

ATLAS Experiment
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Particle Physics An Introduction

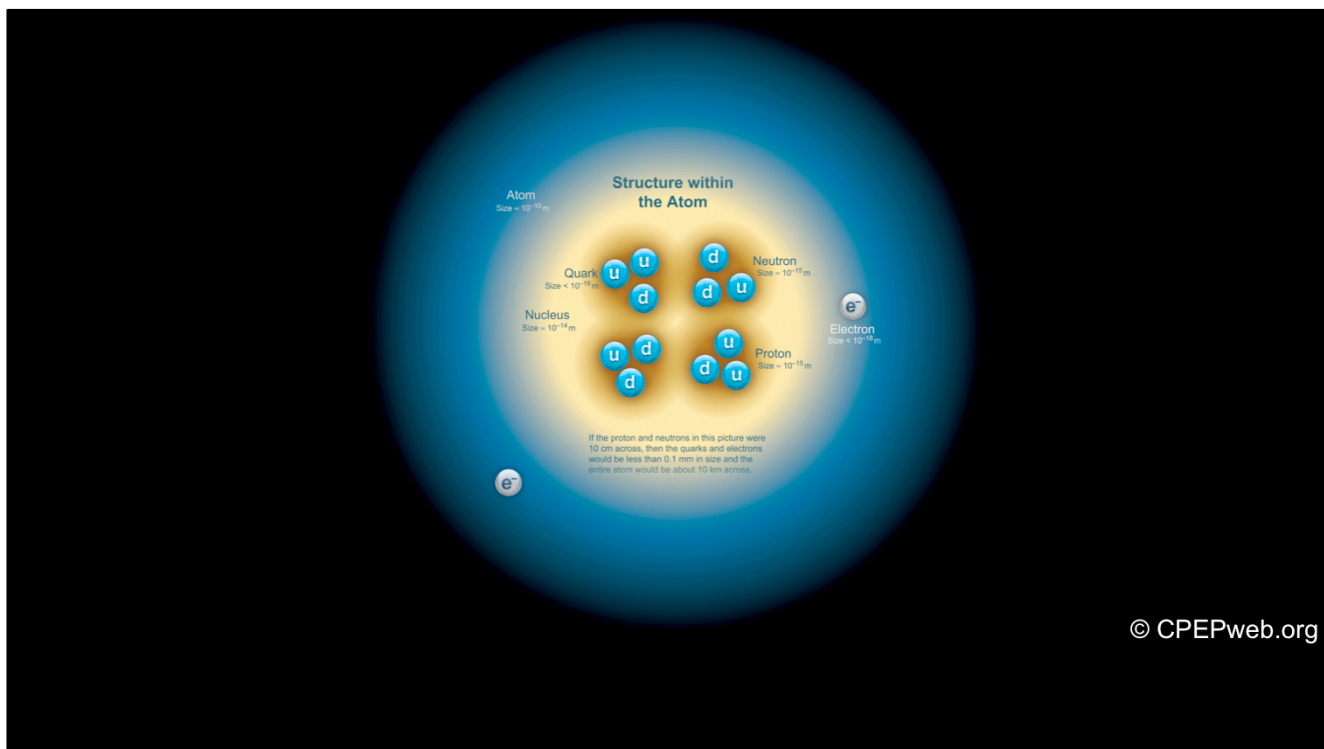
Module 2: Nuclear physics
Part 2.1: Nuclear mass and binding energy

During this second module, we deal with nuclear physics and its applications. At the end of the module we will visit the Tokamak of the Swiss Institute of Technology in Lausanne and the Beznau nuclear power plant, the oldest one still in operation.

This is a rather self-contained module. If your main interest is nuclear physics, you will be well served. You will also notice that the module is somewhat longer than others. Just take your time to digest the material without pressure.

In this first video, we will review what is known about the mass of nuclei, The goals for you are the following:

- To know the nomenclature of atomic nuclei and their periodic system;
- To be able to qualitatively describe the mass and binding energy of nuclei.



