AC32008 Theory of Computation
Class Test 2 - Friday 2 April 2021 - 11.00-12.15
Answer ALL 5 Questions

Total marks: 30

1. Explain what does it mean for a language L to be decidable in polynomial time? **[5 marks]**

2. Suppose we are given a Turing Machine with

   - $Q = \{q_0, q_1, q_2, q_3, q_4\}$,
   - $\Sigma = \{0, 1\}$,
   - $\Gamma = \{0, 1, X, Y, B\}$,
   - $F = \{q_4\}$.

δ	q_0	q_1	q_2	q_3	q_4
0	(q_1, X, R)	$(q_1, 0, R)$	$(q_2, 0, L)$	–	–
1	–	(q_2, Y, L)	–	–	–
X	–	–	(q_0, X, R)	–	–
Y	(q_3, Y, R)	(q_1, Y, R)	(q_2, Y, L)	(q_3, Y, R)	–
B	–	–	–	(q_4, B, R)	–

   Simulate the computation of this Turing Machine on a string 1100 by writing a serious of instantaneous descriptions the machine goes through. Is the string accepted by the machine? **[6 marks]**

3. Let M be a Turing Machine with states q_1, q_2, where q_1 is the initial state, $F = \{q_2\}$, input alphabet $\{0, 1\}$ and tape alphabet $\{0, 1, B\}$. The transition function δ for M is as follows:

δ	q_1	q_2
0	$(q_1, 0, R)$	–
1	–	–
B	(q_2, B, L)	–

   Let w be the string 00000.

   (a) Determine a code for the machine M.

   (b) What is $\langle M, w\rangle$?

   (c) Is $\langle M, w\rangle \in L_{\text{halt}}$?

   [Recall that $X_1 = 0$, $X_2 = 1$, $X_3 = B$, $D_1 = L$, $D_2 = R$.] **[7 marks]**

4. Suppose that M is a nondeterministic Turing machine (NDTM), and that, for some n, there are just two strings, x and y, of length n, that M accepts.

   - on input x, with guess g_1, the computation accepts after 181 steps;
   - on input y, with guess g_2, the computation accepts after 203 steps;
   - on input y, with guess g_3, the computation accepts after 171 steps;
   - there are no other accepting computations on inputs of length n.

   What are (i) $t_M(x)$, (ii) $t_M(y)$, (iii) $T_M(n)$? **[6 marks]**

5. a Let x be an input to a (standard) non-deterministic Turing Machine (NDTM) M. What does it mean to say that M accepts x?

   b Say informally what it means for there to be a polynomial transformation (or reduction) from one problem to another.

   [6 marks]