# vio_benchmark

May 10, 2024

Notebook for two-view reconstruction with inertial data.

This code is written upon vo_benchmark.ipynb.

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[1]:
```
import symforce
symforce.set_epsilon_to_symbol()

import cv2
import numpy as np
import matplotlib.pyplot as plt
from pathlib import Path
from scipy.spatial.transform import Rotation as R
import time

import vo, vio
# pose_metrics requires evo (see top of pose_metrics.py for install␣
 ↪instructions)
import pose_metrics
import utils

# Note: this notebook requires pandas in addition to all of Prof. Bretl's␣
 ↪dependencies
import pandas as pd
```

### 0.0.1 Read data

[2]:
```
# Specify the dataset (should be 'kitti' or 'euroc')
# chosen_dataset = 'euroc'
chosen_dataset = 'kitti'

assert(chosen_dataset in ['kitti', 'euroc'])
```

### 0.0.2 Provide settings

[3]:
```
# When matching (max threshold for ratio test)
if chosen_dataset == 'euroc':
    matching_threshold = 0.5
else:
    matching_threshold = 0.3

# When deciding if triangulated points are invalid
max_reprojection_err = 0.75

# Temporary folder for evo metrics
temporary_folder = Path('./temp')
temporary_folder.mkdir(parents=True, exist_ok=True)
```

[4]:
```
if chosen_dataset == 'euroc': # Note: euroc takes a bit longer to load.
    # Use EuRoC MAV

    # MAV video folder
    mav_video_folder = Path('./data/mav0')


    # Read MAV data
    dataset_info = utils.read_data_mav(mav_video_folder)
    print("Read dataset with keys: {}".format(sorted(list(dataset_info.
 ↪keys()))))

    # Extract relevant data
    cam0_K = dataset_info['cam0_K']
    cam0_distortion = dataset_info['cam0_distortion']
    visual_inertial_data = dataset_info['visual_inertial_data']

    sigma_acc_wn = dataset_info['imu_accelerometer_noise_density']
    sigma_gyr_wn = dataset_info['imu_gyroscope_noise_density']
    sigma_acc_rw = dataset_info['imu_accelerometer_noise_density']
    sigma_gyr_rw = dataset_info['imu_gyroscope_random_walk']

    dt = 1/200 # IMU frequency

    # Get extrinsics
    T_inB_ofC = dataset_info['cam0_extrinsics']
    T_inC_ofB = np.block([[T_inB_ofC[:3,:3].T, (-T_inB_ofC[:3,:3].T @␣
 ↪T_inB_ofC[:3,-1])[:,np.newaxis]], [np.zeros(3), 1]])

    # Collate
    acc_meas, gyr_meas = utils.imu_collate(dataset_info['visual_inertial_data'])
```

```
    R_inR_ofB, v_inR_ofB, p_inR_ofB, b_a, b_w = utils.
↪groundtruth_collate(dataset_info['visual_inertial_data'], True)

    # As EuRoC's ground-truth (MoCap) is not aligned with gravity (i.e., in␣
↪world frame), we identify the orientation of MoCap frame in world frame
    gravity = np.array([0., 0., -9.81])

    g_inB = - np.mean(acc_meas[:10], axis=0)
    g_inW = gravity

    def align_vectors(g_inB, g_inW):
        # Normalize input vectors
        g_inB_unit = g_inB / np.linalg.norm(g_inB)
        g_inW_unit = g_inW / np.linalg.norm(g_inW)

        # Compute the axis of rotation
        v = np.cross(g_inB_unit, g_inW_unit)

        # Compute the angle of rotation
        cos_theta = np.dot(g_inB_unit, g_inW_unit)
        sin_theta = np.linalg.norm(v)
        theta = np.arctan2(sin_theta, cos_theta)

        v /= np.linalg.norm(v)

        # Compute the rotation matrix
        Rot = R.from_rotvec(theta*v)
        return Rot

    # Compute rotation matrix
    R_inW_ofB = align_vectors(g_inB, g_inW)
    print("Rotation Matrix:\n", R_inW_ofB.as_matrix())

    R_inW_ofB.apply(g_inB)

    R_inW_ofR = R_inW_ofB * R_inR_ofB[:10].mean().inv()
    R_inW_ofB = R_inW_ofR * R_inR_ofB
    v_inW_ofB = R_inW_ofR.apply(v_inR_ofB)
    p_inW_ofB = R_inW_ofR.apply(p_inR_ofB)




else:
    # Use KITTI
    kitti_base_path = './data/kitti'
    kitti_date = '2011_09_26'
```

```
    kitti_drive = '0022'

    # Read KITTI data
    dataset_info = utils.read_data_kitti(kitti_base_path, kitti_date,␣
↪kitti_drive)

    # Extract relevant data
    cam0_K = dataset_info['cam0_K']
    cam0_distortion = dataset_info['cam0_distortion']
    visual_inertial_data = dataset_info['visual_inertial_data']

    T_inC_ofB = dataset_info['cam0_extrinsics']
    R_inB_of_C = T_inC_ofB[:3, :3].T
    t_inB_of_C = R_inB_of_C @ T_inC_ofB[:3, 3]
    T_inB_ofC = np.block([[R_inB_of_C, t_inB_of_C[:,np.newaxis]], [np.zeros(3),␣
↪1]])

    sigma_acc_wn = 1e-4  # accelerometer white noise sigma
    sigma_gyr_wn = 1e-6  # gyroscope white noise sigma
    sigma_acc_rw = 1e-5  # accelerometer random walk sigma
    sigma_gyr_rw = 1e-7  # gyroscope random walk sigma

    # KITTI has data at 10 Hz
    dt = 0.1

    # Collate
    acc_meas, gyr_meas = utils.imu_collate(visual_inertial_data)
    R_inW_ofB, v_inW_ofB, p_inW_ofB, b_a, b_w = utils.
↪groundtruth_collate(visual_inertial_data, False)
```

### 0.0.3 Create random generator

[5]:
```
rng = utils.create_rng(42)
```

```
seeding RNG with 42
```

### 0.0.4 Create image keypoint feature extractor

[6]:
```
feature_extractor = cv2.SIFT_create() # could also do ORB_create() for ORB␣
↪features
```

### 0.0.5 Two view reconstruction

Get initial solution

[7]:
```
if chosen_dataset == 'euroc': # Note: euroc takes a bit longer to load.
    # Use EuRoC MAV
    chosen_index = 500
    advance = 100
else:
    chosen_index = 50
    advance = 5


# Get first index closest to chosen index
first_frame_idx = utils.get_index_of_next_image(visual_inertial_data,␣
 ↪chosen_index)
# Get second index
second_frame_idx = utils.get_index_of_next_image(visual_inertial_data,␣
 ↪first_frame_idx+advance)

# Create two views
views = [
    vio.create_view_data(utils.
 ↪read_image(visual_inertial_data[first_frame_idx]['image_file']),
                         first_frame_idx, feature_extractor, cam0_K,␣
 ↪cam0_distortion),
    vio.create_view_data(utils.
 ↪read_image(visual_inertial_data[second_frame_idx]['image_file']),
                         second_frame_idx, feature_extractor, cam0_K,␣
 ↪cam0_distortion)
]

# Perform two-view reconstruction
R_inC_ofB = T_inC_ofB[:3,:3]
p_inC_ofB = T_inC_ofB[:3,-1]

R_inC_ofW = R_inC_ofB @ R_inW_ofB[first_frame_idx].as_matrix().T
p_inC_ofW = - R_inC_ofB @ R_inW_ofB[first_frame_idx].as_matrix().T @␣
 ↪p_inW_ofB[first_frame_idx] + p_inC_ofB

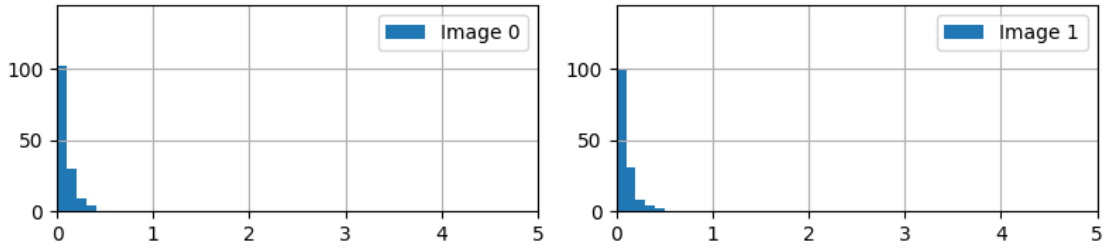
tic = time.time()
tracks = vio.vo_2view(views, matching_threshold, cam0_K, R_inC_ofW, p_inC_ofW,␣
 ↪rng, use_opencv=False)
toc = time.time()

analyctical_guess = toc - tic
print(f"Analytical guess: {analyctical_guess:.2f} [s]")
```

```
found 145 good matches
found 145 inliers
Analytical guess: 2.83 [s]
```

