U.N. Sede Medellín

Una universidad con criterio nacional y presencia regional

Procesamiento de Señales
Correlation of Signals

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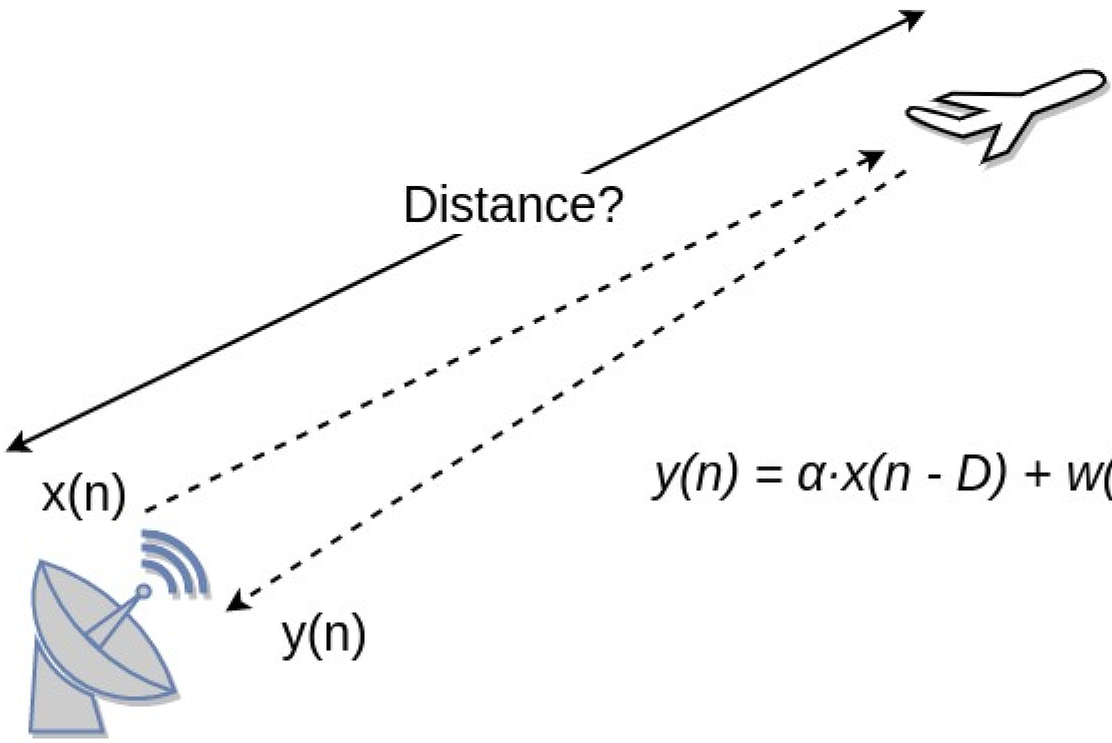
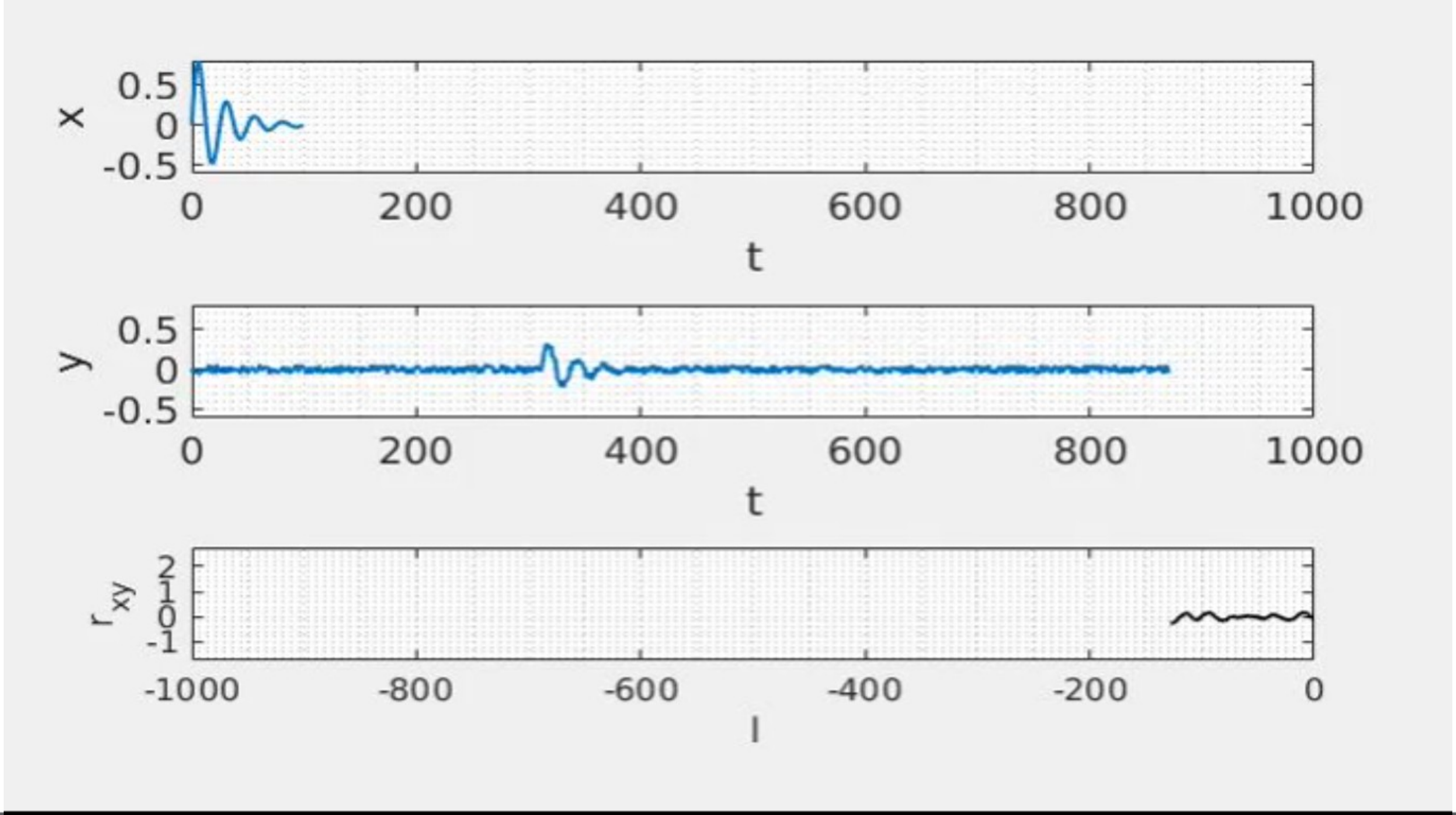
Starting point: A radar system



