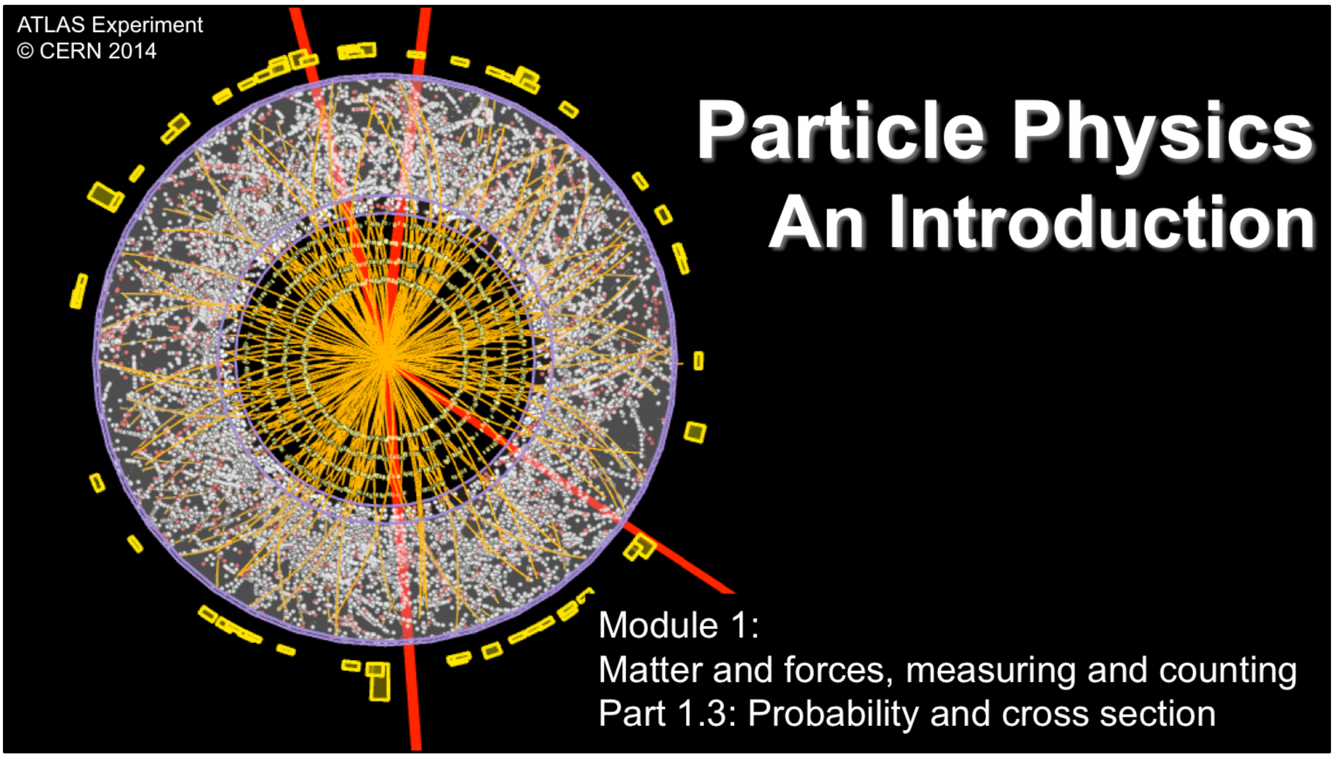


During this first module, we are introducing the objects studied in particle physics, namely matter, forces, space-time and particle reactions.

In this third video we present the **concept of probability** to characterize the strength of fundamental forces. We will also define one of the most important quantities in particle physics: the **cross section**. This quantity allows to **calculate and measure** the strength of subatomic reactions.

After watching this video you will be able to:

- define the cross section in terms of incident flux and number of target particles;
- relate the cross section to the probability and rate of a particle reaction.

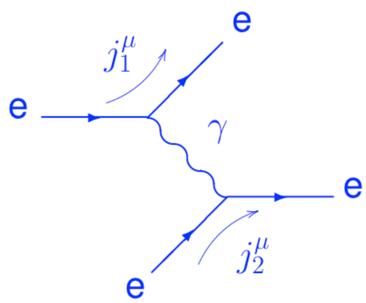
Complex amplitude: Real probability:

$$\psi_f(t,\vec{x}) = \psi_0 e^{-ip^\mu x_\mu}$$

$$\psi_f(t,\vec{x})^2 = \psi_f^* \psi_f$$

$$\mathcal{M} \propto \int j_{\mu 1} \left(\frac{-1}{q^2}\right) j_2^\mu \, d^4x$$

