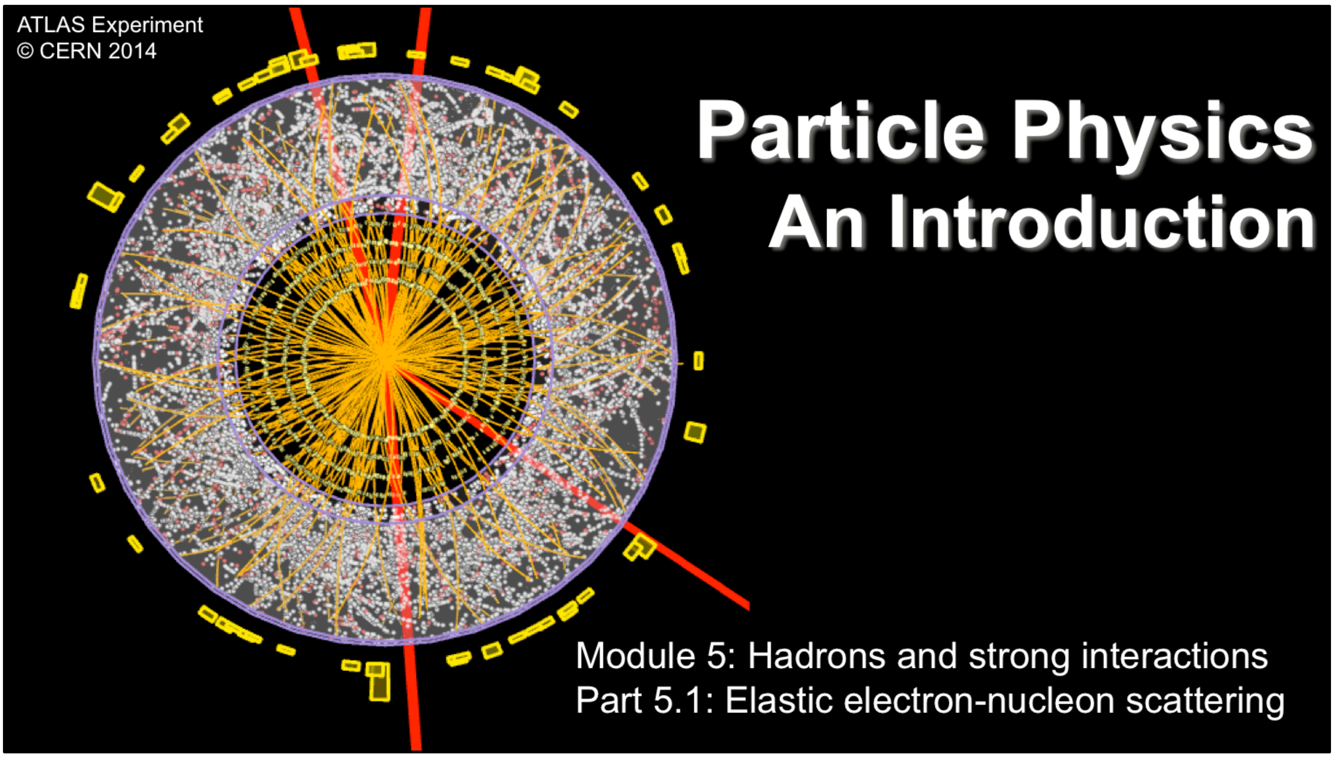
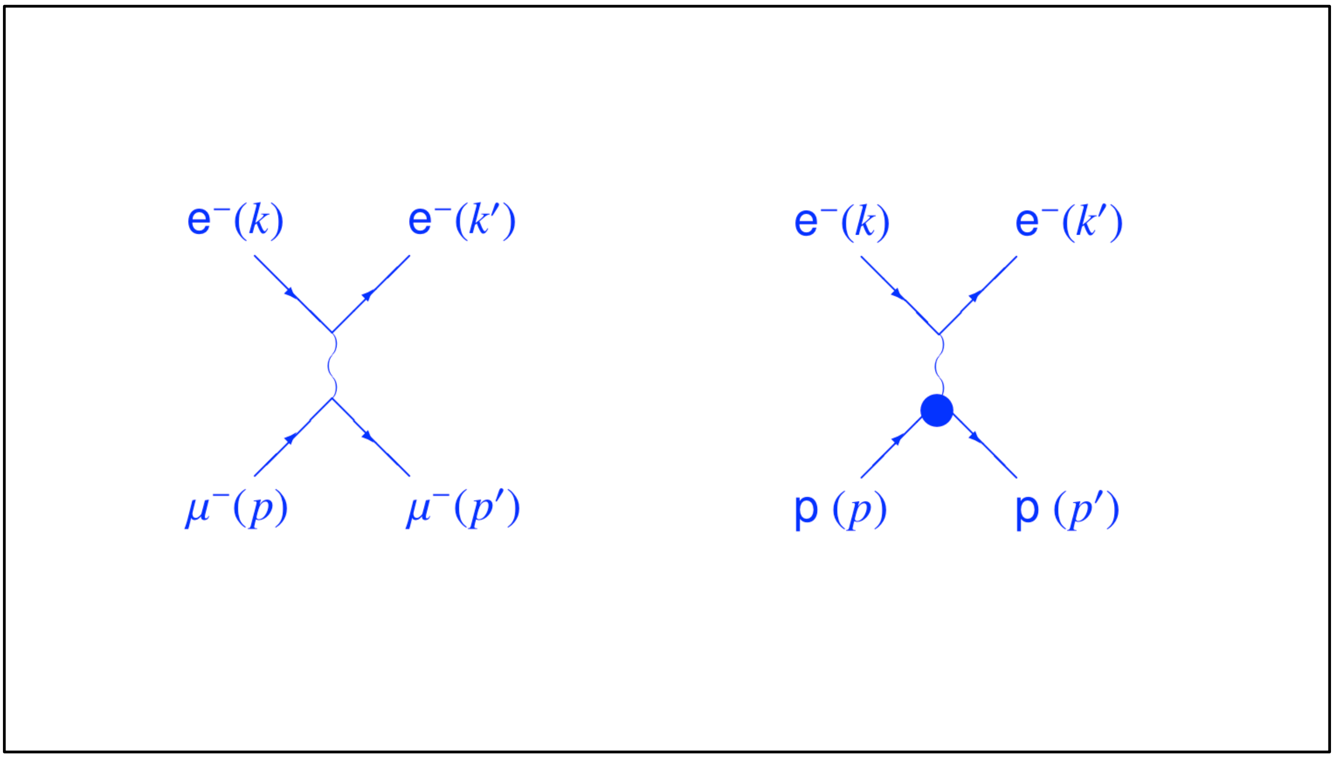


