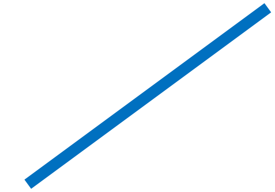
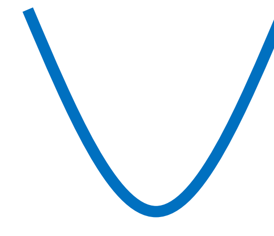
Curvas de Bezier

Funciones Polinomiales

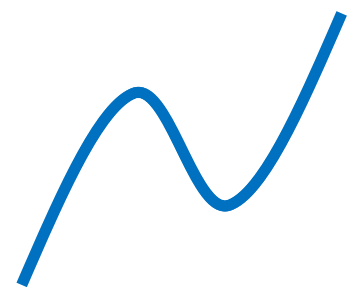
Lineal: $f\left(t\right)=at+b$



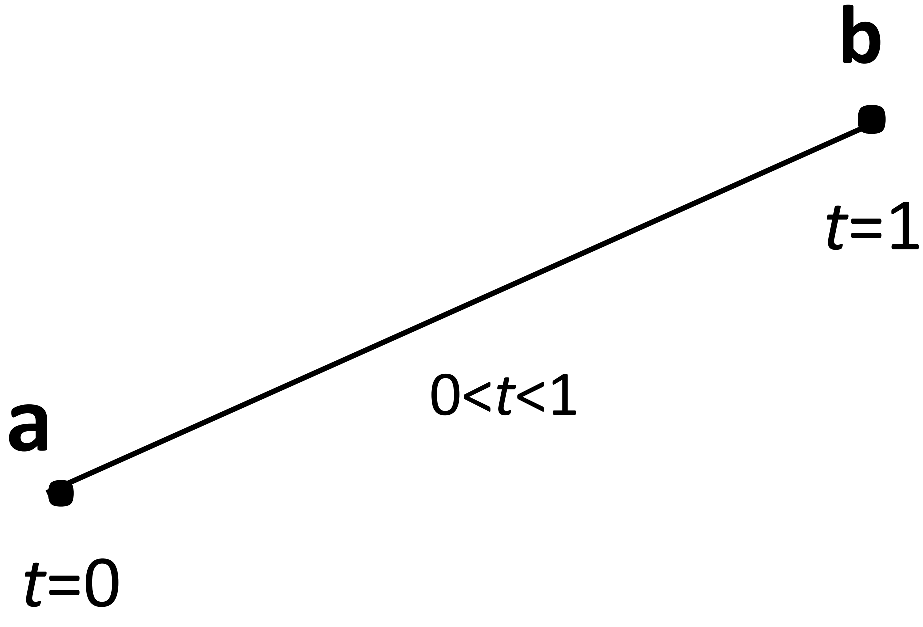
Cuadrática: $f\left(t\right)=at^2+bt+c$



Cúbica: $f\left(t\right)=at^3+bt^2+ct+d$

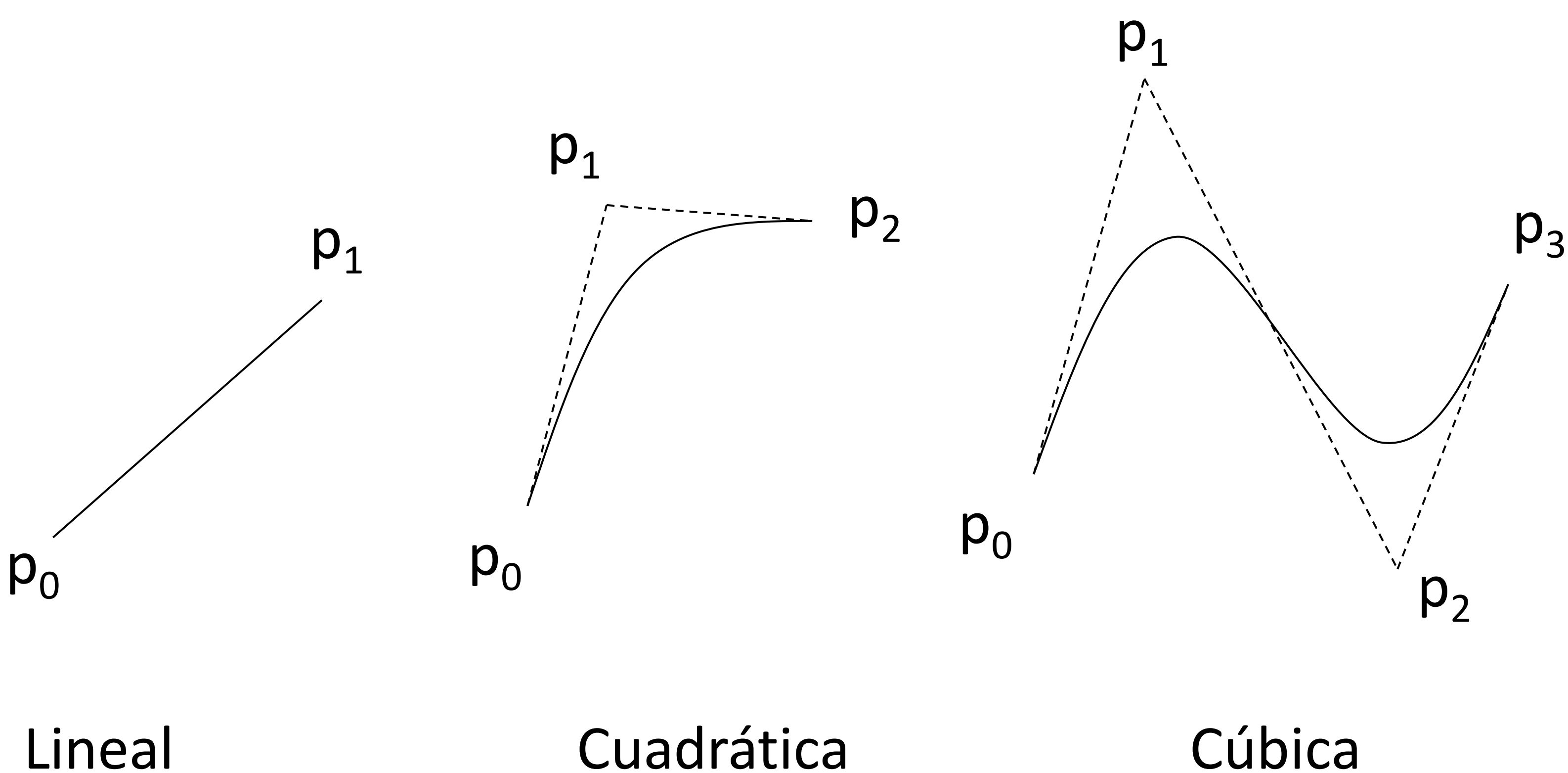


Interpolación lineal

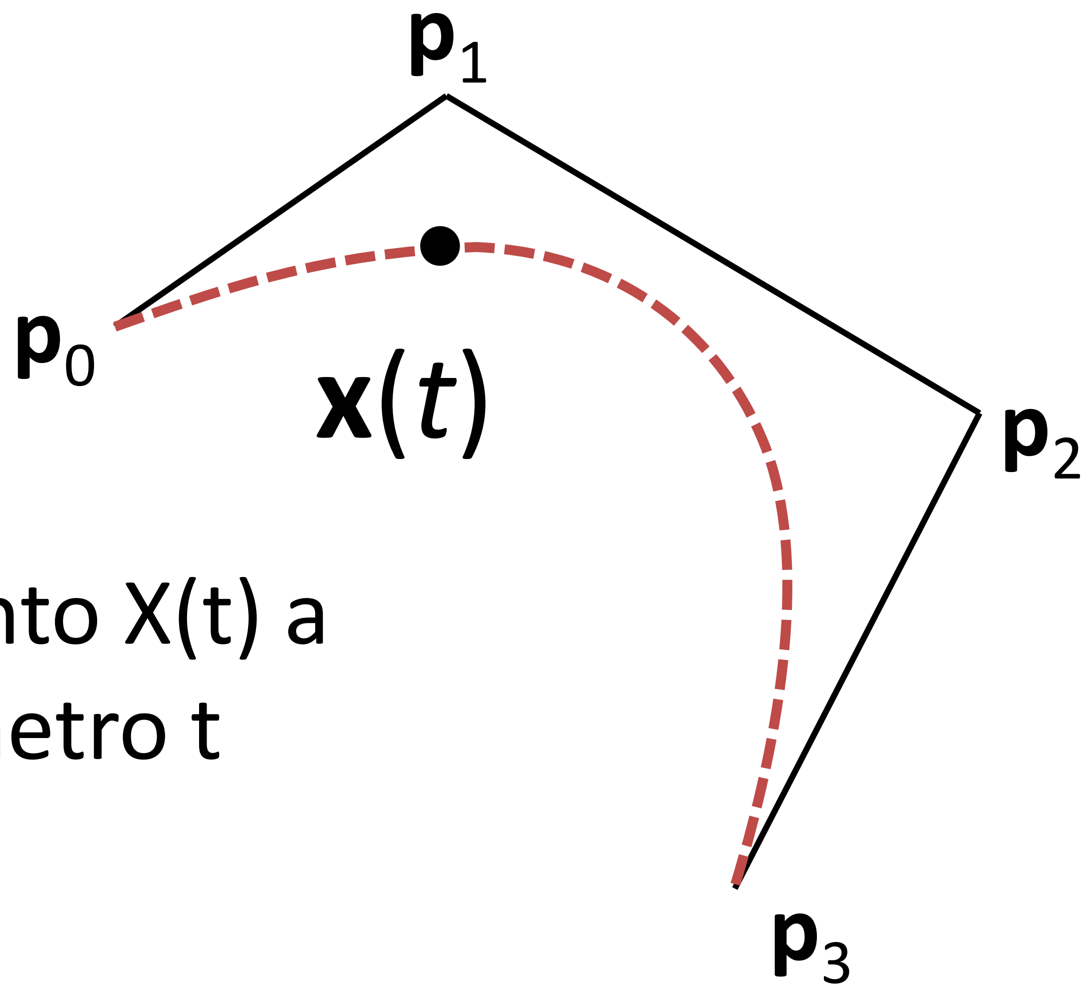


$$Lerp(t,\mathbf{a},\mathbf{b}) = (1-t)\mathbf{a} + t\mathbf{b}$$

Interpolaciones de mayor orden



Curva Bezier cúbica



Encontrar el punto X(t) a partir del parámetro t con $0 \le t \le 1$

Curvas de Bezier cúbicas

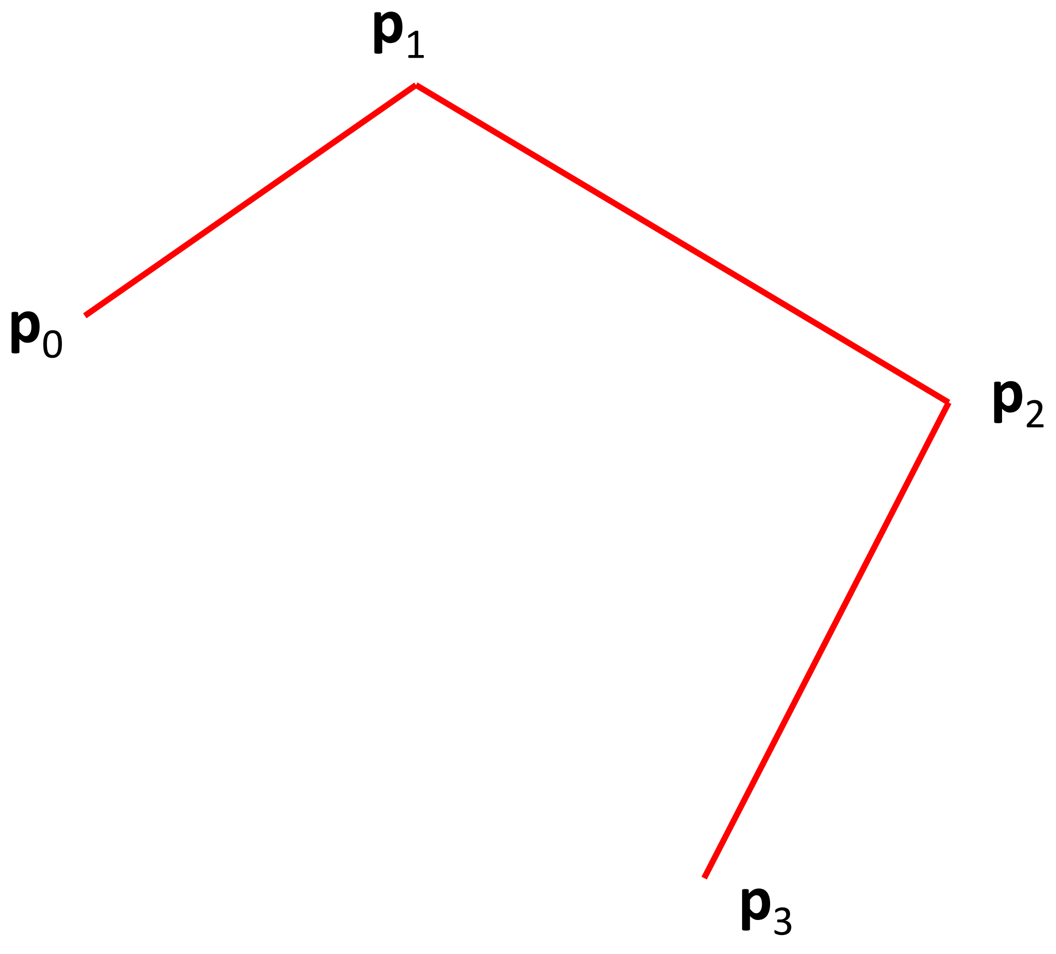
Formulaciones:

