Transformadas de Laplace:

1. Definir la transformada de Laplace de una función ***f(t)***. ¿Que requisitos debe cumplir esta función?
2. Calcular la transformada de Laplace de la función escalón, del impulso unitario, y ***f(t)=t***.
3. Demostrar el Teorema del Valor Final.
4. Demostrar el Teorema del Valor Inicial.
5. Demostrar las propiedades de las transformadas de Laplace dadas en clase.
6. Hallar las transformadas inversas de Laplace de las siguientes Funciones:

7. $F(s) = \dfrac{1}{s(s+1)}$

8. $F(s) = \dfrac{s+1}{(s+2)(s+3)}$

9. $F(s) = \dfrac{(s+3)(s+4)(s+5)}{(s+1)(s+2)}$

10. $F(s) = \dfrac{s}{(s+1)(s+1)(s+2)}$

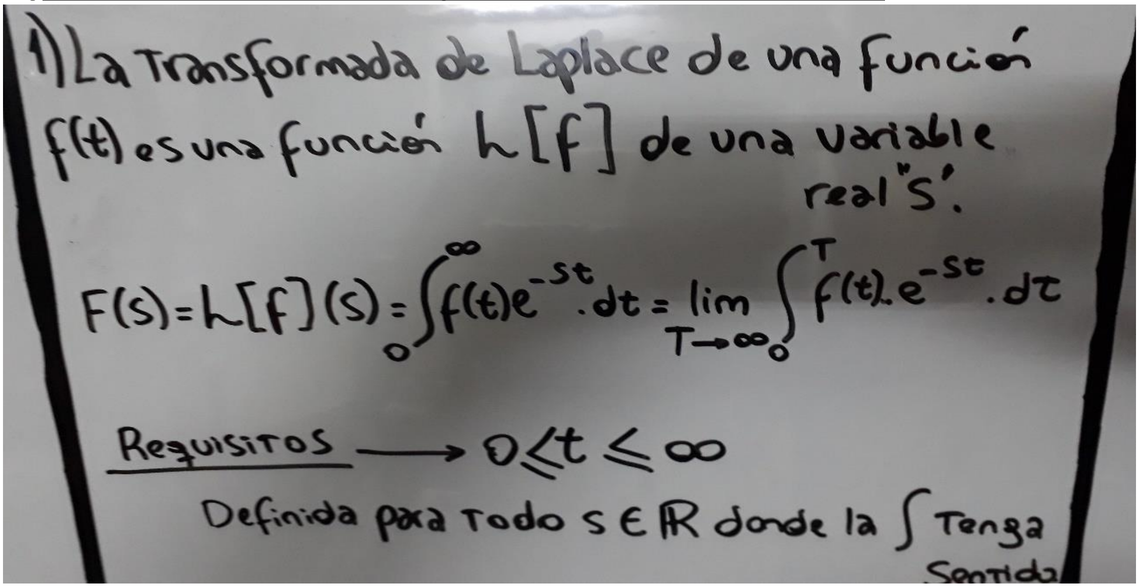
11. Utilizando el método de la Transformada de Laplace (T.L.), resolver las siguientes ecuaciones diferenciales lineales, verificar los resultados mediante la implementación de soluciones por el método de diferencias finitas (DF) y graficar:

$$\ddot{Y}(t) + 3\dot{Y}(t) + 6Y(t) = 0 ----------------C.I.---\dot{Y}(0) = 0; Y(0) = 3$$
$$m\ddot{Y}(t) + kY(t) = \delta(t) -----------------C.I.---\dot{Y}(0) = 0; Y(0) = 0$$
$$2\ddot{Y}(t) + 7\dot{Y}(t) + 3Y(t) = 0 ---------------C.I.---\dot{Y}(0) = 1; Y(0) = 0$$

12. Utilizando el TVF, encontrar el valor final de ***f(t)*** cuya transformada de Laplace es:

$$F(s) = \frac{10}{s(s+1)}$$

1)Transformada de Laplace de la función f(t)



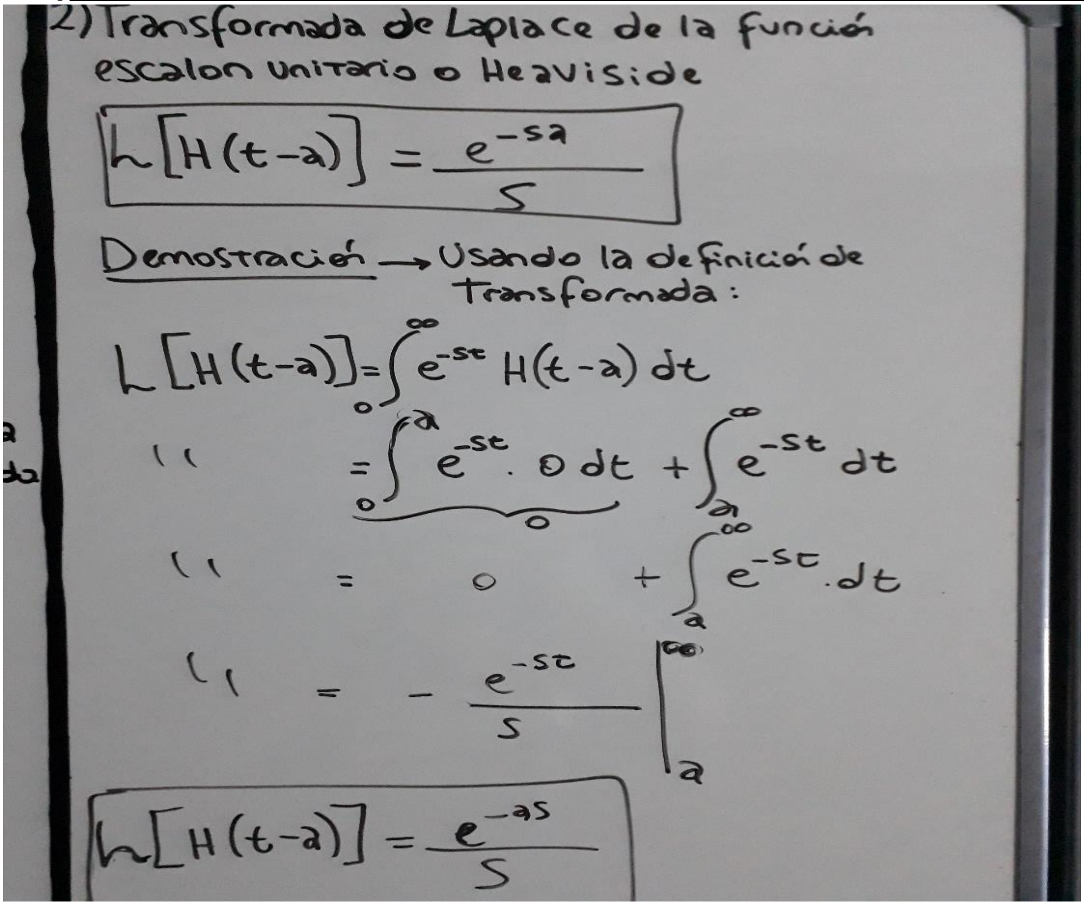
1) La Transformada de Laplace de una función f(t) es una función $\mathcal{L}[f]$ de una variable real "s".