In this fifth module we discuss the structure of hadrons and strong interactions.

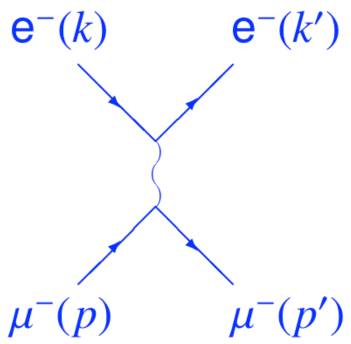
In this 1st video we discuss the elastic electromagnetic scattering between electrons and nucleons, which is one of the ways used to study the structure of hadrons.

After following this video you will know:

- The main properties of elastic scattering between fermions and the meaning of different terms entering the cross section.
- The form factor concept and its interpretation in terms of the size of the nucleon.
- The historic experiments at the Stanford Linear Accelerator Center in this area.



- When the structure of an interaction is known, it can be used as a tool to better understand the particles involved. We'll see in this video how elastic **electron-nucleon scattering** is used to determine the internal **structure of nucleons**.
- We take advantage of the fact that the **electron** − as far as we know - is a **point-like particle**, and that we know **electromagnetic interactions** very well. We can thus elucidate a single but important unknown, which is the **distribution and dynamics of quarks** inside the proton and neutron, represented by the big blue dot in the right diagram.
- We'll see in video 5.3 how **meson resonances** are formed, by an incident photon for example. These can be used to better understand the strong force which binds quarks and antiquarks together.



