cálculo 1, stewart, vol 1, ed5, cap 2.8

13 $f(x) = 3 - 2x + 4x^2 = 4x^2 - 2x + 3$

$$f'(a) = \lim_{h\to 0} \frac{4(a+h)^2 - 2(a+h) + 3 - (4a^2 - 2a + 3)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{4a^2} + 8ah + 4h^2 - \cancel{2a} - 2h \cancel{+3} - \cancel{4a^2} + \cancel{2a} - \cancel{3}}{h}$$

$$= \lim_{h\to 0} \frac{8ah + 4h^2 - 2h}{h} = \lim_{h\to 0} \frac{\cancel{h}(8a + 4h - 2)}{\cancel{h}} = \lim_{h\to 0} 8a + 4h - 2 = 8a - 2$$

14 $f(x) = t^4 - 5t$ $\quad f(a+h) = (a+h)^4 - 5(a+h) = (a+h)^2(a+h)^2 - 5(a+h)$

$= (a^2 + 2ah + h^2)(a^2 + 2ah + h^2) - 5a + 5h$

$= a^4 + 2a^3h + a^2h^3 + 2a^3h + 4a^2h^2 + 2ah^3 + a^2h^2 + 2ah^3$

$+ h^4 - 5a + 5h = \underline{a^4} + 4a^3h + 6a^2h^2 + 4ah^3 + h^4 - \underline{5a} - 5h$

$f(a) = \underline{a^4} - 5a$

$$f'(a) = \lim_{h\to 0} \frac{4a^3h + 6a^2h^2 + 4ah^3 + h^4 - 5h}{h} = \lim_{h\to 0} 4a^3 + 6a^2h + 4ah^2 + h^3 - 5$$

$= 4a^3 - 5$

15 $f(x) = \dfrac{2t+1}{t+3}$ $\quad f' \Rightarrow \dfrac{\dfrac{2(a+h)+1}{a+h+3} - \dfrac{2a+1}{a+3}}{h} = \dfrac{\dfrac{(a+3)(2a+2h+1) - (a+h+3)(2a+1)}{(a+h+3)(a+3)}}{h}$

$$= \frac{(a+3)(2a+2h+1) - (a+h+3)(2a+1)}{h(a+h+3)(a+3)} = \frac{5\cancel{h}}{\cancel{h}(a+h+3)(a+3)} \Rightarrow \lim_{h\to 0} \frac{5}{(a+h+3)(a+3)}$$

$$= \frac{5}{(a+3)(a+3)} = \frac{5}{(a+3)^2}$$

19 $\displaystyle\lim_{h\to 0} \frac{(1+h)^{10} - 1}{h}$ $\quad f(x) = x^{10} \quad a = 1$

20 $\displaystyle\lim_{h\to 0} \frac{\sqrt[4]{16+h} - 2}{h}$ $\quad f(x) = \sqrt[4]{x} \quad a = 16$

21 $\displaystyle\lim_{x\to 5} \frac{2^x - 32}{x-5}$, na forma $f'(a) = \displaystyle\lim_{x\to a} \frac{f(x) - f(a)}{x-a}$

$a = 5 \quad f(x) = 2^x$

22 $\lim_{x \to \frac{\pi}{4}} \frac{tg\, x - 1}{x - \pi/4}$, na forma $f'(a) = \frac{f(x) - f(a)}{x - a}$

$f(x) = tg(x) \qquad a = \frac{\pi}{4}$

23 $\lim_{h \to 0} \frac{\cos(\pi + h) + 1}{h} = f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

$f(x) = \cos(x) \qquad a = \pi$

24 $\lim_{t \to 1} \frac{(t^4 + t) - 2}{t - 1} = f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

$a = 1 \qquad f(t) = t^4 + t$

25 velocidade em $t = 2$ é $f'(2)$

$f(t) = t^2 - 6t - 5 \qquad f'(2) = \lim_{h \to 0} \frac{(2+h)^2 - 6(2+h) - 5 + 13}{h}$

$= \lim_{h \to 0} \frac{\cancel{4} + 4h + h^2 - \cancel{12} - 6h - \cancel{5} + \cancel{13}}{h} = \lim_{h \to 0} \frac{h^2 - 2h}{h} = \lim_{h \to 0} \frac{\cancel{h}(h-2)}{\cancel{h}}$

$= \lim_{h \to 0} h - 2 = -2\, m/s$