$$F(s)=\mathcal{L}[f](s)=\int_0^{\infty} f(t)e^{-st}.dt=\lim_{T\to\infty}\int_0^{T} f(t).e^{-st}.dt$$

Requisitos $\longrightarrow 0\leq t\leq\infty$

Definida para Todo $s\in\mathbb{R}$ donde la $\int$ Tenga Sentido

2A)Transformada de Laplace de la Funcion Escalon Unitario



2) Transformada de Laplace de la función escalon unitario o Heaviside

$$\boxed{\mathcal{L}[H(t-a)]=\frac{e^{-sa}}{s}}$$

Demostración $\to$ Usando la definición de transformada:

$$\mathcal{L}[H(t-a)]=\int_0^{\infty} e^{-st}H(t-a)\,dt$$

$$\text{''}\quad=\underbrace{\int_0^{a} e^{-st}.0\,dt}_{0}+\int_a^{\infty} e^{-st}dt$$

$$\text{''}\quad=0+\int_a^{\infty} e^{-st}.dt$$

$$\text{''}\quad=-\frac{e^{-st}}{s}\Bigg|_a^{\infty}$$

$$\boxed{\mathcal{L}[H(t-a)]=\frac{e^{-as}}{s}}$$

2B)Transformada de Laplace de la función Impulso Unitario

2.b) Transformada de Laplace del impulso Unitario

– En la práctica se aplica otro tipo de impulso llamado la función de Dirac.

Función delta de Dirac

$$\lim_{a \to 0} \delta_a(t-t_0) = \delta(t-t_0).$$

Propiedad

$$\delta(t-t_0) = \begin{cases} \infty & \text{si } t = t_0 \\ 0 & \text{si } t \neq t_0 \end{cases}$$

$$\int_0^{\infty} \delta(t-t_0)\,dt = 1$$

Transformada de la función Delta

Para $t_0 > 0$

$$\boxed{\mathcal{L}[\delta(t-t_0)] = e^{-St_0}}$$

Demostración

Para iniciar la prueba debemos escribir la función impulso unitario en Términos de la función escalon unitario

$$\delta_a(t-t_0)=\frac{1}{2a}\left(H(t-(t_0-a))-H(t-(t_0+a))\right).$$

De donde tenemos que:

$$\mathcal{L}[\delta_a(t-t_0)]=\frac{1}{2a}\mathcal{L}[H(t-(t_0-a))]-\frac{1}{2a}\mathcal{L}[H(t-(t_0+a))]$$

$$\mathcal{L}[\delta_a(t-t_0)]=\frac{1}{2a}\left(\frac{e^{-s(t_0-a)}}{s}\right)-\frac{1}{2a}\left(\frac{e^{-s(t_0+a)}}{s}\right)$$

$$\mathcal{L}[\delta_a(t-t_0)]=e^{-st_0}\left(\frac{e^{sa}-e^{-sa}}{2sa}\right)$$

Con lo cuál

$$\mathcal{L}[\delta(t-t_0)]=\mathcal{L}\left[\lim_{a\to 0}\delta_a(t-t_0)\right]$$

$$\mathcal{L}[\delta(t-t_0)]=\lim_{a\to 0}\mathcal{L}[\delta_a(t-t_0)]$$

$$\mathcal{L}[\delta(t-t_0)]=\lim_{a\to 0}e^{-st_0}\underbrace{\left(\frac{e^{sa}-e^{-sa}}{2sa}\right)}_{\text{L'Hopital}}$$

$$\mathcal{L}[\delta(t-t_0)]=e^{-st_0}\lim_{a\to 0}\frac{se^{sa}+se^{-sa}}{2s}$$

$$\boxed{\mathcal{L}[\delta(t-t_0)] = e^{-st_0}}$$

$$\mathcal{L}[\delta(t-t_0)] = e^{-st_0}$$

Se concluye

$$\mathcal{L}[\delta(t)] = 1$$

$\delta(t)$ = No es una funció Ordinaria

Se espera que

$\mathcal{L}[f(t)]$ cuando $s \longrightarrow \infty$

2C)Transformada de Laplace de la función f(t)=t

2.C) Transformada de Laplace de la funció $f(t) = t$

(Integración por partes)

$$\mathcal{L}[t] = \int_0^{\infty} t e^{-st} \cdot dt$$

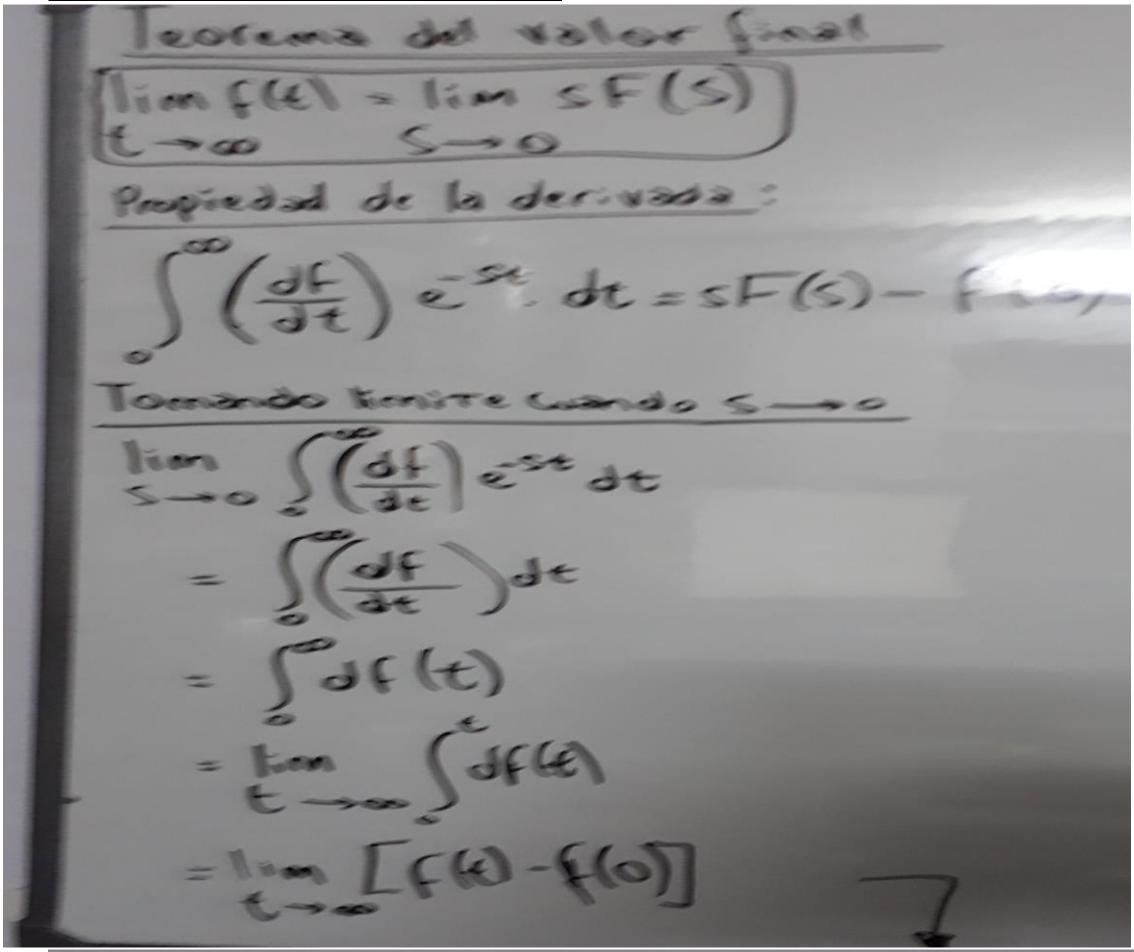
$$\mathcal{L}[t] = -\frac{t e^{-st}}{s}\Big|_0^{\infty} - \int_0^{\infty} \left(-\frac{e^{-st}}{s}\right) dt$$

