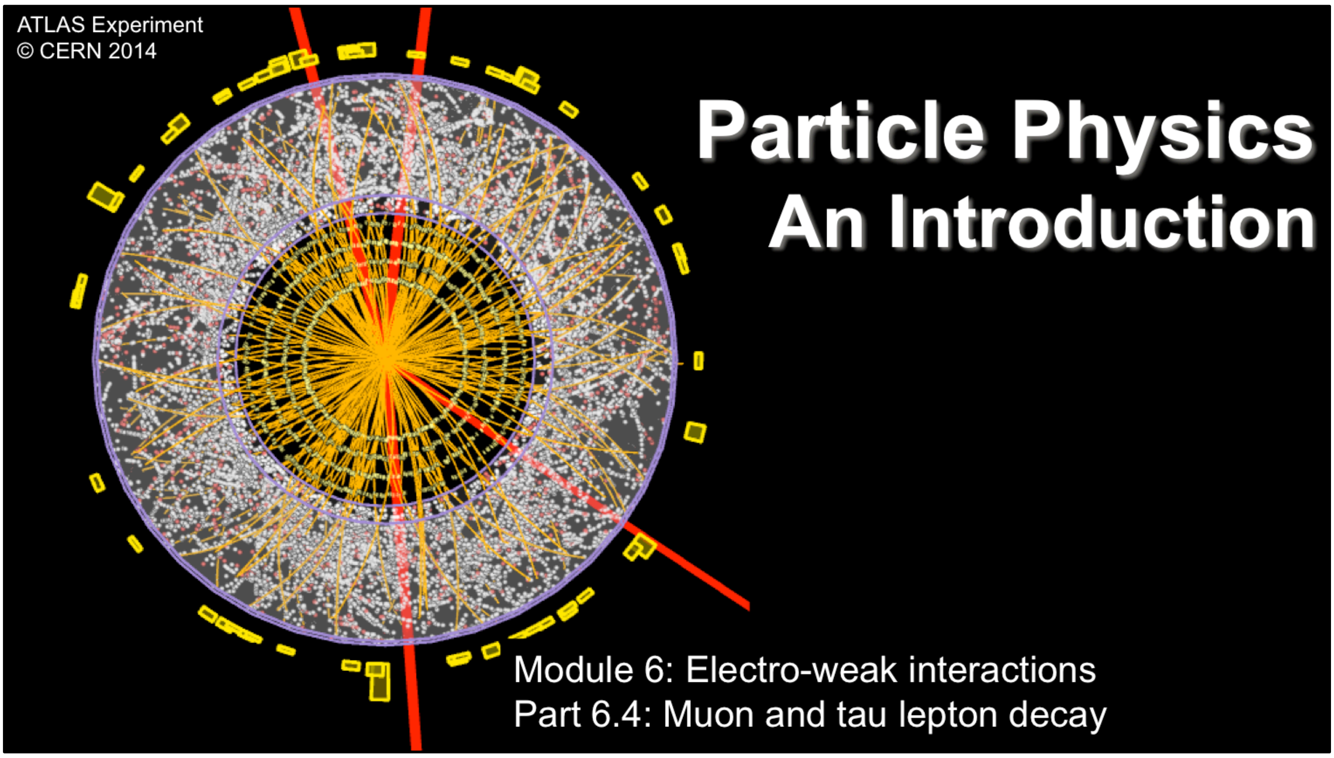


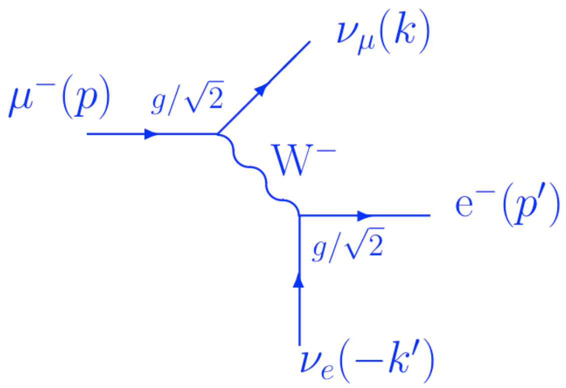
In this 6th module, we are discussing weak interactions.

