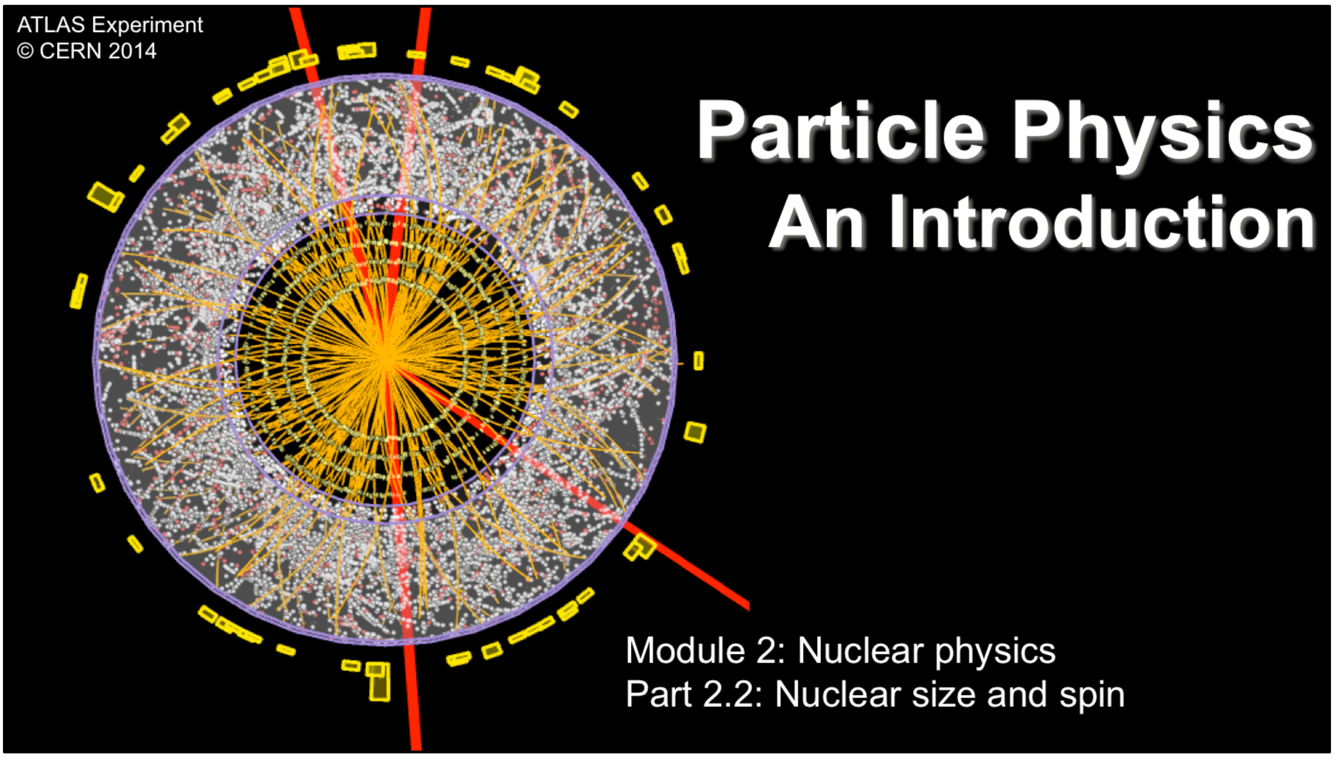


During this second module, we deal with nuclear physics and its applications.

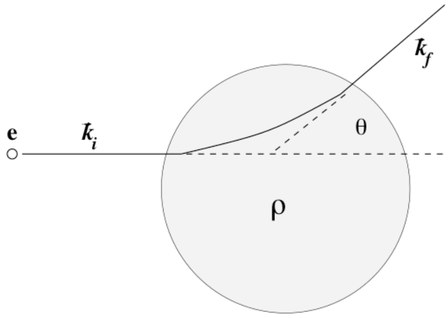
In this second video we will summarize what is known about the size and spin of nuclei. The goals for you are:

- To know how nuclear size is measured and what the results are;
- To know general facts about the spin of nuclei;
- To be able to describe the valley of stability of nuclei as a function of the number of protons and neutrons.

