Assignment 2

Network measures

Theoretical part

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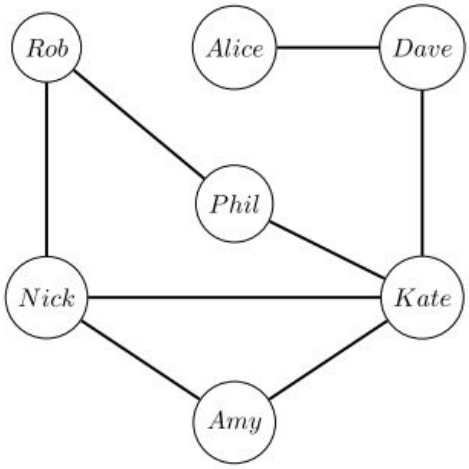
Task 1. Given the social network in Figure 1, nodes represent users and edges represent message communication between them.



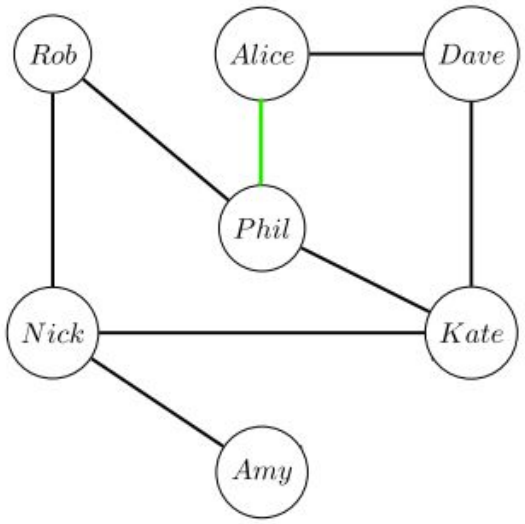
1. Which user has the highest Betweenness and Closeness centrality values in the graph?

Closeness Centrality	**Betweenness Centrality**
$C^c(Amy) = \frac{1}{((1+1+2+2+3)/6)}$ = 0.545	$C^b(Amy)$ = 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 = 0
$C^c(Nick) = \frac{1}{((1+1+1+2+3)/6)}$ = 0.6	$C^b(Nick)$ = 0 + 0 + 0 + 0 + 1 + 0 + 0.5 + 0.5 + 0.5 + 0 + 0 + 0 + 0 + 0 + 0 = 2.5
$C^c(Kate) = \frac{1}{((1+1+1+1+2+2)/6)}$ = **0.75**	$C^b(Kate)$ = 0 + 0 + 1 + 1 + 1 + 0 + 0.5 + 1 + 1 + 0 + 1 + 1 + 1 + 1 + 0 = **9.5**
$C^c(Phil) = \frac{1}{((1+1+2+2+2+3)/6)}$ = 0.545	$C^b(Phil)$ = 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0.5 + 0.5 + 0.5 + 0 = 1.5

$C^c(Rob) = \frac{1}{((1+1+2+2+3+4)/6)}$ = 0.461	$C^b(Rob)$ = 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0.5 + 0.5 + 0.5 + 0 = 1.5
$C^c(Dave) = \frac{1}{((1+1+2+2+2+3)/6)}$ = 0.545	$C^b(Dave)$ = 0 + 0 + 0 + 0 + 1 + 0 + 0 + 0 + 1 + 0 + 0 + 1 + 0 + 1 + 1 = 5
$C^c(Alice) = \frac{1}{((1+2+3+3+3+4)/6)}$ = 0.375	$C^b(Alice)$ = 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 = 0

The node with the highest Betweenness and Closeness is Kate

2. Modify the graph by performing at most two edge operations, such that user Kate becomes the node with the highest Closeness centrality and user Nick becomes the node with the highest Betweenness centrality. An edge operation is either an edge addition or a removal.



