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Performance evaluation in BRT systems: An analysis to predict the BRT systems planning

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ABSTRACT

Mobility issues may create various inconveniences in urban areas due to traffic light delays, traffic jams and acidents. Alternatives have been created to improve public transportation. One of them is the Bus rapid transit system (BRT), delivering convincing results from a cost-effective perspective; i.e., lower deployment costs and a greater number of passengers. This paper proposes a Stochastic Petri Net (SPN) model for performance evaluation of the BRT system, focusing on the mean system size, mean queue size, mean queue time and the probability that the user will miss the bus (discard probability). Scenarios based on the BRT system were created, improving a set of solutions being with a variation in headways and in the number of vehicles on the route. The results show that, from a management perspective, it is up to the decision maker to define which is the most important metric at a given moment and, thus, to define the start intervals, as well as the number of vehicles to have on the route. Under the perspective in study, the best scenario is presented with headways of 300s and 5 vehicles in the route, operating as the one that leads to lower waiting times for passengers.

1. Introduction

In developing countries, greater industry incentives and higher economic power encourage a greater number of people to buy their own vehicles, resulting in chaotic traffic conditions, as roads are unable to cope with higher volume of cars. In most cases, bus services are perceived as unsafe, unhealthy and unreliable (Beirão and Cabral, 2007). From this perspective, it is necessary to invest in transportation planning, aiming at alternatives that can ensure comfort, confidence and safety to users.

Many studies have been conducted to evaluate the performance of the public transportation system. Ingvardson and Jensen (2012) describe that an efficient and attractive public transport system should provide high speed, high frequency of operations, user comfort and predictable services. Avila (2018) describes that the urban transport system operates under uncertain traffic conditions, with several parameters being involved in the planning process, namely passengers, vehicles, drivers, routes, time (travel, waiting and holding times), policies of urban transport enterprises, operation costs and others (Avila-Torres et al., 2018). Uncertain travel times can potentially affect the passenger's departure time, the choice of transport mode and route options (Cats and Gkioulou, 2017).

Bus Rapid Transit (BRT) systems have gained worldwide popularity as they offer fast, green, safe and efficient services. In addition to having design cost in about a third lower than a rail project. BRT can provide performance, quality and good passengers capacity. Silaen et al. ((2018) With this in mind, performance techniques, such as arrival intervals of passengers and vehicles, the volume of people that circulate in the system, as well as waiting times of passengers in each platform-level boarding should be considered. Furthermore, most works available in the literature do not take into account state-based models as evaluation and modeling techniques.

Petri Nets (PNs) present themselves as a formalism that can contribute to the development of intelligent transportation systems (ITS), helping to solve decision problems such as resource management, route planning, and optimization, transport activities control, among others. Besides working with graphical and compact representation, modularity, the possibility of modeling simultaneous and parallel events, Petri Nets, solve various problems via linear integer programming (Cavone et al., 2018).

This research began with Dantas et al. (2018), where hierarchical models are presented, using Continuous Time Markov Chain (CTMC)

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modeling techniques to evaluate metrics, such as performance and performability of BRT Systems. The results show that these models point to peak intervals with a greater probability of reaching the destination in a shorter period of time, besides showing the probability of the vehicle being affected by failure in each interval. The work also establishes bases for replication of the model in different scenarios.

In this context, this paper presents a comprehensive BRT performance model, using Stochastic Petri Nets (SPNs). The model takes into account: (1) the probability that the user will miss the bus - Discard probability; (2) average number of passengers waiting for platform-level boarding - Mean Queue Size (3); and the average number of passengers in the system - Mean System Size. We also present a case study which evaluates the defined metrics based on the interval between bus departures and the number of buses on-route.

The main contribution of this paper are:

- A methodology capable of evaluating a BRT system;
- A stochastic model to evaluate performance metrics based on different scenarios;
- A proposition of BRT system planning based on the interval between bus departures and the number of vehicles on-route.

The remainder of the paper is organized as follows: Section 2 shows related works and Section 3 introduces an overview of basic concepts. Section 4 presents the proposed modeling strategy and Section 4.1 demonstrates the performance evaluation model. Section 5 exposes a case study and the results obtained. Finally, Section 6 shows concluding remarks and future works.

2. Related work

Over the last years, some works have been intended to evaluate transport systems. Zhang et al. (2016) developed a combined evaluation method (CEM) that proposed a view on various perspectives, from the traffic operator to the passenger's perspective. The main contributions of this paper are the construction of a system of indicators based on satisfaction and efficiency to measure the performance of public transportation, besides the application of a CEM of information entropy and a model of super efficient data envelopment analysis (SE- DEA) to measure public transportation.

Wang et al. (2013) worked with route optimization for drivers through the GeoTNavi software system that uses geo-temporal traffic information, suggesting optimized routes. The authors consider historical traffic information to predict patterns for specific segments and moments, working with real-time information, always aiming to offer what they call an optimized route.

In Huo et al. (2014), a reliability analysis was performed on a route from the Changzhou-China BRT System. The authors consider the irregularity of intervals waiting time and the bus travel time. The study considers historical series on the time to reach the destination, besides temporal and spatial factors.

In Chen and Sun (2019), the authors develop a multistate-based model to design travel time schedule for a fixed transit route. The study promotes a multistate model and aims to identify service states and model travel time distribution. In sequence, the authors use an optimization model, followed by a Monte Carlo simulation-based genetic algorithm procedure to obtain the optimal slack time. The case study shows a numerical example from a fixed transit route in the city of Shenzhen, China was used to demonstrate the model applicability. The authors conclude that the multistate model significantly improves model fitting compared with Lognormal distribution. The multistate-distributed travel time samples can well reflect different states in bus operation, especially during peak hours.

In Santos et al. (2020), the authors conclude that the results of this study serve as a reflection on sustainability and resilience policies. Also, indicates deficient conditions which are related to the dichotomic relationship between economic growth and the sustainable development of cities nowadays, especially the continuous geographical expansion of cities in developing countries.

Some papers propose evaluation techniques for BRT systems. In Lopez et al. (2011) Stochastic Petri Nets (SPN) are used to demonstrate the stations of a BRT system, performing rearrangements in order to improve the system. This same work is based on passenger displacement times of the BRT transmilenio system in Bogota, Colombia. A Petri Net was constructed according to stations in a specific route, with changes made aimed at proposing improvements. However, it is not possible to identify the analysis of metrics that can promote system planning.

The present work offers a different approach to those described above. In this paper, a complete representation of the BRT system is made, using stochastic petri net, which considers input parameters such as headways, bus capacity and others. With this approach, it is possible for the system manager to make vehicle scenario composition decisions and departures intervals, which will generate as initial metrics: mean system size, mean queue size, mean queue time and the probability that the user will miss the bus.

3. Preliminaries

This section introduces an overview of relevant concepts for a better understanding of this paper.

