Introduction to Causal Inference

Causal Inference using Machine Learning
Master in Economics, UNT

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Spring 2024

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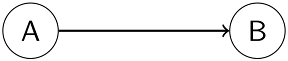
What is a Causal Question?



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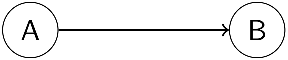

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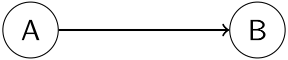

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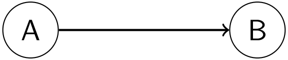

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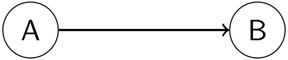

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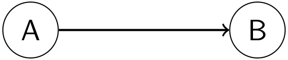

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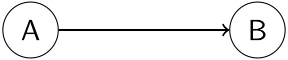

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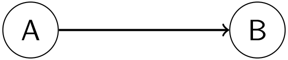

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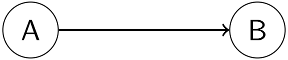

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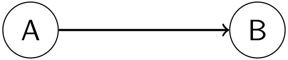

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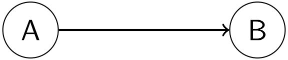

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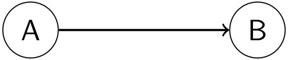

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   - Example: Eating more food can cause weight gain, but if food intake and exercise both increase proportionally, we may observe no correlation between food and weight in the data, even though causation exists.

Causal Inference Tree

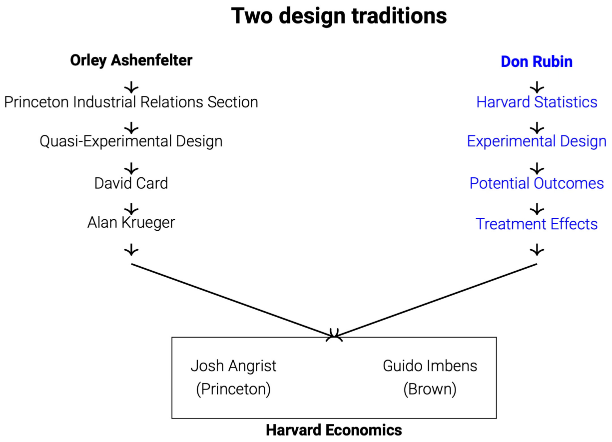

Figure: Source: Scott Cunningham Substack

Experimental Design Tradition

Experimental design relies on randomized controlled trials (RCTs) to establish causality through direct manipulation of the treatment.

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  - Duflo and Banerjee pioneered the use of RCTs in development economics to evaluate the impact of poverty alleviation policies.

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- **Synthetic Control (SCM)**: Abadie and Gardeazabal (2003): developed SCM to run comparative case studies.
  -*CQ*: What is the impact of Terrorism on Economic Performance?

Machine Learning and Causal Inference

What is Machine Learning?

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How Machine Learning is Used in Causal Inference

- Machine Learning can be used to estimate nuisance parameters (e.g., propensity scores, regression functions) in causal inference models.
- Chernozhukov et al. (2018) introduced **Double/Debiased Machine Learning (DML)**, which combines machine learning for estimating high-dimensional nuisance functions with traditional econometric techniques to ensure valid causal inference.
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- Comparative Case Study: How do immigration shocks affect local labor markets? Card (1990)

These questions seek to determine the effect of specific treatments (institutions, education, immigration) on outcomes of interest (development, earnings, labor markets).

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- **Regression Discontinuity (RD)**:
  - Strategy: Use a cutoff or threshold to identify the causal effect.
  - Assumption: **Local Randomization**, **Continuity of Potential Outcomes**.

4. What's the Inference Strategy?

After identifying the causal effect, we need to make valid statistical inferences. Common inference strategies include:

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  - pproximate the variance (or standard error) of a function of an estimator, by using a first-order Taylor expansion.

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- **Bootstrap**:
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- **Influence Functions**:
  - Analyze the sensitivity of the estimator to small changes in the sample. Common in non-parametric estimation.

Each strategy allows for valid inference under different circumstances, depending on the complexity of the model and data.