[8]:
```
vio.show_reproj_results(views, tracks, cam0_K, cam0_distortion,␣
↪print_raw_reproj=True, show_reproj_histogram=True)
vio.visualize_predictions(views, tracks, cam0_K, cam0_distortion)
```

```
REPROJECTION ERRORS
 Image  0 (  145 points) : (mean, std, max, min) = (0.0868, 0.0800, 0.4025,
0.0004)
 Image (raw reprojection)  0 (  145 points) : (mean, std, max, min) = (0.0868,
0.0800, 0.4025, 0.0004)
 Image  1 (  145 points) : (mean, std, max, min) = (0.0931, 0.0883, 0.4770,
0.0004)
 Image (raw reprojection)  1 (  145 points) : (mean, std, max, min) = (0.0931,
0.0883, 0.4770, 0.0004)
```




Get post-optimization solution

Run below to keep the initial views and tracks

[9]:
```
views_ini = views.copy()
tracks_ini = tracks.copy()
```

Run two-view reconstruction with inertial data

[10]:
```
T_inB0_ofW = np.block([[R_inW_ofB[first_frame_idx].as_matrix().T, -␣
↪R_inW_ofB[first_frame_idx].as_matrix().T @ p_inW_ofB[first_frame_idx][:,np.
↪newaxis]],
                       [np.zeros(3), 1]])
T_inC0_ofW = T_inC_ofB @ T_inB0_ofW
```

```
T_inB1_ofW = np.block([[R_inW_ofB[second_frame_idx].as_matrix().T, -␣
 ↪R_inW_ofB[second_frame_idx].as_matrix().T @ p_inW_ofB[second_frame_idx][:,np.
 ↪newaxis]],
                       [np.zeros(3), 1]])
T_inC1_ofW = T_inC_ofB @ T_inB1_ofW

T_inC0_ofW = None   # comment this out when you'd like to use ground truth as␣
 ↪initial guess
T_inC1_ofW = None   # comment this out when you'd like to use ground truth as␣
 ↪initial guess

tic = time.time()



views, tracks, initial_values, results = vio.vio_nonlinear_optimize(views_ini,␣
 ↪tracks_ini, acc_meas[first_frame_idx:second_frame_idx],␣
 ↪gyr_meas[first_frame_idx:second_frame_idx],
                                                                     cam0_K,␣
 ↪T_inC_ofB, max_reprojection_err,
                                                                    ␣
 ↪sigma_acc_wn, sigma_gyr_wn, sigma_acc_rw, sigma_gyr_rw, dt,
                                                                     np.
 ↪mean(b_a, axis=0), np.mean(b_a, axis=0), np.mean(b_w, axis=0), np.mean(b_w,␣
 ↪axis=0),
                                                                    ␣
 ↪v_inW_ofB[first_frame_idx], v_inW_ofB[second_frame_idx],
                                                                     T_inC0_ofW,␣
 ↪T_inC1_ofW)

toc = time.time()

nonlinear = toc - tic
print(f"{nonlinear:.2f} [s]")
```

```
[2024-05-10 16:46:28.388] [info] LM<sym::Optimize> [iter    0] lambda:
1.000e+00, error prev/linear/new: 1.403e+10/0.000e+00/8.029e+08, rel reduction:
9.42758e-01
[2024-05-10 16:46:28.410] [info] LM<sym::Optimize> [iter    1] lambda:
1.000e-01, error prev/linear/new: 8.029e+08/0.000e+00/4.394e+06, rel reduction:
9.94527e-01
[2024-05-10 16:46:28.430] [info] LM<sym::Optimize> [iter    2] lambda:
1.000e-02, error prev/linear/new: 4.394e+06/0.000e+00/1.111e+06, rel reduction:
7.47203e-01
[2024-05-10 16:46:28.451] [info] LM<sym::Optimize> [iter    3] lambda:
1.000e-03, error prev/linear/new: 1.111e+06/0.000e+00/3.419e+05, rel reduction:
6.92208e-01
```

```
[2024-05-10 16:46:28.471] [info] LM<sym::Optimize> [iter    4] lambda:
1.000e-04, error prev/linear/new: 3.419e+05/0.000e+00/1.298e+05, rel reduction:
6.20440e-01
[2024-05-10 16:46:28.492] [info] LM<sym::Optimize> [iter    5] lambda:
1.000e-05, error prev/linear/new: 1.298e+05/0.000e+00/3.118e+03, rel reduction:
9.75970e-01
[2024-05-10 16:46:28.513] [info] LM<sym::Optimize> [iter    6] lambda:
1.000e-06, error prev/linear/new: 3.118e+03/0.000e+00/4.533e+02, rel reduction:
8.54637e-01
[2024-05-10 16:46:28.533] [info] LM<sym::Optimize> [iter    7] lambda:
1.000e-07, error prev/linear/new: 4.533e+02/0.000e+00/4.239e+02, rel reduction:
6.47951e-02
[2024-05-10 16:46:28.554] [info] LM<sym::Optimize> [iter    8] lambda:
1.000e-08, error prev/linear/new: 4.239e+02/0.000e+00/4.120e+02, rel reduction:
2.80140e-02
[2024-05-10 16:46:28.575] [info] LM<sym::Optimize> [iter    9] lambda:
1.000e-09, error prev/linear/new: 4.120e+02/0.000e+00/6.917e+02, rel reduction:
-6.78909e-01
[2024-05-10 16:46:28.596] [info] LM<sym::Optimize> [iter   10] lambda:
5.000e-09, error prev/linear/new: 4.120e+02/0.000e+00/3.936e+02, rel reduction:
4.47207e-02
[2024-05-10 16:46:28.617] [info] LM<sym::Optimize> [iter   11] lambda:
5.000e-10, error prev/linear/new: 3.936e+02/0.000e+00/1.047e+04, rel reduction:
-2.56031e+01
[2024-05-10 16:46:28.639] [info] LM<sym::Optimize> [iter   12] lambda:
2.500e-09, error prev/linear/new: 3.936e+02/0.000e+00/4.642e+02, rel reduction:
-1.79497e-01
[2024-05-10 16:46:28.660] [info] LM<sym::Optimize> [iter   13] lambda:
1.250e-08, error prev/linear/new: 3.936e+02/0.000e+00/3.814e+02, rel reduction:
3.10055e-02
[2024-05-10 16:46:28.681] [info] LM<sym::Optimize> [iter   14] lambda:
1.250e-09, error prev/linear/new: 3.814e+02/0.000e+00/4.158e+02, rel reduction:
-9.02693e-02
[2024-05-10 16:46:28.701] [info] LM<sym::Optimize> [iter   15] lambda:
6.250e-09, error prev/linear/new: 3.814e+02/0.000e+00/3.723e+02, rel reduction:
2.38570e-02
[2024-05-10 16:46:28.722] [info] LM<sym::Optimize> [iter   16] lambda:
6.250e-10, error prev/linear/new: 3.723e+02/0.000e+00/2.145e+03, rel reduction:
-4.76037e+00
[2024-05-10 16:46:28.742] [info] LM<sym::Optimize> [iter   17] lambda:
3.125e-09, error prev/linear/new: 3.723e+02/0.000e+00/5.087e+02, rel reduction:
-3.66440e-01
[2024-05-10 16:46:28.763] [info] LM<sym::Optimize> [iter   18] lambda:
1.563e-08, error prev/linear/new: 3.723e+02/0.000e+00/3.525e+02, rel reduction:
5.31907e-02
[2024-05-10 16:46:28.783] [info] LM<sym::Optimize> [iter   19] lambda:
1.563e-09, error prev/linear/new: 3.525e+02/0.000e+00/3.591e+02, rel reduction:
-1.87896e-02
```

```
[2024-05-10 16:46:28.804] [info] LM<sym::Optimize> [iter   20] lambda:
7.813e-09, error prev/linear/new: 3.525e+02/0.000e+00/3.444e+02, rel reduction:
2.28343e-02
[2024-05-10 16:46:28.825] [info] LM<sym::Optimize> [iter   21] lambda:
7.813e-10, error prev/linear/new: 3.444e+02/0.000e+00/3.725e+02, rel reduction:
-8.13411e-02
[2024-05-10 16:46:28.846] [info] LM<sym::Optimize> [iter   22] lambda:
3.906e-09, error prev/linear/new: 3.444e+02/0.000e+00/3.539e+02, rel reduction:
-2.74886e-02
[2024-05-10 16:46:28.867] [info] LM<sym::Optimize> [iter   23] lambda:
1.953e-08, error prev/linear/new: 3.444e+02/0.000e+00/3.479e+02, rel reduction:
-1.00509e-02
[2024-05-10 16:46:28.887] [info] LM<sym::Optimize> [iter   24] lambda:
9.766e-08, error prev/linear/new: 3.444e+02/0.000e+00/3.443e+02, rel reduction:
3.55419e-04
[2024-05-10 16:46:28.908] [info] LM<sym::Optimize> [iter   25] lambda:
9.766e-09, error prev/linear/new: 3.443e+02/0.000e+00/3.512e+02, rel reduction:
-1.99134e-02
[2024-05-10 16:46:28.929] [info] LM<sym::Optimize> [iter   26] lambda:
4.883e-08, error prev/linear/new: 3.443e+02/0.000e+00/3.453e+02, rel reduction:
-2.75137e-03
[2024-05-10 16:46:28.950] [info] LM<sym::Optimize> [iter   27] lambda:
2.441e-07, error prev/linear/new: 3.443e+02/0.000e+00/3.419e+02, rel reduction:
7.09311e-03
[2024-05-10 16:46:28.971] [info] LM<sym::Optimize> [iter   28] lambda:
2.441e-08, error prev/linear/new: 3.419e+02/0.000e+00/3.444e+02, rel reduction:
-7.29467e-03
[2024-05-10 16:46:28.991] [info] LM<sym::Optimize> [iter   29] lambda:
1.221e-07, error prev/linear/new: 3.419e+02/0.000e+00/3.427e+02, rel reduction:
-2.54460e-03
[2024-05-10 16:46:29.013] [info] LM<sym::Optimize> [iter   30] lambda:
6.104e-07, error prev/linear/new: 3.419e+02/0.000e+00/3.390e+02, rel reduction:
8.54260e-03
[2024-05-10 16:46:29.034] [info] LM<sym::Optimize> [iter   31] lambda:
6.104e-08, error prev/linear/new: 3.390e+02/0.000e+00/3.386e+02, rel reduction:
9.34787e-04
[2024-05-10 16:46:29.055] [info] LM<sym::Optimize> [iter   32] lambda:
6.104e-09, error prev/linear/new: 3.386e+02/0.000e+00/3.437e+02, rel reduction:
-1.49103e-02
[2024-05-10 16:46:29.076] [info] LM<sym::Optimize> [iter   33] lambda:
3.052e-08, error prev/linear/new: 3.386e+02/0.000e+00/3.396e+02, rel reduction:
-2.74348e-03
[2024-05-10 16:46:29.097] [info] LM<sym::Optimize> [iter   34] lambda:
1.526e-07, error prev/linear/new: 3.386e+02/0.000e+00/3.378e+02, rel reduction:
2.61886e-03
[2024-05-10 16:46:29.118] [info] LM<sym::Optimize> [iter   35] lambda:
1.526e-08, error prev/linear/new: 3.378e+02/0.000e+00/3.411e+02, rel reduction:
-9.94762e-03
```

```
[2024-05-10 16:46:29.139] [info] LM<sym::Optimize> [iter   36] lambda:
7.629e-08, error prev/linear/new: 3.378e+02/0.000e+00/3.370e+02, rel reduction:
2.13109e-03
[2024-05-10 16:46:29.159] [info] LM<sym::Optimize> [iter   37] lambda:
7.629e-09, error prev/linear/new: 3.370e+02/0.000e+00/3.407e+02, rel reduction:
-1.07450e-02
[2024-05-10 16:46:29.179] [info] LM<sym::Optimize> [iter   38] lambda:
3.815e-08, error prev/linear/new: 3.370e+02/0.000e+00/3.407e+02, rel reduction:
-1.08468e-02
[2024-05-10 16:46:29.200] [info] LM<sym::Optimize> [iter   39] lambda:
1.907e-07, error prev/linear/new: 3.370e+02/0.000e+00/3.366e+02, rel reduction:
1.35205e-03
[2024-05-10 16:46:29.221] [info] LM<sym::Optimize> [iter   40] lambda:
1.907e-08, error prev/linear/new: 3.366e+02/0.000e+00/3.362e+02, rel reduction:
1.16573e-03
[2024-05-10 16:46:29.241] [info] LM<sym::Optimize> [iter   41] lambda:
1.907e-09, error prev/linear/new: 3.362e+02/0.000e+00/3.400e+02, rel reduction:
-1.13201e-02
[2024-05-10 16:46:29.262] [info] LM<sym::Optimize> [iter   42] lambda:
9.537e-09, error prev/linear/new: 3.362e+02/0.000e+00/3.359e+02, rel reduction:
7.14514e-04
[2024-05-10 16:46:29.283] [info] LM<sym::Optimize> [iter   43] lambda:
9.537e-10, error prev/linear/new: 3.359e+02/0.000e+00/3.606e+02, rel reduction:
-7.33815e-02
[2024-05-10 16:46:29.305] [info] LM<sym::Optimize> [iter   44] lambda:
4.768e-09, error prev/linear/new: 3.359e+02/0.000e+00/3.399e+02, rel reduction:
-1.17373e-02
[2024-05-10 16:46:29.326] [info] LM<sym::Optimize> [iter   45] lambda:
2.384e-08, error prev/linear/new: 3.359e+02/0.000e+00/3.378e+02, rel reduction:
-5.67186e-03
[2024-05-10 16:46:29.347] [info] LM<sym::Optimize> [iter   46] lambda:
1.192e-07, error prev/linear/new: 3.359e+02/0.000e+00/3.358e+02, rel reduction:
5.46960e-04
[2024-05-10 16:46:29.368] [info] LM<sym::Optimize> [iter   47] lambda:
1.192e-08, error prev/linear/new: 3.358e+02/0.000e+00/3.376e+02, rel reduction:
-5.60438e-03
[2024-05-10 16:46:29.388] [info] LM<sym::Optimize> [iter   48] lambda:
5.960e-08, error prev/linear/new: 3.358e+02/0.000e+00/3.356e+02, rel reduction:
5.00844e-04
[2024-05-10 16:46:29.409] [info] LM<sym::Optimize> [iter   49] lambda:
5.960e-09, error prev/linear/new: 3.356e+02/0.000e+00/3.375e+02, rel reduction:
-5.71874e-03
[2024-05-10 16:46:29.430] [info] LM<sym::Optimize> [iter   50] lambda:
2.980e-08, error prev/linear/new: 3.356e+02/0.000e+00/3.375e+02, rel reduction:
-5.81976e-03
[2024-05-10 16:46:29.456] [info] LM<sym::Optimize> [iter   51] lambda:
1.490e-07, error prev/linear/new: 3.356e+02/0.000e+00/3.355e+02, rel reduction:
3.21244e-04
```

