$$\bullet \int_{-1}^{1} P_n(x)\, P_m(x)\, dx = \begin{cases} 0 & m \neq n \\ \frac{2}{2n+1} & m = n \end{cases}$$

$$\rightarrow f(x) = \sum_{n=0}^{\infty} c_n P_n(x)$$

$\rightarrow$ Proyectar sobre la base $\rightarrow \int_{-1}^{1} f(x) P_m(x)\, dx = \int \sum_{n=0}^{\infty} c_n P_n(x) P_m(x)\, dx$

$$\int_{-1}^{1} f(x)\, P_m(x) = \sum_{n=0}^{\infty} c_n \int_{-1}^{1} P_n(x)\, P_m(x)\, dx$$

$\rightarrow$ con $m = n$

$$\int_{-1}^{1} f(x)\, P_n(x)\, dx = \frac{2}{2n+1} c_n$$

$$\boxed{c_n = \frac{2n+1}{2} \int_{-1}^{1} f(x)\, P_n(x)\, dx}$$