Fitting associate learning models to data

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The Boussard *et al* data set

In here I present the statistical model used to estimate reinforcement learning parameters from data from a reversal learning task. In the experimental set up, individuals from two experimental treatments are trained to pick one of two stimuli. Where one of those two options provides reward in the form of food pellets. The experiments is composed of 11 reversal blocks, each of these blocks is composed of 30 trial. In other words, every 30 trials the stimulus that provides reward is switched. Thus, individuals must reverse their estimate of reward in order to make adaptive decisions and choose the rewarding stimulus. In figure 1, I show the proportion of correct choices for both treatment groups along the trials, and reversal blocks.

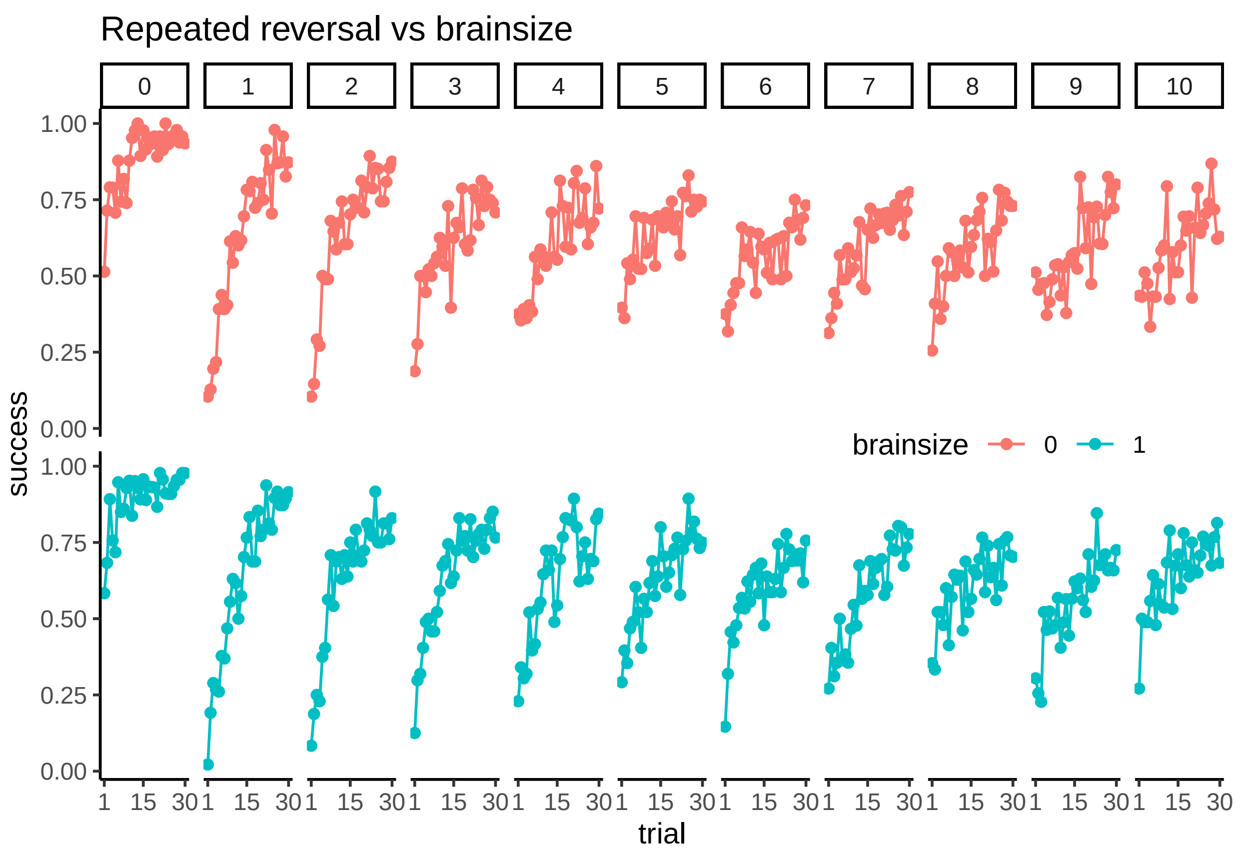

Figure 1: The Boussard *et al* data-set. Points show the proportion of successes achieved by individuals of both treatment groups along trials and reversal blocks.

In order to evaluate the performance of the model in estimating the parameter values. I first fit the model parameters to a data-set obtained by simulation the model with a set of pre-set parameter values. I then evaluate whether the algorithm correctly estimates these parameters. The model is based of the Rescorla-Wagner model of associative learning. Each individual has an estimate of reward for each stimulus. When two stimuli are presented together, the individual chooses one of the two with a probability given by the soft-max distribution. We assume all individuals use the same temperature parameter (τ). This parameter is estimated. Once the individual chooses one of the two stimuli, it updates its estimates of reward associated with the chosen stimulus. The update is performed every trial and is given by the product of the *prediction error* (δ) and the *speed of learning* (α). The *prediction error* is calculated as the difference between the reward triggered by choosing a particular stimulus and the estimation of that reward prior to the choice. We assume each individual expresses a different *speed of learning*, and it's given by fixed and random effects. The treatment group of each individual is the fixed effect. Thus, we estimate a contribution of the treatment group to the *speed of learning* of each individual. As for the random effect, we assume the individual specific contributions to the speed of learning are distributed as a random variable with a normal distribution. We estimate the mean and standard deviation of the distribution. The model uses thus a hierarchical bayesian approach to estimating the parameters of the Rescorla-Wagner model.

The simple fake data set

We first use the model to estimate a simple associative learning task without any reward reversal. In table 1, I show the values of the model parameters to simulate the first data-set. In figure 2, I show the average frequency of success for the two treatment groups. In figure 3, I show the confidence interval of the parameters estimated, as well as the values used for the simulations. In fig 4, I show how the data predicted by the estimated parameters compares with the data simulated from the 'real' parameters.

Table 1: Parameters of the simulated data-set.

parameters	values
Temperature	1.0
Mean speed of learning	0.2
Effect of treatment 0	-5.0
Effect of treatment 1	5.0
St. dev. of speed of learning	2.0

