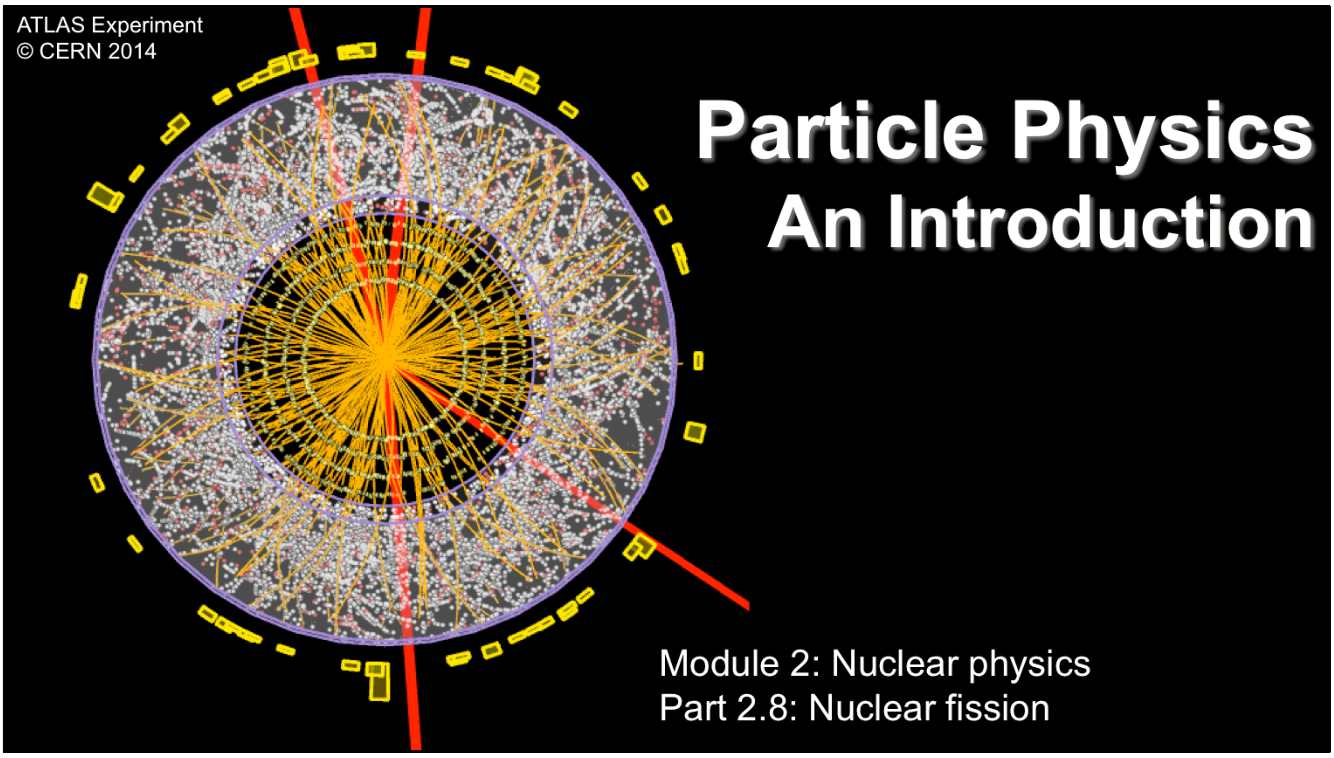
