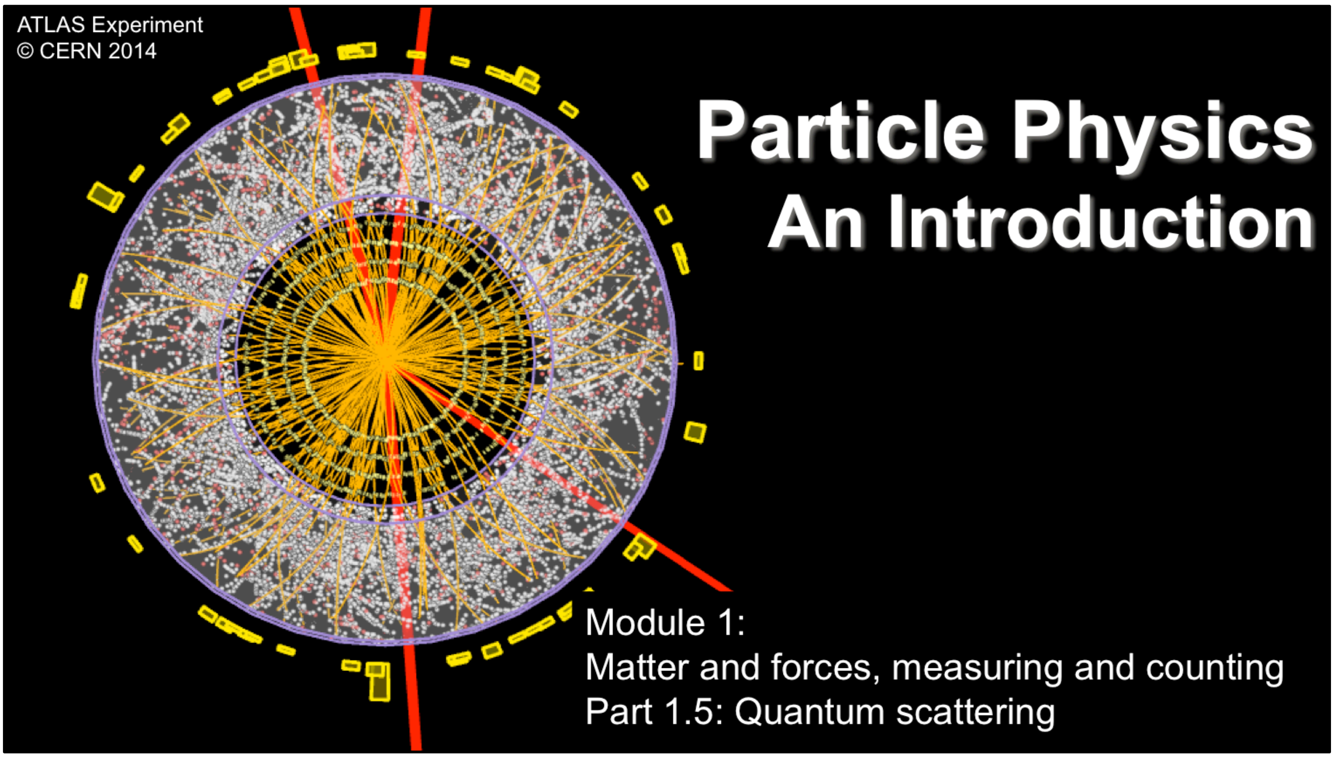


During this first module, we are introducing the objects studied in particle physics, namely matter, forces, space-time and of course scattering processes.

In this fifth video we will show how to approach scattering between particles in a quantum way. The goals for you are:

- To identify the conceptual differences between the classical and the quantum evolution of a system.
- To know how to draw a Feynman diagram for a simple scattering process and explain its ingredients.

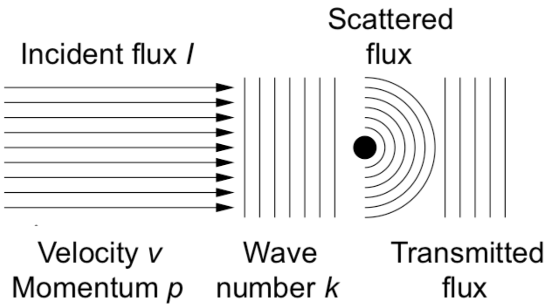
$$\boxed{\frac{d\sigma(\theta)}{d\Omega} = \left(\frac{zZe^2}{8\pi E_{kin}}\right)^2 \frac{1}{4\sin^4\frac{\theta}{2}}}$$

In video 1.4, Mercedes has calculated the differential cross section for Coulomb scattering off a static target. By luck coincidence, the classical physics result is still valid in a relativistic quantum context. The reasons are the following:

- The result of quantum theory contains **no factor of *ħ***. This means that the artificial limit *ħ* ➔ *0*, which normally gives us back the classical result, will not change the answer.
- The classical result is already **valid in the relativistic regime**. This fact is not surprising: Maxwell's equations are valid for relativistic velocities. After all they also describe electromagnetic waves, which move at the speed of light.
- We also do not need to take into account **nuclear interactions** between projectile and target, since the alpha particle will not penetrate into the target nucleus even for a head-on collision. Only the **electromagnetic force** acts outside the nuclear volume.

