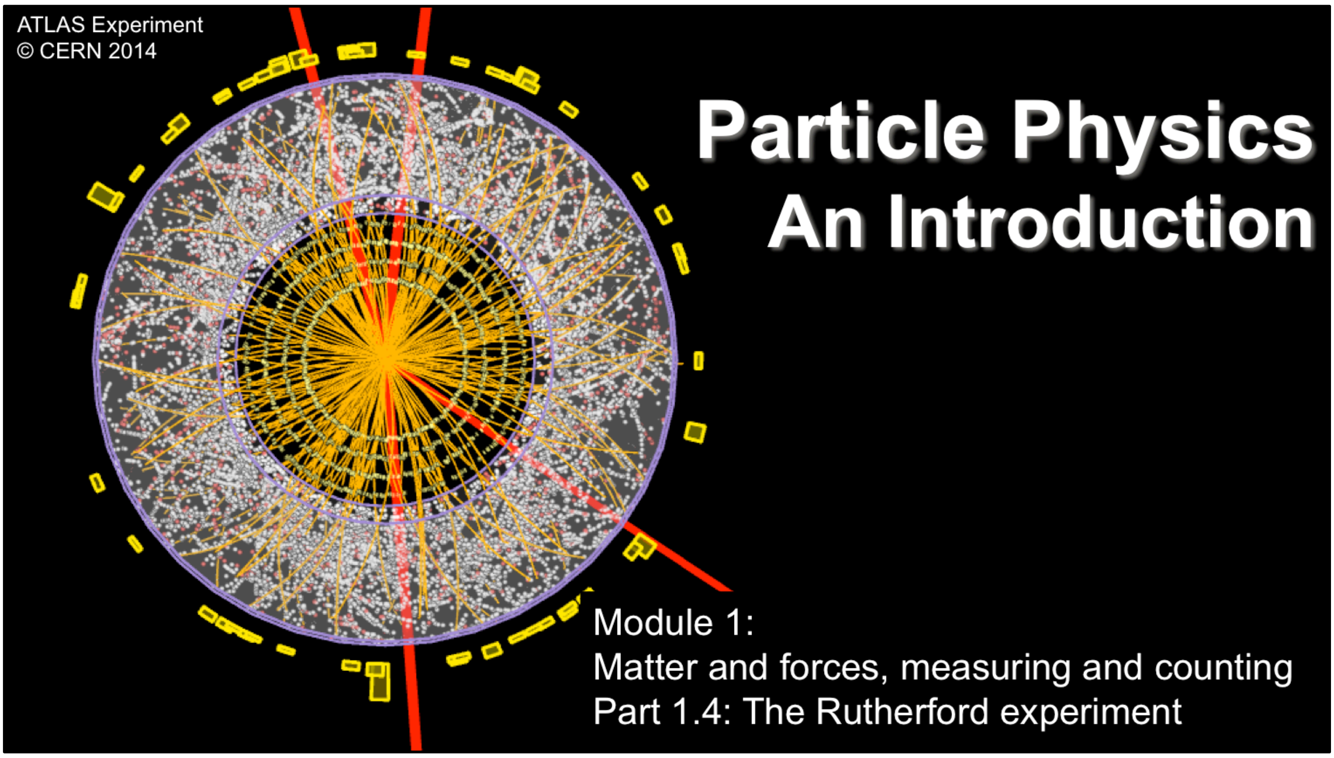
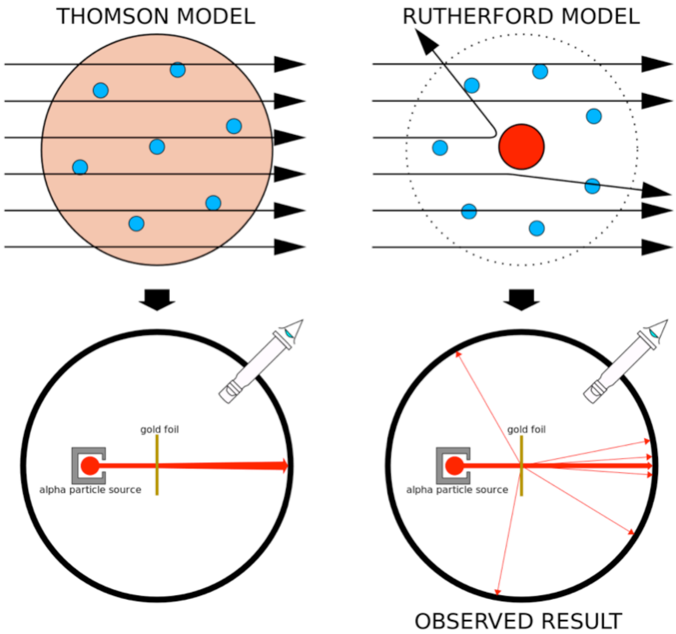


During this first module, we are introducing the objects studied in particle physics, namely matter, forces and space-time.

