Transformadas de Laplace más comunes

	$f(t)$	$F(s)$
1	Unit impulse $\delta(t)$	1
2	Unit step $1(t)$	$\frac{1}{s}$
3	t	$\frac{1}{s^2}$
4	$\frac{t^{n-1}}{(n-1)!} \quad (n = 1, 2, 3, \dots)$	$\frac{1}{s^n}$
5	$t^n \quad (n = 1, 2, 3, \dots)$	$\frac{n!}{s^{n+1}}$
6	e^{-at}	$\frac{1}{s+a}$
7	te^{-at}	$\frac{1}{(s+a)^2}$
8	$\frac{1}{(n-1)!} t^{n-1} e^{-at} \quad (n = 1, 2, 3, \dots)$	$\frac{1}{(s+a)^n}$
9	$t^n e^{-at} \quad (n = 1, 2, 3, \dots)$	$\frac{n!}{(s+a)^{n+1}}$
10	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
11	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
12	$\sinh \omega t$	$\frac{\omega}{s^2-\omega^2}$
13	$\cosh \omega t$	$\frac{s}{s^2-\omega^2}$
14	$\frac{1}{a}(1 - e^{-at})$	$\frac{1}{s(s+a)}$
15	$\frac{1}{b-a}(e^{-at} - e^{-bt})$	$\frac{1}{(s+a)(s+b)}$
16	$\frac{1}{b-a}(be^{-bt} - ae^{-at})$	$\frac{s}{(s+a)(s+b)}$
17	$\frac{1}{ab}\left[1 + \frac{1}{a-b}(be^{-at} - ae^{-bt})\right]$	$\frac{1}{s(s+a)(s+b)}$
18	$\frac{1}{a^2}(1 - e^{-at} - ate^{-at})$	$\frac{1}{s(s+a)^2}$
19	$\frac{1}{a^2}(at - 1 + e^{-at})$	$\frac{1}{s^2(s+a)}$
20	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
21	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
22	$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1-\zeta^2}\, t \quad (0 < \zeta < 1)$	$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

23	$-\dfrac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t-\phi)$ $\phi=\tan^{-1}\dfrac{\sqrt{1-\zeta^2}}{\zeta}$ $(0<\zeta<1,\quad 0<\phi<\pi/2)$	$\dfrac{s}{s^2+2\zeta\omega_n s+\omega_n^2}$
24	$1-\dfrac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\phi)$ $\phi=\tan^{-1}\dfrac{\sqrt{1-\zeta^2}}{\zeta}$ $(0<\zeta<1,\quad 0<\phi<\pi/2)$	$\dfrac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$
25	$1-\cos\omega t$	$\dfrac{\omega^2}{s(s^2+\omega^2)}$
26	$\omega t-\sin\omega t$	$\dfrac{\omega^3}{s^2(s^2+\omega^2)}$
27	$\sin\omega t-\omega t\cos\omega t$	$\dfrac{2\omega^3}{(s^2+\omega^2)^2}$
28	$\dfrac{1}{2\omega}t\sin\omega t$	$\dfrac{s}{(s^2+\omega^2)^2}$
29	$t\cos\omega t$	$\dfrac{s^2-\omega^2}{(s^2+\omega^2)^2}$
30	$\dfrac{1}{\omega_2^2-\omega_1^2}(\cos\omega_1 t-\cos\omega_2 t)\qquad(\omega_1^2\neq\omega_2^2)$	$\dfrac{s}{(s^2+\omega_1^2)(s^2+\omega_2^2)}$
31	$\dfrac{1}{2\omega}(\sin\omega t+\omega t\cos\omega t)$	$\dfrac{s^2}{(s^2+\omega^2)^2}$

Pulse function $f(t)=\dfrac{A}{t_0}1(t)-\dfrac{A}{t_0}1(t-t_0)$	$\mathcal{L}[f(t)]=\dfrac{A}{t_0 s}-\dfrac{A}{t_0 s}e^{-st_0}$
Impulse function $g(t)=\lim\limits_{t_0\to 0}\dfrac{A}{t_0},\quad$ for $0<t<t_0$ $=0,\quad$ for $t<0,\ t_0<t$	$\mathcal{L}[g(t)]=\lim\limits_{t_0\to 0}\left[\dfrac{A}{t_0 s}(1-e^{-st_0})\right]$ $=\lim\limits_{t_0\to 0}\dfrac{\frac{d}{dt_0}[A(1-e^{-st_0})]}{\frac{d}{dt_0}(t_0 s)}$ $=\dfrac{As}{s}=A$