- Experiments of the **Rutherford** type demonstrate the existence of a positively charged nucleus, four orders of magnitude smaller than the size of the atom. These experiments only require to understand **electromagnetic interactions** between the projectile and the target.
- Scattering experiments can also yield information about nuclear properties, and thus establish a catalogue of the properties of the **nuclear interaction** that holds protons and neutrons together inside the nucleus.
- One must not confuse this **nuclear force** with the **strong force** introduced in the first module and more extensively discussed in Module 5.
- The **strong force binds quarks** inside hadrons by gluon exchange. It does not permit quarks to leave their hadrons. So hadrons in general and **nucleons** in particular **do not carry a net color charge**. Thus gluons cannot bind protons and neutrons to form a nucleus.
- The **nuclear force** is more like a long distance residue of the strong force. In that it resembles the Van der Waals force, which is a residue of the electromagnetic interaction which acts between neutral molecules.

Atoms:

- Electromagnetic force, well known properties
- Atomic spectra motivate quantum mechanics
- Perturbative calculations feasible

Nuclei:

- Nuclear force much stronger
- Very short range
- No classical analogue
- Only empirical models

Let us compare some basic properties of **atoms and the electromagnetic force**, to **nuclei and the nuclear force**.

- The **electromagnetic force** is responsible for holding atoms together. Its properties are well known classically and rather easy to extrapolate to quantum distances.
- The study of atomic spectra gave rise to **quantum mechanics**, which qualitatively and quantitatively explains many phenomena of condensed matter physics.
- The **fine structure constant** $\alpha \approx 1/137$ is small, which makes perturbative calculations feasible.
- The **nuclear force** must be much stronger, since it wins over the Coulomb repulsion between tightly packed protons.
- It must be of **short range**, since it does not make itself felt outside the nucleus.
- It has **no classical analogue**, only experimental results can help to understand its properties.
- One thus uses experiments as a guide towards **empirical models** of the nucleus and the nuclear force.

We will come back to the relation between experiments and models as we go along.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	H Hydrogen 1.00794																	2 He Helium 4.002602
3	Li Lithium 6.941	4 Be Beryllium 9.012182																
5	9 B Boron 10.811	10 C Carbon 12.0107	7 N Nitrogen 14.0064	8 O Oxygen 15.9994	9 F Fluorine 18.9984032	10 Ne Neon 20.1797												
11	11 Na Sodium 22.98976928	12 Mg Magnesium 24.3050																
13	13 Al Aluminum 26.9815386	14 Si Silicon 28.0855	15 P Phosphorus 30.973762	16 S Sulfur 32.06	17 Cl Chlorine 35.453	18 Ar Argon 39.948												
19	19 K Potassium 39.0983	20 Ca Calcium 40.078	21 Sc Scandium 44.955912	22 Ti Titanium 47.887	23 V Vanadium 50.9415	24 Cr Chromium 51.9961	25 Mn Manganese 54.938045	26 Fe Iron 55.845	27 Co Cobalt 58.933195	28 Ni Nickel 58.6934	29 Cu Copper 63.546	30 Zn Zinc 65.38	31 Ga Gallium 69.723	32 Ge Germanium 72.64	33 As Arsenic 74.9216	34 Se Selenium 78.96	35 Br Bromine 79.904	36 Kr Krypton 83.798
37	37 Rb Rubidium 85.4678	38 Sr Strontium 87.62	39 Y Yttrium 88.90585	40 Zr Zirconium 91.224	41 Nb Niobium 92.90638	42 Mo Molybdenum 95.94	43 Tc Technetium (97.9072)	44 Ru Ruthenium 101.07	45 Rh Rhodium 102.90550	46 Pd Palladium 106.42	47 Ag Silver 107.8682	48 Cd Cadmium 112.411	49 In Indium 114.818	50 Sn Tin 118.710	51 Sb Antimony 121.757	52 Te Tellurium 127.60	53 I Iodine 126.90547	54 Xe Xenon 131.29
55	55 Cs Cesium 132.9054519	56 Ba Barium 137.327	57-71 Lanthanoids	72 Hf Hafnium 178.49	73 Ta Tantalum 180.94788	74 W Tungsten 183.84	75 Re Rhenium 186.207	76 Os Osmium 190.23	77 Ir Iridium 192.222	78 Pt Platinum 195.084	79 Au Gold 196.966569	80 Hg Mercury 200.59	81 Tl Thallium 204.3833	82 Pb Lead 207.2	83 Bi Bismuth 208.9804	84 Po Polonium (209)	85 At Astatine (210)	86 Rn Radon (222)
87	87 Fr Francium (223)	88 Ra Radium (226)	89-103 Actinoids	104 Rf Rutherfordium (261)	105 Db Dubnium (262)	106 Sg Seaborgium (266)	107 Bh Bohrium (264)	108 Hs Hassium (277)	109 Mt Meitnerium (268)	110 Ds Darmstadtium (271)	111 Rg Roentgenium (272)	112 Uub Ununbium (285)	113 Uut Ununtrium (284)	114 Uuq Ununquadium (289)	115 Uup Ununpentium (288)	116 Uuh Ununhexium (292)	117 Uus Ununseptium (294)	118 Uuo Ununoctium (294)

