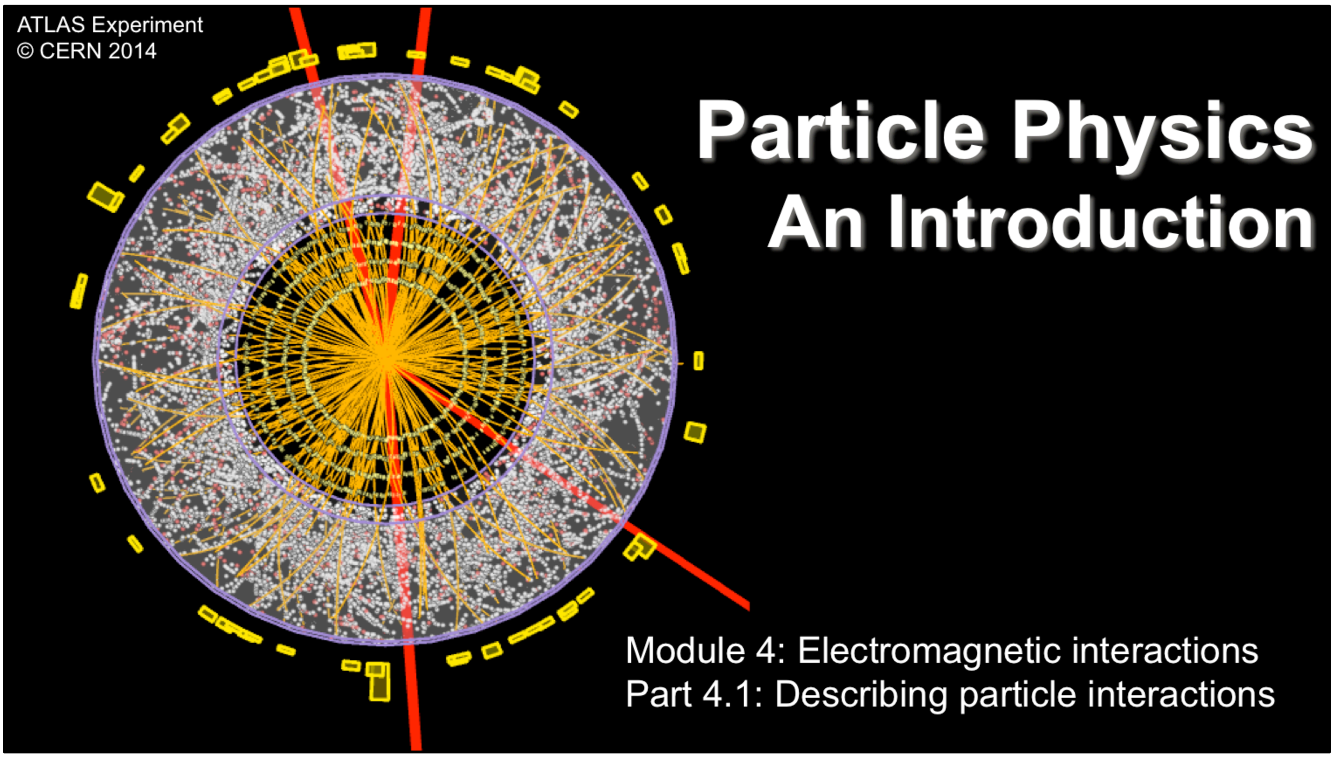
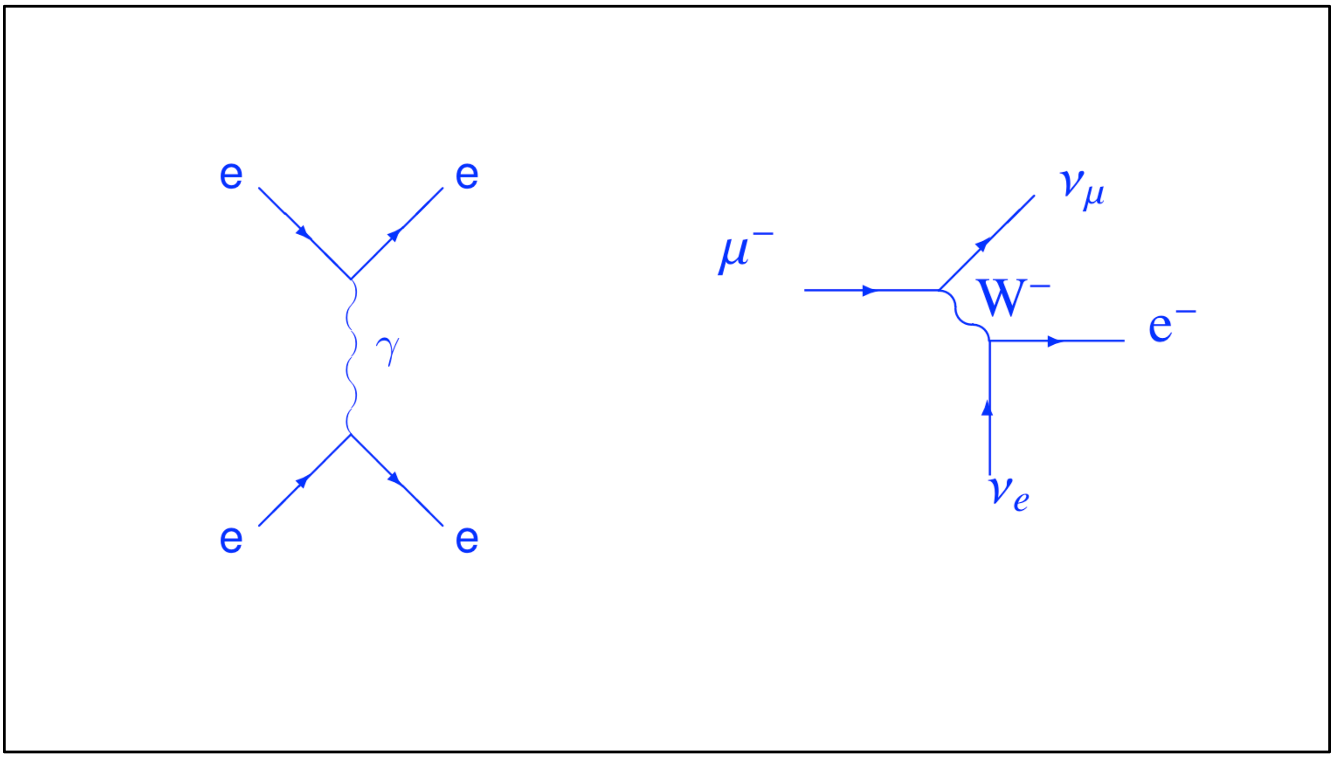


