$\rightarrow$ como $-1 \leq \cos\left(\frac{1}{x^2}\right) \leq 1$, $-x^2 \leq x^2 \cdot \cos\left(\frac{1}{x^2}\right) \leq x^2$

$\lim_{x \to 0} -x^2 = 0$ $\lim_{x \to 0} x^2 = 0$, pelo teorema do confronto

$$\lim_{x \to 0} x^2 \cdot \cos\left(\frac{1}{x^2}\right) = 0$$

⑦ a) $\frac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}(x-1)} = \frac{x-3}{x-1}$ $\lim_{x \to -3} \frac{x-3}{x-1} = \frac{-6}{-4} = \frac{3}{2}$

b) $\lim_{x \to 9^+} \frac{\sqrt{x}}{(x-9)^+} = +\infty$ $\lim_{x \to 9} \frac{\sqrt{x}}{(x-9)^+} = +\infty$

c) $\frac{4-\sqrt{x}}{x-16} = \frac{4-\sqrt{x}}{x-16} \cdot \frac{4+\sqrt{x}}{4+\sqrt{x}} = \frac{16-x}{(x-16)(4+\sqrt{x})} = \frac{-1(16-x)}{-1(x-16)(4+\sqrt{x})}$

$= \frac{\cancel{x-16}}{\cancel{(x-16)}(-4-\sqrt{x})} = \frac{1}{-4-\sqrt{x}}$ $\lim_{x \to 16} \frac{1}{-4-\sqrt{x}} = \frac{1}{-4-4} = \frac{1}{-8}$

d) $\frac{1-\sqrt{1-x^2}}{x} \cdot \frac{(1+\sqrt{1-x^2})}{(1+\sqrt{1-x^2})} = \frac{1-(1-x^2)}{x(1+\sqrt{1-x^2})} = \frac{x^{\cancel{2}}}{\cancel{x}(1+\sqrt{1-x^2})}$

$= \frac{x}{1+\sqrt{1-x^2}}$ $\lim_{x \to 0} \frac{x}{1+\sqrt{1-x^2}} = \frac{0}{1+\sqrt{1-0}} = \frac{0}{2} = 0$

e) $\lim_{x \to +\infty} \frac{\sqrt{x^2-9}}{2x-6} = \lim_{x \to +\infty} \frac{\sqrt{x^2\left(1-9\cdot\frac{1}{x^2}\right)}}{x\left(2-6\cdot\frac{1}{x}\right)} = \lim_{x \to \infty} \frac{\cancel{x}\sqrt{1-9\cdot\frac{1}{x^2}}}{\cancel{x}\left(2-6\cdot\frac{1}{x}\right)}$

$= \frac{\sqrt{1-9\cdot \cancel{\lim_{x\to\infty}\frac{1}{x^2}}^{\to 0}}}{2-6\cdot\cancel{\lim_{x\to\infty}\frac{1}{x}}^{\to 0}} = \frac{1}{2}$

f) quando $x \to 4^+$, $|x-4| = x-4$ $\lim_{x \to 4^+} \frac{\cancel{x-4}}{\cancel{x-4}} = 1$

g) $\frac{(x-2)(x+4)}{(x^2-4)(x^2+4)} = \frac{\cancel{(x-2)}\cdot(x+4)}{\cancel{(x-2)}(x+2)(x^2+4)}$ $\lim_{x \to 2} \frac{x+4}{(x+2)(x^2+4)} = \frac{6}{32} = \frac{3}{16}$

h) $\dfrac{\sqrt{x+2}-\sqrt{2x}}{x^2-2x} \cdot \dfrac{\sqrt{x+2}+\sqrt{2x}}{\sqrt{x+2}+\sqrt{2x}} = \dfrac{(x+2)-(2x)}{(x^2-2x)(\sqrt{x+2}+\sqrt{2x})}$

$= \dfrac{-x+2}{(x^2-2x)(\sqrt{x+2}+\sqrt{2x})} = \dfrac{-\cancel{(x-2)}}{x\cancel{(x-2)}(\sqrt{x+2}+\sqrt{2x})} = \dfrac{-1}{x\cdot(\sqrt{x+2}+\sqrt{2x})}$

$\lim\limits_{x\to 2} \dfrac{-1}{x(\sqrt{x+2}+\sqrt{2x})} = \dfrac{-1}{2(2+2)} = \dfrac{-1}{8}$

i) $\lim\limits_{x\to 0} \dfrac{\tan(x)}{x} = \lim\limits_{x\to 0} \dfrac{\frac{\mathrm{sen}(x)}{\cos(x)}}{x} = \lim\limits_{x\to 0} \dfrac{\mathrm{sen}(x)}{\cos(x)} \cdot \dfrac{1}{x} = \cancel{\lim\limits_{x\to 0} \dfrac{\mathrm{sen}(x)}{x}}^{1} \cdot \lim\limits_{x\to 0} \cos(x)$

$= 1 \cdot \lim\limits_{x\to 0} \cos(x)$ como $\mathrm{sen}^2 + \cos^2 = 1$, $\cos = \sqrt{1-\mathrm{sen}^2}$

$= \lim\limits_{x\to 0} \sqrt{1-\mathrm{sen}^2(x)} = \sqrt{1-\lim\limits_{x\to 0} \mathrm{sen}^2(x)} = \sqrt{1-0^2} = \sqrt{1} = 1$

j) $\lim\limits_{x\to 0} \dfrac{\sqrt{x^2}}{\mathrm{sen}(x)} = \dfrac{|x|}{\mathrm{sen}(x)}$ $\lim\limits_{x\to 0^+} \dfrac{|x|}{\mathrm{sen}(x)} = \lim\limits_{x\to 0^+} \dfrac{x}{\mathrm{sen}\,x}$

