Multivariable Calculus

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1 Linear Equations in Linear Algebra

1.1 Systems of Linear Equations

Definition 1.1. A *linear equation is an equation of the form*

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$$

Definition 1.2. A *linear system* is a set of linear equations involving like variables.

Definition 1.3. A *solution* to a linear system is an ordered set that makes the linear system true.

Definition 1.4. A *solution set* is the set of all possible solutions to the linear system.

Remark. Two linear systems with like solution sets are *equivalent.*

Remark. A linear system is *consistent* if it has at least one solution, and *inconsistent* if it has no solutions.

Definition 1.5. A *coefficient matrix* is a matrix that consists of the coefficients of the variables of a linear system.

Remark. Each column of the coefficient matrix corresponds to a variable in the linear system.

Definition 1.6. An *augmented matrix* consists of the coefficient matrix with an added column containing the constants of the RHS of the linear system.

Definition 1.7. An $m \times n$ *matrix* is a rectangular array of elements with m rows and n columns.

1.1.1 Elementary Row Operations

- add the multiple of one row to another
- switch two rows
- scale a row by a nonzero constant

Remark. Row operations are reversible.

Definition 1.8. Two matrices are *row equivalent* if a sequence of row operations can transform one into the other.

Remark. All row equivalent augmented matrices have the same solution set.

1.1.2 Questions

- does a solution to the linear system exist?
- If it does, is it unique?

1.2 Row Reduction and Echelon Forms

Definition 1.9. The *leading entry* of a row is its left-most non-zero entry.

Definition 1.10. A matrix is in *echelon form* if:

- all non-zero rows are above any all-zero rows
- the leading entry of each row is in a column to the right of the leading entry of the row above it
- all entries in a column below a leading entry are zeros

Definition 1.11. A matrix is in *reduced row echelon form* if:

- it's in echelon form
- all leading entries are 1
- all leading entries are the only non-zero entries in their columns

Remark. A matrix can be row equivalent with many echelon forms but only one reduced echelon form.

Definition 1.12. A *pivot position* corresponds to the position of one of the leading entries of the reduced echelon form of a matrix.

Definition 1.13. A column of the coefficient matrix is a *free column* if it doesn't contain a pivot position.

Definition 1.14. A column of the augmented matrix is a *pivot column* if it contains a pivot position.

Remark. Variables corresponding to free columns are *free variables*. Variables corresponding to pivot columns are *basic variables*.

Remark. The solution set of a consistent linear system has a *parametric representation* in which by convention free variables act as parameters. The solution set of an incosistent linear system is empty and has **no** parametric representation.

Remark. Solving a system amounts to finding a parametric representation of the solution set or determing that the solution set is empty.

Remark. A linear system is consistent iff the right-most column of the augmented matrix is **not** a pivot column.