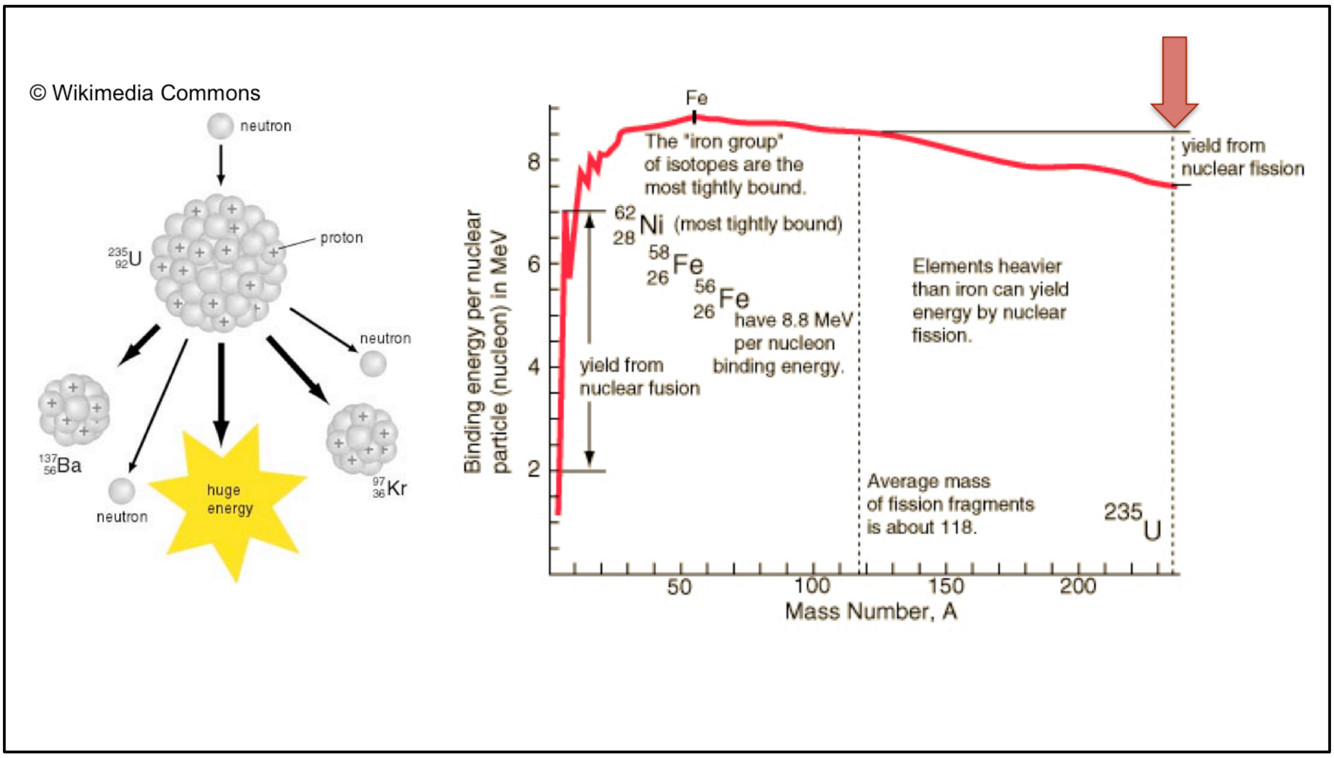


