cálculo 2, stewart vol. 2, ed 8, cap 12.5

3. $P(2, 2.4, 3.5)$ $\quad$ $\vec{v} = (3 \quad 2 \quad -1)$

$\quad x_0 \quad y_0 \quad z_0 \qquad\qquad a \quad b \quad c$

$$\begin{cases} x = x_0 + at \\ y = y_0 + bt \\ z = z_0 + ct \end{cases} \rightarrow \begin{cases} x = 2 + 3t \\ y = 2.4 + 2t \\ z = 3.5 - t \end{cases}$$

4. $P(0, 14, -10)$

$$\begin{cases} x = -1 + 2t \\ y = 6 - 3t \\ z = 3 + 9t \end{cases} \rightarrow \text{vetor diretriz} = \begin{pmatrix} 2 \\ -3 \\ 9 \end{pmatrix}$$

$$\begin{cases} x = 0 + 2t \\ y = 14 - 3t \\ z = -10 + 9t \end{cases}$$ é a paralela

5. se a reta é perpendicular ao plano, ela tem direção igual ao vetor normal do plano

dada a equação $x + 3y + z = 5$ do plano, o vetor normal é $\vec{n} = (1, 3, 1)$

a reta passa por $P(1, 0, 6)$ e é paralela ao vetor $\vec{n} = (1, 3, 1)$

$\qquad x_0 \; y_0 \; z_0 \qquad\qquad a \; b \; c$

$x = 1 + t \quad , \quad y = 0 + 3t \quad , \quad z = 6 - 1t$

7. reta que passa por $P(0, 1/2, 1)$ e $P_0(2, 1, -3)$

tem vetor diretor $\overrightarrow{P_0P} = P - P_0 = (0, 1/2, 1) - (2, 1, -3)$

$= (-2, -1/2, -2)$

ou $\overrightarrow{PP_0} = P_0 - P = (2, 1, -3) - (0, 1/2, 1)$

$= (2, 1/2, -4)$

$\quad a \quad b \quad c$

$$\begin{cases} x = 2 + 2t \\ y = 1 + 1/2 t \\ z = -3 - 4t \end{cases}$$

$P_0(2, 1, -3)$

$\quad x_0 \; y_0 \; z_0$

$$\frac{x-2}{2} = \frac{y-1}{(1/2)} = \frac{z+3}{-4}$$

$x_0 \quad y_0 \quad z_0$

9 $P(8,4,-1) \qquad P_0(5,4,6)$

$\overrightarrow{P_0P} = (8,4,-1)-(5,4,6) = (3,\emptyset,-7)$

$a \quad b \quad c$

$\overrightarrow{PP_0} = (5,4,6)-(8,4,-1)=$

$$\begin{cases} y = y_0 = 4 \quad , \quad y = 4 \\ \dfrac{x-5}{3} = \dfrac{z-6}{-7} \end{cases}$$

11 $P(-6,2,3)$ (x_0, y_0, z_0) $\qquad \dfrac{x}{2} = \dfrac{y}{3} = \dfrac{z+1}{1} \qquad \vec{v} = (2,3,1)$

$$\frac{x+6}{2} = \frac{y-2}{3} = \frac{z-3}{1}$$

13 $P(-4,-6,1) \qquad P_0(-2,0,-3)$

$K(10,18,4) \qquad K_0(5,3,14)$

$P-P_0 = (-2,-6,4) \rightarrow \div 2 = (1,3,-2)$ $\qquad$ sim

$K-K_0 = (5,15,-10) \rightarrow \div 5 = (1,3,-2)$

15A $P(1,-5,6)$ (x_0, y_0, z_0) $\qquad \vec{v} = (-1,2,-3)$ (a, b, c)

$$\frac{x-1}{-1} = \frac{y+5}{2} = \frac{z-6}{-3}$$

15B plano $yz \rightarrow x=0 \rightarrow \dfrac{-1}{-1} = 1 = \dfrac{y+5}{2} = \dfrac{z-6}{-3}$

em $y=-3$ e $z=3$

plano $xz \rightarrow y=0 \rightarrow \dfrac{5}{2} = \dfrac{x-5}{-1} = \dfrac{z-6}{-3}$