Whit the new graph the network measures change

Closeness Centrality	**Betweenness Centrality**
$C^c(Amy) = \frac{1}{((1+2+2+3+3+4)/6)}$ = 0.4	$C^b(Amy)$ = 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 = 0
$C^c(Nick) = \frac{1}{((1+1+1+2+2+3)/6)}$ = 0.6	$C^b(Nick)$ = 1 + 1 + 1 + 1 + 1 + 0 + 0.5 + 0.5 + 0 + 0 + 0 + 0 + 0 + 0 + 0 = **6**
$C^c(Kate) = \frac{1}{((1+1+1+2+2+2)/6)}$ = **0.667**	$C^b(Kate)$ = 0 + 0 + 0.5 + 0.66 + 1 + 0 + 0 + 0 + 0.66 + 0.5 + 0 + 0.5 + 1 + 0.66 + 0 =5.48

$C^c(Phil) = \frac{1}{((1+1+1+2+2+3)/6)}$ = 0.6	$C^b(Phil)$ = 0 + 0 + 0 + 0 + 0.66 + 0 + 0 + 0 + 0.66 + 0.5 + 1 + 0.66 + 0 + 0.5 + 0 = 3.98
$C^c(Rob) = \frac{1}{((1+1+2+2+2+3)/6)}$ = 0.545	$C^b(Rob)$ = 0 + 0 + 0 + 0.5 + 0.5 + 0 + 0 + 0.5 + 0.33 + 0 + 0 + 0 + 0 + 0 + 0 = 1.83
$C^c(Dave) = \frac{1}{((1+1+2+2+3+3)/6)}$ = 0.5	$C^b(Dave)$ = 0 + 0 + 0 + 0 + 0.33 + 0 + 0 + 0 + 0.33 + 0 + 0 + 0 + 0 + 0 + 0.5 = 1.16
$C^c(Alice) = \frac{1}{((1+1+2+2+3+4)/6)}$ = 0.46	$C^b(Alice)$ = 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0.33 + 0.5 + 0 + 0 = 0.83

How it was expected the node Kate keeps the highest Closeness Centrality but now Nick is the highest Betweenness Centrality.

Task 2. Given the social network in Figure 2, nodes represent users and edges represent friendships between them.

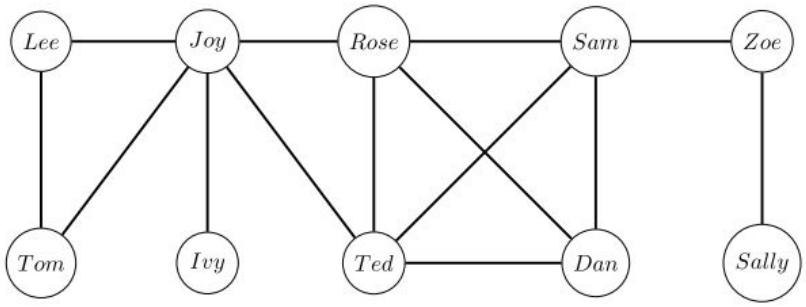
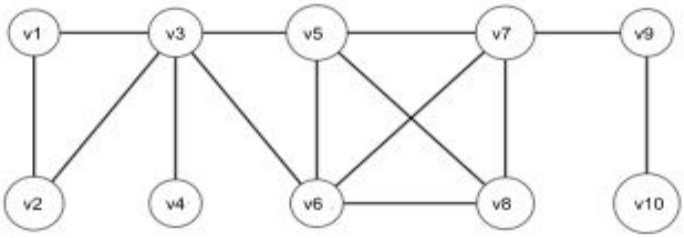


Figure 2 - Friendship social network

1. Calculate the Jaccard similarity between user Joy and all other users. Which is the most structurally equivalent user to Joy?

The first thing to calculation facilities is rename the nodes by single characters:

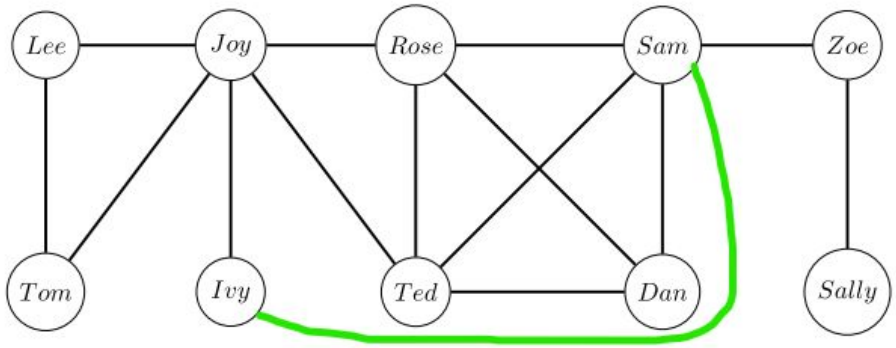


$\sigma_{Jac}(vi, vj) = \frac{|N(vi) \cap N(vj)|}{|N(vi) \cup N(vj)|} \quad \forall\ i \neq j$ Jaccard Similarity