1. Algoritmo “de Castlejau”-interpolación lineal recursiva

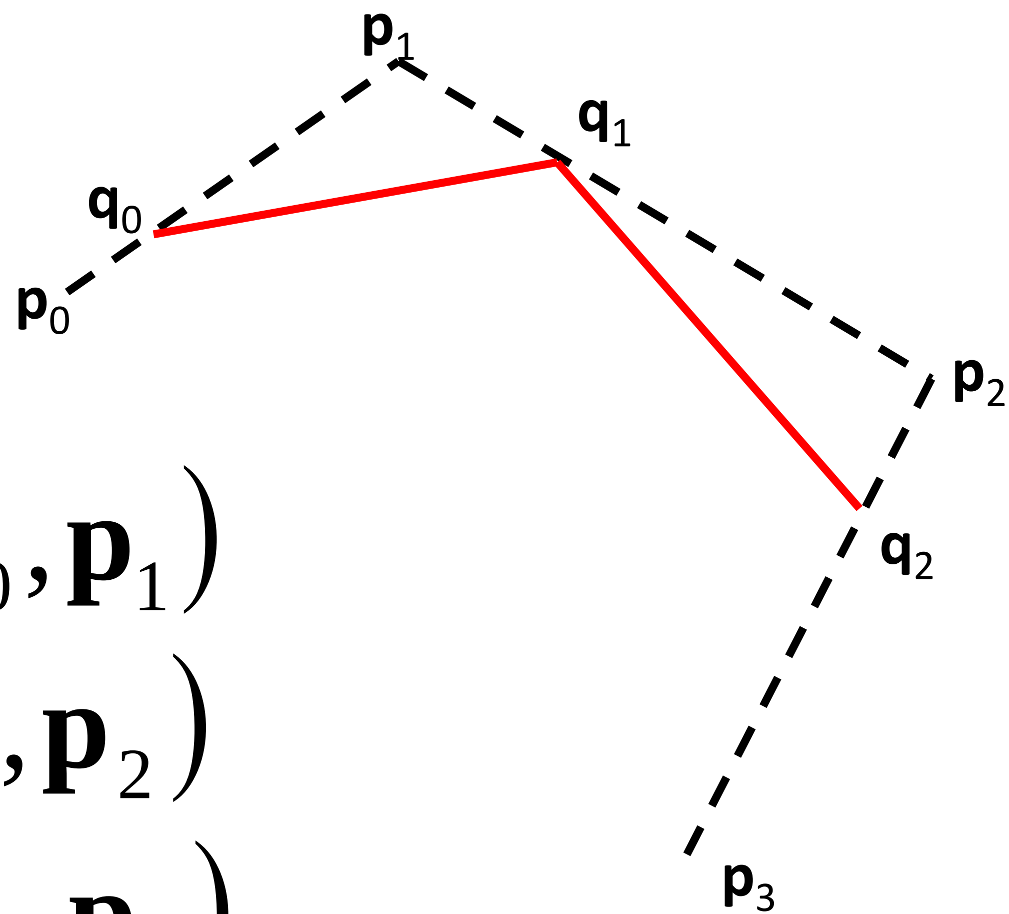
2. Bases de Bernstein

3. Ecuaciones Cúbicas

Algoritmo “de Casteljau” - Paso 1



Algoritmo “de Casteljau” - Paso 2

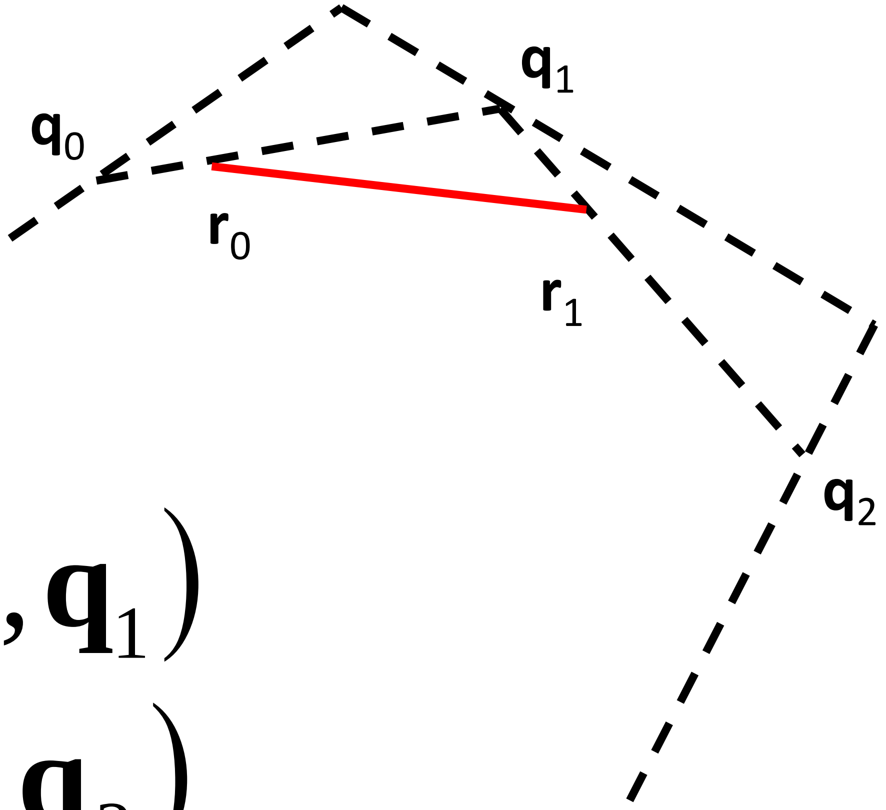


$$\mathbf{q}_0 = Lerp(t, \mathbf{p}_0, \mathbf{p}_1)$$

$$\mathbf{q}_1 = Lerp(t, \mathbf{p}_1, \mathbf{p}_2)$$

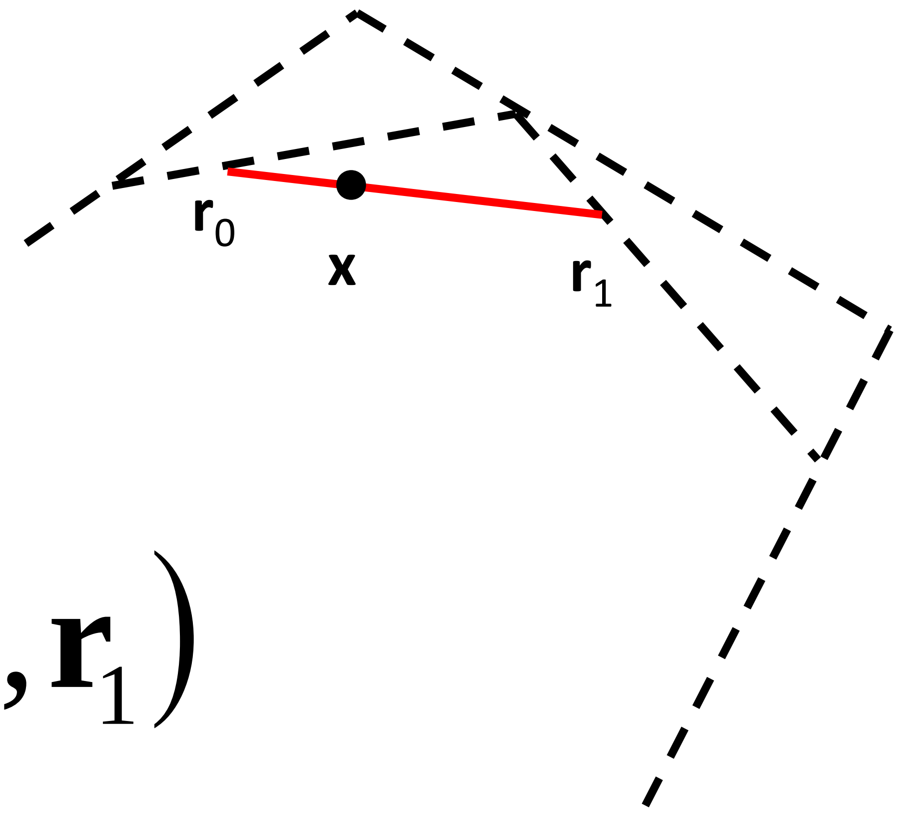
$$\mathbf{q}_2 = Lerp(t, \mathbf{p}_2, \mathbf{p}_3)$$

Algoritmo “de Casteljau” - Paso 3



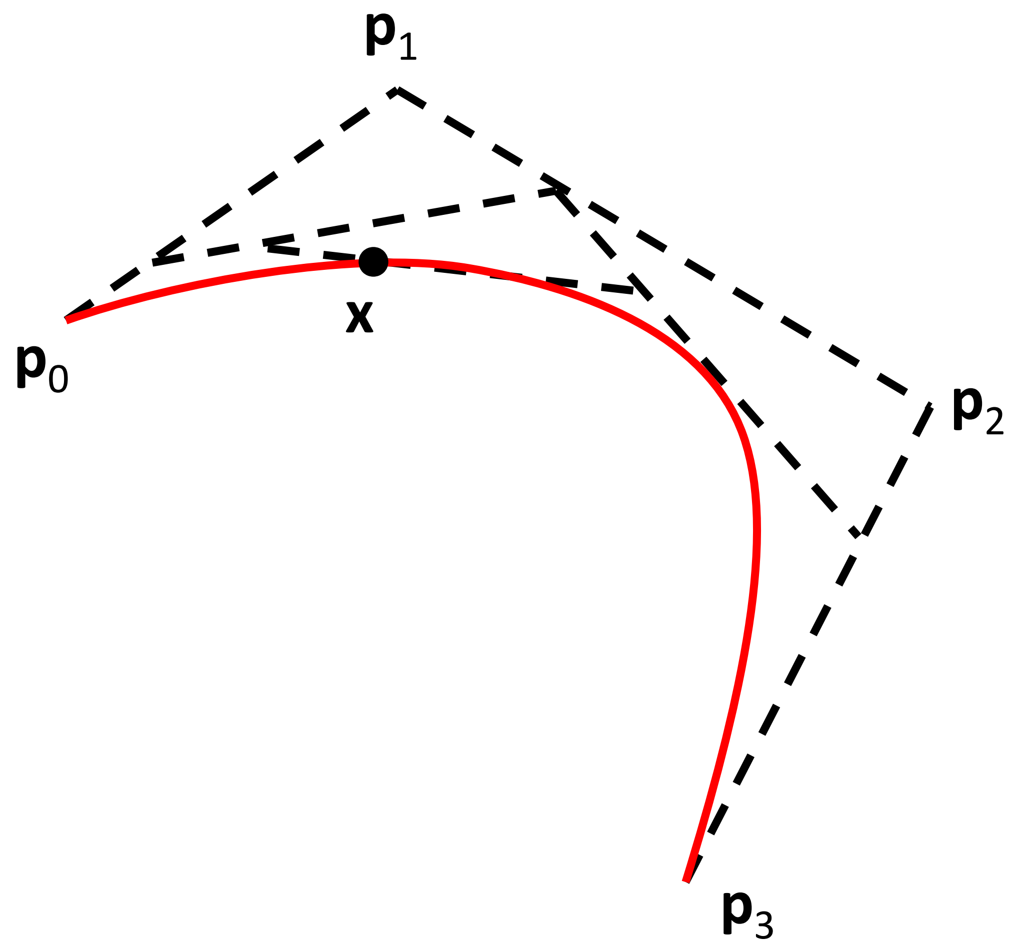
$$\mathbf{r}_0 = Lerp(t, \mathbf{q}_0, \mathbf{q}_1)$$
$$\mathbf{r}_1 = Lerp(t, \mathbf{q}_1, \mathbf{q}_2)$$