But until now we have discussed the scattering process using the language of **classical physics**. Now we will change tool kit to discuss the **quantum approach**.

- Heisenberg principle: $\Delta p \Delta b \geq 1$
- Wave function: $\psi(x) = \sqrt{N} e^{i(\vec{k}\vec{x}-Et)} = \sqrt{N}\left[\cos(\vec{k}\vec{x}-\omega t) + i\sin(\vec{k}\vec{x}-\omega t)\right]$
- Incoming plane wave: $\psi = e^{ik_\mu x^\mu}$, $\rho = \psi^2 = 1$, $I = \rho v = v$
- Huygens' principle: $\psi' = (1 - f(\theta,\phi))\, e^{ikx} + f(\theta,\phi)\frac{e^{ikr}}{r}$



While the classical result stays valid in the quantum regime, the interpretation is totally different. Particles may well be point-like, but they move like a probability wave.

- Because of Heisenberg's principle, the **impact parameter** has no more role in our consideration. We must use a scenario à la Huygens to understand scattering. This means that the target particle will be the origin of a new scattered wave, which will add to the incoming wave of the projectile.
- The classical trajectory is replaced by the **wave function *ψ(x)***. It contains the **particle aspect** *(E,p)*, and the **wave aspect** *(ω,k)* of the kinematics at the same time.
- Before the scattering, the projectile is described by a **plane wave** in the *x* direction. Here we have normalized the amplitude to 1 for simplicity.
- **Huygen's principle** says that a small fraction *f* of the incident amplitude will be **spherically re-emitted** by the target. The factor *1/r* in the scattered amplitude is necessary, it ensures that the number of scattered particles, *ψ*ψ*, remains independent of distance.
- The **remaining amplitude** *(1-f)* continues as an unperturbed plain wave.

$$\psi = e^{ik_\mu x^\mu} \quad \rightarrow \quad \psi' = (1 - f(\theta,\phi))\, e^{ikx} + f(\theta,\phi)\frac{e^{ikr}}{r}$$

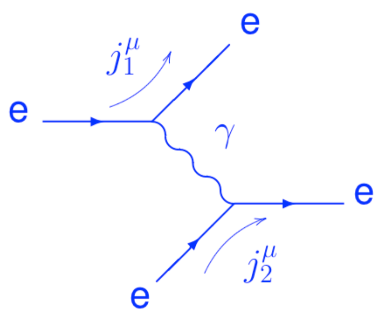
$$\left| f(\theta,\phi)\frac{e^{ikr}}{r} \right|^2 (r^2 d\Omega)dr = |f(\theta,\phi)|^2\, d\Omega dr$$

$$|f(\theta,\phi)|^2\, d\Omega\frac{dr}{dt} = |f(\theta,\phi)|^2\, v\, d\Omega = |f(\theta,\phi)|^2\, I\, d\Omega$$

$$\frac{d\sigma}{d\Omega} = |f(\theta,\phi)|^2$$

As a consequence of the wave approach, we must also re-interpret the cross section in terms of intensities for incoming and scattered wave:

- This means that we now look for a relation between the **cross section and the scattered amplitude *f(ϑ,φ)***.
- The square of the second term is proportional to the number of particles scattered into the a volume subtended by the solid angle element *dΩ* and the radial thickness *dr*. It represents the **scattered flux**.
- For our normalized incident wave, the particle density *ρ=1*, thus the **incident flux** is *I=ρv=v*.
- The ratio between the scattered and the incoming flux is the **cross section**.
- We find a simple result: *f* is the probability amplitude for the scattering process, σ = f^2 is the scattering probability.
- The amplitude *f* is calculable if we know the potential generated by the target, like in the classical case.



