

U.N. Sede Medellín

Una universidad con criterio nacional y presencia regional

# Procesamiento de Señales Correlation of Signals

Freddy Bolaños Martínez  
2022

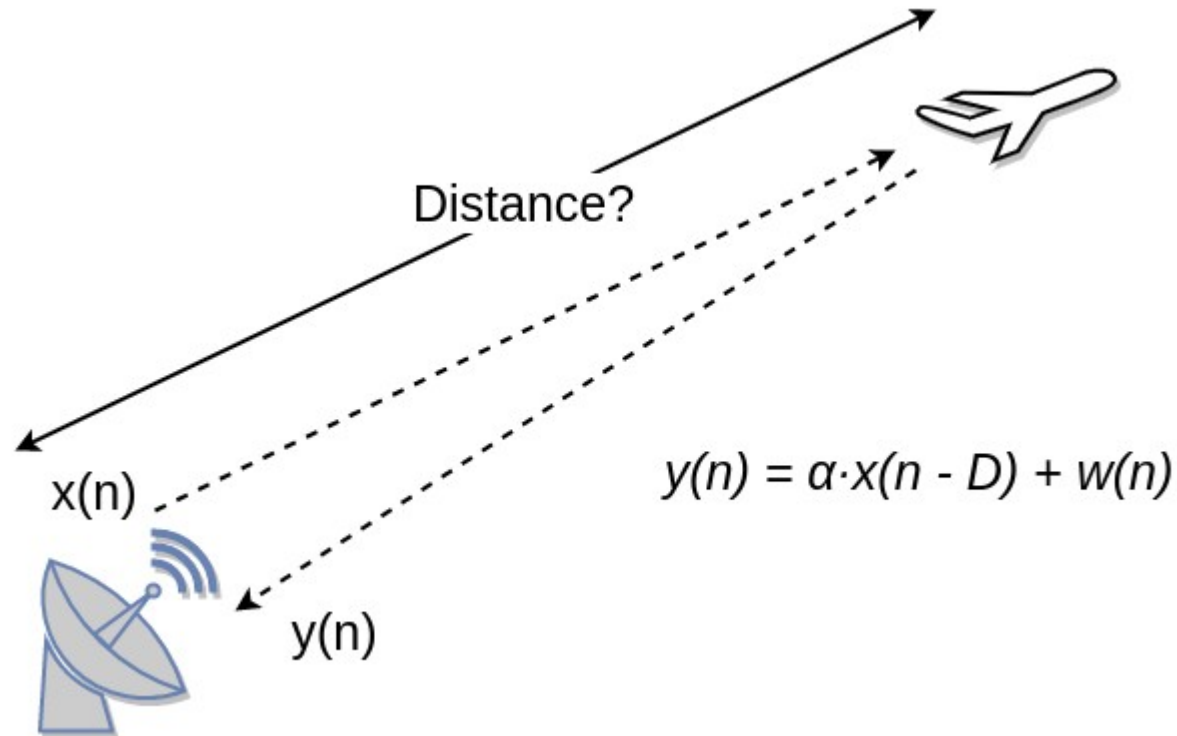
[minas.medellin.unal.edu.co](http://minas.medellin.unal.edu.co)

