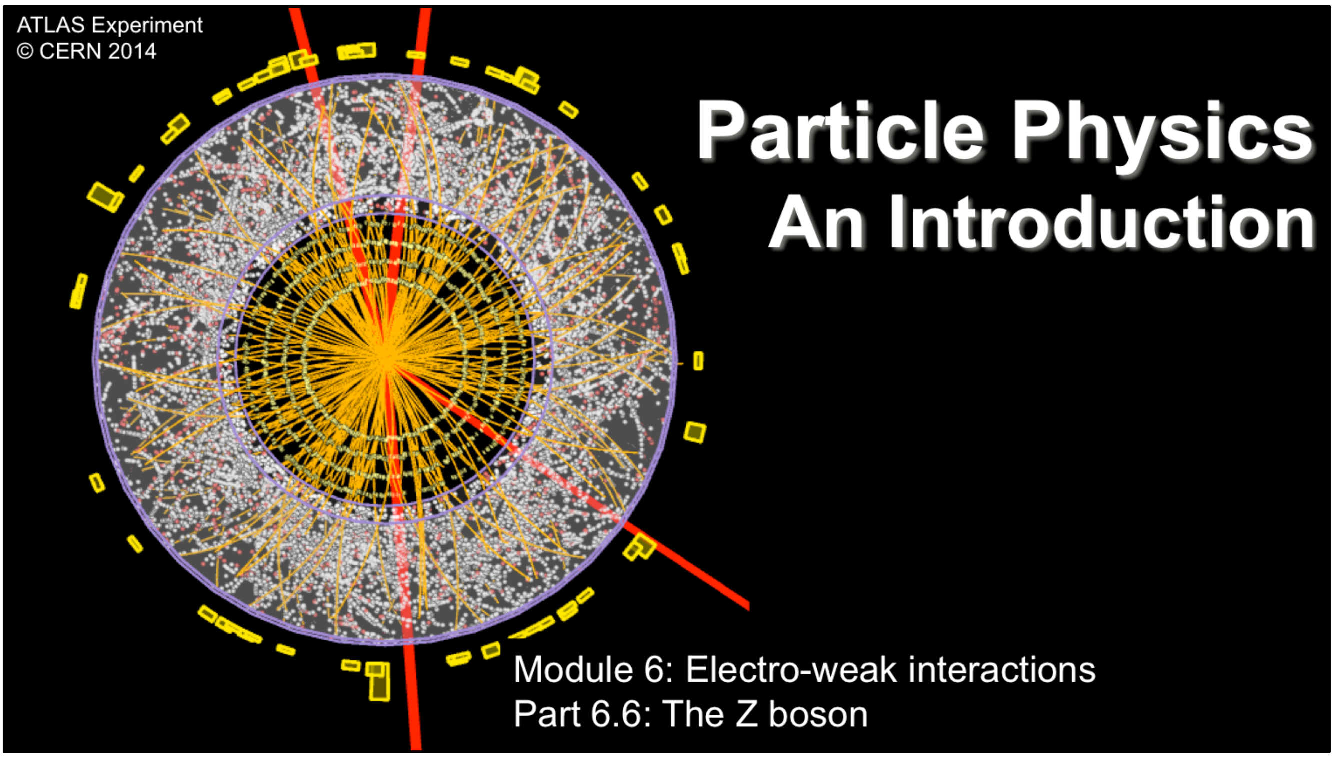
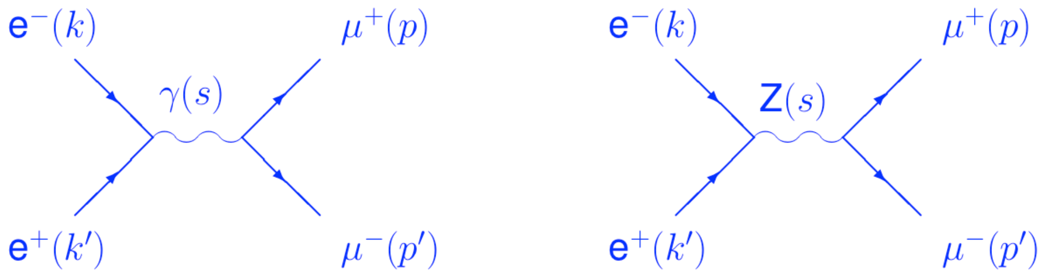



In this 6th module, we are discussing weak interactions.