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[2024-05-10 16:46:29.477] [info] LM<sym::Optimize> [iter   52] lambda:
1.490e-08, error prev/linear/new: 3.355e+02/0.000e+00/3.375e+02, rel reduction:
-5.86540e-03
[2024-05-10 16:46:29.498] [info] LM<sym::Optimize> [iter   53] lambda:
7.451e-08, error prev/linear/new: 3.355e+02/0.000e+00/3.353e+02, rel reduction:
4.63973e-04
[2024-05-10 16:46:29.520] [info] LM<sym::Optimize> [iter   54] lambda:
7.451e-09, error prev/linear/new: 3.353e+02/0.000e+00/3.349e+02, rel reduction:
1.23176e-03
[2024-05-10 16:46:29.541] [info] LM<sym::Optimize> [iter   55] lambda:
7.451e-10, error prev/linear/new: 3.349e+02/0.000e+00/3.594e+02, rel reduction:
-7.31116e-02
[2024-05-10 16:46:29.561] [info] LM<sym::Optimize> [iter   56] lambda:
3.725e-09, error prev/linear/new: 3.349e+02/0.000e+00/3.470e+02, rel reduction:
-3.61090e-02
[2024-05-10 16:46:29.582] [info] LM<sym::Optimize> [iter   57] lambda:
1.863e-08, error prev/linear/new: 3.349e+02/0.000e+00/3.408e+02, rel reduction:
-1.76808e-02
[2024-05-10 16:46:29.603] [info] LM<sym::Optimize> [iter   58] lambda:
9.313e-08, error prev/linear/new: 3.349e+02/0.000e+00/3.367e+02, rel reduction:
-5.38267e-03
[2024-05-10 16:46:29.624] [info] LM<sym::Optimize> [iter   59] lambda:
4.657e-07, error prev/linear/new: 3.349e+02/0.000e+00/3.346e+02, rel reduction:
9.06720e-04
[2024-05-10 16:46:29.650] [info] LM<sym::Optimize> [iter   60] lambda:
4.657e-08, error prev/linear/new: 3.346e+02/0.000e+00/3.334e+02, rel reduction:
3.73563e-03
[2024-05-10 16:46:29.671] [info] LM<sym::Optimize> [iter   61] lambda:
4.657e-09, error prev/linear/new: 3.334e+02/0.000e+00/3.395e+02, rel reduction:
-1.82822e-02
[2024-05-10 16:46:29.691] [info] LM<sym::Optimize> [iter   62] lambda:
2.328e-08, error prev/linear/new: 3.334e+02/0.000e+00/3.332e+02, rel reduction:
4.03921e-04
[2024-05-10 16:46:29.712] [info] LM<sym::Optimize> [iter   63] lambda:
2.328e-09, error prev/linear/new: 3.332e+02/0.000e+00/3.456e+02, rel reduction:
-3.71680e-02
[2024-05-10 16:46:29.734] [info] LM<sym::Optimize> [iter   64] lambda:
1.164e-08, error prev/linear/new: 3.332e+02/0.000e+00/3.393e+02, rel reduction:
-1.83321e-02
[2024-05-10 16:46:29.757] [info] LM<sym::Optimize> [iter   65] lambda:
5.821e-08, error prev/linear/new: 3.332e+02/0.000e+00/3.352e+02, rel reduction:
-5.81071e-03
[2024-05-10 16:46:29.778] [info] LM<sym::Optimize> [iter   66] lambda:
2.910e-07, error prev/linear/new: 3.332e+02/0.000e+00/3.352e+02, rel reduction:
-5.92610e-03
[2024-05-10 16:46:29.799] [info] LM<sym::Optimize> [iter   67] lambda:
1.455e-06, error prev/linear/new: 3.332e+02/0.000e+00/3.330e+02, rel reduction:
5.71680e-04
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[2024-05-10 16:46:29.819] [info] LM<sym::Optimize> [iter   68] lambda:
1.455e-07, error prev/linear/new: 3.330e+02/0.000e+00/3.327e+02, rel reduction:
9.73558e-04
[2024-05-10 16:46:29.840] [info] LM<sym::Optimize> [iter   69] lambda:
1.455e-08, error prev/linear/new: 3.327e+02/0.000e+00/3.366e+02, rel reduction:
-1.18113e-02
[2024-05-10 16:46:29.861] [info] LM<sym::Optimize> [iter   70] lambda:
7.276e-08, error prev/linear/new: 3.327e+02/0.000e+00/3.325e+02, rel reduction:
7.63083e-04
[2024-05-10 16:46:29.882] [info] LM<sym::Optimize> [iter   71] lambda:
7.276e-09, error prev/linear/new: 3.325e+02/0.000e+00/3.347e+02, rel reduction:
-6.59985e-03
[2024-05-10 16:46:29.902] [info] LM<sym::Optimize> [iter   72] lambda:
3.638e-08, error prev/linear/new: 3.325e+02/0.000e+00/3.328e+02, rel reduction:
-8.98677e-04
[2024-05-10 16:46:29.930] [info] LM<sym::Optimize> [iter   73] lambda:
1.819e-07, error prev/linear/new: 3.325e+02/0.000e+00/3.328e+02, rel reduction:
-1.11849e-03
[2024-05-10 16:46:29.951] [info] LM<sym::Optimize> [iter   74] lambda:
9.095e-07, error prev/linear/new: 3.325e+02/0.000e+00/3.329e+02, rel reduction:
-1.19391e-03
[2024-05-10 16:46:29.973] [info] LM<sym::Optimize> [iter   75] lambda:
4.547e-06, error prev/linear/new: 3.325e+02/0.000e+00/3.304e+02, rel reduction:
6.24044e-03
[2024-05-10 16:46:29.995] [info] LM<sym::Optimize> [iter   76] lambda:
4.547e-07, error prev/linear/new: 3.304e+02/0.000e+00/3.298e+02, rel reduction:
1.72502e-03
[2024-05-10 16:46:30.015] [info] LM<sym::Optimize> [iter   77] lambda:
4.547e-08, error prev/linear/new: 3.298e+02/0.000e+00/3.296e+02, rel reduction:
5.83064e-04
[2024-05-10 16:46:30.037] [info] LM<sym::Optimize> [iter   78] lambda:
4.547e-09, error prev/linear/new: 3.296e+02/0.000e+00/3.358e+02, rel reduction:
-1.86057e-02
[2024-05-10 16:46:30.059] [info] LM<sym::Optimize> [iter   79] lambda:
2.274e-08, error prev/linear/new: 3.296e+02/0.000e+00/3.316e+02, rel reduction:
-5.93990e-03
[2024-05-10 16:46:30.080] [info] LM<sym::Optimize> [iter   80] lambda:
1.137e-07, error prev/linear/new: 3.296e+02/0.000e+00/3.295e+02, rel reduction:
3.51877e-04
[2024-05-10 16:46:30.101] [info] LM<sym::Optimize> [iter   81] lambda:
1.137e-08, error prev/linear/new: 3.295e+02/0.000e+00/3.292e+02, rel reduction:
8.31941e-04
[2024-05-10 16:46:30.122] [info] LM<sym::Optimize> [iter   82] lambda:
1.137e-09, error prev/linear/new: 3.292e+02/0.000e+00/3.415e+02, rel reduction:
-3.73469e-02
[2024-05-10 16:46:30.149] [info] LM<sym::Optimize> [iter   83] lambda:
5.684e-09, error prev/linear/new: 3.292e+02/0.000e+00/3.354e+02, rel reduction:
-1.86329e-02
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[2024-05-10 16:46:30.170] [info] LM<sym::Optimize> [iter   84] lambda:
2.842e-08, error prev/linear/new: 3.292e+02/0.000e+00/3.333e+02, rel reduction:
-1.24256e-02
[2024-05-10 16:46:30.192] [info] LM<sym::Optimize> [iter   85] lambda:
1.421e-07, error prev/linear/new: 3.292e+02/0.000e+00/3.313e+02, rel reduction:
-6.14509e-03
[2024-05-10 16:46:30.213] [info] LM<sym::Optimize> [iter   86] lambda:
7.105e-07, error prev/linear/new: 3.292e+02/0.000e+00/3.292e+02, rel reduction:
2.14323e-04
[2024-05-10 16:46:30.233] [info] LM<sym::Optimize> [iter   87] lambda:
7.105e-08, error prev/linear/new: 3.292e+02/0.000e+00/3.312e+02, rel reduction:
-6.12971e-03
[2024-05-10 16:46:30.254] [info] LM<sym::Optimize> [iter   88] lambda:
3.553e-07, error prev/linear/new: 3.292e+02/0.000e+00/3.290e+02, rel reduction:
4.91561e-04
[2024-05-10 16:46:30.275] [info] LM<sym::Optimize> [iter   89] lambda:
3.553e-08, error prev/linear/new: 3.290e+02/0.000e+00/3.307e+02, rel reduction:
-5.27332e-03
[2024-05-10 16:46:30.296] [info] LM<sym::Optimize> [iter   90] lambda:
1.776e-07, error prev/linear/new: 3.290e+02/0.000e+00/3.288e+02, rel reduction:
6.48901e-04
[2024-05-10 16:46:30.316] [info] LM<sym::Optimize> [iter   91] lambda:
1.776e-08, error prev/linear/new: 3.288e+02/0.000e+00/3.303e+02, rel reduction:
-4.71824e-03
[2024-05-10 16:46:30.337] [info] LM<sym::Optimize> [iter   92] lambda:
8.882e-08, error prev/linear/new: 3.288e+02/0.000e+00/3.305e+02, rel reduction:
-5.32834e-03
[2024-05-10 16:46:30.358] [info] LM<sym::Optimize> [iter   93] lambda:
4.441e-07, error prev/linear/new: 3.288e+02/0.000e+00/3.286e+02, rel reduction:
5.86684e-04
[2024-05-10 16:46:30.379] [info] LM<sym::Optimize> [iter   94] lambda:
4.441e-08, error prev/linear/new: 3.286e+02/0.000e+00/3.302e+02, rel reduction:
-4.78595e-03
[2024-05-10 16:46:30.401] [info] LM<sym::Optimize> [iter   95] lambda:
2.220e-07, error prev/linear/new: 3.286e+02/0.000e+00/3.304e+02, rel reduction:
-5.59679e-03
[2024-05-10 16:46:30.423] [info] LM<sym::Optimize> [iter   96] lambda:
1.110e-06, error prev/linear/new: 3.286e+02/0.000e+00/3.281e+02, rel reduction:
1.35377e-03
[2024-05-10 16:46:30.444] [info] LM<sym::Optimize> [iter   97] lambda:
1.110e-07, error prev/linear/new: 3.281e+02/0.000e+00/3.301e+02, rel reduction:
-6.06092e-03
[2024-05-10 16:46:30.464] [info] LM<sym::Optimize> [iter   98] lambda:
5.551e-07, error prev/linear/new: 3.281e+02/0.000e+00/3.304e+02, rel reduction:
-6.83269e-03
[2024-05-10 16:46:30.485] [info] LM<sym::Optimize> [iter   99] lambda:
2.776e-06, error prev/linear/new: 3.281e+02/0.000e+00/3.305e+02, rel reduction:
-7.11393e-03