em $x = -3/2$ e $z = -3/2$

plano $xy \rightarrow z=0 \rightarrow 2 = \dfrac{x-1}{-1} = \dfrac{y+5}{2}$

em $x=-1$ e $y=-1$

tilibra

17 $\vec{r} = \vec{r_0} + \alpha \vec{v}$ $\qquad P_0(6,-1,9) \qquad P(7,6,0)$

$\vec{v} = \overrightarrow{P_0P} = P - P_0 = (1,7,-9)$

$\vec{r_0} = (6,-1,9)$

$r(t) = (6,-1,9) + \alpha \cdot (1,7,-9)$

19 $L_1: \begin{cases} x = 3+2t \\ y = 4-t \\ z = 1+3t \end{cases} \qquad L_2: \begin{cases} x = 5+18t \\ y = 6-9t \\ z = 3-6t \end{cases}$

$\vec{v_1} = (2,-1,3) \qquad A_1(3,4,1) \qquad \overrightarrow{A_1A_2} = (2,2,2)$

$\vec{v_2} = (18,-9,-6) \qquad A_2(5,6,3)$

$\vec{v_1} \cdot (\vec{v_2} \times \overrightarrow{A_1A_2}) = \det \begin{pmatrix} 2, & -1, & 3 \\ 18, & -9, & -6 \\ 2, & 2, & 2 \end{pmatrix} = 198$ não são coplanares

21 $\dfrac{x-2}{1} = \dfrac{y-3}{-2} = \dfrac{z-1}{-3} \qquad \dfrac{x-3}{1} = \dfrac{y+4}{3} = \dfrac{z-2}{-7}$

$\vec{v_1} = (1,-2,-3) \qquad A_1(2,3,1) \qquad \overrightarrow{A_1A_2} = (1,-7,1)$

$\vec{v_2} = (1,3,-7) \qquad A_2(3,-4,2)$

$\vec{v_1} \cdot (\vec{v_2} \times \overrightarrow{A_1A_2}) = \det \begin{pmatrix} 1, & -2, & -3 \\ 1, & 3, & -7 \\ 1, & -7, & 1 \end{pmatrix} = 0$ são coplanares e concorrentes

intersecção

$L_1 = (2,3,1) + t \cdot (1,-2,-3) \qquad t+2 = u+3$

$L_2 = (3,-4,2) + u \cdot (1,3,-7) \qquad t-u=1$

$-2t+3 = -4+3u$

$-2t-3u = -7$

$\begin{cases} t-u=1 \quad (\cdot 2) \\ -2t-3u=-7 \end{cases}$

$-5u = -5, \; u=1 \qquad t=2$

$(2,3,1) + 2 \cdot (1,-2,-3) = (4,-1,-5)$ //

$(3,-4,2) + (1,3,-7) = (4,-1,-5)$

23 $P_0(x_0, y_0, z_0) = P_0(6, 3, 2)$ $\vec{n} = (a, b, c) = (-2, 1, 5)$

$-2\cdot(x-6) + (y-3) + 5\cdot(x-2) = 0$

ou

$d = (-ax_0 - by_0 - cz_0) = (2\cdot 6 - 3 - 10) = -1$

$-2x + y + 5z - 1 = 0$

25 $P_0(x_0, y_0, z_0) = P_0(-1, 1/2, 3)$ $\vec{n} = (a, b, c) = (1, 4, 1)$

$d = (1 - 2 - 3) = -4$

$x + 4y + z - 4 = 0$

27 o plano $5x - y - z = 6$ tem vetor normal $= (5, -1, -1)$

$P_0(x_0, y_0, z_0) = P_0(1, -1, -1)$

$d = (-5 - 1 - 1) = -7$

$5x - y - z = 7$

29 $P_0(x_0, y_0, z_0) = P_0(1, 1/2, 1/3)$ $\vec{n} = (a, b, c) = (1, 1, 1)$

$x + y + z + (-1 - 1/2 - 1/3) = 0$

$x + y + z - 11/6 = 0$

31 $\vec{n} = (1, 1, 1)$ $P_0(1, 0, 1)$

$x + y + z - 2 = 0$ $x + y + z = 2$

33 achando um vetor ortogonal aos vetores

$P(2, 1, 2)$ $Q(3, -8, 6)$ $R(-2, -3, 1)$

$\vec{PQ} = (3, -8, 6) - (2, 1, 2) = (1, -9, 4)$

$\vec{PR} = (-2, -3, 1) - (2, 1, 2) = (-4, -4, -1)$

$$\vec{PQ} \times \vec{PR} = \det \begin{pmatrix} i & j & k \\ 1 & -9 & 4 \\ -4 & -4 & -1 \end{pmatrix} = -15j - 40k + 25i = \begin{pmatrix} 25 \\ -15 \\ 40 \end{pmatrix}$$

