

## Problem set # 1

### Numerical Methods for Data Science 2021/22

#### UC3M — *Master on Statistics for Data Science*

**Due date:** October 15. Value: 50% of the final grade.

Note: This is an individual assignment. Evidence of plagiarism will be penalized. Hand in the assignment as a pdf file, with Gurobi-Python code in the pdf (no python files).

**Problem 1 (50 points).** Consider the linear optimization problem

$$\begin{aligned} & \text{maximize } 2x_1 + 3x_2 - 4x_3 \\ & \text{subject to:} \\ & 3x_1 + 5x_2 + 2x_3 = 15 \\ & x_1 + 3x_2 - 4x_3 = 8 \\ & x_1 \geq 0, x_2 \geq 0, x_3 \geq 0. \end{aligned}$$

- (a, 10 points) Formulate the dual problem, and find all its solutions using the graphical method.
- (b, 10 points) Formulate the optimality conditions that must be satisfied by any optimal primal solution in relation with a dual optimal solution  $\pi^*$ . Apply them, along with part (a), to find all solutions of the primal problem.
- (c, 10 points) Obtain all possible values for the reduced cost of each primal variable. Are reduced costs unique? Interpret the reduced costs obtained.
- (d, 10 points) Carry out a sensitivity analysis with respect to simultaneous changes of constraint right-hand sides for the primal problem. Contrast the results with those obtained through Gurobi-Python.
- (e, 10 points) Carry out a sensitivity analysis with respect to simultaneous changes of objective coefficients for the primal problem. Contrast the results with those obtained through Gurobi-Python.

**Problem 2 (50 points).** In a CSI investigation, the crime suspect left both her/his hand and shoe imprints in the crime scene. From that evidence the investigators want to infer the suspect's height. For that purpose, they plan to obtain a prediction equation for height using hand and shoe size based on the following sample data:

| Hand size (cm) | Shoe size (cm) | Height (cm) |
|----------------|----------------|-------------|
| 17.9           | 30.1           | 176.2       |
| 18.2           | 29.5           | 176.8       |
| 18.5           | 30.4           | 184.2       |
| 16.9           | 31.6           | 173.2       |
| 17.3           | 27.4           | 172.8       |
| 17.9           | 28.3           | 174.1       |
| 18.1           | 33.4           | 180.5       |

- (a, 30 points) Formulate the Linear Optimization model seen in class for estimating the best prediction equation under the Mean Absolute Error (MAE) criterion, and implement it in Gurobi-Python.
- (b, 10 points) Solve the model and give the optimal solution (prediction equation). Is it unique?
- (c, 10 points) Obtain the optimal dual solution and discuss its possible interpretation.