\theta}{\sqrt{7 \sec^2 \theta - 7}}$$

$$= \int \frac{\sqrt{7} \sec^2 \theta \tan \theta d\theta}{\sqrt{7 \cdot \underbrace{\sec^2 \theta - 1}_{\tan^2 \theta}}} = \int \frac{\sqrt{7} \sec^2 \theta \tan \theta d\theta}{\tan \theta} = \int \sqrt{7} \sec^2 \theta d\theta$$

$$\tan^2 \theta, \quad \sqrt{\tan^2 \theta} = |\tan \theta| = \tan \theta$$

$$= \sqrt{7} \tan \theta + c = [\dots]$$

$$x = \sqrt{7} \sec \theta = \frac{\sqrt{7}}{\cos \theta} \quad \cos \theta = \frac{\sqrt{7}}{x} = \frac{\text{adj}}{\text{hyp}}$$



$$\tan \theta = \frac{\text{op}}{\text{adj}} = \frac{\sqrt{x^2-7}}{\sqrt{7}}$$

$$[\dots] = \frac{\sqrt{7}}{\sqrt{7}} \cdot \sqrt{x^2-7} + c = \sqrt{x^2-7} + c //$$

$$19 \int \frac{\sqrt{1+x^2}}{x} dx \quad \begin{cases} x = 1 \cdot \tan \theta & -\frac{\pi}{2} < \theta < \frac{\pi}{2} \\ dx = \sec^2 \theta d\theta & \theta \neq 0 \end{cases}$$

$$= \int \frac{\sqrt{1+\tan^2 \theta} \cdot \sec^2 \theta d\theta}{\tan \theta} = \int \frac{\sec \theta \cdot \sec^2 \theta d\theta}{\tan \theta} = \int \frac{\sec^3 \theta d\theta}{\tan \theta}$$

$$= \int \frac{(1+\tan^2 \theta) \cdot \sec \theta d\theta}{\tan \theta} = \int \frac{\sec \theta + \tan^2 \theta \sec \theta d\theta}{\tan \theta} = \int \frac{\sec \theta d\theta}{\tan \theta} + \int \frac{\tan^2 \theta \sec \theta d\theta}{\tan \theta}$$

$$= \int \frac{\sec \theta d\theta}{\tan \theta} + \int \sec \theta \cdot \tan \theta = [\dots]$$

resolvendo a primeira integral

$$\int \frac{\sec \theta d\theta}{\tan \theta} = \int \frac{1}{\cos \theta} \cdot \frac{\cos \theta}{\sin \theta} d\theta = \int \frac{1}{\sin \theta} d\theta = \int \csc \theta d\theta$$

$$u = \csc \theta + \cot \theta \quad du = -\csc \theta \cdot \cot \theta - \csc^2 \theta$$

$$= \int \frac{\csc \theta}{\csc \theta + \cot \theta} d\theta = \int \frac{\csc^2 \theta + \csc \theta \cdot \cot \theta}{\csc \theta + \cot \theta} d\theta$$

$$= - \int \frac{1}{u} du = - \ln |u| + c = - \ln |\operatorname{cosec} \theta + \cot \theta| + c$$

resolvendo a segunda integral

$$\int \sec \theta \cdot \tan \theta d\theta = \sec \theta + c$$

resultado em theta

$$[\dots] = - \ln |\operatorname{cosec} \theta + \cot \theta| + \sec \theta + c$$

$$\left[\begin{array}{l} x = \tan \theta = \frac{\text{op}}{\text{adj}} \quad \operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{\sqrt{1+x^2}}{x} \quad \sec \theta = \sqrt{1+x^2} \\ \begin{array}{c} \sqrt{1+x^2} \\ \theta \\ 1 \end{array} \quad \sin \theta = \frac{x}{\sqrt{1+x^2}} \quad \cos \theta = \frac{1}{\sqrt{1+x^2}} \\ \cot \theta = \frac{1}{x} \end{array} \right.$$

