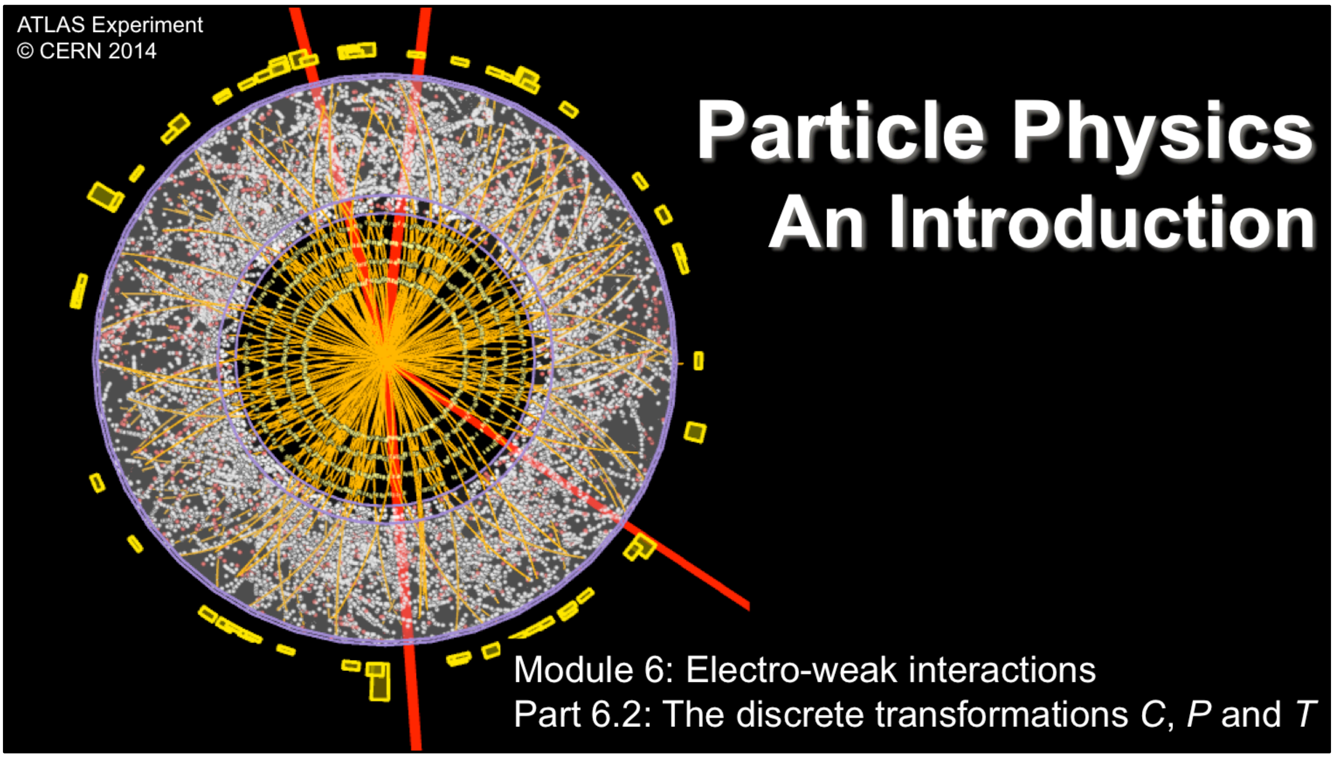


In this 6th module, we are discussing weak interactions.

In this 2nd video we introduce the operations of charge conjugation, parity and time reversal.

After following this video you will know:

- Discrete transformations of space-time, which are parity and time reversal;
- The charge conjugation transformation, which reverses all charges;
- The behavior of forces and matter under these transformations.

$$x = (t, x, y, z) \xrightarrow{P} x' = (t, -x, -y, -z) \quad ; \quad \mathbf{P}\phi(t, \vec{r}) = \phi(t, -\vec{r})$$

$$x = (t, x, y, z) \xrightarrow{T} x' = (-t, x, y, z) \quad ; \quad \mathbf{T}\phi(t, \vec{r}) = \phi^*(-t, \vec{r})$$

- Unitarity: $\mathbf{P}(\mathbf{P}\phi) = \phi \rightarrow \mathbf{P}^2 = \mathbf{1}$
- Eigenstates: $\mathbf{P}\phi = \pm\phi$, eigenvalues $P = \pm 1$

