The role of attention in learning

Andres Quiñones

January 17, 2023

The model

We asses the effect of the dynamics of associability in the learning process by implementing a reinforcement learning model where the value of α (speed of learning) changes according a different sets of rules proposed in the psychology literature. The model assumes that agents use different features of the object they are presented with to estimate a value for each object. Furthermore, they use the estimated value of the object to choose among them when these are presented together. We assume that the features characterizing objects can be classified in different perceptual dimensions. Thus, the combination of features present in an object for all the perceptual dimensions can potentially be used to discriminate among different type of objects. In the model we control the feature composition of the different objects, so we can change how informative they are in the discrimination task.

For illustration purposes, let's say we have two objects denoted by the index k. Each object can be characterized by the feature vector f of length m, where each entry of the vector represents the state of a stimulus dimension. m is then the number of stimuli dimensions available for the discrimination task. We further assume that these stimuli states are a discrete variable. Where the value 0 represents the absence of such stimuli, and all the other values each a different state.

The learning agent stores in memory a value of association for each of the states of each stimuli dimension. Thus, if we assume each stimuli has n possible states, the memory of the learning agent for this task can be represented as a matrix (A) with dimensions $n \times m$. Each entry of this matrix is a real number that quantifies the estimate of value associated with that particular state of that stimulus. The total value predicted for one particular object is given by the sum of the associative values of each of the features present in the object. This computation, performed by the agent, can be summarized as follows

$$V = \sum_{i=1}^{m} \sum_{i=1}^{n} a_{ij} F_{ij}$$

where a_{ij} represents the entries of matrix A, and F_{ij} are the entries of matrix F. Matrix F is a matrix of zeros and ones representing the absence or presence, respectively, of a particular feature in each stimulus dimension for the focal object. The columns of F correspond to the stimulus dimensions and and each raw to a state of the stimulus.

As the agent encounters and chooses different objects it collects rewards from them. The difference between the reward obtained from an object and the estimated reward (the prediction error) is used to update the value estimates of each of the features present in the object. Formally, the prediction error is given by

$$\delta^t = R^t - v^t.$$

where the superscript t denotes the time at which the interaction took place.

The update for each of the features is given by

$$\Delta S_{ij}^t = \alpha_i^t \delta^t,$$

where the subscripts i and j correspond to the state of a given stimulus and the stimulus, respectively; α_j^t is the speed of learning at time t for the stimulus j.

Desicion making

Every time the agent faces two objects (regardless of the type they belong to) it must decide which object to exploit. These decision making process is done by converting the difference in estimated value into a probability with which to choose each object. This conversion is done using the soft-max distribution. Formally the probability of choosing object k is given by

$$p_k^t = \frac{e^{V_k^t/\tau}}{e^{V_k^t/\tau} + e^{V_n^t/\tau}}$$

where V_n^t is the estimated value of object n at time t, and τ is a parameter controlling how strongly value differences determine the agent's choice. In other words, it's a parameter determining the balance between exploration and exploitation in the learning process.

Mechanisms of selective attention

To assess the effect of changes in the speed of learning as a form of selective attention, we test three different scenarios. First as a benchmark, we test a scenario where learning speed remains constant throughout the trial. We then evaluate the effect of changes in the speed of learning dictated by the Mackintosh (1975) and Pearce and Hall (1980) rules. The rules described here determined the changes in the speed of learning in the simulations as long as the rate does not go bellow 0 or above 0.5, otherwise the rate was set to the corresponding limit. This was done because very high values can make the learning algorithm unstable, and lead to erroneous estimation.

The Mackintosh update Mackintosh (1975) proposed a model of attentional dynamics, where the central idea is that stimuli which are the best predictors of reward get an increase in the attention they enjoy during the learning process. Conversely, stimuli that are bad predictors, compared to all other, get a reduction in attention. Formally, Mackintosh (1975) defined the attentional update as

$$\Delta \alpha_i > 0 \text{ if } |\lambda - V_i| < |\lambda - V_P|$$

 $\Delta \alpha_i < 0 \text{ if } |\lambda - V_i| < |\lambda - V_P|$

Where λ is the "real" associative value in a given trial, which translates to RL jargon as the reward; V_i is the associative strength of the focal stimulus; and V_p is sum of the associative strengths of the stimuli other than the focal.

Following a notation more in line with the current model, we implemented the Mackintosh (1975) idea as follows

$$\alpha_j^{t+1} = \alpha_j^t + \hat{\alpha}(|R^t - \sum_{l \neq j}^m \sum_{i=1}^n a_{il} F_{il}| - |R^t - \sum_{i=1}^n a_{ij} F_{ij}|)$$

where $hat\alpha$ is a constant scaling the update on the speed of learning. Here, the update is proportional to the difference between a partial prediction error given by all other stimuli and the partial prediction error given by the focal stimuli.

The Pearce and Hall

Pearce and Hall (1980) proposed a model of attentional dynamics, where the central idea is that the level of attention is proportional to the difference between the real associative strength and that predicted by the stimulus. Formally, Pearce and Hall (1980) defined the attentional update as

$$\alpha^{t} = \rho |\lambda^{t-1} - V_{\sum}^{t-1}| + (1 - \rho)\alpha^{t-1},$$

where V_{\sum}^{t-1} represents the total associative strength triggered by the stimulus; and ρ is a constant that measure how fast are the jumps in attention.

Following a notation more in line with the current model, we implemented the Pearce and Hall (1980) idea as follows

$$\alpha_i^{t+1} = \hat{\alpha}|R^t - a_{ij}^t| + (1 - \hat{\alpha})\alpha_i^t$$

Mackintosh 2

The idea of Mackintosh is that more attention is paid to features that are better predictors of reward. However, the classic implementation of this idea includes in the update rule a reduction in attention to all other features that are not the best predictor of reward (ref). This leads to individuals focusing their learning in only one of the available features, rather distributing their attention in proportion how well features predict reward. In order to allow for a continuous distribution of attention among the different features Esber and Haselgrove (2011) propose a alternative implementation of Mackintosh's rule where the speed of learning in the next time step is simply given by the estimated associative value of the focal feature. In our model, we implement this by setting the speed of learning for stimulus i as the estimated reward of the last feature from that stimulus:

$$\alpha_i^{t+1} = \hat{\alpha} S_{ij}^t$$

Pearce and Hall 2

Just as the idea of Mackintosh, Pearce and Hall's idea has been implemented in different ways. One important difference between alternative implementations is whether the update is dependent on the total prediction error (δ) , or on the prediction error triggered by just the focal stimulus dimension (lepelley_Role_2004?). Thus, in our model we implement a second version of Pearce and Hall's idea, in the second implementation we let the be driven by the total prediction error, formally:

$$\alpha_i^{t+1} = \hat{\alpha}\delta^t + (1 - \hat{\alpha}\alpha_i^t)$$

Hybrid model

Given the different phenomena that Mackintosh's, as well as Pearce and Hall's, ideas fit from learning experiments, a different set of models combine both of the ideas in an attempt to unify the mechanisms of attentional update. One of such models proposed by (pearce_Two_2010?), combines the two ideas by having to different terms for the speed of learning, whose product determine the

combines the two ideas by having to different terms for the speed of learning, whose product determine the realized speed. Each one of these terms corresponds to one of the two ideas. Following this hybrid model we implement a combination of Mackintosh and Pearce and Hall's ideas bu letting the speed of learning be the product of the speed of learning computed in the first two implementations.

All the source code necessary to run de simulations can be found at https://github.com/andreseqp/CleanSarsa.

Preliminary results

In order to asses the dynamic behavior of the three discrimination mechanisms described above we run the model under three alternative scenarios where we vary how informative the object features are, bellow we describe each scenario. In all cases, we assume object 1 is twice as rewarding for the agent. Thus, the optimal choice should be to go for that first object.

Perfect information for two stimuli dimensions:

Here we assume that objects are characterized by two features, each from different stimuli dimensions. Both of this features have useful information for the discrimination task. That means for the first dimension object 1 has almost always feature 1, while object 2 has almost always feature two. Similarly for the second dimension, object 1 has feature 2, while object 2 has feature 1.

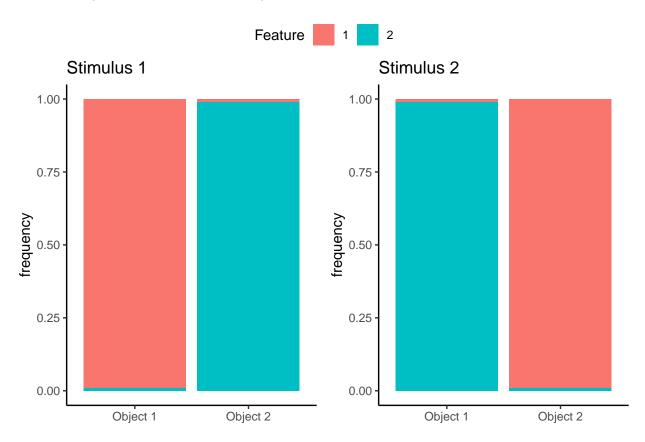


Figure 1: Frequency of features of the two different stimuli in the two different objects for the perfect information scenario.

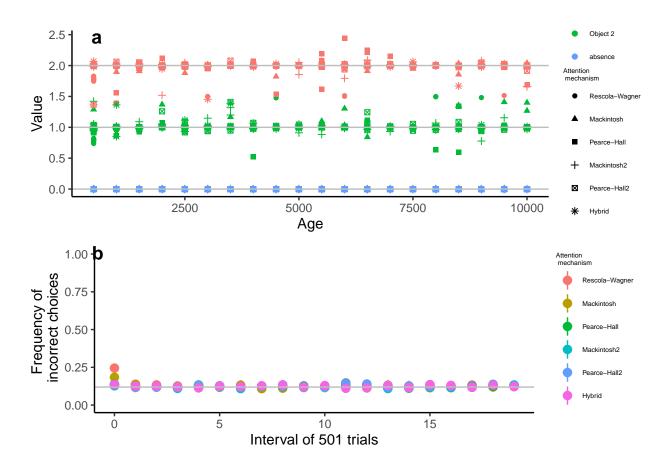
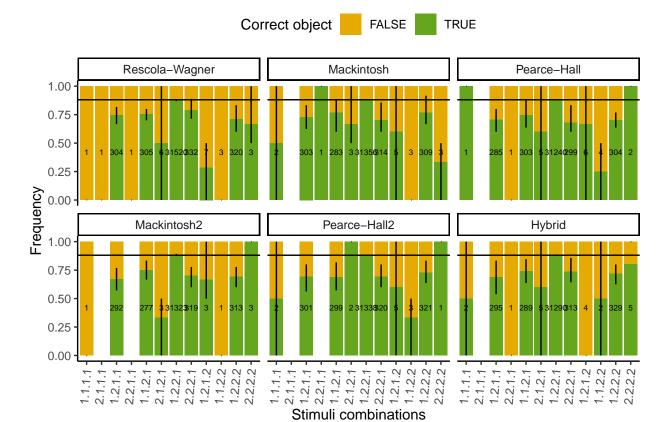


Figure 2: Dynamics of value estimation for the two objects (a) and performance (b) in the scenario with full information in both stimuli dimensions. Grey lines in a correspond to the real value of the two objects. Grey line in b correspond to the expected proportion of wrong choices given the exploration parameter au in the desicion making rule.



Partial information for one stimulus out of two:

Here one of the stimuli contains information to distinguish the two object types. However, the information does not allow perfect discrimination. Specifically, object 1 has the feature 1, in the first stimulus dimension, with probability 0.67 and the alternative feature with the complementary probability (0.33). In contrast, object 2 has feature 2 in the first stimulus dimension, with probability 0.99. Thus, associating object 2 with feature two in the first dimension will lead to some errors where object 1 will be identifies as object 2. The features of the second stimulus have even probability for both objects.