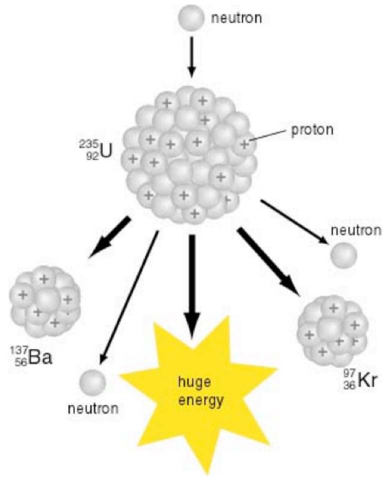
During this second module, we deal with nuclear physics.

In this eighth video we will explain how to transform a part of nuclear binding energy into heat by nuclear fission.

After viewing this video you will know the nuclear fission mechanism and its necessary conditions.



- A very important application of nuclear physics is the transformation of nuclear binding energy into thermal and then electrical energy by the **fission mechanism.**
- **Low energy neutrons** can penetrate inside the nucleus, since they are not subject to Coulomb repulsion. This way, they increase the nuclear mass, the excited nucleus may break up into two daughter nuclei of smaller mass.
- This fragmentation of a heavy nucleus into two medium mass fragments (and other accessory products) is called **nuclear fission**.
- The α decay discussed in video 2.4 is an extreme example of a very asymmetric fission. It takes place spontaneously.
- More important is the **fission induced by the absorption of a neutron**, but nuclear fission stays in the domain of **heavy mother nuclei**.
- This is due to the shape of the **binding energy per nucleon** as a function of mass number, first shown in video 2.1. Beyond the iron group, it decreases steadily, such that binding energy can ultimately be gained by splitting a heavy nucleus into two in a more or less symmetric way.

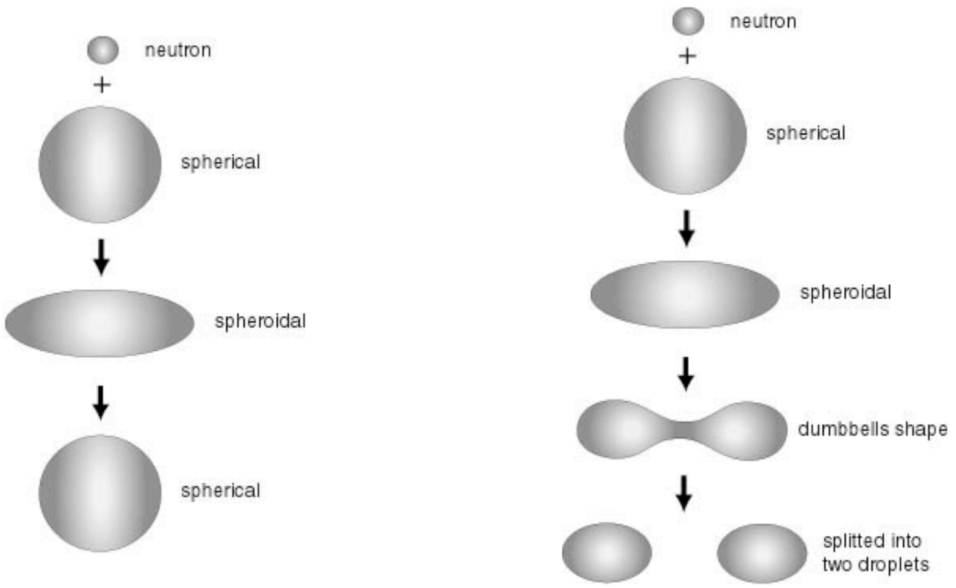


$$\frac{\mathrm{BE}}{A}(^{235}\mathrm{U}) \simeq -7.5\ \mathrm{MeV}$$

$$\frac{\mathrm{BE}}{A}(^{137}\mathrm{Ba}, ^{97}\mathrm{Kr}) \simeq -8.4\ \mathrm{MeV}$$

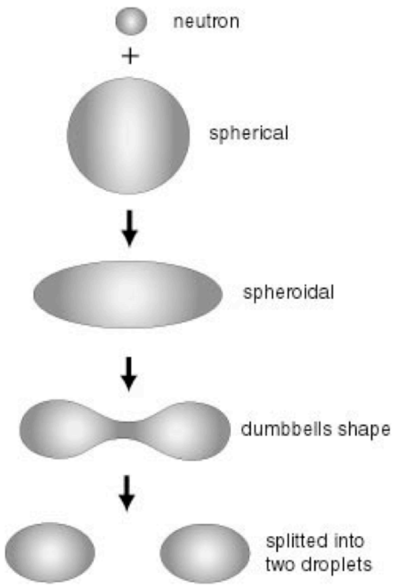
- 1g ^{235}U contains $\simeq 3 \times 10^{21}$ nuclei
- Fission yields 10^{11} Joules, roughly 1 MW-day
- Burning 1 t of coal yields 0.36 MW-day