Algoritmo "de Casteljau" - Paso 4



$$\mathbf{x} = Lerp\left(t, \mathbf{r}_0, \mathbf{r}_1\right)$$

Algoritmo “de Casteljau”



Interpolación lineal recursiva

$$\mathbf{p}_0$$

$$\mathbf{q}_0 = Lerp(t, \mathbf{p}_0, \mathbf{p}_1)$$

$$\mathbf{r}_0 = Lerp(t, \mathbf{q}_0, \mathbf{q}_1)$$

$$\mathbf{p}_1$$

$$\mathbf{x} = Lerp(t, \mathbf{r}_0, \mathbf{r}_1)$$

$$\mathbf{q}_1 = Lerp(t, \mathbf{p}_1, \mathbf{p}_2)$$

$$\mathbf{r}_1 = Lerp(t, \mathbf{q}_1, \mathbf{q}_2)$$

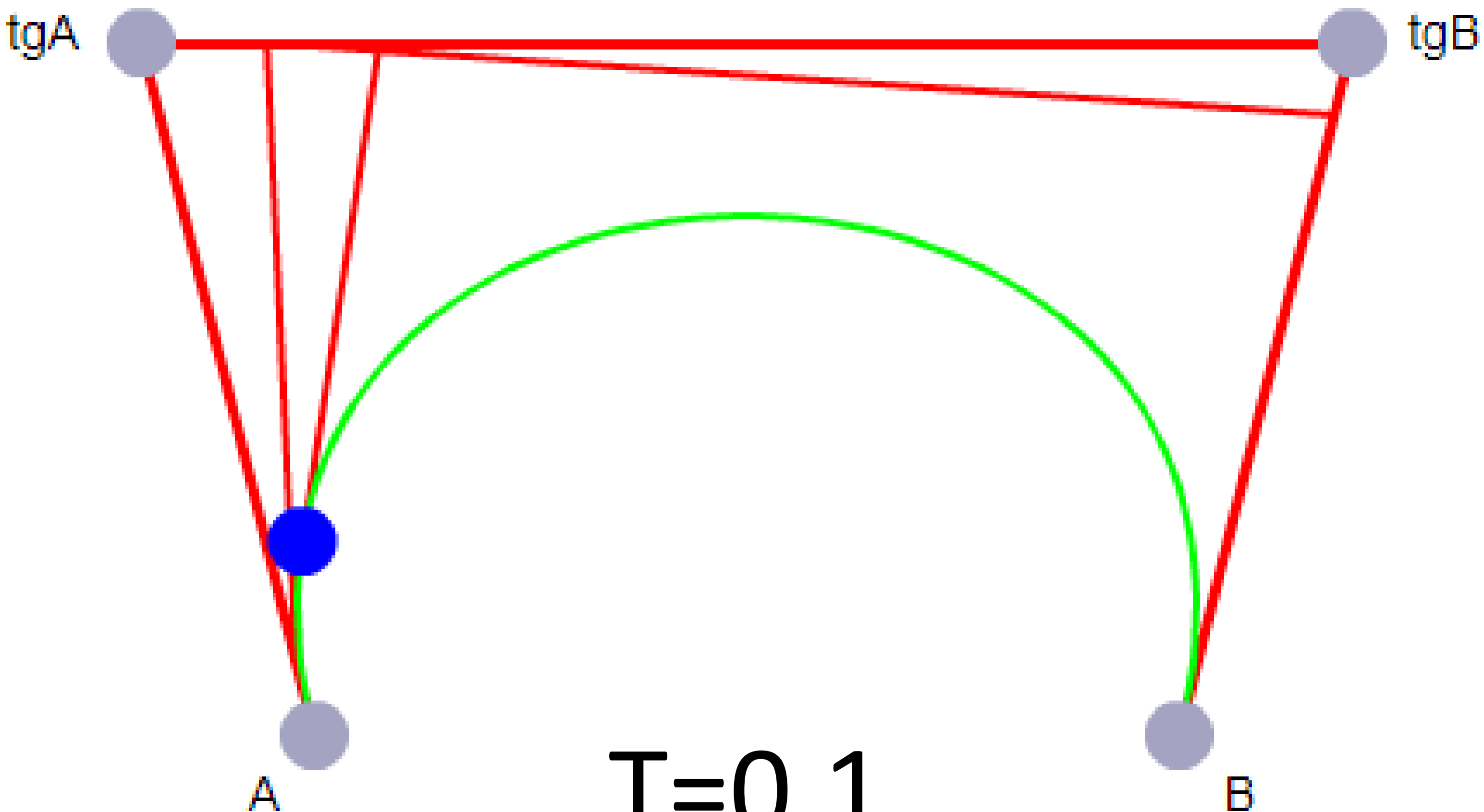
$$\mathbf{p}_2$$

$$\mathbf{q}_2 = Lerp(t, \mathbf{p}_2, \mathbf{p}_3)$$

$$\mathbf{p}_3$$

$$\begin{tikzcd}
 & & & \mathbf{p}_0 \\
 & & \mathbf{q}_0 \arrow[ur, dash] \arrow[dr, dash] & \\
 & \mathbf{r}_0 \arrow[ur, dash] \arrow[dr, dash] & & \mathbf{p}_1 \\
\mathbf{x} \arrow[ur, dash] \arrow[dr, dash] & & \mathbf{q}_1 \arrow[ur, dash] \arrow[dr, dash] & \\
 & \mathbf{r}_1 \arrow[ur, dash] \arrow[dr, dash] & & \mathbf{p}_2 \\
 & & \mathbf{q}_2 \arrow[ur, dash] \arrow[dr, dash] & \\
 & & & \mathbf{p}_3
\end{tikzcd}$$

