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Tabla A-1. Parejas de la transformada de Laplace.

	$f(t)$	$F(s)$
1	Unidad de impulso $\delta(t)$	1
2	Unidad de paso $1(t)$	$\frac{1}{s}$
3	t	$\frac{1}{s^2}$
4	$\frac{t^{n-1}}{(n-1)!} \quad (n = 1, 2, 3, \ldots)$	$\frac{1}{s^n}$
5	$t^n \quad (n = 1, 2, 3, \ldots)$	$\frac{n!}{s^{n+1}}$
6	e^{-at}	$\frac{1}{s+a}$
7	te^{-at}	$\frac{1}{(s+a)^2}$
8	$\frac{1}{(n-1)!} t^{n-1} e^{-at} \quad (n = 1, 2, 3, \ldots)$	$\frac{1}{(s+a)^n}$
9	$t^n e^{-at} \quad (n = 1, 2, 3, \ldots)$	$\frac{n!}{(s+a)^{n+1}}$
10	$\operatorname{sen} \omega \mathrm{t}$	$\frac{\omega}{\mathrm{s}^2 + \omega^2}$
11	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
12	$\operatorname{senh} \omega t$	$\frac{\omega}{s^2 - \omega^2}$
13	$\cosh \omega t$	$\frac{s}{s^2 - \omega 2}$
14	$\frac{1}{a}(1 - e^{-at)}$	$\frac{1}{s(s+a)}$
15	$\frac{1}{b-a}(e^{-at} - e^{-bt})$	$\frac{1}{(s+a)(s+b)}$
16	$\frac{1}{b-a}(be^{-bt} - ae^{-at})$	$\frac{s}{(s+a)(s+b)}$
17	$\frac{1}{ab}\left[1 + \frac{1}{a-b}(be^{-at} - ae^{-bt})\right]$	$\frac{1}{s(s+a)(s+b)}$

(*continúa*)

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Tabla A-1. (*Continuación*).

	$f(t)$	$F(s)$
18	$\frac{1}{a^2}(1 - e^{-at} - ate^{-at})$	$\frac{1}{s(s+a)^2}$
19	$\frac{1}{a^2}(at - 1 + e^{-at})$	$\frac{1}{s^2(s+a)}$
20	$e^{-at}\,\mathrm{sen}\,\omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
21	$e^{-at}\cos\omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
22	$\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\,\mathrm{sen}\,\omega_n\sqrt{1-\zeta^2}\,t \quad (0<\zeta<1)$	$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$
23	$-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\,\mathrm{sen}\,(\omega_n\sqrt{1-\zeta^2}\,t-\phi)$ $\phi = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$ $(0<\zeta<1,\ 0<\phi<\pi/2)$	$\frac{s}{s^2+2\zeta\omega_n s+\omega_n^2}$
24	$-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\,\mathrm{sen}\,(\omega_n\sqrt{1-\zeta^2}\,t+\phi)$ $\phi = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$ $(0<\zeta<1,\ 0<\phi<\pi/2)$	$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$
25	$1-\cos\omega t$	$\frac{\omega^2}{s(s^2+\omega^2)}$
26	$\omega t - \mathrm{sen}\,\omega t$	$\frac{\omega^3}{s^2(s^2+\omega^2)}$
27	$\mathrm{sen}\,\omega t - \omega t\cos\omega t$	$\frac{2\omega^3}{(s^2+\omega^2)^2}$
28	$\frac{1}{2\omega}t\,\mathrm{sen}\,\omega t$	$\frac{s}{(s^2+\omega^2)^2}$
29	$t\cos\omega t$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
30	$\frac{1}{\omega_2^2-\omega_1^2}(\cos\omega_1 t - \cos\omega_2 t) \qquad (\omega_1^2 \neq \omega_2^2)$	$\frac{s}{(s^2+\omega_1^2)(s^2+\omega_2^2)}$
31	$\frac{1}{2\omega}(\mathrm{sen}\,\omega t + \omega t\cos\omega t)$	$\frac{s^2}{(s^2+\omega^2)^2}$

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Tabla A-2. Propiedades de la transformada de Laplace.