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[2024-05-10 16:46:30.505] [info] LM<sym::Optimize> [iter  100] lambda:
1.388e-05, error prev/linear/new: 3.281e+02/0.000e+00/3.305e+02, rel reduction:
-7.18505e-03
[2024-05-10 16:46:30.526] [info] LM<sym::Optimize> [iter  101] lambda:
6.939e-05, error prev/linear/new: 3.281e+02/0.000e+00/3.269e+02, rel reduction:
3.73420e-03
[2024-05-10 16:46:30.554] [info] LM<sym::Optimize> [iter  102] lambda:
6.939e-06, error prev/linear/new: 3.269e+02/0.000e+00/3.265e+02, rel reduction:
1.34774e-03
[2024-05-10 16:46:30.576] [info] LM<sym::Optimize> [iter  103] lambda:
6.939e-07, error prev/linear/new: 3.265e+02/0.000e+00/3.263e+02, rel reduction:
4.96098e-04
[2024-05-10 16:46:30.598] [info] LM<sym::Optimize> [iter  104] lambda:
6.939e-08, error prev/linear/new: 3.263e+02/0.000e+00/3.257e+02, rel reduction:
1.98180e-03
[2024-05-10 16:46:30.621] [info] LM<sym::Optimize> [iter  105] lambda:
6.939e-09, error prev/linear/new: 3.257e+02/0.000e+00/3.261e+02, rel reduction:
-1.21796e-03
[2024-05-10 16:46:30.642] [info] LM<sym::Optimize> [iter  106] lambda:
3.469e-08, error prev/linear/new: 3.257e+02/0.000e+00/3.267e+02, rel reduction:
-3.16741e-03
[2024-05-10 16:46:30.663] [info] LM<sym::Optimize> [iter  107] lambda:
1.735e-07, error prev/linear/new: 3.257e+02/0.000e+00/3.252e+02, rel reduction:
1.52438e-03
[2024-05-10 16:46:30.684] [info] LM<sym::Optimize> [iter  108] lambda:
1.735e-08, error prev/linear/new: 3.252e+02/0.000e+00/3.254e+02, rel reduction:
-5.34118e-04
[2024-05-10 16:46:30.705] [info] LM<sym::Optimize> [iter  109] lambda:
8.674e-08, error prev/linear/new: 3.252e+02/0.000e+00/3.263e+02, rel reduction:
-3.60031e-03
[2024-05-10 16:46:30.726] [info] LM<sym::Optimize> [iter  110] lambda:
4.337e-07, error prev/linear/new: 3.252e+02/0.000e+00/3.249e+02, rel reduction:
9.74520e-04
[2024-05-10 16:46:30.747] [info] LM<sym::Optimize> [iter  111] lambda:
4.337e-08, error prev/linear/new: 3.249e+02/0.000e+00/3.252e+02, rel reduction:
-9.65839e-04
[2024-05-10 16:46:30.767] [info] LM<sym::Optimize> [iter  112] lambda:
2.168e-07, error prev/linear/new: 3.249e+02/0.000e+00/3.264e+02, rel reduction:
-4.64698e-03
[2024-05-10 16:46:30.788] [info] LM<sym::Optimize> [iter  113] lambda:
1.084e-06, error prev/linear/new: 3.249e+02/0.000e+00/3.247e+02, rel reduction:
5.33026e-04
[2024-05-10 16:46:30.810] [info] LM<sym::Optimize> [iter  114] lambda:
1.084e-07, error prev/linear/new: 3.247e+02/0.000e+00/3.257e+02, rel reduction:
-3.04592e-03
[2024-05-10 16:46:30.832] [info] LM<sym::Optimize> [iter  115] lambda:
5.421e-07, error prev/linear/new: 3.247e+02/0.000e+00/3.249e+02, rel reduction:
-5.19715e-04
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[2024-05-10 16:46:30.854] [info] LM<sym::Optimize> [iter  116] lambda:
2.711e-06, error prev/linear/new: 3.247e+02/0.000e+00/3.246e+02, rel reduction:
2.69262e-04
[2024-05-10 16:46:30.875] [info] LM<sym::Optimize> [iter  117] lambda:
2.711e-07, error prev/linear/new: 3.246e+02/0.000e+00/3.262e+02, rel reduction:
-4.82226e-03
[2024-05-10 16:46:30.896] [info] LM<sym::Optimize> [iter  118] lambda:
1.355e-06, error prev/linear/new: 3.246e+02/0.000e+00/3.245e+02, rel reduction:
4.44599e-04
[2024-05-10 16:46:30.917] [info] LM<sym::Optimize> [iter  119] lambda:
1.355e-07, error prev/linear/new: 3.245e+02/0.000e+00/3.232e+02, rel reduction:
3.84000e-03
[2024-05-10 16:46:30.937] [info] LM<sym::Optimize> [iter  120] lambda:
1.355e-08, error prev/linear/new: 3.232e+02/0.000e+00/3.169e+02, rel reduction:
1.95988e-02
[2024-05-10 16:46:30.958] [info] LM<sym::Optimize> [iter  121] lambda:
1.355e-09, error prev/linear/new: 3.169e+02/0.000e+00/3.673e+02, rel reduction:
-1.59057e-01
[2024-05-10 16:46:30.979] [info] LM<sym::Optimize> [iter  122] lambda:
6.776e-09, error prev/linear/new: 3.169e+02/0.000e+00/3.253e+02, rel reduction:
-2.66890e-02
[2024-05-10 16:46:31.000] [info] LM<sym::Optimize> [iter  123] lambda:
3.388e-08, error prev/linear/new: 3.169e+02/0.000e+00/3.112e+02, rel reduction:
1.77608e-02
[2024-05-10 16:46:31.022] [info] LM<sym::Optimize> [iter  124] lambda:
3.388e-09, error prev/linear/new: 3.112e+02/0.000e+00/3.300e+02, rel reduction:
-6.02119e-02
[2024-05-10 16:46:31.044] [info] LM<sym::Optimize> [iter  125] lambda:
1.694e-08, error prev/linear/new: 3.112e+02/0.000e+00/3.101e+02, rel reduction:
3.71732e-03
[2024-05-10 16:46:31.066] [info] LM<sym::Optimize> [iter  126] lambda:
1.694e-09, error prev/linear/new: 3.101e+02/0.000e+00/3.358e+02, rel reduction:
-8.30643e-02
[2024-05-10 16:46:31.088] [info] LM<sym::Optimize> [iter  127] lambda:
8.470e-09, error prev/linear/new: 3.101e+02/0.000e+00/3.205e+02, rel reduction:
-3.36747e-02
[2024-05-10 16:46:31.110] [info] LM<sym::Optimize> [iter  128] lambda:
4.235e-08, error prev/linear/new: 3.101e+02/0.000e+00/3.122e+02, rel reduction:
-6.84289e-03
[2024-05-10 16:46:31.131] [info] LM<sym::Optimize> [iter  129] lambda:
2.118e-07, error prev/linear/new: 3.101e+02/0.000e+00/3.105e+02, rel reduction:
-1.39363e-03
[2024-05-10 16:46:31.151] [info] LM<sym::Optimize> [iter  130] lambda:
1.059e-06, error prev/linear/new: 3.101e+02/0.000e+00/3.108e+02, rel reduction:
-2.38495e-03
[2024-05-10 16:46:31.179] [info] LM<sym::Optimize> [iter  131] lambda:
5.294e-06, error prev/linear/new: 3.101e+02/0.000e+00/3.109e+02, rel reduction:
-2.53396e-03
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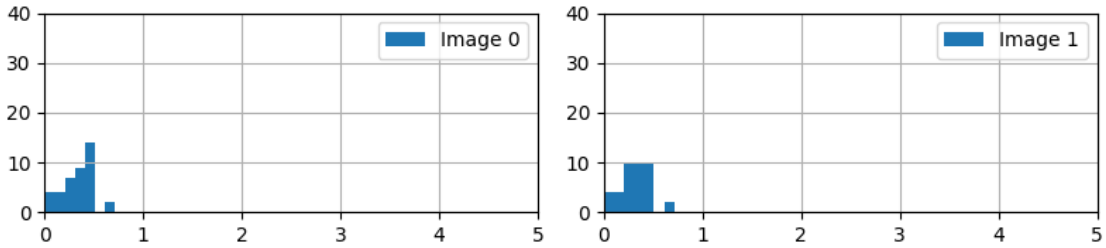
[2024-05-10 16:46:31.200] [info] LM<sym::Optimize> [iter  132] lambda:
2.647e-05, error prev/linear/new: 3.101e+02/0.000e+00/3.080e+02, rel reduction:
6.69753e-03
[2024-05-10 16:46:31.221] [info] LM<sym::Optimize> [iter  133] lambda:
2.647e-06, error prev/linear/new: 3.080e+02/0.000e+00/3.069e+02, rel reduction:
3.50871e-03
[2024-05-10 16:46:31.243] [info] LM<sym::Optimize> [iter  134] lambda:
2.647e-07, error prev/linear/new: 3.069e+02/0.000e+00/3.065e+02, rel reduction:
1.36376e-03
[2024-05-10 16:46:31.265] [info] LM<sym::Optimize> [iter  135] lambda:
2.647e-08, error prev/linear/new: 3.065e+02/0.000e+00/3.104e+02, rel reduction:
-1.25860e-02
[2024-05-10 16:46:31.286] [info] LM<sym::Optimize> [iter  136] lambda:
1.323e-07, error prev/linear/new: 3.065e+02/0.000e+00/3.087e+02, rel reduction:
-7.11421e-03
[2024-05-10 16:46:31.307] [info] LM<sym::Optimize> [iter  137] lambda:
6.617e-07, error prev/linear/new: 3.065e+02/0.000e+00/3.089e+02, rel reduction:
-7.72049e-03
[2024-05-10 16:46:31.328] [info] LM<sym::Optimize> [iter  138] lambda:
3.309e-06, error prev/linear/new: 3.065e+02/0.000e+00/3.089e+02, rel reduction:
-7.86306e-03
[2024-05-10 16:46:31.349] [info] LM<sym::Optimize> [iter  139] lambda:
1.654e-05, error prev/linear/new: 3.065e+02/0.000e+00/3.064e+02, rel reduction:
4.54098e-04
[2024-05-10 16:46:31.370] [info] LM<sym::Optimize> [iter  140] lambda:
1.654e-06, error prev/linear/new: 3.064e+02/0.000e+00/3.089e+02, rel reduction:
-8.26117e-03
[2024-05-10 16:46:31.391] [info] LM<sym::Optimize> [iter  141] lambda:
8.272e-06, error prev/linear/new: 3.064e+02/0.000e+00/3.067e+02, rel reduction:
-1.11709e-03
[2024-05-10 16:46:31.412] [info] LM<sym::Optimize> [iter  142] lambda:
4.136e-05, error prev/linear/new: 3.064e+02/0.000e+00/3.062e+02, rel reduction:
5.62841e-04
[2024-05-10 16:46:31.433] [info] LM<sym::Optimize> [iter  143] lambda:
4.136e-06, error prev/linear/new: 3.062e+02/0.000e+00/3.089e+02, rel reduction:
-8.86091e-03
[2024-05-10 16:46:31.454] [info] LM<sym::Optimize> [iter  144] lambda:
2.068e-05, error prev/linear/new: 3.062e+02/0.000e+00/3.050e+02, rel reduction:
3.79383e-03
[2024-05-10 16:46:31.476] [info] LM<sym::Optimize> [iter  145] lambda:
2.068e-06, error prev/linear/new: 3.050e+02/0.000e+00/3.049e+02, rel reduction:
3.44571e-04
[2024-05-10 16:46:31.497] [info] LM<sym::Optimize> [iter  146] lambda:
2.068e-07, error prev/linear/new: 3.049e+02/0.000e+00/3.048e+02, rel reduction:
4.61985e-04
[2024-05-10 16:46:31.519] [info] LM<sym::Optimize> [iter  147] lambda:
2.068e-08, error prev/linear/new: 3.048e+02/0.000e+00/3.087e+02, rel reduction:
-1.28274e-02
```