3.1. Performance evaluation

The evaluation of systems is a combination of measurements and interpretations of various characteristics, such as performance, speed communication, data, and information size (Jain, 1991). The analyses usually depend on the situation given, the owner's interest and the analyst's skill (Lilja, 2000Maciel et al., 2011).

Three techniques for performance evaluation are the most used: analytical modeling, simulation and measurement. Measurements are only possible if something similar to the proposed system already exists, such as when designing an improved version of a system. When considering a new concept, analytical modeling and simulation are the only techniques to choose from, as these can only be used for situations in which measurement is not possible (Jain, 1991).

As measuring the metropolitan public transportation system is an unaffordable task, our focus is in a modeling methodology. This methodology is often carried out through mathematical and analytical techniques or by the system's behavioral evaluation. We opted for a behavioral evaluation, which may be employed through Stochastic Petri Nets (SPN), Continuous Time Markov Chains (CTMC), Fault Trees (FT) and Reliability Block Diagrams (RBD).

3.2. Petri nets

The Petri nets (PN) (Murata, 1989) are a family of formalisms for modeling various types of systems, including simultaneity, synchronization, communication mechanisms, also supporting deterministic and probabilistic delays. This work takes a particular length, Stochastic Petri Nets (SPN).

In Stochastic Petri Nets (SPN) the triggering of a transition is an atomic operation and two types of transitions are worked: transitions, which trigger without delay (immediate), and timed transitions, which are triggered after a random delay. The triggering of immediate transitions takes precedence over the triggering of timed transitions. Each immediate transition associated a weight that determines its probability of firing if this transition conflicts with some other immediate transition (German, 2000).

SPNs also allow the adoption of simulation, as an alternative to generate a CTMC. Regarding SPN formal definitions and semantics, the reader is referred to Marsan et al. (1998).

Timed transitions in SPNs are supposed to follow exponential distributions, which is an acceptable approximation in many cases, but it might not be suitable in several circumstances. A well-established method that considers expolynomial distribution with random variables is based on moment matching distribution and aims at representing a generally distributed delay as an Erlangian, Hypoexponential, or Hyperexponential subnet, denoted by s-transitions (Desrochers, 1995).

3.3. BRT system

The BRT System can be understood as a systematic combination of infrastructure (vehicle routes, stations and terminals) with organized operations and intelligent technologies, geared towards providing a higher quality experience than travel operations on traditional bus routes. This combination depends on the local market, the physical and operational application environment, as well as on the resources available (Hidalgo and Graftieaux, 2008).

Data from 2017, available in Global BRT Data (Global, 2018), describe the growth of the BRT system, noting that there are currently 169 cities worldwide where the system is being deployed, with 5,017 km of total extension, benefitting in the daily transportation of 33,329,284 people. Thus, it is possible to realize the importance of BRT systems, given the impact they generate and the size they has achieved.

According to a study carried out by the Canadian Urban Transit Association, cited in Filipe and Macario (2013), there is consensus on the advantages that BRT has over other mass transit systems, such as: good service speed and reliability, greater ease of financing, lower costs, higher capacity, operational flexibility and the possibility of incremental implementation (part by part). The BRT System has, thus, attracted many developing cities interested in improving their mass transportation systems, leading to satisfactory results, greater user satisfaction and lower costs when compared to other systems with similar capacity.

4. Modeling strategy for performance evaluation in BRT system

In order to evaluate the performance of the BRT System, the methodology has been developed, which shows each activity performed until the verification of scenarios that point to the feasibility of using the model to achieve the results of the proposed metrics.

Initially, the **operation mode of the BRT system** was defined, which, as described in Dantas et al. (2018), considers that the vehicle travels by a unique route between a set of stations, with the starting point being a central station and the destination of the vehicle as the final station.

Starting from the definition of the operational mode, **interest metrics for the study are identified**, which will take into account the perspective of the users of the BRT system, considering mean system size, mean queue size, mean queue time and discard probability, that in the specific case of transportation systems, represent the users who need to wait for more than one vehicle to proceed to boarding. We consider that these factors can provide the managers with information on the number of vehicles and exit time between vehicles, which, in balance, can lead to more satisfactory factors on the cost of operating the system.

With the definition of the metrics to be evaluated, **the Stochastic Petri Nets modeling was chosen** for the approximate representation of the system reality, given the complexity of the factors involved in the analysis, which involve the relation between stations, vehicles and passengers in the BRT system. In other words, Stochastic Modeling of Petri Nets provides the appropriate resources to obtain results for the interest metrics.

Once the modeling strategy and the interest metrics were defined, the **SPN model was designed** to promote the achievement of the desired results. The model will be presented in Section 4.1.

For the evaluation, we used the **scenario** already worked on Dantas et al. (2018), described by Huo et al. (2014) about Changzhouś BRT. Data on passenger input and output (Matrix Origin/Destination) of a certain route (Line 40) from the public transportation system of the city of Recife – Pernambuco – Brazil (Pesquisa, 2010) were then added to the initial data.

From the **results verification** obtained in the metrics, and after verifying its usability, or lack thereof, the result of the model or reassessment of the scenarios is constructed. In this way, the result can be validated and can generate subsidies to facilitate the decision making of the government transport managers.

4.1. SPN modeling

The SPN model depicted in Fig. 1 was conducted to evaluate the desired metrics. As the system behaviour is modelled as an SPN, it is assumed that all model transitions are exponentially distributed. The proposed SPN model can also be adapted to represent other probability distribution functions (e.g., Erlang, Weibull, and Cox), using the Mercury tool (Oliveira et al., 2017; Maciel et al., 2017). Considering a large state space, we used the simulation analysis.

For better understanding, we separated it in three blocks, which are represented as: 1 – shows the Central Station, 2 – representing intermediate stations and 3 – representing the Final Station. We emphasize that blocks 1 and 3 will be individual, and block 2 can be replicated according to the number of stations in the BRT system.

The logic of the model arises from the need to achieve the relationship between vehicles, stations and passengers, in order to achieve results for metrics that impact the improvement of the BRT system. Thus, in Table 1, the functions of each place of the SPN model are presented (Fig. 1).

Considering that in this model, many of the arcs assume values other than 1, as it has associated probabilities of landing at the stations, we will also present the descriptions of weights of the arcs (Table 2).