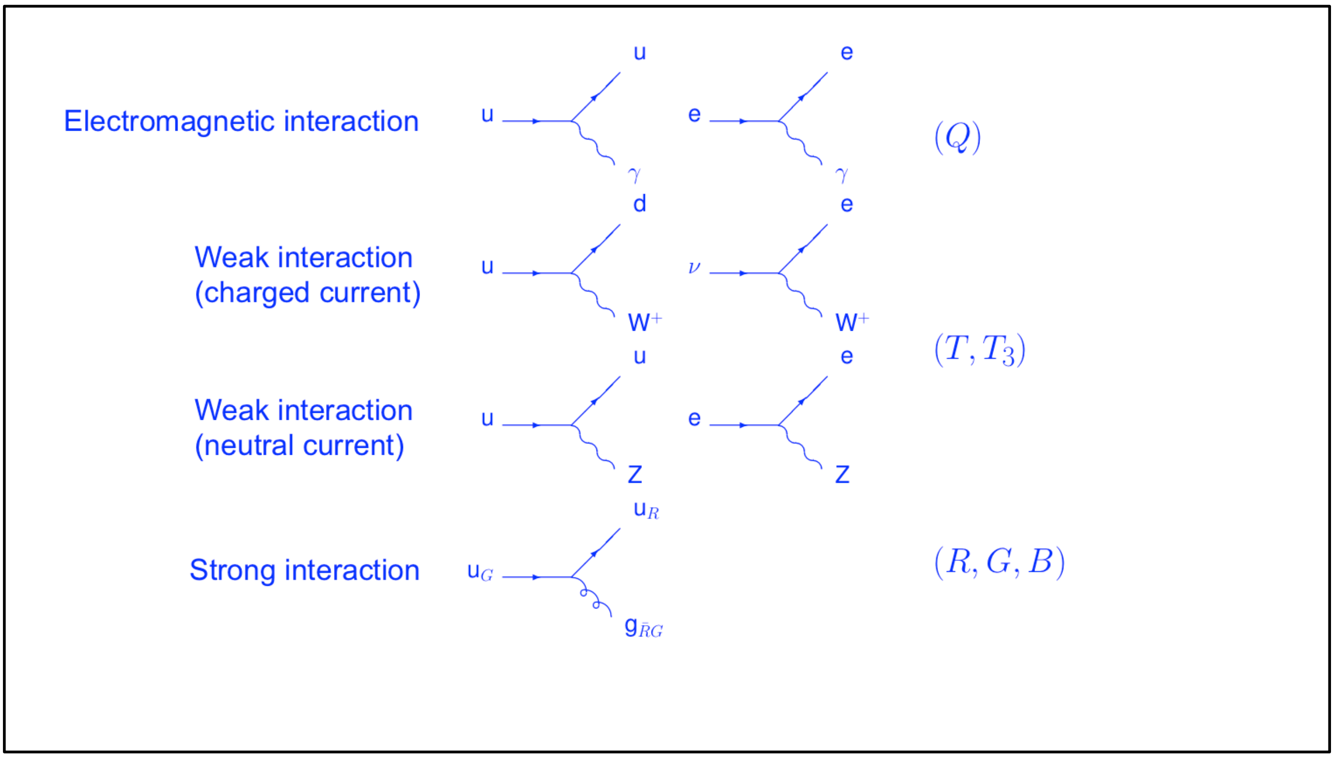
Feynman diagrams:

- lines of propagation for particles in energy-momentum space
- interaction vertices
- visualisation of particle reaction, but also calculation rules for probability amplitudes
- at each vertex, energy-momentum and quantum numbers are strictly conserved
- lines connecting two vertices correspond to virtual particles

To visualize reactions between particles and calculate their probability amplitude, we use **Feynman diagrams**.