```
[2024-05-10 16:46:31.540] [info] LM<sym::Optimize> [iter  148] lambda:
1.034e-07, error prev/linear/new: 3.048e+02/0.000e+00/3.067e+02, rel reduction:
-6.31912e-03
[2024-05-10 16:46:31.561] [info] LM<sym::Optimize> [iter  149] lambda:
5.170e-07, error prev/linear/new: 3.048e+02/0.000e+00/3.047e+02, rel reduction:
4.03495e-04
[2024-05-10 16:46:31.582] [info] LM<sym::Optimize> [iter  150] lambda:
5.170e-08, error prev/linear/new: 3.047e+02/0.000e+00/3.065e+02, rel reduction:
-6.15873e-03
[2024-05-10 16:46:31.603] [info] LM<sym::Optimize> [iter  151] lambda:
2.585e-07, error prev/linear/new: 3.047e+02/0.000e+00/3.068e+02, rel reduction:
-6.90570e-03
[2024-05-10 16:46:31.623] [info] LM<sym::Optimize> [iter  152] lambda:
1.292e-06, error prev/linear/new: 3.047e+02/0.000e+00/3.068e+02, rel reduction:
-7.11637e-03
[2024-05-10 16:46:31.644] [info] LM<sym::Optimize> [iter  153] lambda:
6.462e-06, error prev/linear/new: 3.047e+02/0.000e+00/3.047e+02, rel reduction:
-1.26049e-04
[2024-05-10 16:46:31.665] [info] LM<sym::Optimize> [iter  154] lambda:
3.231e-05, error prev/linear/new: 3.047e+02/0.000e+00/3.046e+02, rel reduction:
1.47954e-04
[2024-05-10 16:46:31.687] [info] LM<sym::Optimize> [iter  155] lambda:
3.231e-06, error prev/linear/new: 3.046e+02/0.000e+00/3.068e+02, rel reduction:
-7.29073e-03
[2024-05-10 16:46:31.709] [info] LM<sym::Optimize> [iter  156] lambda:
1.616e-05, error prev/linear/new: 3.046e+02/0.000e+00/3.031e+02, rel reduction:
5.09471e-03
[2024-05-10 16:46:31.730] [info] LM<sym::Optimize> [iter  157] lambda:
1.616e-06, error prev/linear/new: 3.031e+02/0.000e+00/3.030e+02, rel reduction:
3.06659e-04
[2024-05-10 16:46:31.751] [info] LM<sym::Optimize> [iter  158] lambda:
1.616e-07, error prev/linear/new: 3.030e+02/0.000e+00/3.028e+02, rel reduction:
4.44433e-04
[2024-05-10 16:46:31.772] [info] LM<sym::Optimize> [iter  159] lambda:
1.616e-08, error prev/linear/new: 3.028e+02/0.000e+00/3.089e+02, rel reduction:
-2.00058e-02
[2024-05-10 16:46:31.797] [info] LM<sym::Optimize> [iter  160] lambda:
8.078e-08, error prev/linear/new: 3.028e+02/0.000e+00/3.024e+02, rel reduction:
1.62330e-03
[2024-05-10 16:46:31.818] [info] LM<sym::Optimize> [iter  161] lambda:
8.078e-09, error prev/linear/new: 3.024e+02/0.000e+00/3.150e+02, rel reduction:
-4.19178e-02
[2024-05-10 16:46:31.839] [info] LM<sym::Optimize> [iter  162] lambda:
4.039e-08, error prev/linear/new: 3.024e+02/0.000e+00/3.089e+02, rel reduction:
-2.18072e-02
[2024-05-10 16:46:31.860] [info] LM<sym::Optimize> [iter  163] lambda:
2.019e-07, error prev/linear/new: 3.024e+02/0.000e+00/3.070e+02, rel reduction:
-1.52558e-02
```