$$y(n) = \alpha \cdot x(n - D) + w(n)$$

Starting point: A radar system



It’s all about time…

How to compute the distance after finding the integer delay given by l?

Cross Correlation formulation

$$r_{xy}(l)=\sum_{n=-\infty}^{\infty} x(n)\cdot y(n-l)$$

$$r_{xy}(l)=\sum_{n=-\infty}^{\infty} x(n+l)\cdot y(n)$$

The l value represents the relative delay among the x and y signals. So it is clear that:

$$\boxed{r_{xy}(l)\neq r_{yx}(l)}$$ Any Ideas?

Cross Correlation formulation

$$r_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n+l) \cdot y(n)$$

$$r_{yx}(l) = \sum_{n=-\infty}^{\infty} y(n) \cdot x(n-l)$$

$$\boxed{r_{xy}(l) = r_{yx}(-l)}$$

Let’s perform a Cross Correlation *”by hand”*

$$x(n)=\{-4,2,\underline{7},0,-1\}$$

$$y(n)=\{0,1,3,0,-2,\underline{4},-4,8,-10\}$$

$$r_{xy}(l)=?$$

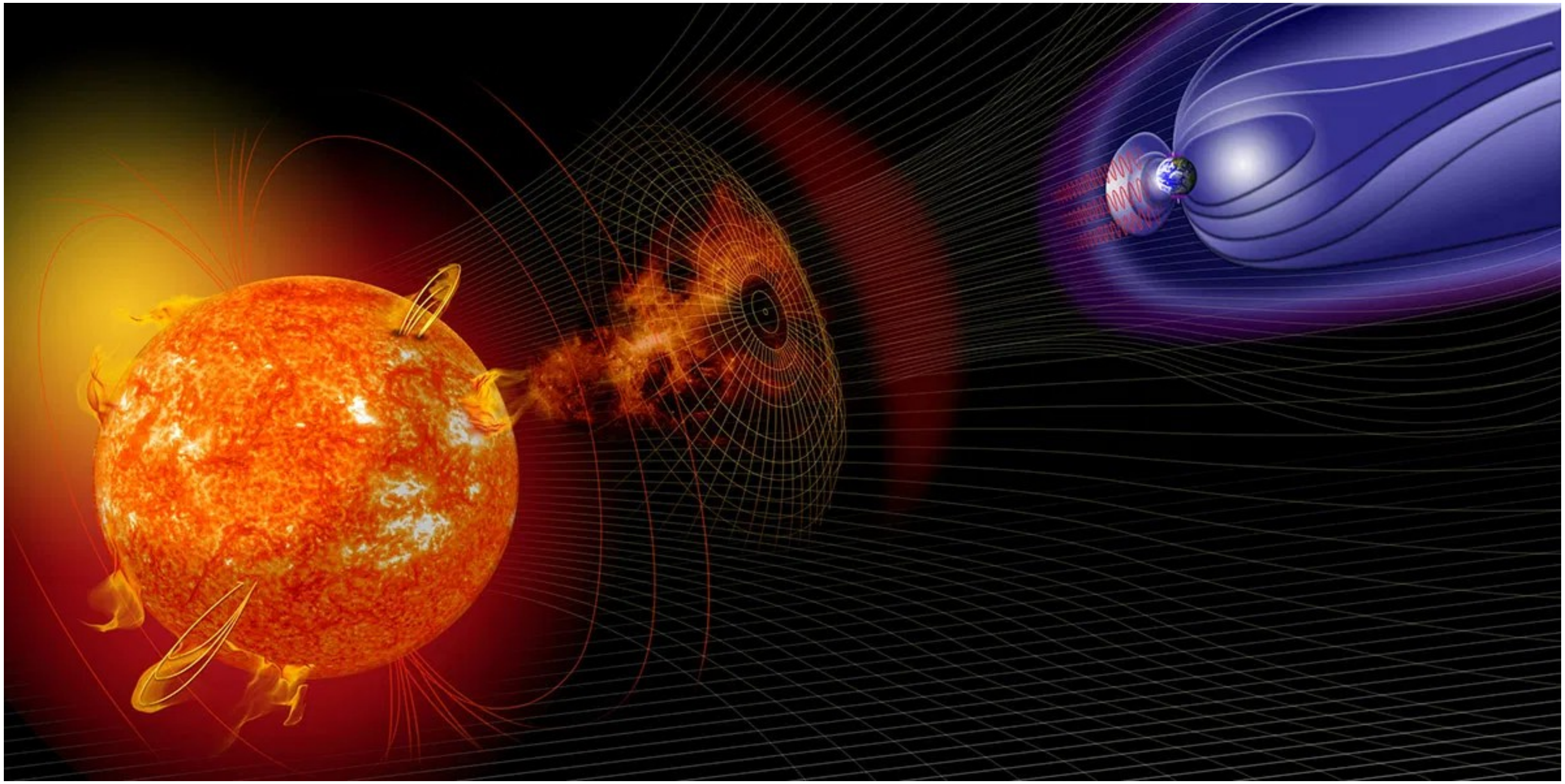
Autocorrelation of a signal

- Autocorrelation is often used to estimate the periodicity of a repetitive signal.
- Autocorrelation is also useful to compute the *spectral energy density* of a given signal.

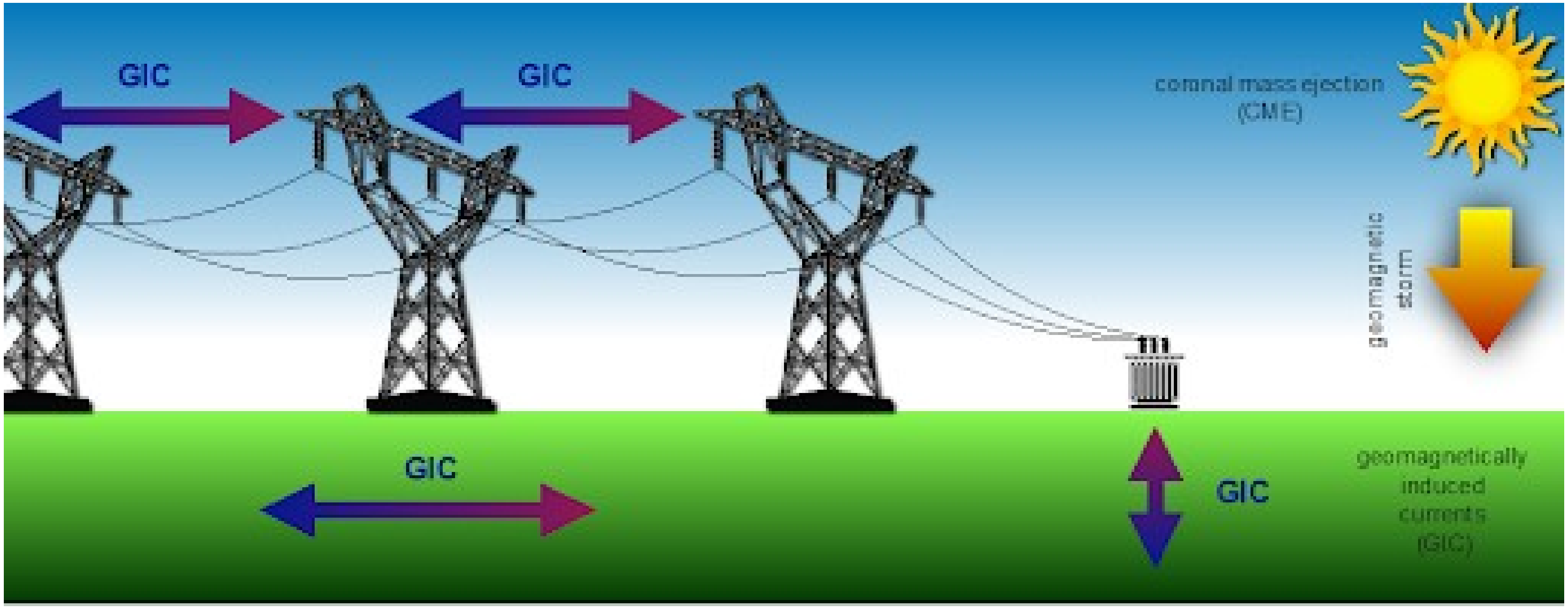
$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n) \cdot x(n-l)$$