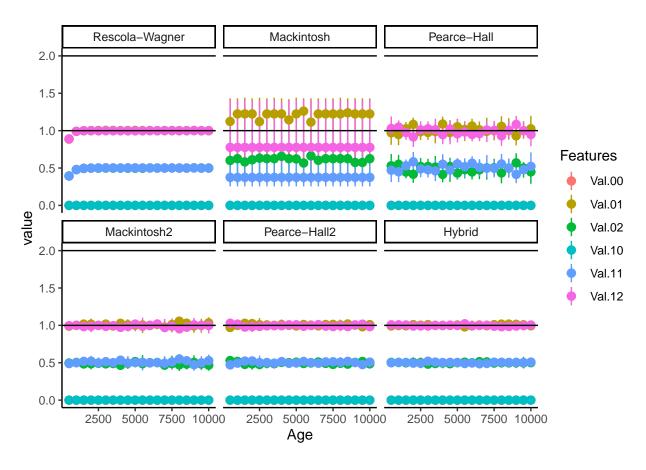


Figure 3: Dynamics of the values associated with the different features of the two stimuli dimensions for the full information scenario. In the legend the first number of the labels corresponds to the stimuli dimension index, and the second to the feature index. The black lines show the real value of the objects.

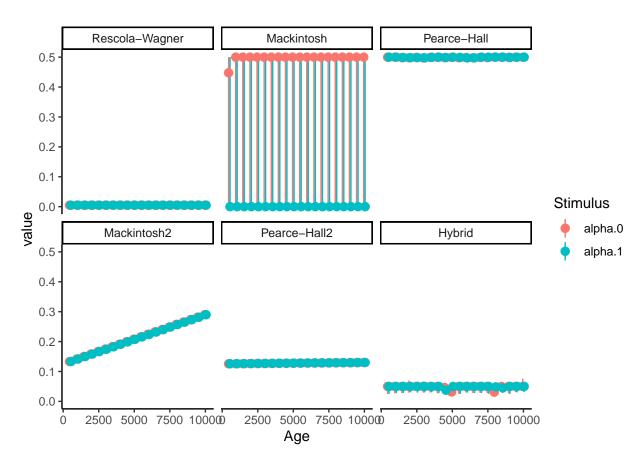


Figure 4: Dynamics of the learning speeds for each stimuli dimension discriminated by the attention mechanisms in the escenario with full information.

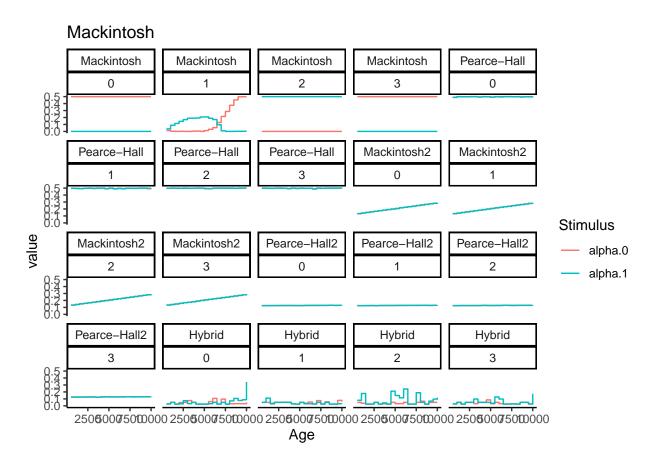


Figure 5: Examples of the dynamics of learning rates in a set of 4 replicates in the attention mechanisms for which learning rate changes in the scenario with full information.

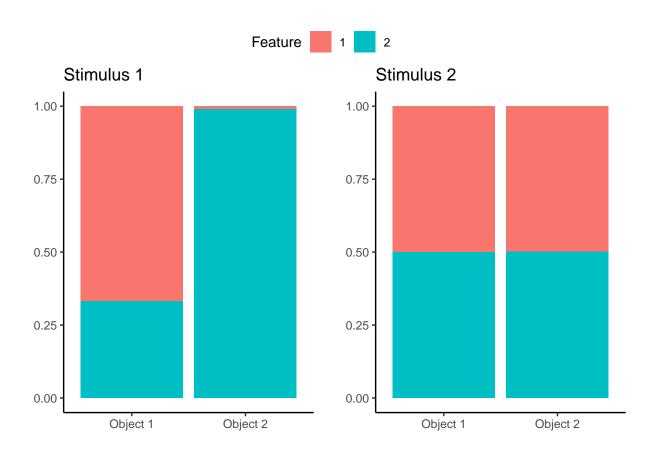


Figure 6: Frequency of features of the two different stimuli in the two different objects for the scenario with partial information for one stimulus.

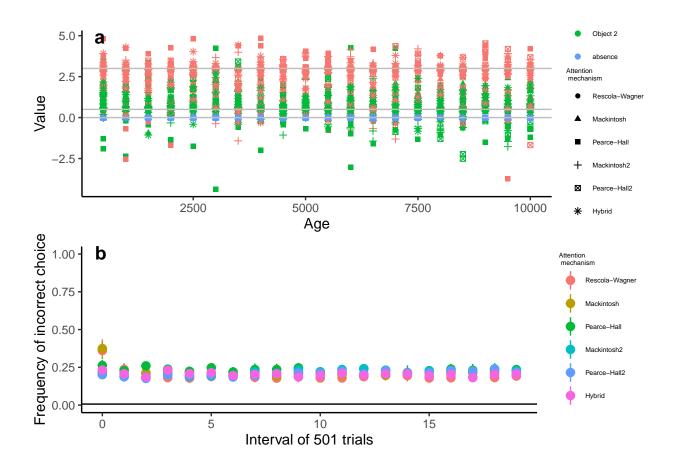
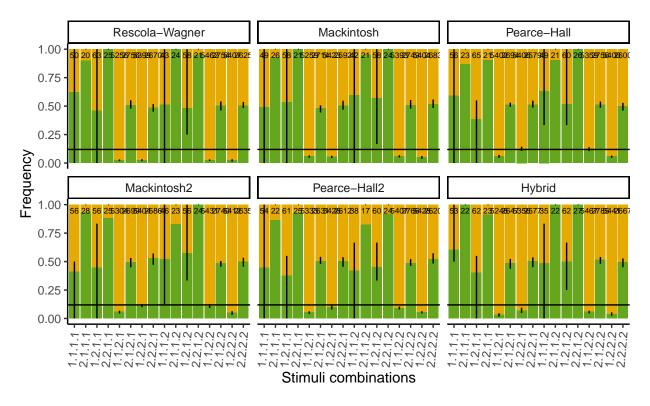


Figure 7: Dynamics of value estimation for the two objects (a) and performance (b) in the scenario with partial information in one stimuli dimensions. Grey lines in a correspond to the real value of the two objects. Grey line in b correspond to the expected proportion of wrong choices given the exploration parameter au in the desicion making rule.





Partial information for both stimuli:

Here both of the stimuli contains information to distinguish the two object types. However, each individually does not allow for perfect discrimination. Specifically, object 1 has the feature 1, in the first stimulus dimension, with probability 0.67 and the alternative feature with the complementary probability (0.33). In contrast, object 2 has feature 2 in the first stimulus dimension, with probability 0.99. Thus, associating object 2 with feature 2 in the first dimension will lead to some errors where object 1 will be identified as object 2. This same behavior applies in this scenario for both stimuli dimensions.

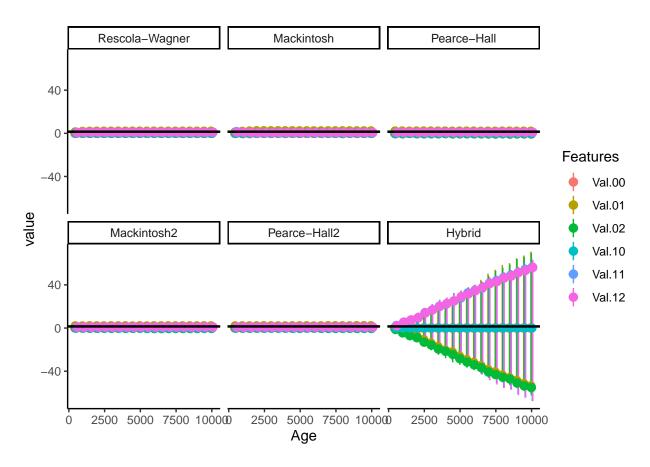


Figure 8: Dynamics of the values associated with the different features of the two stimuli dimensions for the scenario with partial information in one stimuli. In the legend the first number of the labels corresponds to the stimuli dimension index, and the second to the feature index. The black lines show the real value of the objects.

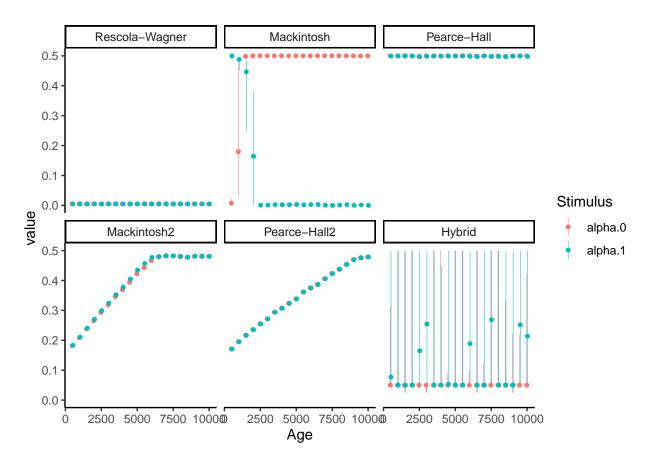


Figure 9: Dynamics of the learning speeds for each stimuli dimension discriminated by the attention mechanisms in the escenario with partial information for one stimuli.

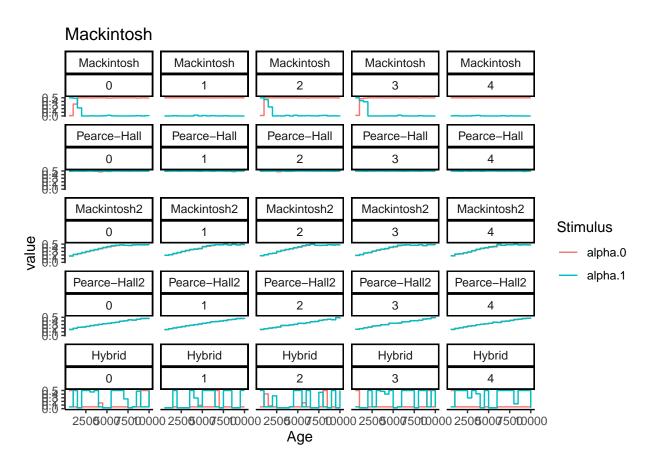


Figure 10: Examples of the dynamics of learning rates in a set of 4 replicates in the attention mechanisms for which learning rate changes in the scenario with partial information for one stimuli.

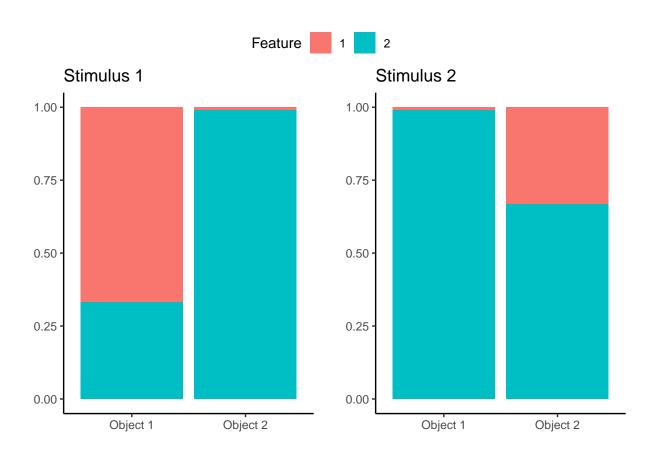


Figure 11: Frequency of features of the two different stimuli in the two different objects for the scenario with partial information for two stimulus.