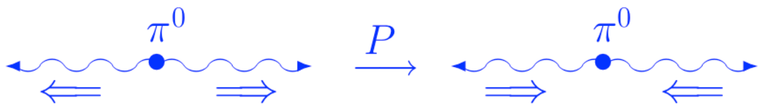
- The reversals of the space and time directions are **discrete transformations** of space-time coordinates.
- For the space coordinates, the transformation is also called **parity *P***.
- Both transformations can be applied as **operators** to a quantum state, such as a particle.
- These operators are **unitary**, because a double application to the initial state reproduces the original state. Therefore, the **eigenvalues** of the two operators are ± 1, if they are applied to **eigenstates** of parity or time reversal.
- However, there are many wave functions which are **not eigenstates of *P***. For example, a function $\varphi = \cos x + \sin x$ becomes $P\varphi = \cos x - \sin x$, which is neither $+\varphi$, nor $-\varphi$.

quantity Q	$\mathbf{P}(Q)$	J^P
scalar s	$\mathbf{P}(s) = s$	0^+
pseudoscalar p	$\mathbf{P}(p) = -p$	0^-
vector $\vec{v}$	$\mathbf{P}(\vec{v}) = -\vec{v}$	1^-
axial vector $\vec{a}$	$\mathbf{P}(\vec{a}) = \vec{a}$	1^+

- **Eigenstates of *P*** can be classified like in this table.
- **Scalar** wave functions describe spin 0 particles with parity +. Pseudoscalar ones describe spin 0 particles with parity -.
- **Vector** and **axial vector** wave function describe spin 1 particles with parity - and +, respectively.
- Parity is a **multiplicative quantum number**, so the total parity of a system in its ground state is the product of the individual parities of its components. If there is an angular momentum between the components, characterized by an *l*-type quantum number, it must be taken into account by a factor $(-1)^l$.
- Parity is **conserved** in reactions due to electromagnetic and strong forces, but not in weak interactions. Conservation means that if a system is in an eigenstate before the reaction, it will be in an eigenstate with the same eigenvalue after the reaction.
- For **bound states**, the total angular momentum $J = l+s$ comes in, multiplying the intrinsic parity of the components. Consequently, the notation J^P is used to characterize particles. You find it in the PDG tables for each particle.
- Electromagnetic transitions in bound states (as the hydrogen atom for example) require $\Delta l = \pm 1$, so the parity of the photon must be (-1). The **photon** is thus indeed a **vector particle**, as we noted in Module 4.

Particle	J^P
q, l, ν $\bar{\text{q}}$, $\bar{l}$, $\bar{\nu}$	$\frac{1}{2}^+$ $\frac{1}{2}^-$
γ	1^-
p, n $\bar{\text{p}}$, $\bar{\text{n}}$	$\frac{1}{2}^+$ $\frac{1}{2}^-$
π, K ρ, K*, ω, ϕ	0^- 1^-

- **Hadrons** are parity eigenstates, thus their parity eigenvalues can be used to characterize a particle, just like spin, electric charge or baryon number.
- The parity of **fermions** is opposite to that of antifermions. By convention, parity is arbitrarily fixed to **(+1) for leptons and quarks**, antiquarks and antileptons therefore have parity (-1).
- For **bosons** the parity is the same for particles and antiparticles. The photon parity is (-1).
- **Baryons,** which contain three quarks without relative angular momentum (such as proton (uud) and neutron (udd)), have a parity $(+1)^3 = (+1)$, so $J^P = ½^+$.
- The **lightest mesons**, like pions (ud-bar), (du-bar) and (uu-bar + dd-bar)/√2, as well as light kaons, contain quark and antiquark with antiparallel spins ↑↓. They have parity (+1) (-1) = (-1) and are called pseudoscalar mesons.
- The states with same quark contents, but parallel spins ↑↑, like the mesons ρ, K*, ω, ϕ, are called **vector mesons,** they have $J^P = 1^-$.



$$
\begin{aligned}
\psi_1(2\gamma) &= A\,(\vec{\epsilon}_1 \cdot \vec{\epsilon}_2) && \propto \cos\Phi \\
\psi_2(2\gamma) &= B\,(\vec{\epsilon}_1 \times \vec{\epsilon}_2) \cdot \vec{k} && \propto \sin\Phi
\end{aligned}
$$

