Support Vector Machine (SVM)

Machine Learning and Pattern Recognition

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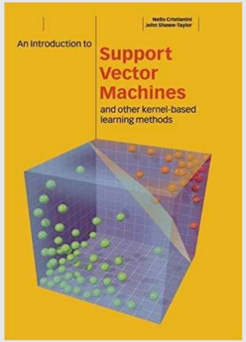
Institute of Computing (IC/Unicamp)

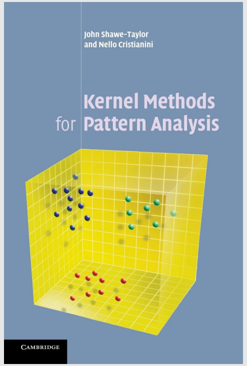
MC886/MO444, November 13, 2018

SVMs are among the best "off-the-shelf" supervised learning algorithm.

Andrew Ng

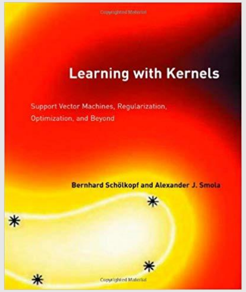
"**An Introduction to Support Vector Machines**: And Other Kernel-based Learning Methods", Cristianini & Shawe-Taylor, 2000.



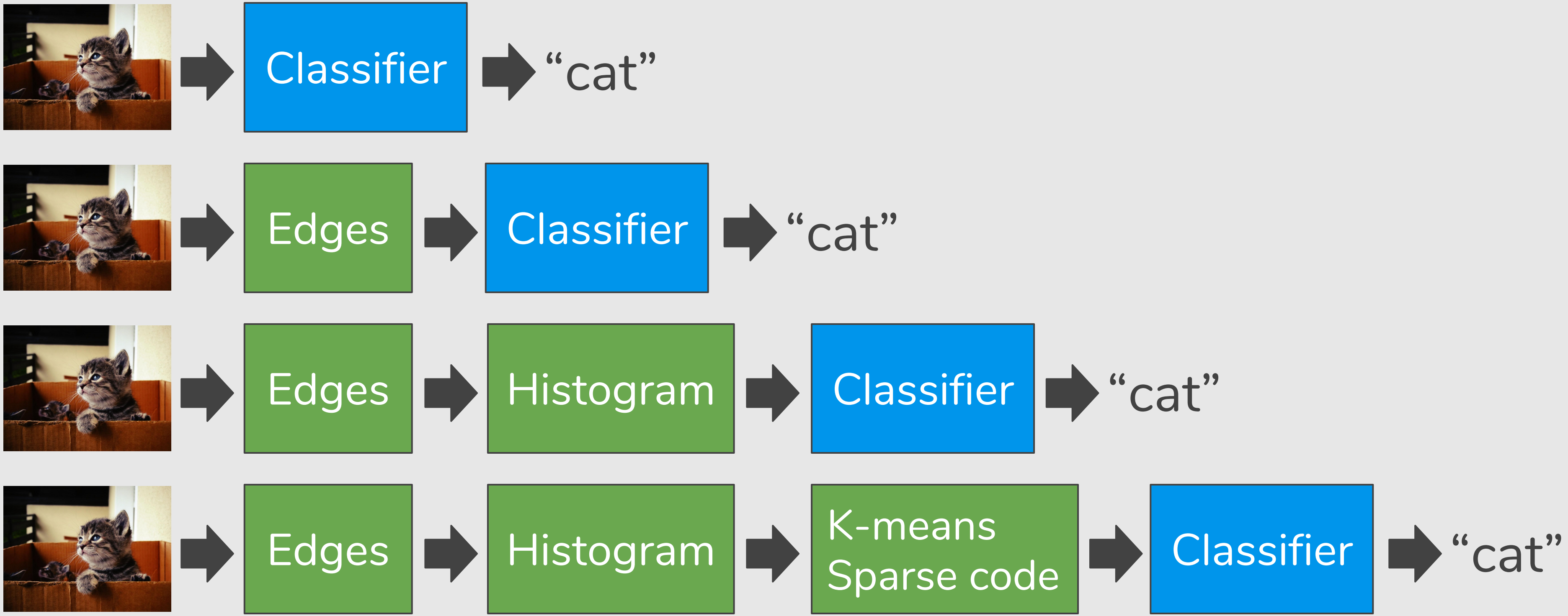


"**Kernel Methods for Pattern Analysis**", Shawe-Taylor & Cristianini, 2004.

"**Learning with Kernels**: Support Vector Machines, Regularization, Optimization, and Beyond", Scholkopf & Smola, 2001.



Traditional Recognition



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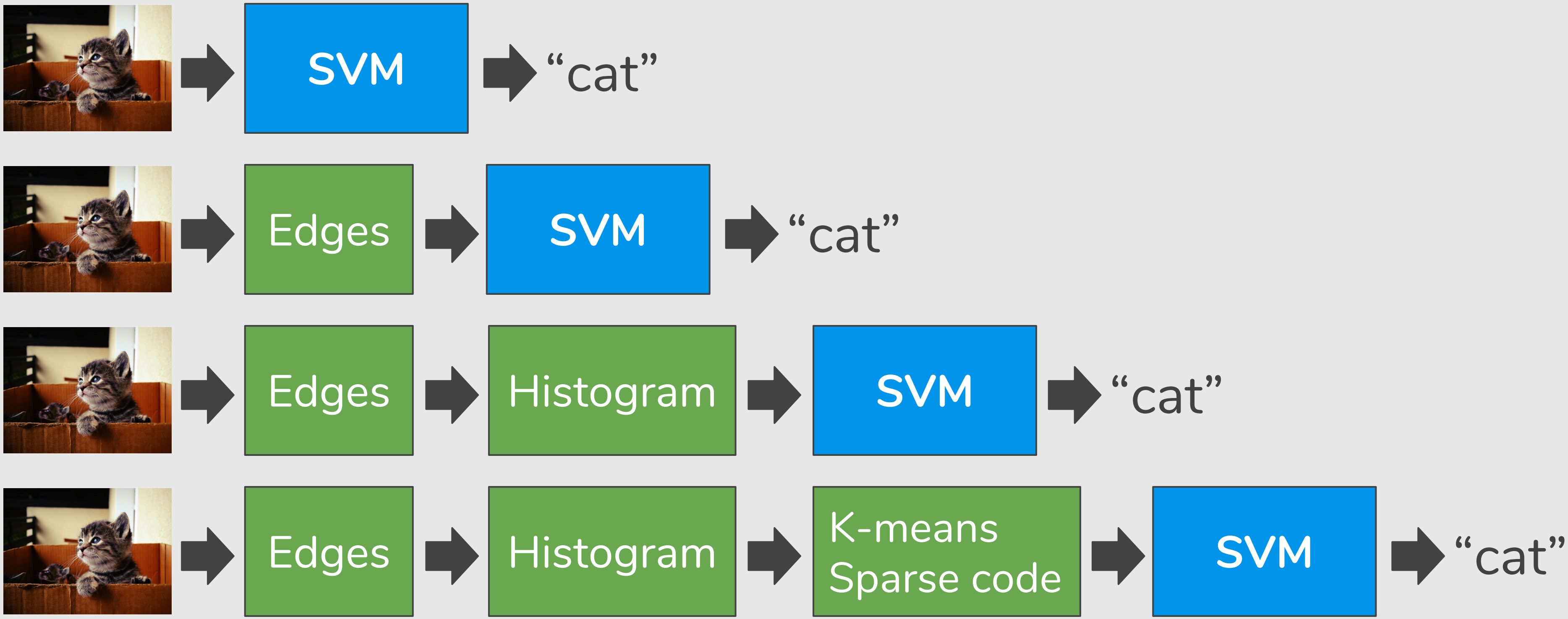
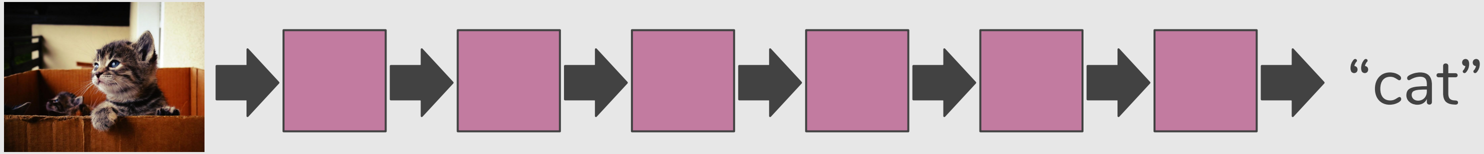


