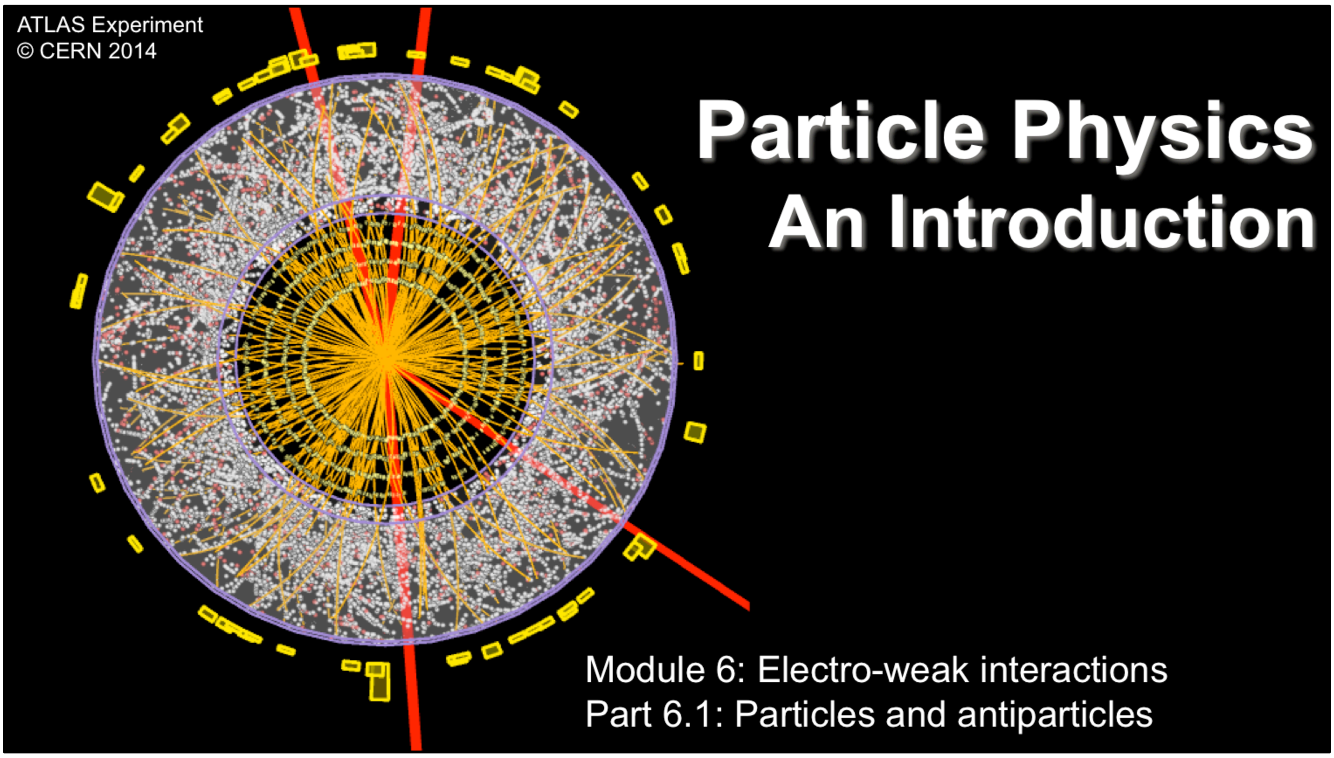


In this 6th module, we discuss weak interactions and the Higgs mechanism.

You will notice that this module is again larger that average. This is due to the rich phenomenology of electro-weak interactions We recommend that you take 2 weeks to digest the contents.

Before entering into our subject, in this first video we go into more depth on the subject of antiparticles.

After following this video, you will know:

- The difference between particles and antiparticles;
- The connection between antiparticles and energy-momentum reversal.

$$E^2 = \vec{p}^2 + m^2$$
$$-\frac{\partial^2}{\partial t^2}\phi + \vec{\nabla}^2\phi = m^2\phi$$
$$\left(\partial_\mu \partial^\mu + m^2\right)\phi = 0$$

$$\partial_\mu j^\mu \;=\; 0$$
$$j^\mu = (\rho, \vec{j}) \;=\; i(\phi^* \partial^\mu \phi - \phi \partial^\mu \phi^*)$$

How do **antiparticles** come into play in the Standard Model?