In this 4th video we give an example for calculating and measuring a cross section, for the so-called Rutherford experiment. This is the scattering process which demonstrated the existence of the atomic nucleus as we know it today.

The goals of this video are to show you how:

- To apply the concept of cross section in a process that can be treated in a semi-classical manner;
- To compute an interaction rate from the characteristics of an experiment.



- We will use a **seminal experiment** to demonstrate how the cross section serves to understand the unknown subatomic structure of a target. **Geiger and Marsden** measured in 1909 the angular distribution of α particles, i.e. $^{4}He^{++}$, scattering off thin foils of a heavy metal like gold.
- The preferred model of the time, due to **Thomson** who had discovered the electron in 1897, described the atom as a homogeneous positive substance in which electrons are embedded. It thus has a very modest charge density. In this case the alpha particles, being much smaller than the atoms, should be able to penetrate the foil with only minor perturbation.
- If on the other hand the atom consists of a **tiny, dense and positively charged nucleus**, surrounded by electrons to fill the atomic volume, as proposed by **Rutherford**, large scattering angles would be possible.

Newton's law:

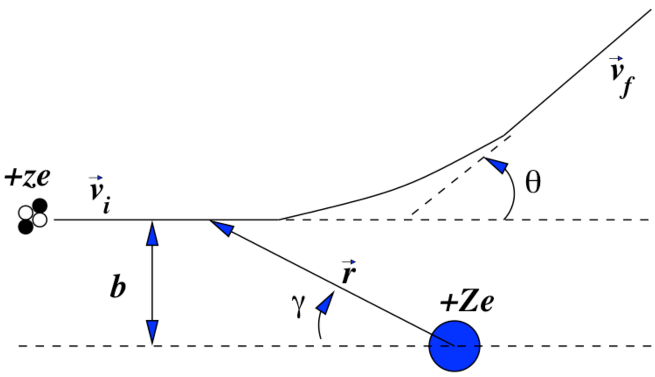
$$\vec{F} = m\frac{d\vec{v}}{dt} = m\frac{d\vec{v}}{d\gamma}\frac{d\gamma}{dt} = \frac{(ze)(Ze)}{4\pi}\frac{\hat{r}}{r^2}$$

Conservation of kinetic energy:

$$\frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 \quad ; \quad |\vec{v}_i| = |\vec{v}_f|$$

Conservation of angular momentum:

$$L = mr^2\frac{d\gamma}{dt} = mv_i b$$



As we have seen in the previous video, the **classical calculation** of the cross section requires to find a relation between the impact parameter *b* and the scattering angle *ϑ*. We will calculate this relation for non-relativistic velocities of the incident alpha particle.

We base the calculation on three laws:

- The first one is **Newton's law** which relates the rate at which the momentum changes to the Coulomb force between projectile and target nucleus. It is quoted here using the variables indicated in the sketch on the right.
- The second law is the **conservation of kinetic energy** characteristic for elastic scattering on a heavy target, which does not recoil when hit by the projectile.
- The third one is the **conservation of angular momentum**, which introduces the angular velocity *dγ/dt*.

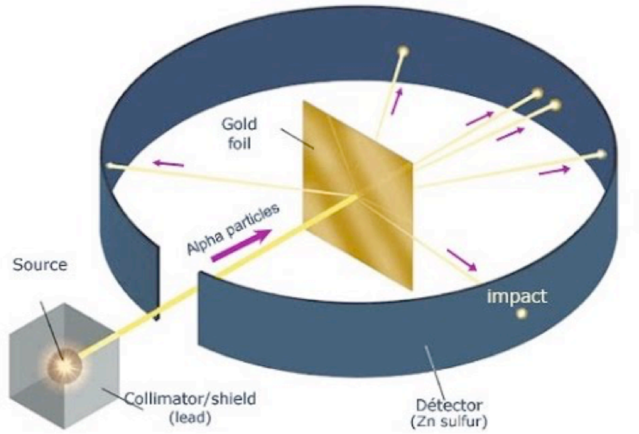
Combining these three equations, we obtain the relation between *b* and *ϑ* which we are looking for. If you are interested in the details of the calculation, we refer you to video 1.4a.

$$\frac{d\sigma(\theta)}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

$$b = \frac{zZe^2}{8\pi E_{kin}}\cot\frac{\theta}{2}$$

$$db = \frac{zZe^2}{8\pi E_{kin}}\left(\frac{-1}{2\sin^2\frac{\theta}{2}}\right)d\theta$$