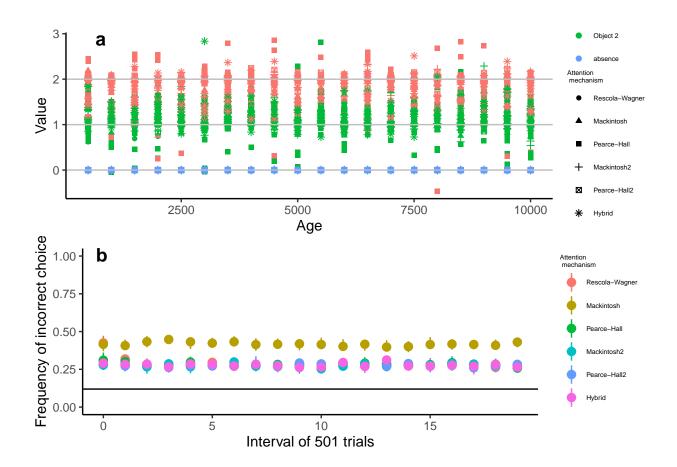
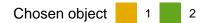
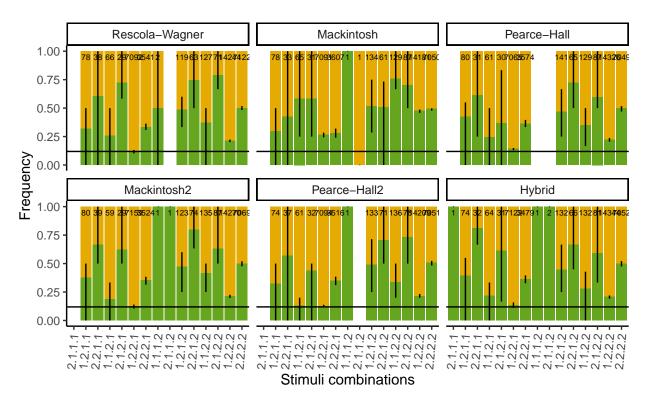


Figure 12: Dynamics of value estimation for the two objects (a) and performance (b) in the scenario with partial information in both stimuli dimensions. Grey lines in a correspond to the real value of the two objects. Grey line in b correspond to the expected proportion of wrong choices given the exploration parameter au in the desicion making rule.





Partial information for one stimulus and full for the other one:

Here both of the stimuli contains information to distinguish the two object types. However, each individually does not allow for perfect discrimination. Specifically, object 1 has the feature 1, in the first stimulus dimension, with probability 0.67 and the alternative feature with the complementary probability (0.33). In contrast, object 2 has feature 2 in the first stimulus dimension, with probability 0.99. Thus, associating object 2 with feature 2 in the first dimension will lead to some errors where object 1 will be identified as object 2. This same behavior applies in this scenario for both stimuli dimensions.

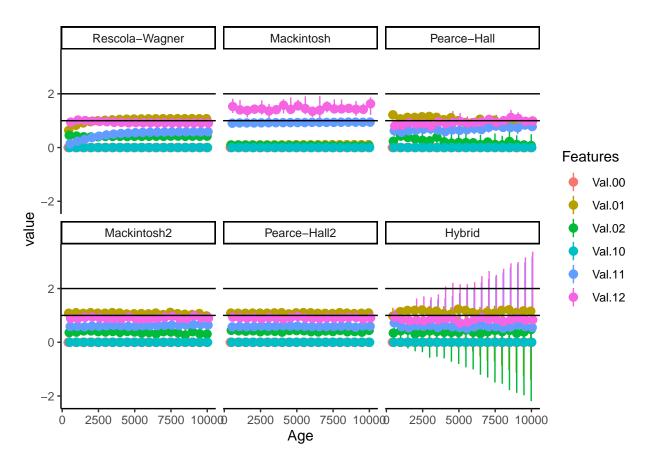


Figure 13: Dynamics of the values associated with the different features of the two stimuli dimensions for the scenario with partial information in one stimuli. In the legend the first number of the labels corresponds to the stimuli dimension index, and the second to the feature index. The black lines show the real value of the objects.

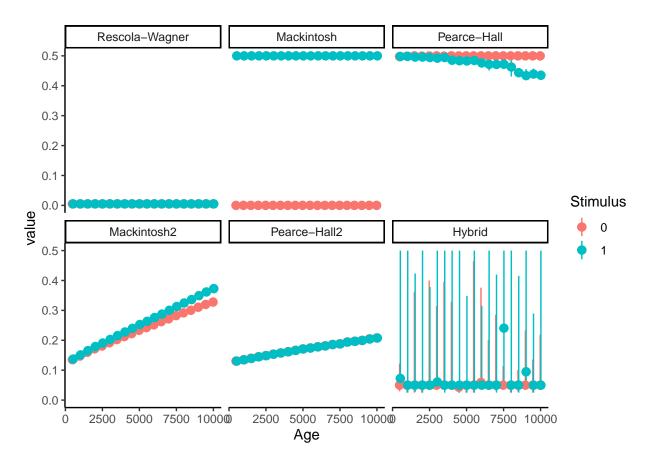


Figure 14: Dynamics of the learning speeds for each stimuli dimension discriminated by the attention mechanisms in the escenario with partial information for both stimuli.

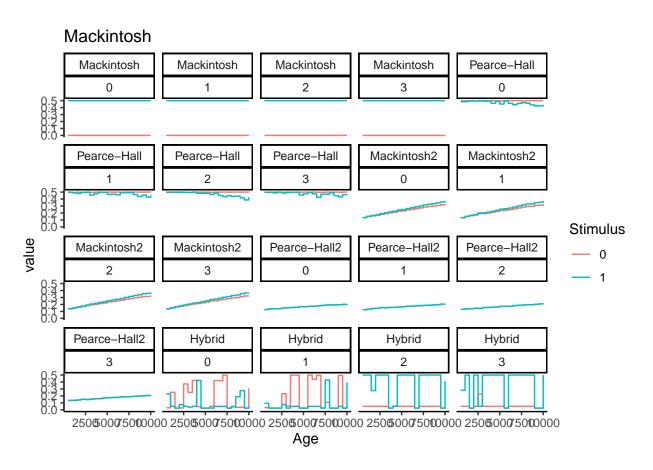


Figure 15: Examples of the dynamics of learning rates in a set of 4 replicates in the attention mechanisms for which learning rate changes in the scenario with partial information for one stimuli.

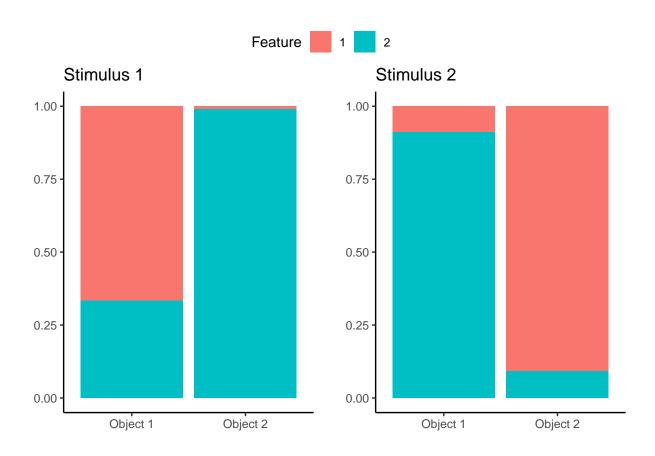


Figure 16: Frequency of features of the two different stimuli in the two different objects for the scenario with partial information for two stimulus.

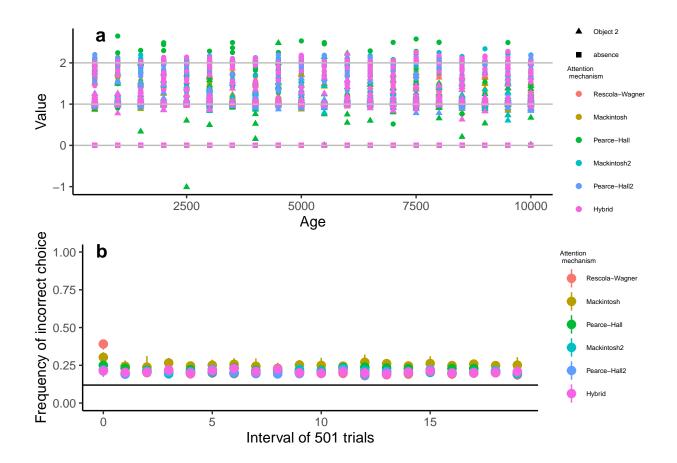
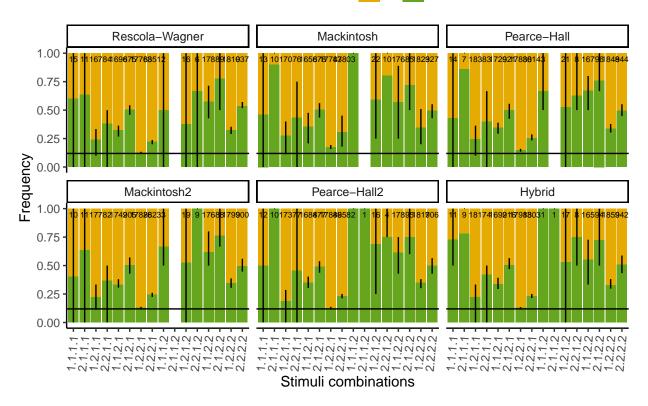
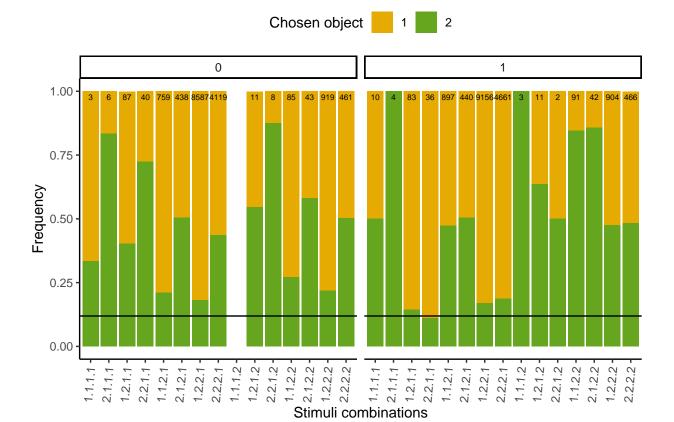


Figure 17: Dynamics of value estimation for the two objects (a) and performance (b) in the scenario with partial information in both stimuli dimensions. Grey lines in a correspond to the real value of the two objects. Grey line in b correspond to the expected proportion of wrong choices given the exploration parameter au in the desicion making rule.

Chosen object 1





Partial information for one stimulus and full for the other one (the second stimulus s only added half way the simulation):

Here both of the stimuli contains information to distinguish the two object types. However, each individually does not allow for perfect discrimination. Specifically, object 1 has the feature 1, in the first stimulus dimension, with probability 0.17 and the alternative feature with the complementary probability (0.83). In contrast, object 2 has feature 2 in the first stimulus dimension, with probability 0.17. Thus, associating object 2 with feature 2 in the first dimension will lead to some errors where object 1 will be identified as object 2. This same behavior applies in this scenario for both stimuli dimensions.

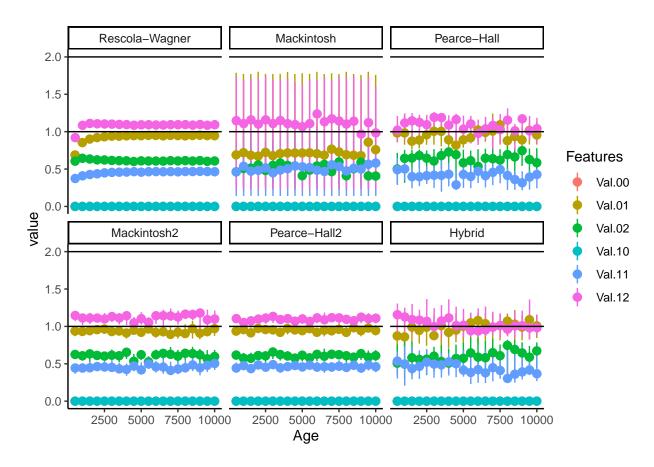


Figure 18: Dynamics of the values associated with the different features of the two stimuli dimensions for the scenario with partial information in one stimuli. In the legend the first number of the labels corresponds to the stimuli dimension index, and the second to the feature index. The black lines show the real value of the objects.