Block 1, in its upper part, represents the Central Station, Place **A_CS** represents the arrival of the passenger, having a marking token in this Place. The **T_ACS** Transition represents the interval between passenger arrivals at the station, which will be fired whenever it is enabled, consuming a token and depositing it in Place of **I_CS**, which represents the passenger's entry into the station. The **TI_CS** Immediate Transition is enabled if there is a token in Places **I_CS** and **B_CS**, that represent the capacity of the station, i.e., the passenger queue limit. The shooting of **TI_CS** transition deposits a token to Place **A_CS**, representing that there will be the arrival of another passenger, with a token being deposited in Place **Q_CS**, where the passenger will be in the queue, limited to the capacity defined by the variable SC in Place **B_CS**. The passenger will leave the station and enter the bus in the **T_IB** Transition shot, which counts the time of entry of each passenger in the vehicle, represented as a single server. However, this transition is enabled if there is also a token in Place **B_P_SC**. To have a token in this Place, it is necessary that the vehicle has entered the route. For this, Place **Buffer_Bus**, that represents the number of vehicles available for the system, must have a value other than 0; that is, the variable **bb**, that represents the number of vehicles, must be greater than or equal to 1 in its initial marking. This is due to the fact that this value enables the **T_BD** transition to be triggered, which will deposit in Place **B_P_SC** the number of tokens determined by the weight of arc n. Variable n represents the capacity of the vehicle, which will be the seats available for boarding passengers. Thus, if there are tokens in Place **B_P_SC** and Place **Q_CS**, the transition **T_IB** can be shot, which shall deposit those tokens in the **Station_Central**, limited to the capacity of the vehicle, **B_P_SC**, and the number of passengers in the queue, **Q_CS**. Place Central Station represents the number of passengers that are inside the vehicle.

In turn, Place **B_ISC** represents that the vehicle is at the station. This Place will have tokens deposited by the shot of the transition **T_BD**, which will be enabled if there are tokens in Places **Buffer_Bus** and **B_B_ISC**. Place **B_B_ISC** represents the number of vehicles capable of stopping at the station. In the case of BRT stations, this limit is restricted

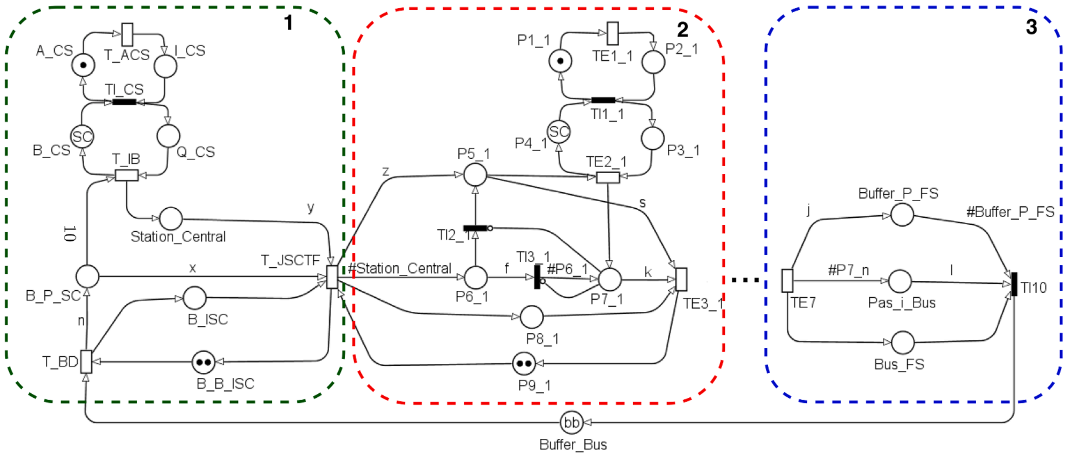


Fig. 1. Proposed SPN model of the BRT System.

Table 1
Places definition of the SPN Model.

Place	Function in the System
Buffer_Bus	Waiting buses
A_CS	Passengers Waiting to Access Central Station
I_CS	Passengers inside the Central Station
B_CS	Central Station Passenger Capacity
Q_CS	Queue at Central Station
B_P_SC	Passenger Capacity in the Vehicle at the Central Station
Station_Central	Passengers on the Vehicle at the Central Station
B_ISC	Central Central Station Capacity for vehicles
B_B_ISC	Vehicles at Central Station
P1_1	Passengers Waiting to Access the Station
P2_1	Passengers inside the Central
P3_1	Station Passengers Capacity
P4_1	Queue at Station
P5_1	Passenger Capacity in the Vehicle at the Station
P6_1	Passengers on the Vehicle when it arrives at the Station
P7_1	Passengers on the Vehicle at the Station
P8_1	Station Capacity for vehicles
P9_1	Vehicles at Station
Buffer_P_FS	Vehicle capacity at the Final Station
Pas_i_Bus	Passengers on the Vehicle at the Final Station
Bus_Fs	Vehicle in the Final Station

Table 2
Arcs Weights of the SPN Model.

Arc	Weight
x	IF(#B_ISC > 1): ((#B_P_SC)-n) ELSE(#B_P_SC)
n	Number of assents on the bus
y	IF(#Station_Central>=1): (#Station_Central) ELSE(0);
z	IF(#B_ISC > 1): ((#B_P_SC)-50) ELSE(#B_P_SC);
f	IF(#P6_1>=1): (#P6_1) ELSE(1);
s	IF(#P8_1 > 1): ((#P5_1)-50) ELSE(#P5_1);
k	IF(#P7_1>=1): (#P7_1) ELSE(0);
j	IF(#P8_5 > 1): ((#P5_5)-50) ELSE(#P5_5)
l	IF(#Buffer_P_FS>=1): (#Buffer_P_FS) ELSE(1);

to two vehicles per station, so the initial marking of the model is 2 tokens in Place **B_B_ISC** and each shot of the T_BD a token is consumed and deposited in Place **B_ISC**.

The transition **T_JSCTF** represents the travel time from **Central_Station** to **Station_1**. The shot transition **T_JSCTF** precedes a necessary condition for it to be enabled, which is the need to have at least one bus at the station; i.e., that Place **B_ISC** is larger or equal to 1. The transition is still fed by two arches, the one coming from Place **Central_Station**, that has weight **y** and represents the condition of consumption of all tokens of Place **Central_Station**, in case there are tokens in this Place; or it will have weight of consumption 0 if there is no passenger in the station, leaving the bus free to proceed without any passengers. The other arc that enables the **T_JSCTF** transition precedes Place **B_P_SC**, which has weight **x**. This variable represents that if Place **B_ISC** has more than one token, the weight of the arc will be **B_P_SC** (the number of seats remaining in the vehicle, since some vacancies may have been subtracted by **T_IB**), subtracted from n, which is the capacity of a vehicle. This is due to the fact that disPlacement occurs from one vehicle at a time, with this condition limiting the number of spaces that can be occupied per vehicle. If there is only one token in **B_ISC** the weight of the arc will be equal to the number of tokens of **B_P_SC**; that is, all vacancies will be subtracted by the shot of the transition, which represents the passage of the vehicle to the next station, in this case, Block 2.