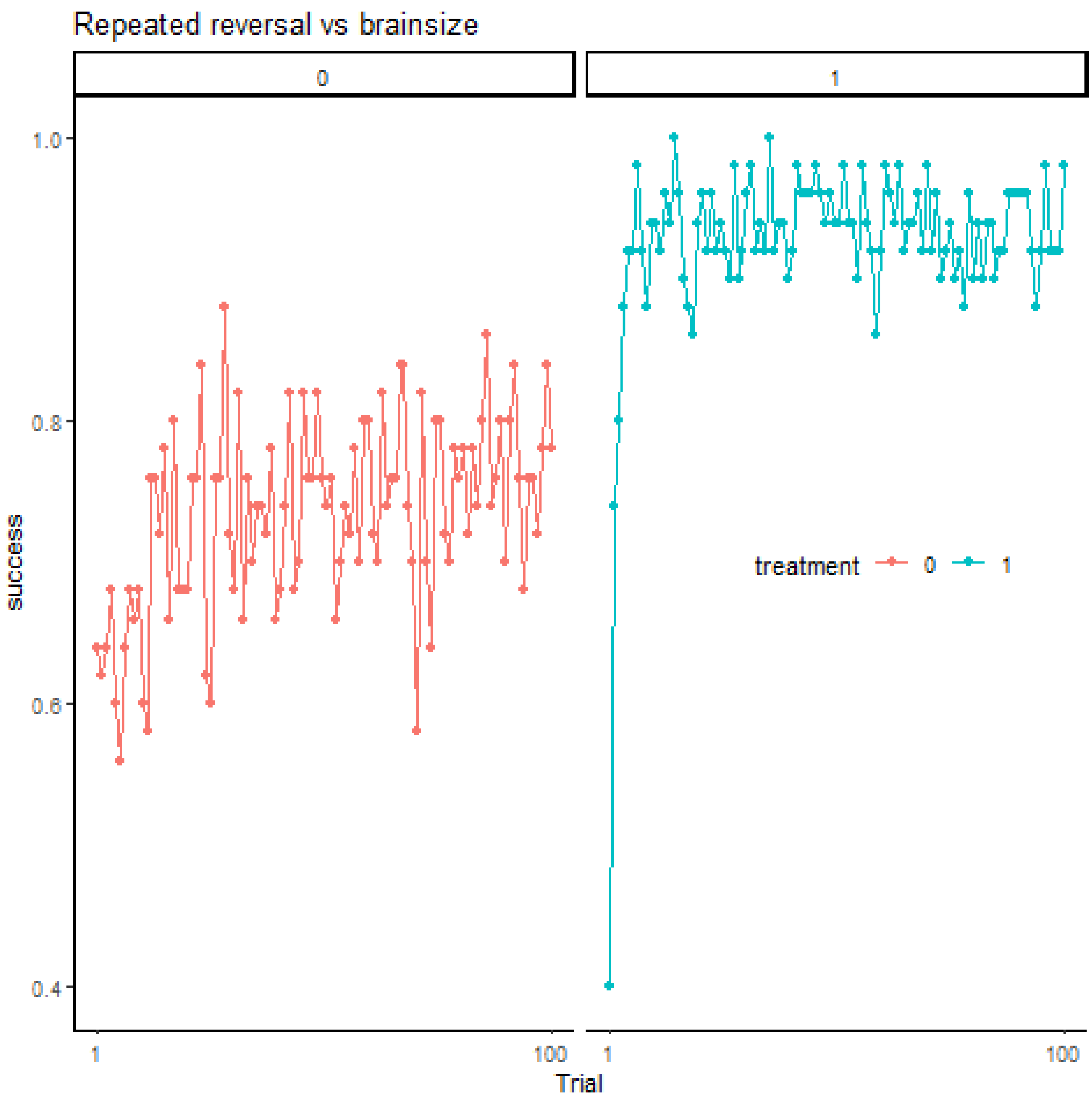

Figure 2: Simulated data from a simple associative learning model with two treatment effects. Points represent the frequency of successes among 50 individuals.

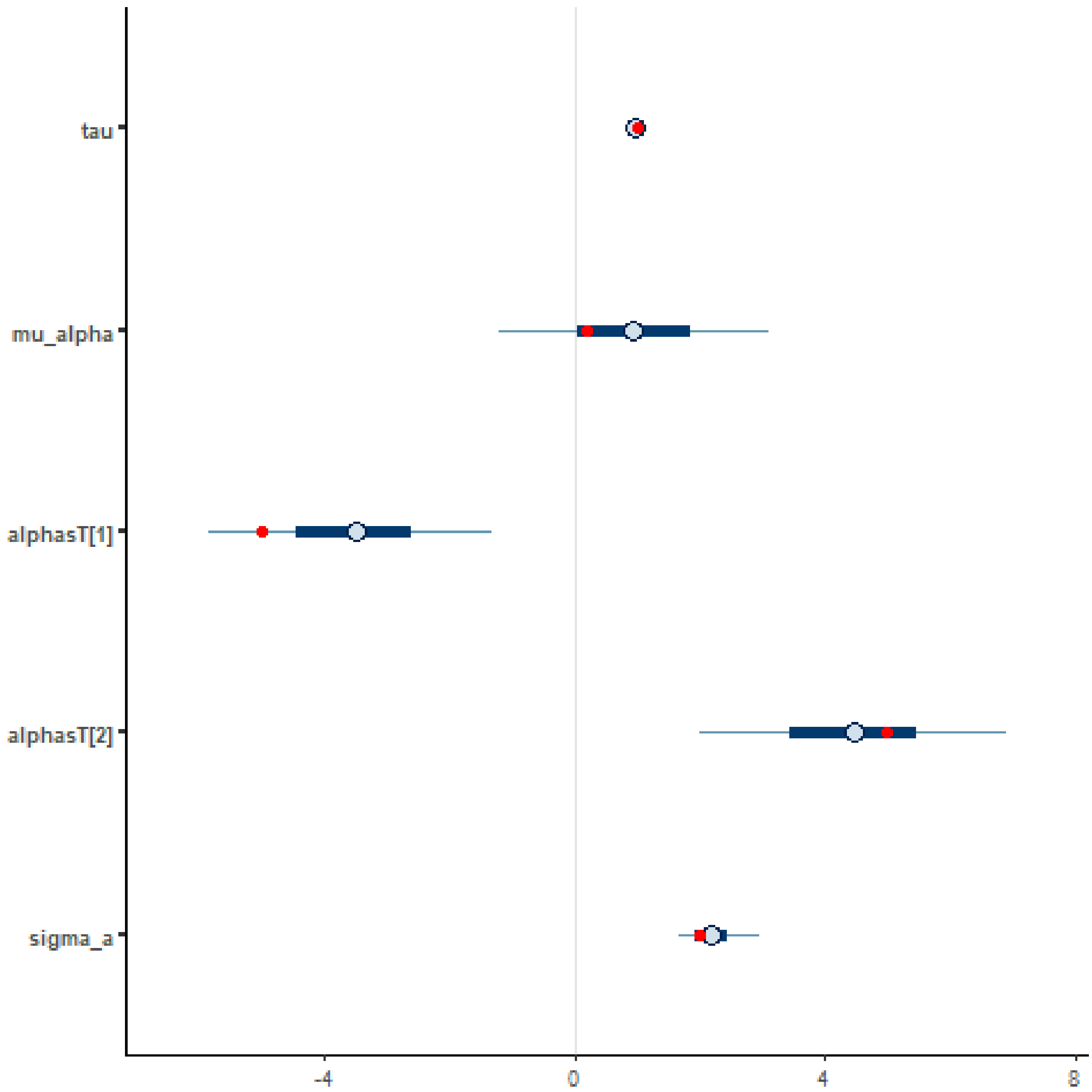

Figure 3: Estimation of the paramerters used to generate the simulated data. Blue bars and lines correspond to the 50 and 90% credible intervals, respectively. Red points correspond to the real values of the parameteres used in the simulation.