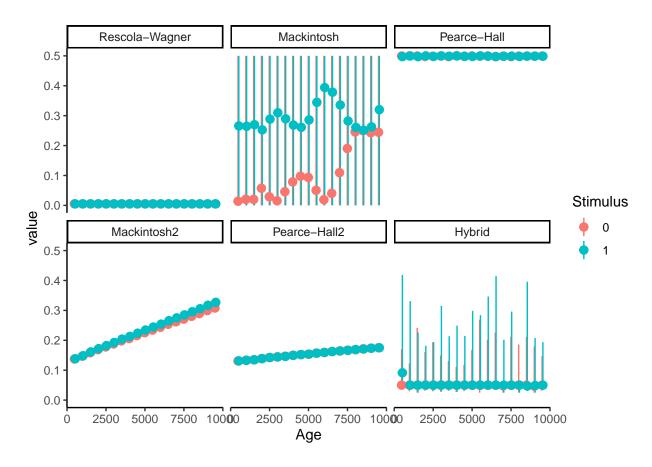


Figure 19: Dynamics of the learning speeds for each stimuli dimension discriminated by the attention mechanisms in the escenario with partial information for both stimuli.

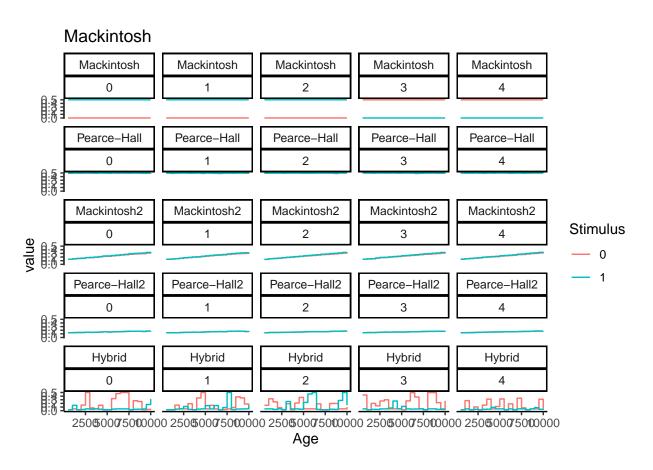


Figure 20: Examples of the dynamics of learning rates in a set of 4 replicates in the attention mechanisms for which learning rate changes in the scenario with partial information for one stimuli.

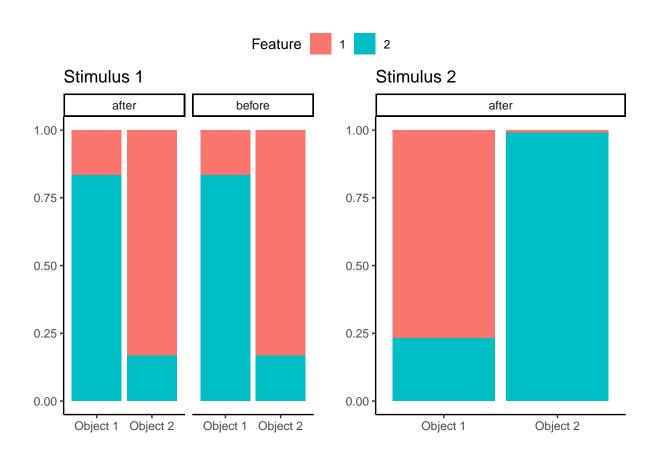


Figure 21: Frequency of features of the two different stimuli in the two different objects for the scenario with partial information for two stimulus.

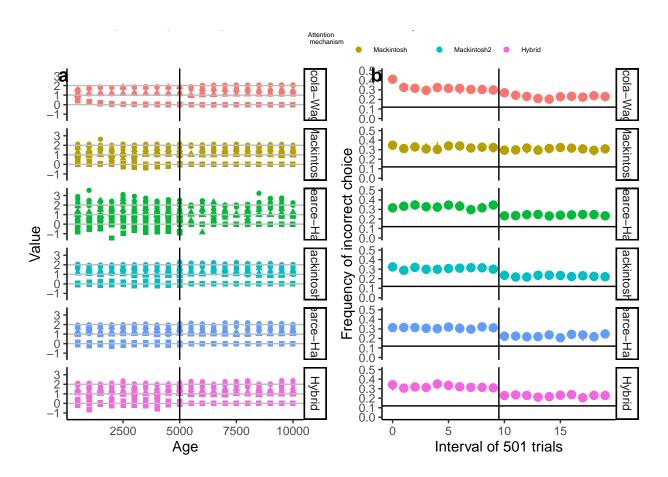
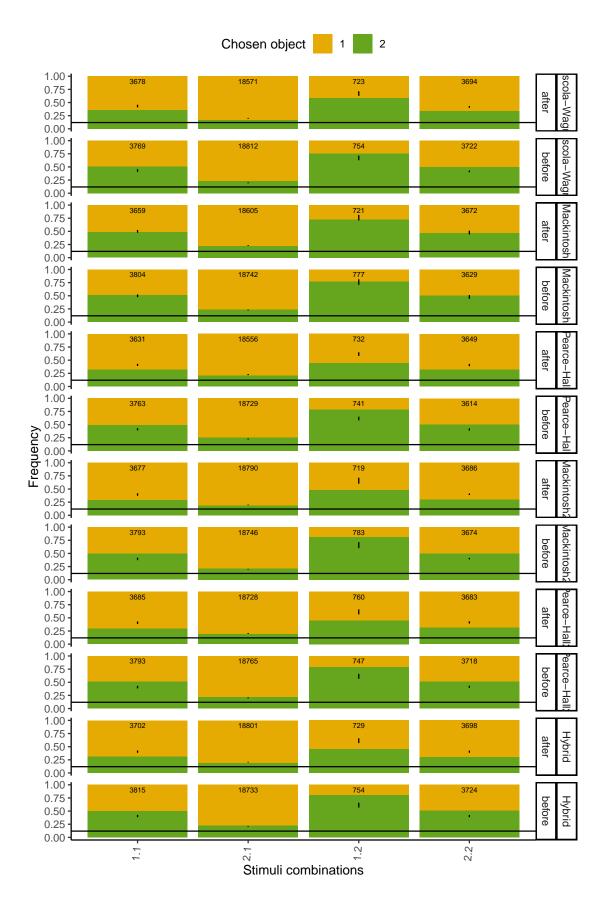


Figure 22: Dynamics of value estimation for the two objects (a) and performance (b) in the scenario with partial information in both stimuli dimensions. Grey lines in a correspond to the real value of the two objects. Grey line in b correspond to the expected proportion of wrong choices given the exploration parameter au in the desicion making rule.



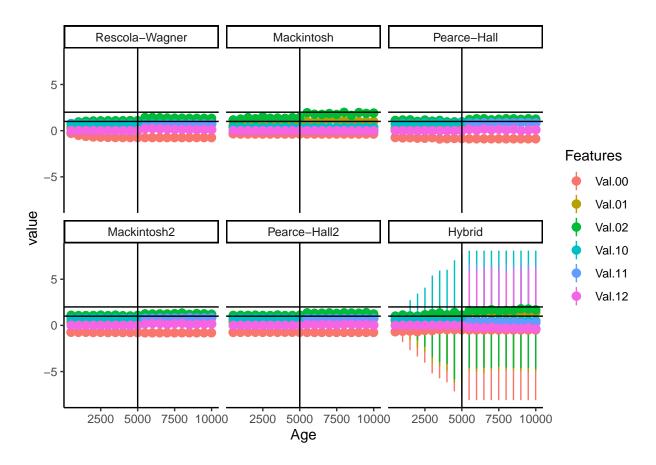


Figure 23: Dynamics of the values associated with the different features of the two stimuli dimensions for the scenario with partial information in one stimuli. In the legend the first number of the labels corresponds to the stimuli dimension index, and the second to the feature index. The black lines show the real value of the objects.

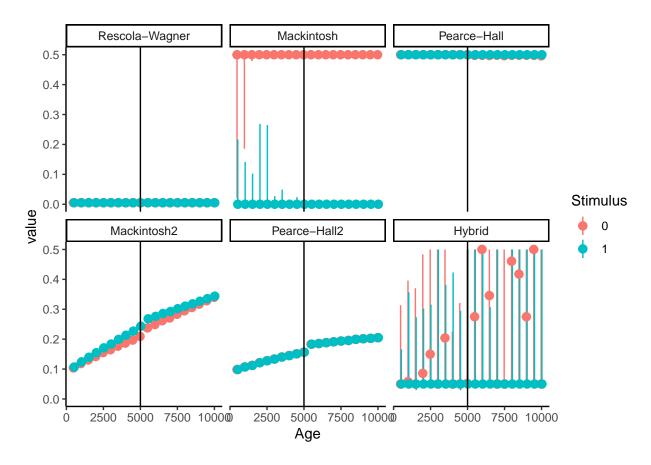


Figure 24: Dynamics of the learning speeds for each stimuli dimension discriminated by the attention mechanisms in the escenario with partial information for both stimuli.

Partial information for 5 stimuli:

Here both of the stimuli contains information to distinguish the two object types. However, each individually does not allow for perfect discrimination. Specifically, object 1 has the feature 1, in the first stimulus dimension, with probability 0.17 and the alternative feature with the complementary probability (0.83). In contrast, object 2 has feature 2 in the first stimulus dimension, with probability 0.17. Thus, associating object 2 with feature 2 in the first dimension will lead to some errors where object 1 will be identified as object 2. This same behavior applies in this scenario for both stimuli dimensions.

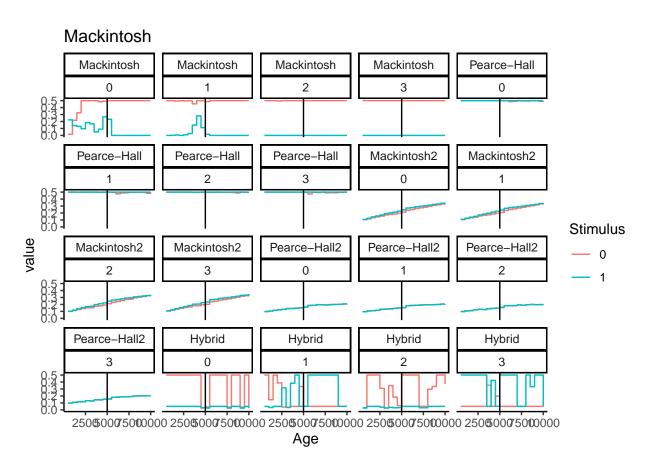


Figure 25: Examples of the dynamics of learning rates in a set of 4 replicates in the attention mechanisms for which learning rate changes in the scenario with partial information for one stimuli.

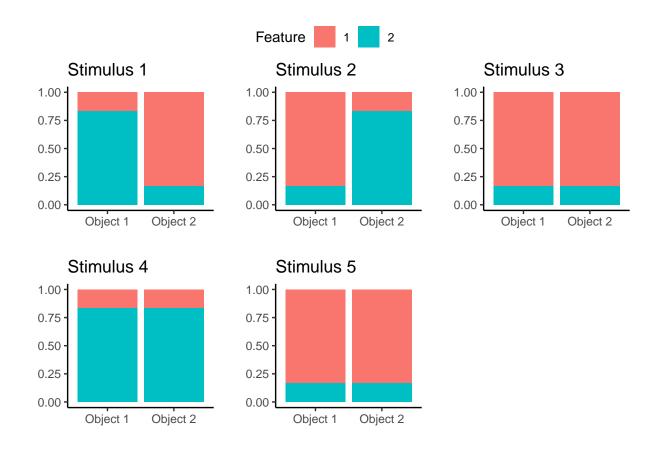


Figure 26: Frequency of features of the two different stimuli in the two different objects for the scenario with partial information for two stimulus.