Resumen de Propiedades de Laplace

1	$\mathscr{L}[Af(t)] = AF(s)$
2	$\mathscr{L}[f_1(t) \pm f_2(t)] = F_1(s) \pm F_2(s)$
3	$\mathscr{L}_\pm\left[\frac{d}{dt}f(t)\right] = sF(s) - f(0\pm)$
4	$\mathscr{L}_\pm\left[\frac{d^2}{dt^2}f(t)\right] = s^2F(s) - sf(0\pm) - \dot{f}(0\pm)$
5	$\mathscr{L}_\pm\left[\frac{d^n}{dt^n}f(t)\right] = s^nF(s) - \sum_{k=1}^{n} s^{n-k}\overset{(k-1)}{f}(0\pm)$ where $\overset{(k-1)}{f}(t) = \frac{d^{k-1}}{dt^{k-1}}f(t)$
6	$\mathscr{L}_\pm\left[\int f(t)\,dt\right] = \frac{F(s)}{s} + \frac{1}{s}\left[\int f(t)\,dt\right]_{t=0\pm}$
7	$\mathscr{L}_\pm\left[\int\cdots\int f(t)(dt)^n\right] = \frac{F(s)}{s^n} + \sum_{k=1}^{n}\frac{1}{s^{n-k+1}}\left[\int\cdots\int f(t)(dt)^k\right]_{t=0\pm}$
8	$\mathscr{L}\left[\int_0^t f(t)\,dt\right] = \frac{F(s)}{s}$
9	$\int_0^\infty f(t)\,dt = \lim_{s\to 0}F(s) \qquad \text{if } \int_0^\infty f(t)\,dt \text{ exists}$
10	$\mathscr{L}[e^{-\alpha t}f(t)] = F(s+\alpha)$
11	$\mathscr{L}[f(t-\alpha)1(t-\alpha)] = e^{-\alpha s}F(s) \qquad \alpha \geq 0$
12	$\mathscr{L}[tf(t)] = -\frac{dF(s)}{ds}$
13	$\mathscr{L}[t^2f(t)] = \frac{d^2}{ds^2}F(s)$
14	$\mathscr{L}[t^nf(t)] = (-1)^n\frac{d^n}{ds^n}F(s) \qquad (n = 1, 2, 3, \dots)$
15	$\mathscr{L}\left[\frac{1}{t}f(t)\right] = \int_s^\infty F(s)\,ds \qquad \text{if } \lim_{t\to 0}\frac{1}{t}f(t) \text{ exists}$
16	$\mathscr{L}\left[f\left(\frac{1}{a}\right)\right] = aF(as)$
17	$\mathscr{L}\left[\int_0^t f_1(t-\tau)f_2(\tau)\,d\tau\right] = F_1(s)F_2(s)$
18	$\mathscr{L}[f(t)g(t)] = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty}F(p)G(s-p)\,dp$

Initial value theorem	$f(0+) = \lim_{t\to 0+}f(t) = \lim_{s\to\infty}sF(s)$
Final value theorem	$f(\infty) = \lim_{t\to\infty}f(t) = \lim_{s\to 0}sF(s)$

Expansiones en fracciones parciales más comunes

(i) Factored roots

$$\frac{K}{s(s+a)} = \frac{A}{s} + \frac{B}{(s+a)}$$

(ii) Repeated roots

$$\frac{K}{s^2(s+a)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{(s+a)}$$

(iii) Second-order real roots ($b^2 > 4ac$)

$$\frac{K}{s(as^2+bs+c)} = \frac{K}{s(s+d)(s+e)} = \frac{A}{s} + \frac{B}{(s+d)} + \frac{C}{(s+\mathrm{e})}$$

(iv) Second-order complex roots ($b^2 < 4ac$)

$$\frac{K}{s(as^2+bs+c)} = \frac{A}{s} + \frac{Bs+C}{as^2+bs+c}$$

Completing the square gives

$$\frac{A}{s} + \frac{Bs+C}{(s+\alpha)^2+\omega^2}\ ; \text{ being } s_{1,2} = -\alpha \pm j\,\omega$$

Note: In (iii) and (iv) the coefficient a is usually factored to a unity value.