This 4th video discusses a prototype process of charged weak interactions, the leptonic interactions of the W boson.

After following this video you will know:

- How to describe the decays of muon and tau leptons in terms of a Feynman diagram;
- How to discuss the properties of these interactions quantitatively.

$$\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$$



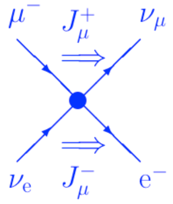
W propagator:

$$\sim \frac{1}{q^2 - M_W^2} \underset{q^2 \leq m_\mu^2 \ll M_W^2}{\propto} \frac{1}{M_W^2}$$

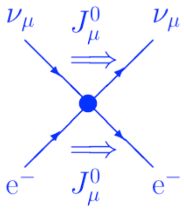
Fermi coupling constant:

$$\frac{G_F}{\sqrt{2}} = \left(\frac{g}{\sqrt{2}}\right)^2 \frac{1}{4M_W^2}$$

- Here is the Feynman diagram for **muon decay**. The emission of the W^- converts the muon into a muon neutrino, the virtual W then decays into an electron-electron-antineutrino pair. This way, the lepton number, the weak isospin and other quantum numbers are conserved.
- In the diagram, we already converted the **outgoing antineutrino** into an **incoming neutrino** by inverting its charge and four-momentum $k' \rightarrow -k'$.
- In the amplitude, the **W propagator** contributes a factor of $1/(q^2 - M_W^2)$, which weakens the amplitude for $q^2 << M_W^2$. At low momentum transfers, like for light particle decays where $q^2 \leq m_\mu^2$ is negligible, the propagator is constant and equal to $1/M_W^2$.
- We can then treat this process in the so-called **Fermi approximation**.
- By convention, one absorbs the residue of the W propagator with the square of the coupling constant g^2 to form the **Fermi constant** G_F. It takes the role of the fine structure constant for weak interactions.



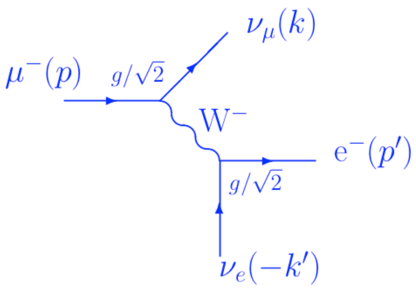
$$\mathcal{M}^{CC} = \left(\frac{g}{\sqrt{2}}\right)^2 \frac{1}{M_W^2} J_\mu^+ J^{-\mu} = \frac{4G_F}{\sqrt{2}} J_\mu^+ J^{-\mu}$$



$$\mathcal{M}^{NC} = \left(\frac{g'}{\sin\theta_W}\right)^2 \frac{1}{M_Z^2} J_\mu^0 J^{0\mu} = \frac{4G_F}{\sqrt{2}} 2\rho J_\mu^0 J^{0\mu}$$

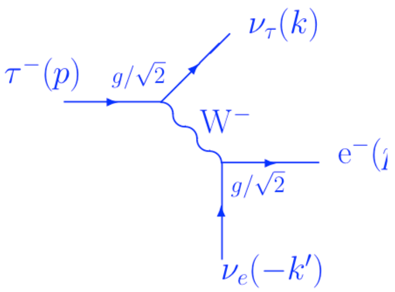
$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} \quad ; \quad \tan\theta_W = \frac{g'}{g}$$

- In this approximation valid at low momentum transfers, weak interactions can be described by a **direct interaction between two current densities**.
- The transition currents corresponding to charged weak interactions are similar to the electromagnetic ones. One denotes them − in an imprecise manner – as **charged currents** (CC).
- Analogously, the interaction of the Z boson at low energy, $q^2 << M_Z^2$, can be approximated by a **direct interaction of two neutral currents** (NC).
- The constant ρ involves the **Weinberg angle**, ϑ_W, which measures the ratio between the coupling constants of the weak charged and neutral interactions.
- The constant ρ is equal to 1 with good precision. This is due to the fact that in the standard theory of electro-weak interactions, the masses of the vector bosons and the coupling constants are connected by the **Higgs mechanism**, which we will discuss in video 6.11. High precision experimental results verify this relationship.