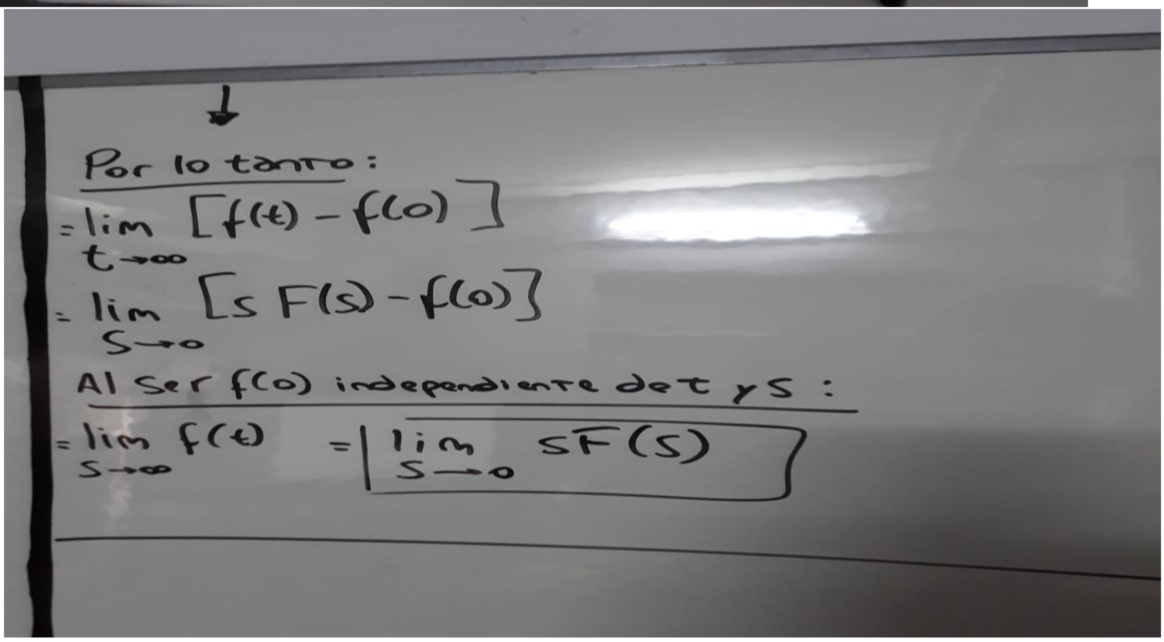
$$\mathcal{L}[t] = -\frac{t e^{-st}}{s}\Big|_0^{\infty} + \frac{1}{s}\int_0^{\infty} e^{-st} \cdot dt$$

Donde

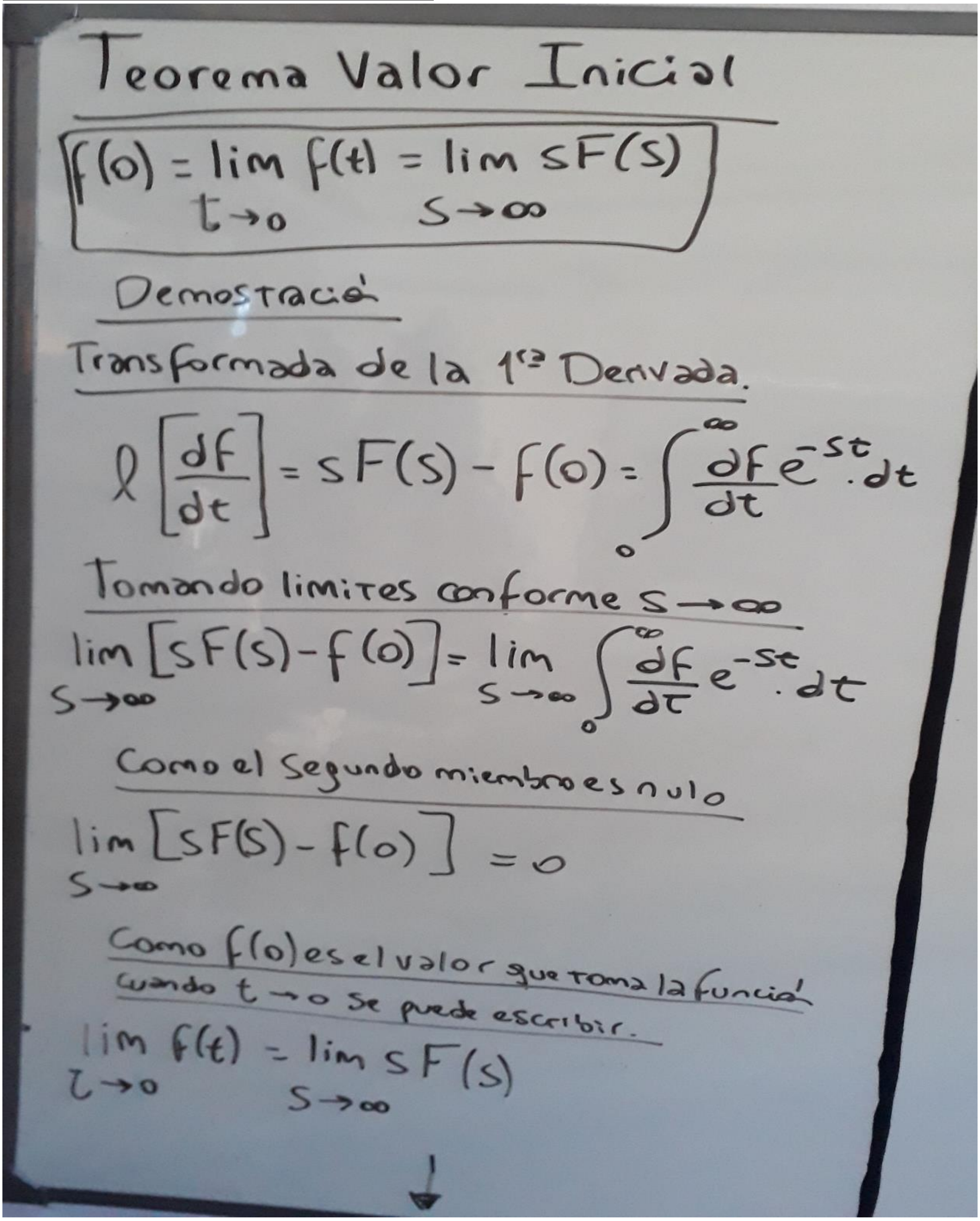
$-\frac{t e^{-st}}{s}\Big|_0^{\infty}$ indica $\lim_{a \to \infty}\left(-\frac{t e^{-st}}{s}\Big|_0^{a}\right)$

3)Teorema del Valor Final





4)Teorema del Valor Inicial



Teorema Valor Inicial

$$\boxed{f(0) = \lim_{t \to 0} f(t) = \lim_{s \to \infty} sF(s)}$$

Demostración

Transformada de la 1ra Derivada.

