Tópicos en Matemáticas Discretas III - Online Algorithms and Scheduling
Professor: Andreas Wiese.
Teaching Assistant: Andrés Cristi.

Homework #2

P1. a) Consider an arbitrary two-person zero-sum game defined by the $(n \times m)$-matrix $M = (m_{ij})$. Let

$$V_R = \max_i \min_j m_{ij}$$
$$V_C = \min_j \max_i m_{ij}$$

Show that $V_R \leq V_C$.

b) Give an example in which $V_R < V_C$.

P2. a) Formulate the Ski Rental Problem as a Metrical Task System.

b) Formulate the list accessing problem as a Metrical Task System.

c) Consider the greedy algorithm for an MTS, this is, one that from state s_i moves to $s_{i+1} \in \arg\min_x d(s_i, x) + \tau_{i+1}(x)$ to process task τ_{i+1}. Show that this algorithm is not competitive at all.

P3. In this exercise we compute a lower bound for the competitivity ratio of any randomized algorithm for the list accessing problem. For that purpose, consider an instance where the requests are drawn uniformly at random from $\{1, \dots n\}$.

a) Calculate the expected cost of serving one request for a deterministic algorithm.

b) Calculate the expected optimal offline cost if $n = 2$ and there are only 3 requests.

c) Conclude a lower bound of 12/11 for the (asymptotic) competitivity of any randomized algorithm.

d) Get a strictly better bound.