- It all starts with the **evolution equation** for fields, which takes the role of an equation of motion for particles. It follows from **relativistic energy-momentum conservation** by operator substitution. The result is the **Klein-Gordon equation** for free particles.
- Its solutions follow a **continuity equation** as seen here. It describes the conservation of a four-vector-current j_μ of particles. The time component is the **probability density** of finding a particle at a location (x,t). Its spatial component is the **particle flow density** at this same position. Imagine an infinitely small volume that surrounds the point. The density in the volume can only change, $\partial\rho/\partial t \neq 0$, if particles enter or exit through the boundaries, $div\, j \neq 0$. This ensures that the number of particles is conserved.
- It might surprise you that the probability density contains the **time derivative** of the wave function, we will understand this fact shortly.

$$\left(\partial_\mu \partial^\mu + m^2\right)\phi = 0$$

$$\begin{aligned}
\phi &= \sqrt{N} e^{i(\vec{p}\vec{x} - Et)} \\
\rho &= i\left(\phi^* \frac{\partial \phi}{\partial t} - \phi \frac{\partial \phi^*}{\partial t}\right) = 2EN \\
\vec{j} &= -i\left(\phi^* \vec{\nabla}\phi - \phi \vec{\nabla}\phi^*\right) = 2\vec{p}N
\end{aligned}$$

$$\begin{aligned}
d^3\vec{x} &\rightarrow d^3\vec{x}/\gamma \\
\rho &\rightarrow \gamma\rho = \frac{E}{m}\rho
\end{aligned}$$

- The Klein-Gordon equation is manifestly **covariant**, it contains only scalars under Lorentz transformation.
- **Plane waves** must be a solution of the Klein-Gordon equation, and describe a free particle. We use a normalization proportional to an arbitrary constant $\sqrt{N}$. You may take N as the number density of particles.
- Their **current density** is proportional to the four-momentum. Its temporal component is proportional to the energy.
- This is due to the covariance of the quantity: it requires that the probability ρd^3x is invariant. The volume element changes like d^3x ➔ d^3x/γ because of the **Lorentz contraction** in the direction of motion. For invariance, the probability density must change as ρ ➔ $\gamma\rho$. With $\gamma = E/m$, ρ must thus be proportional to energy.
- It is for this reason that probability density and current density contain, respectively, a **time and space derivative** of the field.

$$\left(\partial_\mu \partial^\mu + m^2\right)\phi = 0$$
$$E = \pm(\vec{p}^2 + m^2)^{\frac{1}{2}}$$

$$j^\mu = (\rho, \vec{j}) = i(\phi^* \partial^\mu \phi - \phi \partial^\mu \phi^*) \quad \rightarrow \quad j^\mu = iQ(\phi^* \partial^\mu \phi - \phi \partial^\mu \phi^*)$$

- Let us now consider the **energy eigenvalues** of relativistic free particles, inserting a plane wave solution into the Klein-Gordon equation.
- We find solutions with **positive and negative energy.** And since the density of probability is proportional to the energy, it is no more positive definite either. This contradicts the definition: a probability is a real number between 0 and 1.

- To solve this problem, we must interpret the continuity equation in an innovative way, introducing the **electromagnetic current density.**
- Proportional to the electric charge Q, j^0 becomes the **local charge density** and thus may be negative as well as positive, in agreement with intuition.
- It is obvious that **solutions of positive and negative energy** are distinguished by the charge of the considered particle, if one wants to keep the probability density always positive.
- It is thus not the probability density, $2Np^\mu$, which is invariant, but the **electromagnetic current density** $2QNp^\mu$, which obeys the rules for physically sensible probabilities.
- The current density is **conserved** by the continuity equation. With this concept, we can very well spontaneously **create and annihilate particles**, provided that the total charge remains constant. All of this is a consequence of relativistic covariance!

$$\begin{aligned} j^\mu &= iQ(\phi^* \partial^\mu \phi - \phi \partial^\mu \phi^*) \\ &= -2eNp^\mu = -2eN(E, \vec{p}) \end{aligned}$$