$$\mathcal{L}\left[\frac{df}{dt}\right] = sF(s) - f(0) = \int_0^{\infty} \frac{df}{dt} e^{-st} \cdot dt$$

Tomando limites conforme $s \to \infty$

$$\lim_{s \to \infty} [sF(s) - f(0)] = \lim_{s \to \infty} \int_0^{\infty} \frac{df}{dt} e^{-st} \cdot dt$$

Como el segundo miembro es nulo

$$\lim_{s \to \infty} [sF(s) - f(0)] = 0$$

Como $f(0)$ es el valor que toma la función cuando $t \to 0$ se puede escribir.

$$\lim_{t \to 0} f(t) = \lim_{s \to \infty} sF(s)$$

Por ejemplo, sea:

$$F(s) = \frac{-2s^3 + 7s^2 + 2s + 9}{3s^4 + 3s^3 - 2s^2 + 6}$$

Donde:

$$f(0) = \lim_{s \to \infty} sF(s) = \lim_{s \to \infty} s\left(\frac{-2s^3}{3s^4}\right) = \boxed{\frac{-2}{3}}$$

5)Propiedades de las Transf. de Laplace vistas

(5) $$\boxed{\mathcal{L}[C f(t)] = C. F(s)}$$

Demostración

$$\mathcal{L}[c f(t)] = \int_0^{\infty} c f(t). e^{-st}. dt$$

$$= C \int_0^{\infty} f(t). e^{-st}. dt$$

$$= \boxed{c F(s)}$$

$$\boxed{\mathcal{L}[C_1 f(t) \pm C_2 g(t)] = C_1 F(s) \pm C_2 G(s).}$$

Demostración

$$\mathcal{L}[c_1 f(t) \pm C_2 g(t)] = \int_0^{\infty} [C_1 f(t). C_2 g(t)]. e^{-st}. dt$$

$$= \int_0^{\infty} c_1 f(t) dt \pm \int_0^{\infty} C_2 g(t) dt.$$

$$= C_1\int_0^\infty f(t)\,dt \pm C_2\int_0^\infty g(t)\,dt$$

$$= \boxed{C_1F(s) \pm C_2G(s)}$$

Derivada de una función

$$\boxed{\mathcal{L}[f'(t)] = s\mathcal{L}[f(t)] - f(0)}$$

Demostración

$$\mathcal{L}[f'(t)] = \int_0^\infty f'(t).e^{-st}.dt$$

$$= u.v\Big|_0^\infty - \int_0^\infty v.du$$

$$= e^{-st}\Big|_0^\infty - \int_0^\infty f(t).(-se^{-st}.dt)$$

$$= \lim_{t\to\infty} e^{-st}f(t) - e^{-s(0)}f(0) + s\int_0^\infty f(t)\,e^{-st}.dt$$

Integramos por partes

$u = e^{-st}$

$du = -se^{-st}.dt$

$dv = f'(t).dt$

$dv = \int f'(t).dt$

$v = f(t)$

$$= \underbrace{\cancel{\lim_{t\to\infty} e^{-st} f(t)}}_{0} - \underbrace{\underbrace{e^{-s(0)}}_{1} f(0)}_{-f(0)} + s\int_0^\infty f(t)\, e^{-st}.dt.$$

$$= -f(0) + s\int_0^\infty f(t)\, e^{-st}.dt$$

$$\boxed{-f(0) + s\,\mathcal{L}[f(t)]}$$ ✓

Segunda Derivada de una función

$$\boxed{\mathcal{L}[f''(t)] = s^2\mathcal{L}[f(t)] - s\,\mathcal{L}[f(t)] - f(0)}$$

<u>Demostración</u>

→ Mismo procedimiento del de Derivada de una función

7)AntiTransformadas(Metodo Fracciones Parciales)

7) $F(s)=\frac{1}{s(s+1)} \longrightarrow \mathcal{L}^{-1}[F(s)]$

$$\frac{A}{s}+\frac{B}{s+1}$$

$$\frac{1}{s(s+1)}=\frac{A}{s}+\frac{B}{s+1}$$

$$1=s(s+1)\left(\frac{A}{s}+\frac{B}{s+1}\right)$$

$$1=\frac{\cancel{s}(s+1)A}{\cancel{s}}+\frac{s\cancel{(s+1)}B}{\cancel{s+1}}$$

$$1=(s+1)A+s.B$$

Tenemos una ecuación con 2 incognitas, entonces $s=1 \wedge s=-1$

$\boxed{s=1}$

$1=((1)+1)A+(1)B$

$\boxed{1=2A+B}$

$1=2A-1$

$\frac{2}{2}=A \rightarrow \boxed{A=1}$

$\boxed{s=-1}$

$1=(\cancel{(-1)+1})A+(-1)B$

$1=-B \longrightarrow \boxed{B=-1}$

Tenemos $\rightarrow A=1 \wedge B=-1$

$$\frac{A}{s}+\frac{B}{s+1} \longrightarrow \frac{1}{s}+\frac{-1}{s+1}$$

$$\mathcal{L}^{-1}\left[\frac{1}{s}+\frac{-1}{s+1}\right]$$

$$\mathcal{L}^{-1}\left[\frac{1}{s}\right]+\mathcal{L}^{-1}\left[\frac{-1}{s+1}\right]$$

$$\mathcal{L}^{-1}\left[\frac{1}{s}\right]-\mathcal{L}^{-1}\left[\frac{1}{s+1}\right]$$

$$\boxed{1-e^{-1t}}\ \checkmark$$

$$\mathcal{L}^{-1}\left[\frac{1}{s(s+1)}\right]$$

Formulas

$\mathcal{L}^{-1}\left[\frac{a}{s}\right]=a$

$\mathcal{L}^{-1}\left[\frac{1}{s-a}\right]=e^{at}$

8)AntiTranformadas(Metodo Fracciones Parciales)

8) $\boxed{F(S)=\frac{S+1}{(S+2)(S+3)}} \rightarrow \mathcal{L}^{-1}[F(S)]$

$=\frac{A}{S+2}+\frac{B}{S+3}$ //Aplicamos fracciones parciales

$\frac{S+1}{(S+2)(S+3)}=\frac{A}{S+2}+\frac{B}{S+3}$ //Igualamos con la ecuación y hallamos A y B

$S+1=(S+2)(S+3)\left(\frac{A}{S+2}+\frac{B}{S+3}\right)$

$S+1=\frac{(\cancel{S+2})(S+3)A}{\cancel{S+2}}+\frac{(S+2)(\cancel{S+3})B}{\cancel{S+3}}$

$S+1=(S+3)A+(S+2)B$ //Le damos valores convenientes a S

$S=0$

$(0)+1=((0)+3)A+((0)+2)B$

$1=3A+2B$ //Como tenemos una ec. con 2 incógnitas, planteamos lo siguiente. $S=1 \wedge S=-1$