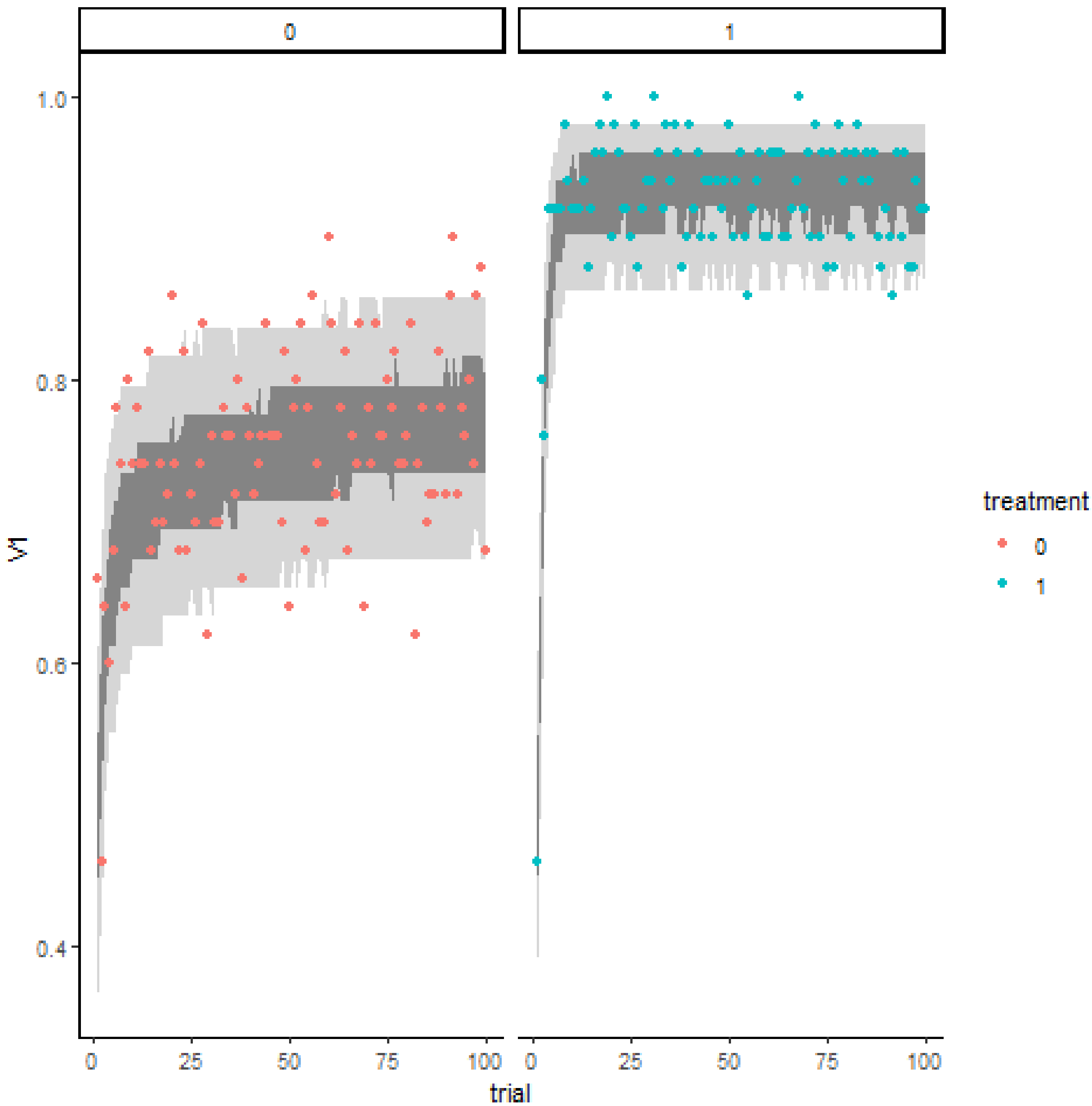

Figure 4: Out of sample posterior predictive checks of the bayesian estimation. Points correspond to a simulated data-set using the parameter values shown earlier. Light and dark grey bands correspond to the 50 and 90% credible interval of predictions made from the posterior distributions.

The reversal fake data set

Now I presented a date-set simulatd from the model, but including reversal learning structured in the same way as the Boussard *et al* experiments. Table 2 shows the parameter values used for the simulations, fig. 5 shows the data simulated from the model, fig. 6 shows the estimation of the parameters together with the'real' value. Finally, fig. 7 shows the predictions from the estimated parameters together with data simulated with the 'real' values.

Table 2: Parameters of the simulated reversal learning data-set

parameters	values
Temperature	1.0
Mean speed of learning	0.2
Effect of treatment 0	-2.0
Effect of treatment 1	2.0
St. dev. of speed of learning	0.5

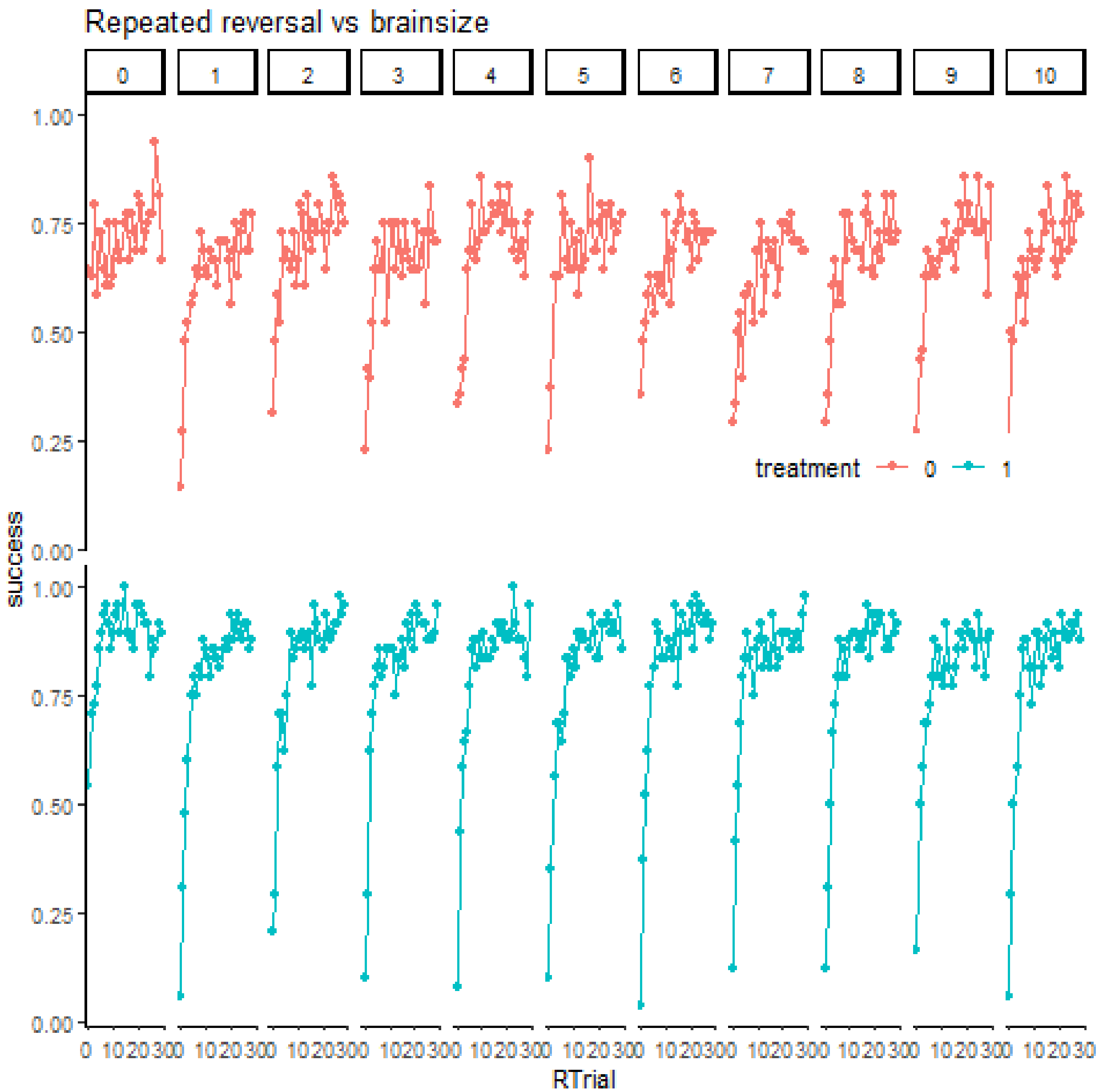

Figure 5: Simulated data of the reversal learning model.