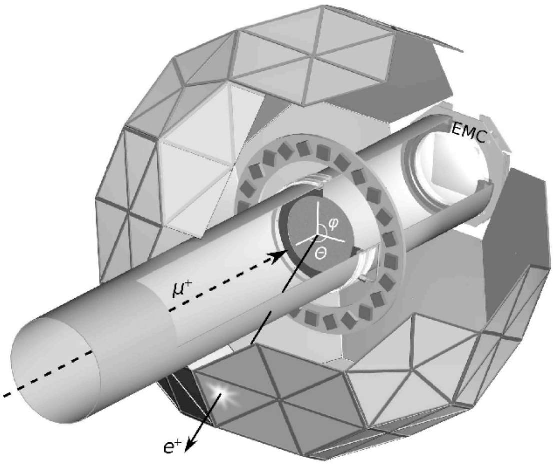
$$\frac{d\Gamma}{dE'} = \frac{G_F^2}{12\pi^3} m_\mu^2 E'^2 \left(3 - \frac{4E'}{m_\mu}\right)$$

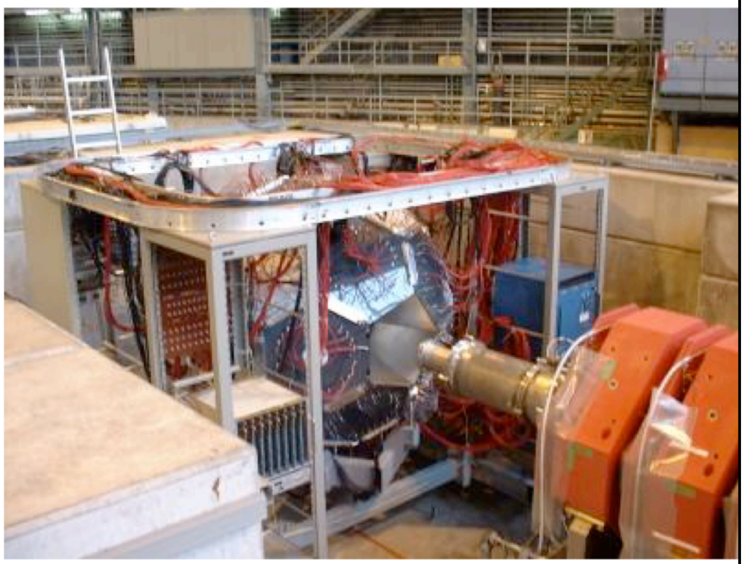
$$\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu) = \frac{1}{\tau_\mu} = \int_0^{m_\mu/2} dE' \frac{d\Gamma}{dE'} = \frac{G_F^2 m_\mu^5}{192\pi^3}$$



$$\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau) = \frac{\mathrm{BR}(\tau^- \to e^- \bar{\nu}_e \nu_\tau)}{\tau_\tau} = \frac{G_F^2 m_\tau^5}{192\pi^3}$$

- The **differential muon decay rate** (neglecting $m_e \ll m_\mu$) is given here as a function of the energy *E'* of the outgoing electron. This energy can vary between $0 \leq E' \leq m_\mu/2$.
- As this process is virtually the **only decay channel**, the total rate is obtained by integrating over *E'*.
- The measured **lifetime** of the muon is close to 2.2×10^{-6}s, we thus find a FermI constant of $G_F \approx 10^{-5}$ [1/ GeV^2].
- The factor m^5 comes in through the **phase space factors**, which enter into the calculation of Γ.
- Therefore, we find the same formula for other weak decays, especially for **tau decay**.
- It does, on the other hand, not directly correspond to the total decay rate because the **tau has other important decay channels,** due to its large mass. Γ for this specific decay is thus just a partial width, we must thus take into account the **branching fraction** in the numerator.