$\boxed{S=1}$

$1+1=(1+3)A+(1+2)B$

$2=4A+3B$

$2=4\left(-\frac{B}{2}\right)+3B$

$2=-2B+3B$

$\boxed{2=B}$

$\boxed{S=-1}$

$\cancel{(-1)+1}=((-1)+3)A+((-1)+2)B$

$0=2A+1B$

$\boxed{-\frac{B}{2}=A}$

$-\frac{2}{2}=A \rightarrow \boxed{A=-1}$

Tenemos $\rightarrow B=2 \wedge A=-1$

$=\frac{A}{S+2}+\frac{B}{S+3}$ //Sustituimos

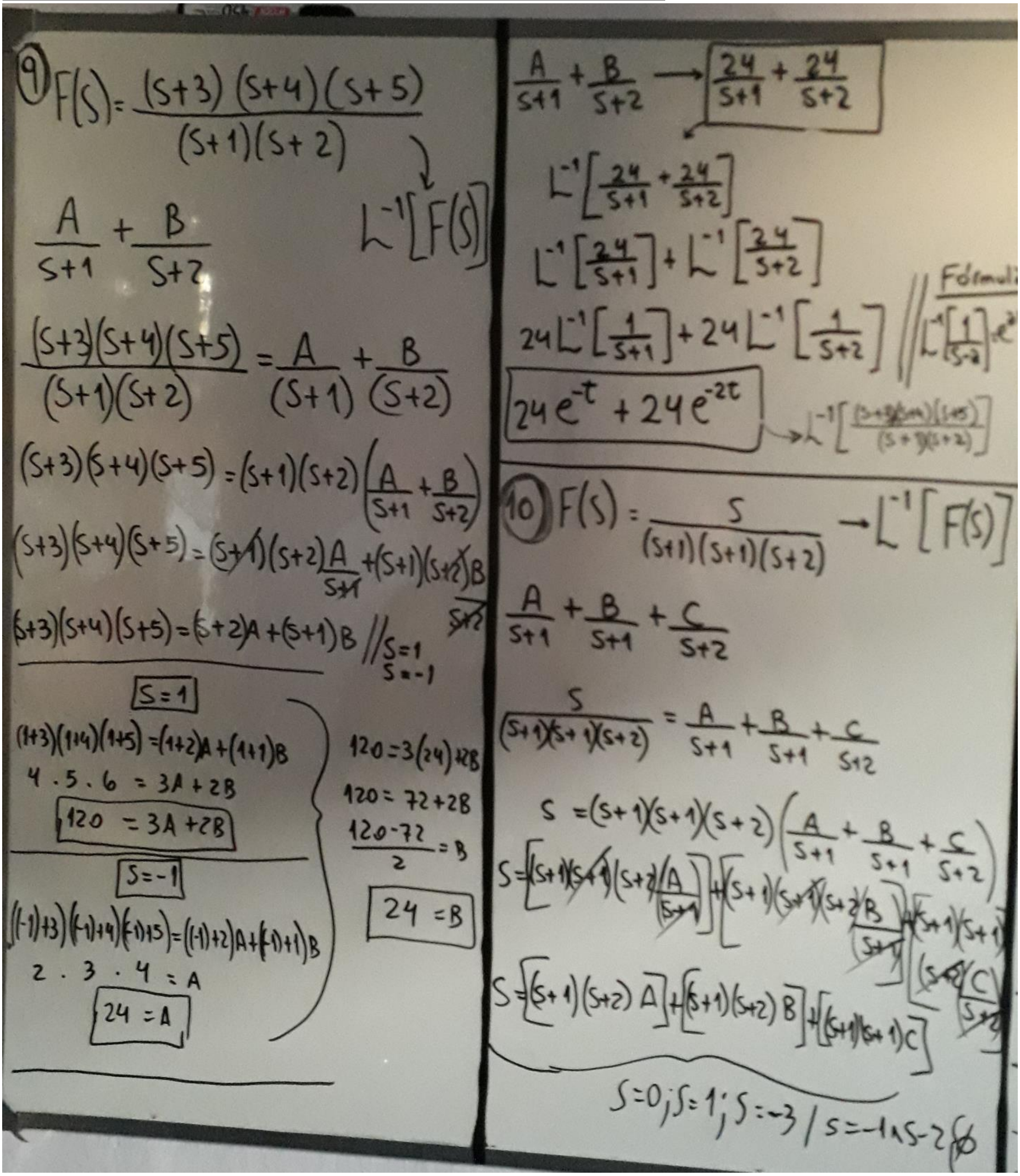
$=\frac{-1}{S+2}+\frac{2}{S+3}$ //Aplicamos $\mathcal{L}^{-1}$ a c/u de los términos

$=\mathcal{L}^{-1}\left[-\frac{1}{S+2}\right]+\mathcal{L}^{-1}\left[\frac{2}{S+3}\right]$ //Sacamos las C de la antitransformada

$=-\mathcal{L}^{-1}\left[\frac{1}{S+2}\right]+2\mathcal{L}^{-1}\left[\frac{1}{S+3}\right]$ //Fórmula $\mathcal{L}^{-1}\left[\frac{1}{S-a}\right]=e^{at}$

$=\boxed{-e^{2t}+2e^{-3t}} \rightarrow \mathcal{L}^{-1}\left[\frac{S+1}{(S+2)(S+3)}\right]$

9/10)Antitransformadas(Fracciones Parciales)



9) $F(s) = \dfrac{(s+3)(s+4)(s+5)}{(s+1)(s+2)} \rightarrow L^{-1}[F(s)]$

$\dfrac{A}{s+1} + \dfrac{B}{s+2}$

$\dfrac{(s+3)(s+4)(s+5)}{(s+1)(s+2)} = \dfrac{A}{(s+1)} + \dfrac{B}{(s+2)}$

$(s+3)(s+4)(s+5) = (s+1)(s+2)\left(\dfrac{A}{s+1} + \dfrac{B}{s+2}\right)$

$(s+3)(s+4)(s+5) = \cancel{(s+1)}(s+2)\dfrac{A}{\cancel{s+1}} + (s+1)\cancel{(s+2)}\dfrac{B}{\cancel{s+2}}$

$(s+3)(s+4)(s+5) = (s+2)A + (s+1)B \quad // \; s=1,\; s=-1$

$\boxed{s=1}$

$(1+3)(1+4)(1+5) = (1+2)A + (1+1)B$

$4 \cdot 5 \cdot 6 = 3A + 2B$

$\boxed{120 = 3A + 2B}$

$\boxed{s=-1}$

$((-1)+3)((-1)+4)((-1)+5) = ((-1)+2)A + ((-1)+1)B$

$2 \cdot 3 \cdot 4 = A$

$\boxed{24 = A}$

$120 = 3(24) + 2B$

$120 = 72 + 2B$

$\dfrac{120-72}{2} = B$

$\boxed{24 = B}$

$\dfrac{A}{s+1} + \dfrac{B}{s+2} \rightarrow \boxed{\dfrac{24}{s+1} + \dfrac{24}{s+2}}$

$L^{-1}\left[\dfrac{24}{s+1} + \dfrac{24}{s+2}\right]$

$L^{-1}\left[\dfrac{24}{s+1}\right] + L^{-1}\left[\dfrac{24}{s+2}\right]$

$24 L^{-1}\left[\dfrac{1}{s+1}\right] + 24 L^{-1}\left[\dfrac{1}{s+2}\right]$ // Fórmula $L^{-1}\left[\dfrac{1}{s-a}\right] e^{at}$

$\boxed{24e^{-t} + 24e^{-2t}} \rightarrow L^{-1}\left[\dfrac{(s+3)(s+4)(s+5)}{(s+1)(s+2)}\right]$