$\vec{n} = (25, -15, 40)$

$25x - 15y + 40z \; (-50 + 15 - 80) = 0$

$25x - 15y + 40z = 115$

35 $P_0(3,5,-1)$ L: $\begin{cases} x = 4 - t \\ y = 2t - 1 \\ z = -3t \end{cases}$ $\vec{v} = (-1,2,-3)$ é o vetor diretor

é preciso de outro vetor nesse plano para achar a normal

encontrando um ponto qualquer na reta:

$t=0 \quad x=4 \quad y=-1 \quad z=0 \rightarrow P(4,-1,0)$

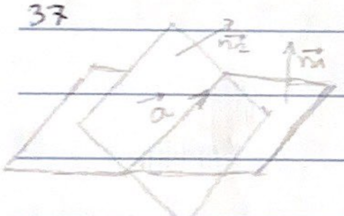
$\overrightarrow{P_0P} = (4,-1,0) - (3,5,-1) = (1,-6,1)$

$$\begin{pmatrix} -1 \\ 2 \\ -3 \end{pmatrix} \times \begin{pmatrix} 1 \\ -6 \\ 1 \end{pmatrix} = \det \begin{vmatrix} i & j & k \\ -1 & 2 & -3 \\ 1 & -6 & 1 \end{vmatrix} = 2i - 3j + 6k - 2k - 18i + j = -16i - 2j + 4k$$

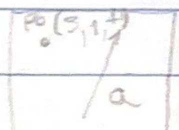
$\vec{n} = (-16, -2, -4)$

$-16x - 2y + 4z = -62$

37



$$\vec{a} = \vec{n_1} \times \vec{n_2}$$
$$= (1,2,3) \times (2,-1,1)$$
$$= \det \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 2 & -1 & 1 \end{vmatrix} = 2i + 6j - k - 4k + 3i - j = 5i + 5j - 5k = (5,5,-5)$$



precisamos da normal do plano

achando outro ponto no plano (um ponto da interseção)

em $z=0$ $\begin{cases} x + 2y = 1 \\ 2x - y = -3 \quad (\cdot 2) \end{cases}$

$5x = -5 \quad x = -1 \quad y = 1$

$P(-1,1,0)$

achando a normal

$\overrightarrow{PP_0} = (3,1,4) - (-1,1,0) = (4,0,4)$

$$\vec{a} \times \overrightarrow{PP_0} = \det \begin{vmatrix} i & j & k \\ 5 & 5 & -5 \\ 4 & 0 & 4 \end{vmatrix} = 20i - 40j - 20k = i - 2j - k$$

$\vec{n} = (1,-2,-1)$ $P_0(-1,1,0)$

$x - 2y - z = -(1+2) = -3$

53 $\vec{n_1} = (9, -3, 6)$ não paralelos

$\vec{n_2} = (2, -2, 1)$ não perpendiculares

$\vec{n_1} \cdot \vec{n_2} = 18 + 6 + 6 \neq 0$

55 $\vec{n_1} = (2, -3, -1)$ $\vec{n_2} = (4, -6, -2)$

paralelos

57 o vetor diretor da reta é $\vec{n_1} \times \vec{n_2}$

$$\vec{n_1} \times \vec{n_2} = (1, 1, 1) \times (1, 2, 2) = \det \begin{pmatrix} i & j & k \\ 1 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

$$= 2i + j + 2k - k - 2i - 2j = 0i - j + k = (0, -1, 1)$$

precisa-se de um ponto da reta

em $z = 0$ $\begin{cases} x + y = 1 & (\cdot -1) \\ x + 2y = 1 \end{cases}$ $P_0(1, 0, 0)$

$y = 0$ $x = 1$

$$r = \begin{cases} x = 1 \\ y = -t \\ z = t \end{cases}$$