Deep Learning



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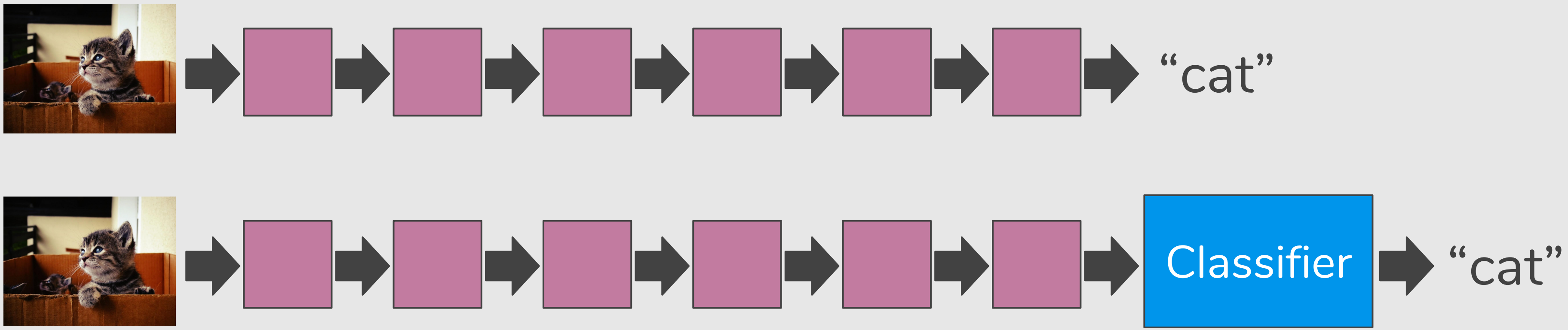

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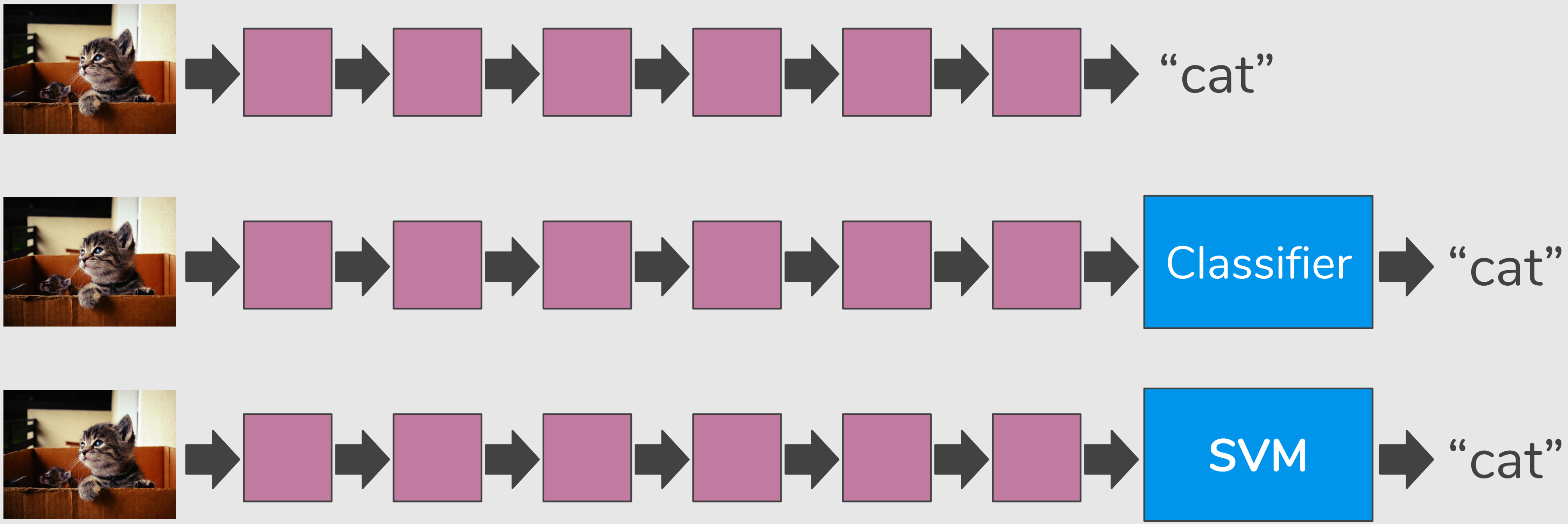
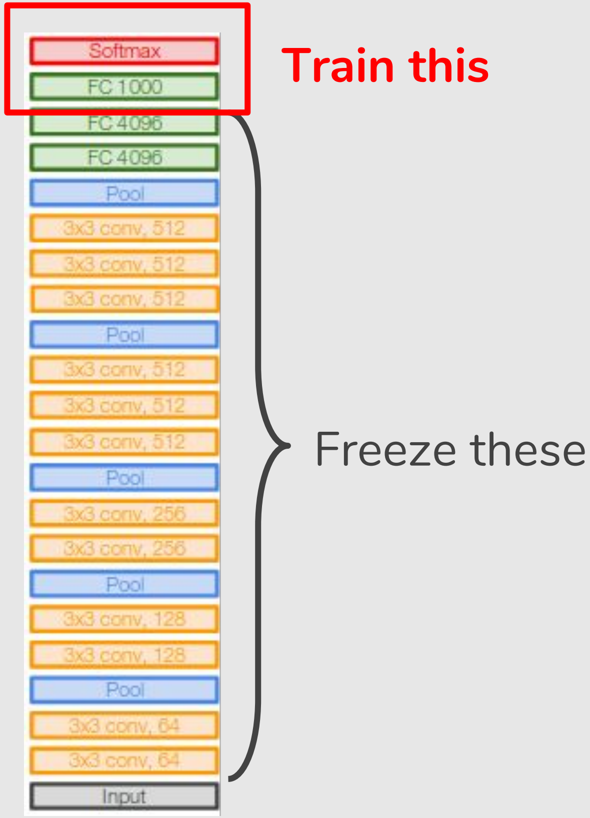


