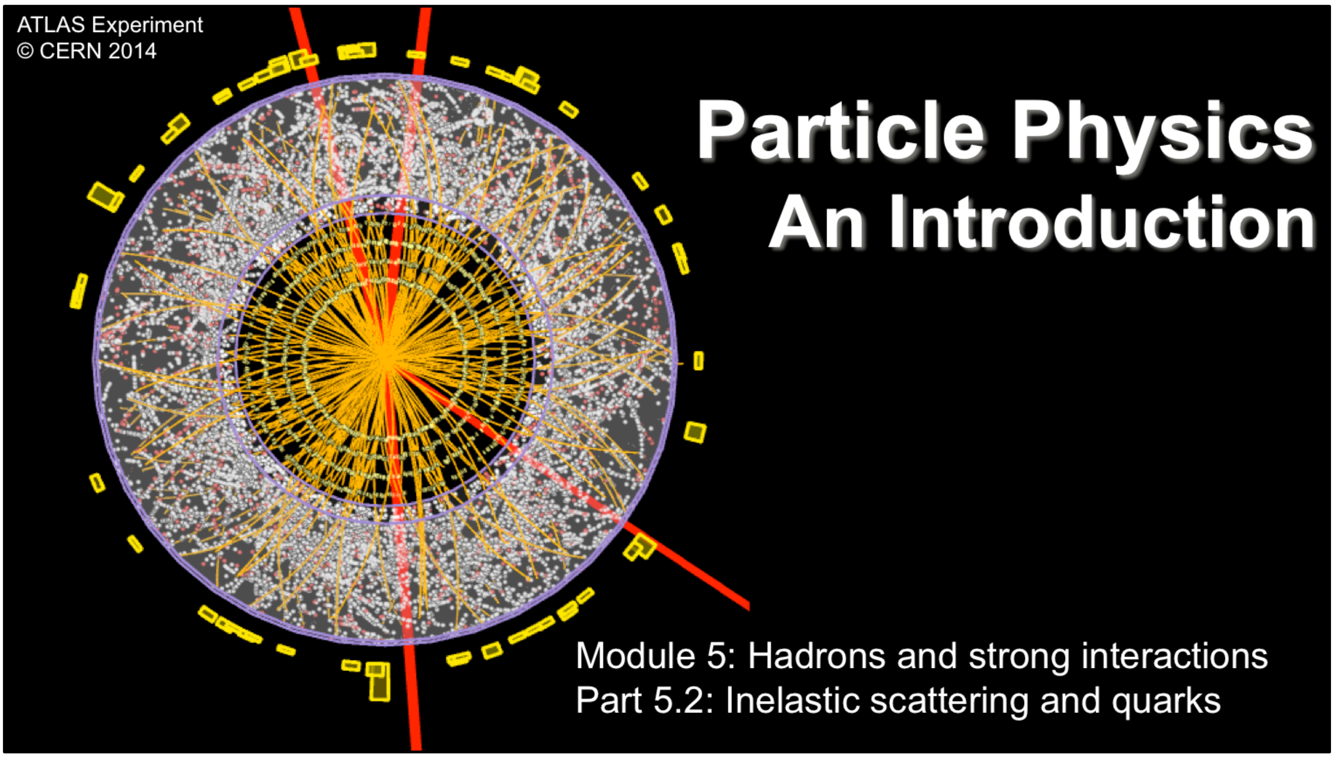
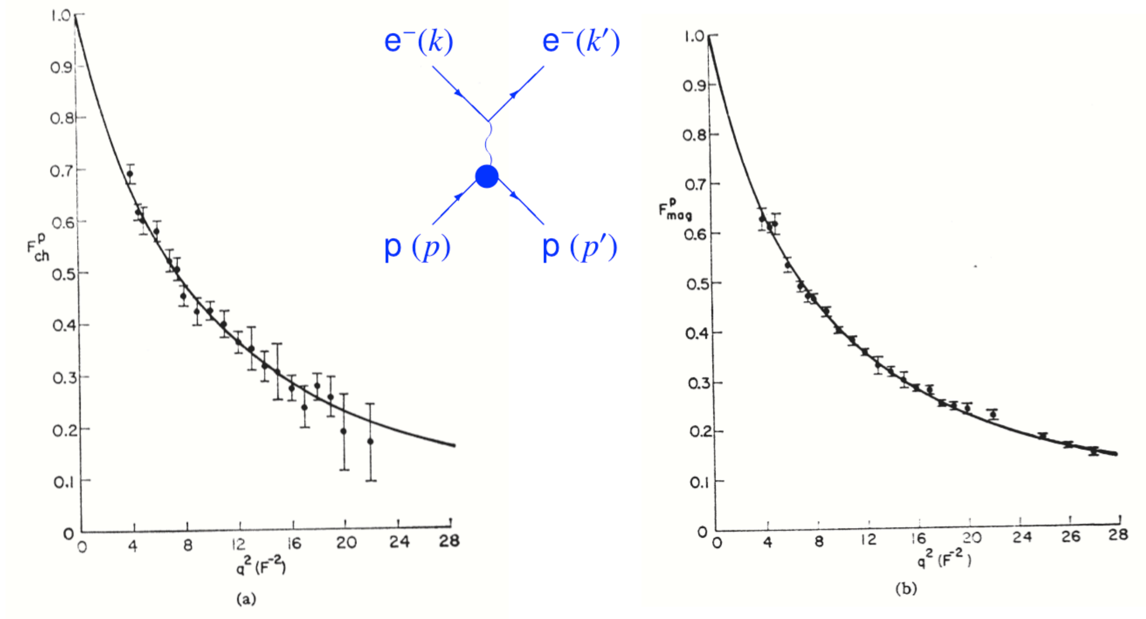