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Random Variable and Probability Measure

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1. **Non-negativity**: $P(A) \geq 0$ for any event $A \in \mathcal{F}$.
2. **Normalization**: $P(\Omega) = 1$.
3. **Additivity**: For disjoint events $A_1, A_2, \ldots$,

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$$

Binary Outcome and Sigma Algebra

Binary Outcome and Probability Measure:

- Define the sample space $\Omega = \{High, Low\}$.
- The sigma-algebra $\mathcal{F}$ is the power set of Ω, i.e.,

$$\mathcal{F} = \{\emptyset, \{Low\}, \{High\}, \{Low, High\}\}$$

- Define $Y \in \{High, Low\}$, with the following values and probabilities:

i	Y
1	Low
2	Low
3	High
4	High
5	High
6	High

- $P(Y = Low) = 0.33$, $P(Y = High) = 0.66$

Verification of Probability Axioms

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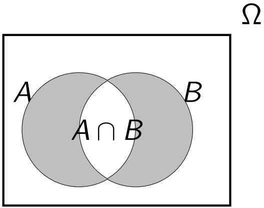
Venn Diagram, Conditional Probability, and Independence



Conditional Probability:

- The conditional probability of A given B is defined as:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \quad \text{for } P(B) > 0$$

Independence:

- Two events A and B are independent if:

$$P(A \cap B) = P(A)P(B)$$

Exogenous D

Binary Outcome with New Variable D: $D \sim$ **Bernoulli**(0.5)

i	Y	D
1	Low	0
2	Low	1
3	Low	0
4	High	1
5	High	0
6	Low	1

- $P(D = 1) = 0.5$, $P(Y = High) = 0.33$
- Joint probability: $P(Y = High \cap D = 1) = \frac{1}{6} = 0.167$
- Independence check:

$$P(Y = High)P(D = 1) = 0.33 \times 0.5 = 0.165$$

- Independence check:

$$P(Y = High|P(D = 1) = P(Y = High) = 0.33$$

Endogenous Assignment of D

Binary Outcome with Endogenous D: $D \sim$ Bernoulli$(p(Y))$, where $p(Y^{High}) > p(Y^{Low})$

i	Y	D
1	Low	0
2	High	1
3	Low	0
4	High	1
5	Low	1
6	High	0

- $P(Y = High) = 0.5, \; P(D = 1) = 0.5$
-
- $P(Y = High \cap D = 1) = 0.33 \neq 0.5 * 0.5$
- $P(Y = High | D = 1) = \frac{0.33}{0.5} = 0.67 = \frac{2}{3}$

Law of Total Probability:

$$P(Y = H) = P(Y = H|D = 1)P(D = 1) + P(Y = H|D = 0)P(D = 0)$$

Using the values from our example:

- $P(Y = High|D = 1) = \frac{2}{3} = 0.67$
- $P(D = 1) = 0.5$
- $P(Y = High|D = 0) = \frac{1}{3} = 0.33$
- $P(D = 0) = 0.5$

Therefore, applying the law of total probability:

$$P(Y = High) = 0.67 \times 0.5 + 0.33 \times 0.5 = 0.335 + 0.165 = 0.5$$

This matches the marginal probability $P(Y = High) = 0.5$ calculated earlier.

Expectation

Expectation (Discrete Variable):

- For a discrete random variable X with probability mass function $p(x)$:

$$\mathbb{E}[X] = \sum_{x \in \mathcal{X}} x \cdot p(x)$$

where $\mathcal{X}$ is the set of possible values of X.

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Example: Bernoulli(0,1)

- $X \in \{0, 1\} \quad P(X = 1) = 0.5$
- $\mathbb{E}[X] = 1 \cdot 0.5 + 0 \cdot 0.5 = 0.5$

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- For a continuous random variable X with probability density function $f(x)$:

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where $f(x)$ is the PDF of X.

Example: Normal(0,1)

- $X \sim N(0, 1)$, where the PDF is $f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$
- $\mathbb{E}[X] = \int_{-\infty}^{\infty} x \cdot \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx = 0$

Conditional Expectation

Conditional Expectation (Discrete Variable):

- For a discrete random variable X with conditional probability $P(X = x|Y = y)$:

$$\mathbb{E}[X|Y = y] = \sum_{x \in \mathcal{X}} x \cdot P(X = x|Y = y)$$

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Example: Bernoulli(0,1)

- $X \in \{0, 1\}$ with $P(X = 1|Y = y) = p(y)$
- $\mathbb{E}[X|Y = y] = 1 \cdot p(y) + 0 \cdot (1 - p(y)) = p(y)$

Conditional Expectation

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$$\mathbb{E}[X|Y = y] = \int_{-\infty}^{\infty} x \cdot f(x|y)\, dx$$

Example: Normal(0,1)

- Suppose $X \sim N(0, 1)$, and $f(x|y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-y)^2}{2}}$ (a shifted normal)
- $\mathbb{E}[X|Y = y] = y$