An Autocorrelation example

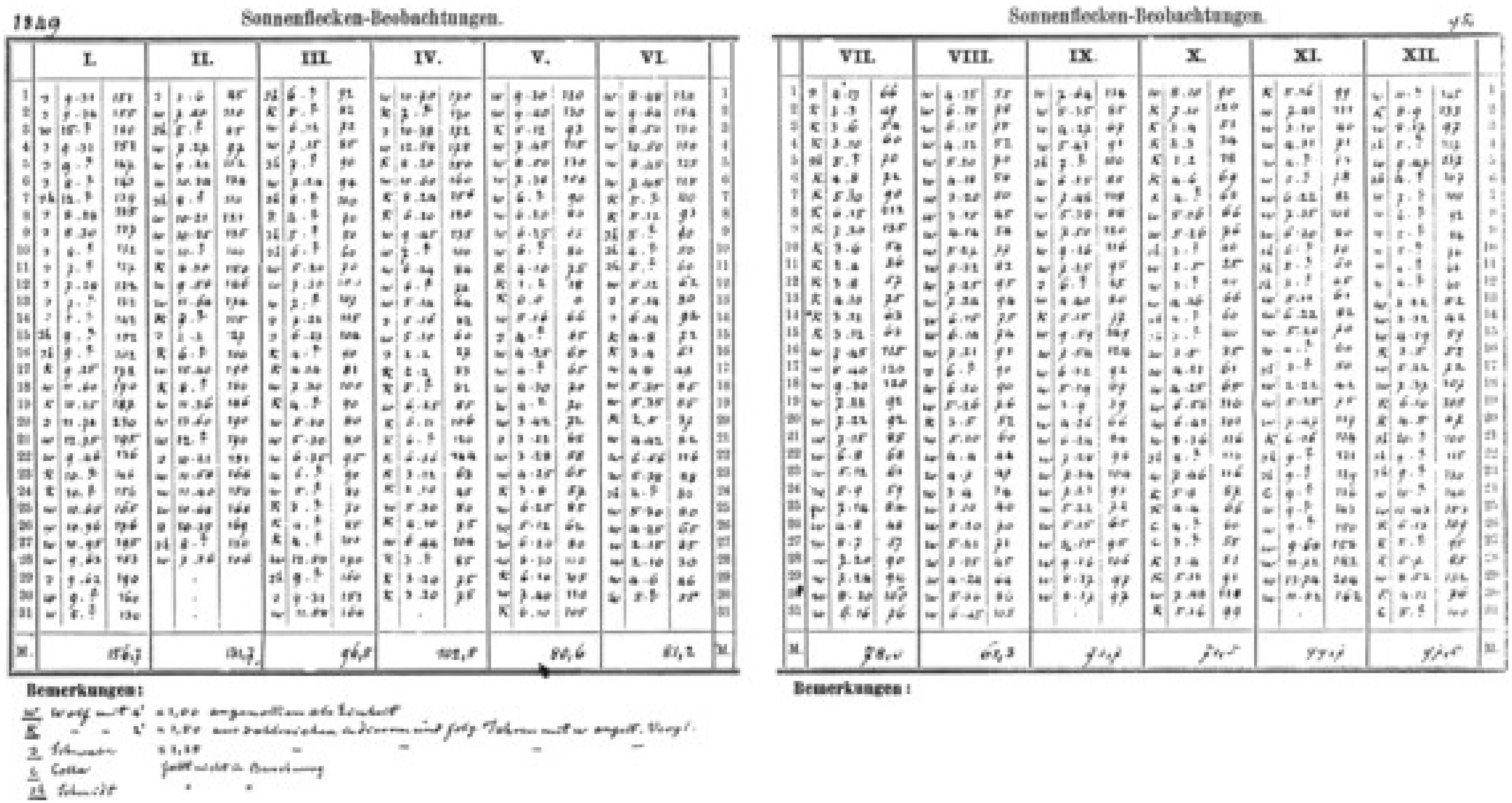
An example: The Wölfer Sunspots (I)



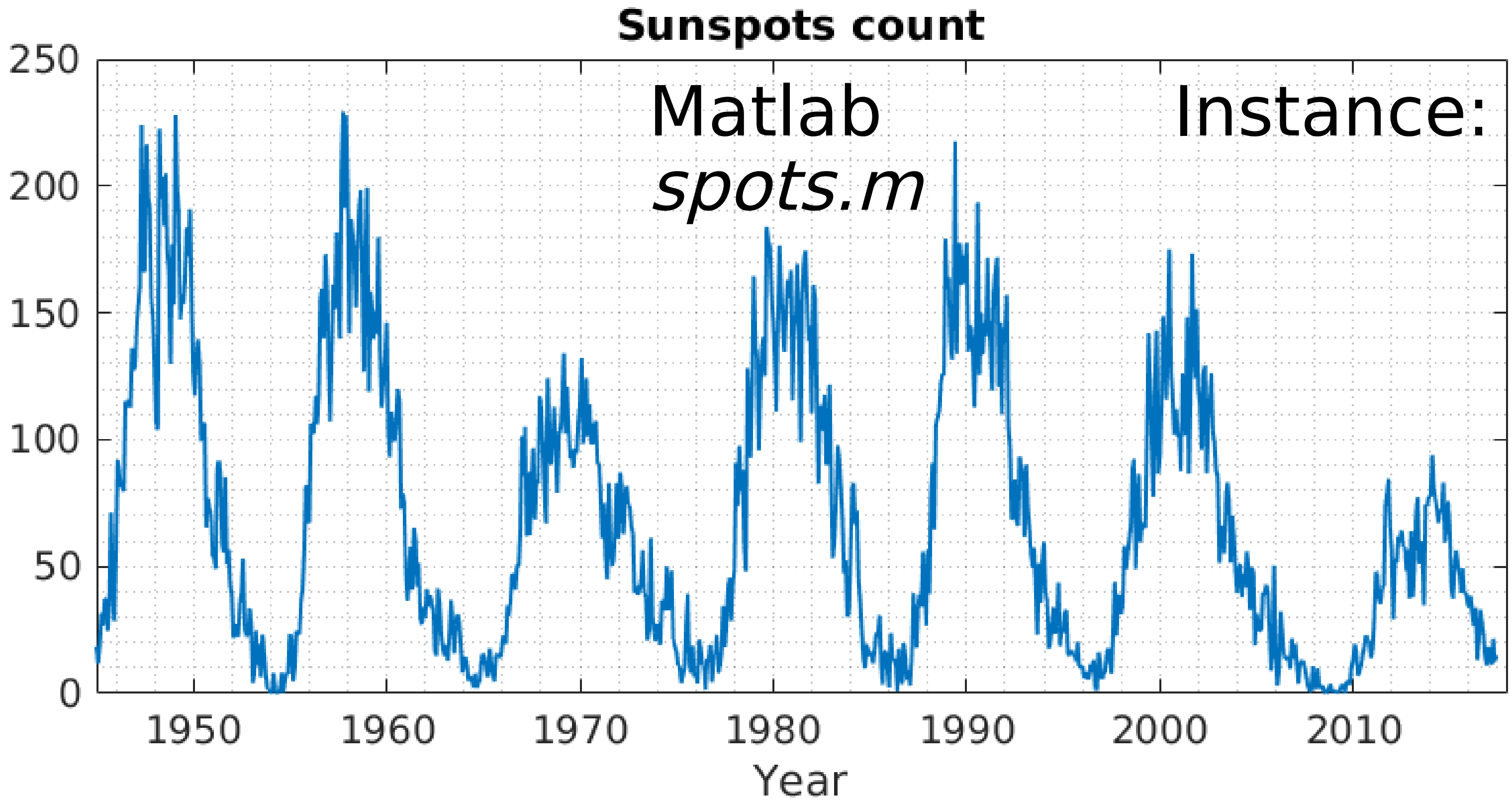
An example: The Wölfer Sunspots (I)