Elastic scattering of an electron by a point-like target without spin, Mott scattering:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} = \frac{Z^2\alpha^2 E^2}{4k^4 \sin^4 \frac{\Theta}{2}} \left(1 - v^2 \sin^2 \frac{\Theta}{2}\right)$$

- $k = |\vec{k}_i| = |\vec{k}_f|$: electron momentum
- $v = k/E$: electron velocity
- $\Theta = \angle(\vec{k}_i, \vec{k}_f)$: scattering angle



Charge distribution $\rho(\vec{x})$ of the target :

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} |F(\vec{q})|^2$$

- $F(\vec{q}) < 1$: form factor, depends on momentum transfer $\vec{q} = \vec{k}_i - \vec{k}_f$

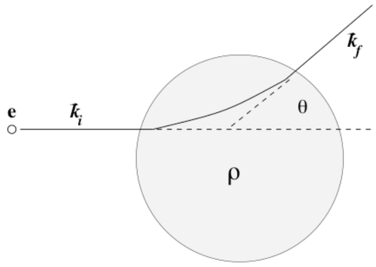
- The **size of a subatomic object** must be carefully defined. In a quantum system, it is given by the root mean square of the eigenvalue of the coordinate operator in its ground state.
- For an atom, this is the root mean square of the radial position of the electron which is farthest away from the nucleus. This distance is calculable, because we know perfectly the binding electromagnetic force, and because it is defined with respect to a static reference point, the position of the nucleus.
- For the nuclear force we do not have a simple description of its action. We must thus interpret the results of **experiments** which probe the distribution of nucleons inside the nucleus.
- It would be unwise to use hadronic probes to do so, which are themselves sensitive to the nuclear force. **High energy electrons**, on the contrary, can penetrate inside the nucleus and their scattering maps out the charge distribution of the target.
- The cross section for a point-like target without spin, is given by the **Mott formula** displayed here. If the charge is instead distributed according to a volume density ***ρ(x)***, leaving the total charge intact, the cross section will be reduced by a **form factor *F(q)*** . The form factor is a function of the momentum transfer *q*.

Static charge distribution of the target:

$$F(\vec{q}) \;=\; \int e^{i\vec{q}\vec{x}}\rho(\vec{x})d^3x \simeq \int\left(1 + i\vec{q}\vec{x} - \frac{(\vec{q}\vec{x})^2}{2} + ...\right)\rho(\vec{x})d^3x$$

- $\rho(\vec{r}) = \rho(r)$: spherically symmetric charge distribution:

$$F(|\vec{q}|) = 1 - \frac{1}{6}|\vec{q}|^2\,\langle r^2\rangle + ...$$

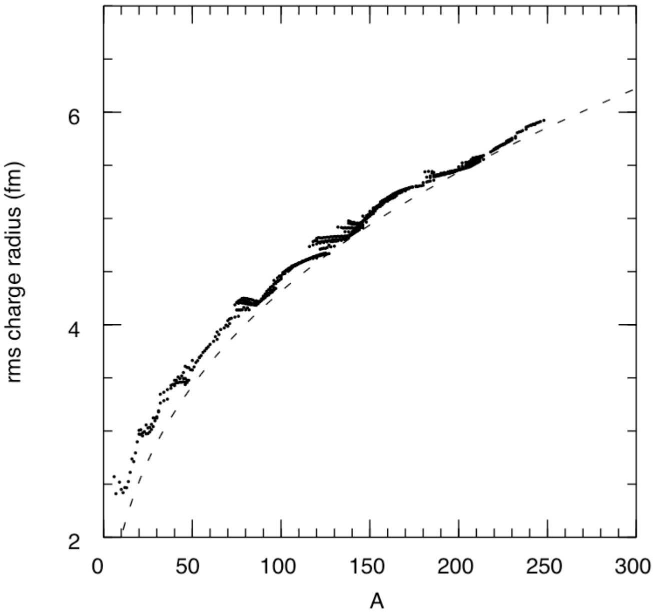


- $\langle r^2\rangle$: mean square radius of the charge distribution

- $\rho(r) \sim \exp(-\Lambda r)$: exponential charge distribution:

$$F(|\vec{q}|) \sim \left(1 + \frac{|\vec{q}|^2}{\Lambda^2}\right)^{-2}$$

- For a **static target**, *F(q)* is the Fourier transform of the spatial charge distribution .
- For **small momentum transfers**, we can develop the form factor in a **Taylor series**.
- If the distribution is **spherically symmetric**, the terms with an odd power drop out. The dominant second term is proportional to the **mean square radius $<r^2>$** of the charge distribution. It can thus serve as a size estimator for the nucleus.
- For an **exponential** charge distribution, $\rho(r) \sim e^{-\Lambda r}$, the form factor takes what is called a **dipolar** form.
- The dependence of the electron-nucleus cross section on momentum transfer for **small-angle scattering at high electron energies** is thus used to measure the size of the nucleus.