$$d\sigma = \frac{|\mathcal{M}|^2}{F} \, dQ$$



- In quantum physics, the motion of fields and their way of interacting is described by **probability amplitudes**, and probabilities themselves. They are defined in a frequentist way by multiple measurements:
  - For the movement of a particle, the wave function ψ describes the probability amplitude as a function of space-time and the four-momentum of the particle. The probability density to find the particle at time t and location x is given by the square of the wave function, $\psi^*\psi$. Integrating this density over a finite volume gives us the probability that the particle is in this volume.
  - The probability amplitude M for a reaction is the joint probability amplitude that two current densities j_1 and j_2 meet and exchange a virtual photon of four-momentum q. The probability for the reaction is then given by the cross section, which is proportional to the square of the amplitude $|M|^2$.
  - The physics of particle motion and particle reactions is thus expressed via statistical probabilities.

Probability $p \in \mathbb{R}$:

$$0 \leq p \leq 1$$

Frequentist approach:

$$p = \frac{\#\,\text{successes}}{\#\,\text{trials}}$$

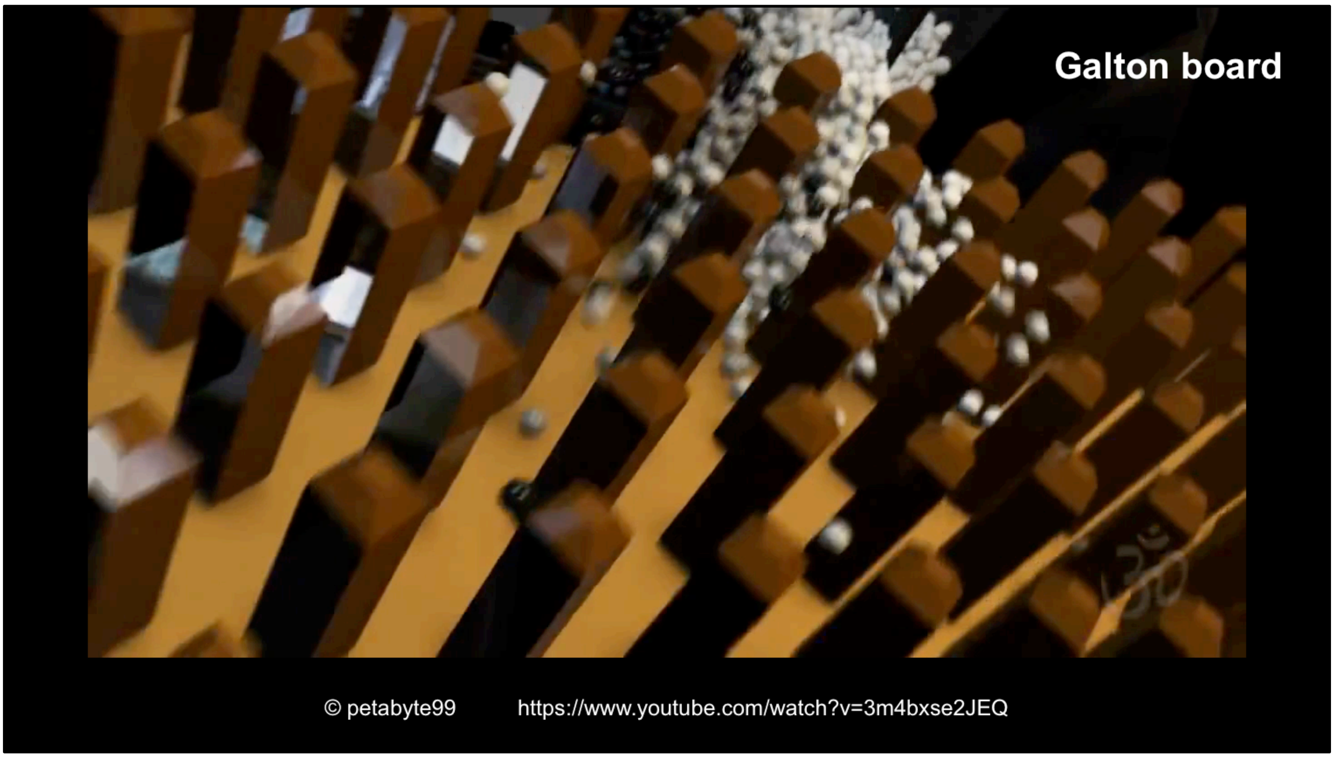
Events A and B independent:

$$p(A \wedge B) = p(A) \cdot p(B)$$

Events A and B incompatible:

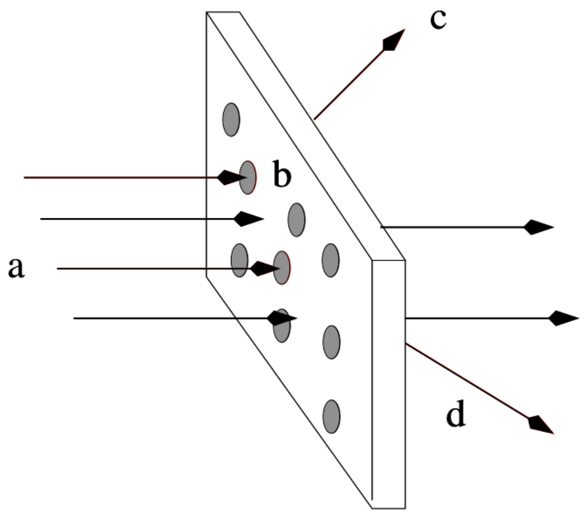
$$p(A \vee B) = p(A) + p(B)$$

- Mathematically speaking, a **probability** is a real number between 0 and 1.
- In a **frequentist approach**, the probability is defined as the ratio between the number of desired outcomes of an experiment, the "successes", and the number of trials. In terms of physics, this is the ratio between the number of times an event happens and the number of times it could have happened. This event can for example be the localization of a particle in a given volume.
- The ratio can also be the number of times a reaction takes place, divided by the number of times it could have happened. This means in particular that reactions do not always happen, in fact we will find that they happen rather rarely.
- Probabilities follow their own rather simple algebra. For our purposes it suffices to know two rules:
    - The joint probability that two **independent** events happen is the product of the individual probabilities.
    - The probability that at least one of two **incompatible** events happen is the sum of the two probabilities. Incompatible means that the two events cannot be realized at the same time.
    - Analogous rules apply to the probability amplitudes we have introduced earlier.
- The two limiting cases are *p=0* for an impossible event and *p=1* for a certain event.
- In general we will find that probabilities for particle reactions are rather small,



- Processes which follow probabilistic laws are not certain individually, but can still be predicted collectively.
- This little simulation shows a **Galton board** as an example. The small balls are introduced in the middle at the top of the board. The obstacles scatter them either left or right with 50% probability. The multiple scatters are independent of each other, their result does not depend on a previous one. At the bottom of the board, the balls show a **Gaussian distribution** around the middle position.
- For those of you who know a little about statistics: This is an example of how chains of multiple events with an arbitrary individual distribution – Boolean in this case – asymptotically produce a Gaussian distribution, which is perfectly calculable and characterized by just two parameters, the mean and the width.

$$\boxed{\text{Flux} = \frac{\#\ \text{particles a}}{\text{s} \cdot \text{m}^2}}$$



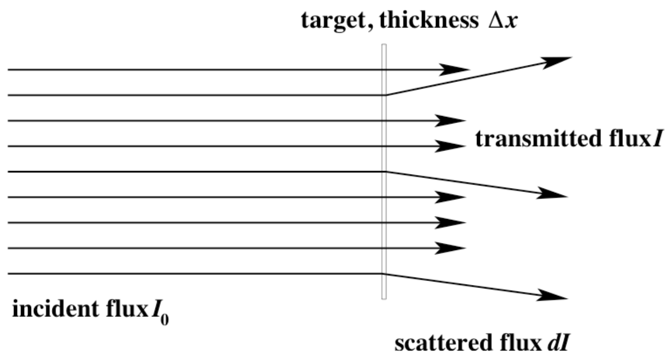
$$\boxed{p_{int} = n\sigma}$$

- But let us come back to particle reactions. Particle physics explores matter and forces via **scattering experiments**.
- For this purpose, a beam of **projectiles** is directed towards a **target**. One counts the number of scatters which take place and measures the scattered particles. The sketch shows a fictitious situation where each target particle is represented by a **small grey surface**. Let us consider a simple model where a reaction takes place if and only if a projectile hits a grey surface.
- Let us make the assumption that the distance between target particles is large compared to their "size", such that the **grey surfaces do not overlap**. The interactions thus stay rare. In real life, this corresponds to a target consisting of a **thin foil** or a **rarified gas**.
- The **number of possible scatters** is evidently proportional to the **flux** of projectiles, i.e. their number per unit time which pass through a perpendicular surface.
- The **number of actual scatters** is proportional to the number of target particles per m^2, i.e. their surface density n, and their individual surface σ. In fact the probability to interact is simply the product of the surface density and the surface size σ .