An example: The Wölfer Sunspots (II)

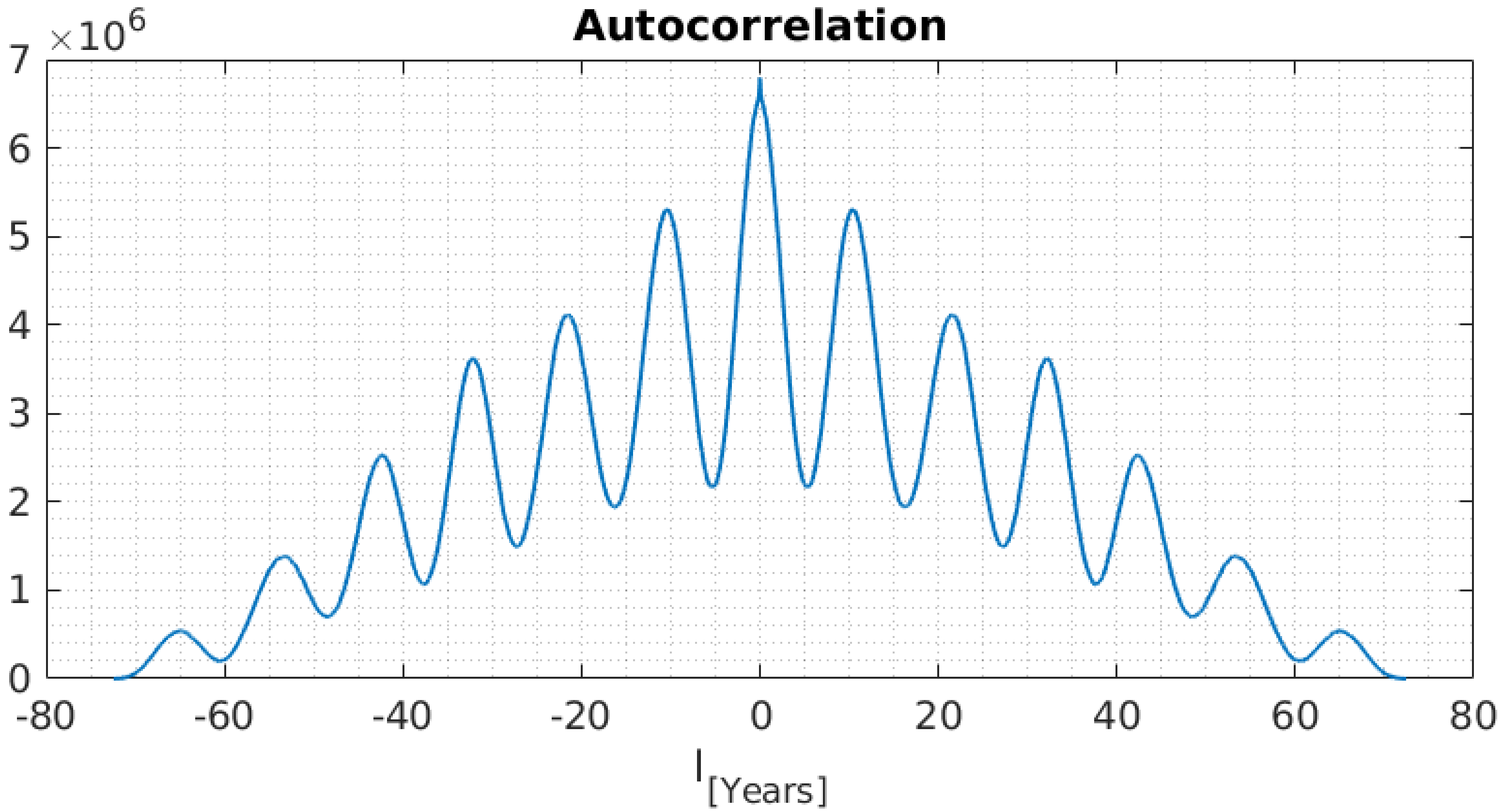


An example: The Wölfer Sunspots (III)