Exogenous D

Discrete Outcome with Exogenous D: $D \sim$ Bernoulli(0.5)

i	Y	D
1	3	0
2	8	1
3	5	1
4	7	0
5	4	1
6	9	0

Exogenous D

Discrete Outcome with Exogenous D: $D \sim$ **Bernoulli**(0.5)

i	Y	D
1	3	0
2	8	1
3	5	1
4	7	0
5	4	1
6	9	0

- **Expectation:**

$$\mathbb{E}[Y] = \frac{3+8+5+9+4+7}{6} = 6$$

Discrete Outcome with Exogenous D: $D \sim$ **Bernoulli**(0.5)

i	Y	D
1	3	0
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3	5	1
4	7	0
5	4	1
6	9	0

- **Expectation:**

$$\mathbb{E}[Y] = \frac{3+8+5+9+4+7}{6} = 6$$

- **Conditional Expectation $\mathbb{E}[Y|D=1]$:**

$$\mathbb{E}[Y|D=1] = \frac{8+5+4}{3} \approx 6$$

Endogenous D

Endogenous Selection D**:** $D \sim$ **Bernoulli**$(p(Y))$**, where** $p(Y^{High}) > p(Y^{Low})$

i	Y	D
1	3	0
2	8	1
3	5	0
4	9	1
5	4	0
6	7	1

- **Expectation:**

$$\mathbb{E}[Y] = \frac{3+8+5+9+4+7}{6} = 6$$

- **Conditional Expectation $\mathbb{E}[Y|D=1]$:**

$$\mathbb{E}[Y|D=1] = \frac{8+9+7}{3} = 8$$

Law of Iterated Expectations:

$$\mathbb{E}[Y] = \mathbb{E}[\mathbb{E}[Y|D]] = \sum_{d \in \mathcal{D}} \mathbb{E}[Y|D = d] \cdot P(D = d) \quad \text{(for discrete D)}$$

i	Y	D
1	3	0
2	8	1
3	5	0
4	9	1
5	4	0
6	7	1

Given $P(D = 1) = 0.5$, the conditional and total expectations are:

$$\mathbb{E}[Y|D = 0] = \frac{3 + 5 + 4}{3} = 4, \quad \mathbb{E}[Y|D = 1] = \frac{8 + 9 + 7}{3} = 8$$

$$\mathbb{E}[Y] = 0.5 \cdot 4 + 0.5 \cdot 8 = 6$$

Content

Potential Outcomes and Parallel Futures

Borges Quote

"Cada vez que un hombre se enfrenta a diversas alternativas, opta por una y elimina las otras; [...] Crea, así, diversos futuros, diversos tiempos, que también proliferan y se bifurcan."
— *Borges, El jardín de los senderos que se bifurcan*

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Potential Outcomes Framework:

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Potential Outcomes Framework:

- $Y(0)$: The outcome that would occur if the individual does not receive the treatment ($D = 0$).

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- $Y(0)$: The outcome that would occur if the individual does not receive the treatment ($D = 0$).
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- The observed outcome Y_i is:

$$Y_i = D_i Y_i(1) + (1 - D_i) Y_i(0)$$

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Treatment Effects and the Fundamental Problem of Causal Inference

i	$Y(0)$	$Y(1)$	D	Y	τ_i
1	3	5	0	3	2
2	4	8	1	8	4
3	5	6	0	5	1
4	7	9	1	9	2
5	4	5	0	4	1
6	3	7	1	7	4

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$$\tau_i = Y_i(1) - Y_i(0)$$

Treatment Effects and the Fundamental Problem of Causal Inference

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5	4	5	0	4	1
6	3	7	1	7	4

$$\tau_i = Y_i(1) - Y_i(0)$$

Fundamental Problem of Causal Inference

We can never observe both potential outcomes $Y(0)$ and $Y(1)$ for the same individual at the same time making it impossible to directly observe the true treatment effect τ_i for any single individual.