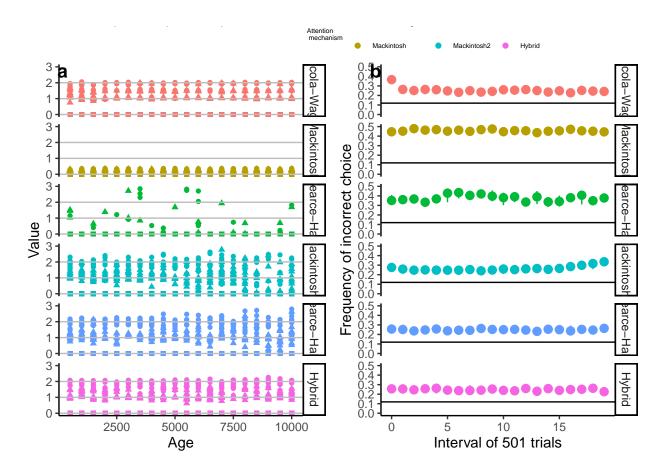
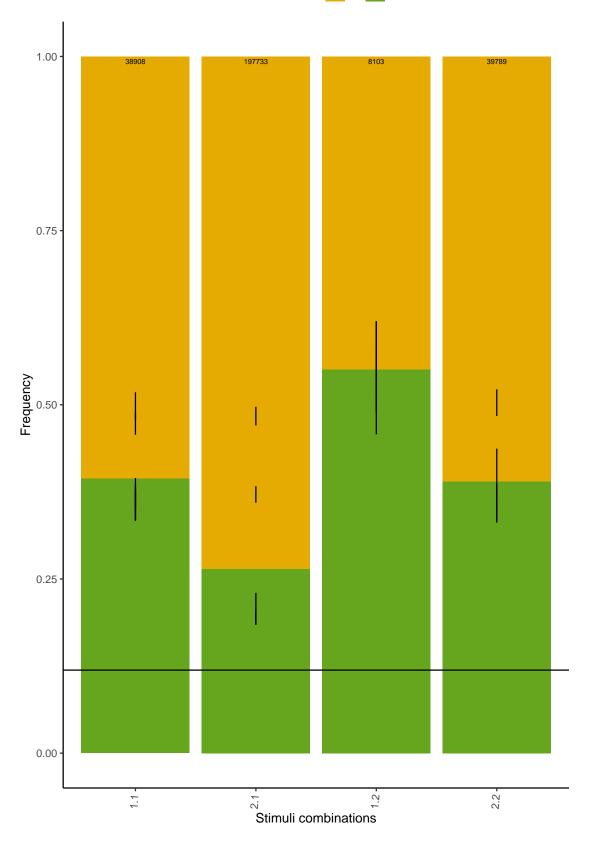


Figure 27: Dynamics of value estimation for the two objects (a) and performance (b) in the scenario with partial information in both stimuli dimensions. Grey lines in a correspond to the real value of the two objects. Grey line in b correspond to the expected proportion of wrong choices given the exploration parameter au in the desicion making rule.





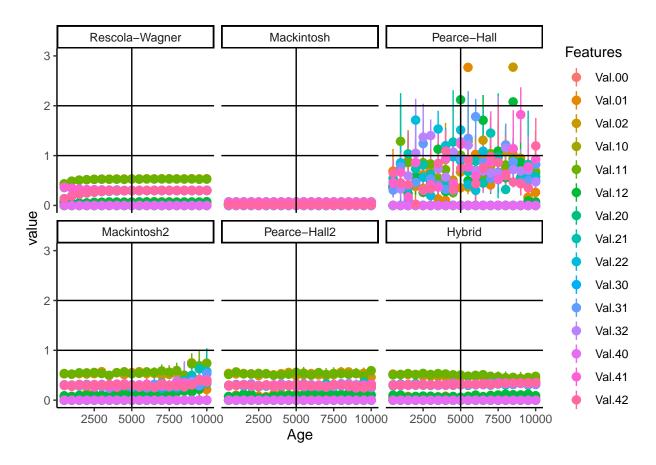


Figure 28: Dynamics of the values associated with the different features of the two stimuli dimensions for the scenario with partial information in one stimuli. In the legend the first number of the labels corresponds to the stimuli dimension index, and the second to the feature index. The black lines show the real value of the objects.

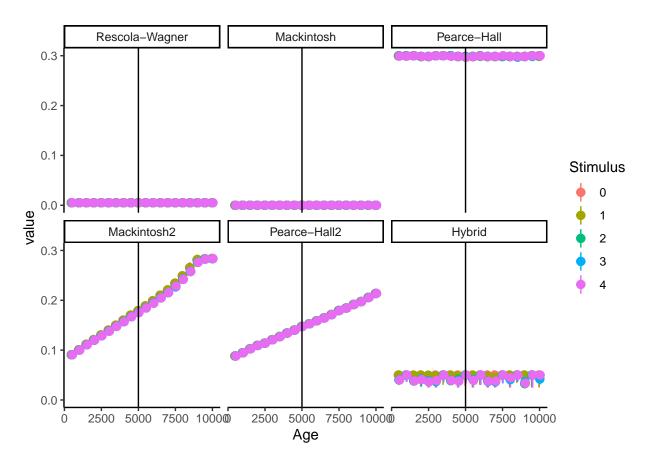


Figure 29: Dynamics of the learning speeds for each stimuli dimension discriminated by the attention mechanisms in the escenario with partial information for both stimuli.

Partial information for 2 stimuli (the second stimulus is only added half way the simulation, Redouan's #1):

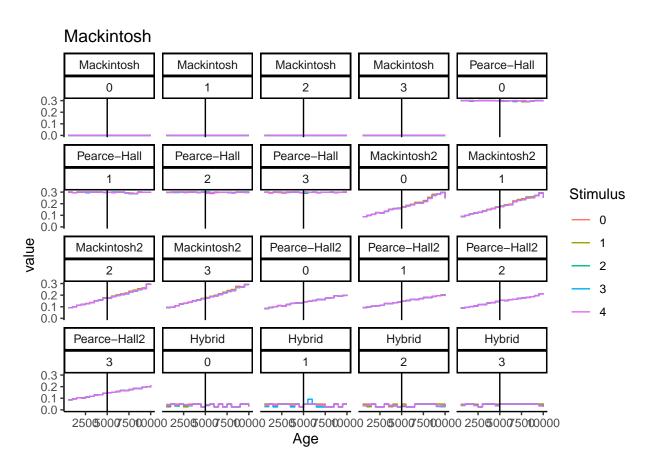


Figure 30: Examples of the dynamics of learning rates in a set of 4 replicates in the attention mechanisms for which learning rate changes in the scenario with partial information for one stimuli.

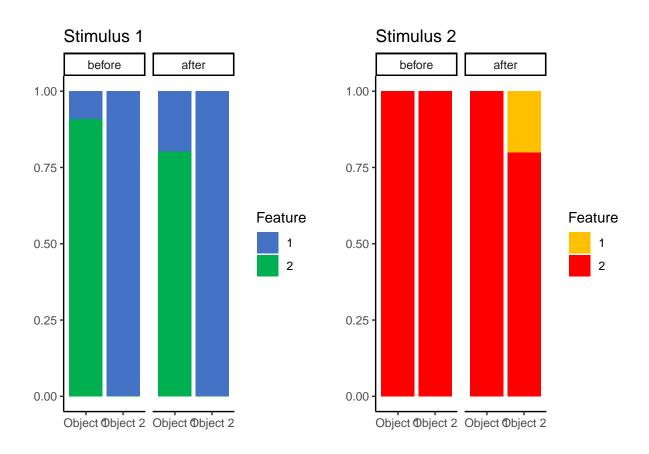


Figure 31: Frequency of features of the two different stimuli in the two different objects for the scenario with partial information for two stimulus.

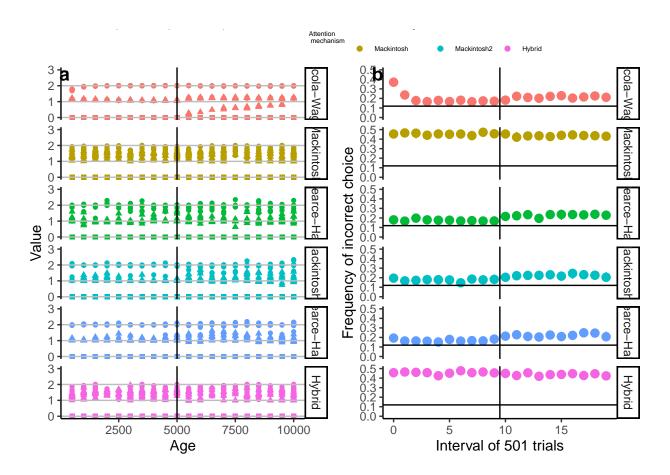
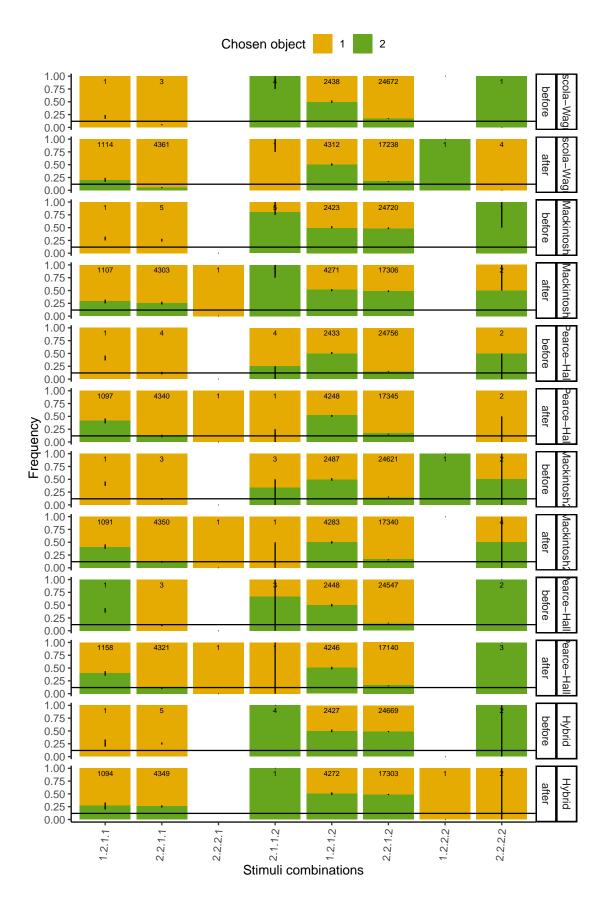


Figure 32: Dynamics of value estimation for the two objects (a) and performance (b) in the scenario with partial information in both stimuli dimensions. Grey lines in a correspond to the real value of the two objects. Grey line in b correspond to the expected proportion of wrong choices given the exploration parameter au in the desicion making rule.



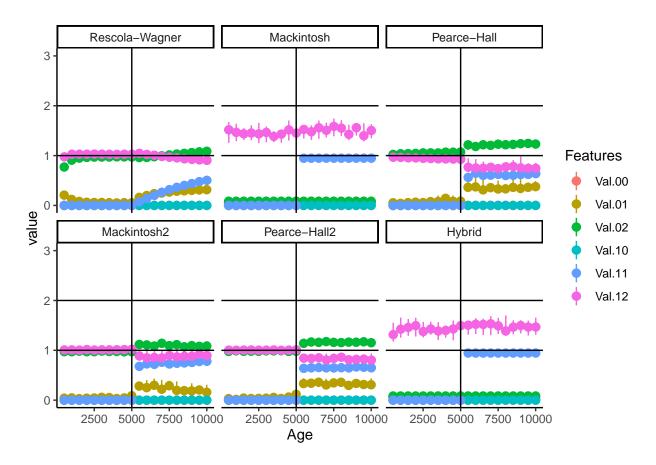


Figure 33: Dynamics of the values associated with the different features of the two stimuli dimensions for the scenario with partial information in one stimuli. In the legend the first number of the labels corresponds to the stimuli dimension index, and the second to the feature index. The black lines show the real value of the objects.

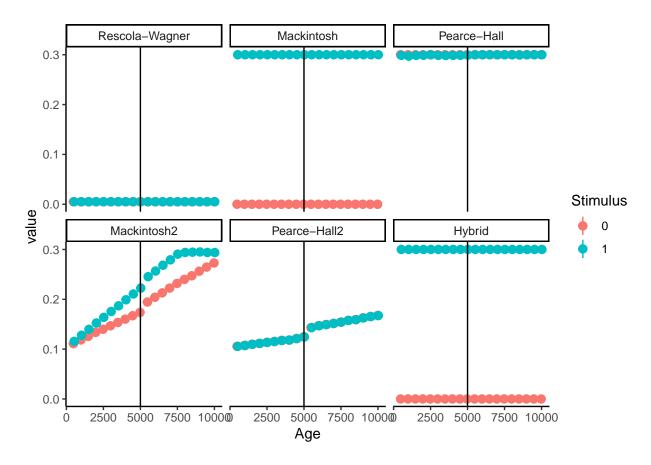


Figure 34: Dynamics of the learning speeds for each stimuli dimension discriminated by the attention mechanisms in the escenario with partial information for both stimuli.