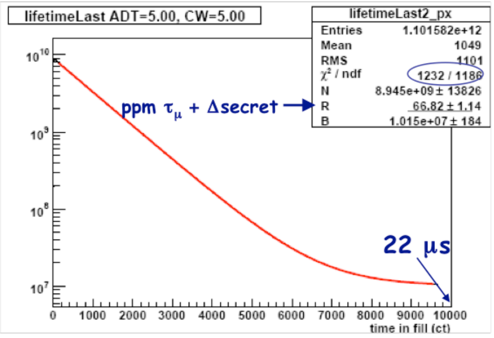


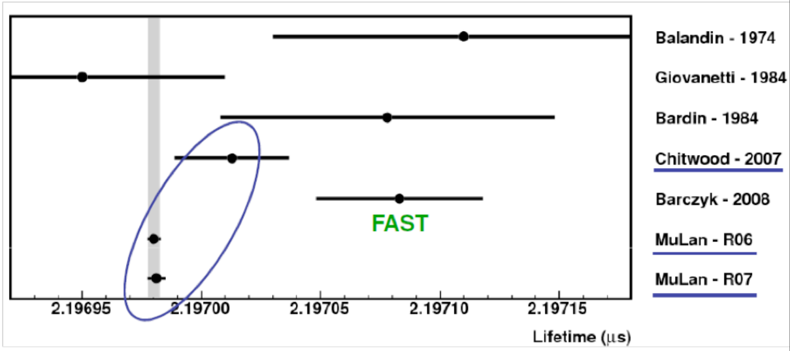


MuLan Coll., D.M. Webber et al., Phys. Rev. Lett. 106 (2011) 041803

- One of the most precise experiments that measured the muon lifetime is **MuLan** at the Paul Scherrer Institute at Villigen, Switzerland.
- A μ^+ beam enters from the left, and is stopped in the target. The **scintillator ball** surrounding the target registers the positrons from the decay and measures their emission times.

MuLan Coll., V. Tishchenko et al., Phys. Rev. D87 (2013) 052003





$$\tau_{\mu^+}^{PDG} = (2.1969811 \pm 0.0000022) \times 10^{-6}\ \text{s}$$

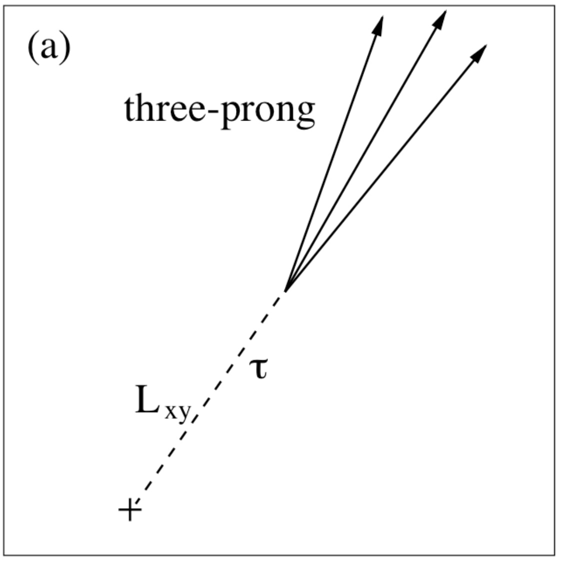
$$G_F = (1.1663787 \pm 0.0000006) \times 10^{-5}\ \text{GeV}^{-2}$$

http://pdg.lbl.gov/2016/tables/rpp2016-sum-leptons.pdf

- The number of decays per unit time follows an **exponential law** to a very good approximation, with a **low background** from accidental coincidences between beam arrival and electron signal.
- The **slope** of the curve gives the muon lifetime. The MuLan collaboration finds τ_μ = 2196980.3 ± 2.2 ps.
- Their result dominates the **global average** of Particle Data Group of τ_μ = 2.1969811 ± 0.0000022 microseconds. This gives a value of the **Fermi constant** accurate to 0.6 ppm.