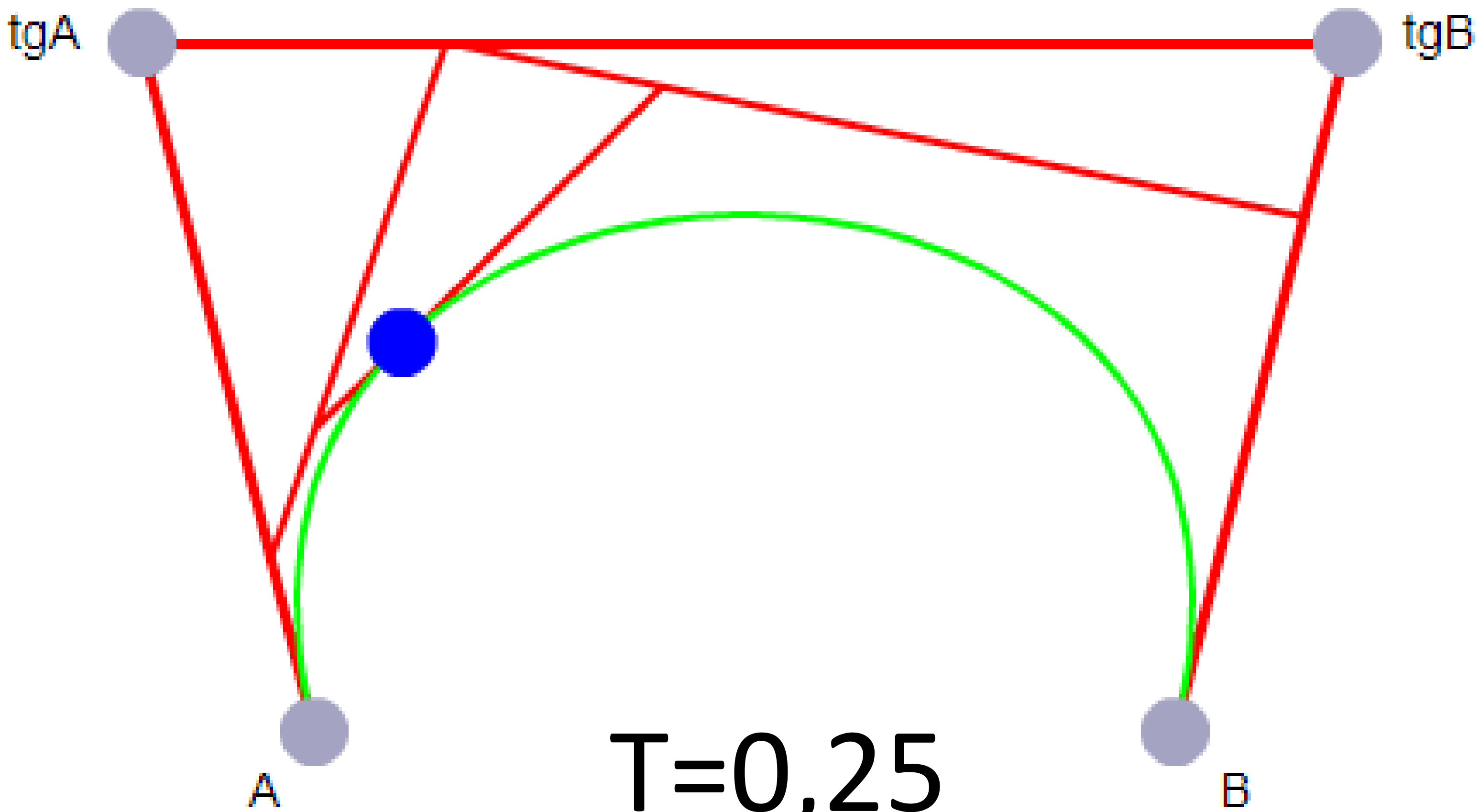
Algoritmo "de Casteljau"



T=0,1

http://aescripts.com/decasteljau-bezier-curve/

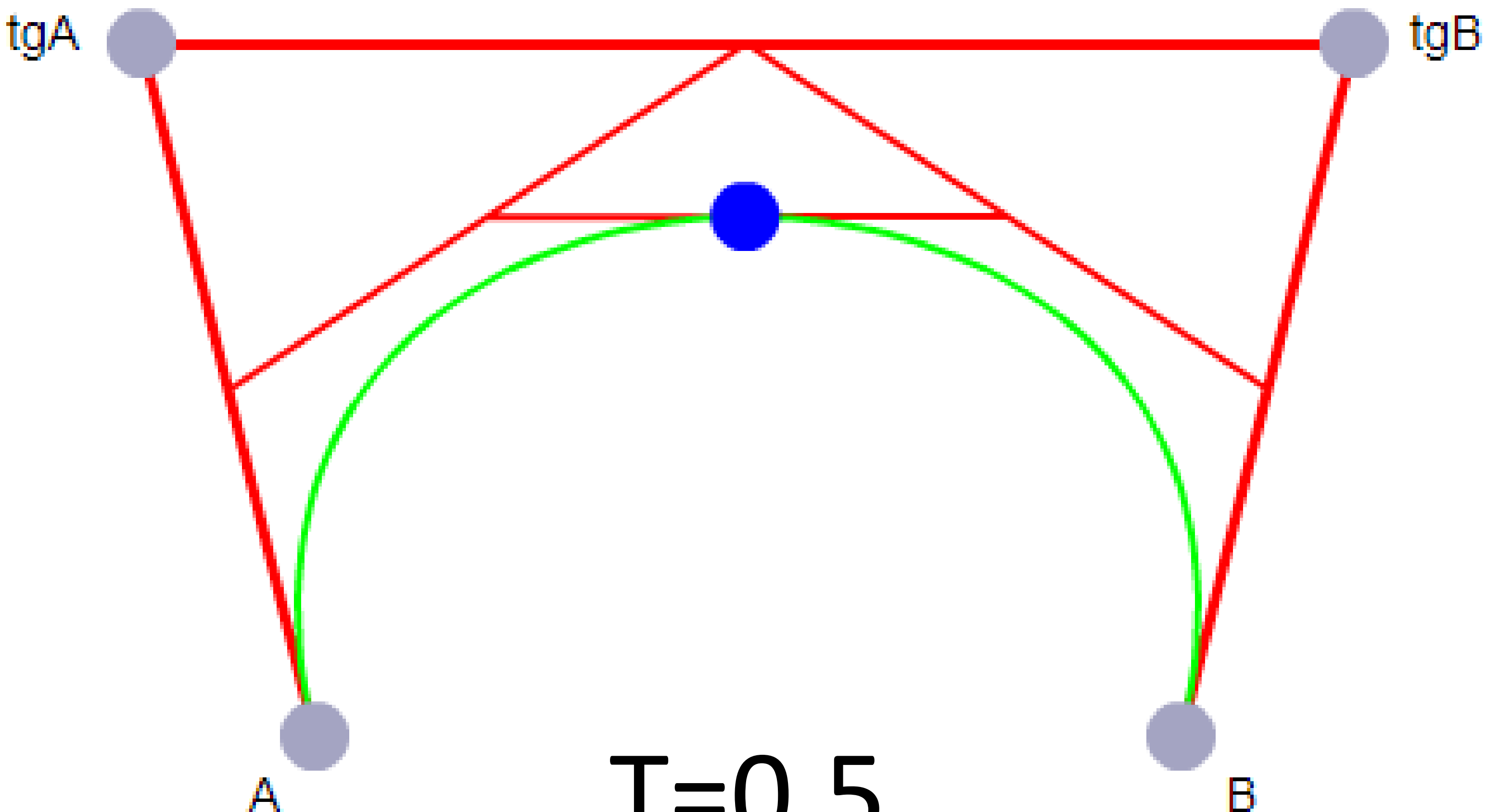
Algoritmo "de Casteljau"