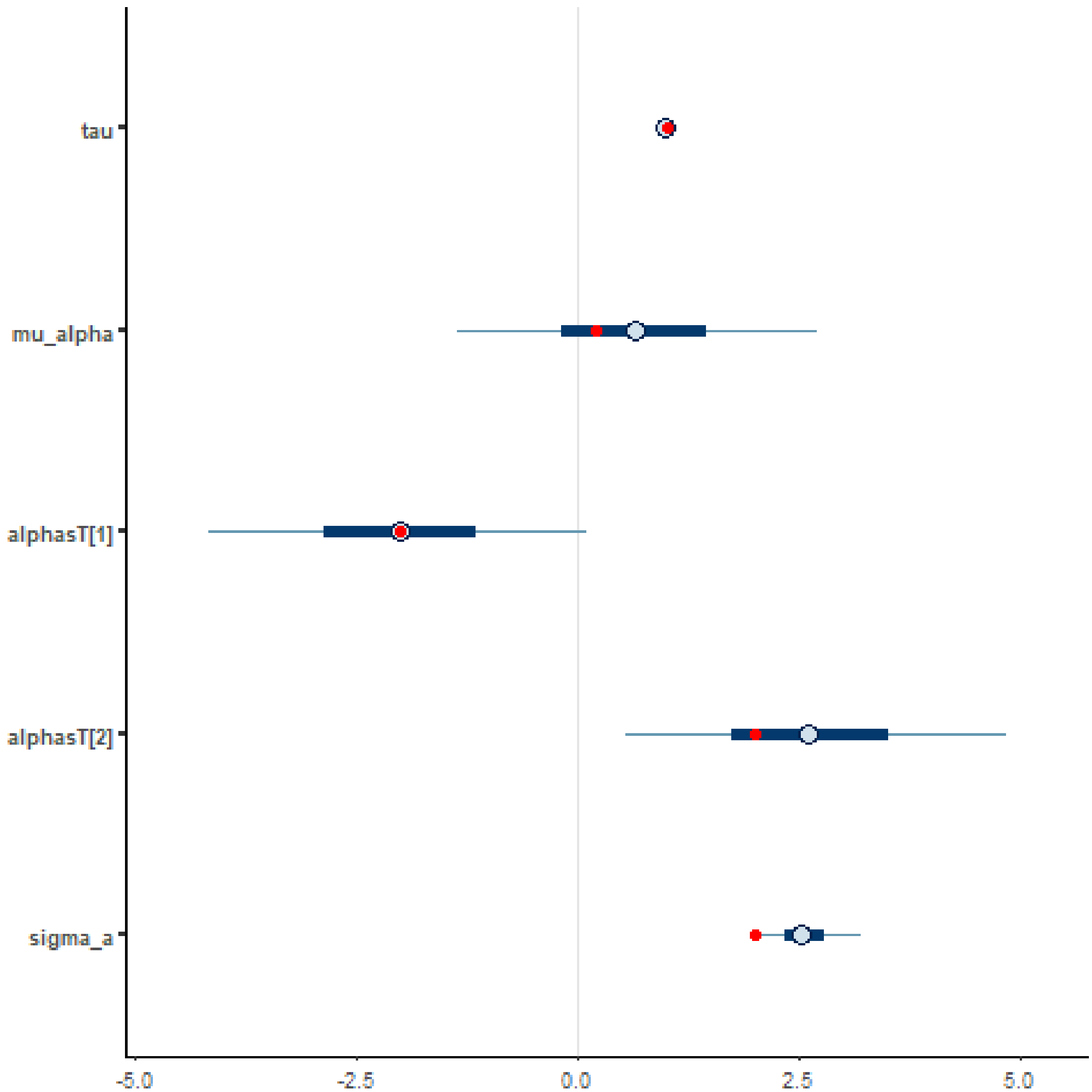

Figure 6: Estimated credible intervals and real values of the parameters of the reversal learning model. Blue bars and lines correspond to the 50 and 90% credible intervals, respectively. Red points correspond to the real values of the parameteres used in the simulation of reversal learning.

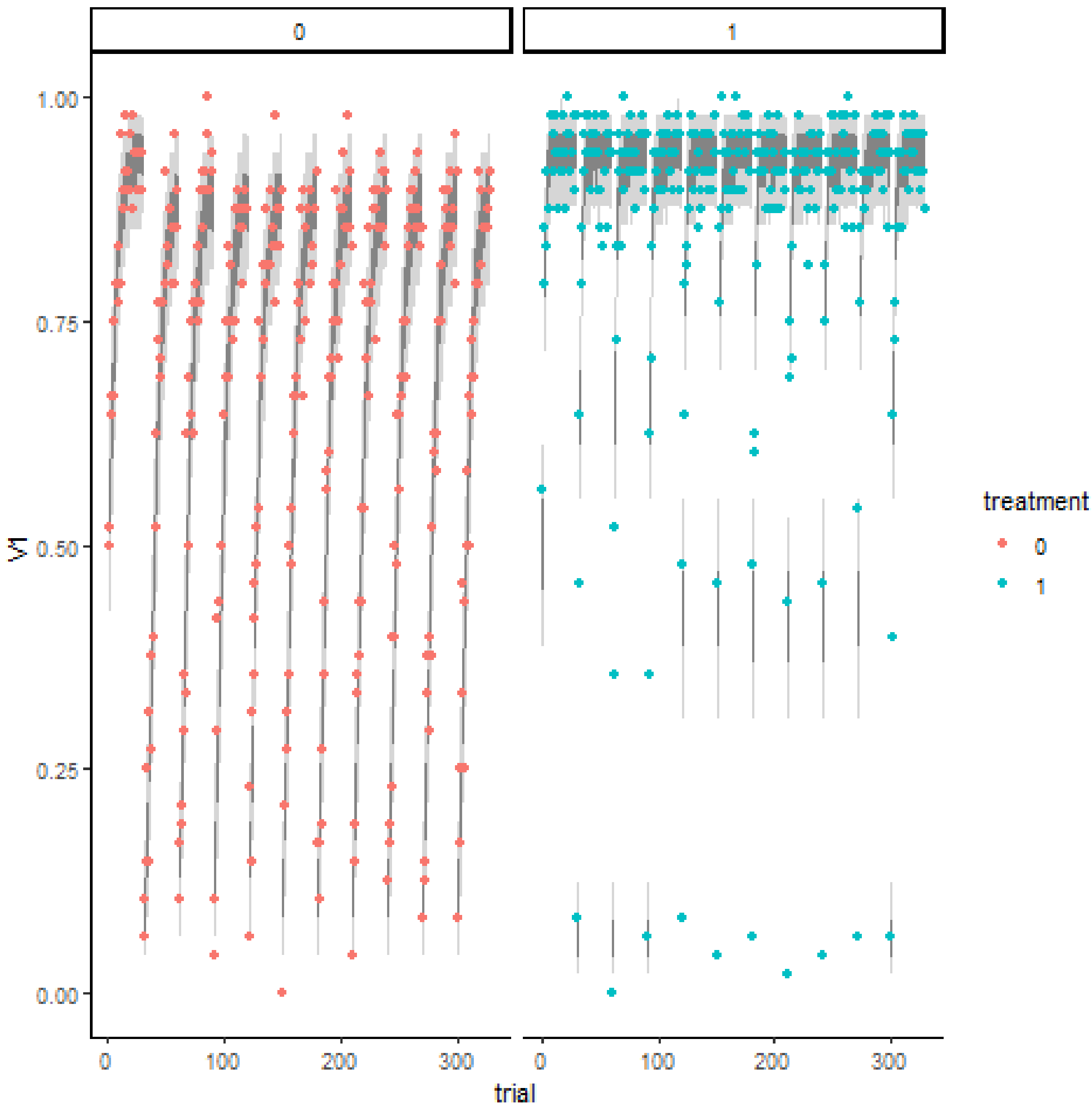

Figure 7: Out of sample posterior predictive checks of the bayesian estimation. Points correspond to a simulated data-set using the parameter values shown earlier from the simulation of reversal learning. Light and dark grey bands correspond to the 50 and 90% credible interval

Fitting the model to the Boussard *et al* data set

Now we use the model to fit the parameters to the Boussard *et al* data-set. Figure 8 shows the parameters estimated and figure 9 shows how well the data fits the predictions from the model. Clearly a model with constant speed of learning throughout the reversal blocks does not capture well the desicion-making dynamics.

