calculo 1, lista de exercícios 7

1a $x_0 = 36$ $\quad f(x) = \sqrt{x}$

$dx = 0,7$ $\quad f'(x) = \frac{1}{2\sqrt{x}}$

$\Delta y \simeq dy = f'(x_0) \cdot dx$

$dy = \frac{1}{2\sqrt{36}} \cdot 0,7 = \frac{1}{12} \cdot 0,7 = \frac{1}{12} \cdot \frac{7}{10} = \frac{7}{120}$

$dy \simeq \Delta y = f(x_0 + \Delta x) - f(x_0)$

$= f(36 + 0,7) - f(x_0) = \sqrt{36,7} - \sqrt{36}$

$\frac{7}{120} + \sqrt{36} = \frac{7}{120} + 6 = 6,058$

1b $f(x_0 + \Delta x) \simeq f'(x_0) \cdot \Delta x + f(x_0)$

$x_0 = 16$ $\quad f(x) = \sqrt[4]{x}$

$dx = -1$ $\quad f'(x) = \frac{1}{4} x^{-\frac{3}{4}} = \frac{1}{4 \cdot \sqrt[4]{x^3}}$

$f(16-1) \simeq f'(x_0) \cdot \Delta x + f(x_0) = \frac{1}{4\sqrt[4]{16^3}} \cdot -1 + \sqrt[4]{16}$

$= \frac{1}{32} \cdot -1 + 2 = 1,96875$

1c $x_0 = 100$ $\quad f(x) = \sqrt{x}$

$\Delta x = 3$ $\quad f'(x) = \frac{1}{2\sqrt{x}}$

$f(100+3) \simeq f'(x_0) \cdot \Delta x + f(x_0) = \frac{1}{2\sqrt{100}} \cdot 3 + 10 = \frac{3}{20} + 10 = 10,15$

1d $\cos(43°) \rightarrow$ $x_0 = \pi/4 = 45°$ $\quad \Delta x = -\pi/90 = -2°$ $\quad f(x) = \cos x$ $\quad f'(x) = -\operatorname{sen} x$

$f(x_0 + \Delta x) = f(\pi/4 - \pi/90) = f(43°) \simeq f'(\pi/4) \cdot -\frac{\pi}{90} + f(\pi/4)$

$$= -\operatorname{sen}\left(\frac{\pi}{4}\right) \cdot -\frac{\pi}{90} + \cos\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} \cdot -\frac{\pi}{90} + \frac{\sqrt{2}}{2}$$

$$= \frac{+\sqrt{2} \cdot \pi}{180} + \frac{\sqrt{2}}{2}$$

2a $V = f(x) = x^3$ $\quad x_0 = 15$ $\quad f'(x) = 3x^2$

$\Delta v \simeq dv$ $\quad \Delta x = 0,01$

A variação de volume dv = $f'(x_0) \cdot \Delta x$

$$dv = 3 \cdot 15^2 \cdot 0,01 = 6,75$$

2b $A = f(x) = x^2$ $\quad x_0 = 15$ $\quad f'(x) = 2x$

$\Delta v \simeq dv$ $\quad \Delta x = 0,01$

$$dv = f'(x_0) \cdot \Delta x$$

$$dv = 2 \cdot 15 \cdot 0,01 = 0,3$$