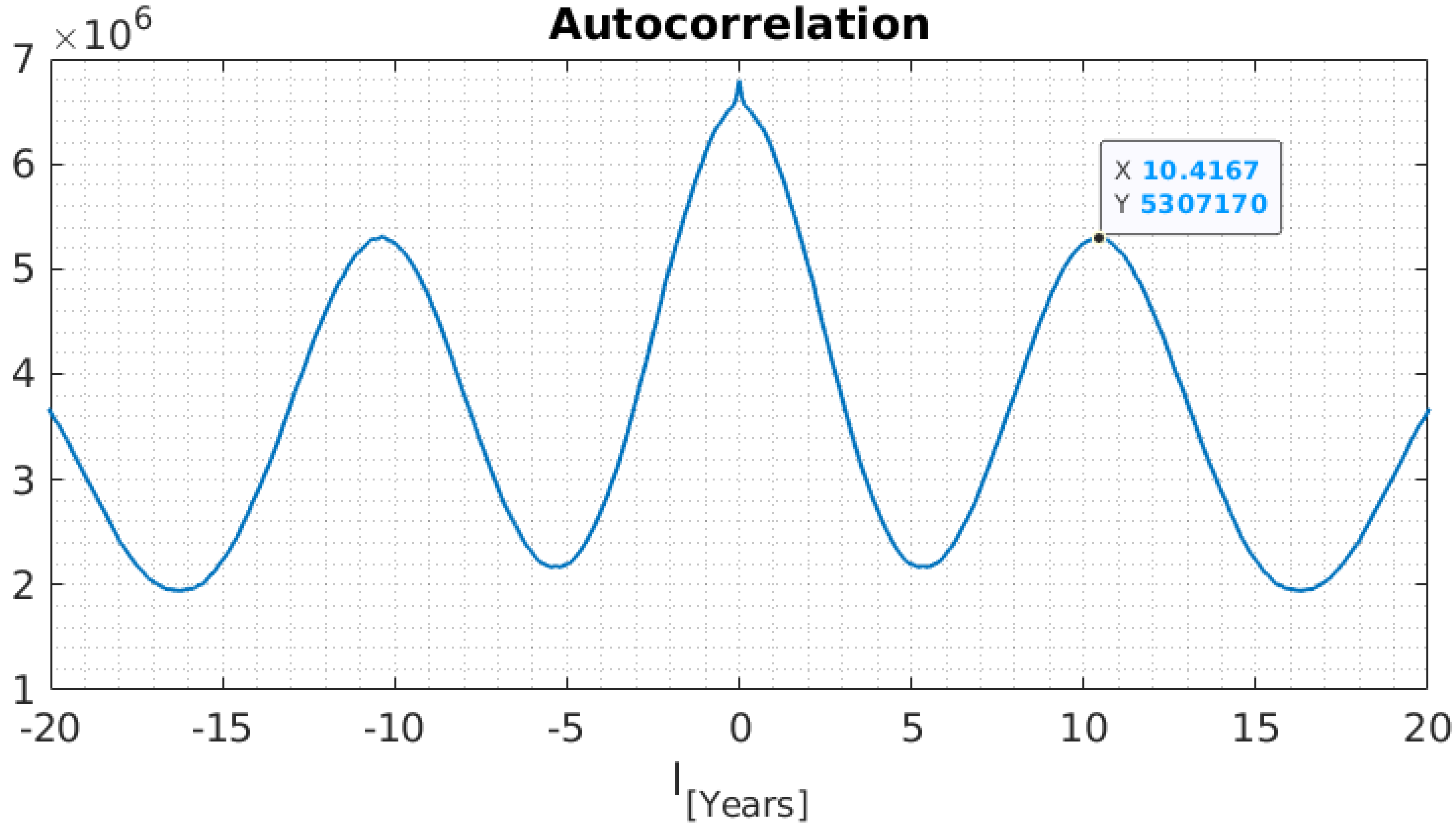


Matlab Instance: *spots.m*

An example: The Wölfer Sunspots (IV)



An example: The Wölfer Sunspots (IV)



Properties of Correlation (I)

$$r_{xx}(l) = rxx(-l)$$

$$E_x = r_{xx}(l)\big|_{l=0}$$

$$|r_{xx}(l)| \leq r_{xx}(0) = E_x; \quad \forall -\infty < l < \infty$$

$$|r_{xy}(l)| \leq \sqrt{r_{xx}(0) \cdot r_{yy}(0)} = \sqrt{E_x \cdot E_y}; \quad \forall -\infty < l < \infty$$

Properties of Correlation (II)

Autocorrelation of a periodic signal:

$$r_{xx}(l) = \frac{1}{N} \cdot \sum_{n=0}^{N-1} x(n) \cdot x(n-l)$$

How to compute correlation by means of a convolution:

$$r_{xy}(l) = x(l) * y(-l)$$

Normalized Correlation

- Cross Correlation computations might vary very much as a function of signals energy.
- In order to provide a more standard measure of *likeness,* a normalized computation is proposed with magnitudes among 0 and 1.