$$\frac{-dI}{I} = \sigma n = \sigma\rho\, dx \quad ; \quad I = I_0 e^{-\sigma\rho\Delta x}$$

$$[\sigma] = \text{barn} = 10^{-28}\text{m}^2$$

- n: surface density of targets
- ρ: volume density of targets



$$\sigma_{tot} = \sigma_{el} + \sigma_{inel} + \sigma_{abs}$$

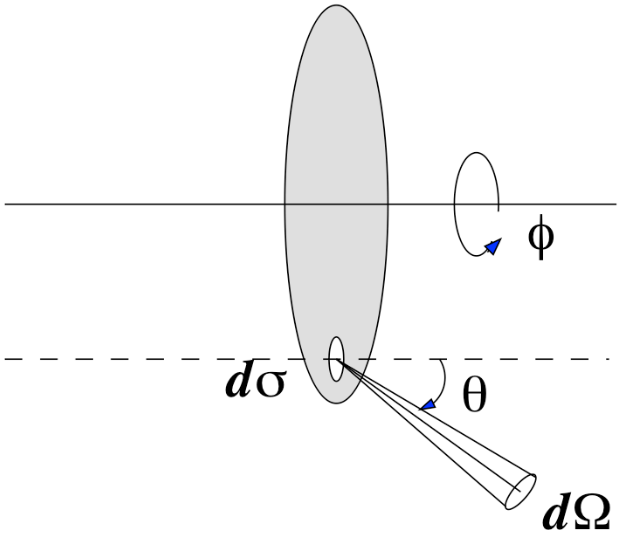
- A small part $-dI$ of the initial flux I_0 is thus scattered, the rest of the flux I is transmitted. The fraction of the scattered flux, $-dI/I$, is thus equal to the interaction probability $n \bullet \sigma$.
- The density of targets can also be expressed as the volume density ρ if one multiplies by the infinitesimal target thickness dx. Integrating over the thickness up to Δx one obtains an exponential law for the attenuation of the incident flux.
- We call σ the **cross section** for the reaction, because it has the dimension of a surface. It has nothing to do with the **geometric size** of the target particles, it rather represents the **probability of interaction** between an individual projectile and an individual target particle. The cross section is often measured in the enormous units of **1 barn = 10^{-28}m^2**.
- The cross section is thus a sort of probability, expressed with strange units. But the rules for probability calculation apply: If two processes are independent, their probabilities add up. One distinguishes for example processes of
    - Elastic scattering where the kinetic energy of the projectile does not change;
    - Inelastic scattering where the projectile looses energy and the target recoils or changes mass;
    - Absorption, where the projectile disappears in the target.
- For these **mutually exclusive** categories, the cross section can be added to form a **total cross section**.
- The cross section must be a **positive real number**. It has a maximum which corresponds to a reaction that always takes place.

An application is calculated in video 1.3a. It deals with the attenuation of photons by a sheet of lead. The result will surprise you and hopefully encourage you to be very careful the next time you expose yourself to energetic photons!

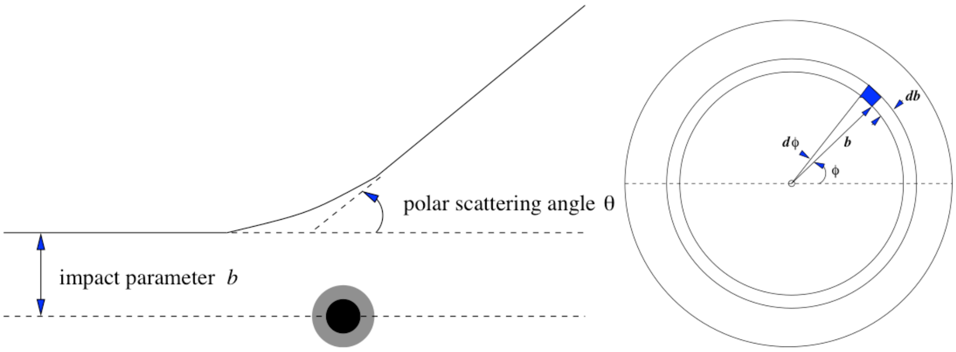
Differential cross section:

$$\frac{-dI}{I} = (\rho\Delta x)\,\frac{d\sigma(\theta,\phi)}{d\Omega}\,d\Omega$$

$$\sigma = \int_0^{2\pi}\int_0^{\pi}\frac{d\sigma(\theta,\phi)}{d\Omega}\sin\theta\,d\theta\,d\phi$$