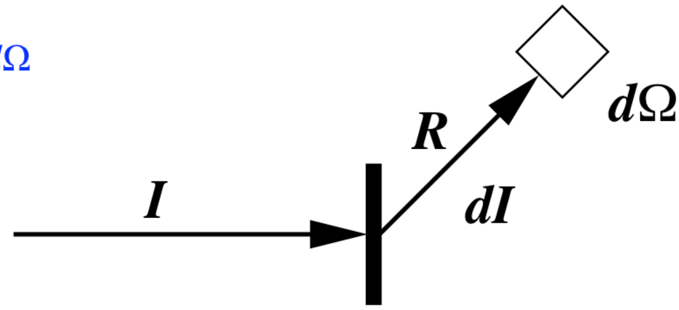
$$\boxed{\frac{d\sigma(\theta)}{d\Omega} = \left(\frac{zZe^2}{8\pi E_{kin}}\right)^2\frac{1}{4\sin^4\frac{\theta}{2}}}$$



- Inserting our result into the general relation for the classical cross section, we obtain the **differential cross section** for a Coulomb interaction with a point-like heavy target.
- The cross section is proportional to the **square of the product of the two charges** *(zZe^2)*. This factor determines the order of magnitude of the cross section.
- The cross section is strongly peaked in the forward direction, according to *$1/\sin^4(\vartheta/2)$,* and inversely proportional to the square of the kinetic energy of the projectile. These factors are due to the propagator of the photon exchanged between the two reaction partners.
- These basic properties have indeed been experimentally established by Geiger and Marsden, **counting the impact of the scattered α particles** on a screen covered by zinc sulfite. This molecule emits a small flash of light when hit by a charged particle. The experimenters counted these flashes by watching the screen through a microscope.

Is it realistic to count scatters by eye in such a set-up? The small video 1.4b will convince you by actually calculating the rate.

$$-\frac{dI}{I} = n\frac{d\sigma}{d\Omega}d\Omega$$



$$\frac{\text{\# interactions}}{\text{sec}} = \left(\frac{\text{\# projectiles}}{\text{sec}} \times \frac{\text{\# targets}}{\text{m}^2}\right) \times \text{cross section}$$
$$= \left(\frac{\text{\# projectiles}}{\text{sec m}^2} \times \text{\# targets}\right) \times \text{cross section}$$

The procedure to measure a cross section is thus clear:

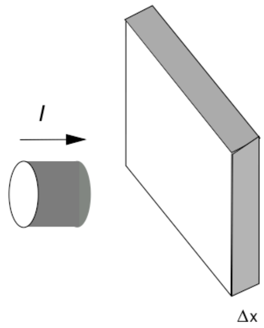
- One must count the **rate of interactions** per second. For that we need a **detector** at a distance *R* covering a surface $R^2 d\Omega$.
- One must then **normalize** the counting rate by the maximum possible interaction rate to obtain a probability. The maximum rate is obtained when every projectile interacts with the target, it is thus the product of the incident beam rate and the surface density of the target particles. It can also be expressed by the flux of projectiles multiplied by the number of target particles covered by the incident beam.
- The proportionality factor between the counting rate and the maximum counting rate is the **cross section**.

Luminosity:

$$\frac{dI}{d\Omega} = (\rho\,\Delta x\, I)\,\frac{d\sigma(\theta,\phi)}{d\Omega}$$

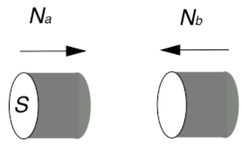
- Laboratory system:

$$L = n\,I = \rho\,\Delta x\; I$$



- Centre of mass system with two beams a and b:

$$L = \frac{N_a N_b f}{S}$$



- The proportionality factor between counting rate and cross section is called **luminosity**.
- In the **laboratory frame**, where the target is at rest, it is given by the rate of the incoming beam times the surface density of target particles. If we want to use the volume density of the target, we multiply by the target thickness.
- In the **center-of-mass frame**, where two beams collide head-on like in a collider, the luminosity is proportional to the population of the two beams, N_a and N_b, and to the frequency f at which they cross. It is inversely proportional to the common surface S of the two beams.
- Counting rate and luminosity thus depend on the details of the experiment we are conducting. Their ratio, the cross section, characterizes the physical process independent of these details.

Until now, we have only considered processes using the **tool kit of classical physics**. But we already know that this will not suffice. In the next video we will see how **quantum physics** approaches scattering processes. Further on we will introduce an indispensable tool of particle physics: **Feynman diagrams**.