In this 6th video we will talk about the properties of the Z boson, which transmits the neutral weak force and can be considered as the heavy brother of the photon.

After following this video you will know its properties:

- Its mass and the ways to measure it;
- Its couplings to matter.




$$\sigma_{\text{tot}}(e^+e^- \to \mu^+\mu^-) = \sigma_\gamma + \sigma_Z + \sigma_{\text{int}}$$

- In video 4.5 we have discussed the prototype process $e^+ e^- \rightarrow \mu^+ \mu^-$ for the production of **fermion pairs by electron-positron annihilation**.
- In addition to electromagnetic interactions as shown in the left graph, this process can be mediated by **neutral weak interactions** as in the right graph.
- Since initial and final states are the same, one must add the amplitudes. In the cross section we thus find three terms at lowest order. The first term, $\sigma_\gamma$,contains the square of the **photon exchange** amplitude, the second one, $\sigma_Z$, that of the **Z exchange** and the 3rd one, $\sigma_{int}$, the **interference** between the two.

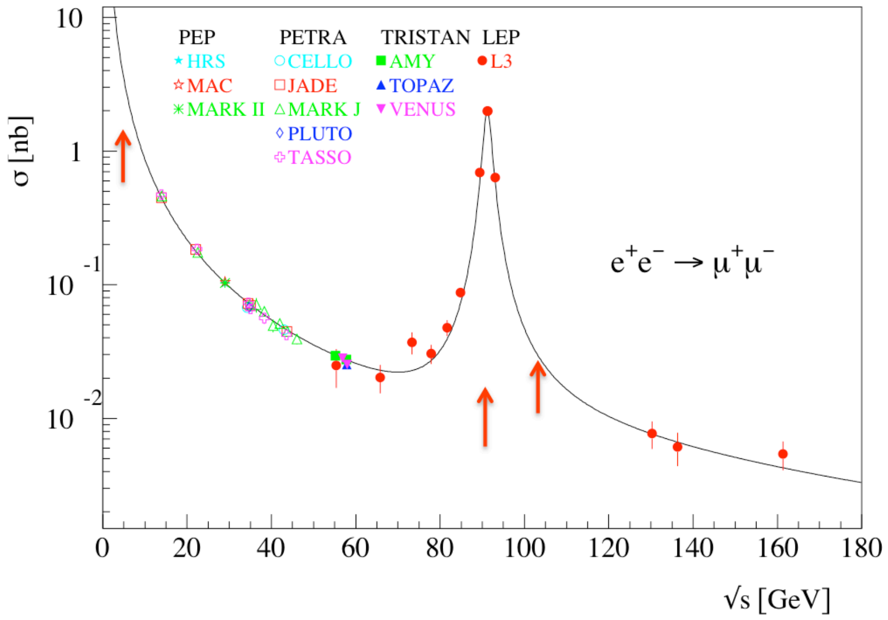
$$\frac{d\sigma_\gamma}{d\Omega} = \frac{\alpha^2}{4s}\left(1+\cos^2\theta\right) \quad ; \quad \sigma_\gamma = \frac{4\pi\alpha^2}{3s}$$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\left[A_0\left(1+\cos^2\theta\right) + A_1\cos\theta\right] \quad ; \quad \sigma = \frac{4\pi\alpha^2}{3s}A_0$$