Block 2 (red) represents an intermediate station of a BRT system. The fire of the **T_JSCTF** transition will consume the tokens described and deposit them in the following Places: In **P5_1**, which represents the number of seats available in the vehicle; the arc connecting the transition to the Place will have weight z, describing that the number of available seats in the transition will be conditioned to Place **B_ISC**; and, if this Place is larger than 1 the weight of the arc will be (**B_P_CS**-n), since it will have more than one vehicle. In turn, n represents the seats of a vehicle, and, if it is less than or equal to 1, it will be the very amount of **B_P_SC**. Another Place that will receive a token will be **P8_1**, that given the weight of the arc, it will receive 1 token, representing the arrival of the vehicle the station. It is worth mentioning that the number of tokens of **P8_1** is limited to the number of vacancies in the station, which is represented by **P9_1**. The last Place to receive tokens from the **T_JSCTF** transition will be **P6_1**, which actually represents the number of passengers that have accessed the station through the vehicle, so much so that the weight of the arc connecting the transition to the Place is determined by the amount tokens at Place **Central_Station**.

However, when the vehicle arrives at station *P6_1*, there is the possibility of the passenger getting off the vehicle, which in this case is represented by the immediate transition **TI2_1**, that will have a weight associated with depending on the percentage of passengers disembarking at the station, or remaining in the vehicle, represented by the immediate transition **TI3_1**, which will also have an associated weight depending on the landing rates of the stations. For the immediate transitions **TI2_1** and **TI3_1**, there are associated inhibitory arcs, since for the construction of the model it was considered that the procedure must be to follow the landing order first and then the shipment. Firing **TI2_1** transition consumes tokens from Place **P6_1** and deposits them in Place **P5_1**, which represents the number of Places available in the vehicle; i.e., the landing of the passengers will generate new vacancies in the vehicle. Firing **TI3_1** transition is preceded by the consumption of tokens of Place **P6_1**, which will be represented by arc **f**, considering the consumption of all tokens of the Place; that is, weight equal to **P6_1**, which would deposit the same quantity in **P7_1**, representing the passengers inside the vehicle at the station.

The arrival at the station in block 2 is represented similarly to Station Central: **P1_1** represents the arrival of the passenger to the station, with a mark token in Place; the **TE1_1** transition represents the passenger arrivals interval, which consumes the token of **P1_1** and deposits it in **P2_1**, representing the passenger's entry into the station. The Immediate Transition **TI1_1** is enabled if there is a token in Places **P2_1** and **P4_1**, which is the capacity of the station queue; in this case, represented by variable **SC**. Thus, when the passenger arrives at the station and is in the queue, the transition will be triggered by consuming a token of the respective seats and depositing them in **P3_1**, which represents the passengers at the station waiting for the vehicle.

Firing **TE2_1** transition is limited to the number of tokens in **P5_1**, which represents the seats in the vehicle, and in **P3_1**, the number of passengers in the queue. If enabled, the **TE2_1** transition will consume tokens of **P3_1** and **P5_1**, with the time associated with it determined by the average time each passenger takes to enter the vehicle. The **TE2_1** transition will deposit tokens in Place **P7_1**, which shows the number of passengers in the vehicle.

TE3_1 transition firing, which represents the time of travel between stations, denotes the consumption of three-place tokens: **P5_1**, where there will be the consumption of tokens defined by variable f. This variable represents that if there is more than one token in Place **P5_1**, all will be consumed; otherwise, the arc will assume weight 1; **P7_1**, where there will be the consumption of tokens defined by the variable k, which represents that if the Place **P7_1** is greater or equal to 1, the weight of the arc will be the number of tokens of the place, if 0, the weight of the arc will be 0; and, finally, **P8_1**, which has a weight of 1 and represents that the vehicle has moved to the next station.

The last block, 3, represents the final station. Firing Transition **TE** consumes the tokens of the places associated with the last station, Final Station, and deposits the token in those places: **Buffer_P_FS**, which represents the number of free seats remaining in the vehicle at the end of the course, conditioned by the weight of arc **j**. Arc **j** is defined by the number of vehicles in the previous station $P8\_x$, where, if it is greater than 1, the weight of the arc will be the number of seats in the vehicle in the previous station minus the vehicle capacity n, $P5\_1 - n$. Otherwise, if it is less than or equal to 1, it will be the number of available vacancies, **P5_1**; **Bus_FS**, which represents that the bus is at the Final Station, having deposited 1 token there. The immediate transition fire **TI10** is conditioned by the consumption of tokens of places: **Buffer_P_FS**, from where all the tokens will be consumed since the vehicle will return to the garage, that is, all the vacancies will be available; **Pas_I_Bus**, which will feed the transition with arc **l**, where in case the place has one or more tokens, it will be the total number of tokens to be consumed and, and if it is 0, it will have weight 1 to enable the transition. The transition fire deposits one token into Place **Buffer_Bus**, which means that the vehicle is ready for use again.

From this model, it is possible to evaluate several metrics. However, in this work, the metrics presented in the following subsections will be evaluated.

4.2. Metrics definition

In this paper, the metrics worked are: mean system size (MSS), mean queue size (MQS), mean queue time (MQT) and discard probability (DP). For the analysis, these metrics can be mathematically evaluated through the model, as will be seen below.

Mean System Size

The MSS represents the number of people who are in the BRT system, either in the vehicle or waiting at the station; that is, it represents the total number of passengers for each scenario. For the evaluation, Eq. 1 was used.

$$MSS = E[S\_C] + E[\#Q\_CS] + E[\#P3\_1] + E[\#P7\_1] + E[\#Ppas\_i\_Bus], \tag{1}$$

The Eq. 1 is given by the sum of the expected value of tokens in the places representing passengers in the system, ie, in the queues of the stations (Q_CS, P3_1, or, if new stations are added, any place representing the queue), or in the vehicle (Station_Central, P7_1, or if new stations are added, the seats representing the passengers are in the vehicle) and the passengers that are in the final station (Pas_i_Bus). The $E[S\_C]$ corresponding of the place *Station_Central*.

Mean Queue Size

The MQS represents the number of passengers waiting to access the vehicle at the station. For transportation system planners, the minimization of queues present itself as an important factor, since, besides interfering in the image of what the system represents for society, it ensures lower costs for their own system and to passengers. In this paper, for the SPN Model, it is evaluated by Eq. 2.

$$MQS\_BRT = E[\#Q\_CS] + E[\#P3\_1] + \ldots, \tag{2}$$

The MQS is described as the sum of the expected values of tokens in the places where it is possible to generate queues, in the presented model Q_CS and P3_1. Thus, given the number of stations that are put in the model, the MQS will be increased by the expected value of tokens in the place that is able to generate queue, until reaching the Final Station where there are no queues for shipment, there is only the landing.