Average Treatment Effect (ATE)

Definition of ATE:

$$ATE = \mathbb{E}[Y(1) - Y(0)]$$

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Computation in the Example:

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4	7	9	1	9	2
5	4	5	0	4	1
6	3	7	1	7	4

$$ATE = \frac{2 + 4 + 1 + 2 + 1 + 4}{6} = 2.34$$

Average Treatment Effect on the Treated (ATT)

Definition of ATT:

$$ATT = \mathbb{E}[Y(1) - Y(0) | D = 1]$$

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3	5	6	0	5	1
4	7	9	1	9	2
5	4	5	0	4	1
6	3	7	1	7	4

$$ATT = \frac{4 + 2 + 4}{3} = 3.33$$

Average Treatment Effect on the Untreated (ATU)

Definition of ATU:

$$ATU = \mathbb{E}[Y(1) - Y(0)|D = 0]$$

Average Treatment Effect on the Untreated (ATU)

Definition of ATU:

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Computation in the Example:

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1	3	5	0	3	2
2	4	8	1	8	4
3	5	6	0	5	1
4	7	9	1	9	2
5	4	5	0	4	1
6	3	7	1	7	4

Average Treatment Effect on the Untreated (ATU)

Definition of ATU:

$$ATU = \mathbb{E}[Y(1) - Y(0)|D = 0]$$

Computation in the Example:

i	$Y(0)$	$Y(1)$	D	Y	τ_i
1	3	5	0	3	2
2	4	8	1	8	4
3	5	6	0	5	1
4	7	9	1	9	2
5	4	5	0	4	1
6	3	7	1	7	4

$$ATU = \frac{2+1+1}{3} = 1.33$$

Selection Bias

Naive Comparison:

$$\tau_{naive} = \mathbb{E}[Y|D=1] - \mathbb{E}[Y|D=0]$$

Selection Bias

Naive Comparison:

$$\tau_{naive} = \mathbb{E}[Y|D = 1] - \mathbb{E}[Y|D = 0]$$

Naive Comparison Decomposition:

$$\begin{aligned}\tau_{naive} &= \mathbb{E}[Y(1)|D = 1] - \mathbb{E}[Y(0)|D = 0] \\ &= \underbrace{(\mathbb{E}[Y(1)|D = 1] - \mathbb{E}[Y(0)|D = 1])}_{\text{ATT}} + \underbrace{(\mathbb{E}[Y(0)|D = 1] - \mathbb{E}[Y(0)|D = 0])}_{\text{Selection Bias}}\end{aligned}$$

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ATT identification

$$\mathbb{E}[Y(1)|D=1] - \mathbb{E}[Y(0)|D=1] = \mathbb{E}[Y(1) - Y(0)|D=1] = ATT$$

Causal Inference = How to overcome Selection Bias?

Naive Estimator and Selection Bias - Example

i	$Y(0)$	$Y(1)$	D	Y	$\tau_i = Y(1) - Y(0)$
1	3	5	0	3	2
2	4	8	1	8	4
3	5	6	0	5	1
4	7	9	1	9	2
5	4	5	0	4	1
6	3	7	1	7	4

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4	7	9	1	9	2
5	4	5	0	4	1
6	3	7	1	7	4

$$\tau_{naive} = \frac{8+9+7}{3} - \frac{3+5+4}{3} = 8 - 4 = 4$$

Naive Estimator and Selection Bias - Example

i	$Y(0)$	$Y(1)$	D	Y	$\tau_i = Y(1) - Y(0)$
1	3	5	0	3	2
2	4	8	1	8	4
3	5	6	0	5	1
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5	4	5	0	4	1
6	3	7	1	7	4

$$\tau_{naive} = \frac{8+9+7}{3} - \frac{3+5+4}{3} = 8 - 4 = 4$$

$$Selection\ Bias = \mathbb{E}[Y(0)|D=1] - \mathbb{E}[Y(0)|D=0] = \frac{4+7+3}{3} - 4 = 4.67 - 4 = 0.67$$

Naive Estimator and Selection Bias - Example

i	$Y(0)$	$Y(1)$	D	Y	$\tau_i = Y(1) - Y(0)$
1	3	5	0	3	2
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6	3	7	1	7	4

$$\tau_{naive} = \frac{8+9+7}{3} - \frac{3+5+4}{3} = 8 - 4 = 4$$

$$Selection\ Bias = \mathbb{E}[Y(0)|D=1] - \mathbb{E}[Y(0)|D=0] = \frac{4+7+3}{3} - 4 = 4.67 - 4 = 0.67$$

$$\tau_{ATT} = 4 - 0.67 = 3.33$$

Randomization Solves Selection Bias

Naive Estimator:

$$\tau_{naive} = \mathbb{E}[Y(1)|D = 1] - \mathbb{E}[Y(0)|D = 0]$$

Randomization Solves Selection Bias

Naive Estimator:

$$\tau_{naive} = \mathbb{E}[Y(1)|D = 1] - \mathbb{E}[Y(0)|D = 0]$$

If $Y(0)$ is independent of D:

$$\mathbb{E}[Y(0)|D = 0] = \mathbb{E}[Y(0)|D = 1]$$

Randomization Solves Selection Bias

Naive Estimator:

$$\tau_{naive} = \mathbb{E}[Y(1)|D=1] - \mathbb{E}[Y(0)|D=0]$$

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Then:

$$\tau_{naive} = \mathbb{E}[Y(1)|D=1] - \mathbb{E}[Y(0)|D=1] = \mathbb{E}[Y(1) - Y(0)|D=1] = ATT$$