T=0,25

http://aescripts.com/decasteljau-bezier-curve/

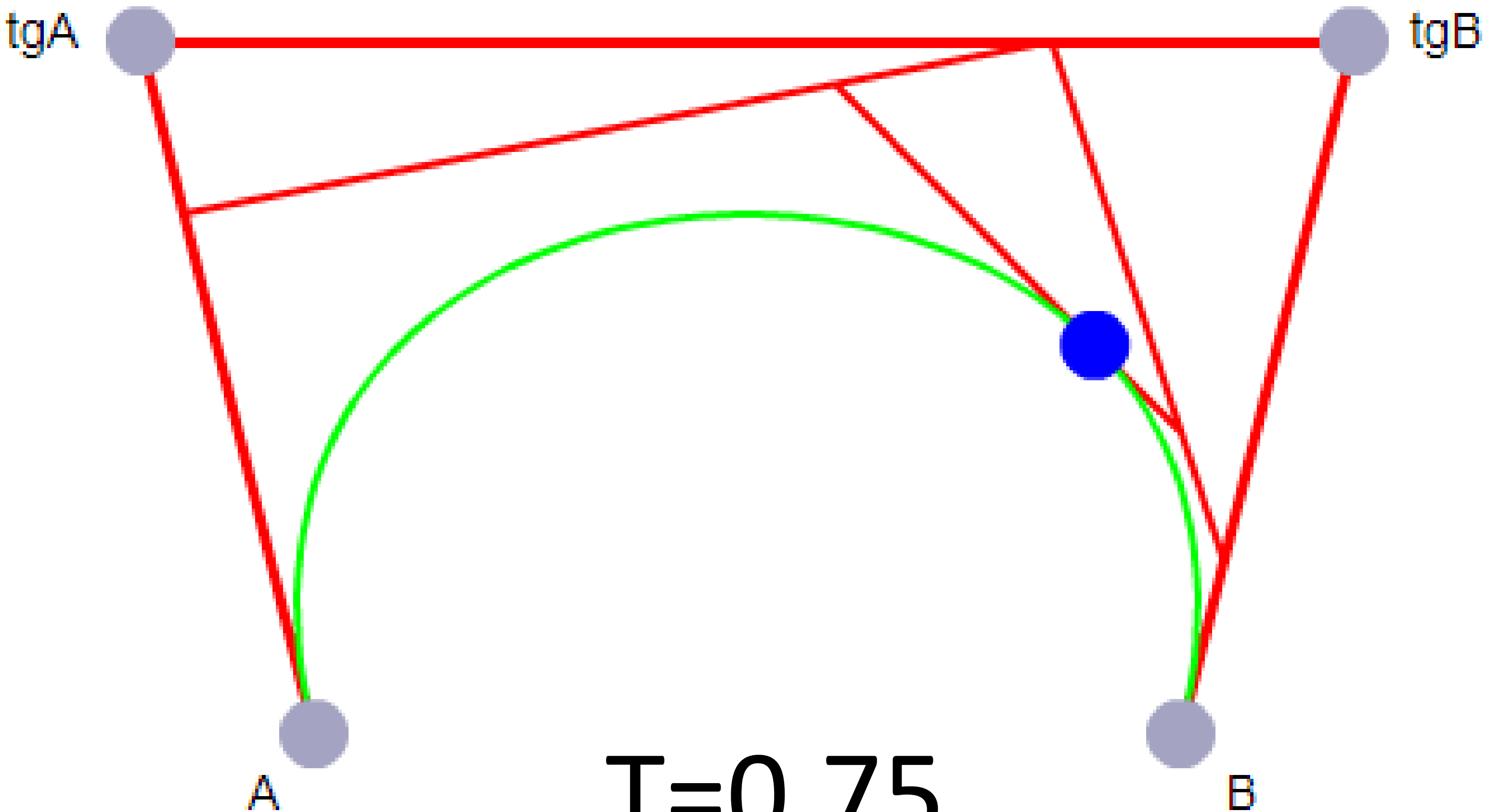
Algoritmo “de Casteljau”



T=0,5

http://aescripts.com/decasteljau-bezier-curve/

Algoritmo “de Casteljau”



T=0,75

http://aescripts.com/decasteljau-bezier-curve/

Expresión recursiva

$$B_i^n(t) = \binom{n}{i} (1-t)^{n-i} (t)^i$$

Bases de Berstein

$$\mathbf{x}(t) = \sum_{i=0}^{n} B_i^n(t)\,\mathbf{p}_i$$

curva

N= grado de los polinomios

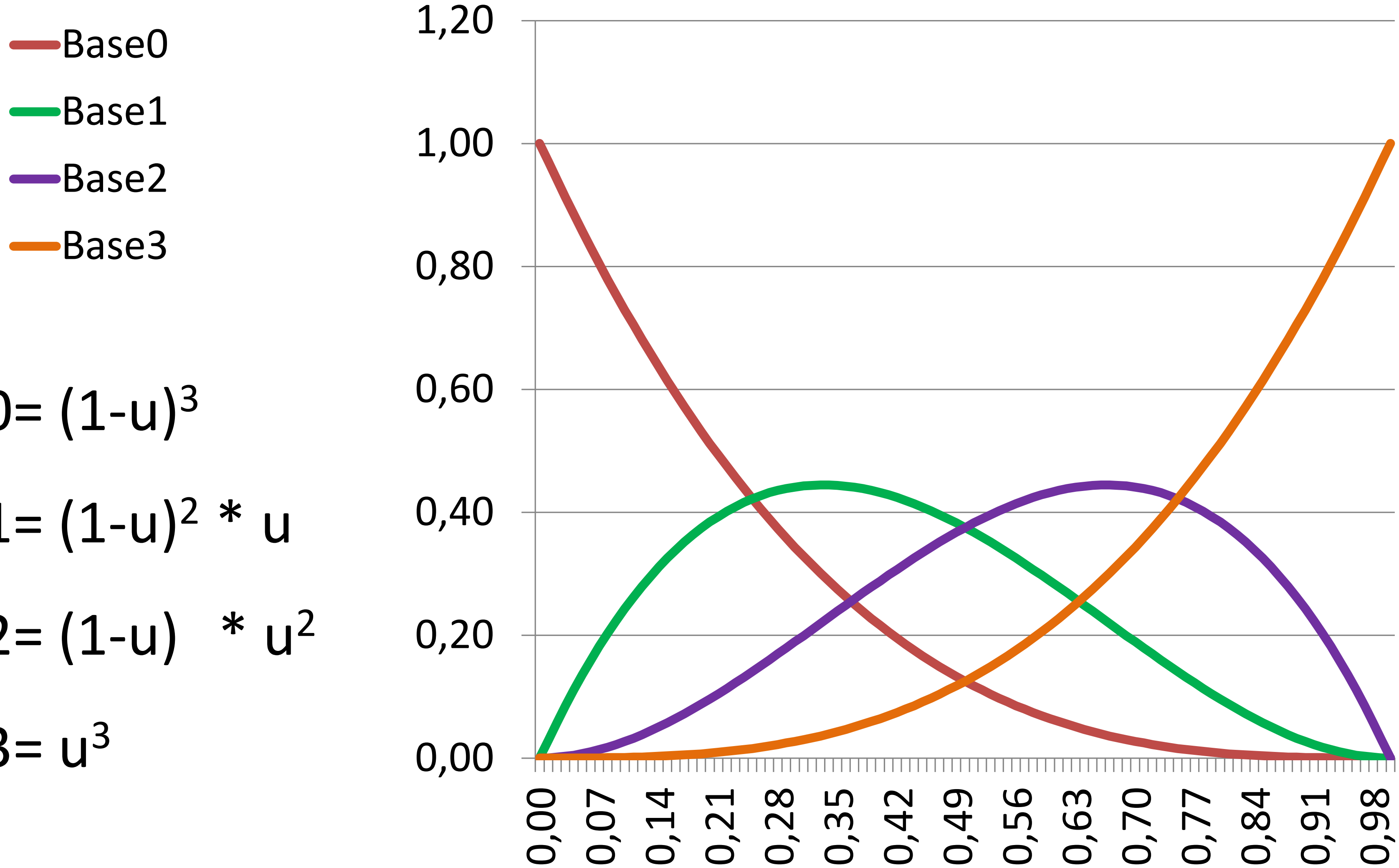
Bases de Berstein cúbicas

X(t)=B0*P0 +B1*P1+B2*P2+B3*P3

$$\mathbf{x} = \underbrace{(1-t)^3}_{B0}\mathbf{p}_0 + \underbrace{3(1-t)^2 t}_{B1}\mathbf{p}_1 + \underbrace{3(1-t)t^2}_{B2}\mathbf{p}_2 + \underbrace{t^3}_{B3}\mathbf{p}_3$$

$$\mathbf{x} = (1-t)\big((1-t)\big((1-t)\mathbf{p}_0 + t\mathbf{p}_1\big) + t\big((1-t)\mathbf{p}_1 + t\mathbf{p}_2\big)\big)$$
$$+ t\big((1-t)\big((1-t)\mathbf{p}_1 + t\mathbf{p}_2\big) + t\big((1-t)\mathbf{p}_2 + t\mathbf{p}_3\big)\big)$$