Partial information for 2 stimuli (the second stimulus is only added half way the simulation - Redouan's #2):

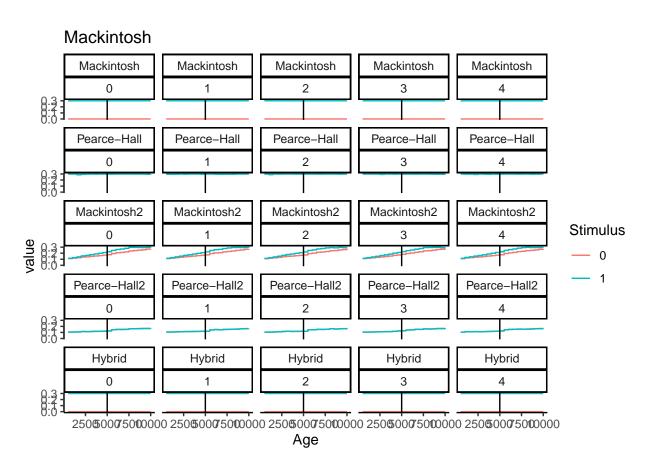


Figure 35: Examples of the dynamics of learning rates in a set of 4 replicates in the attention mechanisms for which learning rate changes in the scenario with partial information for one stimuli.

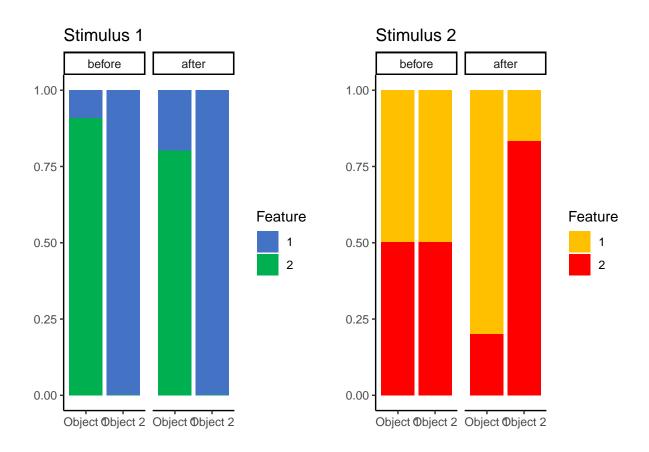


Figure 36: Frequency of features of the two different stimuli in the two different objects for the scenario with partial information for two stimulus.

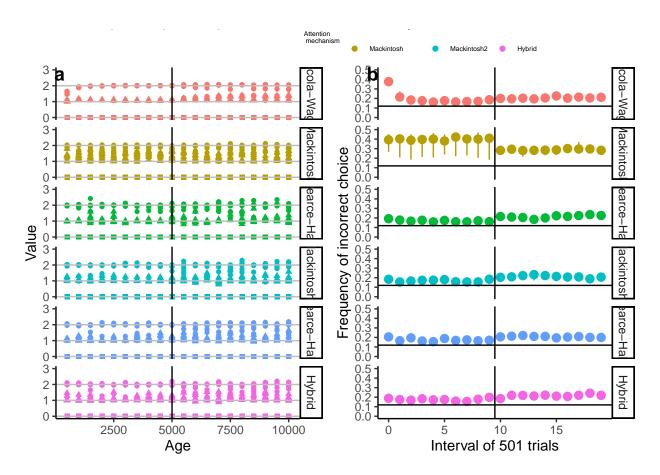


Figure 37: Dynamics of value estimation for the two objects (a) and performance (b) in the scenario with partial information in both stimuli dimensions. Grey lines in a correspond to the real value of the two objects. Grey line in b correspond to the expected proportion of wrong choices given the exploration parameter au in the desicion making rule.



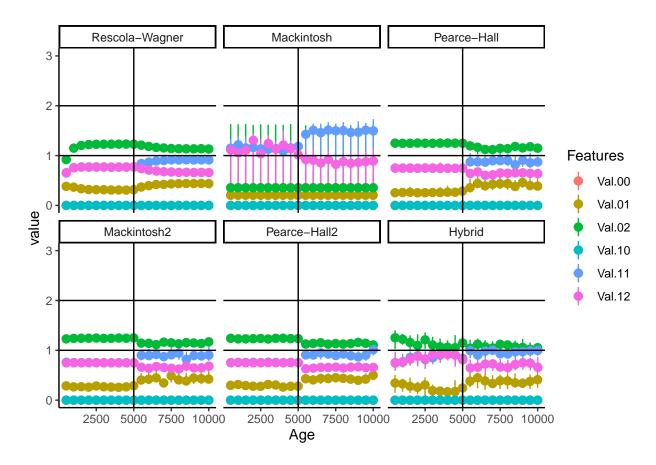


Figure 38: Dynamics of the values associated with the different features of the two stimuli dimensions for the scenario with partial information in one stimuli. In the legend the first number of the labels corresponds to the stimuli dimension index, and the second to the feature index. The black lines show the real value of the objects.

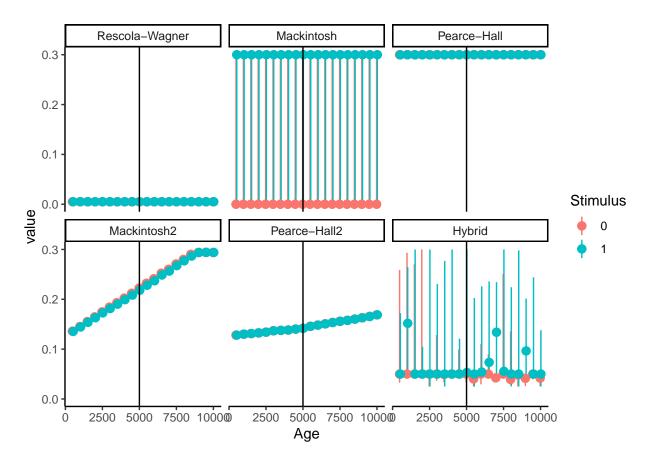


Figure 39: Dynamics of the learning speeds for each stimuli dimension discriminated by the attention mechanisms in the escenario with partial information for both stimuli.

Partial information for 3 stimuli (the second and third stimuli are only added half way the simulation - Redouan's #3):

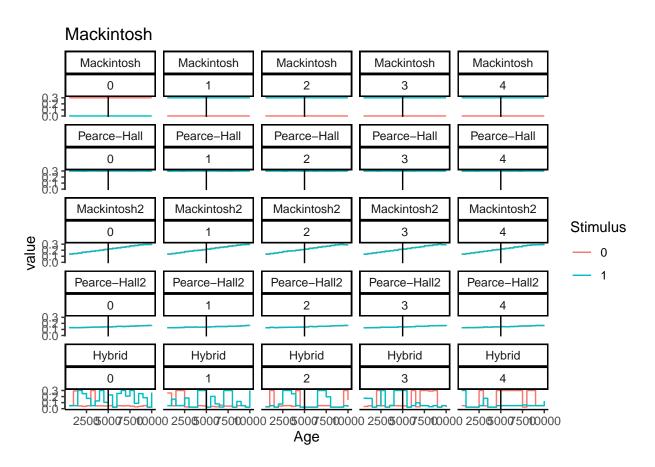


Figure 40: Examples of the dynamics of learning rates in a set of 4 replicates in the attention mechanisms for which learning rate changes in the scenario with partial information for one stimuli.

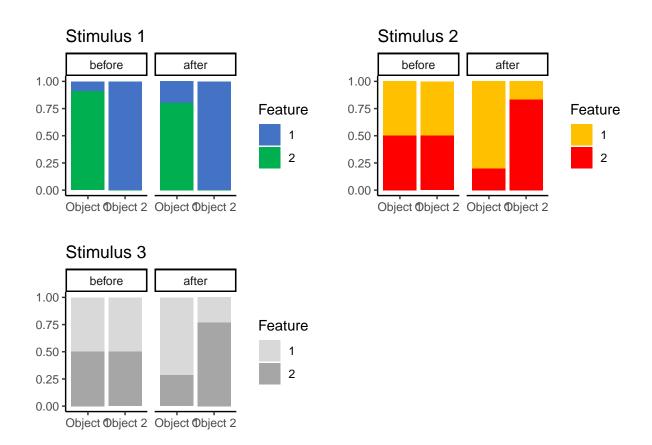


Figure 41: Frequency of features of the two different stimuli in the two different objects for the scenario with partial information for two stimulus.

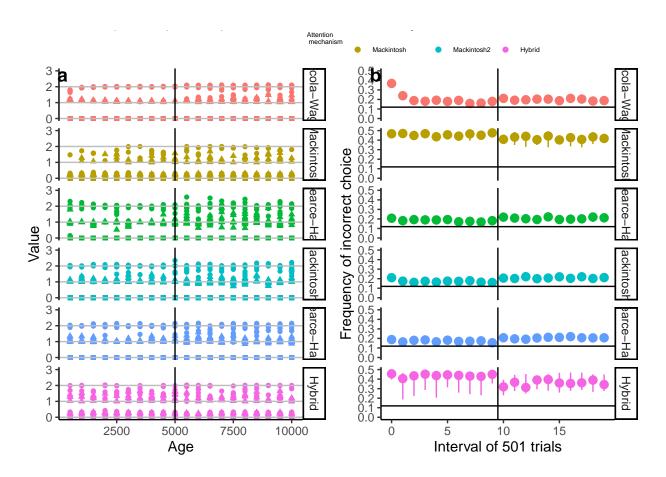
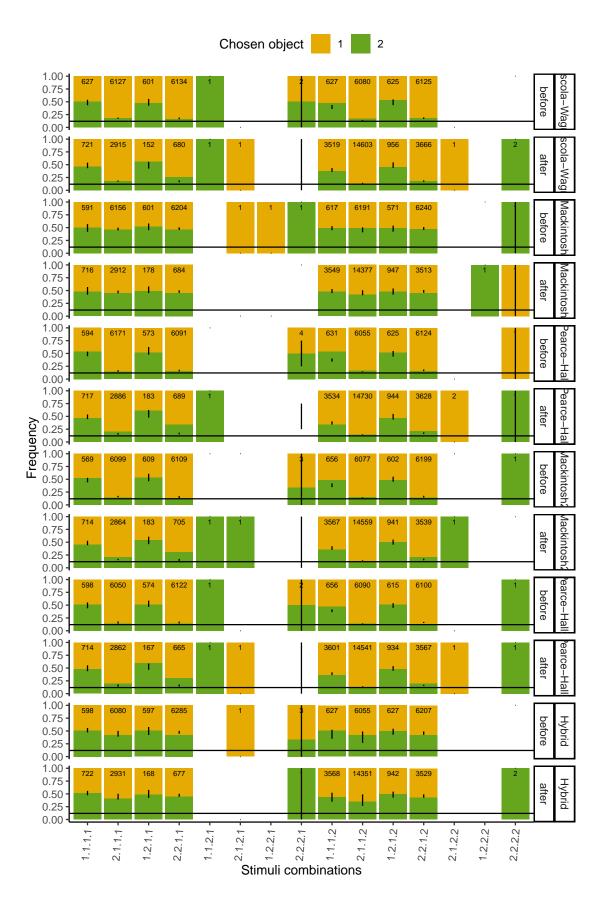


Figure 42: Dynamics of value estimation for the two objects (a) and performance (b) in the scenario with partial information in both stimuli dimensions. Grey lines in a correspond to the real value of the two objects. Grey line in b correspond to the expected proportion of wrong choices given the exploration parameter au in the desicion making rule.



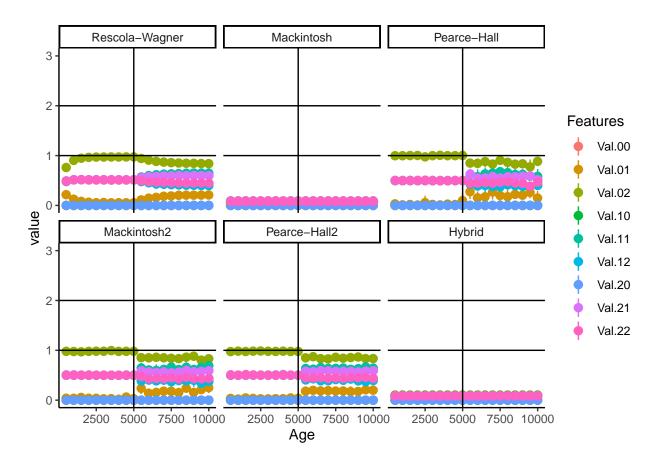


Figure 43: Dynamics of the values associated with the different features of the two stimuli dimensions for the scenario with partial information in one stimuli. In the legend the first number of the labels corresponds to the stimuli dimension index, and the second to the feature index. The black lines show the real value of the objects.