Mean Queue Time

The MQT represents, the average time that each passenger is waiting to board the vehicle. Thus, the relationship of queuing time is a very interesting metric for evaluation, since it directly impacts the image that the BRT system may have on users, as long queues cause discomfort and inconvenience for those waiting. In the SPN Model, it can be calculated by Eq. 3.

$$MQT\_BRT = [E(\#Q\_CS)] \times T\_IB + [E(\#P3\_1)] \times TE2\_1 + \ldots, \tag{3}$$

The Mean Queue Time is given by the sum of the expected value of tokens in the queue-loading places multiplied by the boarding time in the vehicle. Thus, E (#Q_CS) represents the expected value of tokens in Q_CS, and T_IB the time of boarding at the station. Multiplying E(#Q_CS) *x* T_IB, the Mean Queue Time at the Central Station will be found. Each station will do the same process to know the queue time in the station, and the sum of these values will represent the mean queue time.

Discard Probability

The **discard probability** in transport systems, corresponding due to a probability that the user will miss the bus, is related to the possibility of the passenger not being able to enter the vehicle the first time that it reaches the station; that is, the vehicle will be at its maximum capacity and the passenger will have to wait for the next vehicle on the route. The probability of missing the bus is going to be, for planning purposes, closer to zero; which means that there is a low probability that some passengers will not follow the route in the first vehicle that arrives at the station. In addition, if the probability that the passenger will miss the bus is low, it represents that the waiting is inferior; consequently improving the image of the system. Thus, the probability of the user not accessing the vehicle will be computed to each station. This probability is calculated, in this model, by Eqs. 4 and 5.

$$Discard\_Probability[Station\_Central] = P\{(\#B\_P\_SC = 0) \wedge (\#Q\_CS \geqslant 1)\}, \tag{4}$$

Eq. 4 represents that in the Central Station the passenger could not access the vehicle, is calculated by the probability of not having tokens in the Local B_P _SC, that is, this being empty (#B_P_SC = 0) and that there are passengers waiting in the queue (#Q_CS⩾1).

$$Discard\_Probability[Station\_1] = P\{(\#P5\_1 = 0) \wedge (\#P3\_1 \geqslant 1)\}, \tag{5}$$

Eq. 5 represents the probability that in an intermediate station the passenger cannot access the vehicle, being calculated by the probability that a certain place, in this case, $P5\_1$, is empty, not having tokens ($\#P5\_1 = 0$), having passengers also waiting in the queue ($\#P3\_1 \geqslant 1$). If there are other stations, the procedure for the calculation will be the same, except for the Final Station, that there will be no discard.

5. Case study

BRT systems have been expanding quickly in terms of the number of cities in the last decade, especially in developing countries, but not necessarily at an appropriate quality and pace (Lindau et al., 2014). This poses many challenges for transportation planners, since the quality of services must be guaranteed.

This section presents a case study where the metrics described are analyzed. The chosen metrics refer to those that directly impact passenger waiting and queuing times. Thus, the following metrics will be evaluated: mean system size (MSS), mean queue size (MQS), mean queue time (MQT) and the probability that the user will miss the bus.

These studies aim to assist managers in the decision making process, considering that from the models, it is possible to combine input parameters, which will result in data on the metrics evaluated. Thus, it will be up to the manager to define the scenarios that seek to evaluate and collect the data to support the decision on the best relation between headways, size of the fleet, waiting time at the station, among others.

For better understanding, the scenario used for evaluation will be described, with a subsequent evaluation of each metric used.

5.1. Scenario evaluation

The proposed architecture considers a compound BRT system with two central stations and five intermediate stations. This architecture was designed according to real-world BRT systems, where the main goal is to link suburbs to the center of the city, including, within this parameter, numerous of stops in an attempt to satisfy passenger demand.

The scenarios presented in Dantas et al. (2018) consider five peak displacement times. However, in this study, we only considered an early peak scenario, which has a displacement time between Stations of 267 s and is the first peak in the morning.

In order to estimate passengers arrival at the station, the Origin/Destination matrix of line 40 from the City of Recife was used (Pesquisa, 2010) (chosen randomly in order to bring the study closer to reality), with the route being then consulted. However, it had more stations than the required scenario; thus, we opted for a route division. In this case, these were data from each 8 real stations that will compose the input of a single station in the proposed study, considering the quantities of inputs and outputs in the system. In this way, the times of arrival at each station were distributed according to Table 3, which represents the input parameters for the model.

This scenario works with the possibility that two parameters can be altered by managers, corresponding to the vehicles in use at the station. Thus, the interval between the vehicles leaving the garage and the number of available vehicles for the route can be varied, generating a series of possible combinations to be evaluated, as can be seen in Fig. 2.

Table 3
Input parameters for arriving and dropping passengers at the station.

Station	Average arrival Time at the station (s)	Probability of dropping passenger at the Station
SC	46	–
St_1	56	0.01
St_2	71	0.12
St_3	44	0.22
St_4	88	0.15
St_5	360	0.25
FS	-	1

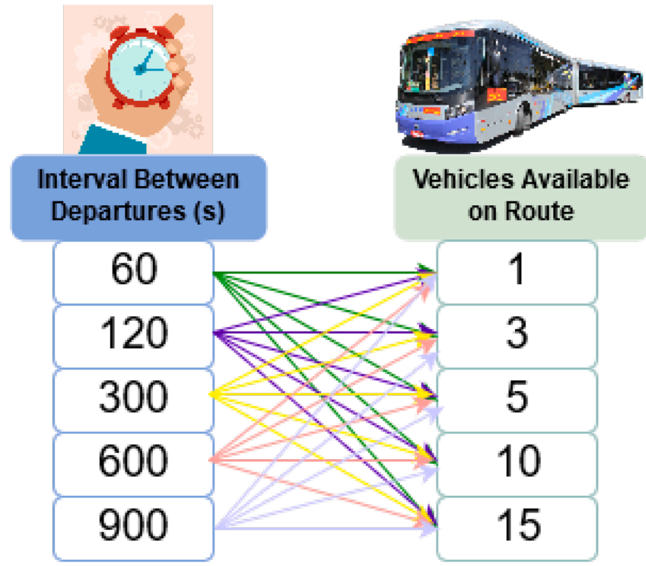


Fig. 2. Scenario Parameters combinations.

Fig. 2 shows the relationship between the number of vehicles in the route and the interval between bus departures. In this case, we consider the possibility of a number of buses on the given route, which may be 1, 3, 5, 10 and 15 vehicles. For each interval between departures, the following Headways were considered: 60s, 120s, 300s, 600s and 900s. Thus, the model was evaluated 25 times to make the expected composition as a result for each scenario metric.

It is worth noting that, as we are working with a reduced number of stations, we limited the number of seats in the vehicle to 50, approximating the reduced number of stations with a lower capacity of passengers in order to minimize the complexity of the study.

This paper seeks to assess the impact of metrics that may directly interfere with the passenger. Thus, the first metric to be evaluated was the mean system size (MSS).

5.2. Mean system size (MSS)

The results shown in Fig. 3 depicts the number of passengers in the system, considering the number of buses on the route.