Transfer Learning



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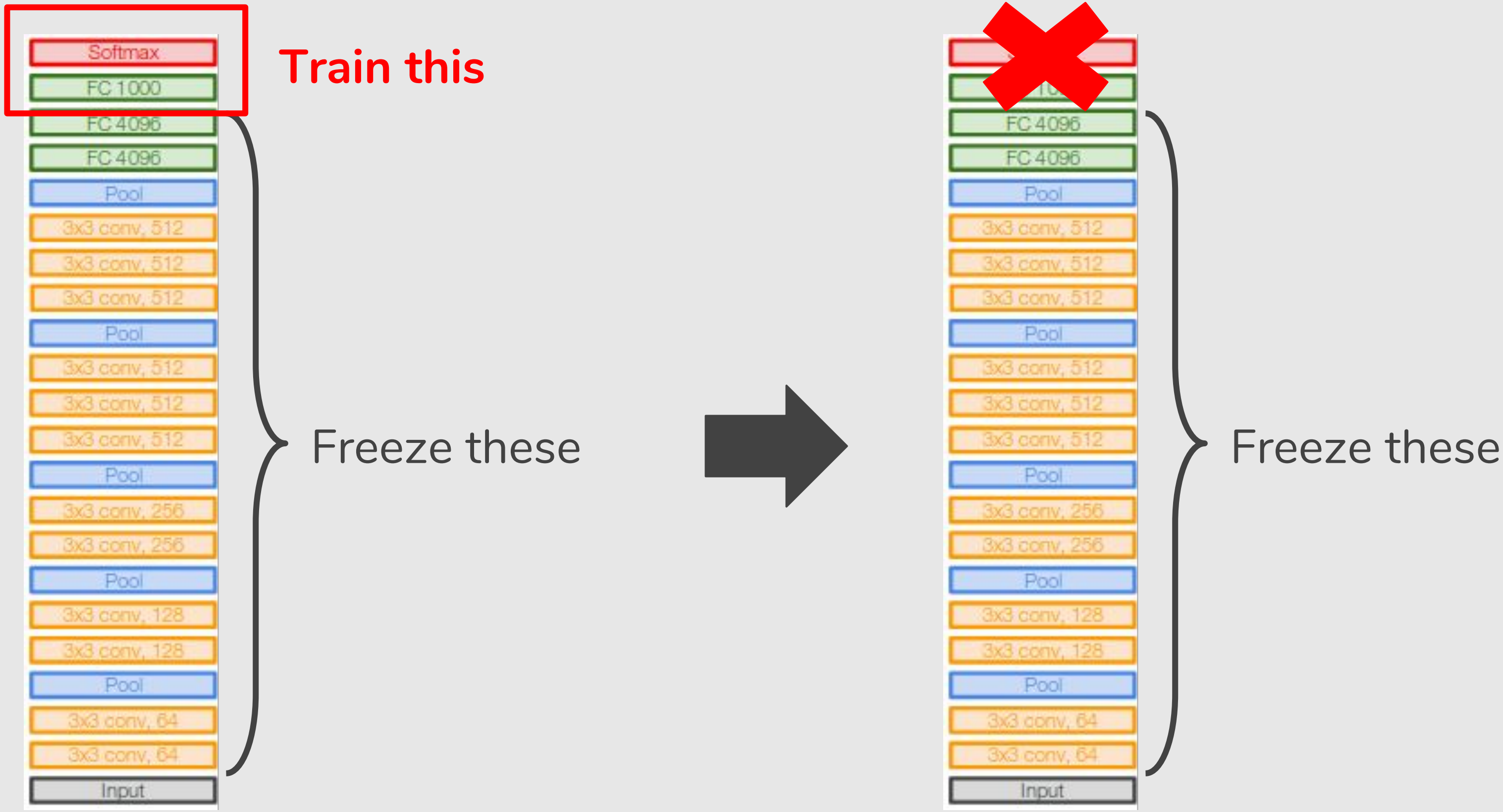

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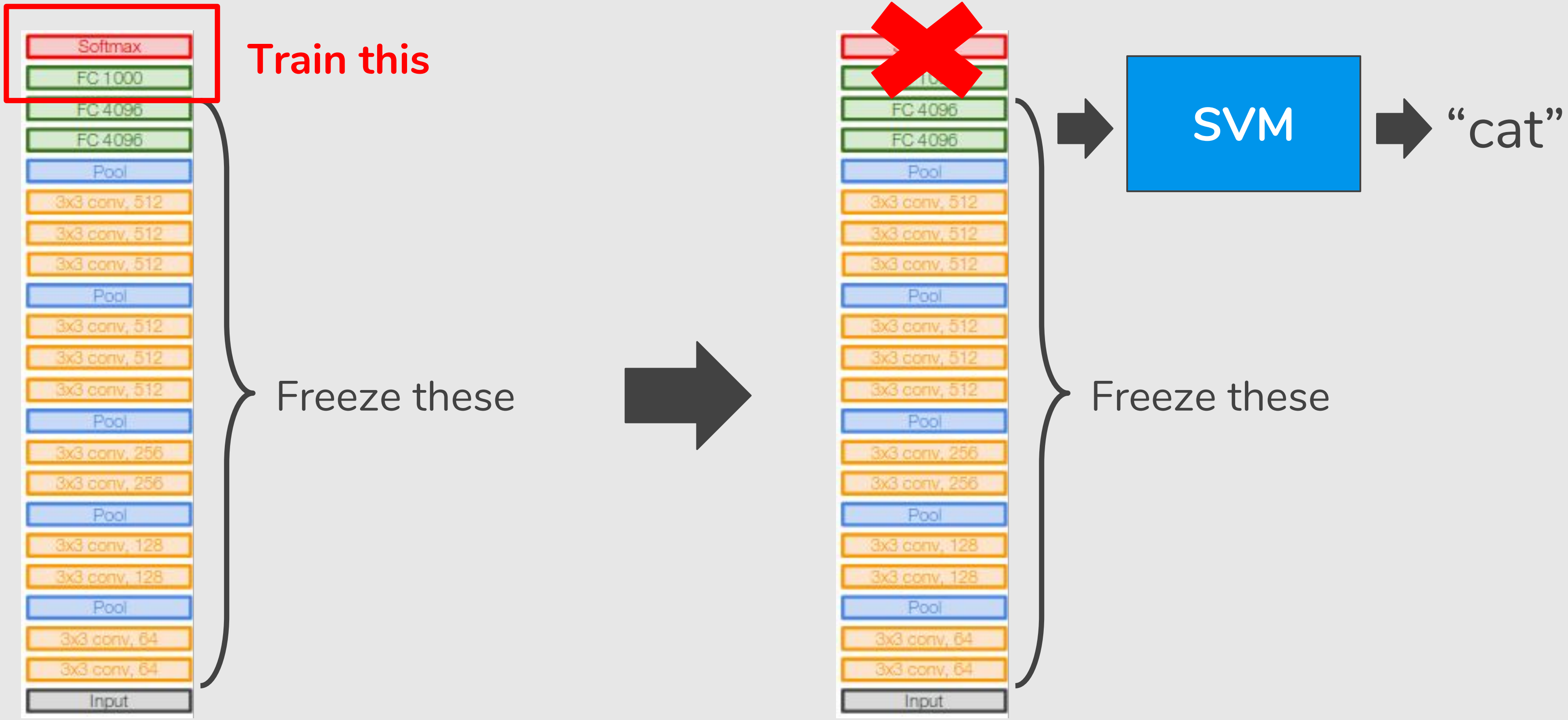
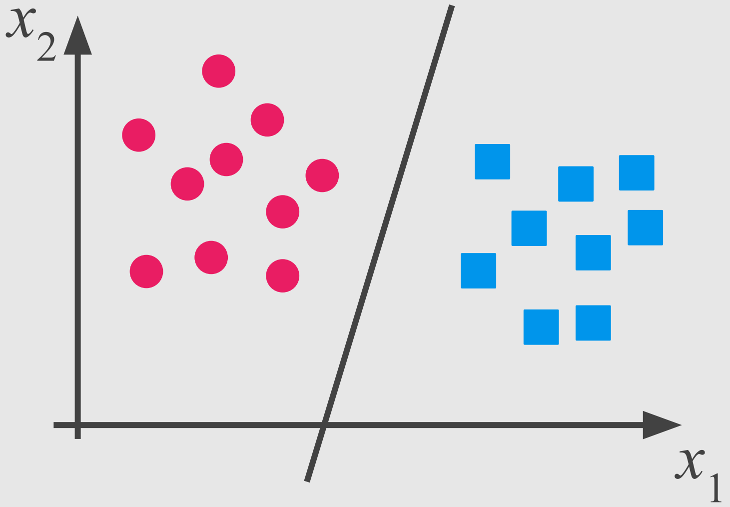


What is Support Vector Machine?