For elements with no stable isotopes, the mass number of the isotope with the longest half-life is in parentheses.

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57 La Lanthanum 138.9047	58 Ce Cerium 140.116	59 Pr Praseodymium 140.90768	60 Nd Neodymium 144.242	61 Pm Promethium (145)	62 Sm Samarium 150.36	63 Eu Europium 151.964	64 Gd Gadolinium 157.25	65 Tb Terbium 158.92535	66 Dy Dysprosium 162.500	67 Ho Holmium 164.93032	68 Er Erbium 167.259	69 Tm Thulium 168.93421	70 Yb Ytterbium 173.054	71 Lu Lutetium 174.967
89 Ac Actinium (227)	90 Th Thorium 232.03806	91 Pa Protactinium 231.03688	92 U Uranium 238.02891	93 Np Neptunium (237)	94 Pu Plutonium (244)	95 Am Americium (243)	96 Cm Curium (247)	97 Bk Berkelium (247)	98 Cf Californium (251)	99 Es Einsteinium (252)	100 Fm Fermium (257)	101 Md Mendelevium (258)	102 No Nobelium (259)	103 Lr Lawrencium (262)

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Let us first summarize how we identify nuclei:

- We denote by **Z** the **nuclear charge**, which is equal to the atomic number in the periodic table. It is given by the number of protons in the nucleus.
- **A** is the **number of nucleons**, the sum of protons and neutrons. It is also called mass number.
- Nuclei are thus completely identified by their electric charge and their number of nucleons, with a notation ${}^A_Z\text{X}$. The element name denotes Z, normally supplemented by A, like in ${}^{14}\text{C}$.
- The number of neutrons is evidently **N=A-Z**.
- The **chemical properties** of the elements are determined by the electron cloud surrounding the nucleus, the **periodic table** is therefore organised according to Z.
- Nuclei with the same number of protons, but a different number of neutrons, ${}^A_Z\text{X}$ and ${}^{A'}_Z\text{X}$, are called **isotopes**. They have very similar chemical properties, but not necessarily similar nuclear properties. Examples are ${}^{235}\text{U}$ and ${}^{238}\text{U}$ (92 protons), ${}^1\text{H}$, ${}^2\text{H}$ et ${}^3\text{H}$ (hydrogen, deuterium and tritium: 1 proton with 0, 1 and 2 neutrons).
- Nuclei with the same number of nucleons, but a different number of protons, ${}^A_Z\text{X}$ et ${}^A_{Z'}\text{Y}$, are called **isobars**. Examples are ${}^{12}\text{C}$ and ${}^{12}\text{B}$ (12 nucleons).
- Nuclei can also be excited to higher energy states keeping the number of protons and neutrons constant. These are called resonances or **isomers**.

Nuclear mass:

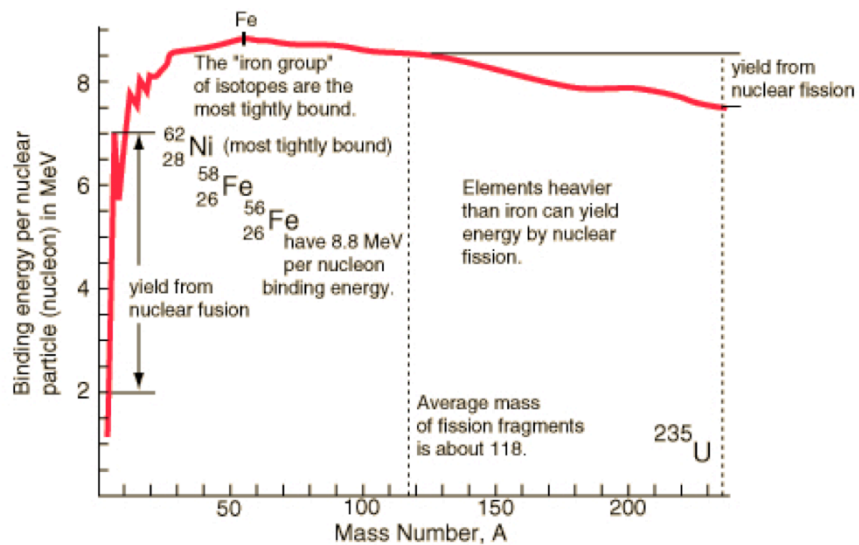
$$M(A, Z) < Zm_p + (A - Z)m_n$$
$$m_p \simeq 938.28 \text{ MeV} \quad ; \quad m_n \simeq 939.56 \text{ MeV}$$

Binding energy:

$$\Delta M(A, Z) = M(A, Z) - Zm_p - (A - Z)m_n < 0$$

$$\frac{\Delta M(A, Z)}{A} = \frac{M(A, Z)}{A} - \frac{Z}{A}m_p - \left(1 - \frac{Z}{A}\right)m_n \ll m_{p/n}$$

- Naively one might assume that the mass of a nucleus is simply given by the **sum of the proton and neutron masses**. But in reality it is of course diminished by the **binding energy** between the nucleons.
- The **mass deficit ΔM** must always be negative so that the nucleus is in a bound state. We call it **binding energy**, it has been measured for practically all nuclei.
- The absolute value of the binding energy is the energy required to decompose the nucleus into separate nucleons.
- The **binding energy per nucleon $\Delta M/A$** is the energy required to separate the average nucleon from its nucleus. It is much smaller than the mass of the nucleons p and n. The mass of the nucleus is thus indeed dominated by the mass of its constituents.
- For the nucleons themselves, the situation is very different: the total mass of the quarks inside a nucleon is only about 1% of the nucleon mass. It is their binding energy which dominates the nucleon mass.



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When one analyses the dependence of the binding energy B per nucleon on the mass number A , one notices the following:

- $A < 20$: $\Delta M/A$ oscillates, but rises rapidly with increasing A
- $20 < A < 60$: $\Delta M/A$ saturates
- $A \approx 60$: $\Delta M/A$ has a broad maximum for the iron group (Ni, Fe, Co) with about 9 MeV/nucleon
- $A > 60$: $\Delta M/A$ decreases slowly
- The general mean is ≈ 8 MeV/nucleon. The kinetic energy of nucleons inside the nucleus must be relatively small, otherwise they would not stay bound. The velocities of bound nucleons are thus non-relativistic.

Wave length of a non-relativistic nucleon confined inside the nucleus:

$$\lambda = \frac{\hbar}{p} = \frac{\hbar}{\sqrt{2mT}} \simeq \frac{197 \text{ MeV fm}}{\sqrt{2 \cdot 940 \cdot 8} \text{ MeV}} \simeq 1.6 \text{ fm} = 1.6 \times 10^{-13} \text{ cm}$$

Wave length of a hypothetic electron confined inside the nucleus:

$$\lambda = \frac{\hbar}{T} \simeq \frac{197 \text{ MeV fm}}{8 \text{ MeV}} \simeq 25 \text{ fm} = 2.5 \times 10^{-12} \text{ cm}$$

- The binding energy corresponds to a **wave length** of nucleons inside the nucleus. This wavelength is **less than 2 fm**, of the order of the nuclear size. It is thus plausible that a nucleus can contain nucleons with a **maximum kinetic energy T of 8 MeV**, or a maximum momentum p of 120 MeV.
- If, on the other hand, the nucleus would contain **electrons**, they would be relativistic and their wave length would be **$2.5 \times 10^{-12} \text{ cm}$, much bigger than the nuclear size**. The nucleus thus contains no bound electrons, they need a much larger volume to be contained. This is obviously in agreement with the findings of Geiger and Marsden.
- In the next video we will talk about the size and spin of nuclei.