It can be seen in Fig. 3 that, with only one vehicle, the average number of passengers in the system reaches its highest peaks, in all departure intervals. This is mainly due to the low capacity of a vehicle to absorb the entire passenger demand; with queues consequently being formed. When placing three vehicles in the route, it is verified that for short intervals, such as 60s and 120s, the MSS reaches values of approximately 200 people. Considering that in the study each vehicle has 50 passengers, there are 150 passengers inside the vehicles and 50 distributed in the stations waiting for boarding.

Starting from five vehicles on the route, the system begins to reach its steady state, i.e., without major changes in system's size; which shows a balance between the quantity of passengers and vehicles. Significant changes are noticed only for output intervals of 600s; which, starting from five vehicles, causes reduction of approximately 400 people. In turn, for 10 and 15 vehicles, there is a reduction to only 200 people in the system. The other changeable range is for the 900s departure interval, which, with 5 vehicles on the route, has an MSS of approximately 700 passengers, reducing to approximately 600 passengers when there are between 10 and 15 vehicles on the route.

Fig. 2 shows that for each interval between departures (60s, 120s, 300s, 600s and 900s), five evaluations were made considering the possibility of a certain number of vehicles; in this case 1, 3, 5, 10 and 15 vehicles. Thus, the model was evaluated 25 times to make the expected composition, with results being reached for each scenario metric.

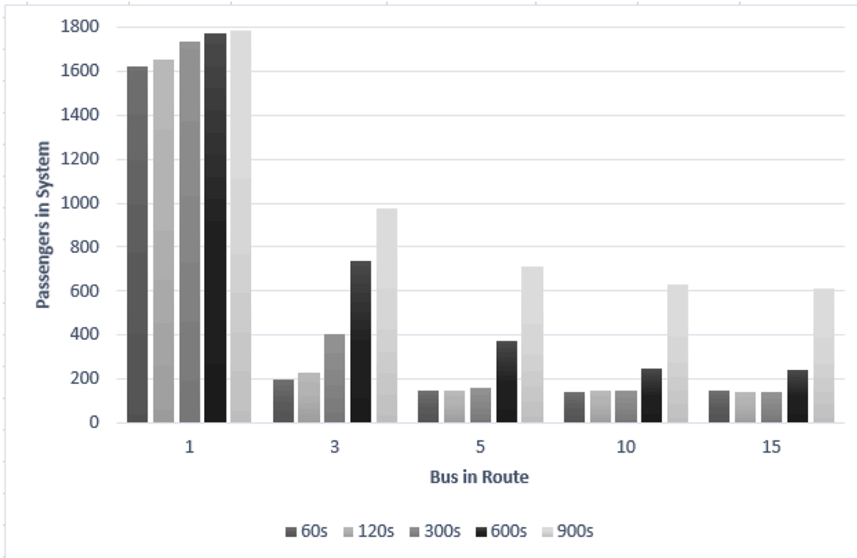


Fig. 3. Mean System Size Results.

Therefore, the manager's demand was well met, with an adequate quality of service and average cost.

5.3. Mean queue size (MQS)

Fig. 4 shows the number of passengers waiting under each varied parameter, i.e., between departure interval and number of vehicles on the route.

In queue size evaluation, it is possible to notice that only one vehicle on the route generates a high volume of passengers waiting, in agreement with what was highlighted in Fig. 3 considering a high-volume scenario of passengers in the system. Thus, the time factor for departure causes minimal interference on the queue size. However, as Fig. 4 shows, as the number of vehicles increases to 3, a significant reduction in queue size is achieved, especially if one considers the 60s departure interval, where the queue size is almost zero. When increasing departure time to 120s, the queue size already approaches 200 passengers. In the worst variation scenario of departure intervals, which are for the 900s intervals, the queue size is reduced to approximately half of the initial size, to one single vehicle. Thus, the impact of the increase in vehicles on the route has positive consequences for the system, with a decrease in passenger waiting time.

In Fig. 4, it is possible to notice that, for the scenario presented, the system remains without any significant changes when there are 5 or more vehicles on-route. In turn, for the departure intervals, there are no big changes even with the addition of more vehicles, such as the inclusion of 10 and 15 vehicles on the route. Thus, when there are 5 vehicles on-route, for starting intervals of 60s, 120s and 300s, the queue becomes smaller.

From the planning perspective, as noticed with MSS, the size of the fleet that would bring benefits would be of 5 vehicles, with the variation of departures range of 60s, 120s, or even 300s, being a consideration to be made by the manager; which may take into account random factors, such as day peak, to enable better results for the system.

5.4. Mean queue time (MQT)

Fig. 5 presents the MQT for the scenario evaluated, showing the average queue times for each departure time configuration and the number of vehicles on the route.

For the evaluated scenario, we notice that when there is only one vehicle on the route, the queue time is high. This is due to the fact that passengers, besides having to wait for the vehicle to make the entire course of the system, will still have to wait for a long time at the stations

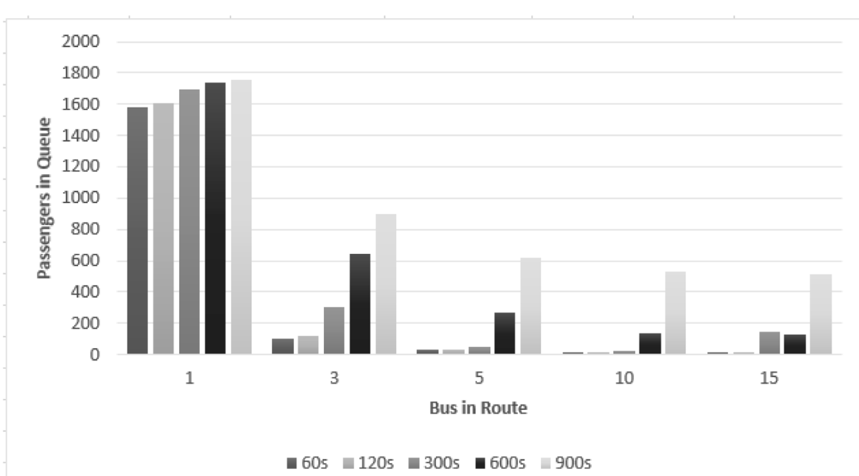


Fig. 4. Mean Queue Size Results.