Support Vector Machine (SVM)

[Vapnik and Chervonenkis, 1964; Vapnik, 1982; Vapnik, 1995]

Idea of separating data with **a large "gap"**.



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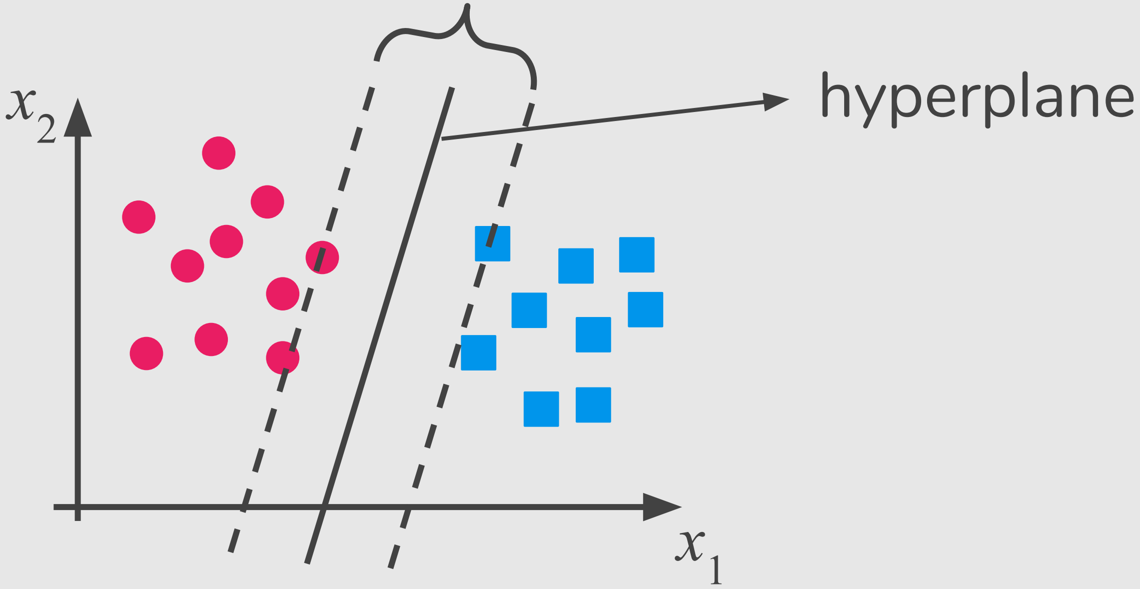
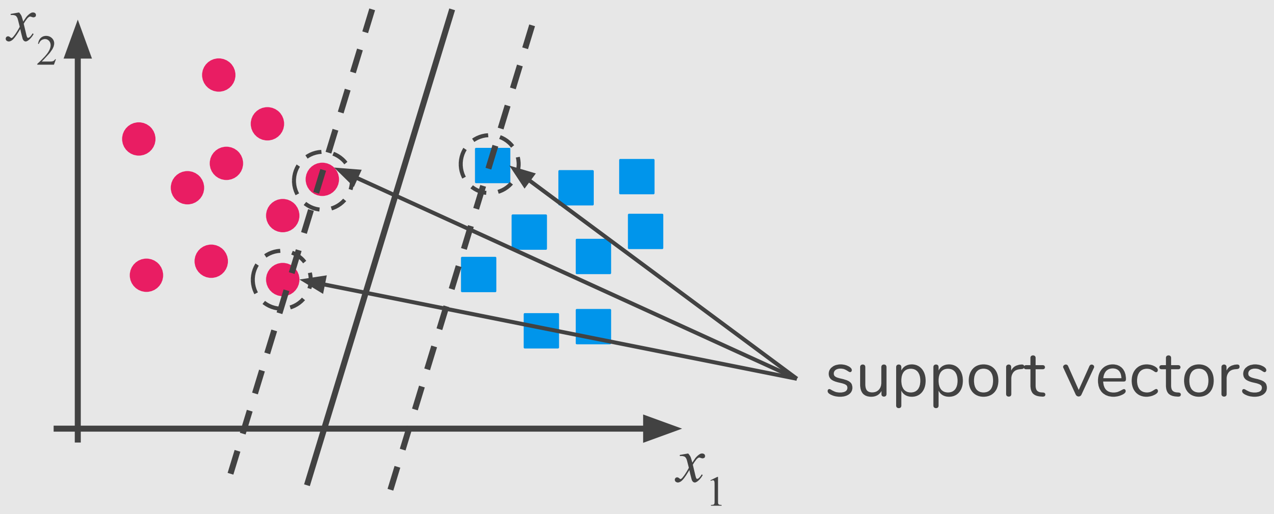


Support Vector Machine (SVM)

[Vapnik and Chervonenkis, 1964; Vapnik, 1982; Vapnik, 1995]

Examples closest to the hyperplane are support vectors.



Support Vector Machine (SVM)

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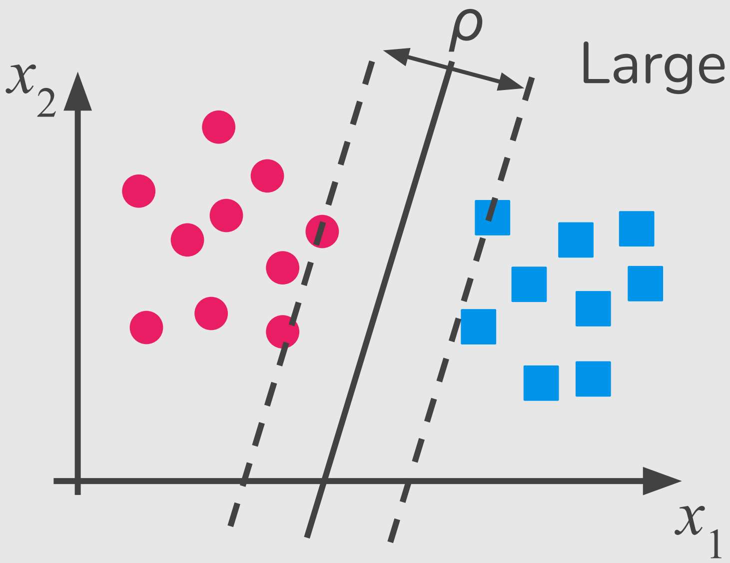
Margin ρ of the separator is the distance between support vectors.

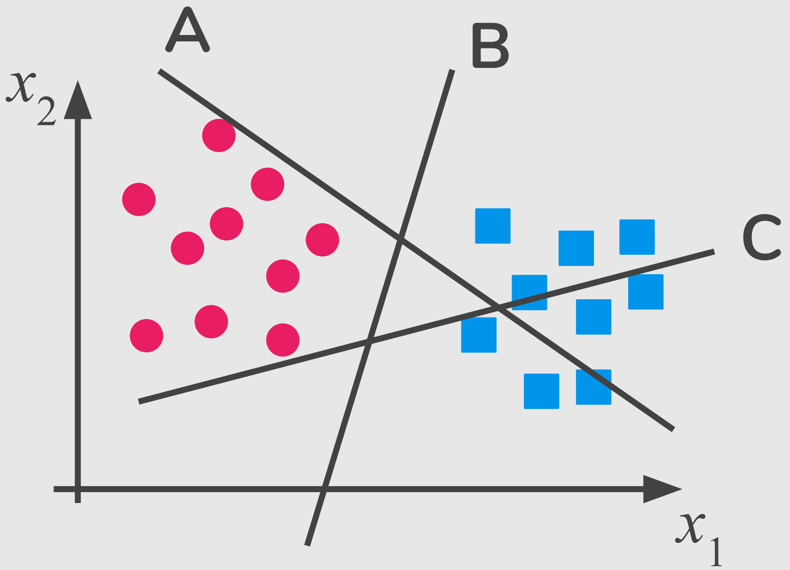


Large margin classifier

How does SVM work?

