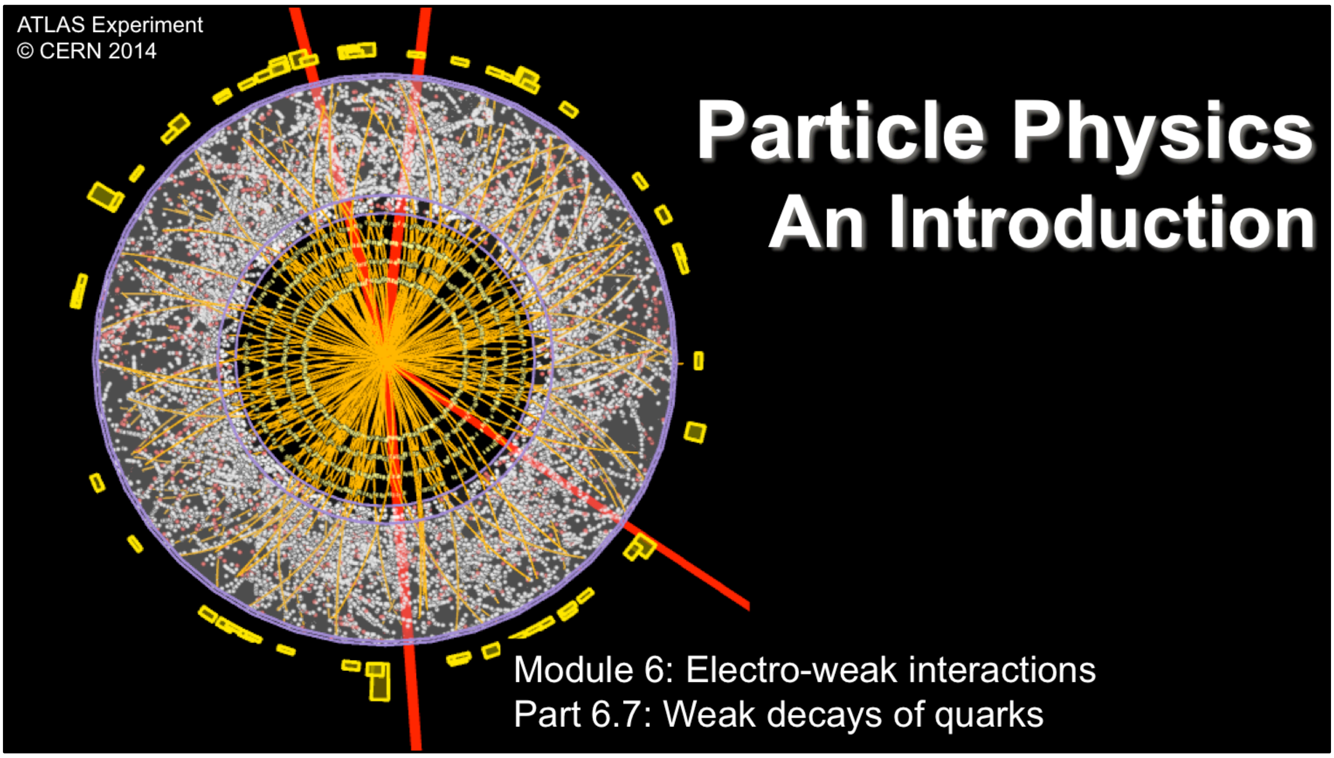


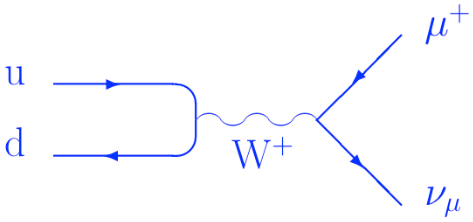
In this sixth module, weak interactions are discussed.

In this 7th video, we talk about quark decays mediated by weak interactions.

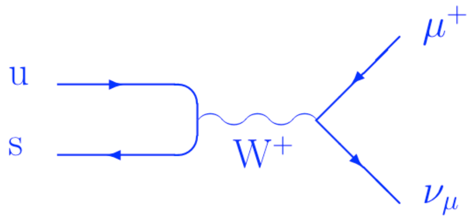
After following this video, you will know:

- The quark decay mechanism, which is analogous to muon decay discussed in video 6.4;
- The quantum mixture of quark states which interacts with W bosons;
- The Cabibbo-Kobayashi-Maskawa matrix, which describes this mixture in the Standard Model.