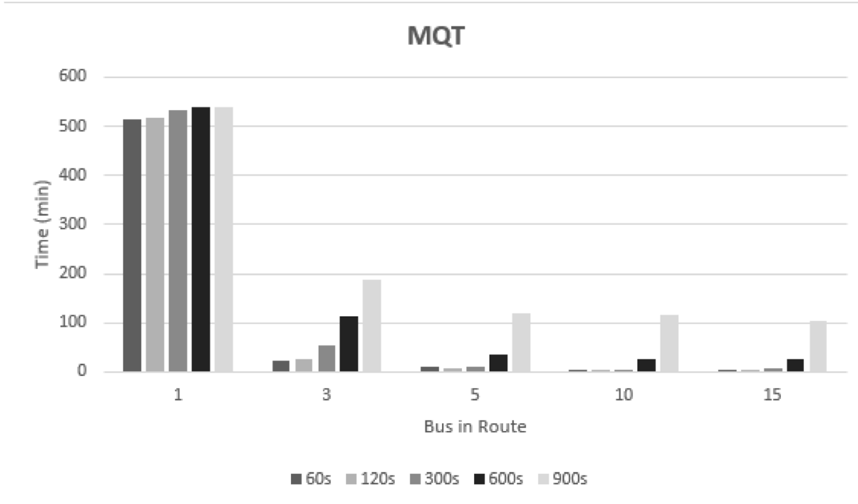


Fig. 5. Mean Queue Time Results.

close to the Final Station, as there is the possibility that the vehicle will be crowded. Thus, in any configuration, this alternative is presented as not favorable, because there will always be longer queues. A scenario that is already expected, as there is only one vehicle.

However, it is possible to notice in Fig. 5 that when there are 3 vehicles in the route, the waiting time reduces; especially for departures with intervals of 60s and 120s, with the system entering into a stationary state. The result for departure intervals of 300s also shows a reduction, decreasing from approximately 550s of queuing time to approximately 100s, which can already be considered satisfactory for the user.

We may also see in Fig. 8 that, for vehicle departures intervals of 900s, a no vehicle configuration should be a satisfactory choice due to long waiting times. This can happen because of the low number of stations, where, in the scenario presented, the queue time, even with more than ten vehicles on the route, still has a waiting time of approximately 200s.

Thus, from the planning perspective, the most feasible possibility would be a longer time interval for departures, a smaller number of vehicles and a shorter waiting time in the queue, considering a ratio of 3 vehicles on the route, with departure intervals of up to 300s. This alternative could reduce costs with a lower number of vehicles circulating, with departures being still organized with brief intervals of 5 min between journeys.

5.5. *Probability that the user will miss the bus (discard probability)*

According to the proposed scenario, described in SubSection 5.1, the analysis of the probability that the user will miss the bus is carried out per station. This is due to the fact that in each station there will be the possibility that the passenger will not be able to access the BRT system. Fig. 6 presents these results.

In Fig. 6a, when evaluating the probability that the passenger will not be able to enter the first bus that stops at the Central Station, it is possible to notice that when there is only one vehicle on the route, this probability approaches 0.9; i.e., almost 90% of passengers will have to wait for more than one vehicle to be able to make their trip, regardless of the range of departures. When there are three vehicles in the route, the probability of having to wait improves, dropping to approximately 60% when the departure intervals are 60s, 120s and 300s; reaching 70% and 80% for 600s and 900s intervals, respectively. The relationship between the departures intervals of vehicles and the number of vehicles in the route is given when there are ten vehicles on the road, where the system assumes its stationary state, and for exit intervals of 60s and 120s, reaching the level of 20% of passengers unable to board the first vehicle arriving at the station (at 300s departures intervals).

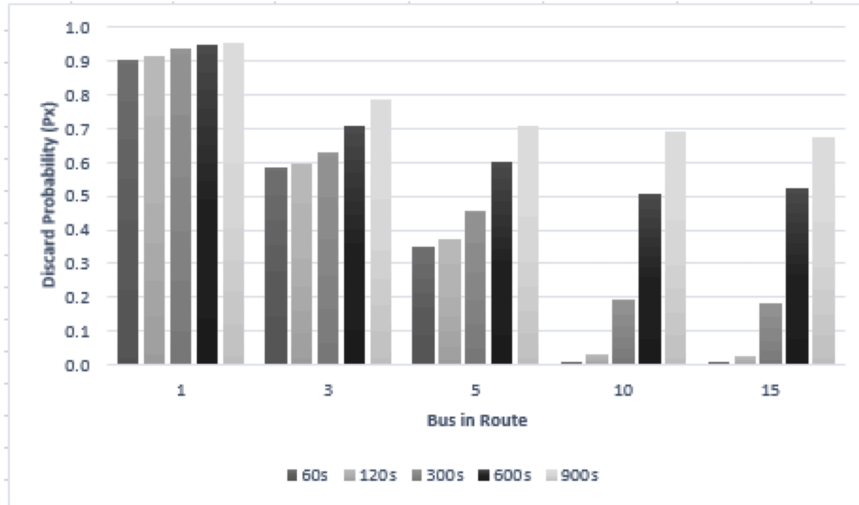
In Fig. 6b, it is possible to notice that the probability of missing the bus, when only one vehicle remains on the route, is close to 100%. In turn, the probability of not being able to board the first vehicle that stops at the station improves, being reduced with the increase of vehicles on the route. However, the behavior that is perceived is that, as the vehicle will already be with some places occupied, given that it came from the Central Station, even with 10 or 15 vehicles, the probability of not being able to board the vehicle increases. At the Central Station, this probability was close to zero, while at the first station it is already at about 20%, for departure intervals of 60s and 120s; reaching 30% for the departure interval of 300s and being greater than 50% for departure intervals higher than 600s. We omitted the other stations for showing similar behavior and had limited space to present our research.

In the context presented, for planners, the best composition of vehicles in the route, intervals of departures and the probability of waiting for more than one vehicle in the set of stations is of 5 vehicles and intervals of departure of up to 300s, respectively. This composition brings results with less probability of waiting, which endorses the choices in MSS, MQS and, MQT.

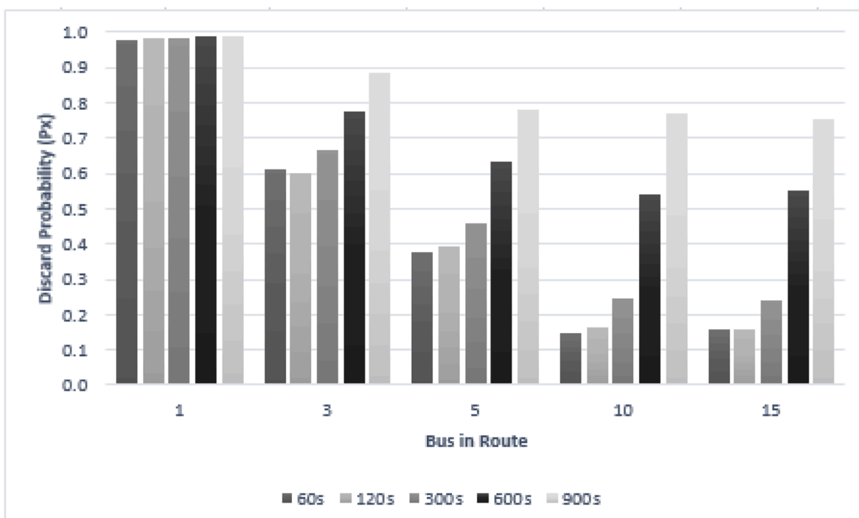
6. Conclusion and future works

This paper addressed fundamental factors involved in the BRT systems, through the use of Stochastic Petri Net, aiming at subsidizing the analysis of the system and to promote instruments that may assist the managers in the planning process. Essentially, the metrics used offer better alternatives to user comfort, especially in the confidence of reaching the destination in the shortest possible time, without waiting in queues. The BRT performance metrics results provide perspectives of combinations of headways and vehicle quantities in the system, which give managers the possibility of decision making based on the expectation of what is a priority in each system. To the user, it is possible to get minimized waiting, with queue reductions and, consequently, greater comfort when traveling to the destination. In the scenario presented, the best relationship that would bring the least wait and lowest number people in the system is tied to a 5 min (300s) headway, with five vehicles down the route. But with the composition of the model, it is possible to study out other scenarios.