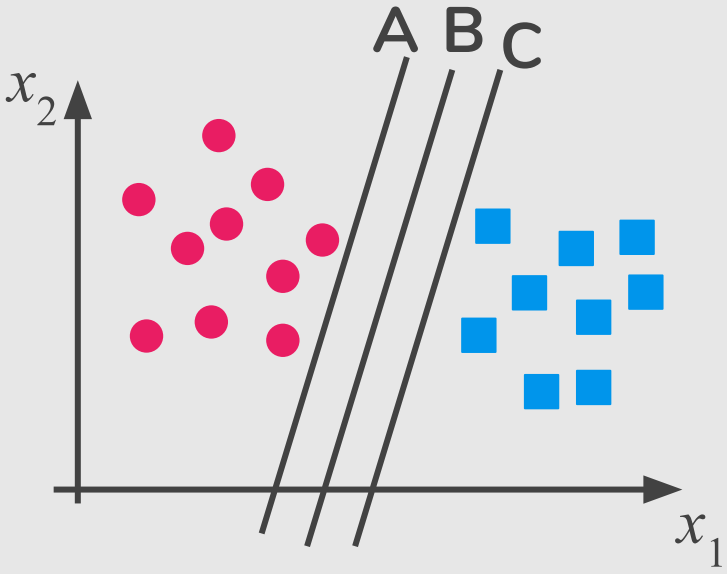
How can we identify the right hyperplane?

Scenario 1



How can we identify the right hyperplane?

Scenario 2



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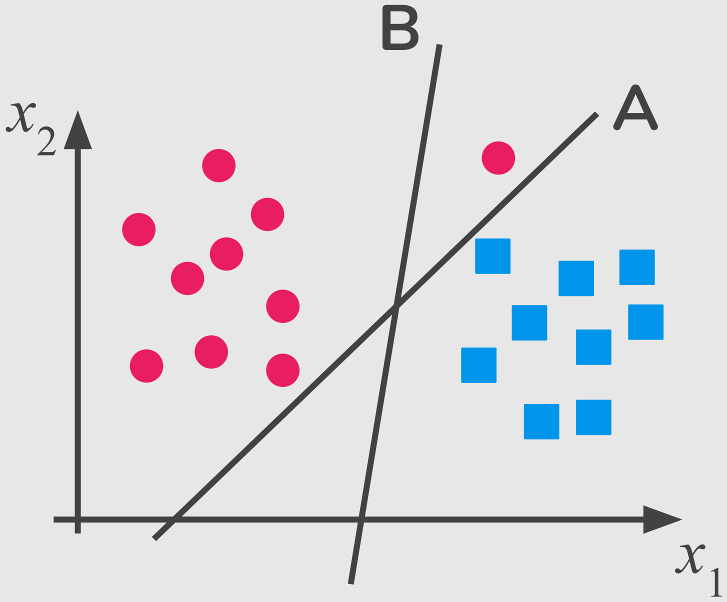
Scenario 3



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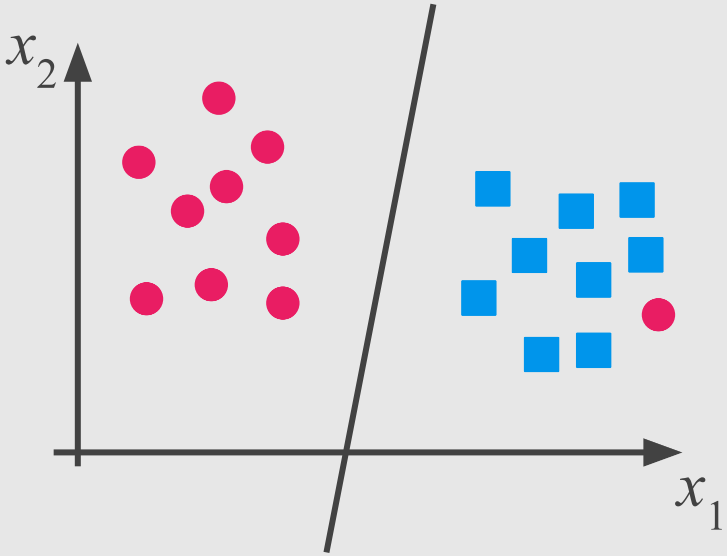
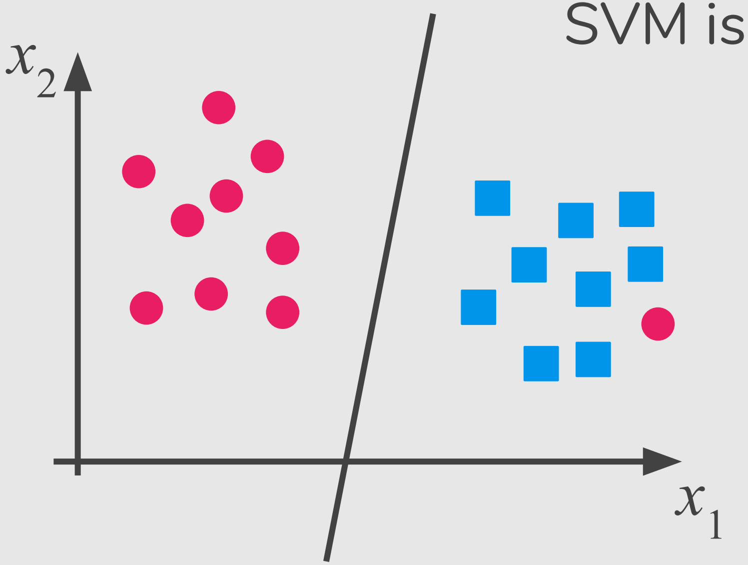
Scenario 4



How can we identify the right hyperplane?

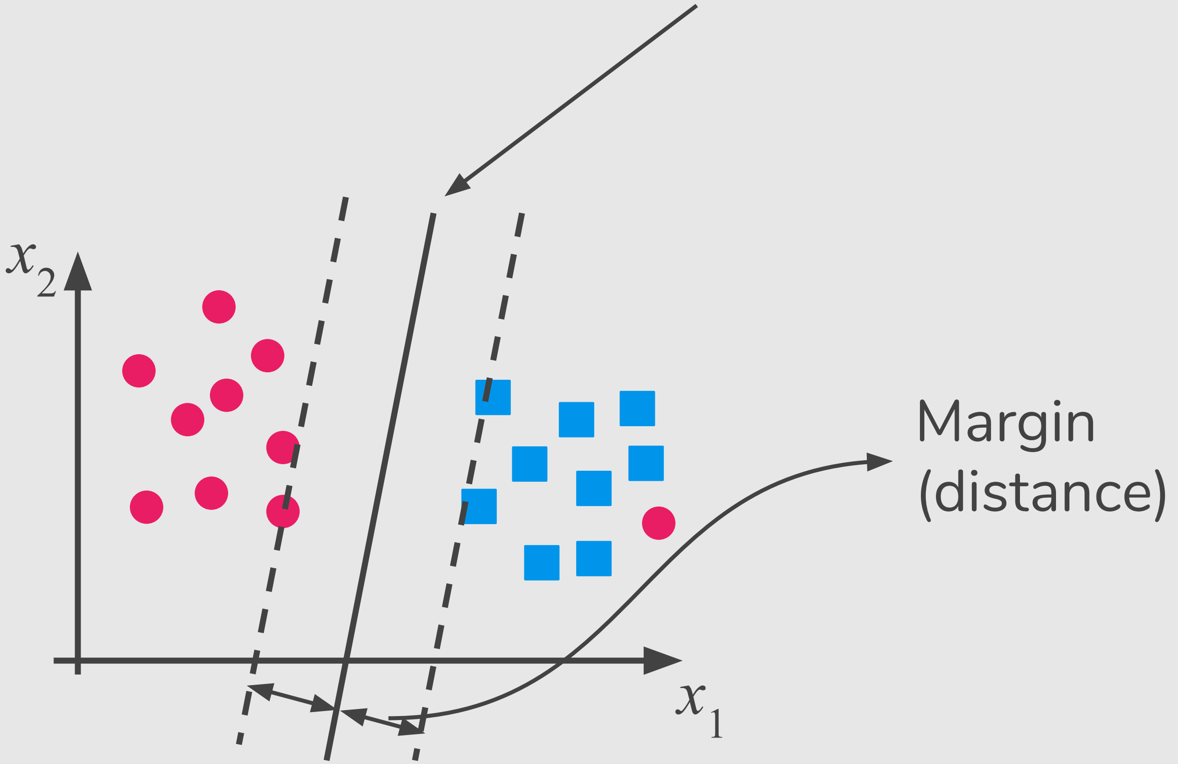
Scenario 4

SVM is robust to outliers



How can we identify the right hyperplane?