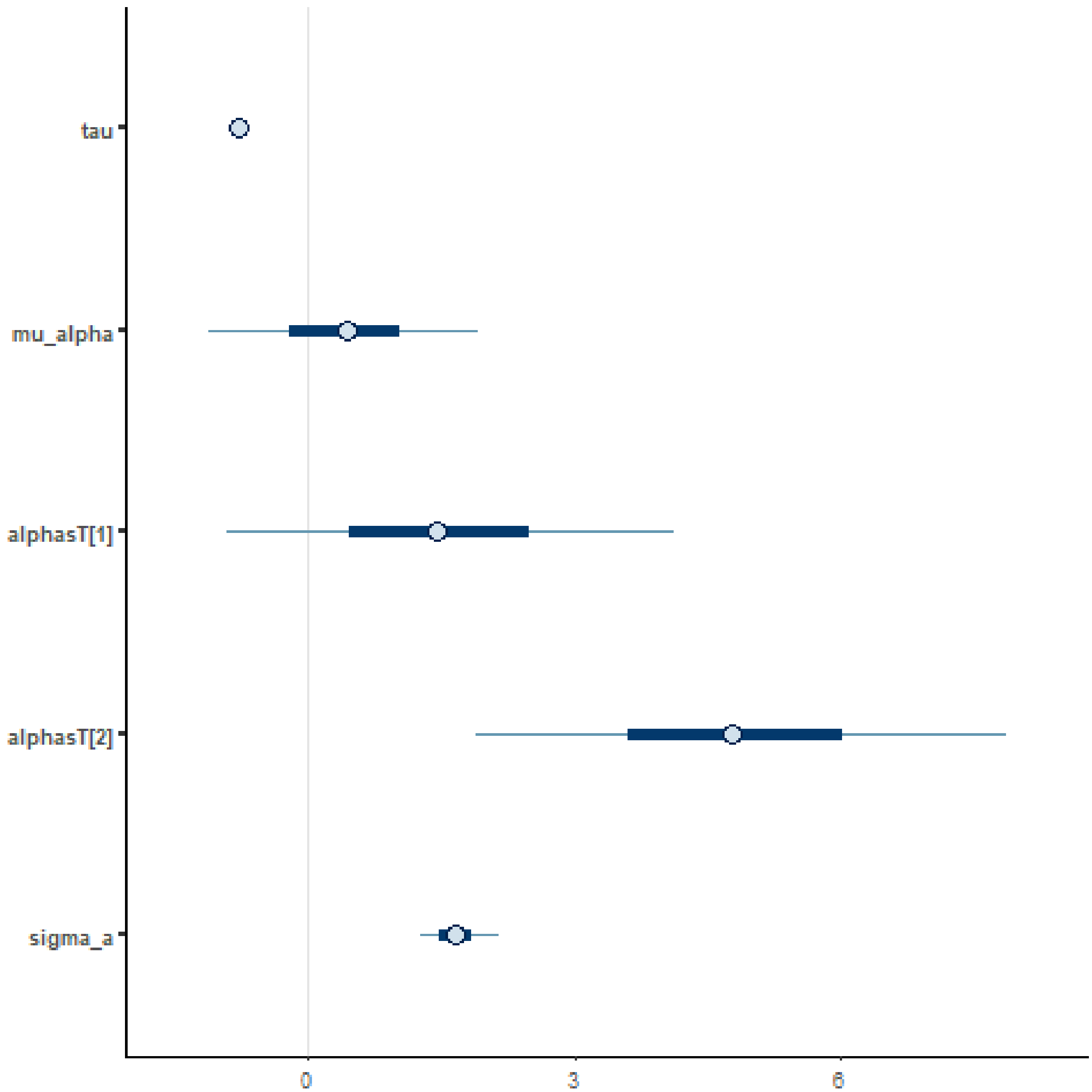

Figure 8: Bayesian estimation of paramteres to the Boussard *et al* data set. Light and dark grey bands correspond to the 50 and 90% credible interval