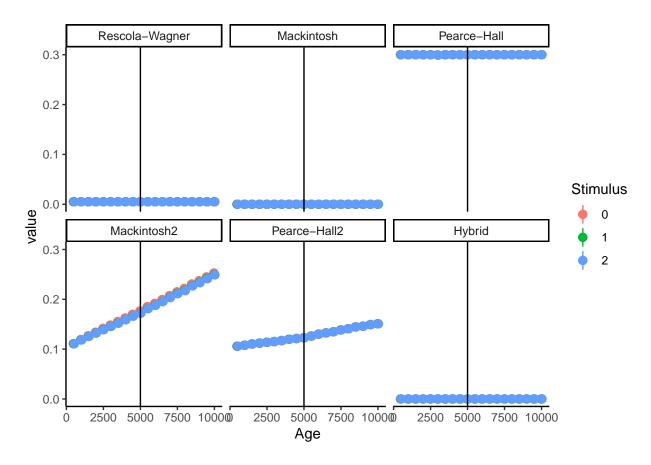


Figure 44: Dynamics of the learning speeds for each stimuli dimension discriminated by the attention mechanisms in the escenario with partial information for both stimuli.

Partial information for 2 stimuli (the second stimuli is only added half way the simulation - Redouan's #4):

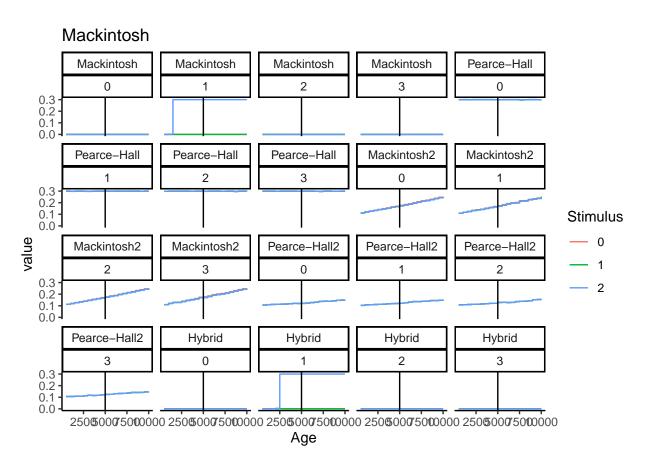


Figure 45: Examples of the dynamics of learning rates in a set of 4 replicates in the attention mechanisms for which learning rate changes in the scenario with partial information for one stimuli.

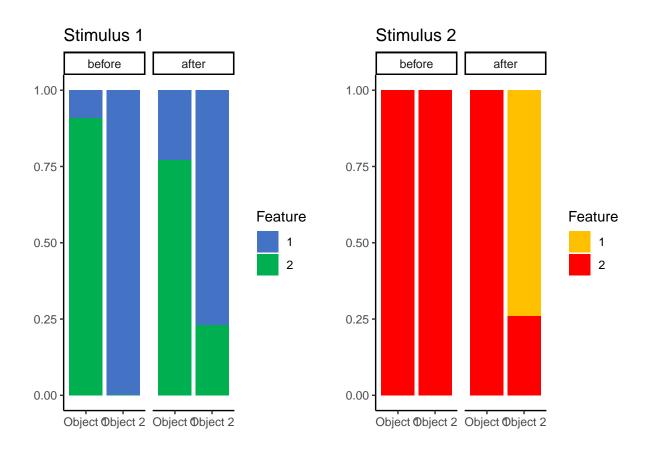


Figure 46: Frequency of features of the two different stimuli in the two different objects for the scenario with partial information for two stimulus.

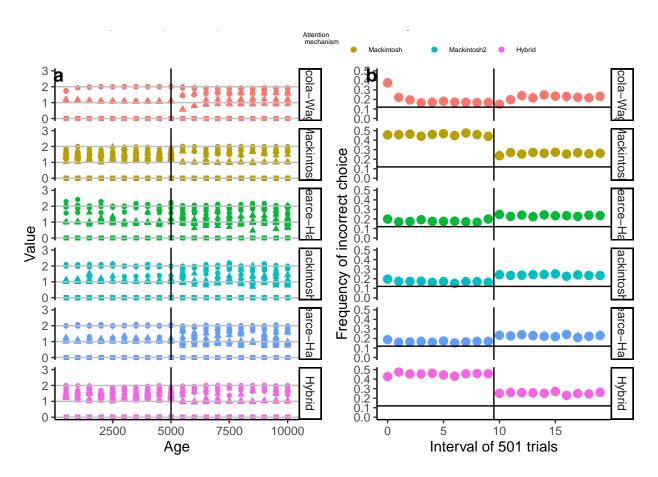


Figure 47: Dynamics of value estimation for the two objects (a) and performance (b) in the scenario with partial information in both stimuli dimensions. Grey lines in a correspond to the real value of the two objects. Grey line in b correspond to the expected proportion of wrong choices given the exploration parameter au in the desicion making rule.



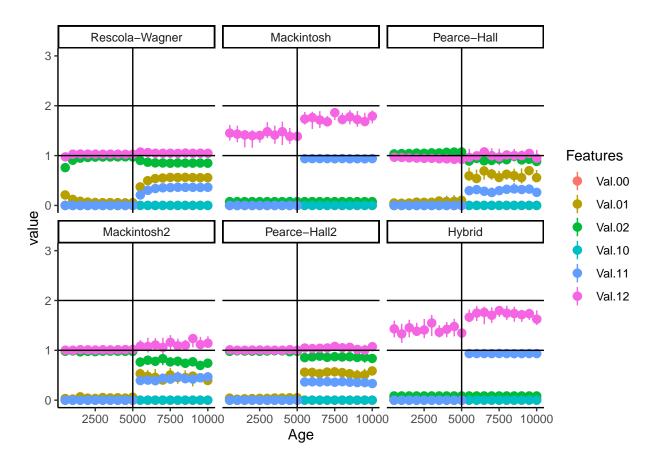


Figure 48: Dynamics of the values associated with the different features of the two stimuli dimensions for the scenario with partial information in one stimuli. In the legend the first number of the labels corresponds to the stimuli dimension index, and the second to the feature index. The black lines show the real value of the objects.

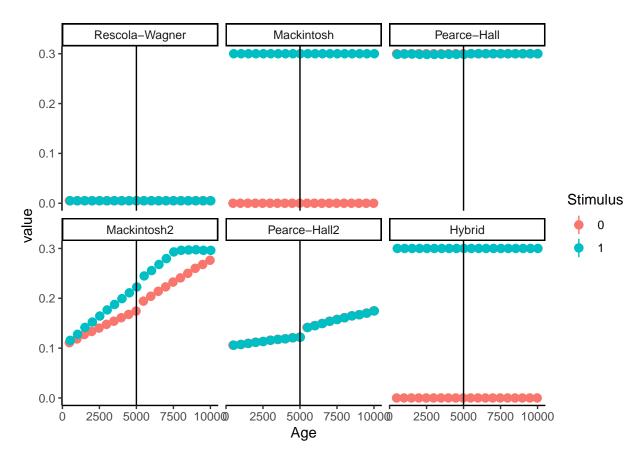


Figure 49: Dynamics of the learning speeds for each stimuli dimension discriminated by the attention mechanisms in the escenario with partial information for both stimuli.

Full info for one Partial information for the other one:

Here one of the stimuli contains information to distinguish the two object types. However, the information does not allow perfect discrimination. Specifically, object 1 has the feature 1, in the first stimulus dimension, with probability 0.99 and the alternative feature with the complementary probability (0.01). In contrast, object 2 has feature 2 in the first stimulus dimension, with probability 0.99. Thus, associating object 2 with feature two in the first dimension will lead to some errors where object 1 will be identifies as object 2. The features of the second stimulus have even probability for both objects.

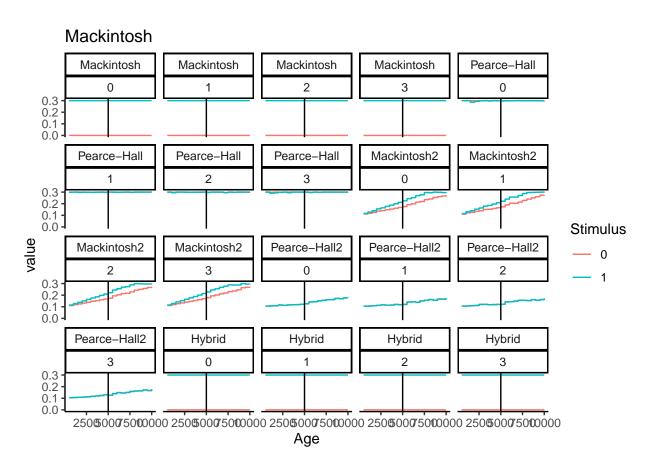


Figure 50: Examples of the dynamics of learning rates in a set of 4 replicates in the attention mechanisms for which learning rate changes in the scenario with partial information for one stimuli.

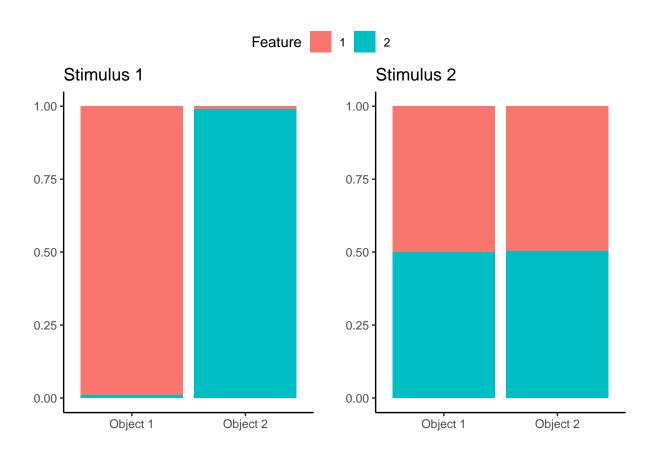


Figure 51: Frequency of features of the two different stimuli in the two different objects for the scenario with partial information for one stimulus.

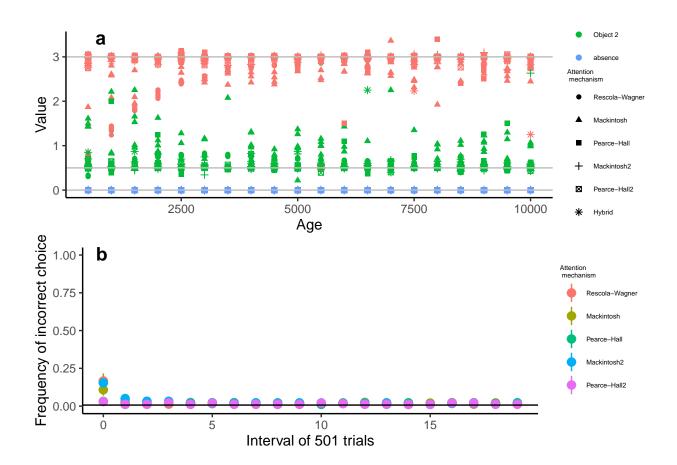
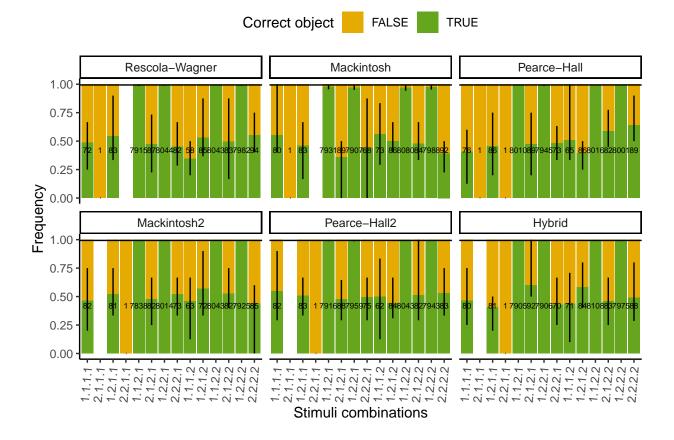


Figure 52: Dynamics of value estimation for the two objects (a) and performance (b) in the scenario with partial information in one stimuli dimensions. Grey lines in a correspond to the real value of the two objects. Grey line in b correspond to the expected proportion of wrong choices given the exploration parameter au in the desicion making rule.



