TAREA CORTA I

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1. $T(n) = 8T(\frac{n}{2}) + \theta(1)$ siendo $T(1) = 1$, recuerde que $a^{\log_b c} = c^{\log_b a}$

- $T(n) = 8T(n/2) + \Theta(1)$ $\quad T(1) = 1$

→ Tomamos $T(n/2)$ como $n = 2^K$

- $T(2^K) = 8T(2^{K-1}) + \Theta(1)$
- $T(2^K) = 8[8T(2^{K-2}) + \Theta(1)] + \Theta(1)$
- $T(2^K) = 8^2 T(2^{K-2}) + 8\Theta(1) + \Theta(1)$
- $T(2^K) = 8^3 T(2^{K-3}) + 8^2\Theta(1) + 8\Theta(1) + \Theta(1)$
- $T(2^K) = 8^i T(2^{K-i}) + 8^i\Theta(1) + 8\Theta(1) + \Theta(1)$
- $T(2^K) = 8^K T(2^{K-K}) + \underbrace{8^K\Theta(1) + 8\Theta(1) + \Theta(1)}$ → Sacamos la sumatoria. $\left[\sum_{i=0}^{K-1} 8^i \Theta(1)\right]$

- $\sum_{i=0}^{K-1} 8^i = \frac{8^{K+1} - 1}{8-1}$

Recordar $T(1) = 1$

- $T(2^K) = 8^K \cdot 1 \left(\frac{8^{K+1} - 1}{8-1}\right) \Theta(1)$

- Como expresamos 8^K en terminos de n usando $8^K = n^3$ porque $K = \log_2(n)$ y $8 = 2^3$

- $T(n) = n^3 + \left(\frac{n^3 - 1}{7}\right) \Theta(1)$

- $T(n) = \Theta(n^3)$ ⟶ Resultado Final

2. $T(n) = T(n-1) + 2^{n-1}$ siendo $T(n) = \theta(1)$ para $n \leq 1$.

- $T(n) = T(n-1) + 2^{n-1}$; $T(n) = \Theta(1)$
- $T(n) = T(n-1) + 2^{n-1}$
- $T(n-1) = T(n-2) + 2^{n-2}$
- $T(n-2) = T(n-3) + 2^{n-3}$
- $T(n) = T(\underbrace{n-i}_{\text{Termina siendo } n-n}) + \underbrace{(2^{n-i} + 2^{n-(i-1)} + 2^{n-(i-2)} + \ldots + 2^{n-1})}$ ⟶ Tambien se puede ver como $2^1 + 2^2 + 2^3 + \ldots + 2^{n-1}$
- $T(n) = T(1) + \underbrace{(2^1 + 2^2 + 2^3 + \ldots + 2^{n-1})}$ → Sacamos la sumatoria. $\sum_{i=1}^{n-1} 2^i = 2 \cdot \frac{2^{n-1} - 1}{2-1} = 2^n - 2$

- $T(n) = T(1) + (2^n - 2)$
- $T(n) = \Theta(1) + (2^n - 2)$
- $T(n) = \Theta(2^n)$ ⟶ Resultado Final.

⑤ $T(n) = 2T(\frac{n}{2} + 17) + n$ siendo $T(1) = 1$

$T(n) = 2T(n/2 + 17) + n$ suponemos $n = 2^K$

- $T(2^K) = 2T(2^K + 17) + 2^K$
- $T(2^K) = 2T(2^{K-1} + 17) + 2^K$
- $T(2^K) = 2[2T(2^{K-2} + 17) + 2^{K-1}] + 2^K$
- $T(2^K) = 2^2T(2^{K-2} + 17) + 2^K + 2^K$
- $T(2^K) = 2^2T(2^{K-2} + 17) + 2 \cdot 2^K$
- $T(2^K) = 2^2[2T(2^{K-3} + 17) + 2^{K-2}] + 2 \cdot 2^K$
- $T(2^K) = 2^3T(2^{K-3} + 17) + 2^K + 2 \cdot 2^K$
- $T(2^K) = 2^3T(2^{K-3} + 17) + 3 \cdot 2^K$

→ Entonces

- $T(2^K) = 2^iT(2^{K-i} + 17) + i \cdot 2^K \longrightarrow$ Finaliza en $i = K$
- $T(2^K) = 2^KT(2^{K-K} + 17) + K \cdot 2^K$

→ Como tenemos $n = 2^K$ y $K = \log n$

- $T(2^K) = 2^{\log n}\, T(18) + \log n \cdot 2^{\log n}$
- $T(n) = n^{\log 2}\, T(18) + \log n \cdot n^{\log 2}$
- $T(n) = \Theta(n^{\log 2} + \log n \cdot n^{\log 2})$
- $T(n) = \Theta(n \log n) \longrightarrow$ Respuesta final.