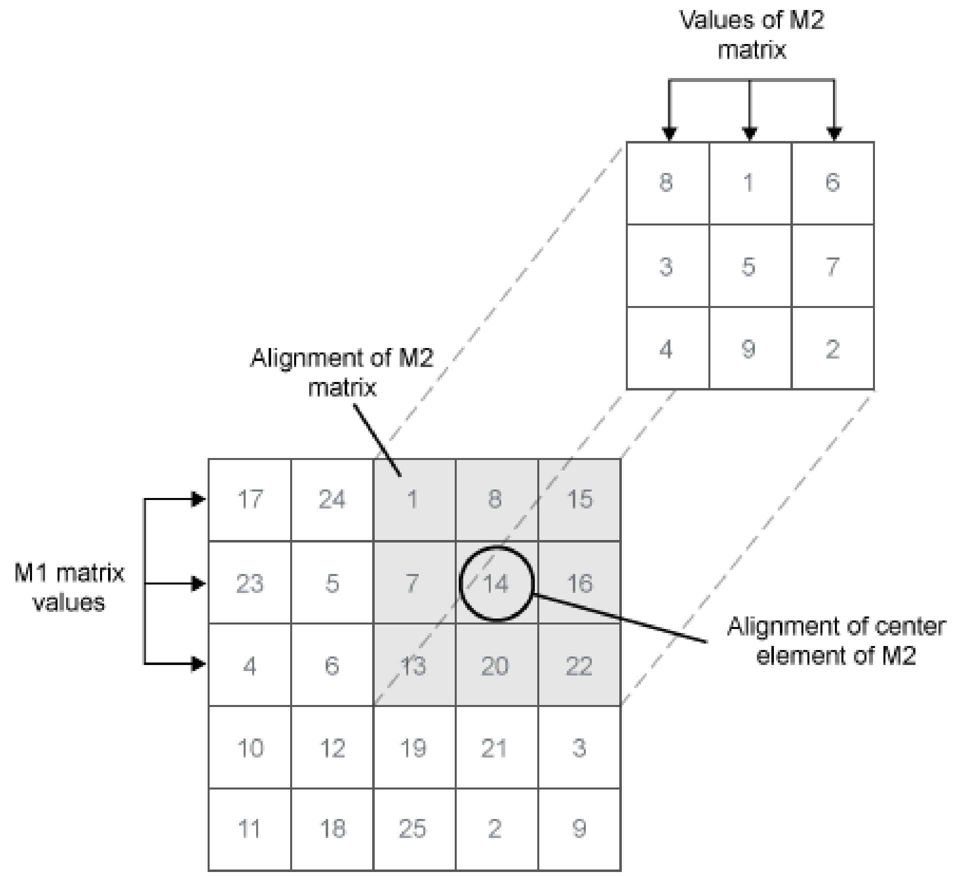
$$\rho_{xy}(l) = \frac{r_{xy}(l)}{\sqrt{E_x \cdot E_y}} = \frac{r_{xy}(l)}{\sqrt{r_{xx}(0) \cdot r_{yy}(0)}}$$

Bonus: Cross Correlation of images

```
M1 = [17 24  1  8 15;
      23  5  7 14 16;
       4  6 13 20 22;
      10 12 19 21  3;
      11 18 25  2  9];

M2 = [8 1 6;
      3 5 7;
      4 9 2];
```