$\pi^+ \to \mu^+\nu_\mu$



$K^+ \to \mu^+\nu_\mu$

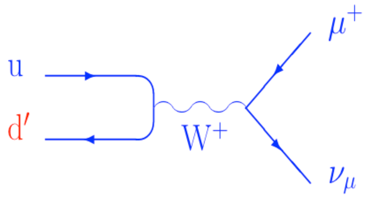


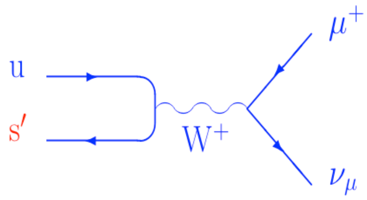
$$\frac{\Gamma(K^+ \to \mu^+\nu_\mu)}{\Gamma(\pi^+ \to \mu^+\nu_\mu)} = \frac{\tau_{\pi^+}}{\tau_{K^+}}\frac{BR(K^+ \to \mu^+\nu_\mu)}{BR(\pi^+ \to \mu^+\nu_\mu)} = \frac{2.60\times 10^{-8}}{1.23\times 10^{-8}} \times \frac{63.5\%}{99.99\%} \simeq \mathcal{O}(1) \quad \text{Experiment}$$

$$\frac{\Gamma(K^+ \to \mu^+\nu_\mu)}{\Gamma(\pi^+ \to \mu^+\nu_\mu)} \simeq \left(\frac{m_{K^+}}{m_{\pi^+}}\right)^5 \simeq \left(\frac{494}{140}\right)^5 \simeq \mathcal{O}(100) \quad \text{Phase space}$$

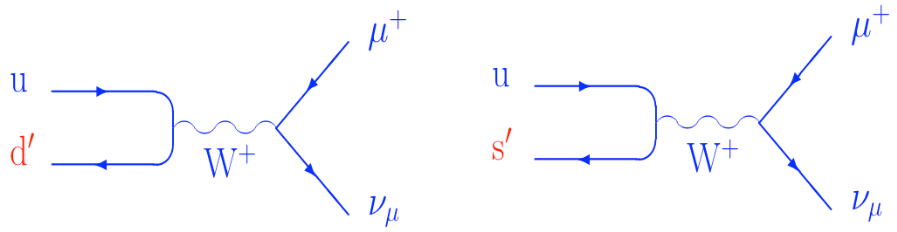
- The **couplings of quarks** to the $W^\pm$ privilege those between members of the same generation, but they also exist across generation boundaries. Pion decay is an example for the dominant case, $\pi^+ \rightarrow \mu^+\nu_\mu$, an example of an inter-generation decay is $K^+ \rightarrow \mu^+ \nu_\mu$.
- In the **Feynman diagrams** shown here, we have again replaced incoming antiquarks by outgoing quarks.
- The **ratio of the two partial rates** is measured to be of order 1. This experimental result shows that kaon decay is very much disfavored.
- The kaon **phase space** is large compared to that of the pion, due to its large mass. Neglecting muon and neutrino masses, the decay width is proportional to the parent particle mass to the fifth power. The ratio is thus expected to be 100 times larger.
- It must be concluded that **inter-generation interactions are disfavored** compared to intra-generation interactions, by at least a factor of 100 in the rate, or a factor of 10 in amplitude.

	Spin	# bar.	# lept.	Q elec.	T, T_3 weak[1]	C strong
Leptons:						
ν_L, ν_M, ν_H	$1/2$	0	$+1$	0	$1/2, +1/2$	0
e^-, μ^-, τ^-	$1/2$	0	$+1$	-1	$1/2, -1/2$	0
Quarks:						
u, c, t	$1/2$	$+1/3$	0	$+2/3$	$1/2, +1/2$	R, G, B
d, s, b	$1/2$	$+1/3$	0	$-1/3$	$1/2, -1/2$	R, G, B





- In 1963, N. **Cabibbo** proposed a way to describe these processes. States, which interact with the $W^{\pm}$, are not the quarks which are listed in the periodic table of matter particles presented in Module 1.
- The particles, which interact with the $W^{\pm}$, are the **mixtures (d', s', b')**, quantum superpositions of the real quarks d, s and b.