④ $T(n) = k + n^2 + 5T(\frac{n}{3})$ siendo $T(1) = k$

→ Tomando $T(n/3)$ como $n = 3^K$

- $T(3^K) = K + (3^K)^2 + 5T(3^{K-1})$
- $T(3^K) = K + 9^K + 5(K + 9^{K-1} + 5T(3^{K-2}))$
- $T(3^K) = K + 9^K + 5K + 5 \cdot 9^{K-1} + 25(K + 9^{K-2} + 5T(3^{K-3}))$
- $T(3^K) = K\underbrace{(1 + 5 + 5^2 + \dots + 5^{K-1})}_{\sum_{i=0}^{K-1} 5^i = \frac{5^K - 1}{5 - 1}} + 9^K\underbrace{(1 + 5/9 + (5/9)^2 + \dots + (5/9)^{K-1})}_{\sum_{i=0}^{K-1} (5/9)^i = \frac{1 - (5/9)^K}{1 - 5/9}}$

→ Entonces:

$$T(3^K) = K \cdot \frac{5^{K+1} - 1}{4} + 9^K \cdot \frac{1 - (5/9)^{K+1}}{4/9}$$

→ Recordar que $3^K = n$ y $K = \log_3 n$ entonces:

$$T(n) = \log_3 n \cdot \frac{5^{\log_3 n + 1} - 1}{4} + n^2 \cdot \frac{1 - (5/9)^{\log_3 n + 1}}{4/9}$$

$T(n) = \Theta(n^2) \longrightarrow$ Respuesta Final

⑥ $T(n) = 4T(\frac{n}{2}) + \theta(n)$ siendo $T(1) = 1$

→ Suponemos $n = 2^K$

- $T(2^K) = 4T(2^{K-1}) + \Theta(2^K)$
- $T(2^K) = 4[4T(2^{K-1}) + \Theta(2^{K-1})] + \Theta(2^K)$
- $T(2^K) = 4^2T(2^{K-2}) + 4\Theta(2^{K-1}) + \Theta(2^K)$
- $T(2^K) = 4^3T(2^{K-3}) + 4^2\Theta(2^{K-2}) + 4\Theta(2^{K-1}) + \Theta(2^K)$
- $T(2^K) = 4^iT(2^{K-i}) + 4^{i-1}\Theta(2^{K-(i-1)}) + 4^{i-2}\Theta(2^{K-(i-2)}) + \Theta(2^{K-0})$

→ Esto termina cuando $i = K$

- $T(2^K) = 4^K T(2^{K-K}) + \sum_{i=0}^{K-1} 4^i \Theta(2^{K-i})$
- $T(2^K) = 4^K T(1) + \sum_{i=0}^{K-1} 4^i \Theta(2^{K-i})$

$\Rightarrow 2^{2i} \cdot 2^{K-i}$

$\Rightarrow 2^{2i+(K-i)}$ → $\Rightarrow 2i+(K-i)$

$\Rightarrow 2i+K-i$

$\Rightarrow i+K$

$\Rightarrow 2^{i+K}$

$\Rightarrow 2^i \cdot 2^K$

Procedimientos

→ Simplificar sumatoria

- $\sum_{i=0}^{K-1} 4^i \underline{\Theta(2^{K-i})}$ → $\Theta(2^{K-i}) = 2^{K-i}$

Entonces → $4^i \cdot 2^{K-i} = 2^{K+i}$

Simplificado → $2^{2i} \cdot 2^{K-i} = 2^K \cdot 2^i$

→ Entonces:

$$\sum_{i=0}^{K-1} 2^K \cdot 2^i \Rightarrow 2^K \sum_{i=0}^{K-1} 2^i = 2^K \cdot \left(\frac{2^K - 1}{2-1}\right)$$

→ Continuando

- $T(2^K) = 4^K T(1) + 2^K \cdot \left(\frac{2^K-1}{2-1}\right)$
- $T(2^K) = 4^K + 2^K \cdot (2^K - 1)$

Dado que $n = 2^K$, entonces $K = \log n$

- $T(n) = 4^{\log n} + 2^{\log n}(2^{\log n} - 1)$
- $T(n) = n^{\log 4} + n^{\log 2}(n^{\log 2} - 1)$
- $T(n) = n^2 + (n^2 - n)$
- $T(n) = \Theta(n^2)$ → Respuesta Final