$$R \simeq r_0 A^{1/3} \simeq 1.2 A^{1/3} 10^{-15}\ \mathrm{m}$$
$$= 1.2 A^{1/3}\ \mathrm{fm}$$

B. Alex Brown, Lecture Notes in Nuclear Structure Physics, Michigan State University (2005), http://www.nscl.msu.edu/~brown/Jina-workshop/BAB-lecture-notes.pdf

- Scattering experiments establish a simple relation between the radius of a nucleus and the number of nucleons: ***R ~ A$^{1/3}$***.
- The radius R is proportional to the cubic root of the number of nucleons A, with a universal **proportionality constant of 1.2 fm**. Nuclei are thus indeed small compared to the atomic size.
- The nuclear volume is proportional to A. This corresponds to **densely packed incompressible nucleons**, which do not fuse.
- The **mass density** of nuclear matter is of the order of 10^{14} g/cm^3.

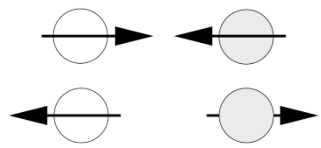
Nuclear spin:

- Proton and neutron : spin $S = 1/2$, orbital angular momentum $L \in \mathbb{I}$
- Total spin : $\vec{L} + \vec{S}$
- Half integer if A is odd, integer if A is even

Observation:

- All nuclei with even N and Z have nuclear spin 0
- Heavy nuclei have small spins in their ground state

$\Rightarrow$ Nucleons form pairs with opposite spins



Nuclear spin is the sum of the spins *S* of all individual nucleons and their relative angular momentum *L.*

- Protons and neutrons are **fermions** with spin ½.
- Like in atoms, the **nuclear angular momentum** *L* follows an integer quantum number.
- The **total** should therefore be a half-integer number if *A* is odd, a integer if *A* is even.
- Indeed, all nuclei with both **N and Z even** have nuclear spin 0. Heavy nuclei have rather small nuclear spin in their ground state. We conclude that neutrons and protons tend to arrange in **pairs of opposite spin** direction.

- Magnetic dipole moment associated to spin :

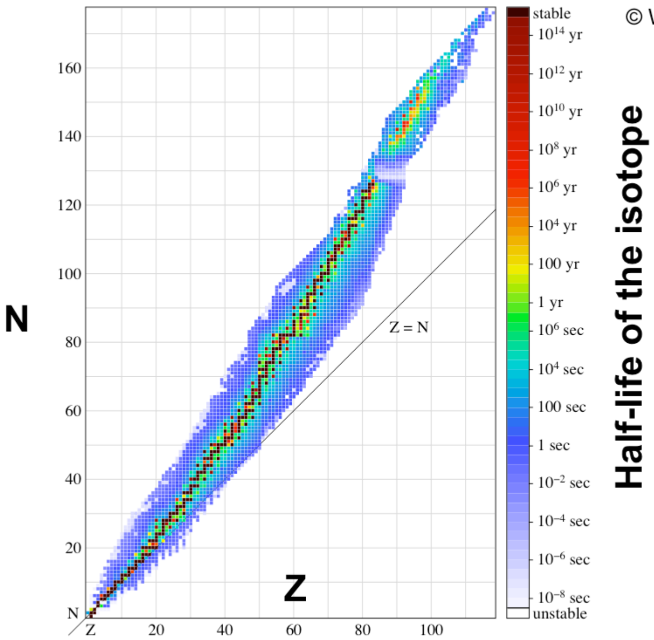
$$\vec{\mu} = g\frac{e}{2m}\vec{S}$$

  - $\mu_B = e/2m_e = 5.79 \times 10^{-11}$ MeV/Tesla: Bohr's magneton
  - $e/2m_{\rm p} \ll \mu_B$: nuclear magneton
  - g: gyromagnetic factor, for a point charge $g = 2$
  - $g \neq 2$: anomalous magnetic moment

- Magnetic moment of proton and neutron:

$$\mu_{\rm p} \simeq 2.79\frac{e}{2m_{\rm p}} \quad ; \quad \mu_{\rm n} \simeq -1.91\frac{e}{2m_{\rm p}}$$

- Every charged particle has a **magnetic dipole moment** associated with its spin, including nuclei.
- The order of magnitude for an **electron** is given by the **Bohr magneton**, $\mu_B = e/2m_e$ = 5.79 ×10^{-11} MeV/Tesla.
- The **nuclear magneton**, $e/2m_p$, is three orders of magnitude smaller, due to the large proton mass.
- The **gyromagnetic factor** g measures the ratio between the angular momentum and the magnetic moment.
- For a **point charge**, $g \cong 2$, with small deviations of order 10^{-3} for electron and muon, as we will see in Module 4.
- The magnetic moments of **proton and neutron** are considerably different, +2.79 and -1.91 nuclear magnetons, respectively. This is a first indication of a charged substructure of the nucleons. In fact, since the neutron has zero net charge, it must contain charged particles.
- All nuclei have **measured magnetic moments between -3 and 10 nuclear magnetons**, thus rather small ones. This is a consequence of the spin pairing of nucleons leading to a limited total nuclear angular momentum.