Facultad de Minas  
Sede Medellín



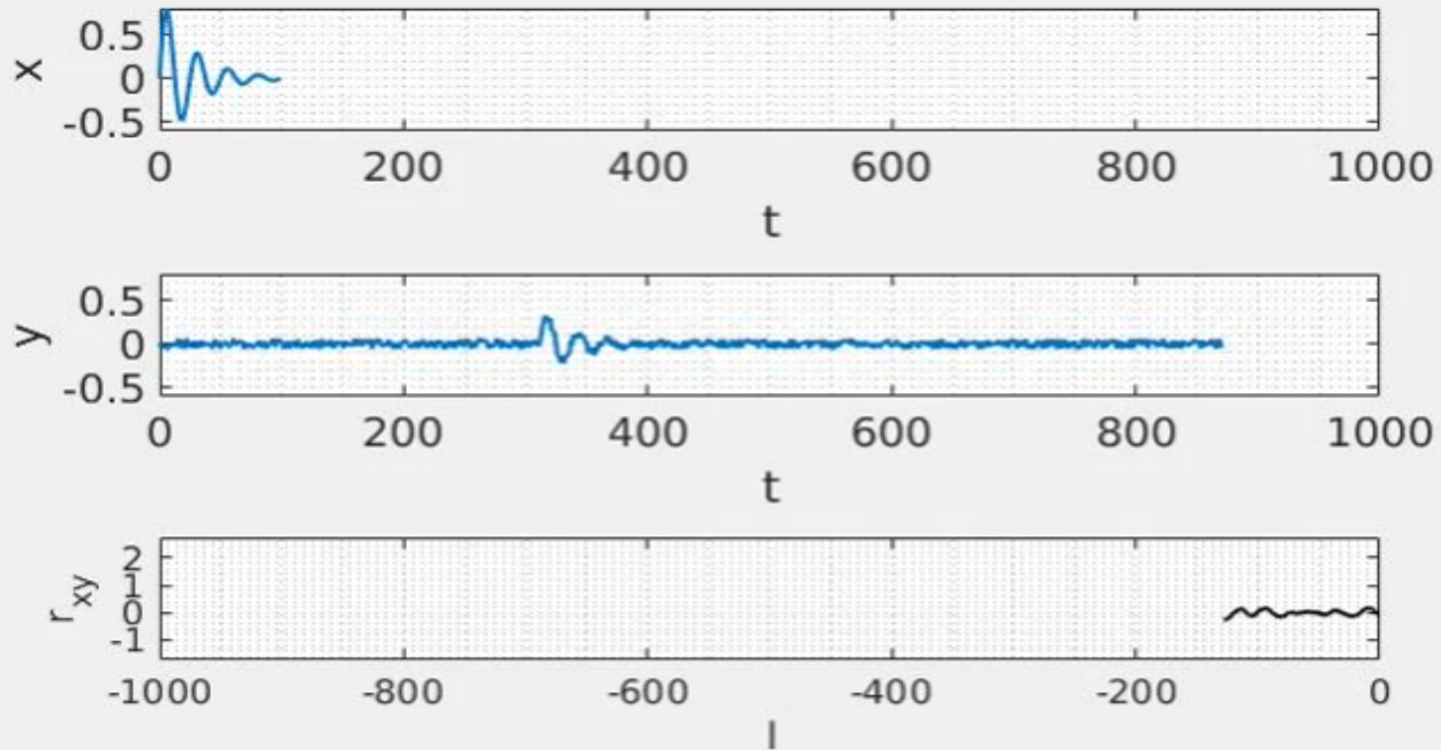
UNIVERSIDAD  
NACIONAL  
DE COLOMBIA

# Starting point: A radar system





# Starting point: A radar system



# It's all about time...

How to  
compute  
the distance  
after finding  
the integer  
delay given  
by  $I$ ?



# Cross Correlation formulation

$$r_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n) \cdot y(n-l)$$

$$r_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n+l) \cdot y(n)$$

The  $l$  value represents the relative delay among the  $x$  and  $y$  signals. So it is clear that:

$$r_{xy}(l) \neq r_{yx}(l) \quad \text{Any Ideas?}$$

# Cross Correlation formulation

$$r_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n+l) \cdot y(n)$$

$$r_{yx}(l) = \sum_{n=-\infty}^{\infty} y(n) \cdot x(n-l)$$

$$r_{xy}(l) = r_{yx}(-l)$$

# Let's perform a Cross Correlation *"by hand"*



$$x(n) = \{-4, 2, \underline{7}, 0, -1\}$$

$$y(n) = \{0, 1, 3, 0, -2, \underline{4}, -4, 8, -10\}$$

$$r_{xy}(l) = ?$$

# Autocorrelation of a signal

- Autocorrelation is often used to estimate the periodicity of a repetitive signal.
- Autocorrelation is also useful to compute the *spectral energy density* of a given signal.