In this fifth module we are discussing the structure of hadrons and strong interactions.

In this 2nd video we discuss the inelastic scattering between electrons and nucleons and what one can learn from that.

After following this video you will know about:

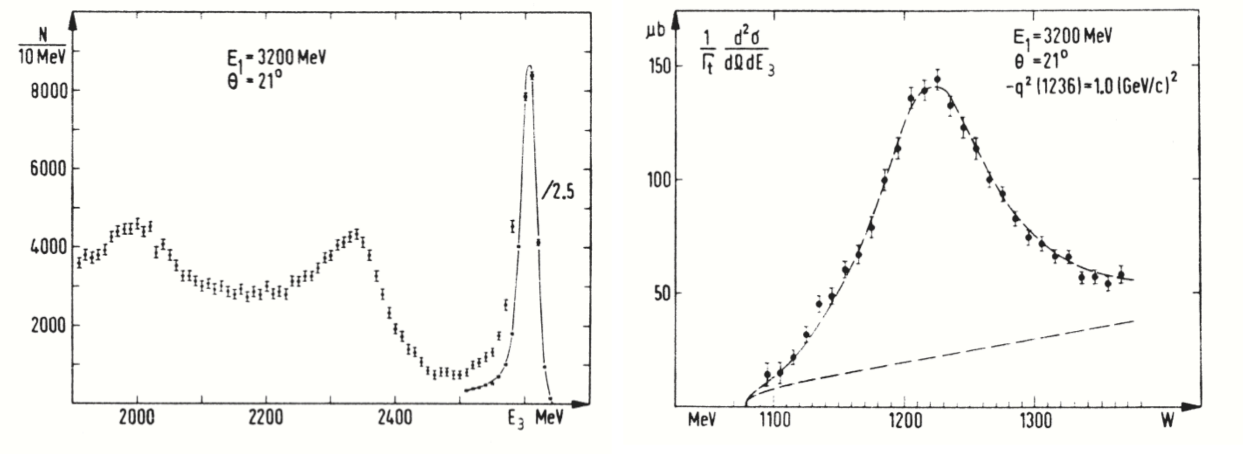
- Resonances as excited states of nucleons;
- Deep inelastic scattering and nucleon structure functions;
- The so-called scaling hypothesis as evidence for substructures inside the nucleon;
- As well as the role and distribution of quarks in the nucleon.