```
[2024-05-10 16:46:31.882] [info] LM<sym::Optimize> [iter  164] lambda:
1.010e-06, error prev/linear/new: 3.024e+02/0.000e+00/3.047e+02, rel reduction:
-7.87887e-03
[2024-05-10 16:46:31.905] [info] LM<sym::Optimize> [iter  165] lambda:
5.049e-06, error prev/linear/new: 3.024e+02/0.000e+00/3.048e+02, rel reduction:
-7.96969e-03
[2024-05-10 16:46:31.926] [info] LM<sym::Optimize> [iter  166] lambda:
2.524e-05, error prev/linear/new: 3.024e+02/0.000e+00/3.029e+02, rel reduction:
-1.73921e-03
[2024-05-10 16:46:31.949] [info] LM<sym::Optimize> [iter  167] lambda:
1.262e-04, error prev/linear/new: 3.024e+02/0.000e+00/3.020e+02, rel reduction:
1.15769e-03
[2024-05-10 16:46:31.970] [info] LM<sym::Optimize> [iter  168] lambda:
1.262e-05, error prev/linear/new: 3.020e+02/0.000e+00/3.028e+02, rel reduction:
-2.79485e-03
[2024-05-10 16:46:31.991] [info] LM<sym::Optimize> [iter  169] lambda:
6.311e-05, error prev/linear/new: 3.020e+02/0.000e+00/3.020e+02, rel reduction:
3.03881e-05
12.50 [s]
```

[11]:
```
vio.show_reproj_results(views, tracks, cam0_K, cam0_distortion,␣
↪print_raw_reproj=True, show_reproj_histogram=True)
vio.visualize_predictions(views, tracks, cam0_K, cam0_distortion)
```