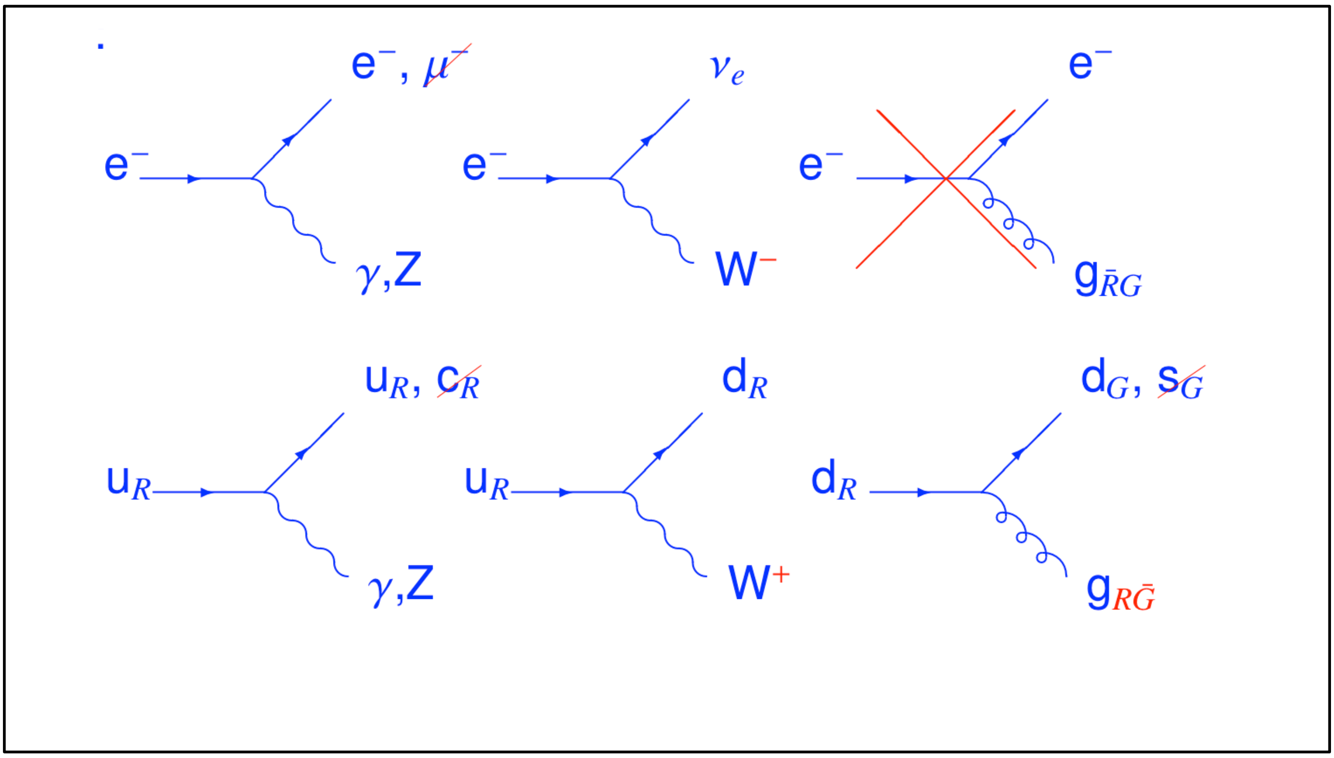
- They represent **lines of propagation** of particles in coordinates *E* and *p*, and not *t* and *x*. The diagrams are located in energy-momentum space.
- They also represent the **vertices** of an interaction, where a force particle is emitted or absorbed to transmit energy and momentum.
- Finally they represent the virtual particles living between two vertices. Those have the properties of their real counterpart, but not the mass of a free particle.

- Feynman diagrams are a useful visualization of a reaction, but also a prescription for calculating their probability amplitude. On does that by applying **Feynman rules**.
- At each vertex, **energy-momentum** and **quantum numbers** are rigorously conserved.
- The virtual particle transfers energy-momentum, thus a **force** acts between projectile and target.
- Details on **how to construct a Feynman diagram** will be discussed in Module 4, when we talk about quantum electrodynamics.



- For each type of elementary interaction, there is a spin 1 boson transmitting the force. They are collectively called gauge bosons:
    - The **photon** transmits electromagnetic interactions.
    - The **W and Z bosons** transmit the two forms of weak interactions.
    - The eight different **gluons** are responsible for the strong interactions.

- To absorb or emit one of these bosons, a particle must carry the required type of charge. In our jargon we say that the **bosons couple to a given charge**. The charge is a **coupling constant**:
    - It needs **electrical charge** Q to couple to the photon; it has 1 component.
    - It needs weak isospin *(T,T_3)* to couple to W or Z; this charge has 2 components.
    - It needs **color** charge *(R,G,B)* to couple to gluons; it has 3 components.
- The probability amplitude to emit or absorb a gauge boson is proportional to the charge of the particle, the probability is thus proportional to the square of the charge, as we found for Rutherford scattering.



- In a Feynman diagram, at each vertex **energy and momentum are conserved**. But do not forget that virtual particles, which relate two vertices, do not necessarily have the mass of their real counterpart, nor even a real number as their mass.
- At each vertex, **charges as well as baryon and lepton number** are conserved. **Flavor** is conserved by electromagnetic and strong interactions, but not by weak ones: charged weak interactions transmitted by W bosons change the particle flavor.
- Here are a few examples of **allowed and forbidden vertices**:
    - In the top left diagram, neither the photon, nor the Z boson can change the flavor of the lepton. The electron must stay an electron.
    - In the second top diagram from the left, conservation of charge requires that the emitted W boson has negative charge. This transforms the electron into a neutrino.
    - The electron does not carry color charge, the rightmost diagram thus does not exist
    - In the bottom left diagram, neither photon nor Z change the flavor of the color of a quark. The red u quark thus stays what it was.
    - To turn into a d quark, the u quark must emit a positively charged W. This way, electric charge is conserved in the middle diagram at the bottom.
    - Color charge is conserved, to change its color from red to green, the d quark in the bottom right diagram must thus emit a gluon of red-anti-green color. It must conserve its flavor, the strong force does not change that property.

- In the **next video**, we will visit the laboratory of the nuclear physics course at University of Geneva to see how our students go about to measure the Rutherford cross section.