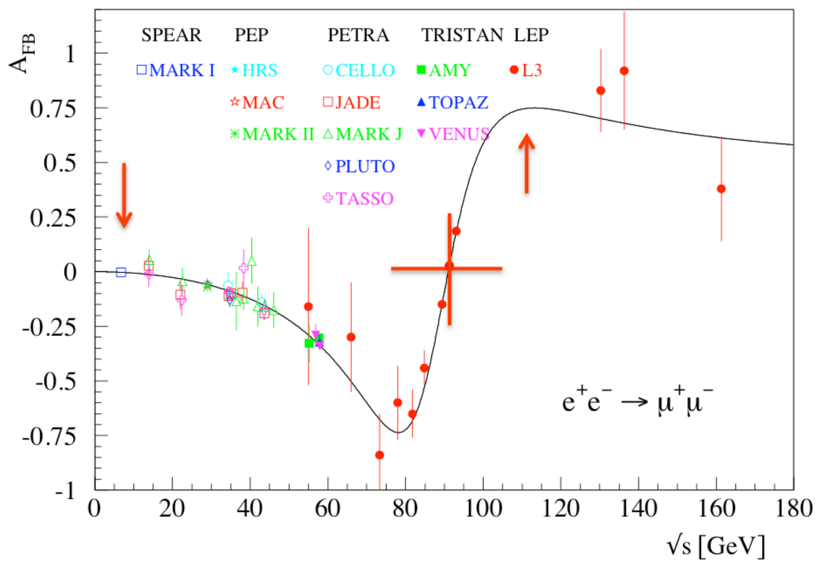
$$\begin{aligned} A_0 &= 1 + 2\Re(r)g_V^2 + |r|^2\left(g_V^2 + g_A^2\right)^2 \\ A_1 &= \phantom{1 +{}} 4\Re(r)g_A^2 + 8|r|^2 g_V^2 g_A^2 \end{aligned}$$

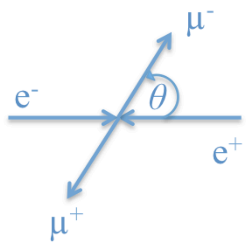
$$r = \frac{\sqrt{2}G_F}{e^2}\frac{sM_Z^2}{s - M_Z^2 + iM_Z\Gamma_Z} \quad ; \quad g_V = T_3 - 2Q\sin^2\theta_W \quad ; \quad g_A = T_3$$

- For the **photon** alone, we already quoted the cross section in video 4.5.
- The total **differential cross section**, which also takes into account the other two contributions, contains a term symmetric in the scattering angle $\vartheta$, like the one we had found for the photon. The scattering angle is the angle between the incoming $e^-$ and the outgoing $\mu^-$. The coefficient, $A_{0,}$ of this angular term now contains a new contribution due to weak interactions. There is also an additional, asymmetric term proportional to the coefficient $A_1$.
- The coefficients contain the real part and the square of a **resonance term**, *r*, which comes from the resonant production of Z bosons when the $e^+e^-$ total energy approaches the Z mass. It contains the Breit-Wigner function characteristic for the propagator of massive virtual bosons.
- The **coupling constants**, $g_V$ and $\mathbf{g_A}$, depend on the electric charge and the weak isospin of the matter particles involved in each vertex. Due to the value of the weak mixing angle, $\sin^2\vartheta_W \cong ¼$, $g_V$ has a small value for charged leptons, , $g_A$ is simply -½.
- At **low energies**, $\sqrt{s} << M_Z$, where $r \approx 0$ and the influence of weak interactions is negligible, we fall back onto the result found by electrodynamics, i.e. $A_0 = 1$ et $A_1 = 0$.
- The terms proportional to *Re(r)* are due to **electro-weak interference** and are important in the intermediate region, where neither the photon, nor the Z exchange are dominant.
- And finally, **close to the Z mass,** where the terms proportional to $|r|^2$ dominate, we find that the weak interaction is two orders of magnitude stronger than the electromagnetic one, due to the amplification by the resonance.




- The Z resonance manifests itself by an enormous **peak in the total cross section**, electro-weak interference causes a large **angular asymmetry** on the flanks of the resonance.
- At **low energies**, the cross section depends on the electromagnetic coupling, $\alpha$, and the square of the center-of-mass energy, $s$, like $\sigma=4\pi\alpha^2/3s$, the characteristic size and shape of purely electromagnetic interactions.
- As the energy approaches the mass of the **Z boson**, we see an enormous **peak** and weak interactions dominate the process, thanks to the resonant exchange of Z bosons that we may call “almost real” instead of virtual.
- Far **above the resonance**, the two processes add up and are of comparable size.




