

cálculo 1, stewart vol.1, ed 8, cap 7.8

1 a) a função $\frac{x}{x-1}$ tem assíntota vertical em $x=1$

b) intervalo é infinito

c) intervalo é infinito

d) a função $\frac{1}{\tan(x)} = \cot(x)$ tem assíntota vertical em $x=0$.

2 a) não é imprópria

b) é imprópria, $x \rightarrow \pi/2$, $\tan(x) \rightarrow \pm \infty$

c) é imprópria, assíntota vertical em $x=-1$

d) é imprópria, intervalo infinito

5
$$\int_3^{\infty} \frac{1}{(x-2)^{3/2}} dx = \lim_{t \rightarrow \infty} \int_3^t \frac{1}{(x-2)^{3/2}} dx$$

$$\Rightarrow \int \frac{1}{(x-2)^{3/2}} dx = \int \frac{du}{u^{3/2}} = \int u^{-3/2} du = \frac{u^{-1/2}}{-1/2} = -\frac{2}{\sqrt{u}} = -\frac{2}{\sqrt{x-2}} + c$$

$$u = x-2 \quad du = 1 dx$$

$$\Rightarrow \left(-\frac{2}{\sqrt{x-2}} + c \right) \Big|_3^t = -\frac{2}{\sqrt{t-2}} + \frac{2}{\sqrt{3-2}} = -\frac{2}{\sqrt{t-2}} + 2$$

$$\Rightarrow \lim_{t \rightarrow \infty} -\frac{2}{\sqrt{t-2}} + 2, \text{ quando } t \rightarrow \infty, \sqrt{t-2} \rightarrow \infty$$

$$= 0 + \lim_{t \rightarrow \infty} 2 = 2$$

7
$$\int_{-\infty}^0 \frac{1}{3-4x} dx = \lim_{s \rightarrow -\infty} \int_s^0 \frac{1}{3-4x} dx$$

$$\Rightarrow \int \frac{1}{3-4x} dx = \int \frac{1}{u} \cdot \frac{-4}{-4} du = -\frac{1}{4} \int \frac{1}{u} du = -\frac{1}{4} \ln|3-4x| + c$$

$$u = 3-4x \quad du = -4 dx$$

aplicando $\Big|_s^0$

$$\Rightarrow \left(-\frac{1}{4} \cdot \ln |3-4x| + c \right) \Big|_s^0$$

$$= -\frac{\ln |3-0|}{4} + \frac{\ln |3-4s|}{4} = \frac{\ln \left| \frac{3-4s}{3} \right|}{4}$$

aplicando o limite

$$\Rightarrow \lim_{s \rightarrow -\infty} \frac{\ln \left| \frac{3-4s}{3} \right|}{4} = +\infty, \text{ diverge}$$

$$9 \int_2^{\infty} e^{-5x} dx = \lim_{t \rightarrow +\infty} \int_2^t e^{-5x} dx$$

$$\Rightarrow \int e^{-5x} dx = -\frac{1}{5} \int e^u du = -\frac{1}{5} e^{-5x}$$

$$u = -5x \quad du = -5$$

$$\Rightarrow \left(-\frac{e^{-5x}}{5} \right) \Big|_2^t = \left(-\frac{e^{-5t}}{5} \right) - \left(-\frac{e^{-10}}{5} \right) = -\frac{e^{-5t}}{5} + \frac{e^{-10}}{5}$$

$$\Rightarrow \lim_{t \rightarrow \infty} -\frac{e^{-5t}}{5} + \frac{e^{-10}}{5} = \lim_{t \rightarrow \infty} -\frac{e^{-5t}}{1} \cdot \frac{1}{5} + \lim_{t \rightarrow \infty} \frac{e^{-10}}{5} = \left(\lim_{t \rightarrow \infty} -\frac{1}{e^{5t} \cdot 5} \right) + \frac{1}{5e^{10}} = 1$$

$$11 \int_0^{\infty} \frac{x^2}{\sqrt{1+x^3}} dx = \lim_{t \rightarrow \infty} \int_0^t \frac{x^2}{\sqrt{1+x^3}} dx$$

$$\Rightarrow \int \frac{x^2}{\sqrt{1+x^3}} dx = \frac{1}{3} \int \frac{du}{\sqrt{u}} = \frac{1}{3} \int u^{-1/2} du = \frac{1}{3} \cdot \frac{\sqrt{u} \cdot 2}{1} + c$$

$$u = 1+x^3 \quad du = 3x^2 dx$$

$$\Rightarrow \left(\frac{2\sqrt{1+x^3}}{3} + c \right) \Big|_0^t = \left(\frac{2\sqrt{1+t^3}}{3} - \frac{2\sqrt{1+0}}{3} \right) = \frac{2\sqrt{1+t^3}}{3} - \frac{2}{3}$$

$$\Rightarrow \left(\lim_{t \rightarrow \infty} \frac{2\sqrt{1+t^3}}{3} \right) - \frac{2}{3} = +\infty, \text{ diverge}$$