- Parity is not just a theoretical concept, it has **measurable effects** whenever it corresponds to a conserved observable. Let us look at the dominant decay of the π^0 ➔ $\gamma\gamma$. The study of this process has allowed to conclude that the π^0 is a pseudoscalar boson, $J^P = 0^-$. Its decay products are two photons, vector bosons with $J^P = 1^-$.
- We project spins on the only natural axis of the system, given by the direction of the two photons in the rest frame of the π^0. **Conservation of angular momentum** thus requires one of the two configurations sketched here, or a combination of both.
- The 1rst configuration has two photons in a **right circular polarization state**, ψ_R, the second has **left polarized photons**, ψ_L.
- Under *P*, the **momenta** of the two photons change sign, but their **spins**, being pseudovectors, do not change. The parity operation therefore changes the 1rst configuration into the 2nd one and vice versa.
- The photon wave function is characterized by its **polarization vector** ε, which points in the direction of the electric field. For real photons, this vector is normal to the direction of movement.
- As **parity is conserved** in the decay process, this requires that the total wave function, ψ, be an eigenstate of *P*. Up to a normalization constant, the two possibilities correspond to the wave functions ψ_1 and ψ_2 given here.
    - The 1rst is proportional to a scalar, so it has parity (+1). The **angle Φ between the two polarization planes** will be preferentially **zero,** with an intensity distribution $\sim\cos^2\Phi$.
    - The second is proportional to the product between a pseudovector and a vector, so it has parity (-1). The angle Φ between the polarization planes of the two photons will be preferentially **90°**. Experimentally it is found that they are indeed orthogonal, the parity of the π^0 is therefore (-1).

$$\mathbf{C}\,\phi_p(t,\vec{x}) = \phi_{\bar{p}}(t,\vec{x})$$

- The **charge conjugation** operator, ***C*,** is an example for a transformation of the field itself, and not of the coordinates. It transforms the wave function of a particle, φ_{p}, into the wave function of its antiparticle, $\varphi_{p\text{-}bar}$.
- The operation therefore changes the **sign of electric charge, color, baryon and lepton number**, in short, all the quantum numbers of the charge type.
- Thus, the operator *C* applied to a fermion gives an antifermion, with all charges opposite, but with the **same mass, same spin and same momentum.**
- Again, **electromagnetic and strong interactions conserve *C***, that is to say, these interactions have the same intensity for particles and antiparticles.
- **Weak interactions**, on the contrary, **violate** *C* symmetry, so they distinguish between particles and antiparticles.
- Particles which do not have any charge can be **eigenstates** of *C*, they are their own antiparticles. The photon and the Z boson are examples.

$$\begin{aligned} \mathbf{C}\,\phi_{\pi^0} &= \phi_{\pi^0} \\ \mathbf{C}\,A^\mu &= -A^\mu \end{aligned}$$

$$\pi^0 \to \gamma\gamma \quad ; \quad \pi^0 \not\to \gamma\gamma\gamma$$

- The **pseudoscalar meson** π^0, which is also its own antiparticle, has $C\varphi_\pi=\varphi_\pi$, with eigenvalue $C = (+1)$.
- Since the photon is generated by moving charges, which change sign of their electric charge under C, $C\,A^\mu = -A^\mu$.
- Charge conjugation is another **multiplicative quantum number**, a system of n photons has $C = (-1)^n$.
- For example, π^0 decay into two photons respects the conservation of C, but **π^0 decay into 3 photons is forbidden**. Indeed, the branching ratio of this decay is measured to be less than 3.1×10^{-8}.

$$x = (t, x, y, z) \xrightarrow{T} x' = (-t, x, y, z)$$

$$\mathbf{T}\phi(t, \vec{r}) = \phi^*(-t, \vec{r})$$

- The **time reversal** operation, *T***,** transforms the coordinate four-vector *x = (t, x, y, z)* into *x'= (- t, x, y, z)*.
- When applied to fields, *T* also turns them into their **complex conjugate**. This is necessary because of the transformation of the equation of motion.
- Except for real boson fields, there are no **eigenstates** of *T* alone, so no conserved quantum number. The importance of *T* is rather in combination with the other discrete transformations, parity *P*, and charge conjugation, *C.* One one can easily understand that all local field theories must be invariant under the **joint action of *CPT*,** transforming a process into itself.
- We will see in what follows that **weak interactions** violate to a maximum extent the symmetries *C* and *P*. And even the combined operation *CP***,** which transforms a left-handed fermion into a right-handed antifermion, is not respected by weak interactions. This introduces a small difference between a reaction and its reverse; it gives an objective direction to time.

We will introduce basic properties of weak interactions in the next video.