T. Janssen et al., Phys Rev. 142 (1966) 922

- **Form factors decrease rapidly with $q^2$**, like we showed in video 5.1. Consequently, the probability to observe an elastic scattering becomes low at high energy-momentum transfer.
- This is not surprising. Large $q^2$ correspond to a **short photon wavelength** which is then more and more able to resolve the internal structure of the target particle. This structure will then not be insensitive to the energy-momentum transfer: the target will be excited or even destroyed. In other words, **inelastic processes** take over.

$$
\begin{aligned}
e^- p \rightarrow e^- \Delta^+ &\rightarrow e^- p \pi^0 \\
&\rightarrow e^- n \pi^+
\end{aligned}
$$



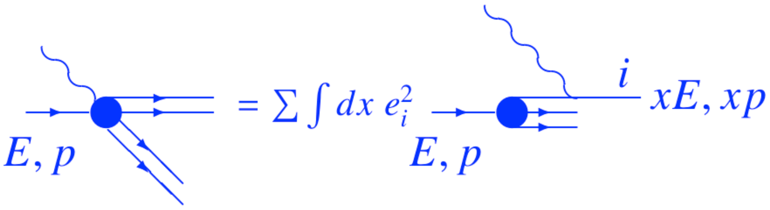

S. Galston et al, Phys. Rev. D5 (1972) 519

- At moderate $q^2$, inelastic processes produce **excited states** of the nucleon, also called **resonances**, such as $\Delta^+$, which have the same quantum numbers as the proton. These resonances have an extremely **short lifetime** and therefore a **broad mass distribution**.
- The figures show the results of an **electron-proton scattering experiment** at an initial energy of $E \approx 3$ GeV and at a fixed scattering angle.
- The inelasticity of the reaction is first visible in the **energy distribution** of the outgoing electron (left), which is no longer fixed as in the elastic case. Broad secondary maxima are formed, which correspond to **excited states** of the nucleon, also clearly visible in the **invariant mass distribution** of the outgoing hadronic state (right). Its kinematics is completely determined by measuring the outgoing electron, as in the case of elastic scattering. The position and inverse width of the large maximum indicate the mass and lifetime of the **resonance $\Delta^+$(1236).**

$$\left.\frac{d^2\sigma}{dE'd\Omega}\right|_{\text{lab}} = \left(\frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\right)\left\{W_2(\nu,q^2)\cos^2\frac{\theta}{2} + 2W_1(\nu,q^2)\sin^2\frac{\theta}{2}\right\}$$

- $\nu = E - E'$ : energy transfer e → p = photon energy
- $q^2$ : square of the photon invariant mass

- At higher $q^2$, beyond the resonance region, we enter into the region of the so-called **deep inelastic scattering**. The final state consists of multiple hadrons, with at least one baryon to conserve quantum numbers.
- This is the region where the photon interacts individually with the **charged constituents** of the nucleon. The cross section can be parameterized with two terms as before, but with two unknown **structure functions** $W_1$ and $W_2$ replacing the form factors. They parameterize the distribution and dynamics of the nucleon constituents. They depend on the transferred energy $\nu = E - E'$, which is the energy of the photon, and its invariant mass, $q^2$.
- The kinematics now requires a second, additional kinematic parameter, because the **mass of the hadronic system** is no longer fixed, as was the case with a single proton or resonance in the final state. In addition to the scattering angle, we can thus choose a second variable, here it is the energy $E'$ of the outgoing electron. The structure functions are measured experimentally analyzing the dependence of the cross section on the scattering angle, like in the elastic case.



