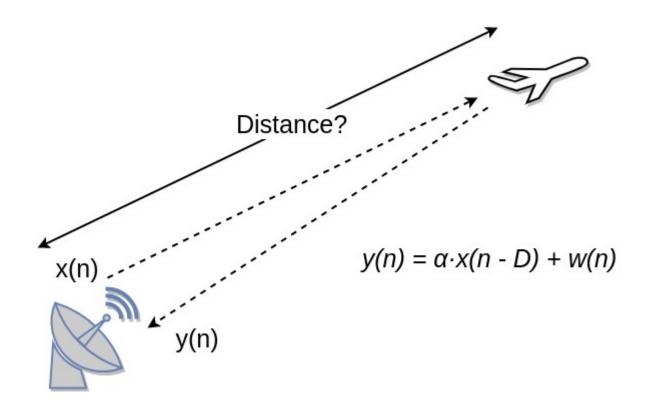


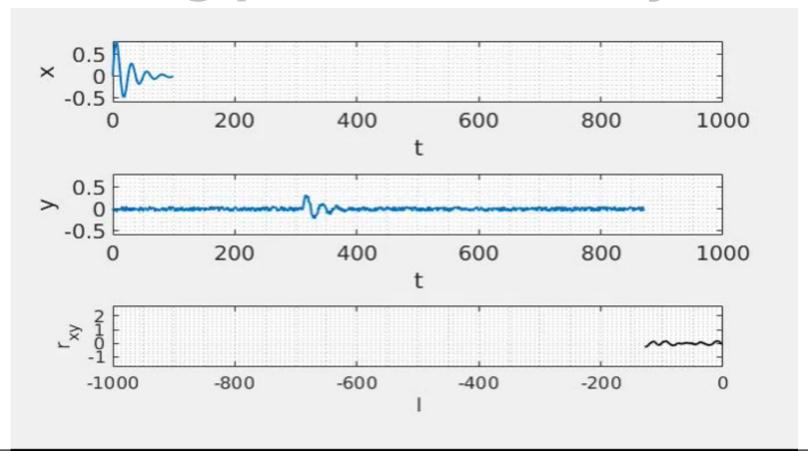
minas.medellin.unal.edu.co



Starting point: A radar system



Starting point: A radar system



It's all about time...



How to compute the distance after finding the integer delay given by I?



Cross Correlation formulation

$$r_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n) \cdot y(n-l)$$

$$r_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n+l) \cdot y(n)$$

The *l* value represents the relative delay among the *x* and *y* signals. So it is clear that:

$$r_{xy}(l) \neq r_{yx}(l)$$
 Any Ideas?

Cross Correlation formulation

$$r_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n+l) \cdot y(n)$$

$$r_{yx}(l) = \sum_{n=-\infty}^{\infty} y(n) \cdot x(n-l)$$

$$r_{xy}(l)=r_{yx}(-l)$$

Let's perform a Cross Correlation "by hand"



$$x(n) = \{-4, 2, 7, 0, -1\}$$

$$y(n)=[0,1,3,0,-2,4,-4,8,-10]$$

$$r_{xy}(l) = ?$$



Autocorrelation of a signal

- Autocorrelation is often used to estimate the periodicity of a repetitive signal.
- Autocorrelation is also useful to compute the spectral energy density of a given signal.

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n) \cdot x(n-l)$$

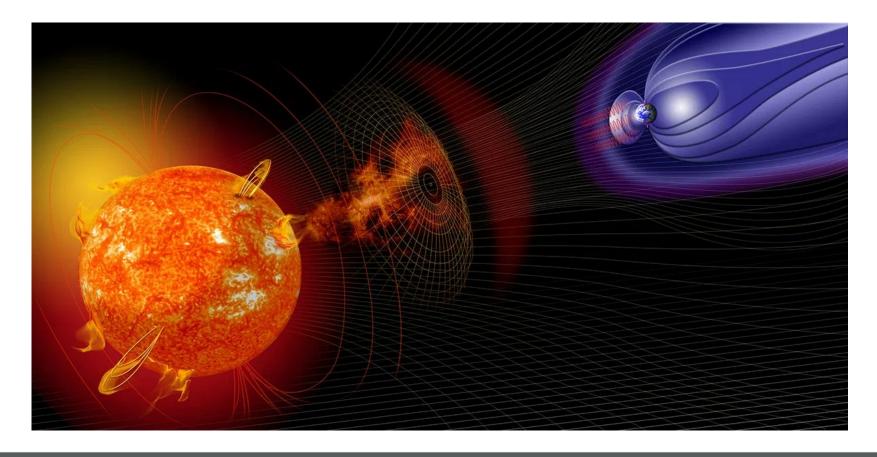


An Autocorrelation example



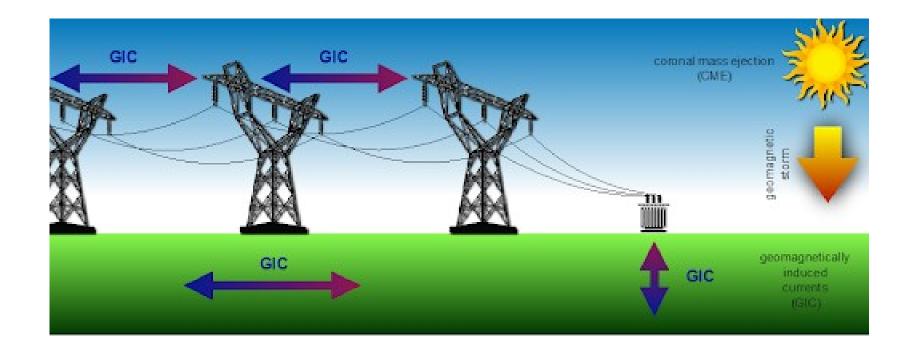


An example: The Wölfer Sunspots (I)