$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3A_1}{8A_0}$$

$$\sigma_F = \int_0^1 \frac{d\sigma}{d\Omega} d\cos\theta$$

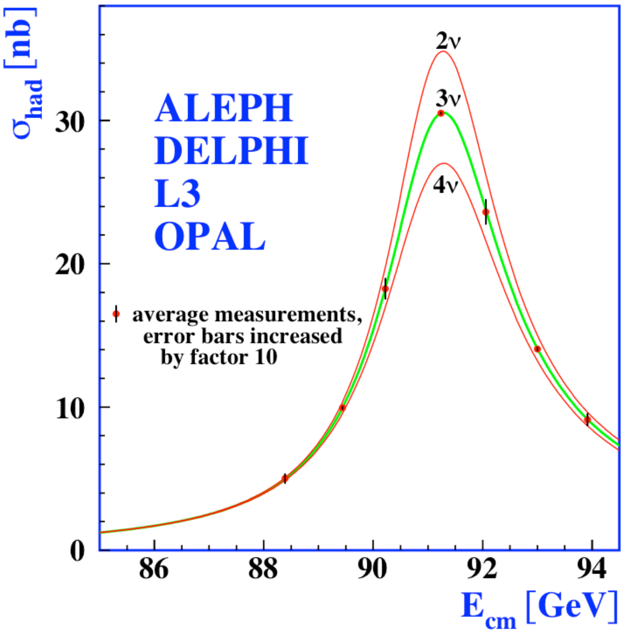
$$\sigma_B = \int_{-1}^0 \frac{d\sigma}{d\Omega} d\cos\theta$$

- The interference term introduces an **asymmetry** into the angular distribution, which we had already noted in video 4.5.
- To quantify it, one defines a **forward-backward asymmetry,** $A_{FB}$, comparing the rate of forward ($\vartheta < 90°$) and backward ($\vartheta > 90°$) scattering.
- This asymmetry is large and **negative below the resonance**, goes through zero at $s=M_Z^2$ and **becomes large and positive above the resonance.**

$$\sigma_Z(s) = \sigma_Z^{\max} \frac{s\Gamma_Z^2}{\left(s - M_Z^2\right)^2 + M_Z^2\Gamma_Z^2}$$

$$\sigma_Z^{\max} = 12\pi \frac{\Gamma_{Z\to e\bar{e}}\Gamma_{Z\to f\bar{f}}}{M_Z^2\Gamma_Z^2}$$

$$\Gamma_{Z\to f\bar{f}} = N_c^f \left(T_3^f - 2Q_f \sin^2\theta_f\right) \frac{G_F M_Z^3}{2\pi\sqrt{2}}$$

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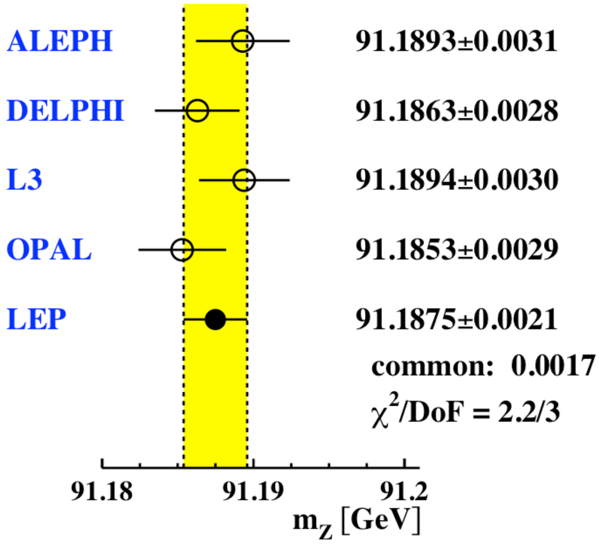
- In the immediate **vicinity of the Z resonance**, i.e. a few Z widths away from the Z mass, we can neglect both electromagnetic interactions and electroweak interference. For all $e^+e^-$ annihilations into a pair of point-like fermions, we then obtain a cross section with the peak given by $r^2$. In this region, the Z boson is quasi real.
- The **maximum of the cross section** can be expressed in terms of the decay width, $\Gamma_Z$, of a real Z boson.
- The **partial widths** of the Z decay into electrons and other fermions depend on their weak and electromagnetic charges, $T_3$ and $Q$, as shown. In addition, the number of colors, $N_C$, comes in for quark final states.
- The partial and total widths can be derived from the measured **shape and height of the cross section peak**, for all fermion species with a mass smaller than half the Z boson mass. This even includes neutrinos, not seen in the measured final state, as well as any other invisible particle. In this way, experiments have fixed the number of neutrino species with $m_\nu < M_Z/2$ to exactly 3, with high precision. There are thus **only three generations of light matter**.