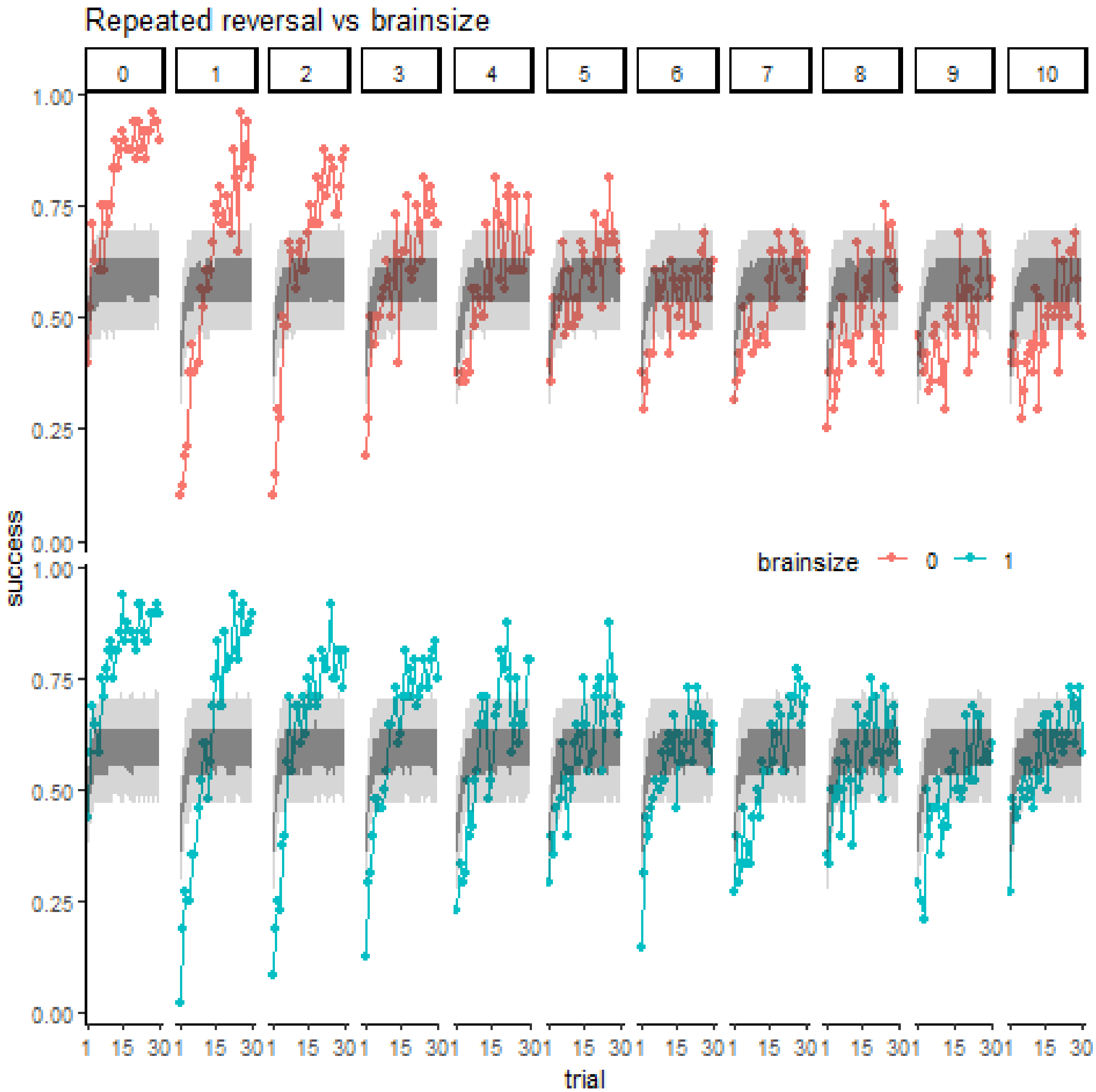

Figure 9: Posterior predictive checks of the bayesian estimation. Points correspond to Boussard data-set . Light and dark grey bands correspond to the 50 and 90% credible interval predicted by the model

Let' fit a model with different learning rates in reversal blocks to the Boussard *et al* data set

Now we use a model where we allow individuals to have a different *speed of learning* on each reversal block. To do that, we estimate a fixed effect of the reversal block on the parameter α. Figure 10 shows the estimated parameters including the effect of reversal block on the *speed of learning*, figure 11 shows how well the model predicts the data. For some reason, the model estimates a very low *speed of learning* in the early blocks of reversal. This causes the model to perform rather poorly in explaining the desicion-making dynamics.

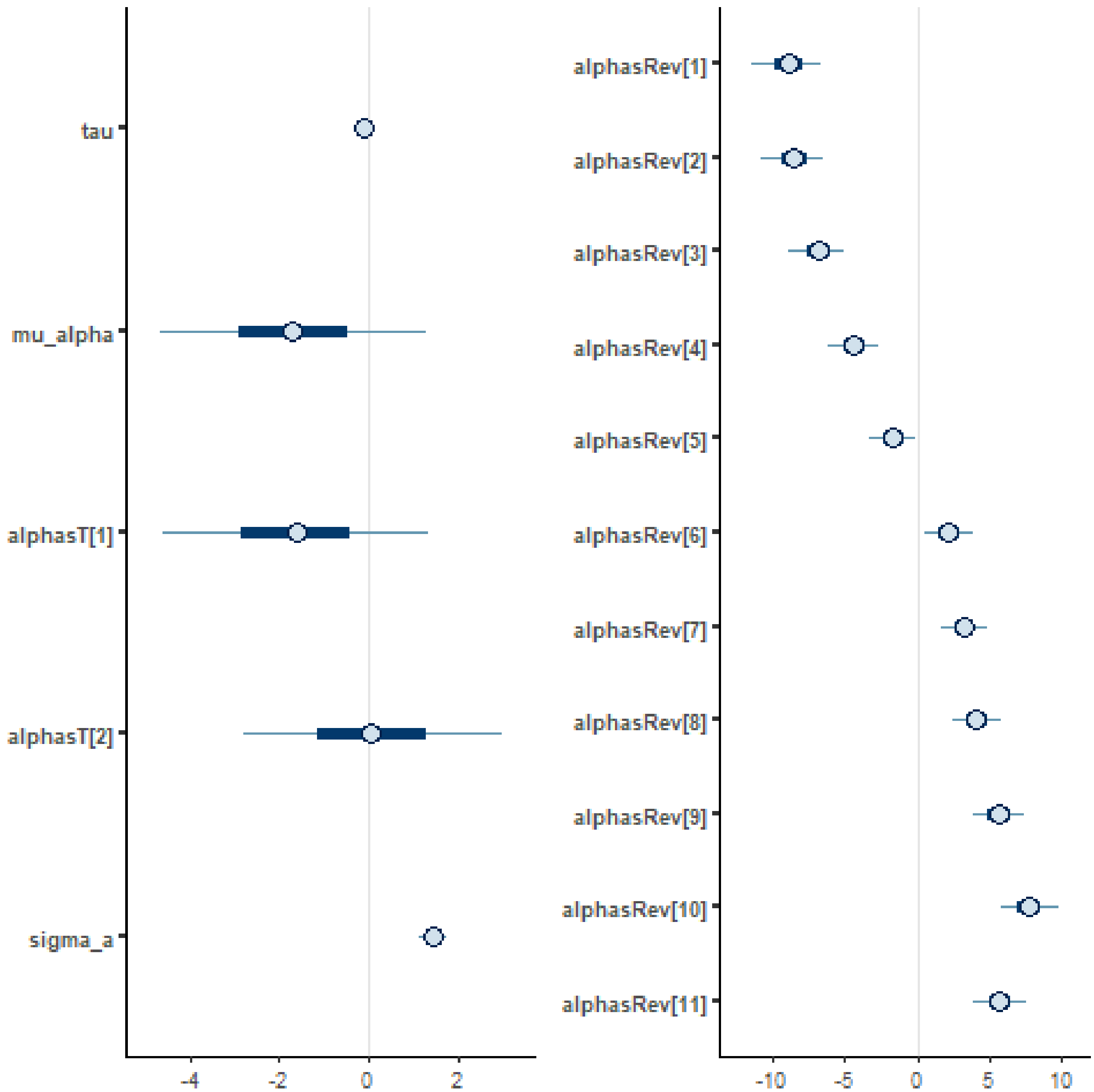

Figure 10: Estimated credible intervals of the parameters of the reversal learning model. Blue bars and lines correspond to the 50 and 90% credible intervals, respectively. Red points correspond to the real values of the parameteres used in the simulation of reversal learning. Panel on the right shows the estimated effect of the reversal blocks on the speed of learning.