Bases de Berstein cúbicas



$B0 = (1-u)^3$

$B1 = (1-u)^2 * u$

$B2 = (1-u) * u^2$

$B3 = u^3$

Bases de Berstein cúbicas

$$B_0^3(u) = -u^3 + 3u^2 - 3u + 1$$

$$B_1^3(u) = 3u^3 - 6u^2 + 3u$$

$$B_2^3(u) = -3u^3 + 3u^2$$

$$B_3^3(u) = u^3$$

Bases de Berstein cúbicas

$$\mathbf{x} = \left(-\mathbf{p}_0 + 3\mathbf{p}_1 - 3\mathbf{p}_2 + \mathbf{p}_3\right)t^3 + \left(3\mathbf{p}_0 - 6\mathbf{p}_1 + 3\mathbf{p}_2\right)t^2 + \left(-3\mathbf{p}_0 + 3\mathbf{p}_1\right)t + \left(\mathbf{p}_0\right)1$$

$$\boxed{\mathbf{x} = \mathbf{a}t^3 + \mathbf{b}t^2 + \mathbf{c}t + \mathbf{d} \qquad \begin{aligned} \mathbf{a} &= \left(-\mathbf{p}_0 + 3\mathbf{p}_1 - 3\mathbf{p}_2 + \mathbf{p}_3\right) \\ \mathbf{b} &= \left(3\mathbf{p}_0 - 6\mathbf{p}_1 + 3\mathbf{p}_2\right) \\ \mathbf{c} &= \left(-3\mathbf{p}_0 + 3\mathbf{p}_1\right) \\ \mathbf{d} &= \left(\mathbf{p}_0\right) \end{aligned}}$$