$$\sigma_Z(s) = \sigma_Z^{\max} \frac{s\Gamma_Z^2}{\left(s - M_Z^2\right)^2 + M_Z^2\Gamma_Z^2}$$

$$\text{PDG}: \quad M_Z = (91.1876 \pm 0.0021)\,\text{GeV}$$

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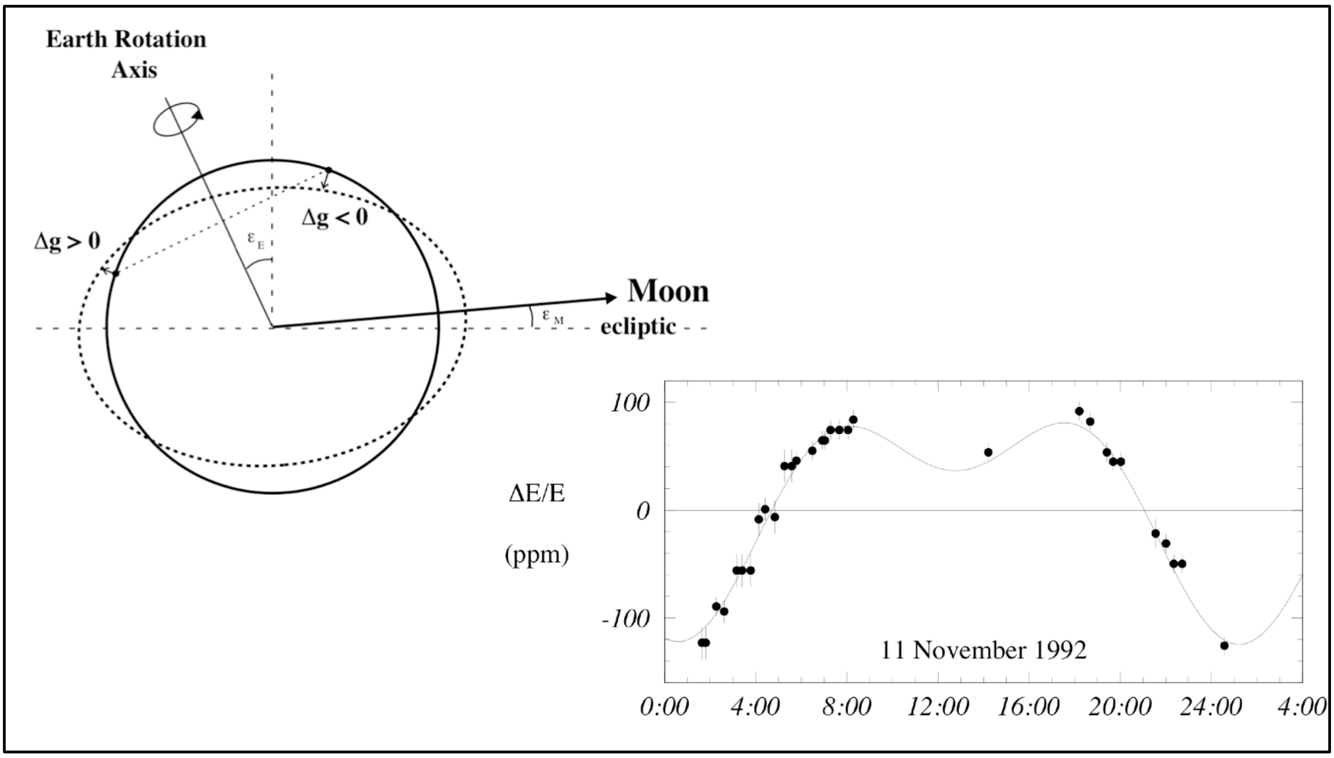
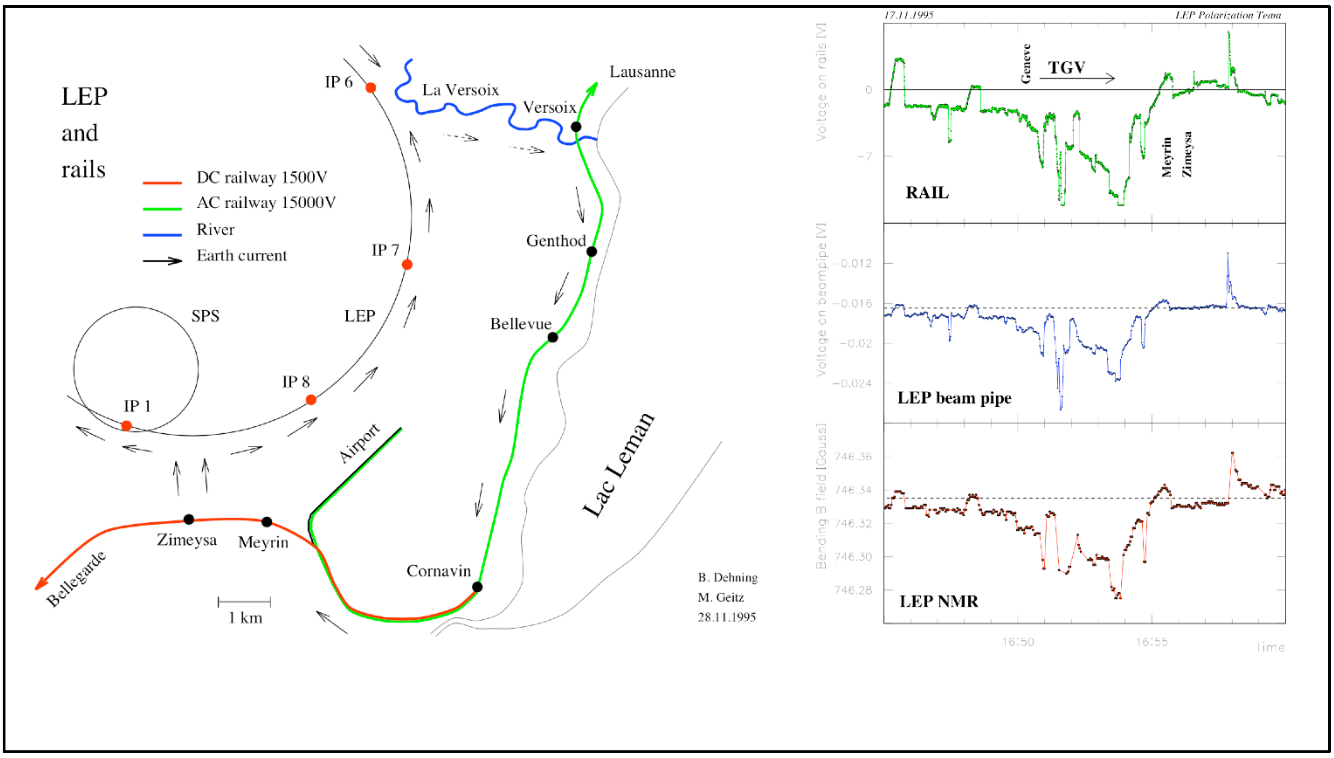
- The position of the peak determines the **mass of the Z boson** with high precision. Here we show the results of the four experiments at the CERN Large Electron Positron collider LEP, the predecessor of the LHC.
- The mass determination by the **resonance** method is precise to $2\times10^{-5}$.
- This accuracy would correspond to a measurement of **my own weight** to about 2g. That may not sound very impressive, but:
  - My lifetime is (hopefully) about $2 \times 10^9$s and I weigh about $10^5$ g;
  - The Z boson lifetime is $3\times10^{-25}$ s it weighs $1.6251\times10^{-22}$ g.
- At such a level of precision, **unexpected effects** influence the measurement.