- It is clear that the properties of the **outgoing particle** are also important to understand the reaction. For example, the angle by which the projectile is scattered gives useful information about the structure of the target and the properties of the interaction.
- Keeping in mind that the cross section has nothing to do with the physical size of the target, let us continue to imagine the cross section as an **effective area** for the interaction. If the projectile hits it, a reaction takes place, otherwise the projectile passes without perturbation. The scattering into an infinitesimal **element *dΩ* of solid angle**, centered on the polar angle ϑ and the azimutal angle φ, then comes from an **element *dσ* of the surface** in our geometrical model.
- The fraction of the incoming beam scattered into this solid angle element is proportional to the **differential cross section *dσ/dΩ***.
- The probability that a projectile is scattered into a solid angle element is **not normally uniform**. Its dependence on the angles ϑ and φ allows to **look into the target**.
- The **total cross section** is obtained **integrating** the differential cross section **over the solid angle**.

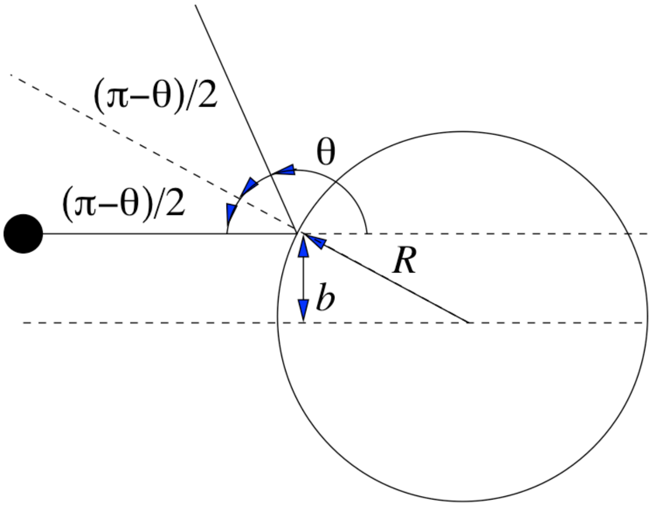


$$\frac{-dI}{I} = (\rho\Delta x)\ (b\ db\ d\phi) = (\rho\Delta x)\frac{d\sigma(\theta,\phi)}{d\Omega}\ d\Omega$$

$$\int \frac{-dI}{I}\ d\phi = (\rho\Delta x)\ (2\pi b\ db) = (\rho\Delta x)\ 2\pi\frac{d\sigma(\theta)}{d\Omega}\sin\theta\ d\theta$$

$$\frac{d\sigma(\theta)}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

- Let us consider a system with **cylindrical symmetry**. The scattering probability in then **independent of the azimutal angle *φ*.**
- An example from classical physics is the scattering by a central force. It shows a fixed relation between the **impact parameter *b*** and the **polar scattering angle *ϑ***.
- All projectiles with an impact parameter between b and $b+db$ and an initial azimuthal angle between φ and $\varphi+d\varphi$ are scattered into an element $d\Omega$ around the direction (ϑ,φ). The **fraction of the incoming beam** which falls into such a region is $dI/I = \rho\Delta x\ (b\ db\ d\varphi)$.
- Integrating over the azimuthal angle on which the force does not depend, we obtain a factor of 2π.
- The result defines the cross section in terms of the impact parameter and the scattering angle. Taking the absolute value guarantees that the cross section is positive definite.
- The scattering angle ϑ depends on the impact parameter b in a specific manner, which is determined by the **distance law of the central force** or the equivalent potential.



Impact parameter:

$$b = R\sin\left(\frac{\pi-\theta}{2}\right) = R\cos\left(\frac{\theta}{2}\right)$$
$$\frac{db}{d\theta} = \frac{R}{2}\sin\left(\frac{\theta}{2}\right)$$
$$\frac{d\sigma(\theta)}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right| = \frac{R^2}{4}$$

Total cross section:

$$\sigma = \pi R^2$$

Let us take a trivial example, the classical shock between two rigid bodies:

- The geometry and the laws of elastic reflection **relate *b* to *ϑ*** in the simple geometrical way sketched here.
- The **differential cross section** is constant and does not depend on the scattering angle. The angular distribution of the scattered particles is **isotropic**.
- The total cross section is $\sigma = \pi R^2$, equal to the geometrical surface of the target, without surprise.

In the **next video** Mercedes will give an example of a more realistic electromagnetic reaction among a ^{4}He nucleus, also known as an alpha particle, and a heavy nucleus like gold. This process is called **Rutherford scattering**. Its experimental observation has revealed the existence of the atomic nucleus.