Bases de Berstein cuadráticas y lineales

$$B_0^2(u) = u^2 - 2u + 1$$

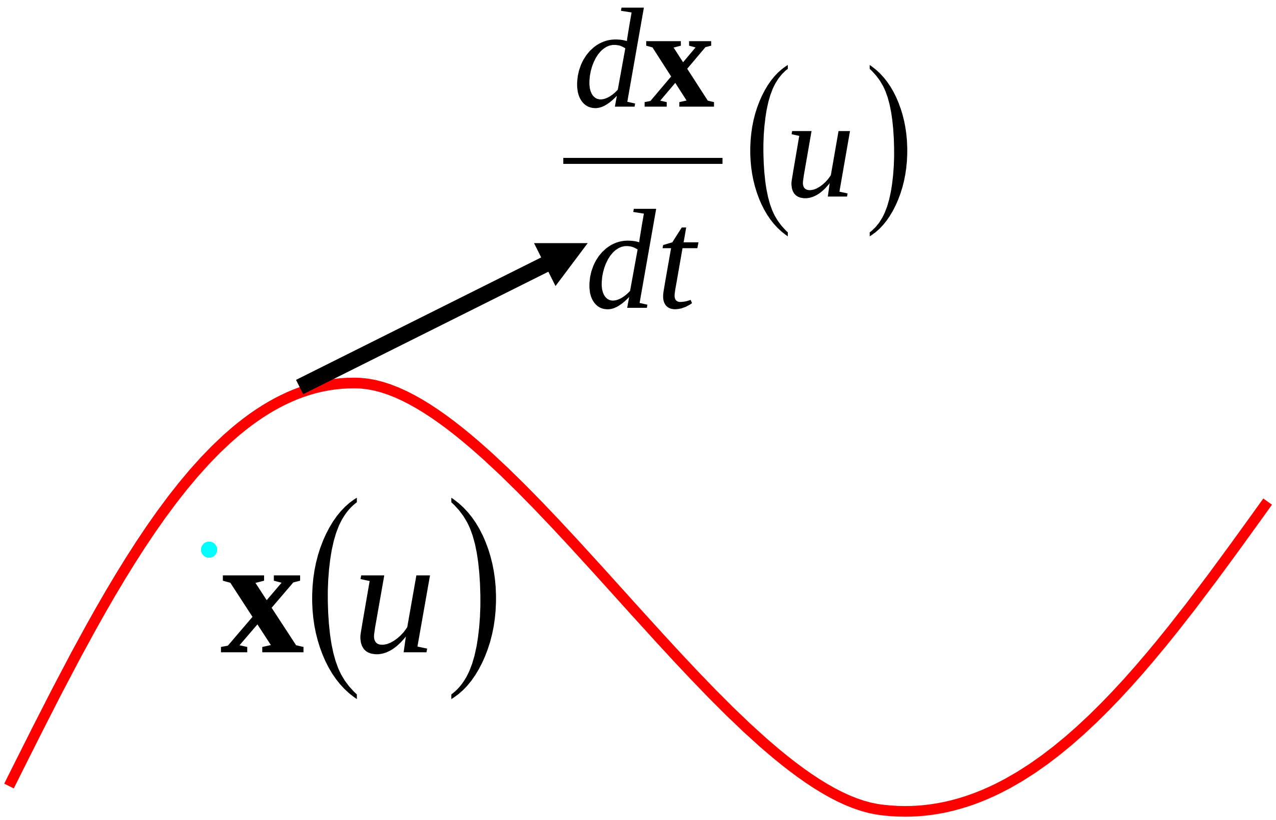
$$B_1^2(u) = -2u^2 + 2u$$

$$B_2^2(u) = u^2$$

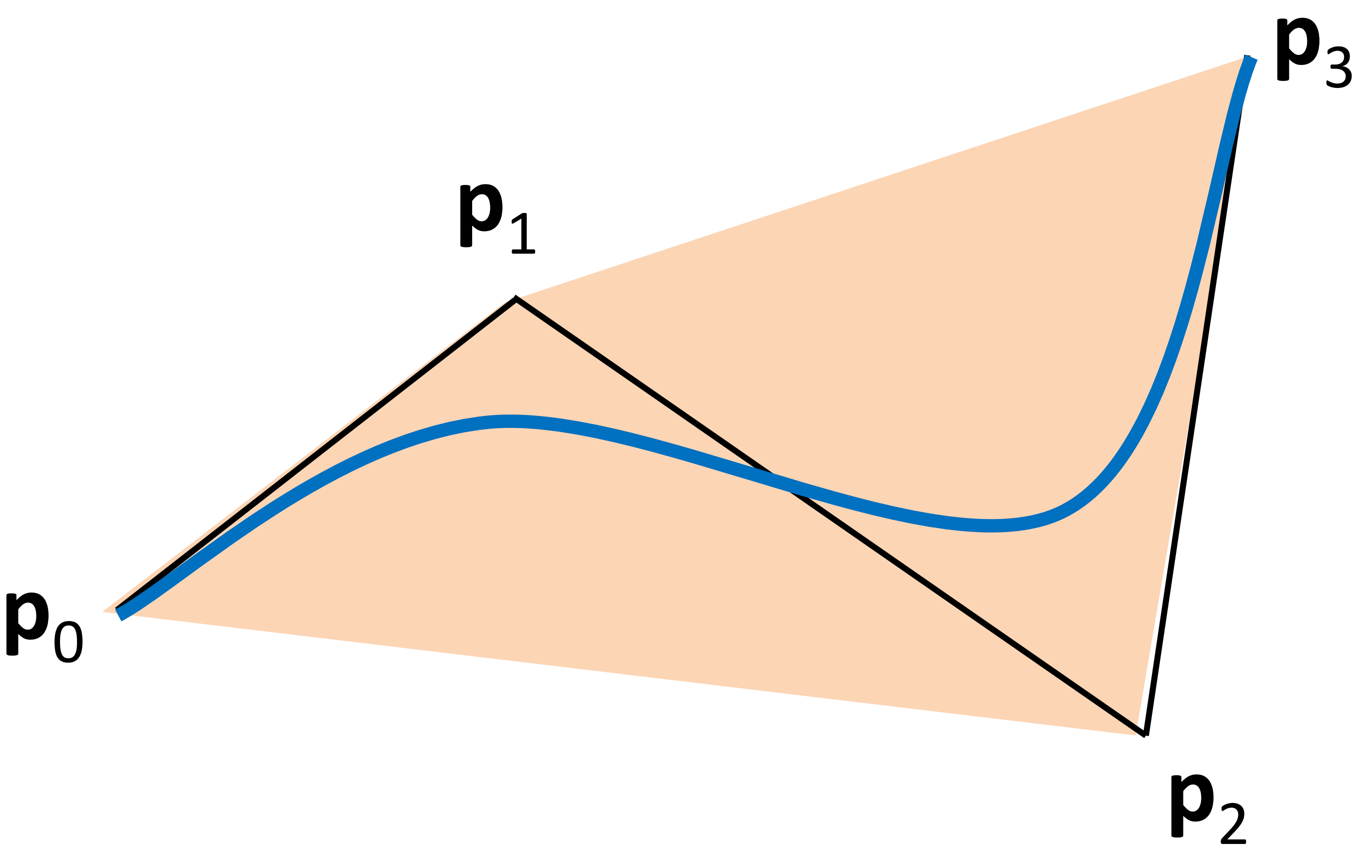
$$B_0^1(u) = -u + 1$$

$$B_1^1(u) = u$$

Tangentes y derivadas



Envolvente convexa



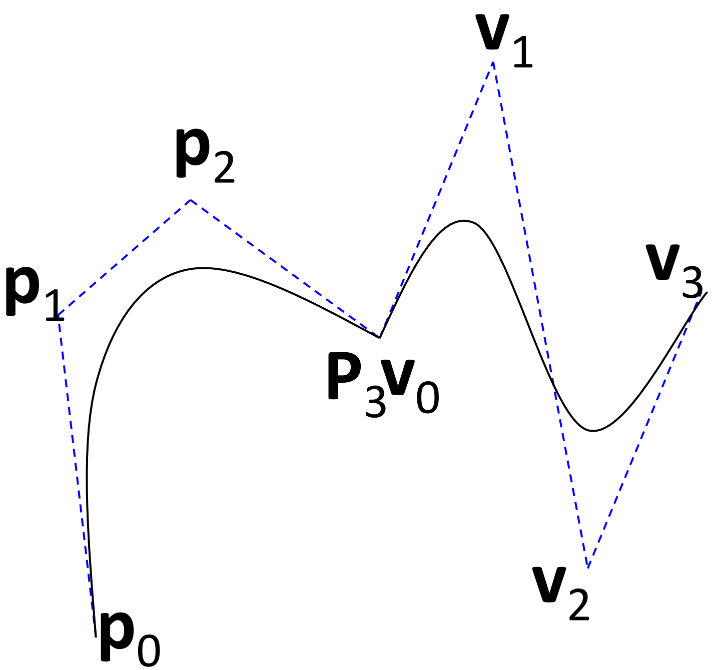
Propiedades

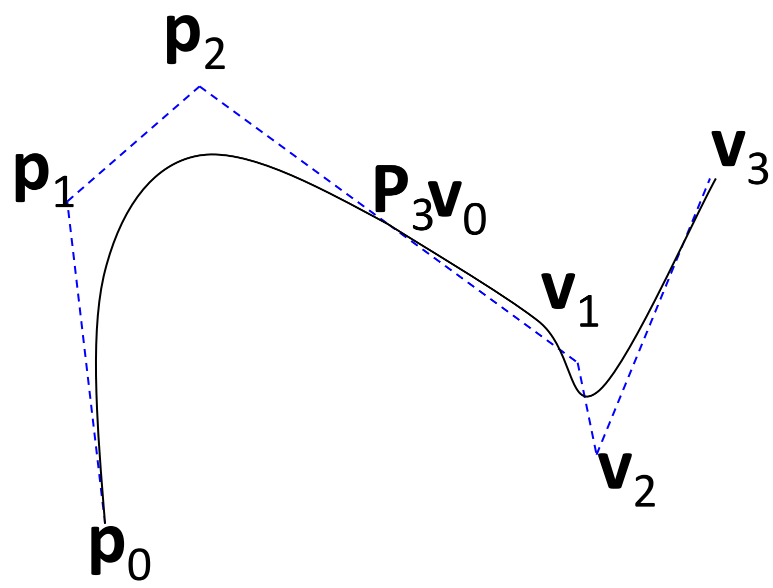
- Continuidad C^0 : una curva Bezier cúbica definida entre t=0 y t=1 será una curva geométricamente continua

- Continuidad C^1: Primera derivada: función cuadrática continua