$\lim\limits_{x\to 0^+} \left(\dfrac{\mathrm{sen}(x)}{x}\right)^{-1} = \lim\limits_{x\to 0^+} 1^{-1} = 1 \longrightarrow$ logo, não existe

$\lim\limits_{x\to 0^-} \dfrac{-x}{\mathrm{sen}(x)} = \lim\limits_{x\to 0^-} -1 \cdot \dfrac{x}{\mathrm{sen}(x)} = -1 \cdot \lim\limits_{x\to 0^-} \left(\dfrac{\mathrm{sen}(x)}{x}\right)^{-1} = -1 \cdot 1 = -1$

k) $\lim_{x\to 0} \frac{\tan(3x)}{\operatorname{sen}(4x)} = \lim_{x\to 0} \frac{\operatorname{sen}(3x)}{\cos(3x)} \cdot \frac{1}{\operatorname{sen}(4x)} \cdot \frac{4x}{4x}$

$= \lim_{x\to 0} \frac{\operatorname{sen}(3x)}{\cos(3x)} \cdot \left(\frac{\operatorname{sen}(4x)}{4x}\right)^{-1} \cdot \frac{1}{4x} \cdot \frac{3x}{3x}$

$= \cancel{\lim_{x\to 0} \frac{\operatorname{sen}(3x)}{3x}}^{\to 1} \cdot \frac{3x}{\cos(3x)} \cdot \left(\cancel{\lim_{x\to 0} \frac{\operatorname{sen}(4x)}{4x}}^{\to 1}\right)^{-1} \cdot \lim_{x\to 0} \frac{1}{4x}$

$= 1 \cdot \lim_{x\to 0} \frac{3x}{\cos(3x)} \cdot 1^{-1} \cdot \lim \frac{1}{4x} = \lim_{x\to 0} \frac{3x}{\cos(3x)\cdot 4x} = \lim_{x\to 0} \frac{3}{\cos(3x)\cdot 4}$

$= \frac{3}{1\cdot 4} = \frac{3}{4}$

l) $\lim_{x\to p} \frac{\tan(x-p)}{x^2-p^2}, \ p \neq 0 \Rightarrow \frac{\operatorname{sen}(x-p)}{\cos(x-p)} \cdot \frac{1}{(x-p)(x+p)} \lim_{x\to p}$

$= \underbrace{\lim_{x\to p} \cancel{\frac{\operatorname{sen}(x-p)}{(x-p)}}^{\to 1}}_{} \cdot \lim_{x\to p} \frac{1}{\cos(x-p)(x+p)} = \frac{1}{1\cdot 2p} = \frac{1}{2p}$

se $x \to p$, $x - p \to 0$

então $\lim_{x-p\to 0} \frac{\operatorname{sen}(x-p)}{(x-p)} = 1$

m) $\lim_{x\to 0} \frac{3x^2}{\operatorname{tg}(x)\cdot\operatorname{sen}(x)} = \lim_{x\to 0} \frac{3x^2}{\frac{\operatorname{sen}^2(x)}{\cos(x)}} = \lim_{x\to 0} \frac{3x^2}{1} \cdot \frac{\cos(x)}{\operatorname{sen}^2(x)}$

$= \lim_{x\to 0} \frac{x}{\operatorname{sen}(x)} \cdot \frac{x}{\operatorname{sen}(x)} \cdot 3\cdot\cos(x) = \left(\cancel{\lim_{x\to 0} \frac{\operatorname{sen}(x)}{x}}^{\to 1}\right)^{-1} \cdot \left(\cancel{\lim_{x\to 0} \frac{\operatorname{sen}(x)}{x}}^{\to 1}\right)^{-1} \cdot \lim_{x\to 0} 3\cos(x)$

$= 1\cdot 1\cdot \lim_{x\to 0} 3\cos(x) = 3$

n) $\lim_{x\to 2} \frac{\sqrt[3]{x}-\sqrt[3]{2}}{x-2}$

$a^3 - b^3 = (a-b)(a^2+ab+b^2)$ $\quad a^3 = x \quad b^3 = 2$

$x - 2 = (\sqrt[3]{x}-\sqrt[3]{2})((\sqrt[3]{x})^2+\sqrt[3]{x}\sqrt[3]{2}+(\sqrt[3]{2})^2)$ $\quad a = \sqrt[3]{x} \quad b = \sqrt[3]{2}$

$\lim_{x\to 2} \frac{(\sqrt[3]{x}-\sqrt[3]{2})((\sqrt[3]{x})^2+\sqrt[3]{x}\sqrt[3]{2}+(\sqrt[3]{2})^2)}{(x-2)((\sqrt[3]{x})^2+\sqrt[3]{x}\sqrt[3]{2}+(\sqrt[3]{2})^2)} = \lim_{x\to 2} \frac{\cancel{x-2}}{\cancel{(x-2)}\cdot[\ldots]}$

$= \lim_{x\to 2} \frac{1}{(\sqrt[3]{x})^2+\sqrt[3]{x}\sqrt[3]{2}+(\sqrt[3]{2})^2} = \frac{1}{3(\sqrt[3]{2})^2}$