$$13 \int_{-\infty}^{\infty} x e^{-x^2} dx = \lim_{t \rightarrow \infty} \int_0^t x e^{-x^2} dx + \lim_{s \rightarrow -\infty} \int_{-\infty}^0 x e^{-x^2} dx$$

resolvendo a integral indefinida

$$\int x e^{-x^2} dx = -\frac{1}{2} \int e^u du = -\frac{1}{2} \cdot e^{-x^2} + c = -\frac{e^{-x^2}}{2} + c$$
$$u = -x^2 \quad du = -2x$$

resolvendo os limites

$$\lim_{t \rightarrow \infty} \frac{-e^{-t^2}}{2} + \frac{e^0}{2} = \left(\lim_{t \rightarrow \infty} \frac{-1}{e^{t^2} \cdot 2} \right) + \frac{1}{2} = \frac{1}{2}$$

$$\lim_{s \rightarrow -\infty} \frac{-e^0}{2} + \lim_{s \rightarrow -\infty} \frac{e^{-s^2}}{2} = \frac{-1}{2} + \lim_{s \rightarrow -\infty} \frac{1}{e^{s^2} \cdot 2} = \frac{-1}{2}$$

resultado final

$$\frac{1}{2} - \frac{1}{2} = 0$$

$$17 \int_1^{\infty} \frac{1}{x^2+x} dx = \lim_{t \rightarrow \infty} \int_1^t \frac{1}{x^2+x} dx$$

$$\Rightarrow \int \frac{1}{x^2+x} dx = \int \frac{1}{x(x+1)} dx = \frac{A}{x} + \frac{B}{x+1} = \frac{1}{x(x+1)}$$

$$A(x+1) + Bx = 1 \quad A = 1$$

$$Ax + A + Bx = 1 \quad B = -1$$

$$= \int \frac{1}{x} dx - \int \frac{1}{x+1} dx = \ln|x| - \ln|x+1| + C$$

$$\Rightarrow \text{aplicando } \Big|_1^t$$

$$\ln|t| - \ln|t+1| - \ln|1| + \ln|2|$$

$$\Rightarrow \lim_{t \rightarrow \infty} \ln \left| \frac{t}{t+1} \right| + \lim_{t \rightarrow \infty} \ln|2| = \ln \left(\lim_{t \rightarrow \infty} \frac{t}{t+1} \right) + \ln|2|$$

$$\stackrel{\text{L'H}}{=} \ln \left(\lim_{t \rightarrow \infty} \frac{1}{1} \right) + \ln|2| = \ln|1| + \ln|2| = \ln(2)$$

$$21 \int_1^{\infty} \frac{\ln x}{x} dx = \lim_{t \rightarrow \infty} \int_1^t \frac{\ln x}{x} dx$$

$$\Rightarrow \int \frac{\ln x}{x} dx = \int u du = \frac{u^2}{2} = \left(\frac{\ln(x)^2}{2} \right) \Big|_1^t$$

$$u = \ln x \quad du = \frac{1}{x} dx$$

$$\Rightarrow \frac{\ln(t)^2}{2} - \frac{\ln(1)^2}{2} = \frac{\ln(t)^2}{2}$$

$$\Rightarrow \lim_{t \rightarrow \infty} \frac{\ln(t)^2}{2} = \lim_{t \rightarrow \infty} \frac{\ln(t)^2}{2} = \ln \left(\lim_{t \rightarrow \infty} t \right)^2 = \ln(\infty)^2 = +\infty$$

ou seja, diverge

$$49 \quad \int_0^{\infty} \frac{x}{x^3+1} dx = \underbrace{\int_0^1 \frac{x}{x^3+1} dx}_{\text{com teste é convergente (intervalo fechado)}} + \int_1^{\infty} \frac{x}{x^3+1} dx \quad \text{testar}$$

$$\text{para } x \gg 1, \quad 0 \leq \frac{x}{x^3+1} \leq \frac{x}{x^3}$$

$$\int_1^{\infty} \frac{x}{x^3} dx = \int_1^{\infty} \frac{1}{x^2} dx = \lim_{t \rightarrow \infty} \left(-\frac{1}{x} \right) \Big|_1^t = \lim_{t \rightarrow \infty} -\frac{1}{t} + \frac{1}{1} = 1$$