As incredible as it may seem, the **phase of the moon** influences the measurement:

- Just like sea water, the Earth crust goes through high and low **tides** with a height difference of about 20cm. The tidal waves deform the LEP storage ring by some millimeters compared to its total length of 27km. At constant frequency, this changes the beam momentum; focalization amplifies the effect.
- Consequently, the **beam energy depends on the moon phase** at the level of 100ppm. This effect has been measured using resonant depolarization, a method to determine the energy of the circulating beam with an impressive precision. You see the result in this graph, which shows the beam energy over large portions of a tidal wave passage.




And even the passage of the high speed **TGV** train in near-by France influences the measurement:

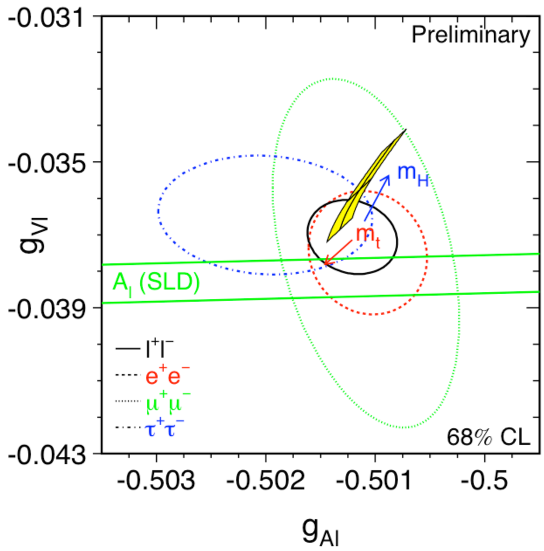
- Since the TGV is powered by a single phase DC current, its passage induces **parasitic currents** into the storage ring. Because of their hysteresis, the magnets remember the train schedule.
- This again leads to a slight **variations of the collider energy**.

- All of these incredible effects had to be taken into account to arrive at the final result for the **Z boson mass.**
- This should allow you to appreciate the impressive precision, which can be reached when measuring a mass through resonant production. The method remains a specialty of **electron-positron colliders**, where the point-like initial state allows an excellent definition of the kinematics.

$$g_V = T_3 - 2Q\sin^2\theta_W \quad ; \quad g_A = T_3$$

$$g_V^l = -0.03772 \pm 0.00041 \quad (l = \mathrm{e}, \mu, \tau)$$
$$g_A^l = -0.50117 \pm 0.00027 \quad (l = \mathrm{e}, \mu, \tau)$$

$$g_V^\nu = g_A^\nu = 0.50085 \pm 0.00075 \quad (\nu = \nu_e, \nu_\mu, \nu_\tau)$$




- The **couplings** of the Z boson to leptons, $g_V$ and $g_A$, are also measured using the total and differential cross sections, as well as the polarization dependence of $e^+e^-$ annihilation into leptons. One finds confirmation of our choice for their weak isospin, as up and down members of a doublet. One also finds that **all generations have the same couplings** to an accuracy of a few times $10^{-3}$.
- For **charged leptons**, the measured values of $g_V$ are close to zero, and $g_A \approx -1/2$, as we expect.
- For **neutrinos**, one finds $g_V = g_A \approx ½$, as we expect for fermions with weak isospin "up" and electric charge zero.

In the next video, we will talk about purely weak reactions, that is to say, weak decays of quarks.