Scenario 4



SVM: Notation

We will be considering a **linear classifier for a binary classification** problem with labels y and features x.

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We will be considering a **linear classifier for a binary classification** problem with labels y and features x.

- Class labels: $y \in \{-1,1\}$ (instead of $\{0,1\}$)
- Parameters: w, b (instead of vector θ)
- Classifier: $h_{w,b}(x) = g(w^T x + b)$
  - $g(z) = 1$ if $z \geq 0$, and $g(z) = -1$ otherwise

SVM: The Optimal Hyperplane

Given a training example $(x^{(i)}, y^{(i)})$, we define the margin of (w, b) with respect to the training example:

$$y^{(i)}(w^T x + b) \geq 1, i = \{1, ..., m\}.$$

SVM: The Optimal Hyperplane

Let $P(x^{(1)}, y^{(1)})$ be a point and l be a line defined by $ax + by + c = 0$. The distance d from P to l is defined by:

$$d(l,P) = \frac{|ax^{(1)} + by^{(1)} + c|}{\sqrt{a^2 + b^2}}$$

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$$\Downarrow$$

$$d(w,b,x) = \frac{|w^T x + b|}{||w||}$$

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$$d(w,b,x) = \frac{|w^T x + b|}{||w||}$$

$$\min_{w,b} \ \tfrac{1}{2}||w||^2$$

$$\text{s.t. } y^{(i)}(w^T x + b) \geq 1, \ i = \{1, \ldots, m\}$$

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http://cs229.stanford.edu/notes/cs229-notes3.pdf

$$\min_{w,b} \ \tfrac{1}{2}||w||^2$$
$$\text{s.t. } y^{(i)}(w^T x + b) \geq 1,\ i = \{1, \ldots, m\}$$

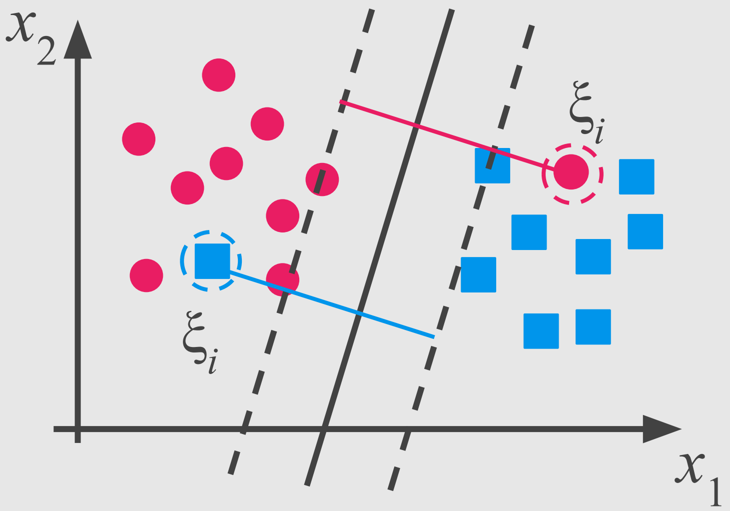
Need to optimize a quadratic function subject to linear constraints.

Soft Margin Classification

What if the training set is not linearly separable?

Soft Margin Classification

Slack variables ξ_i can be added to allow misclassification of difficult or noisy examples, resulting margin called **soft**.



Soft Margin Classification

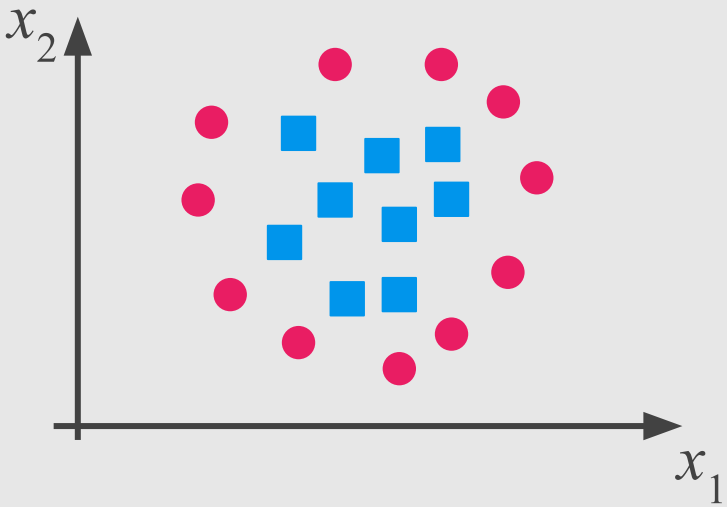
Modified formulation incorporates slack variables:

$$\min_{w,b,\xi} \ \tfrac{1}{2}||w||^2 \ + C\Sigma\xi_i$$
$$\text{s.t. } y_i(w^Tx + b) \geq 1 - \xi_i,\ \xi_i \geq 0,\ i = \{1, \ldots, m\}$$

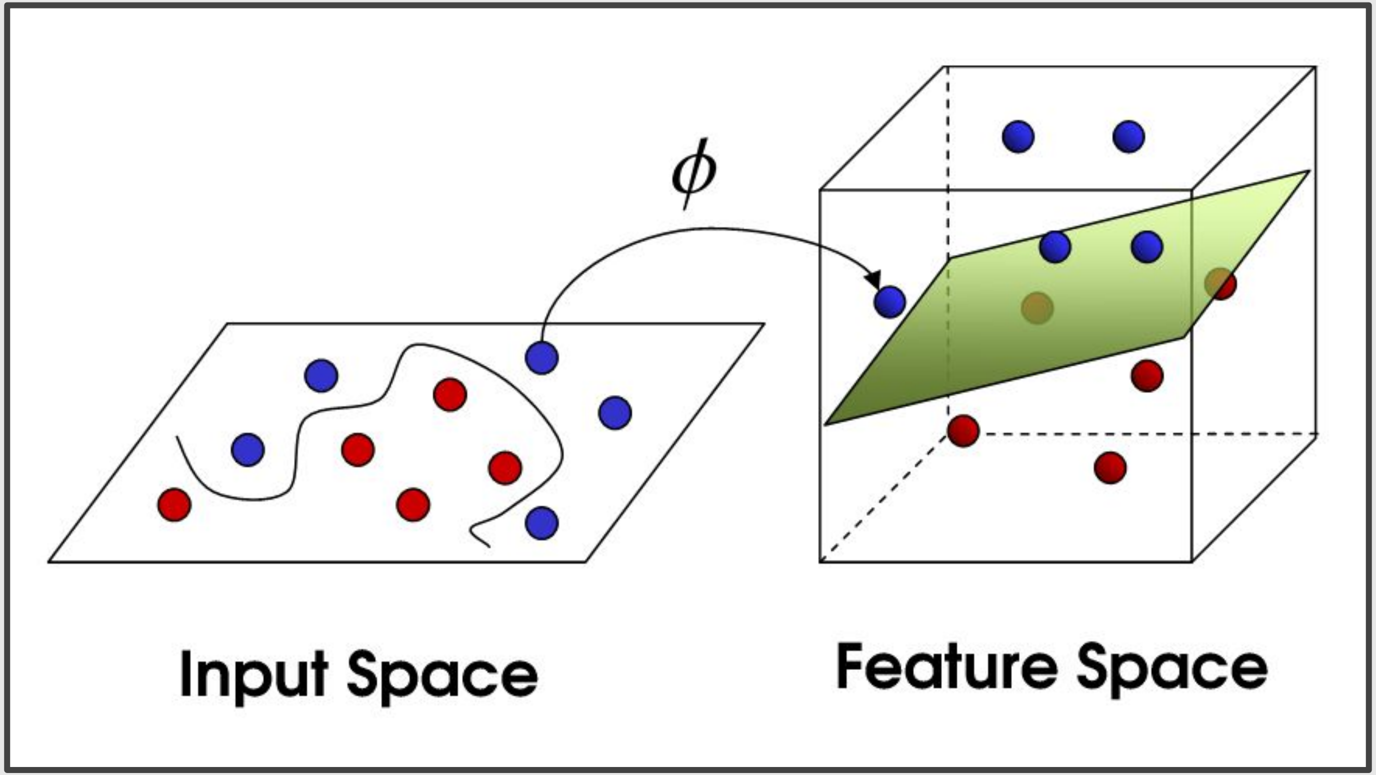
Parameter C can be viewed as a way to control overfitting:
it "trades off" the relative importance of maximizing the margin and fitting the training data.

