Multiple Regression

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1. Show that the properties of least squares estimators are satisfied using the following definitions:

$$\hat{\beta} = (X'X)^{-1}X'Y$$
$$\hat{Y} = X(X'X)^{-1}X'Y = HY$$
$$\hat{\epsilon} = Y - \hat{Y} = (I - H)Y$$

The residuals are orthogonal to the predictors.

The sum of the residuals is zero.

This property is equivalent to $x_i^t\hat{\epsilon} = 0$ or in matrix form $X'\hat{\epsilon} = 0$, this also includes the second result due to the fact that X contains a column of ones in order to account for the intercept term. To see why this is true consider.

$$X'\hat{\epsilon} = X'(I - H)Y = X'(I - X(X'X)^{-1}X')Y = (X' - X'X(X'X)^{-1}X')Y = (X' - X')Y = 0$$

The sum of the observed data is equal to the sum of the fitted values:

To prove this we will use the following descomposition of the Y vector.

$$Y = HY + (I - H)Y = \hat{Y} + \hat{\epsilon}$$

Then multiplying with a column of ones and using the ortogonality of the ones with the error term

$$1^tY = 1^t\hat{Y} + 1^t\hat{\epsilon}$$
$$\sum_{i=1}^n Y_i = \sum_{i=1}^n \hat{Y}_i + 0 = \sum_{i=1}^n \hat{Y}_i$$

The residuals are orthogonal to the fitted values.

Let's calculate the product

$$(\hat{Y})'\hat{\epsilon} = (HY)'(I - H)Y = Y'H(I - H)Y = Y'(H - H)Y = Y'0Y = 0$$

Here we used the fact that H is simetric and H is idempotent ($H^2 = H$).