resultado em x

$$= \sqrt{1+x^2} - \ln \left| \frac{\sqrt{1+x^2}}{x} + \frac{1}{x} \right| + c$$

$$\begin{array}{l} 21 \int_0^{0.6} \frac{x^2}{\sqrt{9-25x^2}} \rightarrow \sqrt{9-25x^2} = \sqrt{9-u^2} = \sqrt{9-9\sin^2 \theta} \\ u = 5x \quad u = 3\sin \theta \quad u^2 = 25x^2 \\ 5x = 3\sin \theta \quad x = \frac{3}{5}\sin \theta \end{array}$$

$$\int \frac{\frac{9}{25}\sin^2 \theta}{\sqrt{9-9\sin^2 \theta}} dx \quad dx = \frac{3}{5}\cos \theta d\theta \quad \text{domínio de } \theta: -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

$$= \int \frac{\frac{9}{25}\sin^2 \theta \cdot \frac{3}{5}\cos \theta d\theta}{\sqrt{9} \cdot \sqrt{1-\sin^2 \theta}} \quad \sqrt{1-\sin^2 \theta} = \sqrt{\cos^2 \theta} = |\cos \theta| = \cos \theta$$

$$= \int \frac{27 \sin^2 \theta \cos \theta}{125 \cdot 3 \cdot \cancel{\cos \theta}} d\theta = \frac{9}{125} \int \sin^2 \theta d\theta$$

resolvendo a integral

$$\int \sin^2 \theta d\theta = \int \frac{1}{2} d\theta - \int \frac{1}{2} \cdot \underbrace{\cos(2\theta)}_{u; du=2} d\theta = \frac{1}{2}\theta - \frac{1}{4}\sin(2\theta)$$

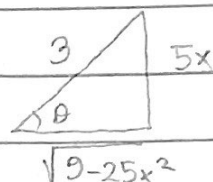
$$= \frac{\theta}{2} - \frac{2 \sin \theta \cos \theta}{4} = \frac{\theta}{2} - \frac{\sin \theta \cos \theta}{2}$$

substituindo a integral resolvida

$$= \frac{9}{125} \cdot \left(\frac{\theta}{2} - \frac{\sin \theta \cos \theta}{2} \right) = [\dots]$$

voltando para x

$$\left[\begin{array}{l} x = \frac{3}{5} \sin \theta \quad \sin \theta = \frac{5x}{3} = \frac{\text{op}}{\text{hip}} \end{array} \right.$$



$$\left[\begin{array}{l} \cos \theta = \frac{\text{adj}}{\text{hip}} = \frac{\sqrt{9-25x^2}}{3} \quad \theta = \arcsin\left(\frac{5x}{3}\right) \end{array} \right.$$

$$[\dots] = \frac{9}{125} \left(\frac{\arcsin\left(\frac{5x}{3}\right)}{2} - \frac{5x/3 \cdot \sqrt{9-25x^2}/3}{2} \right)$$

$$= \frac{9}{125} \left(\frac{\arcsin\left(\frac{5x}{3}\right)}{2} - \frac{5x \cdot \sqrt{9-25x^2}}{9} \cdot \frac{1}{2} \right)$$

$$= \frac{9 \arcsin\left(\frac{5x}{3}\right)}{250} - \frac{45x \sqrt{9-25x^2}}{250} = \frac{9 \arcsin\left(\frac{5x}{3}\right)}{250} - \frac{x \sqrt{9-25x^2}}{50} + C$$

aplicando os limites de integração $|_0^{0.6}$, onde $0.6 = \frac{3}{5}$

$$\frac{9 \arcsin(1)}{250} - \frac{3 \cdot \sqrt{9-25 \cdot \frac{9}{25}}}{50} - \frac{9 \arcsin(0)}{250} + \frac{0 \cdot \sqrt{9-0^2}}{50}$$

$$= \frac{9 \cdot \frac{\pi}{2}}{250} - \frac{3 \sqrt{0}}{50} = \frac{9\pi}{500}$$