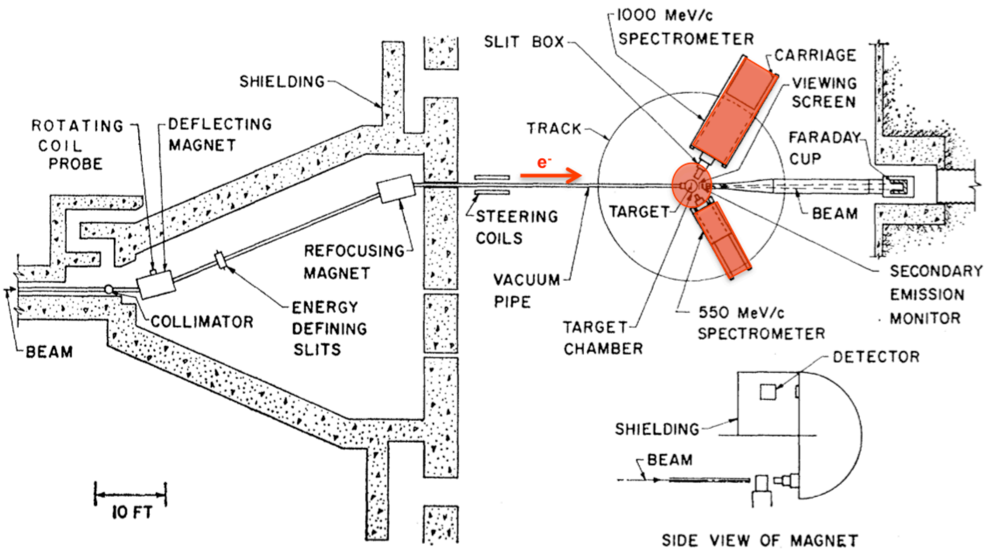
$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Rutherford}} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}}$$

$$\frac{1}{q^2} = \frac{1}{(k'-k)^2} \simeq \frac{1}{2(k_0 k_0' - \vec{k}\vec{k}')} \simeq \frac{1}{4E^2 \sin^2 \frac{\theta}{2}}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Recoil}} \simeq \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \left[\frac{\cos^2 \frac{\theta}{2}}{1 + \frac{2E}{M} \sin^2 \frac{\theta}{2}}\right]$$

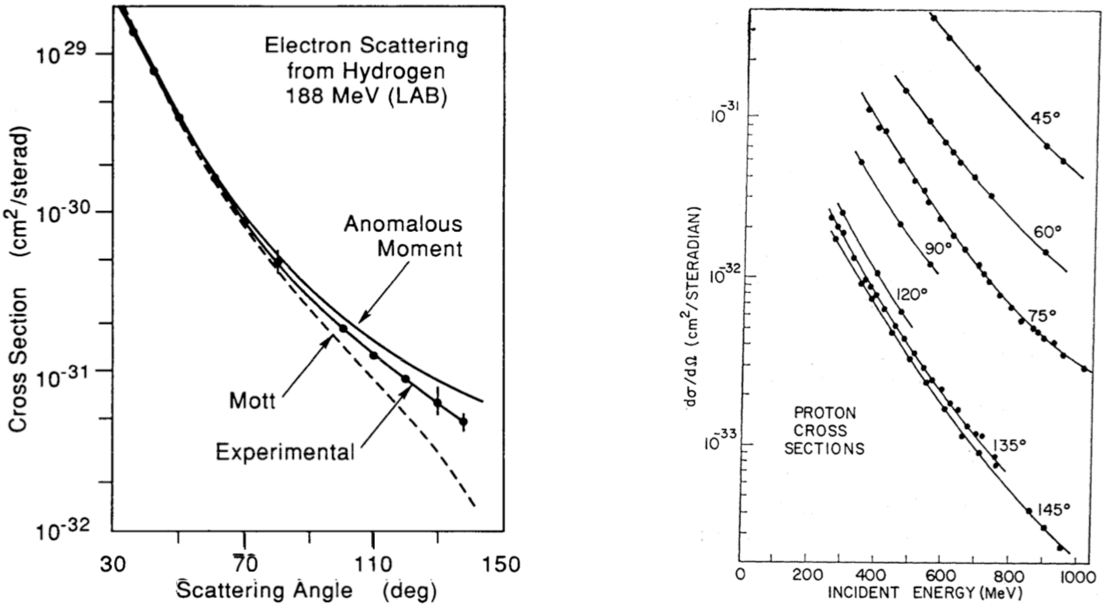
$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \left[\frac{1}{1 + \frac{2E}{M} \sin^2 \frac{\theta}{2}}\right] \left[\cos^2 \frac{\theta}{2} + \frac{q^2}{2M^2} \sin^2 \frac{\theta}{2}\right]$$

