

Ejercicios Final

Parameter Estimation

$$\text{Si } f(x, \mu, \sigma) = \prod_{i=0}^n \frac{1}{\sqrt{2\pi\sigma^2}} e^{-1/2 \left(\frac{x_i - \mu}{\sigma} \right)^2}$$

$$\ln(f) = \ln\left(\prod_{i=0}^n \frac{1}{\sqrt{2\pi\sigma^2}} e^{-1/2 \left(\frac{x_i - \mu}{\sigma} \right)^2} \right)$$

$$= \sum_{i=0}^n \ln\left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-1/2 \left(\frac{x_i - \mu}{\sigma} \right)^2} \right)$$

$$= \sum_{i=0}^n \left(\ln\left(\frac{1}{\sqrt{2\pi\sigma^2}} \right) + \ln\left(e^{-1/2 \left(\frac{x_i - \mu}{\sigma} \right)^2} \right) \right) = \sum_{i=0}^n \left(\ln\left(\frac{1}{\sqrt{2\pi\sigma^2}} \right) - \frac{1}{2} \frac{(x_i - \mu)^2}{\sigma^2} \right)$$

(Derivadas parciales = 0 → máximo)

$$\frac{\partial \ln(f)}{\partial \mu} = \sum_{i=0}^n -\frac{1}{2\sigma^2} \cdot 2(x_i - \mu) \cdot -1$$

$$= \frac{1}{\sigma^2} \sum_{i=0}^n (x_i - \mu) = 0 \rightarrow \sum_{i=0}^n x_i - n\mu = 0$$

$$\sum_{i=0}^n x_i = \sum_{i=0}^n \mu \rightarrow n\mu$$

$$\mu = \frac{1}{n} \sum_{i=0}^n x_i$$

$$\frac{\partial \ln(f)}{\partial \sigma} = \sum_{i=0}^n \left(\frac{1}{\sigma\sqrt{2\pi}} \cdot -\sqrt{2\pi} + \frac{2(x_i - \mu)^2}{\sigma^3} \right)$$

$$\sum_{i=0}^n \frac{1}{\sigma} = \sum_{i=0}^n \frac{(x_i - \mu)^2}{\sigma^3} \rightarrow \sum_{i=0}^n \sigma^2 = \sum_{i=0}^n (x_i - \mu)^2$$

$$\sigma^2 n = \sum_{i=0}^n (x_i - \mu)^2 \rightarrow \sigma^2 = \frac{1}{n} \sum_{i=0}^n (x_i - \mu)^2$$