Full info for one Partial information for the other one with random rewards:

Here one of the stimuli contains information to distinguish the two object types. However, the information does not allow perfect discrimination. Specifically, object 1 has the feature 1, in the first stimulus dimension, with probability 0.99 and the alternative feature with the complementary probability (0.01). In contrast, object 2 has feature 2 in the first stimulus dimension, with probability 0.99. Thus, associating object 2 with feature two in the first dimension will lead to some errors where object 1 will be identifies as object 2. The features of the second stimulus have even probability for both objects.

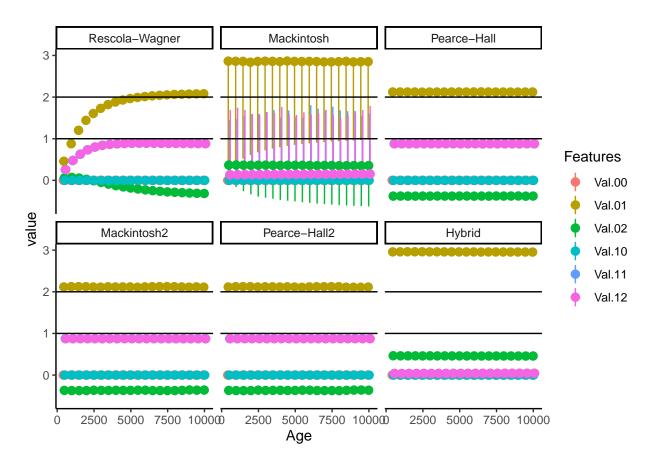


Figure 53: Dynamics of the values associated with the different features of the two stimuli dimensions for the scenario with partial information in one stimuli. In the legend the first number of the labels corresponds to the stimuli dimension index, and the second to the feature index. The black lines show the real value of the objects.

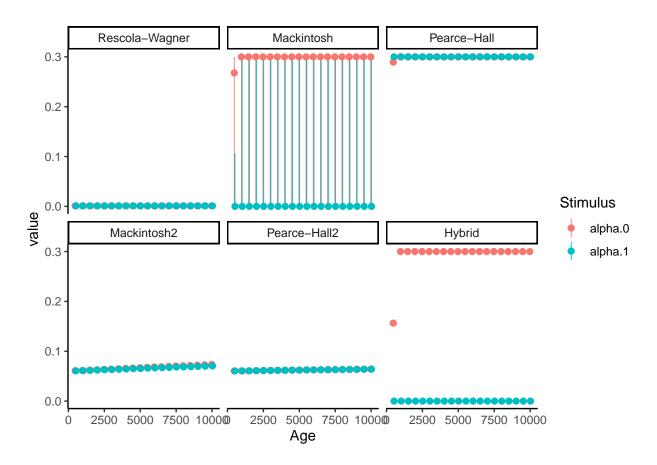


Figure 54: Dynamics of the learning speeds for each stimuli dimension discriminated by the attention mechanisms in the escenario with partial information for one stimuli.

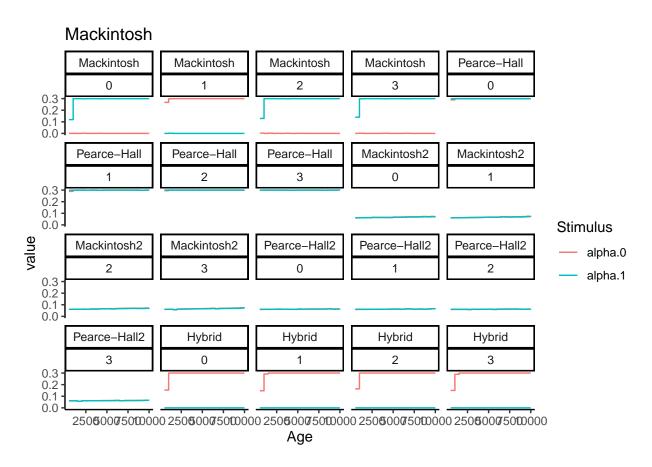


Figure 55: Examples of the dynamics of learning rates in a set of 4 replicates in the attention mechanisms for which learning rate changes in the scenario with partial information for one stimuli.

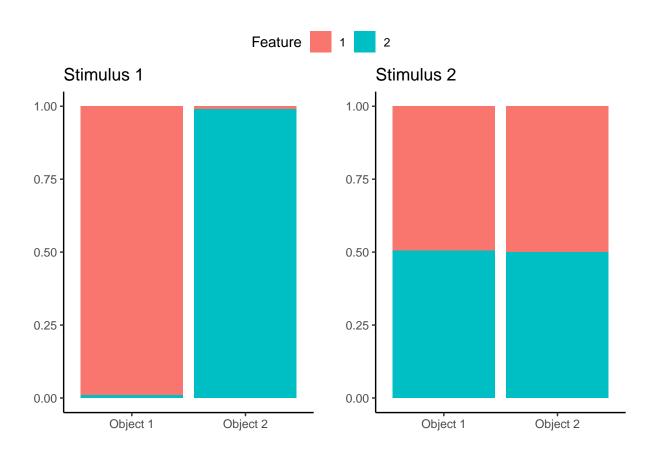


Figure 56: Frequency of features of the two different stimuli in the two different objects for the scenario with partial information for one stimulus.

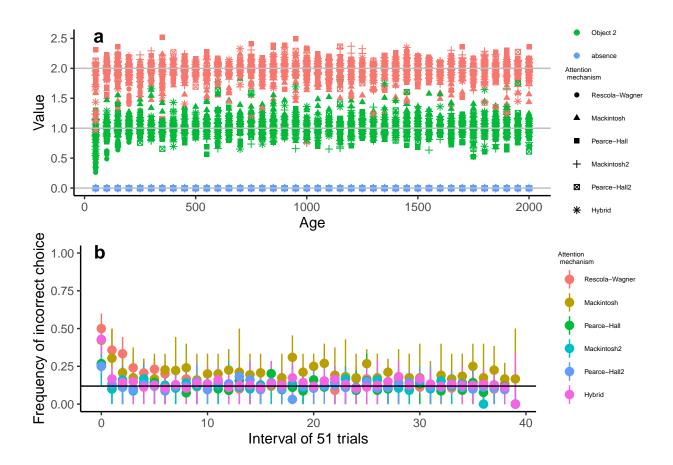
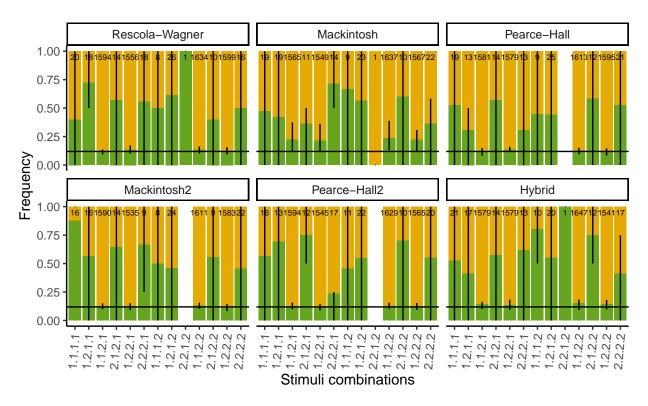


Figure 57: Dynamics of value estimation for the two objects (a) and performance (b) in the scenario with partial information in one stimuli dimensions. Grey lines in a correspond to the real value of the two objects. Grey line in b correspond to the expected proportion of wrong choices given the exploration parameter au in the desicion making rule.





References

Esber, Guillem R., and Mark Haselgrove. 2011. "Reconciling the Influence of Predictiveness and Uncertainty on Stimulus Salience: A Model of Attention in Associative Learning." *Proceedings of the Royal Society of London B: Biological Sciences* 278 (1718): 2553–61. https://doi.org/10.1098/rspb.2011.0836.

Mackintosh, Nicholas J. 1975. "A Theory of Attention: Variations in the Associability of Stimuli with Reinforcement." *Psychological Review* 82 (4): 276.

Pearce, John M., and Geoffrey Hall. 1980. "A Model for Pavlovian Learning: Variations in the Effectiveness of Conditioned but Not of Unconditioned Stimuli." *Psychological Review* 87 (6): 532. http://psycnet.apa.org/journals/rev/87/6/532/.

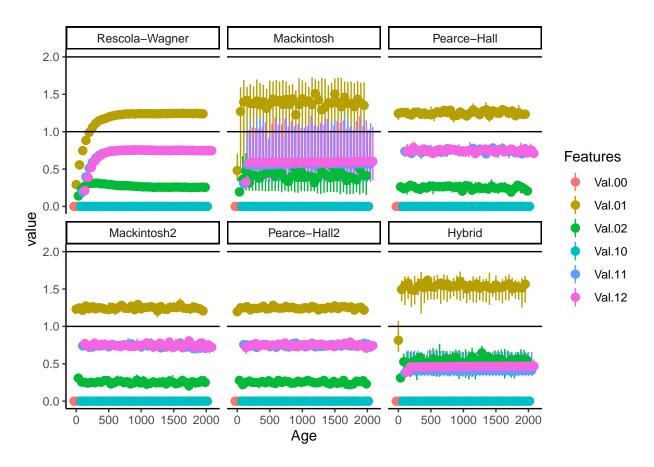


Figure 58: Dynamics of the values associated with the different features of the two stimuli dimensions for the scenario with partial information in one stimuli. In the legend the first number of the labels corresponds to the stimuli dimension index, and the second to the feature index. The black lines show the real value of the objects.

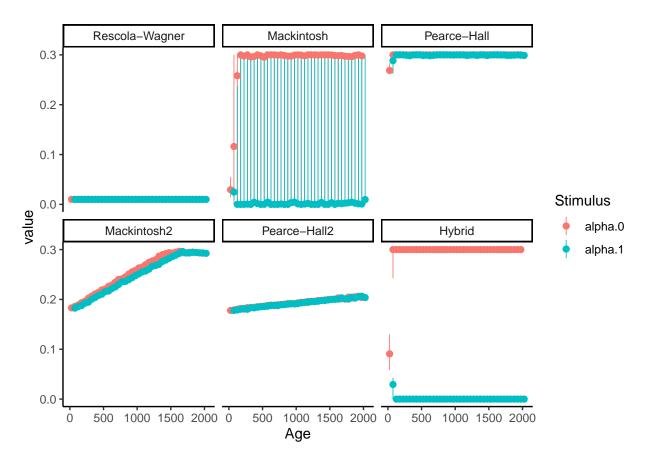


Figure 59: Dynamics of the learning speeds for each stimuli dimension discriminated by the attention mechanisms in the escenario with partial information for one stimuli.

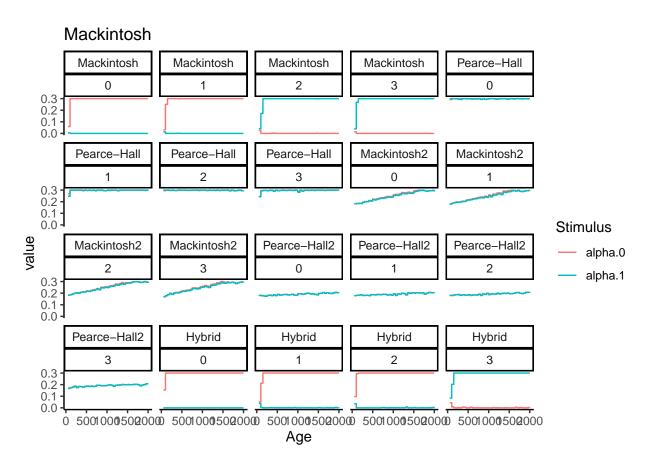


Figure 60: Examples of the dynamics of learning rates in a set of 4 replicates in the attention mechanisms for which learning rate changes in the scenario with partial information for one stimuli.