10) $F(s) = \dfrac{s}{(s+1)(s+1)(s+2)} \rightarrow L^{-1}[F(s)]$

$\dfrac{A}{s+1} + \dfrac{B}{s+1} + \dfrac{C}{s+2}$

$\dfrac{s}{(s+1)(s+1)(s+2)} = \dfrac{A}{s+1} + \dfrac{B}{s+1} + \dfrac{C}{s+2}$

$s = (s+1)(s+1)(s+2)\left(\dfrac{A}{s+1} + \dfrac{B}{s+1} + \dfrac{C}{s+2}\right)$

$s = \left[(s+1)\cancel{(s+1)}(s+2)\dfrac{A}{\cancel{s+1}}\right] + \left[(s+1)\cancel{(s+1)}(s+2)\dfrac{B}{\cancel{s+1}}\right] + \left[(s+1)(s+1)\cancel{(s+2)}\dfrac{C}{\cancel{s+2}}\right]$

$s = [(s+1)(s+2)A] + [(s+1)(s+2)B] + [(s+1)(s+1)C]$

$s=0;\; s=1;\; s=-3 \;/\; s=-1 \wedge s=-2$

$\boxed{S=0}$

$$0=[(0+1)(0+2)A]+[(0+1)(0+2)B]+[(0+1)(0+1)C]$$

$$0=(1.2.A)+(1.2.B)+(1.1.C)$$

$$0=2A+2B+C$$

$$\boxed{-2A-2B=C}$$

$\boxed{S=1}$

$$1=[(1+1)(1+2)A]+[(1+1)(1+2)B]+[(1+1)(1+1)C]$$

$$1=(2.3.A)+(2.3.B)+(2.2.C)$$

$$1=6A+6B+4C. \quad // \text{Sustituimos } C.$$

$$1=6A+6B+4(-2A-2B)$$

$$1=6A+6B-8A-8B$$

$$1=-2A-2B \longrightarrow \boxed{\frac{1+2A}{-2}=B}$$

$\boxed{S=-3}$

$$-1=[((-3)+1)((-3)+2)A]+[((-3)+1)((-3)+2)B]+[((-3)+1).((-3)+1)C]$$

$$-1=(-2.-1.A)+(-2.-1.B)+(-2-2.C)$$

$$-1=2A+2B+4C \quad // \text{Sustituimos } B \text{ y } C$$

$$-1=2A+2\left(\frac{1+2A}{-2}\right)+4\left(-2A-\cancel{2}\left(\frac{1+2A}{\cancel{-2}}\right)\right)$$

$$-1=2A+\cancel{2}\left(-\frac{(1+2A)}{\cancel{2}}\right)+4(-2A+1+A)$$

$$-1=2A-1-A+4(-2A+1+A) \qquad \frac{-1-3}{3}=A$$

$$-1=A-1-8A+4+4A \longrightarrow \boxed{-4/3=A}$$

$$\boxed{A=-4/3} \longrightarrow \frac{1+2A}{-2}=B$$

$$\frac{1+2\left(-\frac{4}{3}\right)}{-2}=B$$

$$\frac{1-\frac{8}{3}}{-2}=B \rightarrow \frac{-\frac{5}{3}}{-2}=B \longrightarrow \boxed{\frac{5}{6}=B}$$

$$C=-2A-2B$$

$$C=-2\left(-\frac{4}{3}\right)-2\left(\frac{5}{6}\right)$$

$$C=+\frac{8}{3}-\frac{10}{6}$$

$$\boxed{C=1}$$

$$\boxed{\begin{array}{l} A=-4/3 \\ B=5/6 \\ C=1 \end{array}}$$

$$\frac{A}{s+1}+\frac{B}{s+1}+\frac{C}{s+2} \longrightarrow \frac{-4/3}{s+1}+\frac{5/6}{s+1}+\frac{1}{s+2}$$

$$L^{-1}\left[\frac{-4/3}{s+1}\right]+L^{-1}\left[\frac{5/6}{s+1}\right]+L^{-1}\left[\frac{1}{s+2}\right]$$

$$-\frac{4}{3}L^{-1}\left[\frac{1}{s+1}\right]+\frac{5}{6}L^{-1}\left[\frac{1}{s+1}\right]+L^{-1}\left[\frac{1}{s+2}\right]$$