How can we identify the right hyperplane?

Scenario 5



Kernel Trick



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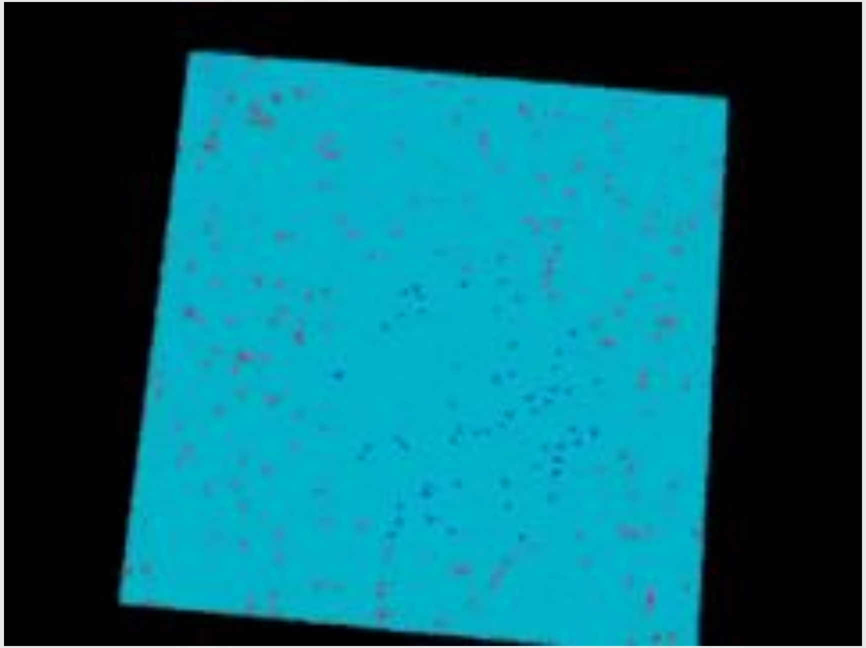

Kernelized SVM

Suppose you want to apply a 2nd degree polynomial transformation to a 2-dimensional training set, then train a linear SVM classifier on the transformed training set.

$$\varphi(x) = \varphi((x_1\ x_2)) = (x_1{}^2\ \ \sqrt{2x_1x_2}\ \ x_2{}^2)$$

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The transformed vector is 3-dimensional instead of 2-dimensional.

Kernelized SVM

Let's look at what happens to a couple 2-dimensional vectors **a** and **b** if we apply this 2nd degree polynomial mapping then compute the dot product of the transformed vectors.

$$\varphi(\mathbf{a})^T \varphi(\mathbf{b}) =$$

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$$\varphi(\mathbf{a})^T\varphi(\mathbf{b}) = (a_1{}^2 \;\; \sqrt{2a_1a_2} \;\; a_2{}^2)^T(b_1{}^2 \;\; \sqrt{2b_1b_2} \;\; b_2{}^2) =$$

Kernelized SVM

Let’s look at what happens to a couple 2-dimensional vectors **a** and **b** if we apply this 2nd degree polynomial mapping then compute the dot product of the transformed vectors.

$$
\begin{aligned}
\varphi(\mathbf{a})^{\mathrm{T}}\varphi(\mathbf{b}) &= (a_1{}^2 \;\; \sqrt{2}a_1a_2 \;\; a_2{}^2)^{\mathrm{T}}(b_1{}^2 \;\; \sqrt{2}b_1b_2 \;\; b_2{}^2) = \\
&= a_1{}^2b_1{}^2 + 2a_1a_2b_1b_2 + a_2{}^2b_2{}^2 = \\
&= (a_1b_1 + a_2b_2)^2 = \\
&= ((a_1 \; a_2)^{\mathrm{T}}(b_1 \; b_2))^2 = \\
&= (\mathbf{a}^{\mathrm{T}} \cdot \mathbf{b})^2
\end{aligned}
$$

Kernelized SVM

The dot product of the transformed vectors is equal to the square of the dot product of the original vectors: $\varphi(\mathbf{a})^{T}\varphi(\mathbf{b}) = (\mathbf{a}^{T} \cdot \mathbf{b})^{2}$

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So **you don't actually need to transform the training instances** at all: just replace the dot product by its square.

This is the essence of the kernel trick.

Kernelized SVM

In Machine Learning, a ***kernel*** is a function capable of computing the dot product $\varphi(\mathbf{a})^T\varphi(\mathbf{b})$ based only on the original vectors **a** and **b**, without having to compute (or even to know about) the transformation φ.