Randomization Solves Selection Bias

Naive Estimator:

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$$\tau_{naive} = \mathbb{E}[Y(1)|D = 1] - \mathbb{E}[Y(0)|D = 1] = \mathbb{E}[Y(1) - Y(0)|D = 1] = ATT$$

If D is also independent of $Y(1)$:

$$\mathbb{E}[Y(1) - Y(0)|D = 1] = \mathbb{E}[Y(1) - Y(0)] = ATE$$

Randomization Solves Selection Bias

Naive Estimator:

$$\tau_{naive} = \mathbb{E}[Y(1)|D=1] - \mathbb{E}[Y(0)|D=0]$$

If $Y(0)$ is independent of D:

$$\mathbb{E}[Y(0)|D=0] = \mathbb{E}[Y(0)|D=1]$$

Then:

$$\tau_{naive} = \mathbb{E}[Y(1)|D=1] - \mathbb{E}[Y(0)|D=1] = \mathbb{E}[Y(1) - Y(0)|D=1] = ATT$$

If D is also independent of $Y(1)$:

$$\mathbb{E}[Y(1) - Y(0)|D=1] = \mathbb{E}[Y(1) - Y(0)] = ATE$$

Conclusion: If $D \perp Y(0), Y(1)$ (Unconfoundedness), then:

$$ATE = ATT = ATU$$

Estimator Results: $\hat{\tau}_{naive}$, ATE, ATT, and ATU

i	$Y(0)$	$Y(1)$	D	Y	$\tau_i = Y(1) - Y(0)$
1	3	5	0	3	2
2	4	8	1	8	4
3	5	6	1	6	1
4	7	9	0	7	2
5	4	5	1	5	1
6	3	7	0	3	4

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3	5	6	1	6	1
4	7	9	0	7	2
5	4	5	1	5	1
6	3	7	0	3	4

$$\hat{\tau}_{naive} = \mathbb{E}[Y|D=1] - \mathbb{E}[Y|D=0] = 6.33 - 4.33 = 2$$

Estimator Results: $\hat{\tau}_{naive}$, ATE, ATT, and ATU

i	$Y(0)$	$Y(1)$	D	Y	$\tau_i = Y(1) - Y(0)$
1	3	5	0	3	2
2	4	8	1	8	4
3	5	6	1	6	1
4	7	9	0	7	2
5	4	5	1	5	1
6	3	7	0	3	4

$$\hat{\tau}_{naive} = \mathbb{E}[Y|D=1] - \mathbb{E}[Y|D=0] = 6.33 - 4.33 = 2$$

$$ATT = \mathbb{E}[Y(1) - Y(0)|D=1] = 2$$

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i	$Y(0)$	$Y(1)$	D	Y	$\tau_i = Y(1) - Y(0)$
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$$\hat{\tau}_{naive} = \mathbb{E}[Y|D=1] - \mathbb{E}[Y|D=0] = 6.33 - 4.33 = 2$$

$$ATT = \mathbb{E}[Y(1) - Y(0)|D=1] = 2$$

$$ATU = \mathbb{E}[Y(1) - Y(0)|D=0] = 2.67$$

Estimator Results: $\hat{\tau}_{naive}$, ATE, ATT, and ATU

i	$Y(0)$	$Y(1)$	D	Y	$\tau_i = Y(1) - Y(0)$
1	3	5	0	3	2
2	4	8	1	8	4
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4	7	9	0	7	2
5	4	5	1	5	1
6	3	7	0	3	4

$$\hat{\tau}_{naive} = \mathbb{E}[Y|D=1] - \mathbb{E}[Y|D=0] = 6.33 - 4.33 = 2$$

$$ATT = \mathbb{E}[Y(1) - Y(0)|D=1] = 2$$

$$ATU = \mathbb{E}[Y(1) - Y(0)|D=0] = 2.67$$

$$ATE = \mathbb{E}[Y(1) - Y(0)] = 2.33$$

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