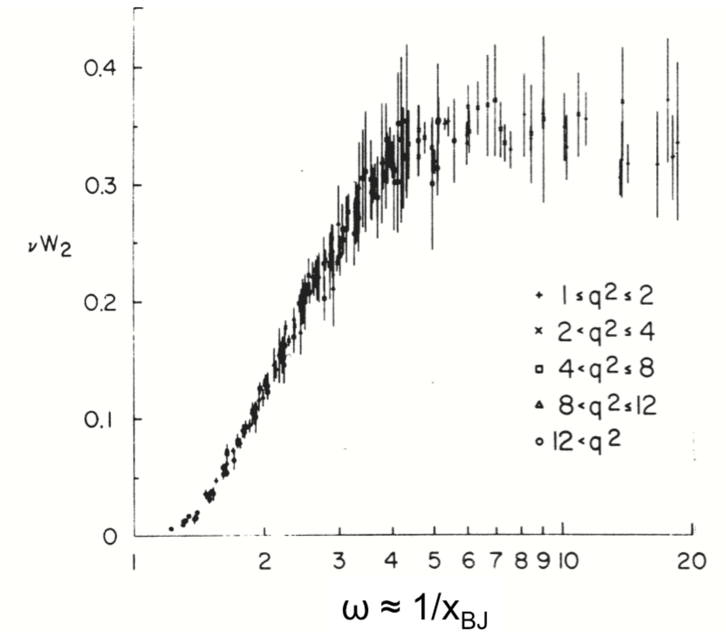

$$W_i(\nu, q^2) \rightarrow W_i(x_{\mathrm{BJ}}) \quad ; \quad x_{\mathrm{BJ}} = \frac{q^2}{2M\nu}$$

$$\nu W_2(\nu, Q^2) \underset{q^2\to\infty}{\longrightarrow} F_2(x) = \sum_i e_i^2 f_i(x) x \quad ; \quad M W_1(\nu, Q^2) \underset{q^2\to\infty}{\longrightarrow} F_1(x) = \frac{1}{2x} F_2(x)$$

$$f_i(x) = dp_i/dx$$

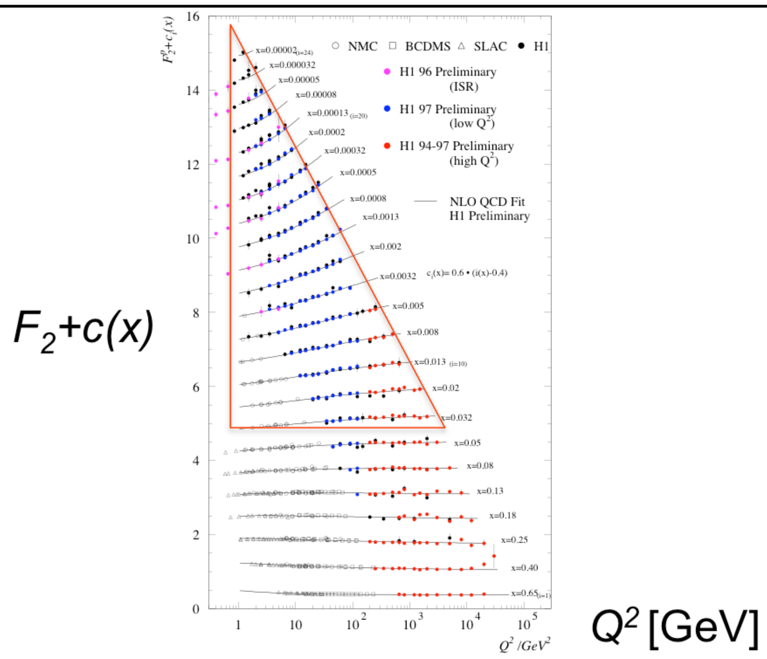
J.D. Bjorken et E.A. Paschos, Phys. Rev. 185 (1969) 1975

- The big discovery at SLAC, in the late 60's, was that the **structure functions do not depend on $\nu$ and $q^2$ separately**, but on their ratio $x_{BJ} = q^2/2M\nu$, with $M$ the proton mass. The variable is named after of its inventor, James D. Bjorken.
- The phenomenon was called **"scaling"** at the time, because of the very general observation that the dimensionless variable $x_{BJ}$ does not depend on any energy nor mass scale.
- Structure functions follow in fact scaling if inside the nucleon there exist **substructures** with which the photon interacts elastically.
- This interpretation of scaling is best demonstrated in the limit of large $q^2$, where the interaction between quarks is negligible compared to the force transmitted by the photon. Quarks then act as **quasi-free particles,** with which the photon interacts in an incoherent manner.
- To respect kinematic constraints in such an **elastic electron-quark scattering**, the photon can only interact with a quark which carries a fraction $x_{BJ}=p_q/p_P$ of the nucleon momentum. Consequently, the cross section is proportional to the probability to find such a quark in the nucleon.
- In the limit $q^2 \rightarrow \infty$, the structure functions $W_i$ thus tend toward functions $\boldsymbol{F_i(x)}$, which are directly related to the **distribution of the fractional momentum** $x_i$ carried by quarks of type $i$, with $f_i(x) = dp_i/dx$.