An example: The Wölfer Sunspots (I)





An example: The Wölfer Sunspots (II)

| T. | | II. III. | | | | | TV | | | V. | | | VI | | | |
|---|--|--|--|---|--|---|-----------------------|--|--|------------------|---|--|-----------------------|--|--|--|
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| 7 | _ | Sonnenflecken-Beobachtungen. | | | | | | | | | | | | _ | 48 | | | | |
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| ı | 96 | 8.5 | 70 | 160 | F.30 | 70 | 26 | 2.3 | stro | 15 | 1.4 | 16 | 100 | 4.7 | 12 | Ser | 4.45 | 100 | F |
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Remerkungen:

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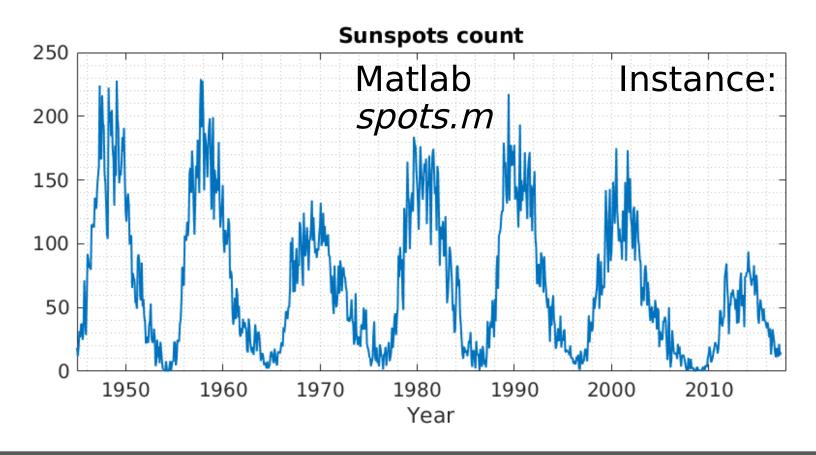
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5 Cotto: Smith at the County

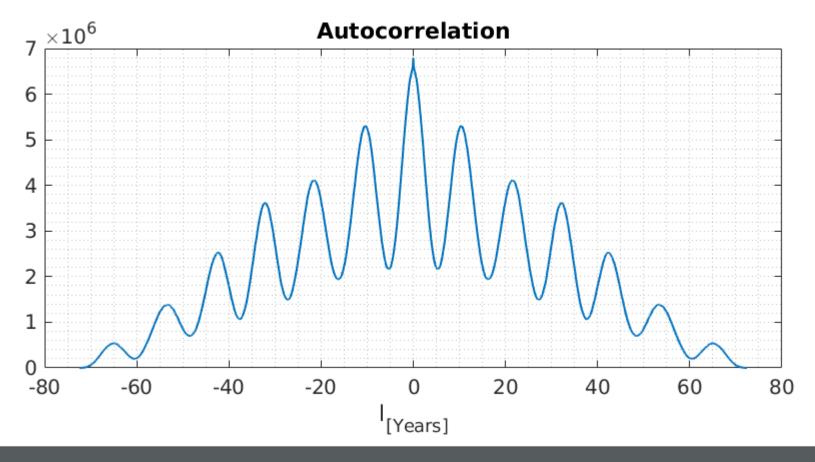
6 Cattle | part of the color

Bemerkungen:

An example: The Wölfer Sunspots (III)

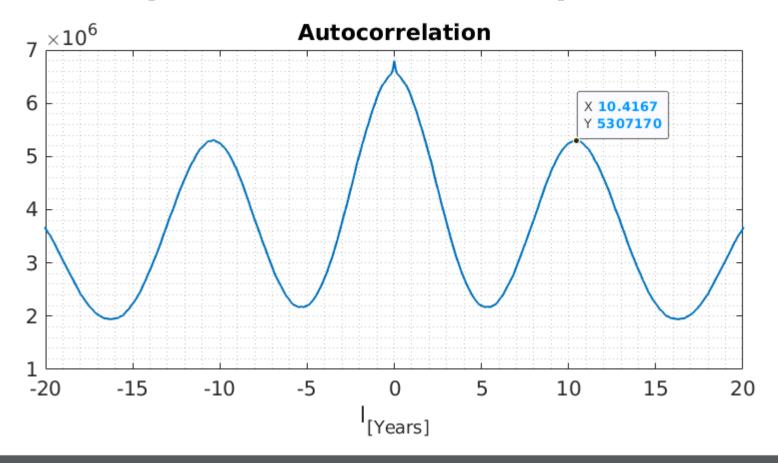


An example: The Wölfer Sunspots (IV)





