Wigner distribution

- General definition: if $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$, then the Wigner distribution is:

$$W(x,p) = \frac{1}{\pi}\sum_i p_i \int dy\, \psi_i^*(x+y)\psi(x-y)e^{2ipy}$$

- It is real valued and maps the wave function to phase space. The following holds:

$$|\psi(x)|^2 = \int dp\, W(x,p),$$

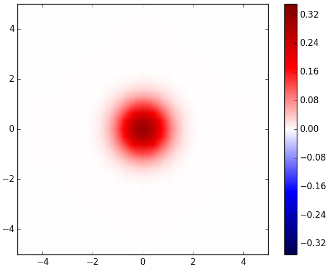
$$|\psi(p)|^2 = \int dx\, W(x,p),$$

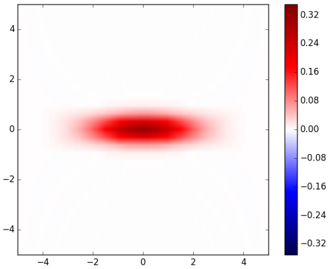
$$< G(x,p) >= \int dx \int dp\, W(x,p)G(x,p).$$

- However it can also take negative values and in quantum mechanics you cannot say there is a probability to have some x and p values at the same time.

Wigner distribution

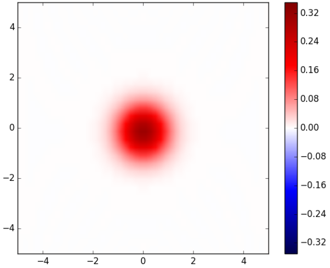
- Wavefunction represented using a basis of Legendre functions.
- Wavefunction sampled on a uniform grid and the transformation done using FFTW.
- Wigner distribution of two Gaussian wavefunctions $\psi(x) = \frac{1}{\sqrt[4]{\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}$, with $\sigma = 1$ and $\sigma = 2$:

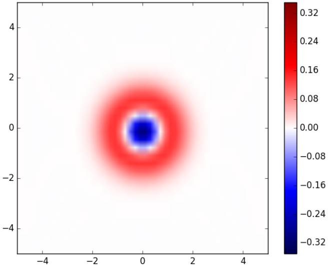




Harmonic oscillator

- For a harmonic potential the Wigner distribution behaves classically. Each point just moves as if a classical particle with that momentum and location.
- Wigner distribution of the first two ground states of the harmonic oscillator $\psi_1(x) = \frac{1}{\sqrt[4]{\pi}} e^{-x^2}$, $\psi_2(x) = \frac{\sqrt{2}x}{\sqrt[4]{\pi}} e^{-x^2}$:

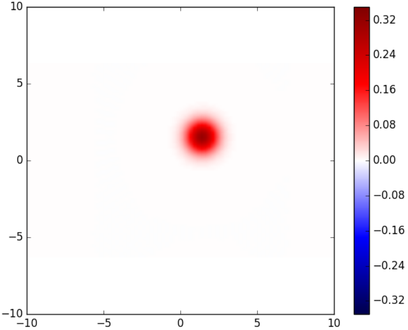




- We notice negative values of the distribution.

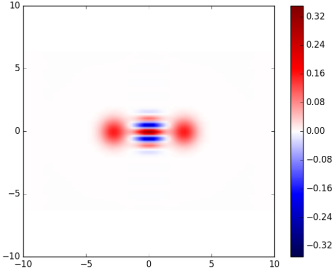
Coherent states

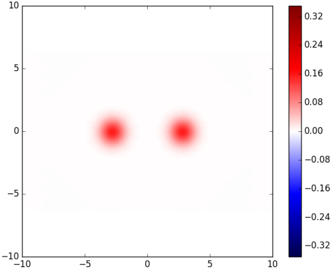
- For a coherent state the graph of $|\psi(x)|^2$ oscillates back and forth as a classical particle.
- Coherent state is characterized by a complex number α. The real and imaginary values of which give the x and p displacement of the Gaussian Wigner distribution. An example for $\alpha = 1 + i$:



Pure and mixed states

- Here I plot the pure and mixed states of two coherent states of the harmonic oscillator:





- The pure state has extra interference terms.

Time evolution

- Time evolution simulations.