```
REPROJECTION ERRORS
 Image  0 (   40 points) : (mean, std, max, min) = (0.3379, 0.1524, 0.7025,
0.0194)
 Image (raw reprojection)  0 (   40 points) : (mean, std, max, min) = (0.3379,
0.1524, 0.7025, 0.0194)
 Image  1 (   40 points) : (mean, std, max, min) = (0.3210, 0.1447, 0.6694,
0.0189)
 Image (raw reprojection)  1 (   40 points) : (mean, std, max, min) = (0.3210,
0.1447, 0.6694, 0.0189)
```



[12]:
```
print(f"\nTiming\n{'='*100}")
print(f"Analytical guess: {analyctical_guess:.2f} [s]")
print(f"Non linear (VO): {nonlinear:.2f} [s]")
```

```
Timing
====================================================================================
====================
Analytical guess: 2.83 [s]
Non linear (VO): 12.50 [s]
```

[13]:
```
# This highlights the scale ambiguity seen in two-view reconstruction
print("Metric pose difference norm (gt) = {:.3f}".format(np.linalg.norm(
    # Note: while this is in world frame, the frames do not have any scaling,
    # so measuring displacement this way still should provide the correct␣
 ↪distance.
    np.linalg.norm(p_inW_ofB[first_frame_idx] - p_inW_ofB[second_frame_idx])
)))
print("Metric pose difference norm (ini) = {:.3f}".format(np.linalg.
 ↪norm(results.initial_values['T_inC0_ofW'].inverse().t - results.
 ↪initial_values['T_inC1_ofW'].inverse().t)))
print("Metric pose difference norm (sf) = {:.3f}".format(np.linalg.norm(results.
 ↪optimized_values['T_inC0_ofW'].inverse().t - results.
 ↪optimized_values['T_inC1_ofW'].inverse().t)))
```

```
Metric pose difference norm (gt) = 2.784
Metric pose difference norm (ini) = 1.000
Metric pose difference norm (sf) = 2.640
```

(ALL: IGNORE BELOW UNLESS IT'S GOING TO BE REPORTED) Evaluate change in position between frames

[14]:
```
# Symforce - Initial values
R_inW_ofC0_ini = results.initial_values['T_inC0_ofW'].R.to_rotation_matrix().T
p_inW_ofC0_ini = - R_inW_ofC0_ini @ results.initial_values['T_inC0_ofW'].t

R_inW_ofB0_ini = R_inW_ofC0_ini @ R_inC_ofB
p_inW_ofB0_ini = R_inW_ofC0_ini @ p_inC_ofB + p_inW_ofC0_ini

R_inW_ofC1_ini = results.initial_values['T_inC1_ofW'].R.to_rotation_matrix().T
```

```
p_inW_ofC1_ini = - R_inW_ofC1_ini @ results.initial_values['T_inC1_ofW'].t

R_inW_ofB1_ini = R_inW_ofC1_ini @ R_inC_ofB
p_inW_ofB1_ini = R_inW_ofC1_ini @ p_inC_ofB + p_inW_ofC1_ini

# Symforce - Optimized values
R_inW_ofC0_sf = results.optimized_values['T_inC0_ofW'].R.to_rotation_matrix().T
p_inW_ofC0_sf = - R_inW_ofC0_sf @ results.optimized_values['T_inC0_ofW'].t