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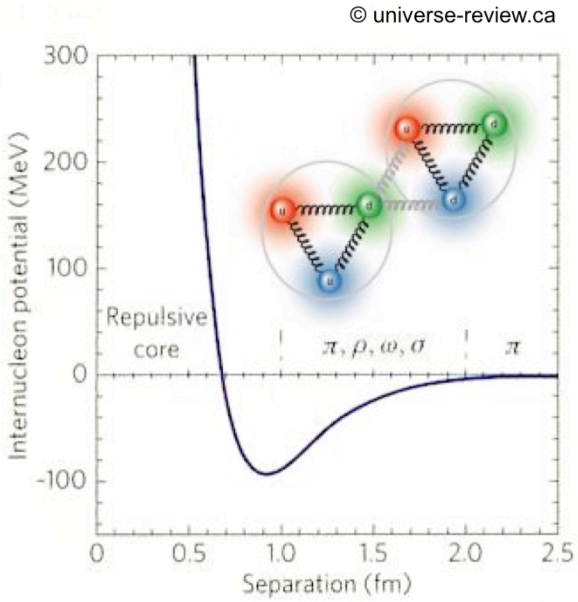
Nuclear decay:

- α: emission of ^{4}He nuclei
- β: emission of $e^{\mp}$
- γ: emission of photons

- Most nuclei and their isotopes are **unstable**. Stable nuclei are found in a narrow band in the *N-Z* diagram, which is called the **valley of stability**.
- The valley has the following shape:
  - For light nuclei $A<40$: $N \cong Z$
  - For heavy nuclei $A>40$: $N \cong 1.7 \times Z$
- This indicates that for **heavy nuclei**, the charge density and thus the Coulomb repulsion must be diluted by **additional neutrons**.
- The decay of unstable nuclei is the source of nuclear radioactivity:
  - α: emission of ^{4}He nuclei, also called alpha particles
  - β: emission of electrons or positrons (together with neutrinos)
  - γ: emission of photons
- Nuclei with a **surplus of neutrons** can be stabilized by converting a neutron into a proton. Those with a **surplus of protons** can convert a proton into a neutron. These are **isobar decays** of the type $\beta^{\pm}$.
- Heavy nuclei often decay into a pair of lighter nuclei. This corresponds to a spontaneous fission, often by emitting a **^{4}He nucleus**. This is the α decay.
- These decays normally lead to an **excited state** of the daughter nucleus and are often followed by a γ decay towards the ground state.
- We will enter into more detail on these processes in the 4th and 5th video of this module.



Properties of the nuclear force:

- Very short range, limited to nuclear size
- Binding energy per nucleon roughly independent of nuclear size
- Attractive and much stronger than Coulomb repulsion between protons
- Repulsive component at distances comparable to the size of a nucleon ($\simeq 1 \times 10^{-15}$ m = 1 fm)

Let us summarize what we have learned about the **nuclear force:**

- It has a **very short range,** limited to the nuclear size**.**
- The binding energy per nucleon it leads to, is independent of the size of the nucleus. The nucleon thus interacts only with its nearest neighbors.
- The nuclear force is **attractive** and much stronger than the Coulomb repulsion between protons. But it must have also a **repulsive component** at distances comparable to the size of the nucleon, ≅ 1 femtometer = 10^{-15}m = 1fm. This is due to the existence of quarks inside the nucleon. The repulsive component is necessary to prevent the fusion among nucleons.
- QCD is a quantum field theory which describes the interactions among colored quarks inside hadrons, via the exchange of equally colored gluons. But the strong force acts only inside hadrons, which thus have a zero net color.
- Nucleons thus cannot exchange gluons. It is by the exchange of colorless objects, like mesons, that they bind together.
- This makes the nucleus a **complex multi-body object** which is difficult to understand. The appropriate theoretical methods are **effective field theories** (like chiral perturbation theory) or **numerical calculations** on a lattice (like lattice QCD). All of this goes much beyond the scope of this course.

- In the next video we will rather concentrate on much simpler models which describe the gross features of nuclei.