Bonus: Cross Correlation of images



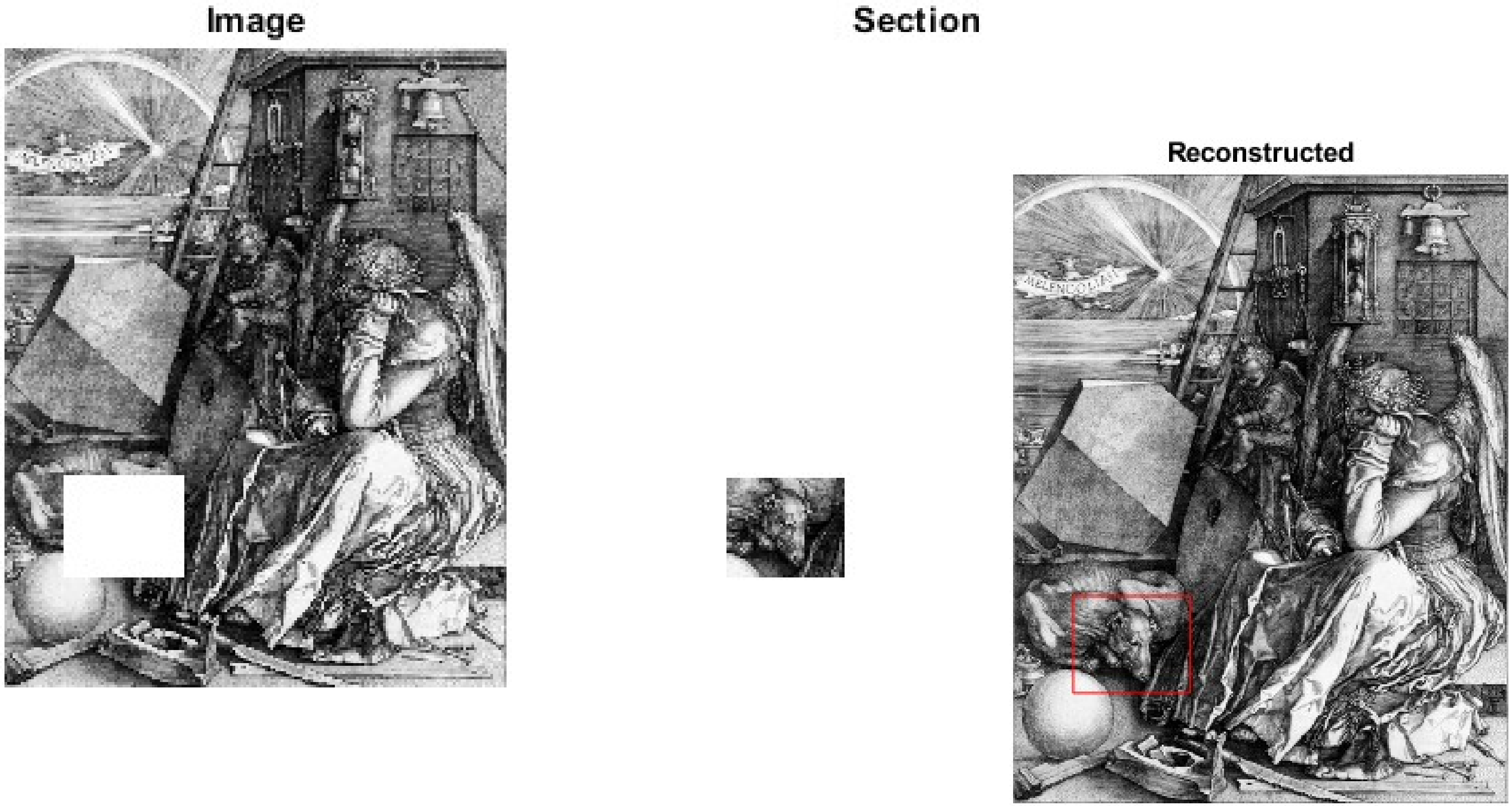
Bonus: Cross Correlation of images

- Cross Correlation computations will work just fine with bipolar signals, but don't work well with only positive values.
- In images, it is better to subtract a mean to the image matrix data, in order to convert it in a bipolar array.

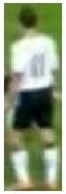