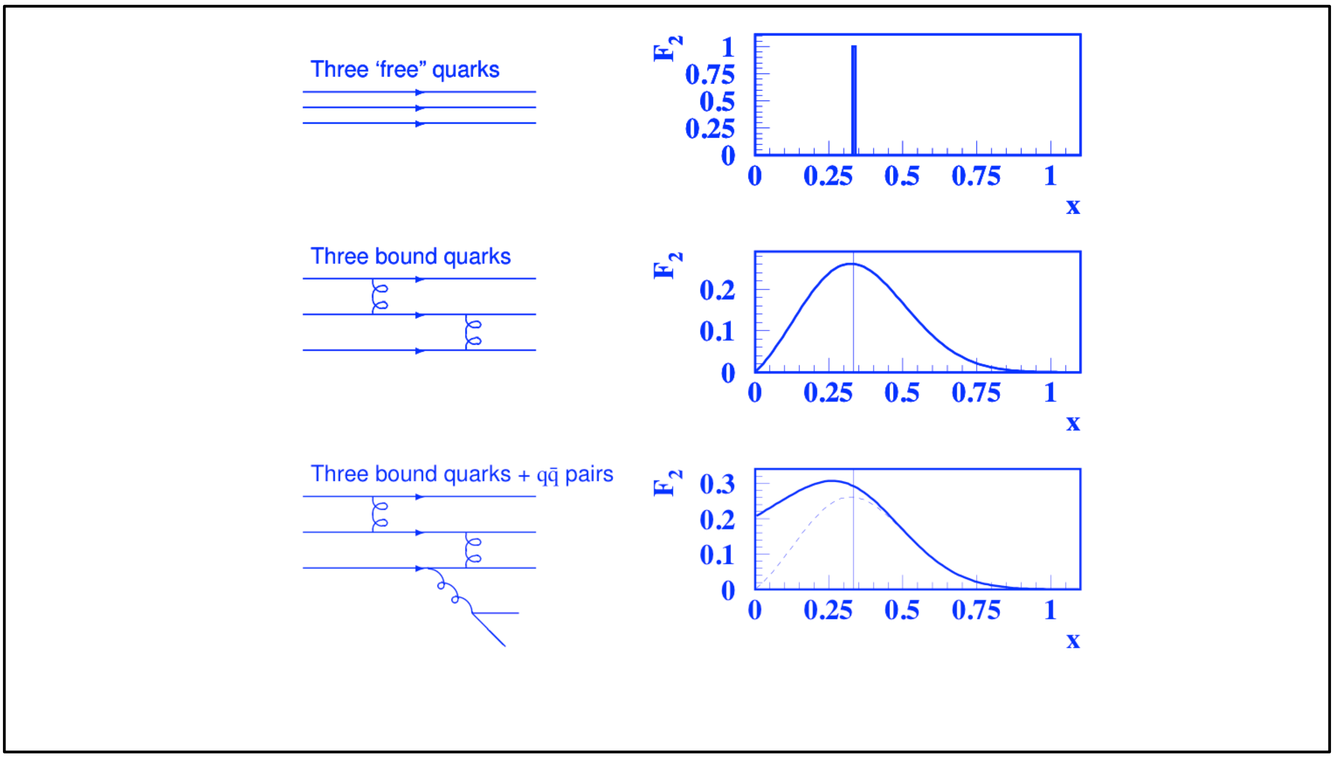

M. Breidenbach et al., Phys. Rev. Lett. 23 (1969) 935; F.J. Gilman, Phys. Rept. 4 (1972) 95

- The experiments at SLAC have in fact shown that the phenomenon of scaling already occurs at **moderate $q^2$.**
- The structure functions $W_2$ as a function of the kinematic variable $\omega \approx 1/x_{BJ}$ smoothen with increasing $q^2$ beyond the resonance region, and then follow a **universal function** of $x_{BJ}$ independent of $q^2$.
- The kinematic constraint imposed by **elastic scattering off point-like particles** inside the proton is indeed respected.