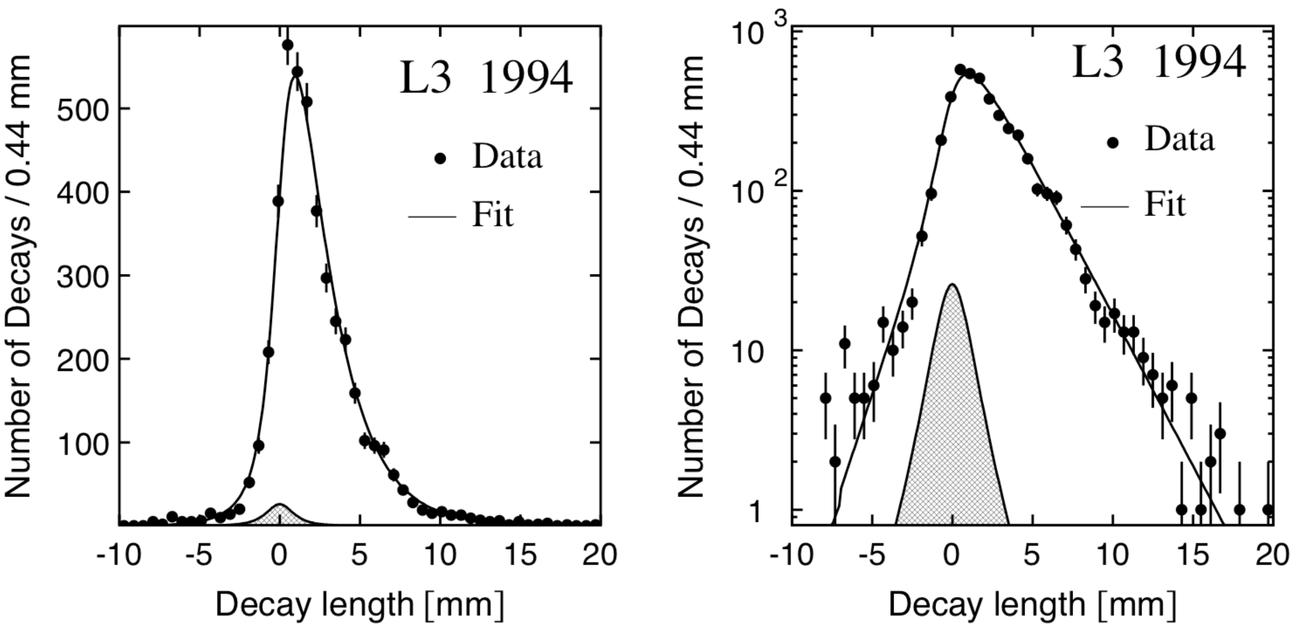
$$\tau^- \rightarrow \pi^-\pi^+\pi^-\nu_\tau$$
$$\tau^+ \rightarrow \pi^+\pi^+\pi^-\bar{\nu}_\tau$$

$$L = \gamma v \tau_\tau = \frac{p}{m_\tau}\tau_\tau$$



L3 Collab., M. Acciarri et al., Phys. Lett. B389 (1996) 187

- The **tau lepton lifetime** is too short for a direct measurement. We rather measure its **path length *L*** at high energies. Tau leptons are produced in large numbers in electron-positron annihilation into τ pairs.
- At high energies, $\sqrt{s} = 2\,E_\tau$, their **lifetime in the laboratory frame** is extended by the relativistic γ factor. Their mean path length, *L*, is therefore longer by a factor p/m_τ.
- With a lifetime of several hundred fs, the path length itself is hardly directly observable even at high energies. At a τ energy of 50 GeV, for example, the average decay distance is about 2.5 mm, and thus completely contained in the **vacuum tube** that contains the beam of a collider experiment.
- One determines the decay point from the vertex of decays into three charged particles, $\tau^\pm \rightarrow \pi^\pm\pi^+\pi^-\nu_\tau$(-bar).
- The trajectories of the three pions are measured with a **tracking detector**, which is typically a silicon detector directly outside the vacuum tube. They are extrapolated back to their common origin, which is the **decay vertex** of the tau lepton. The tau **origin** being known from the beam position, the decay length *L* is thus the distance between the two points.



L3 Collab., M. Acciarri et al., Phys.Lett. B479 (2000) 67

- The **decay length distribution** in logarithmic scale shows the expected exponential behavior, but modified by the experimental resolution, which is significantly worse than for direct time measurement. The **slope** of the distribution measures the average decay length, which in turn gives the life time as $\tau_\tau = L/(\gamma v)$.
- The global average **tau life time**, determined by the Particle Data Group is $\tau = (290.3 \pm 0.5)\times10^{-15}$ s.
- This figure is in excellent agreement with the prediction, based on the Fermi constant as determined by the measured life time of the muon. This shows that the **coupling constants of the electron, muon and tau to the W boson** are the same to within a few per-mille.

- In the next video we'll talk a little more about the properties of $W^\pm$ bosons, and in particular about their mass.