

Operadores de derivada – Punto 1

$$f'(x) = \frac{-f(x+2h) + 4f(x+h) - 3f(x)}{2h}$$

$$f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

1. Desmotración de consisencia del operador dado de derivada y de segunda derivada para  $f(x) = x^2$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \left( \frac{-(x+2h)^2 + 4(x+h)^2 - 3x^2}{2h} \right) \\ &= \lim_{h \rightarrow 0} \left( \frac{-x^2 - 4xh - 4h^2 + 4x^2 + 8xh + 4h^2 - 3x^2}{2h} \right) = \lim_{h \rightarrow 0} \left( \frac{4xh}{2h} \right) \\ &= 2x \end{aligned}$$

$$\begin{aligned} f''(x) &= \lim_{h \rightarrow 0} \left( \frac{(x+h)^2 - 2x^2 + (x-h)^2}{h^2} \right) \\ &= \lim_{h \rightarrow 0} \left( \frac{x^2 + 2xh + h^2 - 2x^2 + x^2 - 2xh + h^2}{h^2} \right) = \lim_{h \rightarrow 0} \left( \frac{2h^2}{h^2} \right) = 2 \end{aligned}$$

2. Demostracion de consistencia del operador dado de derivada y de segunda derivada para  $f(x) = \sin(x)$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \left( \frac{-\sin(x+2h) + 4\sin(x+h) - 3\sin(x)}{2h} \right) = f'(x) \\ &= \lim_{h \rightarrow 0} \left( \frac{-\sin x \cos 2h - \sin 2h \cos x + 4\sin x \cos h + 4\sin h \cos x - 3\sin x}{2h} \right) \\ &= -\sin x \lim_{h \rightarrow 0} \left( \frac{\cos 2h}{2h} \right) - \cos x \lim_{h \rightarrow 0} \left( \frac{\sin 2h}{2h} \right) + 2\sin x \lim_{h \rightarrow 0} \left( \frac{\cos h}{h} \right) + 2\cos x \lim_{h \rightarrow 0} \left( \frac{\sin h}{h} \right) \\ &\quad - 3\sin x \lim_{h \rightarrow 0} \left( \frac{1}{2h} \right) = 0 - \cos x + 0 + 2\cos x - 0 = \cos(x) \end{aligned}$$

$$\begin{aligned}
 f''(x) &= \lim_{h \rightarrow 0} \left( \frac{\sin(x+h) - 2\sin x + \sin(x-h)}{h^2} \right) \\
 &= \lim_{h \rightarrow 0} \left( \frac{\sin x \cos h + \sin h \cos x - 2\sin x + \sin x \cos h - \sin h \cos x}{h^2} \right) \\
 &= \lim_{h \rightarrow 0} \left( \frac{2\sin x \cos h - 2\sin x}{h^2} \right) = 2\sin x \lim_{h \rightarrow 0} \left( \frac{\cos h}{h^2} \right) - 2\lim_{h \rightarrow 0} \left( \frac{\sin x}{h^2} \right) \\
 &= 2\sin x \left( -\frac{1}{2} \right) - 0 = -\sin x
 \end{aligned}$$