- Thus a **less tightly bound heavy nucleus** can gain binding energy by splitting into two lighter, **more tightly bound** ones
- As an example, the binding energy per nucleon is -7.5 MeV/nucleon for ^{235}U and -8.4 MeV on average for the two daughter nuclei. The energy released by fission is typically 0.9 MeV/nucleon. 1g of ^{235}U contains N_A/A atoms, with Avogadro's number $N_A = 6 \times 10^{23}$ atoms/mol. 1g thus contains ~3×10^{21} nuclei.
- The complete fission of all of these would release the impressive energy of 10^{11} Joules, about **1 MegaWatt-day**, ignoring all efficiencies involved.
- To give you an idea for comparison, burning 1 ton of coal liberates about 0.36 MW-day, ignoring efficiencies again. Thus our 1g of ^{235}U contains as much in nuclear energy as 3 tons of coal contain in chemical energy.
- We can understand nuclear fission qualitatively and quantitatively using the liquid drop model of the nucleus introduced in video 2.3.



Let's first look at the process qualitatively:

- The liquid drop model presumes a spherical shape of the nucleus. But for a heavy nucleus, a small **external perturbation**, like an incident neutron, can produce surface waves leading to a change in shape. The drop may thus extend into a **spheroidal shape**.
- If the **perturbation is small enough**, the nucleus will thus be left excited and return to its ground state, by emitting a photon for example. This process is called **radiative neutron capture**.
- If the **perturbation is sufficiently large**, Coulomb repulsion along the axis of the spheroid can split the drop into two droplets, by what is called **induced fission**.



$$\mathrm{BE} = \underbrace{-a_1 A}_{\text{Volume}} + \underbrace{a_2 A^{\frac{2}{3}}}_{\text{Surface}} + \underbrace{a_3 \frac{Z(Z-1)}{A^{\frac{1}{3}}}}_{\text{Coulomb}} + \underbrace{a_4 \frac{(N-Z)^2}{A}}_{\text{Asymmetry}} \pm \underbrace{a_5 A^{-\frac{3}{4}}}_{\text{Pairing}}$$

- Sphere with radius R deformed to ellipsoid of same volume, half-axes a and $b = c$:

$$a = R(1+\epsilon) \quad ; \quad b = R/(1+\epsilon)^{1/2}$$

- Surface and Coulomb energy:

$$a_2 A^{\frac{2}{3}} \to a_2 A^{\frac{2}{3}}\left(1 + \frac{2}{5}\epsilon^2\right) \quad ; \quad a_3 \frac{Z^2}{A^{\frac{1}{3}}} \to a_3 \frac{Z^2}{A^{\frac{1}{3}}}\left(1 - \frac{1}{5}\epsilon^2\right)$$

- Difference in binding energy:

$$\Delta = \mathrm{BE(sphere)} - \mathrm{BE(ellipsoid)} = \frac{1}{5}\epsilon^2 A^{\frac{2}{3}}\left(a_3 \frac{Z^2}{A} - 2a_2\right)$$

- $Z^2 < 47A$: spherical shape generally stable

Now let us calculate under which conditions this may happen:

- A sphere of radius *R* will turn into a spheroid of constant volume due to the incompressibility of nuclear matter. We denote its half-axes by *a*, *b* = *c*. In terms of a small deformation ε, they transform into *b*-ε and *a*+ε .
- For the binding energy, the **volume term** stays constant, but the **surface and Coulomb energy** are different.
- The **deformation increases the surface energy, but decreases the Coulomb term** . The gain or loss in binding energy will depend on the relative importance of the two.
- If ***Δ > 0***, the spherical nucleus is more tightly bound and thus stable against a small perturbation. If ***Δ < 0***, the spherical nucleus is less tightly bound and can undergo fission.
- Putting in the typical values for the coefficients $\boldsymbol{a_2 \sim 16.8}$ and $a_3 \sim 0.72$, one obtains stability for $\boldsymbol{Z^2/A < 47}$.There are small quantum corrections to this classical approach, which come from the two last terms, but they do not change this conclusion.
- We expect on the contrary that nuclei with $\boldsymbol{Z^2 > 47\,A}$ are highly instable against small perturbations and can be easily induced to fission.
- Heavy nuclei have ***Z< A/2, thus all*** $\boldsymbol{Z^2 < 47\,A}$**.** The spherical shape corresponds to maximum binding. But even in that case, fission can be energetically favorable if the **sum of binding energies of the daughter nuclei** is favorable with respect to

