- $I = \int_a^b f(x)\,dx$

- $f(x) \approx P_2(x) = \frac{(x-b)(x-x_m)}{(a-b)(a-x_m)} f(a) + \frac{(x-a)(x-b)}{(x_m-a)(x_m-b)} f(x_m) + \frac{(x-a)(x-x_m)}{(b-a)(b-x_m)} f(b) \quad \forall x \in [a,b]$

- $\int_a^b f(x)\,dx \cong \int_a^b P_2(x)\,dx = \int_a^b \frac{(x-b)(x-x_m)}{(a-b)(a-x_m)} f(a)\,dx + \int_a^b \frac{(x-a)(x-b)}{(x_m-a)(x_m-b)} f(x_m)\,dx$
$+ \int_a^b \frac{(x-a)(x-x_m)}{(b-a)(b-x_m)} f(b)\,dx$

- $I \cong \frac{f(a)}{(a-b)(a-x_m)} \int_a^b x^2 - bx - x x_m + b x_m\,dx + \frac{f(x_m)}{(x_m-a)(x_m-b)} \int_a^b x^2 - ax - bx + ab\,dx$
$+ \frac{f(b)}{(b-a)(b-x_m)} \int_a^b x^2 - ax - x x_m + a x_m\,dx$

- $I \cong \frac{f(a)}{2h^2} \left(\frac{x^3}{3} - \frac{bx^2}{2} - \frac{x_m x^2}{2} + b x_m x \Big|_a^b \right) + \frac{f(x_m)}{-h^2} \left(\frac{x^3}{3} - \frac{ax^2}{2} - \frac{bx^2}{2} + abx \Big|_a^b \right)$
$+ \frac{f(b)}{2h^2} \left(\frac{x^3}{3} - \frac{ax^2}{2} - \frac{x_m x^2}{2} + a x_m x \Big|_a^b \right)$

- $I \cong \frac{f(a)}{2h^2} \left[\left(\frac{b^3}{3} - \frac{b^3}{2} - \frac{x_m b^2}{2} + b^2 x_m \right) - \left(\frac{a^3}{3} - \frac{ba^2}{2} - \frac{x_m a^2}{2} + ba x_m \right) \right]$
$- \frac{f(x_m)}{h^2} \left[\left(\frac{b^3}{3} - \frac{ab^2}{2} - \frac{b^3}{2} + ab^2 \right) - \left(\frac{a^3}{2} - \frac{a^3}{2} - \frac{ab^2}{2} + a^2 b \right) \right]$
$+ \frac{f(b)}{2h^2} \left[\left(\frac{b^3}{3} - \frac{ab^2}{2} - \frac{x_m b^2}{2} + ab x_m \right) - \left(\frac{a^3}{3} - \frac{a^3}{2} - \frac{x_m a^2}{2} + a^2 x_m \right) \right]$

• $a = h \quad x_m = 0 \cdot h \quad b = h$

$$I \approx \left[\left(\frac{h^3}{3} - \frac{h^3}{2}\right) - \left(-\frac{h^3}{3} - \frac{h^3}{2}\right)\right] \frac{F(a)}{2h^2} + \left[\left(\frac{h^3}{3} + \frac{h^3}{2} - \frac{h^3}{2} - h^3\right) - \left(\frac{h^3}{3} + \frac{h^3}{2} + \frac{h^3}{2} + h^3\right)\right] \frac{F(x_m)}{-h^2} + \left[\left(\frac{h^3}{3} + \frac{h^3}{2}\right) - \left(-\frac{h^3}{2} + \frac{h^3}{2}\right)\right] \frac{F(b)}{2h^2}$$

$$I \approx \frac{2/3\,h^3}{2h^2} F(a) - \frac{4/3\,h^3}{-h^2} F(b) + \frac{2/3\,h^3}{2h^2} F(b)$$

$$I \approx \frac{1}{3} h F(a) + \frac{4}{3} h F(x_m) + \frac{1}{3} h^3 F(b)$$

$$I \approx \frac{1}{3} h \left(F(a) + 4F(x_m) + F(b)\right)$$