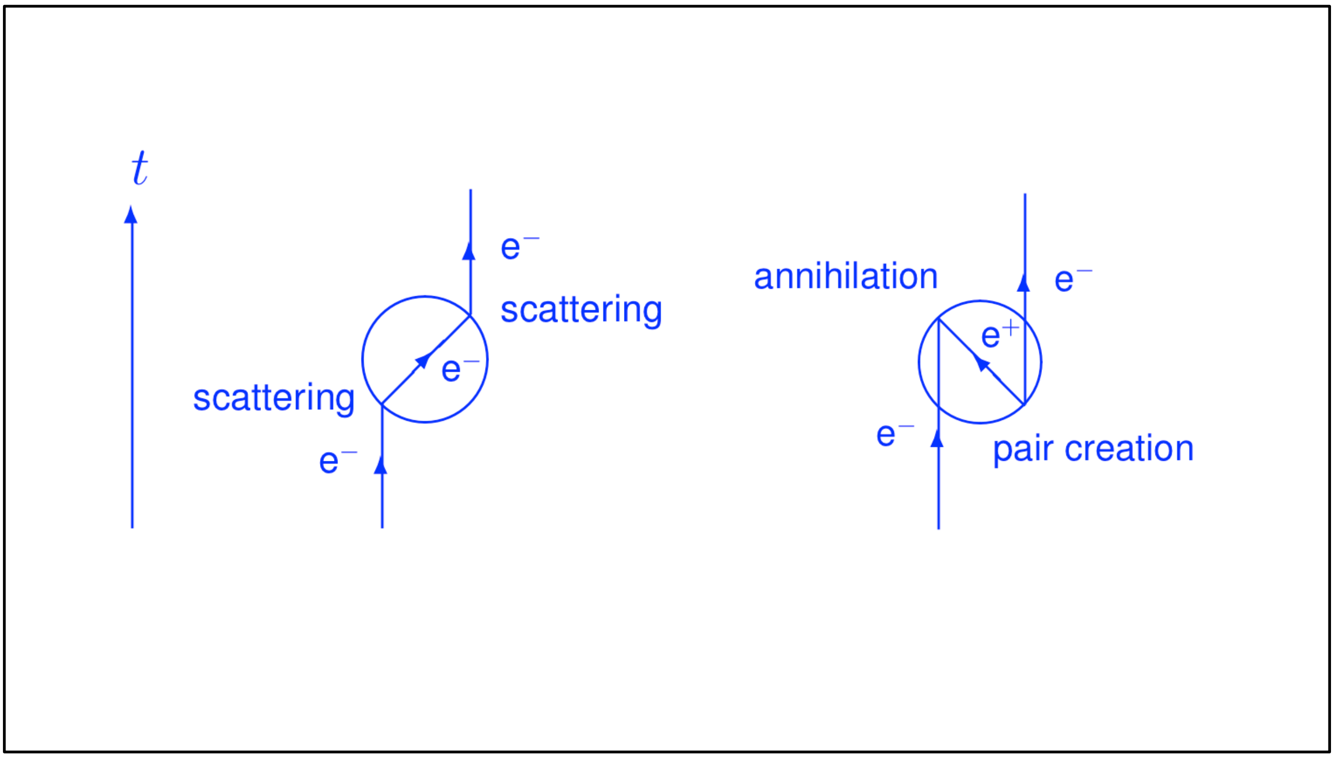
$\mathrm{e}^-(E > 0)$

$$j^\mu = +2eNp^\mu = -2eN(-E, -\vec{p})$$

$\mathrm{e}^+(-E > 0)$

Which are the implications of this re-interpretation for the negative energy solutions? The answer is given by **Feynman's interpretation** of these states:

- The electromagnetic current density of a **free electron** with charge *Q = -e*, ignoring its spin, is given by the first equation. This current is that of a particle with positive energy, advancing in the direction of its momentum vector, let us say that it **emerges from a vertex**.
- For a **free positron**, *Q = +e,* the current is positive. But the current density is the same as for an electron of negative energy, which moves in the opposite direction, i.e. **entering the vertex**.
- That is to say that a particle with negative energy, which comes out of a vertex, is equivalent to an antiparticle of positive energy that enters the vertex. In this way, we can **eliminate the solutions with negative energy** and replace them by antiparticles evolving in the opposite direction.
- This establishes a connection between the transformation between particles and antiparticles, called **charge conjugation**, and the reversal of the **direction of the momentum**, called parity. These two transformations will be analyzed in the next video.



- But first we look at the **consequences of Feynman's interpretation** of negative energy states.
- These consequences are dramatic, as shown by the example of a **double scattering process**. Heisenberg's principle allows us to reverse the order of two scatters in time. The intermediate state will then move backwards in time.
- We replace it by its **antiparticle** moving forward in time. The **double scatter** then becomes a **pair creation** process, followed by a **particle-antiparticle annihilation** process. Both contributions begin with the same initial state and end with the same final state. The intermediate state remains unobserved. Both contributions must thus be summed at the level of amplitudes when calculating the probability of the process.
- When calculating simple scatters, it is sufficient to consider **particles**. The corresponding **antiparticle** processes can be obtained by reversing the directions of the particles.

In the next video we introduce the discrete transformations of space-time—parity and time reversal—and charge conjugation, which transforms the wave functions of particles into antiparticles and vice-versa.