Operadores de derivada – Punto 1

$$f'(x) = \frac{-f(x+2h) + 4f(x+h) - 3f(x)}{2h}$$

$$f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

1. Demostración de consistencia del operador dado de derivada y de segunda derivada para $f(x) = x^2$

$$f'(x) = \lim_{h\to 0}\left(\frac{-(x+2h)^2 + 4(x+h)^2 - 3x^2}{2h}\right)$$
$$= \lim_{h\to 0}\left(\frac{-x^2 - 4xh - 4h^2 + 4x^2 + 8xh + 4h^2 - 3x^2}{2h}\right) = \lim_{h\to 0}\left(\frac{4xh}{2h}\right)$$
$$= 2x$$

$$f''(x) = \lim_{h\to 0}\left(\frac{(x+h)\hat{}2 - 2x^2 + (x-h)^2}{h^2}\right)$$
$$= \lim_{h\to 0}\left(\frac{x^2 + 2xh + h^2 - 2x^2 + x^2 - 2xh + h^2}{h^2}\right) = \lim_{h\to 0}\left(\frac{2h^2}{h^2}\right) = 2$$

2. Demostración de consistencia del operador dado de derivada y de segunda derivada para $f(x) = \sin(x)$

$$f'(x)$$
$$= \lim_{h\to 0}\left(\frac{-\sin(x+2h) + 4\sin(x+h) - 3\sin(x)}{2h}\right) = f'(x)$$
$$= \lim_{h\to 0}\left(\frac{-\sin x\cos 2h - \sin 2h\cos x + 4\sin x\cos h + 4\sin h\cos x - 3\sin x}{2h}\right)$$
$$= -\sin x\lim_{h\to 0}\left(\frac{\cos 2h}{2h}\right) - \cos x\lim_{h\to 0}\left(\frac{\sin 2h}{2h}\right) + 2\sin x\lim_{h\to 0}\left(\frac{\cos h}{h}\right) + 2\cos x\lim_{h\to 0}\left(\frac{\sin h}{h}\right)$$
$$- 3\sin x\lim_{h\to 0}\left(\frac{1}{2h}\right) = 0 - \cos x + 0 + 2\cos x - 0 = \cos(x)$$

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\begin{aligned}
f''(x) &= \lim_{h\to 0}\left(\frac{\sin(x+h) - 2\sin x + \sin(x-h)}{h^2}\right) \\
&= \lim_{h\to 0}\left(\frac{\sin x\cos h + \sin h\cos x - 2\sin x + \sin x\cos h - \sin h\cos x}{h^2}\right) \\
&= \lim_{h\to 0}\left(\frac{2\sin x\cos h - 2\sin x}{h^2}\right) = 2\sin x \lim_{h\to 0}\left(\frac{\cos h}{h^2}\right) - 2\lim_{h\to 0}\left(\frac{\sin x}{h^2}\right) \\
&= 2\sin x\left(-\frac{1}{2}\right) - 0 = -\sin x
\end{aligned}
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