We now start a series of three modules discussing the three fundamental forces described by the Standard Model of particle physics. In this fourth module, we go into more details about the properties of electromagnetic interactions.

You will notice that the intellectual challenge and also the level of mathematical description rises somewhat as we go along. This is why, in this first video we remind you how to describe the intensity of a reaction using the cross section and the decay rate.

After following this video you will know:

- The relation between the reaction rate and the cross section;
- The relation between the decay rate and the lifetime of a particle;
- How the probability amplitude for a process enters into all of these notions.



- We now resume our discussion of scattering and decay processes. To characterize these processes we need a quantity, which measures the intensity of a reaction, or in the context of the quantum mechanics, its **probability**.
- This quantity is the **cross section** $\boldsymbol{\sigma}$ introduced in video 1.3 for scattering processes, and its analogue for decays, the **decay rate**.
- Both quantities measure the ratio of the number of scatters (or decays) **that take place** to the number **that would have been possible**. If this were a game, we would speak of this probability as the ratio between the number of successes and the number of trials.
- We have already seen that the unit for the cross section is a bit surprising, it is a surface. We will see that for the decay rate it is an energy. This is due to using the system of natural units, which we introduced in video 1.2a.

If you are still somewhat uneasy with the construction and interpretation of Feynman diagrams, like the ones shown here, please follow the optional video 4.1a.

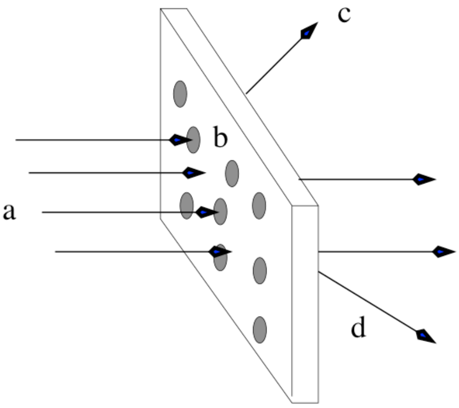
$$W = L\sigma \quad ; \quad \sigma = \int \frac{d\sigma}{d\Omega} d\Omega$$

Fixed target (laboratory) : $L = I_a n_b$

- I_a: projectile rate
- n_b: target surface density

Collider (center-of-mass) : $L = (N_a N_b f)/S$

- N_a, N_b: number of particles per packet
- f: crossing frequency of packets
- S : common surface at collision point