Logo, pelo teste da comparação, $\int_1^{\infty} \frac{x}{x^3+1} dx$, também converge

e como as duas partes da soma convergem, a integral inicial também irá convergir

$$51 \quad \int_1^{\infty} \frac{x+1}{\sqrt{x^4-x}} dx = \int_1^{\infty} \frac{x}{\sqrt{x^4-x}} dx + \int_1^{\infty} \frac{1}{\sqrt{x^4-x}} dx$$

testando se a integral é divergente

$$0 \leq \frac{x+1}{\sqrt{x^4}} \leq \frac{x+1}{\sqrt{x^4-x}} \quad \rightarrow \quad \int_1^{\infty} \frac{x+1}{\sqrt{x^4}} dx = \int_1^{\infty} \frac{x+1}{x^2} dx$$

$$\Rightarrow \int \frac{x+1}{x^2} dx = \int \frac{x}{x^2} dx + \int \frac{1}{x^2} dx = \ln|x| - \frac{1}{x} + c$$

$$\Rightarrow \lim_{t \rightarrow \infty} \left(\ln|x| - \frac{1}{x} \right) \Big|_1^t = \lim_{t \rightarrow \infty} \ln|t| - \frac{1}{t} - \ln|1| + \frac{1}{1} = \lim_{t \rightarrow \infty} \ln|t| + 1$$

$= +\infty + 1$, divergente e, pelo teste da comparação, a integral será também divergente

$$53 \quad \int_0^1 \frac{\sec^2(x)}{x\sqrt{x}} dx, \quad 0 \leq \frac{1}{x^{3/2}} \leq \frac{\sec^2(x)}{x^{3/2}} \quad \text{para } 0 \leq x \leq 1$$

$$\int_0^1 x^{-3/2} = \lim_{t \rightarrow 0^+} \left(-\frac{2}{\sqrt{x}} \right) \Big|_t^1 = -\frac{2}{\sqrt{1}} + \frac{2}{\sqrt{t}} = +\infty, \text{ divergente}$$

Logo, pelo teorema da comparação $\int_0^1 \frac{\sec^2(x)}{x^{3/2}} dx$ também é divergente

59 $\int_0^1 x^p \ln(x) dx$

para $p=1$, $\int_0^1 x \cdot \ln(x) dx = \lim_{t \rightarrow 0^+} \int_t^1 x \cdot \ln(x) dx$

$\Rightarrow \int x \ln(x) dx = \ln(x) \cdot \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2} dx$

$f(x) = \ln(x)$ $f'(x) = \frac{1}{x}$

$g'(x) = x$ $g(x) = \frac{x^2}{2}$

$= \frac{\ln(x) \cdot x^2}{2} - \frac{1}{2} \int x dx = \frac{\ln(x) \cdot x^2}{2} - \frac{1}{2} \cdot \frac{x^2}{2} \Big|_t^1$

$\lim_{t \rightarrow 0^+} \left(\frac{\ln(1) \cdot 1^2}{2} - \frac{1^2}{4} - \frac{\ln(t) \cdot t^2}{2} + \frac{t^2}{4} \right) = -\frac{1}{4}$

$\hookrightarrow \lim_{t \rightarrow 0^+} \ln(t) \cdot t^2 = \lim_{t \rightarrow 0^+} \frac{\ln(t)}{\frac{1}{t^2}}$, $\frac{\infty}{\infty}$, $\stackrel{LH}{=} \lim_{t \rightarrow 0^+} \frac{\frac{1}{t}}{-2 \cdot t^{-3}} = \lim_{t \rightarrow 0^+} \frac{1}{t} \cdot \frac{t^3}{-2} = \lim_{t \rightarrow 0^+} \frac{t^2}{-2} = 0$

$\stackrel{LH}{=} \lim_{t \rightarrow 0^+} 1 \cdot t^2 = \lim_{t \rightarrow 0^+} t^2 = 0$

para $p \neq 1$

$\Rightarrow \int x^p \ln(x) dx$

$f(x) = \ln(x)$ $f'(x) = \frac{1}{x}$

$g'(x) = x^p$ $g(x) = \frac{x^{p+1}}{p+1}$

$= \frac{\ln(x) \cdot x^{p+1}}{p+1} - \int \frac{1}{x} \cdot \frac{x^{p+1}}{p+1} dx = \frac{\ln(x) \cdot x^{p+1}}{p+1} - \frac{1}{p+1} \cdot \frac{x^{p+1}}{p+1}$

$\lim_{t \rightarrow 0^+} \left(\frac{\ln(1) \cdot 1^{p+1}}{p+1} - \frac{1^{p+1}}{(p+1)^2} - \frac{\ln(t) \cdot t^{p+1}}{p+1} + \frac{t^{p+1}}{(p+1)^2} \right) =$

$\hookrightarrow \lim_{t \rightarrow 0^+} \ln(t) \cdot t^{p+1} = \lim_{t \rightarrow 0^+} \frac{\ln(t)}{\frac{1}{t^{p+1}}}$ $\stackrel{LH}{=} \lim_{t \rightarrow 0^+} \frac{\frac{1}{t}}{\frac{-(p+1)}{t^{-(p+2)}}} = \lim_{t \rightarrow 0^+} \frac{1}{t} \cdot \frac{t^{-(p+2)}}{-(p+1)} = \lim_{t \rightarrow 0^+} \frac{t^{-p-2}}{-(p+1)}$

$= \lim_{t \rightarrow 0^+} \frac{t^{-p-2}}{-(p+1)}$ será $= 0$ se $p \neq -1$

ou seja, a função é convergente para $p \neq -1$