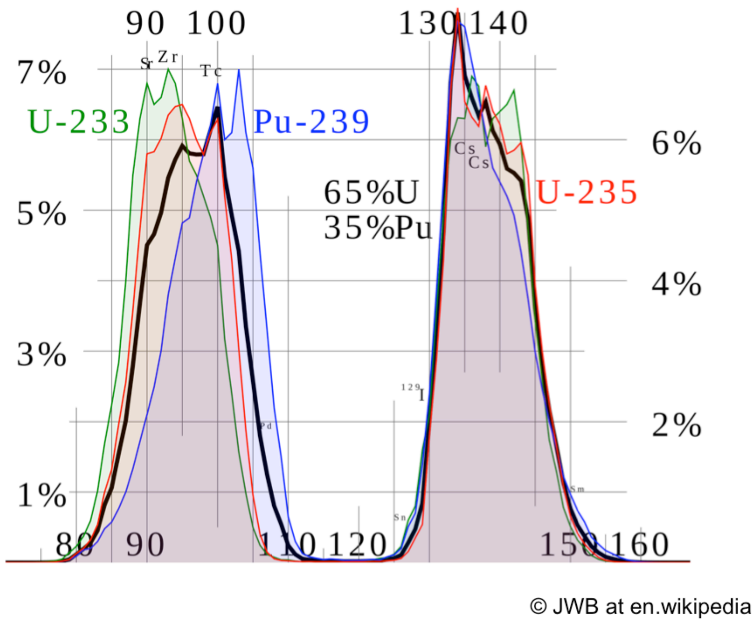
$$\text{BE} = \underbrace{-a_1 A}_{\text{Volume}} + \underbrace{a_2 A^{\frac{2}{3}}}_{\text{Surface}} + \underbrace{a_3 \frac{Z(Z-1)}{A^{\frac{1}{3}}}}_{\text{Coulomb}} + \underbrace{a_4 \frac{(N-Z)^2}{A}}_{\text{Asymmetry}} \pm \underbrace{a_5 A^{-\frac{3}{4}}}_{\text{Pairing}}$$

Gain in binding energy per symmetric fission:

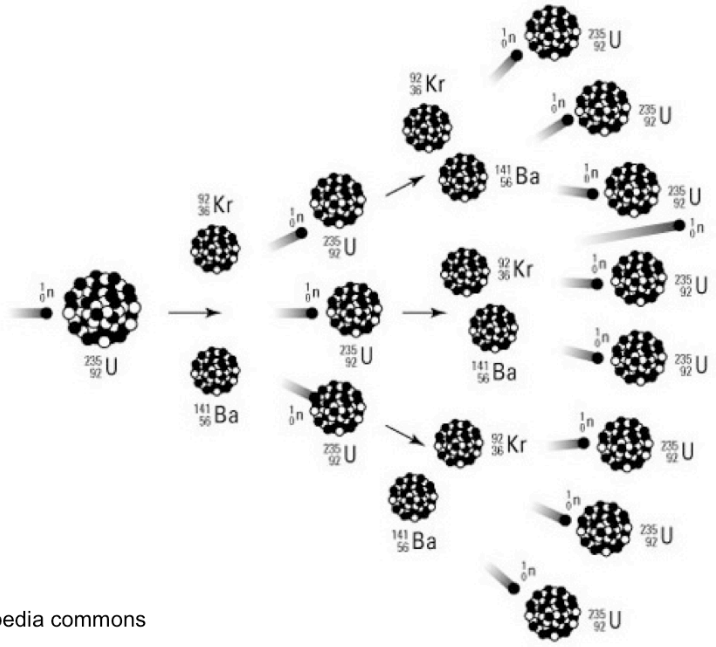
$$\Delta = \text{BE}(A,Z) - 2\text{BE}(A/2, Z/2) = a_2 A^{\frac{2}{3}}\left(1 - 2^{\frac{1}{3}}\right) + a_3 \frac{Z^2}{A^{\frac{1}{3}}}\left(1 - 2^{-\frac{2}{3}}\right) \simeq 0.27 A^{\frac{2}{3}}\left(-16.5 + \frac{Z^2}{A}\right)$$

