Análisis KKT del ejemplo 3.2

$$\mathcal{L}(x,u) = 2x_1 - x_2 + u_1((x_1-1)^2 + x_2 - 5) + u_2(-x_1^2 + x_2) + u_3(-x_2)$$

CONDICIONES DE GRADIENTE

$$L_1: \frac{\partial \mathcal{L}}{\partial x_1} = 0 \iff 2 + 2u_1(x_1 - 1) - 2u_2 x_1 = 0$$

$$L_2: \frac{\partial \mathcal{L}}{\partial x_2} = 0 \iff -1 + u_1 + u_2 - u_3 = 0$$

ANALISIS DE LOS PUNTOS A, O, B, C, D

A: $u_1 \neq 0 \quad u_2 \neq 0 \quad u_3 = 0$

B: $u_1 = 0 \quad u_2 \neq 0 \quad u_3 = 0$

C: $u_1 \neq 0 \quad u_2 \neq 0 \quad u_3 = 0$

D: $u_1 \neq 0 \quad u_2 = 0 \quad u_3 \neq 0$

A, C) $u_1 \neq 0 \Rightarrow (x_1-1)^2 + x_2 = 5$

$u_2 \neq 0 \Rightarrow x_2 = x_1^2$

$$\Big\} (x_1-1)^2 + x_1^2 = 5$$

$$x_1^2 - 2x_1 + 1 + x_1^2 - 5 = 0$$

$$2x_1^2 - 2x_1 - 4 = 0$$

$$x_1 = \frac{2 \pm \sqrt{4+32}}{4} \begin{cases} 2 \\ -1 \end{cases}$$

$$x_1^A = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \qquad x^C = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

A) $L_1: 2 - 4u_1 + 2u_2 = 0$

$L_2: -1 + u_1 + u_2 = 0$

$$\Big\} \xrightarrow{(*4)} -2 + 6u_2 = 0 \qquad u_2^A = \frac{1}{3}$$

$$u_3 = 1 - u_2 \qquad u_1^A = \frac{2}{3}$$

$$A \iff x^A = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad u^A = \begin{pmatrix} 2/3 \\ 1/3 \\ 0 \end{pmatrix} \geq 0 \quad \text{SOL. FACTIBLE (KKT)}$$

C)
$$L_1: \; 2 + 2u_1 - 4u_2 = 0 \qquad -2 + 6u_1 = 0 \qquad u_1^C = 1/3$$
$$L_2: \; -1 + u_1 + u_2 = 0 \quad (*4) \quad +$$
$$u_2^C = 2/3$$

$$C \Rightarrow x^C = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \qquad u^C = \begin{pmatrix} 1/3 \\ 2/3 \\ 0 \end{pmatrix} \geq 0 \qquad \text{SOL. FACTIBLE} \quad \text{(KKT)}$$

B) $u_2 \neq 0 \qquad x_2 = x_1^2$

$$L_1: \; 2 - 2u_2 x_1 = 0 \qquad 1 = u_2 x_1$$
$$L_2: \; -1 + u_2 = 0 \qquad \boxed{u_2 = 1} \qquad \boxed{x_1 = 1} \qquad \boxed{x_2 = 1}$$

$$B \Rightarrow x^B = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad u^B = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad \text{(KKT)}$$

D) $u_1 \neq 0 \quad (x_1 - 1)^2 + x_2 - 5 = 0 \quad \rightarrow \quad (x_1 - 1)^2 - 5 = 0$
$u_3 \neq 0 \qquad x_2 = 0$

$$x_1^2 - 2x_1 + 1 - 5 = 0$$
$$-4$$
$$x_1 = \frac{2 \pm \sqrt{4 + 16}}{2} = \frac{2 \pm 2\sqrt{5}}{2} \quad \begin{matrix} 1 + \sqrt{5} > 0 \\ 1 - \sqrt{5} \end{matrix}$$

D $x^D = \begin{pmatrix} 1 + \sqrt{5} \\ 0 \end{pmatrix}$

$$L_1: \; 2 + 2u_1(x_1 - 1) = 0 \qquad 1 + \sqrt{5} \cdot u_1 = 0 \qquad u_1 = \frac{-1}{\sqrt{5}} < 0$$
$$L_2: \; -1 + u_1 - u_3 = 0 \qquad \boxed{u_1 - u_3 = 1} \qquad u_3 = u_1 - 1 < 0$$

$$D: \; x^D = \begin{pmatrix} 1 + \sqrt{5} \\ 0 \end{pmatrix} \qquad u^D = \begin{pmatrix} -1/\sqrt{5} \\ 0 \\ -1/\sqrt{5} - 1 \end{pmatrix} \not\geq 0 \qquad \text{NO KKT}$$