$$23 \int \frac{1}{\sqrt{x^2+2x+5}} dx \quad x^2+2x+5 = x^2+2x+1+4 = (x+1)^2+4$$

$$= \int \frac{1}{\sqrt{(x+1)^2+4}} dx \quad \left[\begin{array}{l} x+1 = 2 \tan \theta \quad \frac{\pi}{2} < \theta < \frac{3\pi}{2} \\ dx = 2 \sec^2 \theta d\theta \end{array} \right.$$

$$= \int \frac{2 \sec^2 \theta d\theta}{\sqrt{4 \tan^2 \theta + 4}} = \int \frac{2 \sec^2 \theta d\theta}{2 \sqrt{\tan^2 \theta + 1}} = \int \frac{\sec^2 \theta d\theta}{\sqrt{\sec^2 \theta}} = \int \sec \theta d\theta$$

$$= \ln |\sec \theta + \tan \theta| + C = \ln \left| \frac{\sqrt{(x+1)^2+4}}{2} + \frac{(x+1)}{2} \right| + C$$

$$\begin{aligned}
 27 \quad \int \sqrt{x^2+2x} \, dx &= \int \sqrt{x^2+2x+1-1} \, dx \quad \begin{cases} x+1 = \sec \theta & 0 \leq \theta \leq \pi/2 \\ dx = \sec \theta \tan \theta \, d\theta \end{cases} \\
 &= \int \sqrt{(x+1)^2-1} \, dx \\
 &= \int \sqrt{\sec^2 \theta - 1} \cdot \sec \theta \tan \theta \, d\theta = \int \tan \theta \cdot \sec \theta \tan \theta \, d\theta \\
 &= \int \sec \theta \cdot \tan^2 \theta \, d\theta = \int \sec \theta \cdot (\sec^2 \theta - 1) \, d\theta = \int \sec^3 \theta \, d\theta - \int \sec \theta \, d\theta \\
 &= \underbrace{\int \sec^3 \theta \cdot \sec \theta \, d\theta}_{\textcircled{1}} - \ln |\sec \theta + \tan \theta| = [\dots]
 \end{aligned}$$

$$\textcircled{1} \int \sec^3 \theta \cdot \sec \theta \, d\theta$$

$$f(x) = \sec \theta$$

$$f'(x) = \sec \theta \cdot \tan \theta$$

$$g'(x) = \sec^2 \theta$$

$$g(x) = \tan \theta$$

$$\begin{aligned}
 &= \sec \theta \cdot \tan \theta - \int \sec \theta \cdot \tan \theta \cdot \tan \theta \, d\theta \\
 &= \sec \theta \cdot \tan \theta - \int \sec \theta \cdot \tan^2 \theta \, d\theta = \sec \theta \tan \theta - \int (\sec^2 \theta - 1) \cdot \sec \theta \, d\theta \\
 &= \sec \theta \tan \theta - \left(\int \sec^3 \theta \, d\theta - \int \sec \theta \, d\theta \right) \\
 &\Rightarrow 2 \int \sec^3 \theta \, d\theta = \sec \theta \tan \theta + \int \sec \theta \, d\theta - \cancel{\int \sec^3 \theta} + \cancel{\int \sec^3 \theta} \\
 &\Rightarrow \int \sec^3 \theta = \frac{\sec \theta \tan \theta}{2} + \frac{\ln |\sec \theta + \tan \theta|}{2} + c
 \end{aligned}$$

resultado em θ

$$[\dots] = \frac{\sec \theta \tan \theta}{2} + \frac{\ln |\sec \theta + \tan \theta|}{2} - \ln |\sec \theta + \tan \theta|$$

$$\begin{aligned}
 &\left[\begin{array}{l} \sec \theta = x+1 = \frac{1}{\cos \theta} \\ \tan \theta = \end{array} \right. \quad \begin{array}{c} x+1 \\ \text{''} \\ 1 \end{array} \quad \begin{array}{c} \sqrt{(x+1)^2-1} \\ \tan \theta = \sqrt{x^2+2x} \end{array}
 \end{aligned}$$