1	$\mathcal{L}[A f(t)] = AF(s)$
2	$\mathcal{L}[f_1(t) \pm f_2(t)] = F_1(s) \pm F_2(s)$
3	$\mathcal{L}_{\pm}\left[\frac{d}{dt} f(t)\right] = sF(s) - f(0\pm)$
4	$\mathcal{L}_{\pm}\left[\frac{d^2}{dt^2} f(t)\right] = s^2F(s) - sf(0\pm) - \dot{f}(0\pm)$
5	$\mathcal{L}_{\pm}\left[\frac{d^n}{dt^n} f(t)\right] = s^nF(s) - \sum_{k=1}^{n} s^{n-k} \overset{(k-1)}{f}(0\pm)$ donde $\overset{(k-1)}{f(t)} = \frac{d^{k-1}}{dt^{k-1}} f(t)$
6	$\mathcal{L}_{\pm}\left[\int f(t)\,dt\right] = \frac{F(s)}{s} + \frac{1}{s}\left[\int f(t)\,dt\right]_{t=0\pm}$
7	$\mathcal{L}_{\pm\ \pm}\left[\int \cdots \int f(t)\,(dt)^n\right] = \frac{F(s)}{s^n} + \sum_{k=1}^{n} \frac{1}{s^{n-k+1}}\left[\int \cdots \int f(t)\,(dt)^k\right]_{t=0\pm}$
8	$\mathcal{L}\left[\int_0^t f(t)\,dt\right] = \frac{F(s)}{s}$
9	$\int_0^{\infty} f(t)\,dt = \lim_{s\to 0} F(s)$ si $\int_0^{\infty} f(t)\,dt$ salidas
10	$\mathcal{L}[e^{-\alpha t} f(t)] = F(s + a)$
11	$\mathcal{L}[f(t-\alpha)1(t-\alpha)] = e^{-\alpha s} F(s) \qquad a \geqslant 0$
12	$\mathcal{L}[t f(t)] = -\frac{dF(s)}{ds}$
13	$\mathcal{L}[t^2 f(t)] = \frac{d^2}{ds^2} F(s)$
14	$\mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} F(s) \qquad (n = 1, 2, 3, ...)$
15	$\mathcal{L}\left[\frac{1}{t} f(t)\right] = \int_s^{\infty} F(s)\,ds$ si $\lim_{t\to 0} \frac{1}{t} f(t)$ salidas
16	$\mathcal{L}\left[f\left(\frac{1}{a}\right)\right] = aF(as)$
17	$\mathcal{L}\left[\int_0^t f_1(t-\tau) f_2(\tau)\,d\tau\right] = F_1(s)F_2(s)$
18	$\mathcal{L}[f(t)g(t)] = \frac{1}{2\pi j}\int_{c-j\omega}^{c+j\infty} F(p)G(s-p)\,dp$

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Por último se presentan dos teoremas frecuentemente utilizados junto con las transformadas de Laplace de la función pulso y de la función impulso.

Teorema de valor inicial	$f(0+) = \lim_{t \to 0+} f(t) = \lim_{s \to \infty} sF(s)$
Teorema de valor final	$f(\infty) = \lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$
Función pulso $f(t) = \dfrac{A}{t_0} 1(t) - \dfrac{A}{t_0} 1(t - t_0)$	$\mathscr{L}[f(t)] = \dfrac{A}{t_0 s} - \dfrac{A}{t_0 s} e^{-st_0}$
Función impulso $g(t) = \lim_{t_0 \to 0} \dfrac{A}{t_0}, \quad \text{para } 0 < t < t_0$ $\quad = 0, \quad \text{para } t < 0,\ t_0 < t$	$\mathscr{L}[g(t)] = \lim_{t_0 \to 0} \left[\dfrac{A}{t_0 s} (1 - e^{-st_0}) \right]$ $= \lim_{t_0 \to 0} \dfrac{\dfrac{d}{dt_0} [A(1 - e^{-st_0})]}{\dfrac{d}{dt_0} (t_0 s)}$ $= \dfrac{As}{s} = A$

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