Cross section σ, units: barn $1\mathrm{b} = 10^{-28}\mathrm{m}^2$

High energy : nanobarn $1\mathrm{nb} = 10^{-9}\mathrm{b}$, picobarn $1\mathrm{pb} = 10^{-12}\mathrm{b} = 10^{-40}\mathrm{m}^2$

Neutrinos : attobarn $10\mathrm{ab} = 10^{-45}\mathrm{m}^2$ (at 1GeV)

As quantum physics takes a statistical approach, the basic problem of particle physics is the measurement and calculation of the probability for a process. Both calculation and measurement use the cross section to express this probability independently of experimental details. Consider a two-body reaction *a+b* ➔ *c+d*.

- If each target particle has a cross section σ, which represents the probability to "hit" it, the **rate of scatters** per second will be: $W = I_a\, n_b\, \sigma$, with I_a the incident flux of projectiles and n_b the number of targets.
- The **cross section** is thus defined as the reaction rate per target particle normalized to the flux of projectiles. It represents the intensity of a reaction, independent of experimental parameters.
- The proportionality factor between the cross section and the rate of interaction is called **luminosity**.
- In the **laboratory frame**, the flux I_a of projectiles *a*, that is to say their number per unit surface and per unit time, is $I_a = \rho_a v_a$, with their volume density ρ_a and velocity v_a relative to the targets *b* at rest. The luminosity *L* is the product of this flux and the number of targets n_b.
- In a **colliding beam** experiment, the luminosity *L* is the product of the number of projectiles and targets per bunch, N_a and N_b, multiplied by their crossing frequency *f* and divided by the surface *S* common to the two beams.
- The **unit** of the cross section is the barn. In high energy reactions we rather find nanobarn, or even picobarn cross sections. The smallest known cross sections are those of neutrinos, of the order of attobarn.

Decay rate:

$$\Gamma = -\frac{dN_b}{dt}\frac{1}{N_b} = \frac{1}{\tau_b}$$

- N_b: number of b particles
- dN_b/dt: number of decays per second
- τ_b: life time

Number of particles b as a function of time:

$$N_b(t) = N_b(0)e^{-\Gamma t} = N_b(0)e^{-t/\tau_b}$$

Number of decays in a time interval Δt at time t :

$$\Delta N_b(t) = N_b(0)\Delta t\,\Gamma e^{-\Gamma t} = \frac{N_b(0)\Delta t}{\tau_b}e^{-t/\tau_b}$$

- As far as **decays** are concerned, the description is quite similar. There is no projectile, so we consider only the particles *b* in the initial state.
- We normalize the **number of decays per second** to their current number N_b, to find the **decay rate Γ**.
- It is the inverse of the **lifetime τ_b**. The decay rate Γ has the dimension of an energy. It is also called the width of the particle because it corresponds to the uncertainty in the mass of *b* particles.
- Therefore it can be measured either by **measuring the lifetime** of the particle, or by observing the **width of the invariant mass distribution** of its decay products.
- As the decay rate is a constant, we find an **exponential law** for the number of remaining particles at time *t*.
- Consequently, the **absolute number of decays** in an interval Δt decreases also exponentially.

Partial decay rate Γ_i for channel i:

$$\Gamma = \sum_i \Gamma_i = \frac{1}{\tau_b}$$

- Γ_i/Γ: branching fraction
- $d\Gamma/d\Omega$: differential decay rate

If only one decay channel i is observed:

$$\boxed{N_b(t) = N_b(0)e^{-\Gamma t}} = N_b(0)e^{-t/\tau_b} \quad \text{or} \quad N_b(t) = N_b(0)e^{-\Gamma_i t} \quad ?$$

- If **several decay channels** exist, the lifetime observed in each channel is of course the same, but the decay rate depends on the channel.
- One thus defines a **partial decay rate Γ_i** for each channel *i.* By probability conservation, their sum is equal to the total width Γ.
- The **branching ratio Γ_i/Γ** expresses the relative probability of observing channel *i* among all decays.
- Analogous to the differential cross section, one can also measure the **differential decay rate**, $d\Gamma/d\Omega$ depending on the solid angle.

Let us think for a moment:

- If, for a decay with multiple channels, only a single channel *i* is observed, is the time evolution determined by Γ or by Γ_i?

The answer should be obvious from what was said before: the lifetime is the same independent of the decay channel, the time evolution is determined by the total decay width.

In the next video we apply the concept of cross section to elementary electromagnetic interactions.