- The prototype process for **the scattering between point-like fermions** is $e^- \mu^- \rightarrow e^- \mu^-$. It is very similar to e^+e^- annihilation, with the Feynman diagram rotated by 90°. We will approach this process in steps.
- The **first step** is to consider scattering off a very **heavy target**, which is **without structure and without spin**. Also ignoring the spin of the incoming electron and neglecting its mass, we obtain the **Rutherford cross section** which we already encountered in video 1.4.
- As usual, the factor α^2 comes from the two electromagnetic vertices, the denominator contains the **square of the characteristic energy**. It results from the propagator of the virtual photon, $1/q^2$. You can easily verify this in the approximation $m_e = 0$, as I have done here.
- The **angular distribution** is extremely steep for the scattering of point-like particles, and peeks at small angles.
- This remains valid when including the effects of the **target recoil** and the **spin of the particles**. The additional term in red, with M mass of the target, takes into account its recoil. It is the ratio between the incident and outgoing electron energy, E'/E. This ratio is 1 for an infinitely heavy target, but less than 1 if an important recoil occurs.
- This formula still neglects the **magnetic component** of the interaction. For a point-like fermion target with magnetic moment $\mu = e/2M$, one finds the complete **Mott formula**, quoted here in the laboratory frame and for a relativistic electron projectile.
- The **last term in red** is of a magnetic nature, it becomes important at high momentum transfers, $q^2 >> M^2$. The **angular distribution** still roughly follows the form of the Rutherford formula. The third term contains sub-terms proportional to $\cos^2\vartheta$ and $\sin^2\vartheta$, which reduce a little the steepness of the angular distribution.



http://www.nobelprize.org/nobel_prizes/physics/laureates/1990/taylor-lecture.pdf

- The **historic experiments** on elastic scattering between electrons and protons have been conducted at Stanford Linear Accelerator Laboratory SLAC in the 1960's and 70's.
- The experiment consisted of a **liquid hydrogen target**, bombarded by electrons of a few hundred MeV. In the final state, only the electron is observed in two spectrometers positioned at variable scattering angles. The kinematics of the hadronic final state is then deduced from energy-momentum conservation.



http://www.nobelprize.org/nobel_prizes/physics/laureates/1990/taylor-lecture.pdf

- Here are some results on the **cross section as a function of the scattering angle** and **as a function of the incident electron energy**. The measurements confirm the calculation at these modest energies below 1 GeV.
- One observes a **steep angular distribution** in the left plot and **strong reduction with energy** in the right plot, as expected.

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} |F(\vec{q})|^2$$

$$F(\vec{q}) = \int \rho(\vec{x})\, e^{i\vec{q}\vec{x}}\, d^3x \quad ; \quad \int \rho(\vec{x})\, d^3x = 1$$

$$F(\vec{q}) = \int \left(1 + i\vec{q}\vec{x} - \frac{(\vec{q}\vec{x})^2}{2} + \cdots\right) \rho(\vec{x}) d^3x$$

$$\rho(\vec{x}) = \rho(r) \quad \rightarrow \quad F(\vec{q}) = \int \left(1 - \frac{(\vec{q}\vec{x})^2}{2} + \cdots\right) \rho(\vec{x}) d^3x = 1 - \frac{1}{6}|\vec{q}|^2 < r^2 >$$

$$\rho(r) \sim e^{-\Lambda r} \quad \rightarrow \quad F(|\vec{q}|) \sim \left(1 + \frac{|\vec{q}|^2}{\Lambda^2}\right)^{-2}$$

- Let us now consider what happens if the **target is not a point-like particle**.
- Let us first take the model of a **static charge distribution** *ρ(x)*, normalized so that the charge of the target remains elementary, i.e. that the integral of its charge density equals 1. The cross section will then be **reduced** with respect to a point-like target, by a factor $|F|^2$, a so-called **form factor**. We have already used this concept in video 2.2. In the static case, it is simply the Fourier transform of the spatial charge distribution.
- For **small momentum transfers**, one can develop the form factor in powers of *qx*.
- If the distribution possesses **spherical symmetry**, ie if *ρ(x) = ρ(r)*, terms with odd exponent do not contribute. Thus the form factor measures the **mean square radius** of the charge distribution, $<r^2>$, which represents the size of the distribution.
- Take for example an **exponential distribution**. It leads to a so-called dipolar form factor.

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{Lab}} = \left(\frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\right)\frac{E'}{E}\left(\frac{G_E^2+\tau G_M^2}{1+\tau}\cos^2\frac{\theta}{2}+2\tau G_M^2\sin^2\frac{\theta}{2}\right)$$