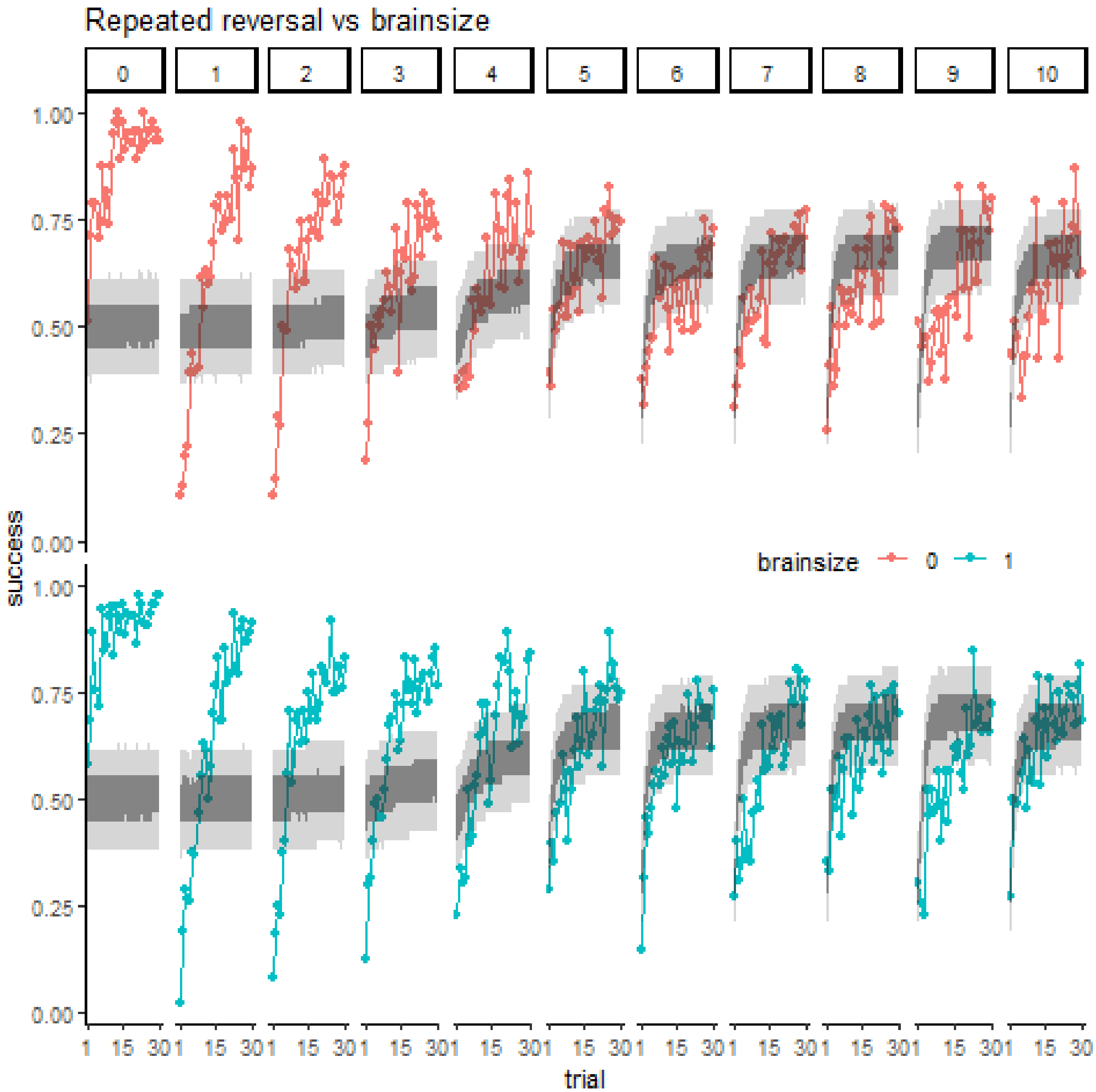

Figure 11: Posterior predictive checks of the bayesian estimation for a model with different *speed of learning* on the reversal blocks. Points correspond to Boussard data-set. Light and dark grey bands correspond to the 50 and 90% credible interval predicted by the model

The reversal fake data set with different learning rates in the different reversal blocks

Table 3: Parameters of the simulated reversal learning data-set

parameters	values
Temperature	1.0
Mean speed of learning	0.0
Effect of treatment 0	-2.0
Effect of treatment 1	0.5
St. dev. of speed of learning	2.0
Effect of reversal block 1	2.0
Effect of reversal block 2	1.6
Effect of reversal block 3	1.2
Effect of reversal block 4	0.8
Effect of reversal block 5	0.4
Effect of reversal block 6	0.0
Effect of reversal block 7	-0.4
Effect of reversal block 8	-0.8
Effect of reversal block 9	-1.2
Effect of reversal block 10	-1.6
Effect of reversal block 11	-2.0

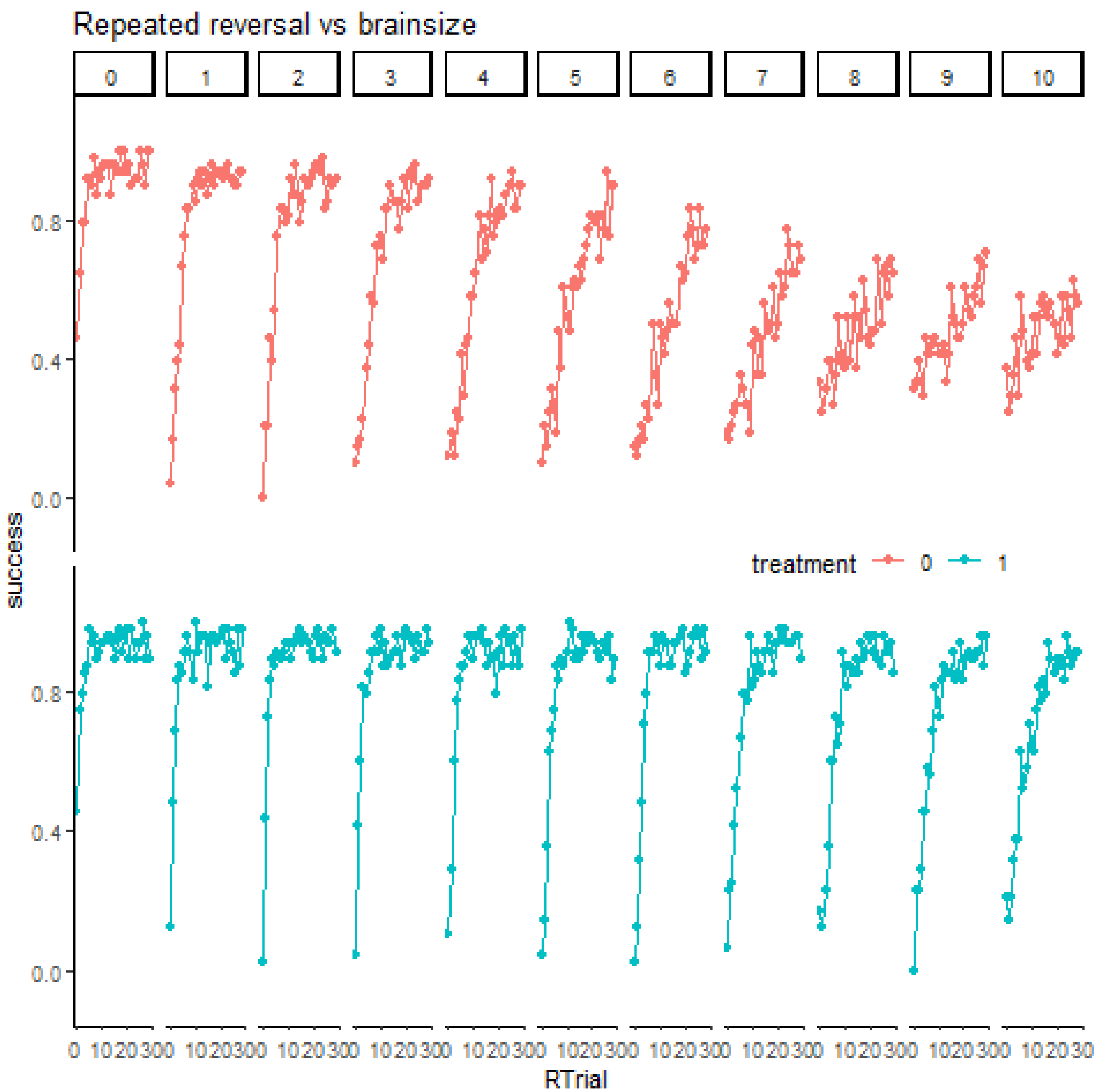

Figure 12: Simulated data of the reversal learning model.