For future studies, other metrics, such as Utilization and Throughput, will be added to reveal the perspective of the system and the possibility of better use of the vehicles in the routes. Other aspects that will be addressed are the expansion of the scenario, increasing the number of stations and vacancies in the vehicles, thus, guaranteeing better



(a) Probability of missing the bus at Central Station



(b) Probability of missing the bus at Station 1

Fig. 6. Results per Station: Comparing the probability of missing the bus.

perspectives for the planning of BRT systems. With this, the more information the manager has, the better the alternatives for achieving expected results for the system.

Acknowledgement

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References

Avila-Torres, P., Caballero, R., Litvinchev, I., Lopez-Irarragorri, F., Vasant, P., 2018. The urban transport planning with uncertainty in demand and travel time: a comparison of two defuzzification methods. J. Ambient Intell. Humanized Comput. 9 (3), 843–856.

Beirão, G., Cabral, J.S., 2007. Understanding attitudes towards public transport and private car: a qualitative study. Transp. Policy 14 (6), 478–489.

Cats, O., Gkioulou, Z., 2017. Modeling the impacts of public transport reliability and travel information on passengers' waiting-time uncertainty. EURO J. Transp. Logist. 6 (3), 247–270.

Cavone, G., Dotoli, M., Seatzu, C., 2018. A survey on petri net models for freight logistics and transportation systems. IEEE Trans. Intell. Transp. Syst. 19 (6), 1795–1813.

Chen, S., Sun, D., 2019. A multistate-based travel time schedule model for fixed transit route. Transp. Lett. 11 (1), 33–42.

Dantas, R., Dantas, J., Melo, C., Maciel, P., Alves, G., 2018. A performability model for the brt system. In: Systems Conference (SysCon), 2018 Annual IEEE International, IEEE, pp. 1–8.

Desrochers, A.A. Applications of petri nets in manufacturing systems: modeling, Control and performance Analysis.

Filipe, L.N., Macario, R., 2013. A first glimpse on policy packaging for implementation of brt projects. Res. Transp. Econ. 39 (1), 150–157.

German, R., 2000. Performance analysis of communication systems with non-Markovian stochastic Petri nets. John Wiley & Sons Inc.

Global brt data. URL: https://brtdata.org/.

Hidalgo, D., Graftieaux, P., 2008. Bus rapid transit systems in latin america and asia: results and difficulties in 11 cities. Transp. Res. Record 2072, 77–88.

Huo, Y., Zhao, J., Li, W., Hu, X., 2014. Measuring bus service reliability: an example of bus rapid transit in changzhou. J. Public Transp. 17 (2), 6.

Ingvardson, J.B., Jensen, J., 2012. Implementation of bus rapid transit in copenhagen based on international experiences. In: Annual Transport Conference at Aalborg University, Kgs. Lyngby, Denmark: Master's Thesis at the Technical University of Denmark, DTU Transport, Aalborg University. p. 15.

Jain, R., 1991. The Art of Computer Systems Performance Analysis: Techniques for Experimental Design, Measurement, Simulation, and Modeling, Wiley Computer Publishing. John Wiley & Sons Inc, New York.

Lilja, D.J., 2000. Measuring Computer Performance: A Practitioner's Guide. Cambridge University Press, New York.

Lindau, L.A., Hidalgo, D., de Almeida Lobo, A., 2014. Barriers to planning and implementing bus rapid transit systems. Res. Transp. Econ. 48, 9–15.

Lopez, D., Triana, A., Chamorro, H., 2011. Simulation model of public transportation system using multiagent approach by means of petri nets: Bogotá study case. In: Robotics symposium, 2011 IEEE IX Latin American and IEEE Colombian conference on automatic control and industry applications (LARC), IEEE, pp. 1–6.
Maciel, P., Trivedi, K., Matias, R., Kim, D., 2011. Dependability modeling. In: Performance and Dependability in Service Computing: Concepts. Techniques and Research Directions.
Maciel, P., Matos, R., Silva, B., Figueiredo, J., Oliveira, D., Fé, I., Maciel, R., Dantas, J., 2017. Mercury: performance and dependability evaluation of systems with exponential, expolynomial, and general distributions. In: 2017 IEEE 22nd Pacific Rim international symposium on dependable computing (PRDC), IEEE. pp. 50–57.
Marsan, M.A., Balbo, G., Conte, G., Donatelli, S., Franceschinis, G., 1998. Modelling with generalized stochastic petri nets. ACM SIGMETRICS Performance Evaluation Review 26 (2), 2.
Murata, T., 1989. Petri nets: properties, analysis and applications. Proc. IEEE 77 (4), 541–580.
Oliveira, D., Matos, R., Dantas, J., Ferreira, J., Silva, B., Callou, G., Maciel, P., Brinkmann, A., 2017. Advanced stochastic petri net modeling with the mercury scripting language. In: Proceedings of the 11th EAI International Conference on Performance Evaluation Methodologies and Tools, pp. 192–197.
Pesquisa origem/destino 2010. URL: http://www.granderecife.pe.gov.br/sitegrctm/transporte/pesquisas/origem-e-destino/, access 08/09/2018.
Santos, T., Silva, M.A., Fernandes, V.A., Marsden, G., 2020. Resilience and vulnerability of public transportation fare systems: the case of the city of Rio de Janeiro, Brazil. Sustainability 12 (2), 647.
Silaen, S.K., Nasution, A.D., Suwantoro, H., 2018. Public preference for new service network plan brt trans mebidang (route: Pancurbatu - sambu market center). IOP Conf. Ser.: Mater. Sci. Eng. 420 (1), 012010. URL: http://stacks.iop.org/1757-899X/420/i=1/a=012010.
Wang, S., Barua, S., Desai, K., Deshmukh, S., 2013. Geotnavi–smart navigation using geo-temporal traffic information. Int. J. Data Min. Model. Manage. 5 (1), 20–36.
Zhang, C., Juan, Z., Luo, Q., Xiao, G., 2016. Performance evaluation of public transit systems using a combined evaluation method. Transp. Policy 45, 156–167.