cálculo 1, stewart, ed5, vol 1, cap 3.2

1 $y = (x^2+1)(x^3+1) = x^5 + x^2 + x^3 + 1$

$y' = (x^2+1)' \cdot (x^3+1) + (x^2+1)(x^3+1)' = 2x \cdot (x^3+1) - (x^2+1)(3x^2)$

$= 2x^4 + 2x + (3x^4 + 3x^2) = 5x^4 + 3x^2 + 2x$

$y' = (x^5)' + (x^2)' + (x^3)' + (1)' = 5x^4 + 2x + 3x^2.$

3 $f(x) = x^2 e^x$ $\qquad f'(x) = (x^2)' \cdot e^x + x^2 \cdot (e^x)' = 2x \cdot e^x + x^2 \cdot e^x$

$= 2 \cdot e^x (2 + x)$

5 $f(x) = \dfrac{e^x}{x^2}$ $\qquad F'(x) = \dfrac{(e^x)' \cdot x^2 - e^x \cdot (x^2)'}{(x^2)^2} = \dfrac{(e^x \cdot x^2 - e^x \cdot 2x)}{x^4}$

$= \dfrac{(e^x - e^x \cdot 2)}{x^3}$

7 $g(x) = \dfrac{3x-1}{2x+1}$ $\qquad g'(x) = \dfrac{(3x-1)' \cdot (2x+1) - (3x-1)(2x+1)'}{4x^2 + 2 \cdot 2x \cdot 1 + 1}$

$= \dfrac{\cancel{3 \cdot (2x+1)} - \cancel{2(3x} - 1)}{4x^2 + 4x + 1} = \dfrac{\cancel{3 - (-2)} \to 5}{4x^2 + 4x + 1}$

9 $V(x) = (2x^3+3)(x^4 - 2x)$

$V'(x) = (6x^2)(x^4 - 2x) + (2x^3 + 3)(4x^3 - 2)$

$= 6x^6 - \cancel{12x^3} + 8x^6 - 4x^3 + \cancel{12x^3} - 6 = 14x^6 - 4x^3 - 6$

11 $F(y) = \left(\dfrac{1}{y^2} - \dfrac{3}{y^4}\right)\left(y + 5y^3\right) = \left(y^{-2} - 3 \cdot y^{-4}\right)(y + 5y^3)$

$F'(y) = (y^{-2} - 3y^{-4})'(y + 5y^3) + (y^{-2} - 3 \cdot y^{-4})(y + 5y^3)'$

$= (-2y^{-3} + 12y^{-5})(y + 5y^3) + (y^{-2} - 3y^{-4})(1 + 15y^2)$

13 $f(x) = \dfrac{t^2}{3t^2 - 2t + 1}$ $\qquad (t^2)' = 2t$

$(3t^2 - 2t + 1)' = 6t - 2$

$f'(x) = \dfrac{2t(3t^2 - 2t + 1) - t^2(6t - 2)}{(3t^2 - 2t + 1)^2} = \dfrac{\cancel{6t^3} - 4t^2 + 2t - \cancel{6t^3} + 2t^2}{[\ldots]}$

$= \dfrac{-2t^2 + 2t}{(3t^2 - 2t + 1)^2}$

15 $y = (r^2 - 2r)e^r$

$y' = (2r-2)(e^r) + (r^2-2r)(e^r) = e^r(\cancel{2r} - 2 + r^2 - \cancel{2r})$

$= e^r(r^2-2)$

17 $y = \dfrac{v^3 - 2v\sqrt{v}}{v}$ $\qquad (v^3 - 2v\sqrt{v})' = 3v^2 - (2 \cdot v^{3/2})'$

$= 3v^2 - \cancel{2} \cdot \dfrac{3}{\cancel{2}} \cdot v^{1/2} = 3v^2 - 3\sqrt{v}$

$y' = \dfrac{(3v^2 - 3\sqrt{v}) \cdot v - 1 \cdot (v^3 - 2v\sqrt{v})}{v^2} = \dfrac{3v^3 - 3v\sqrt{v} - v^3 + 2v\sqrt{v}}{v^2}$

$= \dfrac{\cancel{v}(2v^2 - \sqrt{v})}{v^2} = \dfrac{2v^2 - \sqrt{v}}{v}$ $\qquad (v^2 - 2\sqrt{v})' = 2v - 1v^{-1/2} = \dfrac{2v-1}{\sqrt{v}}$

19 $f(x) = \dfrac{1}{x^4 + x^2 + 1} = (x^4 + x^2 + 1)^{-1}$ $\qquad (x^4 + x^2 + 1)' = 4x^3 + 2x$

$f'(x) = \dfrac{4x^3 + 2x - 0}{(x^4 + x^2 + 1)} = \dfrac{4x^3 + 2x}{(x^4 + x^2 + 1)}$

23 $y = \dfrac{2x}{x+1}$ $\qquad$ punto $(1,1)$

$m = y' = \dfrac{2 \cdot (x+1) - 2x \cdot 1}{(x+1)^2} = \dfrac{\cancel{2x} + 2 - \cancel{2x}}{x^2 + 2x + 1} = \dfrac{2}{x^2 + 2x + 1} = f'(x)$

$f'(1) = \dfrac{2}{4}$

$y - y_0 = m(x - x_0)$

$y - 1 = \dfrac{2}{4}(x-1) = \dfrac{1}{2}x - \dfrac{1}{2}$ $\qquad y = \dfrac{1}{2}x + \dfrac{1}{2}$

25 $y = f(x) = 2xe^x$

$f'(x) = 2 \cdot (x \cdot e^x)' = 2 \cdot ((x)' \cdot e^x + x \cdot (e^x)') = 2(e^x + xe^x)$

$f'(0) = 2(e^0 + 0 \cdot e^0) = 2(1+0) = 2$

$y - y_0 = m(x - x_0)$

$y - 0 = 2(x - 0) \Rightarrow y = 2x$