Kernel Trick

- Linear SVM: $x_i \cdot x_j$

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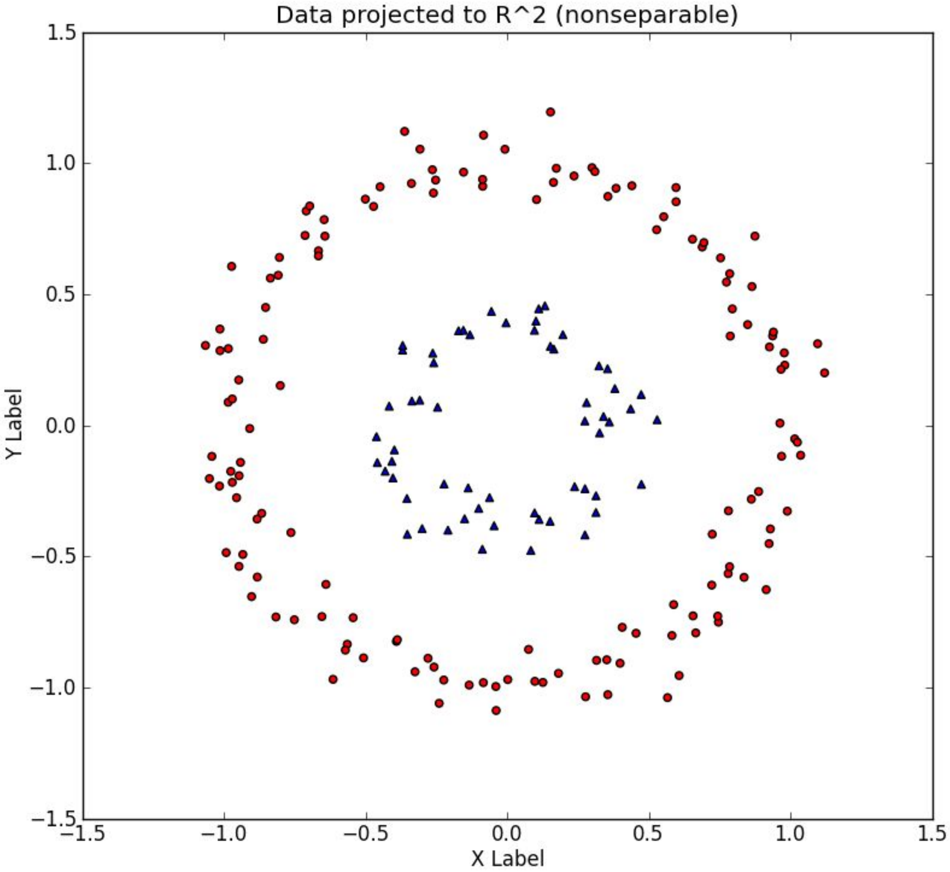
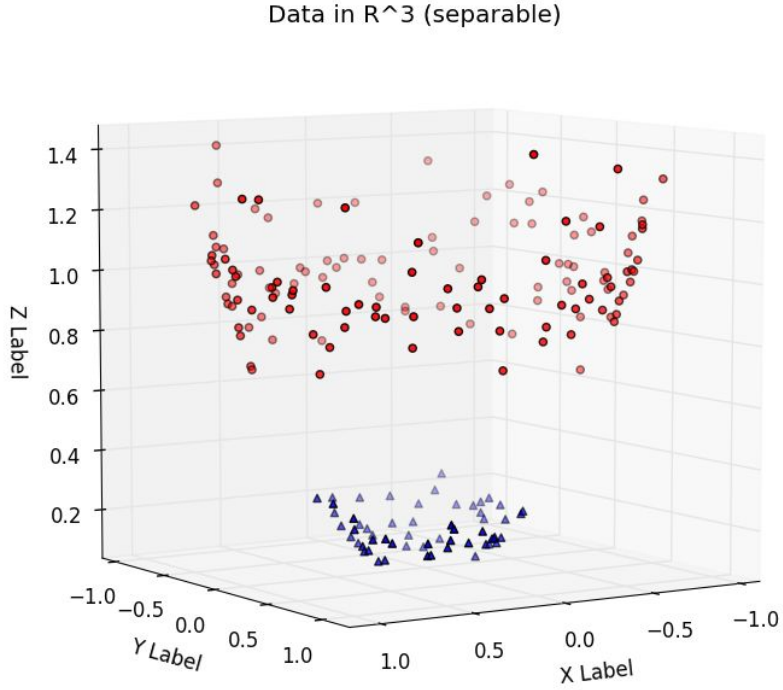
- Radial Basis Function (RBF) kernel: $\exp(-\lambda \|x_i - x_j\|^2)$
- Gaussian kernel: $K(x_i, x_j) = \exp(-\|x_i - x_j\|^2/(2\sigma^2))$
- Polynomial kernel: $K(x_i, x_j) = (x_i \cdot x_j + 1)^{\mathrm{d}}$, d degree
- Chi-square kernel, histogram intersection kernel, string kernel, ...


Data projected to R^2 (nonseparable)
X Label
Y Label


Data in R^3 (separable)
X Label
Y Label
Z Label

SVM & scikit-learn

SVM is also available in scikit-learn library and follow the same structure: import library, object creation, fitting model and prediction.

SVM & scikit-learn

```
#Import Library
from sklearn import svm
#Assumed you have, X (predictor) and Y (target) for training data set and x_test(predictor)
of test_dataset
# Create SVM classification object
model = svm.svc(kernel='linear', c=1, gamma=1)
# there is various option associated with it, like changing kernel, gamma and C value. Will
discuss more # about it in next section.Train the model using the training sets and check s
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model.fit(X, y)
model.score(X, y)
#Predict Output
predicted= model.predict(x_test)
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object creation

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fitting model

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prediction

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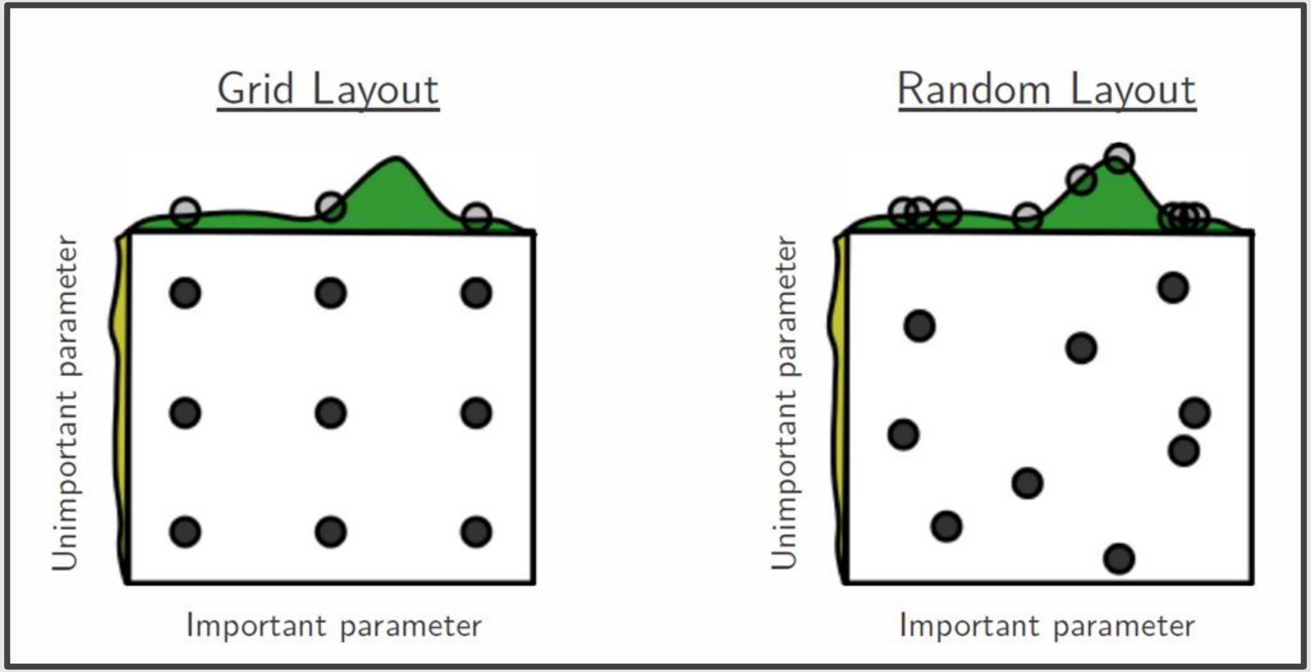
C: Penalty parameter C of the error term. It also controls the trade off between smooth decision boundary and classifying the training points correctly.

The parameters can be tuned using grid-search.

Grid Search



“Random Search for Hyper-Parameter Optimization”: http://www.jmlr.org/papers/volume13/bergstra12a/bergstra12a.pdf

Libraries

- Scikit-learn: https://scikit-learn.org/stable/modules/svm.html
- LIBSVM: https://www.csie.ntu.edu.tw/~cjlin/libsvm
- LIBLINEAR: https://www.csie.ntu.edu.tw/~cjlin/liblinear
- PmSVM: https://sites.google.com/site/wujx2001/home/power-mean-svm

References

Machine Learning Books

- Hands-On Machine Learning with Scikit-Learn and TensorFlow, Chap. 5
- Pattern Recognition and Machine Learning, Chap. 6 & 7

Machine Learning Courses

- https://www.coursera.org/learn/machine-learning, Week 7
- http://cs229.stanford.edu/syllabus.html,
  http://cs229.stanford.edu/notes/cs229-notes3.pdf