- Continuidad C^2: Segunda derivada: funcion lineal continua

Conectando curvas Bezier





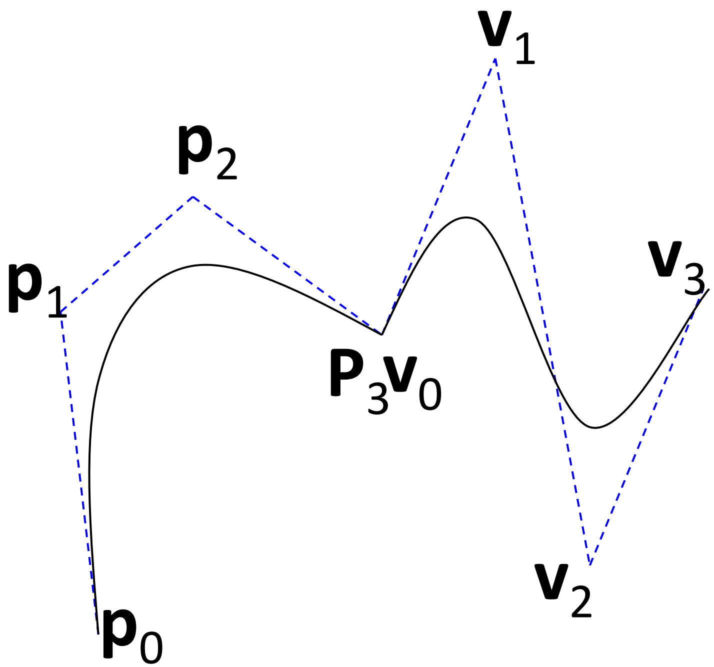
Conectando curvas Bezier

Continuidad de varios tramos:

Si $\mathbf{p}_3=\mathbf{v}_0$ tendrá continuidad C^0

Si $(\mathbf{p}_3-\mathbf{p}_2)=(\mathbf{v}_1-\mathbf{v}_0)$ tendrá continuidad C^1

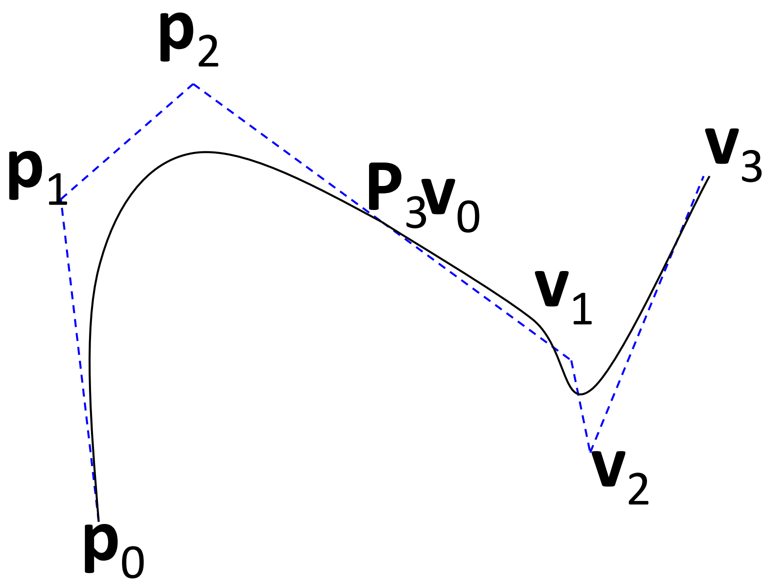
Conectando curvas Bezier



Continuidad C^0 SI

Continuidad C^1 NO

Conectando curvas Bezier



Continuidad C^0 SI

Continuidad C^1 SI