Tópicos en Matemáticas Discretas III - Online Algorithms and Scheduling
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Homework #3

P1.

a) Show that the greedy algorithm for the k-server problem is not c-competitive for any constant c.

b) A space (X, d) is called asymmetric if $d : X^2 \to [0, +\infty]$ satisfies the triangle inequality but not the symmetry condition. Show that the competitive ratio for k-server for such spaces cannot be bounded by a function of k.

P2. Consider a uniform MTS on n states, i.e. a space (X, d) such that $d(x, y) = 1$ for every pair $x, y \in X$; in which the tasks take values only in $\{0, 1\}$.

a) Find a random instance on k tasks such that the expected cost of any deterministic algorithm is k/n.

b) Let $c(k)$ be the expected cost of the optimal offline algorithm. Prove that $\text{lím}_k \frac{c(k)}{k} = \frac{1}{nH_n}$, where H_n is the n-th harmonic number.

c) Conclude that no random algorithm can be better than H_n-competitive.

d) For a state s and a sequence of tasks $\hat{\tau}$, define $\rho(s, \hat{\tau}) = w(s, \hat{\tau}) - \text{mín}_x w(x, \hat{\tau})$, where w is the work function of the MTS. Consider $\rho(\cdot, \hat{\tau})$ as a point in $\mathbb{R}^X$, and describe its possible values.

e) Consider now ρ as a description of the system after a secuence of tasks. Find a reasonable random algorithm that moves to a random state given by a distribution that depends only on ρ, that is H_n-competitive. **Hint:** Consider the potential $\Phi = H_m$, where $m = |\arg\text{mín}_{x \in X} \rho(x, \hat{\tau})|$.

P3. k-server and paging.

P4. SC 1/2.