

8.

$$a) \Omega = \{(x_0, f(x_0)), (x_1, f(x_1)), (x_2, f(x_2))\}$$

$$P_2 = \sum_{i=0}^2 L_i f(x_i)$$

$$L_i = \prod_{\substack{j=0 \\ j \neq i}}^2 \frac{x - x_j}{x_i - x_j}$$

$$\rightarrow P_2 = \left(\frac{x - x_1}{x_0 - x_1} \right) \left(\frac{x - x_2}{x_0 - x_2} \right) f(x_0) + \left(\frac{x - x_0}{x_1 - x_0} \right) \left(\frac{x - x_2}{x_1 - x_2} \right) f(x_1) + \left(\frac{x - x_0}{x_2 - x_0} \right) \left(\frac{x - x_1}{x_2 - x_1} \right) f(x_2)$$

$$P'_2 = \frac{f(x_0)}{(x_1 - x_0)(x_2 - x_0)} \left[(x - x_2) + (x - x_1) \right] + \frac{f(x_1)}{(x_1 - x_0)(x_1 - x_2)} \left[(x - x_2) + (x - x_0) \right] + \frac{f(x_2)}{(x_2 - x_0)(x_2 - x_1)} \left[(x - x_1) + (x - x_0) \right]$$

$$P'_2(x) = \frac{f(x_0)}{2h^2} \left[(x - x_2) + (x - x_1) \right] + \frac{f(x_1)}{-h^2} \left[(x - x_2) + (x - x_0) \right] + \frac{f(x_2)}{2h^2} \left[(x - x_1) + (x - x_0) \right]$$

$$P'_2(x_0) = \frac{f(x_0)}{2h^2} \left[x_0 - x_2 + x_0 - x_1 \right] + \frac{f(x_1)}{-h^2} \left[x_0 - x_2 \right] + \frac{f(x_2)}{2h^2} \left[x_0 - x_1 \right]$$

$$P'_2(x_0) = \frac{-3hf(x_0)}{2h^2} + \frac{-2hf(x_1)}{-h^2} + \frac{hf(x_2)}{2h^2} = \frac{-3f(x_0)}{2h} + \frac{4f(x_1)}{2h} - \frac{f(x_2)}{2h}$$

$$P'_2(x_0) = \frac{1}{2h} \left[-3f(x_0) + 4f(x_1) - f(x_2) \right] = \frac{1}{2h} \left[-3f(x_0) + 4f(x_0+h) - f(x_0+2h) \right]$$