$$\begin{pmatrix} \mathrm{u} \\ \mathrm{d}' \end{pmatrix}_L \begin{pmatrix} \mathrm{c} \\ \mathrm{s}' \end{pmatrix}_L \begin{pmatrix} \mathrm{t} \\ \mathrm{b}' \end{pmatrix}_L$$

$$\mathrm{d}' = \mathrm{d}\cos\theta_C + \mathrm{s}\sin\theta_C \quad ; \quad \mathrm{s}' = -\mathrm{d}\sin\theta_C + \mathrm{s}\cos\theta_C$$

- The states d', s' and b', are **not eigenstates of the mass operator**, so no real particles, which evolve in space-time, but they come from a rotation in flavor space. Since this rotation should not change the number of fermions, the operator must be unitary.
- For **two generations**, the description is simplified; in the formula we symbolically substituted the wave function with the name of the quark. The angle of rotation between the first two generation is called the **Cabibbo angle**, ϑ_C.
- The mixing corresponds to a modification of the **weak charged quark current**: a small strange quark amplitude adds to the down quark wave function, and vice versa.

$$\begin{pmatrix} \mathrm{u} \\ \mathrm{d}' \end{pmatrix}_L \begin{pmatrix} \mathrm{c} \\ \mathrm{s}' \end{pmatrix}_L \begin{pmatrix} \mathrm{t} \\ \mathrm{b}' \end{pmatrix}_L$$

$$\mathrm{d}' = \mathrm{d}\cos\theta_C + \mathrm{s}\sin\theta_C \quad ; \quad \mathrm{s}' = -\mathrm{d}\sin\theta_C + \mathrm{s}\cos\theta_C$$

$$\frac{\Gamma(\mathrm{K}^+ \to \mu^+\nu_\mu)}{\Gamma(\pi^+ \to \mu^+\nu_\mu)} \simeq \left(\frac{m_{K^+}}{m_{\pi^+}}\right)^5 \tan^2\theta_C$$

As a consequence, the **amplitudes of the weak interactions** of quarks change:

- For **dominant interactions** of the W with the u-d current, for example, the rotation induces a factor cos ϑ_C in the amplitude.
- For **disfavored interactions**, such as those with a u-s current, a factor sin ϑ_C appears.
- The **relative rates** of the two types of interactions are thus reduced by a factor $\tan^2 \vartheta_{C.}$ From the observed rate reduction with respect to phase space, one finds a Cabibbo rotation angle $\vartheta_C \cong 13°$.

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_L = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$V_{\mathrm{CKM}} = \begin{pmatrix} \boxed{0.97427 \pm 0.00014} & \boxed{0.22536 \pm 0.00061} & \boxed{0.00355 \pm 0.00015} \\ 0.22522 \pm 0.00061 & \boxed{0.97343 \pm 0.00015} & \boxed{0.0414 \pm 0.0012} \\ 0.00886^{+0.00033}_{-0.00032} & 0.0405^{+0.0011}_{-0.0012} & \boxed{0.99914 \pm 0.00005} \end{pmatrix}$$

$$\begin{array}{llll} t \rightarrow & b\, l^+ \nu_l & & \\ & \hookrightarrow & c\, l^- \bar{\nu}_l & \\ & & \hookrightarrow & s\, l^+ \nu_e \\ & & & \hookrightarrow u\, l^- \bar{\nu}_l \end{array}$$

- These considerations should be generalized to **three generations of quarks**. Matrix notation is introduced between states which appear in the weak charged quark current, i.e. d', s' and b', and the real quarks, d, s and b.
- The unitary matrix $\boldsymbol{V_{CKM}}$ is called the Cabibbo-Kobayashi-Maskawa matrix (CKM) after its inventors. The Particle Data Group compiles the values of its elements from many different measurements.
- These **matrix elements** appear at each vertex of the $W^{\pm}$ with a pair of quarks. Absolute values are measured by comparing decay rates of each type, except for top quark interactions, where they are derived from the unitarity of the matrix.
- The matrix elements become smaller as one deviates from the **diagonal.** This means that intra-generation interactions dominate the weak charged current. Inter-generation interactions jumping a single generation are disfavored, those which bridge two generations even more. The structure of the matrix thus favors **decay cascades**, like the one of the top quark shown here.
- Note that unitarity allows the matrix elements to be **complex**. Therefore, a nontrivial phase angle may appear in the matrix. This factor makes amplitudes **non-invariant under the operation CP**, the symmetry between matter and antimatter.

- We will talk about violations of matter-antimatter symmetry in the next video.