$$= \frac{(x+1) \cdot \sqrt{x^2+2x}}{2} - \frac{\ln |x+1 + \sqrt{x^2+2x}|}{2} + c //$$

$$25 \int x^2 \sqrt{3+2x-x^2} dx, \quad x^2-2x+1 = (x-1)^2$$

$$= \int x^2 \sqrt{-(x^2-2x-3)} dx = \int x^2 \sqrt{-(x^2-2x+1-4)} dx = \int x^2 \sqrt{-((x-1)^2-4)} dx$$

$$= \int x^2 \sqrt{4-(x-1)^2} dx$$

$$\begin{cases} x-1 = 2 \cdot \sin \theta & x = 2 \sin \theta + 1 & -\pi/2 \leq \theta \leq \pi/2 \\ dx = 2 \cos \theta d\theta \end{cases}$$

$$= \int (4 \sin^2 \theta + 4 \sin \theta + 1) \cdot \sqrt{4 - 4 \sin^2 \theta} dx = \int (4 \sin^2 \theta + 4 \sin \theta + 1) \cdot 2 \sqrt{1 - \sin^2 \theta} dx$$

$$\sqrt{1 - \sin^2 \theta} = \sqrt{\cos^2 \theta} = |\cos \theta| = \cos \theta \text{ no domínio de } \theta$$

$$= \int (4 \sin^2 \theta + 4 \sin \theta + 1) \cdot 2 \cos \theta dx \cdot 2 \cos \theta d\theta$$

$$= 16 \underbrace{\int \sin^2 \theta \cos^2 \theta d\theta}_{(3)} + 16 \underbrace{\int \sin \theta \cos^2 \theta d\theta}_{(2)} + 4 \underbrace{\int \cos^2 \theta d\theta}_{(1)}$$

$$\textcircled{1} \int \cos^2 \theta d\theta = \int \frac{1}{2} d\theta + \frac{1}{2} \int \cos(2\theta) d\theta = \theta/2 + \frac{1}{2} \cdot \left(\frac{\sin(2\theta)}{2} \right)$$

$$= \theta/2 + \sin(2\theta)/4$$

$$\textcircled{2} \int \sin \theta \cos^2 \theta d\theta = - \int u^2 du = - \frac{u^3}{3} = - \frac{\cos^3 \theta}{3}$$

$$u = \cos \theta \quad du = -\sin \theta$$

$$\textcircled{3} \int \sin^2 \theta \cos^2 \theta d\theta = \int (1 - \cos^2 \theta) \cos^2 \theta d\theta = \int \cos^2 \theta d\theta - \int \cos^4 \theta d\theta = [\dots]$$

$$\Rightarrow \int \cos^4 \theta d\theta = \int (\cos^2 \theta)^2 d\theta = \int \left(\frac{1}{2} + \frac{1}{2} \cos(2\theta) \right)^2 d\theta = \int \frac{1}{4} d\theta + \int \frac{1}{2} \cos(2\theta) d\theta$$

$$+ \int \frac{1}{4} \cos^2(2\theta) d\theta = \frac{\theta}{4} + \frac{1}{4} \sin(2\theta) + \frac{1}{4} \int \cos^2(2\theta) d\theta = \frac{\theta}{4} + \frac{\sin(2\theta)}{4} + \frac{\theta}{8} + \frac{\sin(4\theta)}{32}$$

$t = 2\theta$

$$[\dots] = \frac{4\theta}{8} + \frac{\sin(2\theta)}{4} - \left(\frac{3\theta}{8} + \frac{\sin(2\theta)}{4} + \frac{\sin(4\theta)}{32} \right) = \frac{\theta}{8} - \frac{\sin(4\theta)}{32}$$

resultado

$$= \frac{2\theta}{2} - \frac{\sin(4\theta)}{32} - \frac{16 \cos^3 \theta}{3} + 2\theta + \sin(2\theta)$$

volta para x...