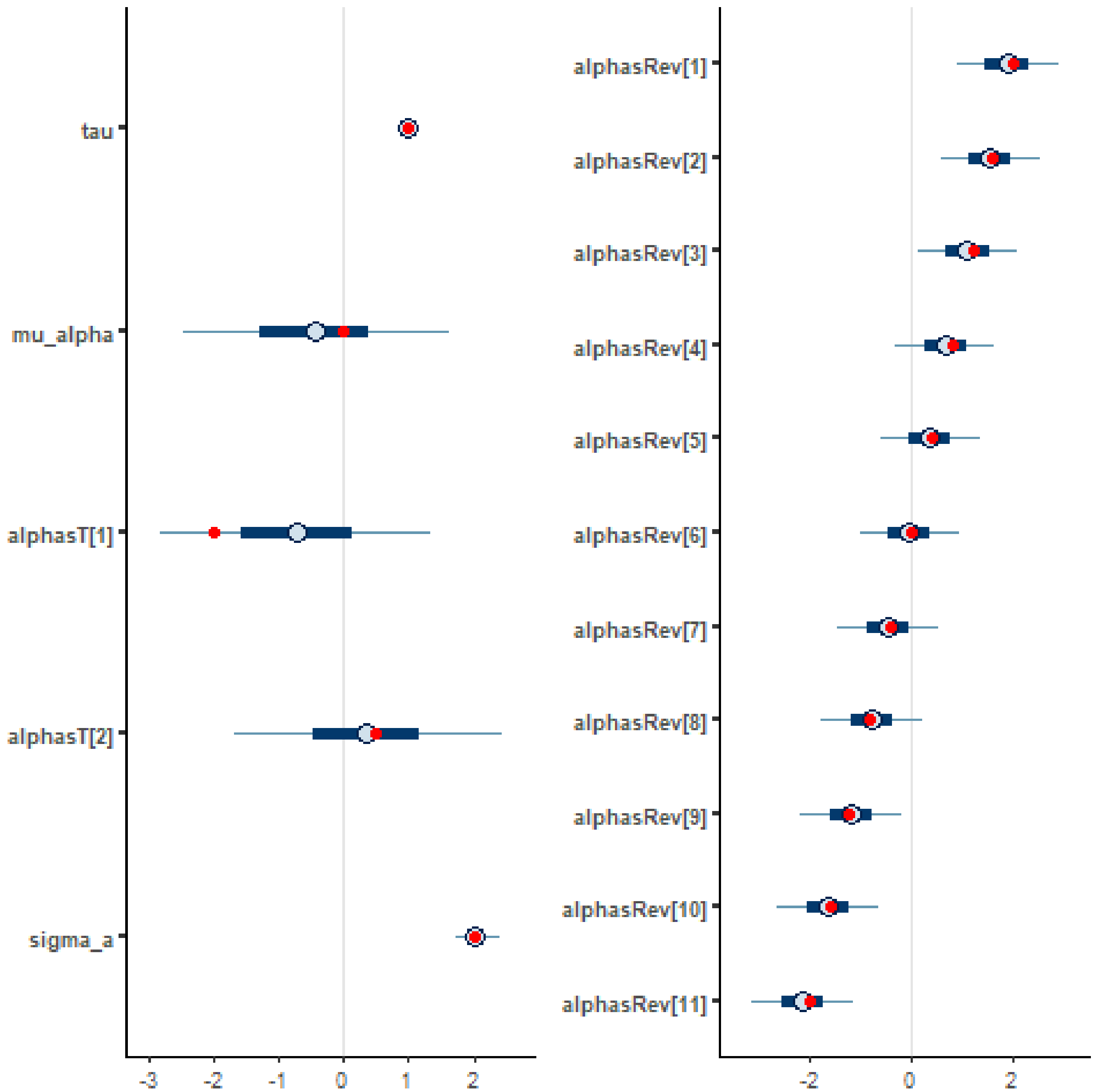

Figure 13: Estimated credible intervals and real values of the parameters of the reversal learning model. Blue bars and lines correspond to the 50 and 90% credible intervals, respectively. Red points correspond to the real values of the parameteres used in the simulation of reversal learning.

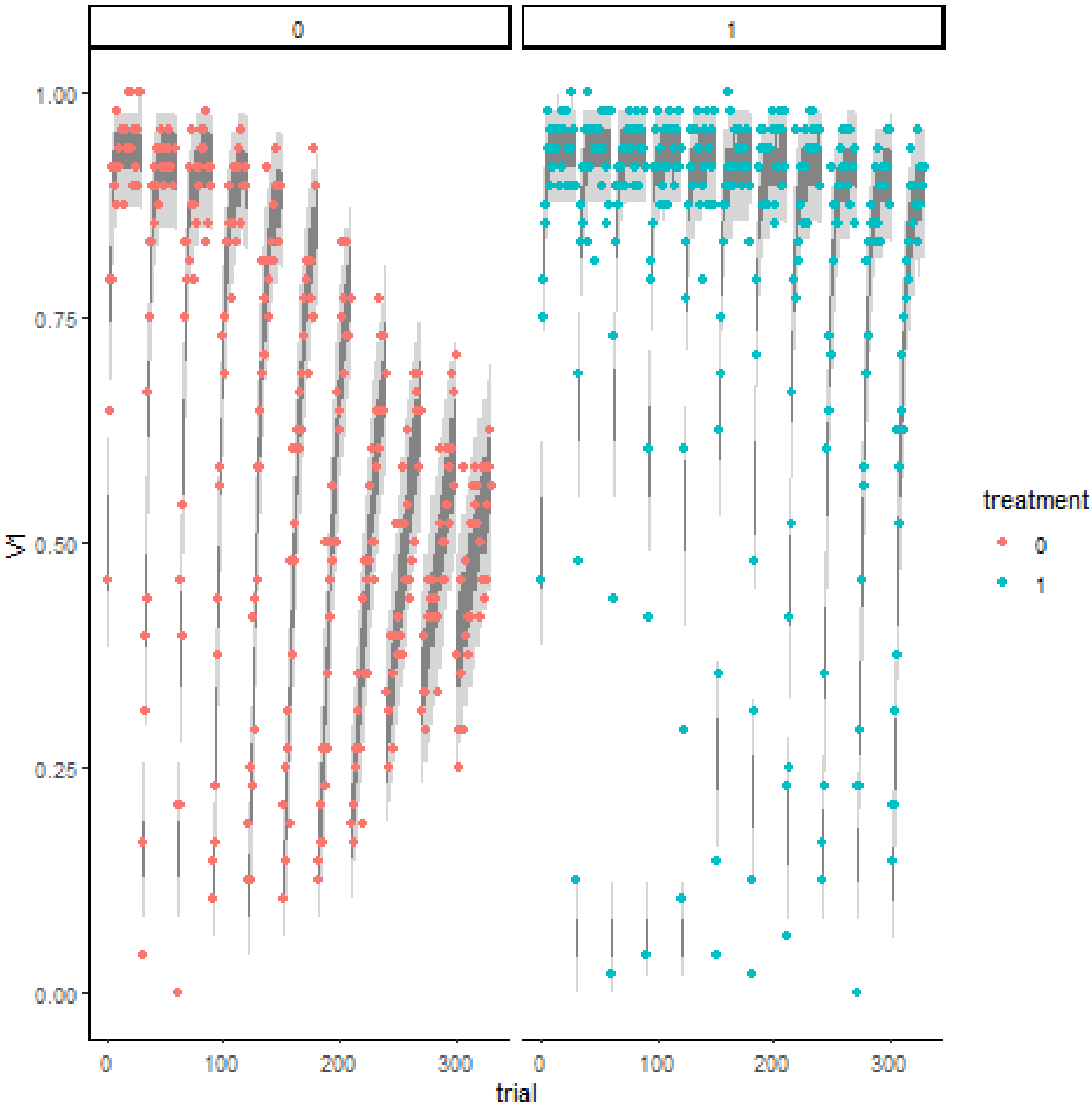

Figure 14: Out of sample posterior predictive checks of the bayesian estimation. Points correspond to a simulated data-set using the parameter values shown earlier from the simulation of reversal learning. Light and dark grey bands correspond to the 50 and 90% credible interval