$$\tau \equiv \frac{-q^2}{4M^2}$$

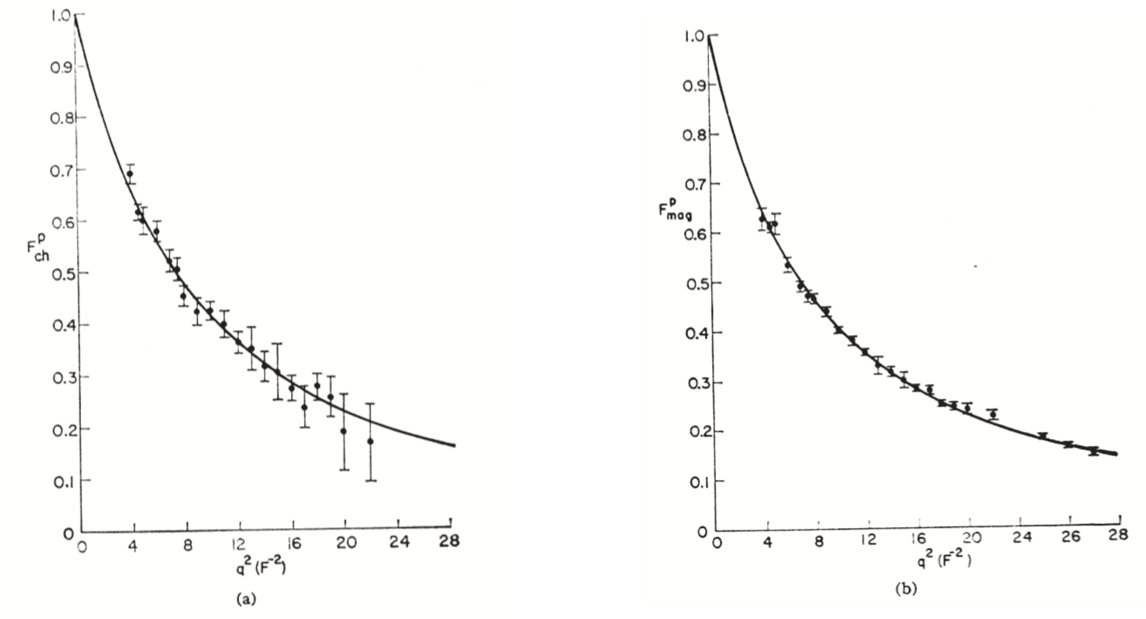
- It is clear that in the case of a **non-static charge distribution** things get more complicated.
- First, **magnetic interactions** come in.
- Second, the charge distribution itself changes during the time represented by the time-like component, q_0, of the four-momentum transfer. By the action of moving charges inside the distribution, **electric and magnetic terms** will be modified and their **functions mixed up**.
- We then find the cross section given here, with the parameter τ, which is the **ratio between momentum transfer and target mass squared**, i.e. the ratio of the two quantities, which characterize the process.
- We call G_E and G_M the **electric and magnetic form factors**, although the distinction between these two obviously depends on the reference frame.
- In the same sense, one can attribute G_E and G_M to the Fourier transforms of the charge and magnetic moment distribution of the target particle, even though this interpretation is only valid in a **very special reference frame**.

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{Lab}} = \left(\frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}}\right) \frac{E'}{E} \left(\frac{G_E^2 + \tau G_M^2}{1+\tau} \cos^2 \frac{\theta}{2} + 2\tau G_M^2 \sin^2 \frac{\theta}{2}\right)$$

$$G_E \sim G_M \sim 1/(1 + q^2/\Lambda^2)^2 \quad ; \quad \Lambda \simeq 0.84 \text{ GeV}$$

$$< r^2 >= 6\left(\frac{dG_E(q^2)}{dq^2}\right)_{q^2=0} \simeq (0.81 \times 10^{-13}\text{cm})^2$$

- The form factors G_E and G_M of the proton are measured by **analyzing the differential cross section** for the reaction $e^- p \rightarrow e^- p$, and separating the terms proportional to $cos^2\vartheta$ and $sin^2\vartheta$ of the angular distribution.
- We observe that G_E and G_M follow the **dipolar shape** predicted for exponential distributions. The parameter Λ that characterizes the size of the distribution, is $\Lambda \approx$ 0.84 GeV in both cases. The **size of the distribution** is thus of order 1 Fermi, and the same for G_E and G_M.
- Finding the same distribution for both form factors is not a miracle: the **quarks** inside the nucleon are at the origin of both charge and magnetic moment distribution.



T. Janssen et al., Phys Rev. 142 (1966) 922

- The figure shows representative results obtained by **Hofstadter and collaborators at SLAC.**
- Form factors G_E (left) and G_M (right) for the proton are measured as a function of momentum transfer squared, and extracted from the cross section for the reaction $e^- p \Rightarrow e^- p$.
- One finds the **dipolar shape**, $G \sim 1/(1 + q^2/\Lambda^2)^2$, with the **same size parameter** $\Lambda \approx$ 0.84 GeV.

- In the next video we will see what happens when we increase q^2 such that the exchanged photon can excite or even break the nucleon.