$v3, v1$	$\frac{\lvert\{v1,v2,v4,v5,v6\}\cap\{v2,v3\}\rvert}{\lvert\{v1,v2,v3,v4,v5,v6\}\rvert}$ = 1/6	$v3, v7$	$\frac{\lvert\{v1,v2,v4,v5,v6\}\cap\{v5,v6,v8,v9\}\rvert}{\lvert\{v1,v2,v4,v5,v6,v8,v9\}\rvert}$ = 2/7
$v3, v2$	$\frac{\lvert\{v1,v2,v4,v5,v6\}\cap\{v1,v3\}\rvert}{\lvert\{v1,v2,v3,v4,v5,v6\}\rvert}$ = 1/6	$v3, v8$	$\frac{\lvert\{v1,v2,v4,v5,v6\}\cap\{v5,v6,v7\}\rvert}{\lvert\{v1,v2,v4,v5,v6,v7\}\rvert}$ = **2/6**
$v3, v4$	$\frac{\lvert\{v1,v2,v4,v5,v6\}\cap\{v3\}\rvert}{\lvert\{v1,v2,v3,v4,v5,v6\}\rvert}$ = 0	$v3, v9$	$\frac{\lvert\{v1,v2,v4,v5,v6\}\cap\{v7,v10\}\rvert}{\lvert\{v1,v2,v4,v5,v6,v7,v10\}\rvert}$ = 0
$v3, v5$	$\frac{\lvert\{v1,v2,v4,v5,v6\}\cap\{v3,v6,v7,v8\}\rvert}{\lvert\{v1,v2,v3,v4,v5,v6,v7,v8\}\rvert}$ = 1/8	$v3, v10$	$\frac{\lvert\{v1,v2,v4,v5,v6\}\cap\{v9\}\rvert}{\lvert\{v1,v2,v4,v5,v6,v9\}\rvert}$ = 0
$v3, v6$	$\frac{\lvert\{v1,v2,v4,v5,v6\}\cap\{v3,v5,v7,v8\}\rvert}{\lvert\{v1,v2,v3,v4,v5,v6,v7,v8\}\rvert}$ = 1/8		

Under this graph configuration and using Jaccard similarity measure, the most similar node to Joy is Dan.

2. Modify the graph by performing one edge operation, such that user Sam becomes the most structurally equivalent user to Joy. An edge operation is either an edge addition or a removal.



The previous graph represents the new edge operation between Sam and Ivy, which modifies the Jaccard Similarity only for Sam, being this now: $\sigma_{Jac}(v3, v7) = \frac{\lvert\{v1,v2,v4,v5,v6\}\cap\{v4,v5,v6,v8,v9\}\rvert}{\lvert\{v1,v2,v4,v5,v6,v8,v9\}\rvert}$ = 3/7, the other modification is over Ivy, but this is negligible, because it is still 0.

Task 3. Create a Python script using NetworkX library, and perform the following tasks:

2. Use the Page Rank centrality function in NetworkX, to print the node with the highest centrality score

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Nodes in order of importance according to PAGERANK algorithm
1- Node: 3437, Centrality Measurement: 0.007614586844749603
```

3. Use the Eigenvector centrality function in NetworkX, to print the centrality score for each node.

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Nodes in order of importance according to EIGENVECTOR algorithm
1- Node: 1912, Centrality Measurement: 0.09540696149067629
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Task 4. Given the friendship social network shown in Figure 3. Implement a script to perform the following tasks:

1. Implement the Adamic Adar similarity measure. Print the similarity score for each pair of nodes. What is the most structurally equivalent pair?

The most structurally equivalent pair according to Adamic-Adar similarity is u2-u3

2. Use the Betweenness centrality function in NetworkX python library, to print the centrality value of each node

The most central node according to Betweenness implementation is u7