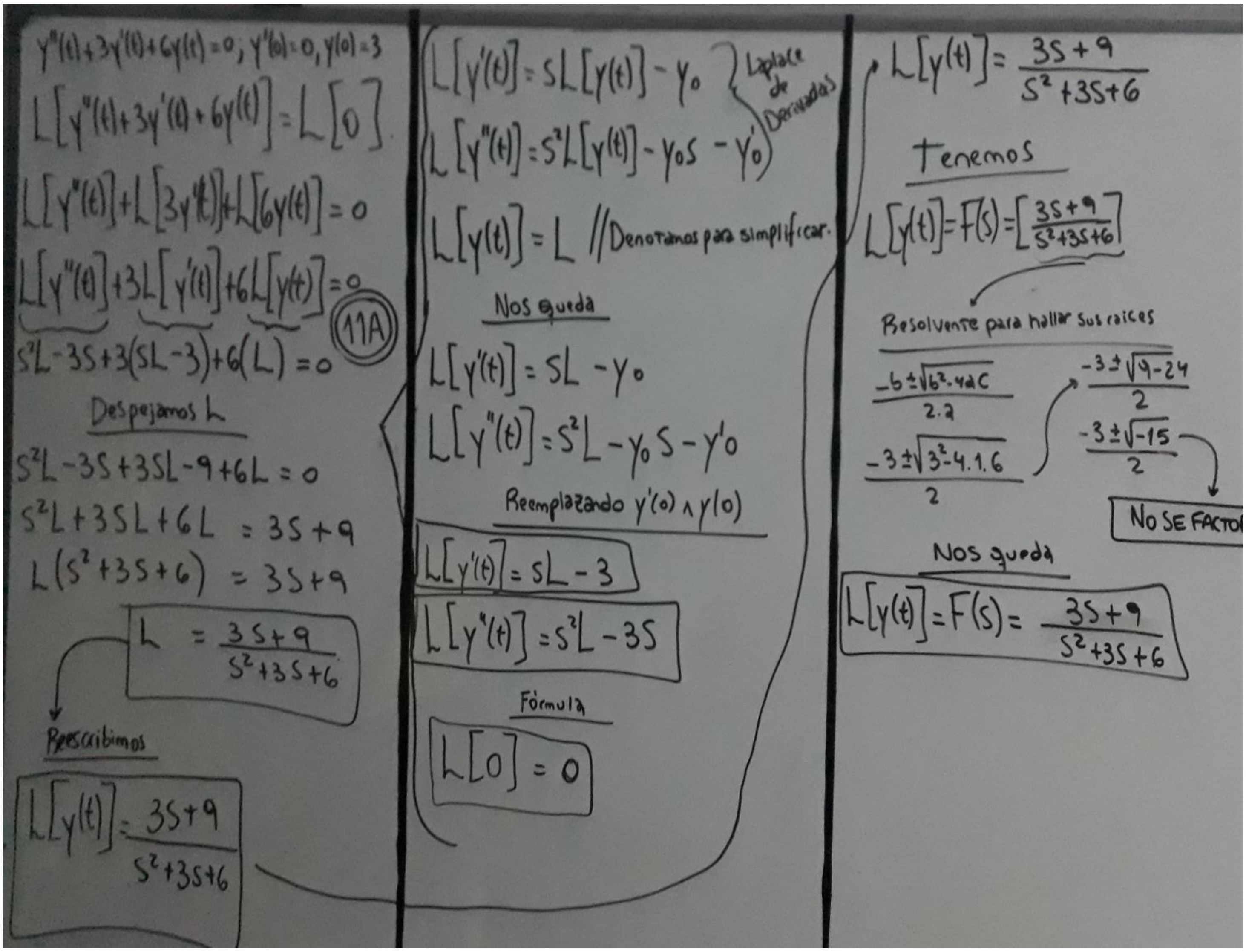
Fórmula

$$L^{-1}\left[\frac{1}{s-a}\right]=e^{at}$$

$$\boxed{-\frac{4}{3}e^{-t}+\frac{5}{6}e^{-t}+e^{-2t}}$$

$$\longrightarrow L^{-1}\left[\frac{s}{(s+1)(s+1)(s+2)}\right]$$

11A)Resolucion de EDL mediante TL



$y''(t)+3y'(t)+6y(t)=0;\ y'(0)=0,\ y(0)=3$

$L[y''(t)+3y'(t)+6y(t)]=L[0]$

$L[y''(t)]+L[3y'(t)]+L[6y(t)]=0$

$\underbrace{L[y''(t)]}+3\underbrace{L[y'(t)]}+6\underbrace{L[y(t)]}=0$ (11A)

$s^2L-3s+3(sL-3)+6(L)=0$

Despejamos L

$s^2L-3s+3sL-9+6L=0$

$s^2L+3sL+6L=3s+9$

$L(s^2+3s+6)=3s+9$

$\boxed{L=\frac{3s+9}{s^2+3s+6}}$

Reescribimos

$\boxed{L[y(t)]=\frac{3s+9}{s^2+3s+6}}$

$L[y'(t)]=sL[y(t)]-y_0$

$L[y''(t)]=s^2L[y(t)]-y_0s-y'_0$

Laplace de Derivadas

$L[y(t)]=L$ //Denotamos para simplificar.