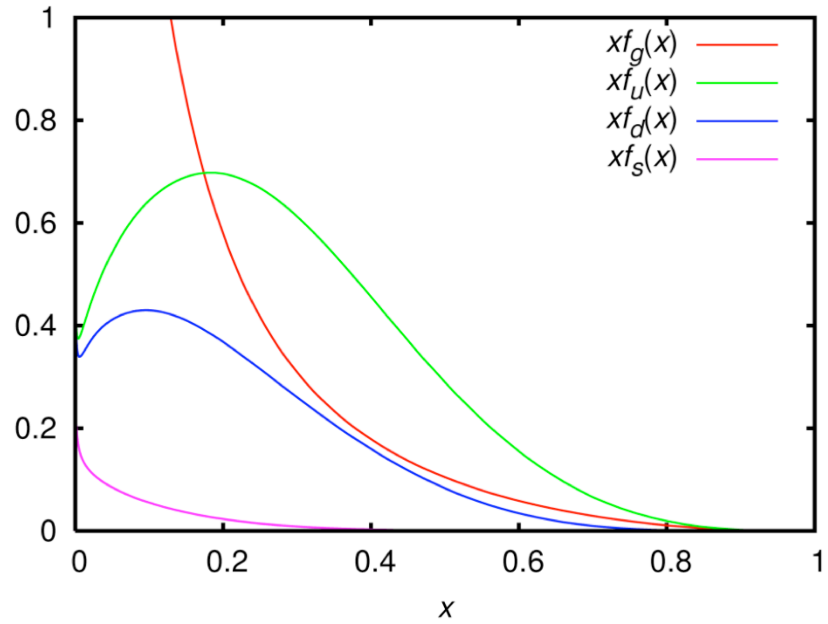



http://www-h1.desy.de/

- However, the **scaling** of structure functions is **not exact**. This figure shows modern measurements that come from experiments at the electron-proton collider **HERA**. The curves and data for different *x* are displaced by a constant in order to be all visible in the same graph.
- At **small *x***, so at small quark momenta, the structure functions vary strongly with $q^2$ because of the contribution from virtual gluons enriching the quark contents. The smaller deviations at **large *x*** are more subtle: the coupling constant of strong interactions depends on the momentum transfer, it is not really constant. We will come back to this astonishing fact in video 5.4.




- Structure functions can thus be used to measure the **momentum distribution of quarks** inside nucleons. We qualitatively demonstrate the behavior of the structure function $F_2(x)$ based on successively refined hypotheses on the dynamics of quarks:
  - If the nucleon consisted of **three quarks without interaction**, $x$ would be fixed to 1/3 for each quark and $F_2$ would be a δ function at this value.
  - If it were a state where **quarks are bound together by gluons**, the average value of $x$ would still be 1/3, but with a wider distribution around this value.
  - If we finally take into account that quarks can emit **virtual gluons**, which can split into a quark-antiquark pair of low energy, we expect that the quark momentum distribution tends to fill up at $x \rightarrow 0$.




http://en.wikipedia.org/wiki/Parton_(particle_physics)

- The **experimental data** indeed follow this qualitative picture.
- For so-called **valence quarks**, i.e. 2 u and 1 d for the proton, the structure function varies around 1/3, with a strong **enhancement at small *x*** by additional quark-antiquark pairs produced by gluons. The integral of the distribution is twice as large for the u quark than for the d quark, as it should be.
- The contribution of virtual gluons is indeed reflected in the **contribution of s and s-bar quarks**, which would not be present in the nucleon otherwise.
- When integrating these distributions, we see that only about half of the proton momentum is carried by quarks, the rest is carried by gluons.

- In the next video we discuss mesons, bound states between quarks and antiquarks.