```
nimg = img-mean(mean(img));
nSec = nimg(szx,szy);

crr = xcorr2(nimg,nSec);
```

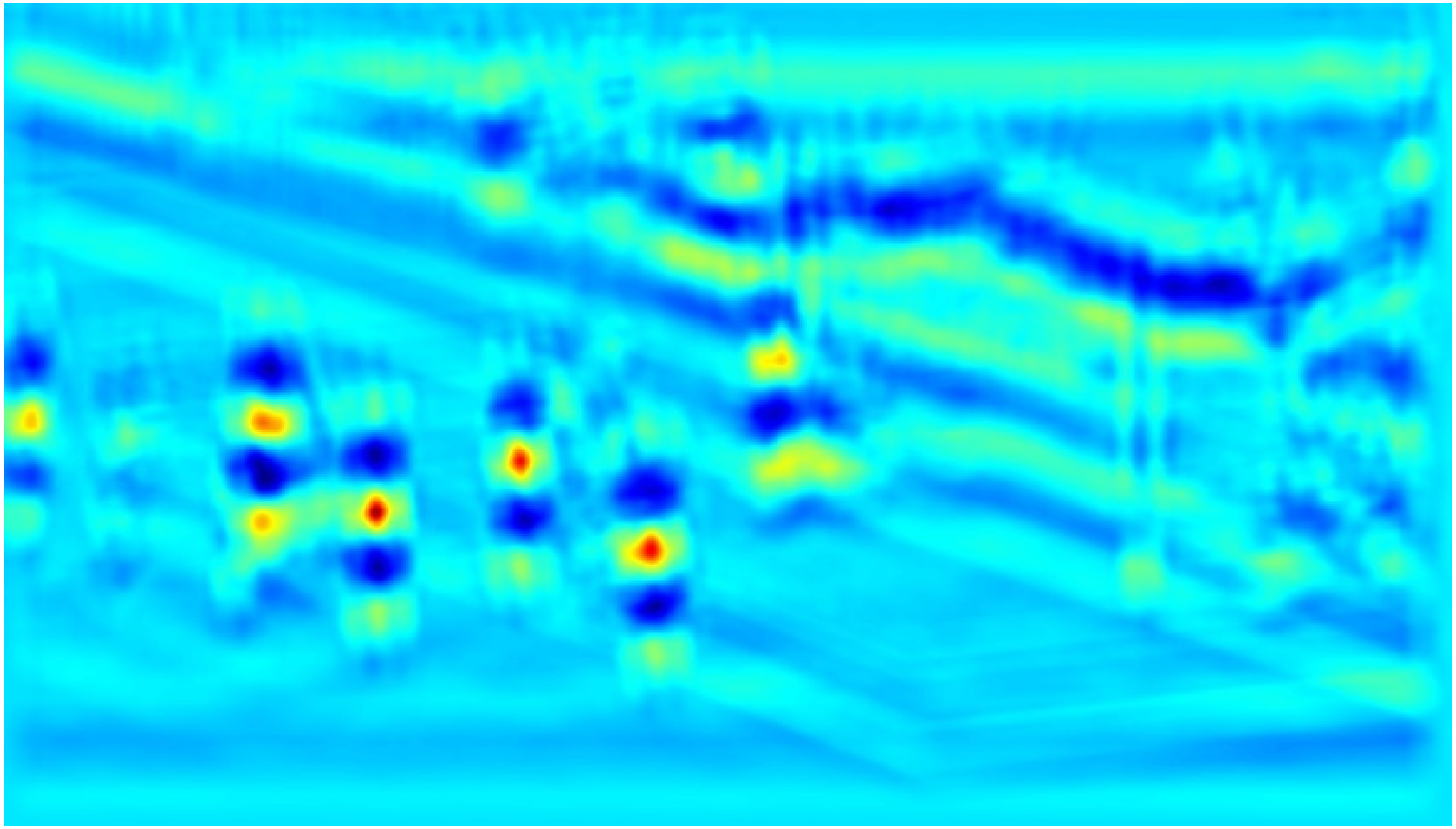
Bonus: Cross Correlation of images



Further Example: Cross Correlation (I)



Further Example: Cross Correlation (II)



Further Example: Cross Correlation (III)

im1.jpg

im2.jpg

im3.jpg

Imagen compuesta

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