R_inW_ofB0_sf = R_inW_ofC0_sf @ R_inC_ofB
p_inW_ofB0_sf = R_inW_ofC0_sf @ p_inC_ofB + p_inW_ofC0_sf

R_inW_ofC1_sf = results.optimized_values['T_inC1_ofW'].R.to_rotation_matrix().T
p_inW_ofC1_sf = - R_inW_ofC1_sf @ results.optimized_values['T_inC1_ofW'].t

R_inW_ofB1_sf = R_inW_ofC1_sf @ R_inC_ofB
p_inW_ofB1_sf = R_inW_ofC1_sf @ p_inC_ofB + p_inW_ofC1_sf

# ground truth
R_inW_ofB0_gt = R_inW_ofB[first_frame_idx].as_matrix()
p_inW_ofB0_gt = p_inW_ofB[first_frame_idx]
v_inW_ofB0_gt = v_inW_ofB[first_frame_idx]

R_inW_ofB1_gt = R_inW_ofB[second_frame_idx].as_matrix()
p_inW_ofB1_gt = p_inW_ofB[second_frame_idx]
v_inW_ofB1_gt = v_inW_ofB[second_frame_idx]
```

Error evaluation on optimized results *w.r.t to Ground Truth*

[15]:
```
p_inW_ofB0toB1_ini = p_inW_ofB1_ini - p_inW_ofB0_ini
R_inB1_ofB0_ini = R_inW_ofB1_ini.T @ R_inW_ofB0_ini

p_inW_ofB0toB1_sf = p_inW_ofB1_sf - p_inW_ofB0_sf
R_inB1_ofB0_sf = R_inW_ofB1_sf.T @ R_inW_ofB0_sf

v_inW_ofB0toB1_gt = v_inW_ofB1_gt - v_inW_ofB0_gt
p_inW_ofB0toB1_gt = p_inW_ofB1_gt - p_inW_ofB0_gt
R_inB1_ofB0_gt = R_inW_ofB1_gt.T @ R_inW_ofB0_gt

print(f"ERROR WRT TO GROUND TRUTH!")
dR_err_ini = R.from_matrix(R_inB1_ofB0_ini.T @ R_inB1_ofB0_gt).as_euler('xyz',␣
 ↪degrees=True)
dR_err_sf = R.from_matrix(R_inB1_ofB0_sf.T @ R_inB1_ofB0_gt).as_euler('xyz',␣
 ↪degrees=True)
print(f'dR: {dR_err_ini} (init.) --> {dR_err_sf} (optm.) [deg] (xyz)')
print('dR scalar: {:.5f} (init.) --> {:.5f} (optm.) [deg]'.format(
    pose_metrics.rotational_error(R_inB1_ofB0_ini, R_inB1_ofB0_gt),
```

```
    pose_metrics.rotational_error(R_inB1_ofB0_sf, R_inB1_ofB0_gt),
))

dp_err_ini = p_inW_ofB0toB1_ini - p_inW_ofB0toB1_gt
dp_err_sf = p_inW_ofB0toB1_sf - p_inW_ofB0toB1_gt
print(f'dp: {np.linalg.norm(dp_err_ini):.2f} (init.) --> {np.linalg.
 ↪norm(dp_err_sf):.2f} (optm.) [m]')
```

```
ERROR WRT TO GROUND TRUTH!
dR: [-0.03649919 -0.03689197 -0.1026139 ] (init.) --> [-0.14346373  0.03172159
0.73365371] (optm.) [deg] (xyz)
dR scalar: 0.11500 (init.) --> 0.74826 (optm.) [deg]
dp: 1.82 (init.) --> 0.53 (optm.) [m]
```

Relative change in position ($\Delta p$) and rotation ($\Delta R$) between frames (**NOT ERROR w.r.t Ground Truth**)

[16]:
```
print(f"RELATIVE CHANGE BETWEEN FRAMES (NOT ERROR)\n")
print(f'dp\n{"="*50}')
print('(Analytical guess) dp: {:.2f} [m]'.format( np.linalg.norm(␣
 ↪p_inW_ofB0_ini -  p_inW_ofB1_ini)))
print('(Non linear - VIO) dp: {:.2f} [m]'.format( np.linalg.norm( p_inW_ofB0_sf␣
 ↪ -  p_inW_ofB1_sf)))
print('(Ground Truth)     dp: {:.2f} [m]'.format( np.linalg.norm( p_inW_ofB0_gt␣
 ↪ -  p_inW_ofB1_gt)))

print(f'\ndR\n{"="*50}')
print('(Analytical guess) dR scalar: {:.5f} [deg]'.format(pose_metrics.
 ↪rotational_error( R_inW_ofB0_ini, R_inW_ofB1_ini)))
print('(Non linear - VIO) dR scalar: {:.5f} [deg]'.format(pose_metrics.
 ↪rotational_error( R_inW_ofB0_sf,  R_inW_ofB1_sf)))
print('(Ground Truth)     dR scalar: {:.5f} [deg]'.format(pose_metrics.
 ↪rotational_error( R_inW_ofB0_gt,  R_inW_ofB1_gt)))
```

```
RELATIVE CHANGE BETWEEN FRAMES (NOT ERROR)

dp
==================================================
(Analytical guess) dp: 0.96 [m]
(Non linear - VIO) dp: 2.63 [m]
(Ground Truth)     dp: 2.78 [m]

dR
==================================================
(Analytical guess) dR scalar: 7.10949 [deg]
(Non linear - VIO) dR scalar: 7.94694 [deg]
(Ground Truth)     dR scalar: 7.21199 [deg]
```

(Optional) Error evaluation on preintegrated results

[17]:
```
gravity = results.initial_values['gravity']
dt_01 = results.initial_values['dt_01']
dR_01 = results.initial_values['dR_01']
dv_01 = results.initial_values['dv_01']
dp_01 = results.initial_values['dp_01']

print(f"Results\n{'='*100}")
dR_err_pi = R.from_matrix(dR_01 @ R_inB1_ofB0_gt).as_euler('xyz', degrees=True)
print(f'dR_preintegrated: {dR_err_pi} [deg] (xyz)')
print('dR_preintegrated scalar: {:.5f} [deg]'.format(
    pose_metrics.rotational_error(dR_01, R_inB1_ofB0_gt)
))

v_inW_ofB0toB1_pi = R_inW_ofB0_gt @ dv_01 + gravity * dt_01
dv_err_pi = v_inW_ofB0toB1_pi - v_inW_ofB0toB1_gt

print(f'dv_preintegrated: {np.linalg.norm(v_inW_ofB0toB1_pi):.2f} [m/s] (abs␣
 ↪err: {np.linalg.norm(dv_err_pi):.2f} [m/s], rel err: {np.linalg.
 ↪norm(dv_err_pi)/np.linalg.norm(v_inW_ofB0toB1_gt)*100:.2f} [%])')

p_inW_ofB0toB1_pi = R_inW_ofB0_gt @ dp_01 + v_inW_ofB0_gt * dt_01 + 0.5 *␣
 ↪gravity * dt_01 ** 2
dp_err_pi = p_inW_ofB0toB1_pi - p_inW_ofB0toB1_gt

print(f'dp_preintegrated: {np.linalg.norm(p_inW_ofB0toB1_pi):.2f} [m] (abs err:␣
 ↪{np.linalg.norm(dp_err_pi):.2f} [m], rel err: {np.linalg.norm(dp_err_pi)/np.
 ↪linalg.norm(p_inW_ofB0toB1_gt)*100:.2f} [%])')


print(f"Timing\n{'='*100}")
print(f"Non linear (VIO): {nonlinear:.2f} [s]")
```

```
Results
====================================================================================================
dR_preintegrated: [-0.14346564  0.03172481  0.73365981] [deg] (xyz)
dR_preintegrated scalar: 15.15829 [deg]
dv_preintegrated: 0.91 [m/s] (abs err: 0.08 [m/s], rel err: 9.07 [%])
dp_preintegrated: 2.64 [m] (abs err: 0.32 [m], rel err: 11.35 [%])
Timing
====================================================================================================
Non linear (VIO): 12.50 [s]
```