Nos queda

$L[y'(t)]=sL-y_0$

$L[y''(t)]=s^2L-y_0s-y'_0$

Reemplazando $y'(0) \wedge y(0)$

$\boxed{L[y'(t)]=sL-3}$

$\boxed{L[y''(t)]=s^2L-3s}$

Fórmula

$\boxed{L[0]=0}$

$L[y(t)]=\frac{3s+9}{s^2+3s+6}$

Tenemos

$L[y(t)]=F(s)=\left[\frac{3s+9}{s^2+3s+6}\right]$

Resolvente para hallar sus raíces

$\frac{-b\pm\sqrt{b^2-4ac}}{2\cdot a}$

$\frac{-3\pm\sqrt{3^2-4.1.6}}{2}$

$\frac{-3\pm\sqrt{9-24}}{2}$

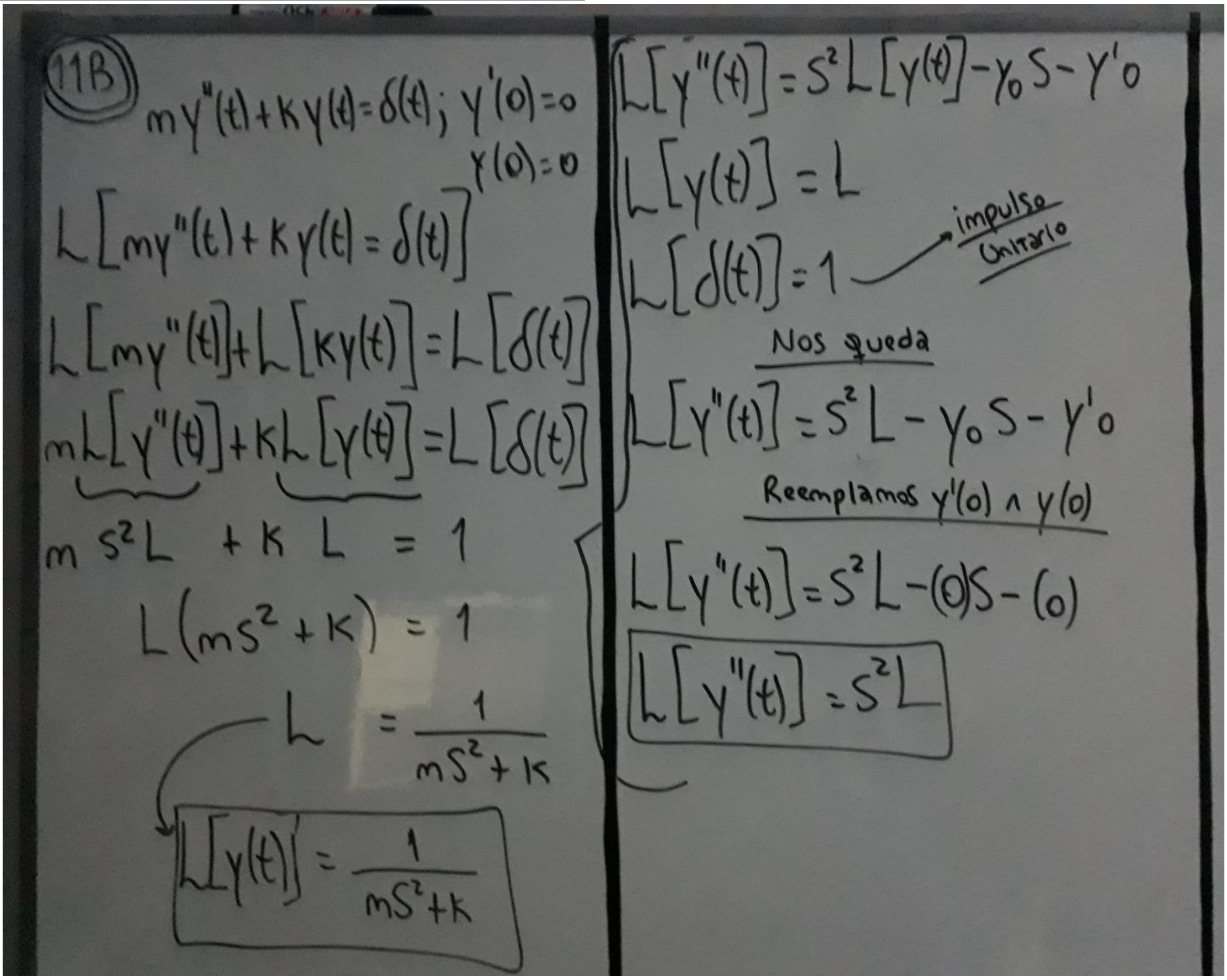
$\frac{-3\pm\sqrt{-15}}{2}$

No se factor

Nos queda

$\boxed{L[y(t)]=F(s)=\frac{3s+9}{s^2+3s+6}}$

11B)Resolucion de EDL mediante TL



11C)Resolucion de EDL mediante TL

$$2y''(t)+7y'(t)+3y(t)=0;\quad y'(0)=1,\quad y(0)=0$$

$$L[2y''(t)+7y'(t)+3y(t)]=L[0]$$

$$L[2y''(t)]+L[7y'(t)]+L[3y(t)]=L[0]$$

$$2\underbrace{L[y''(t)]}+7\underbrace{L[y'(t)]}+3\underbrace{L[y(t)]}=L[0]$$

$$2(s^2L-1)+7sL+3L=0$$

$$2s^2L-2+7sL+3L=0$$

$$L(2s^2+7s+3)=2$$

$$L=\frac{2}{2s^2+7s+3}$$

$$\boxed{L[y(t)]=\frac{2}{2s^2+7s+3}}$$

11C

$$L[y''(t)]=s^2L[y(t)]-y_0s-y'_0$$

$$L[y'(t)]=sL[y(t)]-y_0$$

$$L[y(t)]=L$$

$$L[0]=0 \rightarrow L[0]=\frac{0}{s}=0$$

Nos queda

$$L[y''(t)]=s^2L-y_0s-y'_0$$

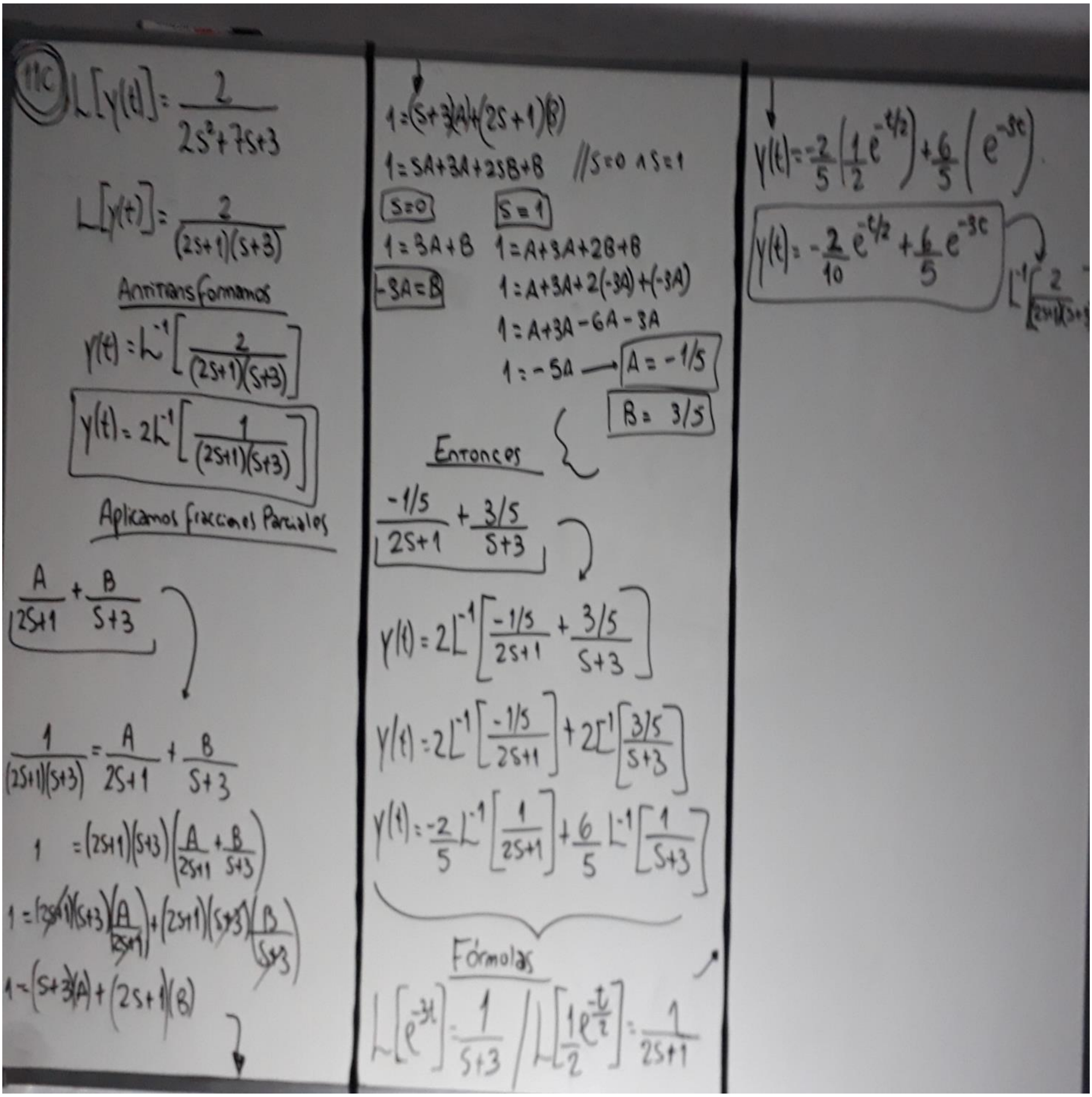
$$L[y'(t)]=sL-y_0$$

Reemplazamos $y'(0) \wedge y(0)$

$$L[y''(t)]=s^2L-\cancel{(0)}s-1$$

$$\boxed{L[y''(t)]=s^2L-1}$$

$$\boxed{L[y'(t)]=sL}$$



11c) $L[y(t)] = \frac{2}{2s^2+7s+3}$
$L[y(t)] = \frac{2}{(2s+1)(s+3)}$
Antitransformamos
$y(t) = L^{-1}\left[\frac{2}{(2s+1)(s+3)}\right]$
$y(t) = 2L^{-1}\left[\frac{1}{(2s+1)(s+3)}\right]$
Aplicamos fracciones Parciales
$\frac{A}{2s+1} + \frac{B}{s+3}$
$\frac{1}{(2s+1)(s+3)} = \frac{A}{2s+1} + \frac{B}{s+3}$
$1 = (2s+1)(s+3)\left(\frac{A}{2s+1} + \frac{B}{s+3}\right)$
$1 = (2s+1)(s+3)\left(\frac{A}{2s+1}\right) + (2s+1)(s+3)\left(\frac{B}{s+3}\right)$
$1 = (s+3)(A) + (2s+1)(B)$
$1 = (s+3)(A) + (2s+1)(B)$
$1 = sA+3A+2sB+B$ // $s=0 \wedge s=1$
$s=0$
$1 = 3A+B$
$-3A = B$
$s=1$
$1 = A+3A+2B+B$
$1 = A+3A+2(-3A)+(-3A)$
$1 = A+3A-6A-3A$
$1 = -5A \longrightarrow A = -1/5$
$B = 3/5$
Entonces
$\frac{-1/5}{2s+1} + \frac{3/5}{s+3}$
$y(t) = 2L^{-1}\left[\frac{-1/5}{2s+1} + \frac{3/5}{s+3}\right]$
$y(t) = 2L^{-1}\left[\frac{-1/5}{2s+1}\right] + 2L^{-1}\left[\frac{3/5}{s+3}\right]$
$y(t) = \frac{-2}{5}L^{-1}\left[\frac{1}{2s+1}\right] + \frac{6}{5}L^{-1}\left[\frac{1}{s+3}\right]$
Fórmolas
$L[e^{-3t}] = \frac{1}{s+3}$ / $L\left[\frac{1}{2}e^{\frac{-t}{2}}\right] = \frac{1}{2s+1}$
$y(t) = \frac{-2}{5}\left(\frac{1}{2}e^{-t/2}\right) + \frac{6}{5}\left(e^{-3t}\right)$
$y(t) = -\frac{2}{10}e^{-t/2} + \frac{6}{5}e^{-3t}$
$L^{-1}\left[\frac{2}{(2s+1)(s+3)}\right]$

12)Aplicación del TVF