An example: The Wölfer Sunspots (IV)





Properties of Correlation (I)

$$r_{xx}(l) = rxx(-l)$$

$$E_x = r_{xx}(l)\big|_{l=0}$$

$$|r_{xx}(l)| \le r_{xx}(0) = E_x; \quad \forall -\infty < l < \infty$$

$$|r_{xy}(l)| \le \sqrt{r_{xx}(0) \cdot r_{yy}(0)} = \sqrt{E_x \cdot E_y}; \quad \forall -\infty < l < \infty$$

Properties of Correlation (II)

Autocorrelation of a periodic signal:

$$r_{xx}(l) = \frac{1}{N} \cdot \sum_{n=0}^{N-1} x(n) \cdot x(n-l)$$

How to compute correlation by means of a convolution:

$$r_{xy}(l) = x(l) * y(-l)$$



Normalized Correlation

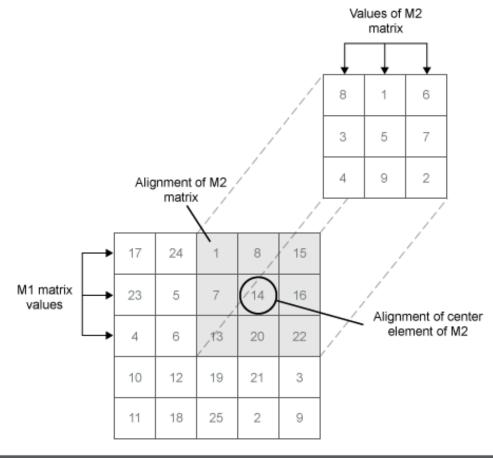
- Cross Correlation computations might vary very much as a function of signals energy.
- In order to provide a more standard measure of *likeness*, a normalized computation is proposed with magnitudes among 0 and 1.

$$\rho_{xy}(l) = \frac{r_{xy}(l)}{\sqrt{E_x \cdot E_y}} = \frac{r_{xy}(l)}{\sqrt{r_{xx}(0) \cdot r_{yy}(0)}}$$

Bonus: Cross Correlation of images

```
M1 = [17 24 1 8 15;
      23 5 7 14 16;
       4 6 13 20 22;
      10 12 19 21 3;
      11 18 25 2 9];
M2 = [8 \ 1 \ 6;
      3 5 7;
      4 9 21;
```

Bonus: Cross Correlation of images





Bonus: Cross Correlation of images

- Cross Correlation computations will work just fine with bipolar signals, but don't work well with only positive values.
- In images, it is better to subtract a mean to the image matrix data, in order to convert it in a bipolar array.

```
nimg = img-mean(mean(img));
nSec = nimg(szx,szy);

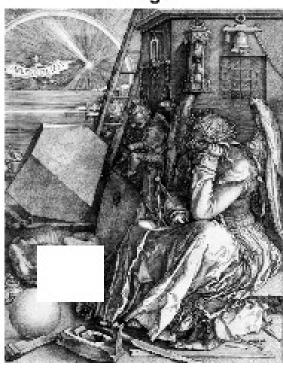
crr = xcorr2(nimg,nSec);
```



Bonus: Cross Correlation of images

Image







Reconstructed



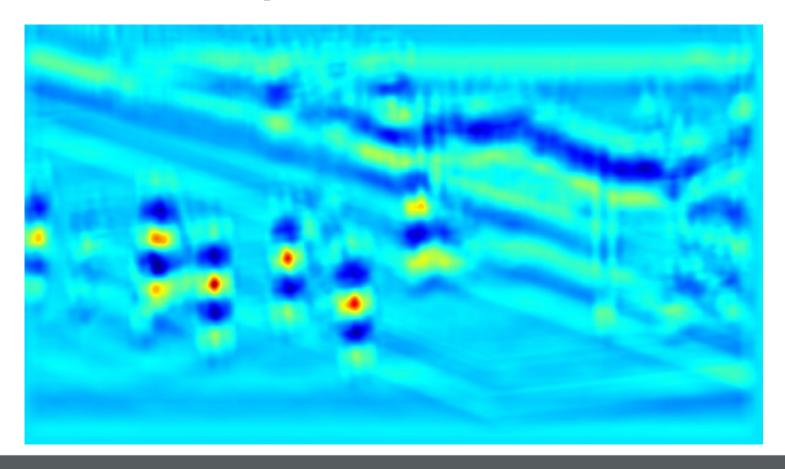
Further Example: Cross Correlation (I)







Further Example: Cross Correlation (II)





Further Example: Cross Correlation (III)

im1.jpg im2.jpg im3.jpg







Imagen compuesta



Facultad de Minas Sede Medellín



Facultad de Minas Departamento de Energía Eléctrica y Automática

Carrera 80 num. 65-223 Medellín, Colombia (+57 4) 430 90 00, Ext. 4 5276 fbolanos@unal.edu.co

minas.medellin.unal.edu.co