

$$x^3 y' = x^4 y^2 - 2x^2 y - 1$$

$$y_1 = x^{-2}$$

$$\text{con } y = y_1 + u^{-1} = x^{-2} + u^{-1}$$

$$y' = -2x^{-3} - u^{-2} u'$$

$$x^3(-2x^{-3} - u^{-2} u') = x^4(x^{-2} + u^{-1})^2 - 2x^2(x^{-2} + u^{-1}) - 1$$

$$-2 - x^3 u^{-2} u' = x^4(x^{-4} + 2x^{-2} u^{-1} + u^{-2}) - 2 - 2x^2 u^{-1} - 1$$

$$\cancel{-2} - x^3 u^{-2} u' = \cancel{1} + \cancel{2x^2 u^{-1}} + x^4 u^{-2} - \cancel{2} - \cancel{2x^2 u^{-1}} - \cancel{1}$$

$$-x^3 u^{-2} u' = x^4 u^{-2}$$

$$u' = -x \rightarrow du = -x dx$$

$$u = -\int x dx = -\frac{x^2}{2} + C$$

$$y = x^{-2} + \left(-\frac{x^2}{2} + C\right)^{-1} = x^{-2} + \frac{1}{-\frac{x^2}{2} + C} = \frac{1}{x^2} + \frac{2}{K - x^2}$$

$$y(\sqrt{2}) = 0 \rightarrow y(\sqrt{2}) = \frac{1}{2} + \frac{2}{K-2} = 0$$

$$\frac{1}{2} = \frac{-2}{K-2} \rightarrow K-2 = -4$$

$$K = -2$$

$$y = \frac{1}{x^2} + \frac{2}{-2-x^2}$$