$Z^2 < 16.5A$	: daughter nuclei more tightly bound than mother nucleus
$16.5A < Z^2 < 47A$	: mother nucleus stable, but fissionable; activation energy $\simeq 6$ to 8 MeV ($A \simeq 240$), induced fission by energetic neutron
$47A < Z^2$	: spontaneous fission by thermal neutron

- Let us calculate this case for a **symmetric fission**, assuming spherical daughter nuclei for simplicity. *A* and *Z* of the parent nucleus must thus be even.
- We again ignore the quantum terms of the Bethe-Weizsäcker formula. The energy difference between parent and daughters is denoted by Δ. With typical coefficients we obtain $\Delta \cong 0.27\, A^{2/3}\,(-16.5 + Z^2/A)$. Thus for **$Z^2 < 16.5$ A**, the two **daughter nuclei are more tightly bound** than the parent nucleus.
- We thus see that for the region **16.5 A < Z^2 < 47 A**, the spherical shape itself may be stable against fission, but it remains energetically favorable for the parent nucleus to split into two smaller ones **when sufficiently perturbed**.
- Beyond that zone, even a **minor perturbation** by a thermal neutron will suffice to cause fission.



- Let us consider **uranium isotopes** as an example.
- ^{238}U is an **even-even** nucleus and thus more tightly bound than ^{235}U, which is **odd-even**. We will thus need a smaller perturbation to induce fission with ^{235}U.
- The **activation energy** is about 5 MeV for ^{235}U and 6 MeV for ^{238}U.
- When capturing a neutron, **^{235}U** becomes an even-even nucleus. This transformation releases **~6.5 MeV binding energy** which is sufficient to activate its fission.
- For **^{238}U** a neutron capture changes the nucleus to a less tightly bound odd-even one. This will be a less exothermal process, it releases only **4.8 MeV** out of the necessary 6 MeV. Thus an **additional 1.2 MeV per neutron** must be provided to induce fission.
- The fragments are often rather asymmetric. One observes a grouping of the **daughter nuclei around *A*~95 and *A*~140,** for a reason yet to be understood. The figure shows the **mass spectrum** of daughter nuclei from the fission of **^{233}U, ^{235}U, ^{239}Pu,** as well as a mixture. The mass spectra are similar.



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$$k = \frac{\#\text{neutrons}\,(n+1)}{\#\text{neutrons}\,(n)}$$

- $k < 1$: under-critical
- $k = 1$: critical
- $k > 1$: over-critical

- After the fission process, the daughter nuclei are often in an **excited state**. They turn into a stable isotope by **neutron emission**. Neutrons are thus abundant among the fission products.
- They can cause a **chain reaction** of nuclear fission processes. ^{235}U produces on average 2.5 neutrons per fission. These neutrons can in turn induce new fission reactions and so forth.
- To see if a chain reaction can be maintained, we consider the **ratio *k* of the number of neutrons** produced in step *n+1* to the one in step *n* of the chain.
- If ***k*<1**, there are not enough neutrons to sustain a chain reaction. The process will stop by itself.
- If ***k*=1**, there are just enough neutrons to maintain a chain reaction with a constant number of fissions in each step. This is the ideal situation for a nuclear fission reactor.
- If ***k*>1**, more and more neutrons will be produced at each stage, thus leading to an exponential growth of the number of fission reactions. One searches such a situation for the production of nuclear weapons.

In the next video, we will talk about how to use the fission process in nuclear power plants.