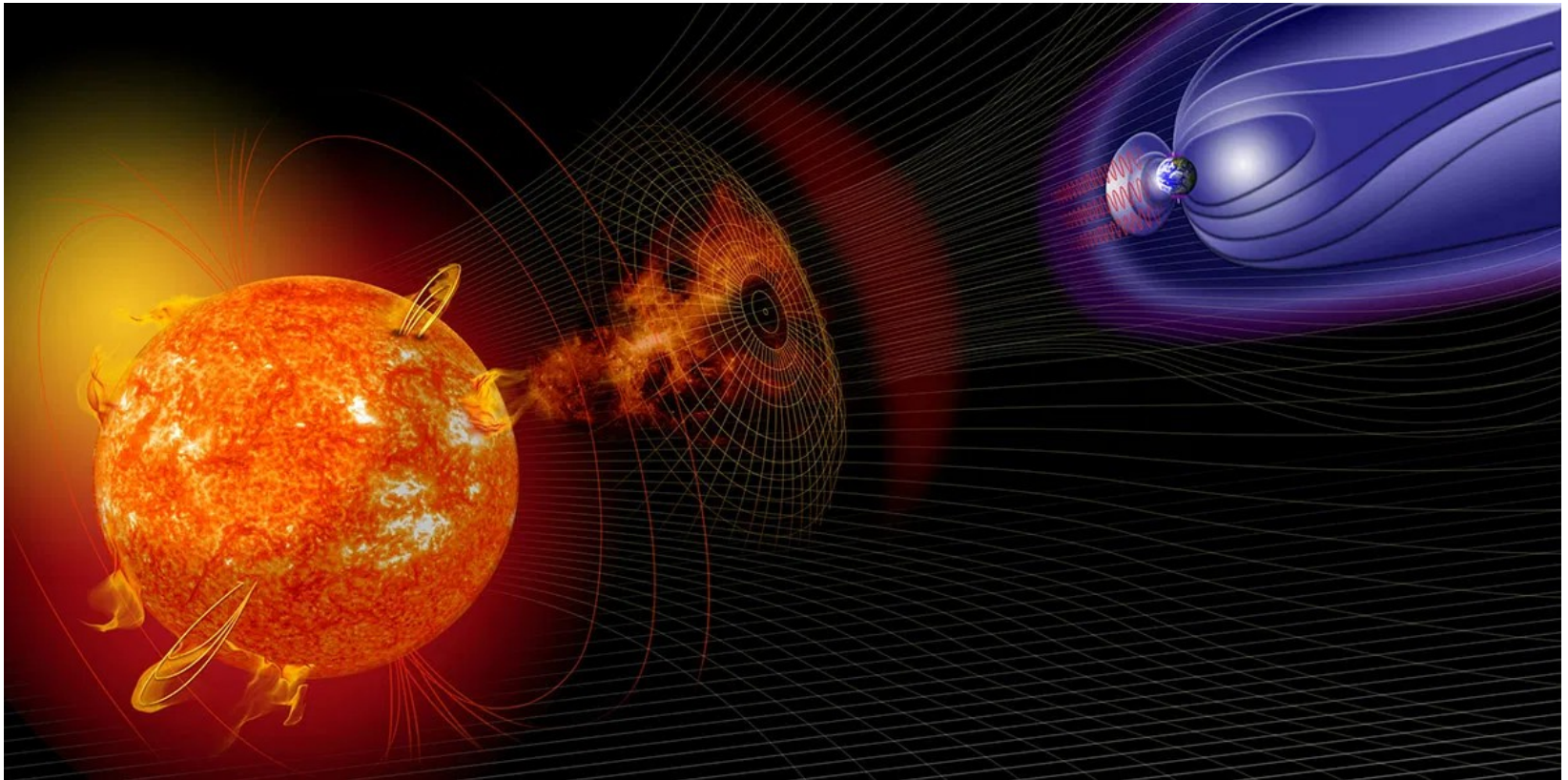
$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n) \cdot x(n-l)$$



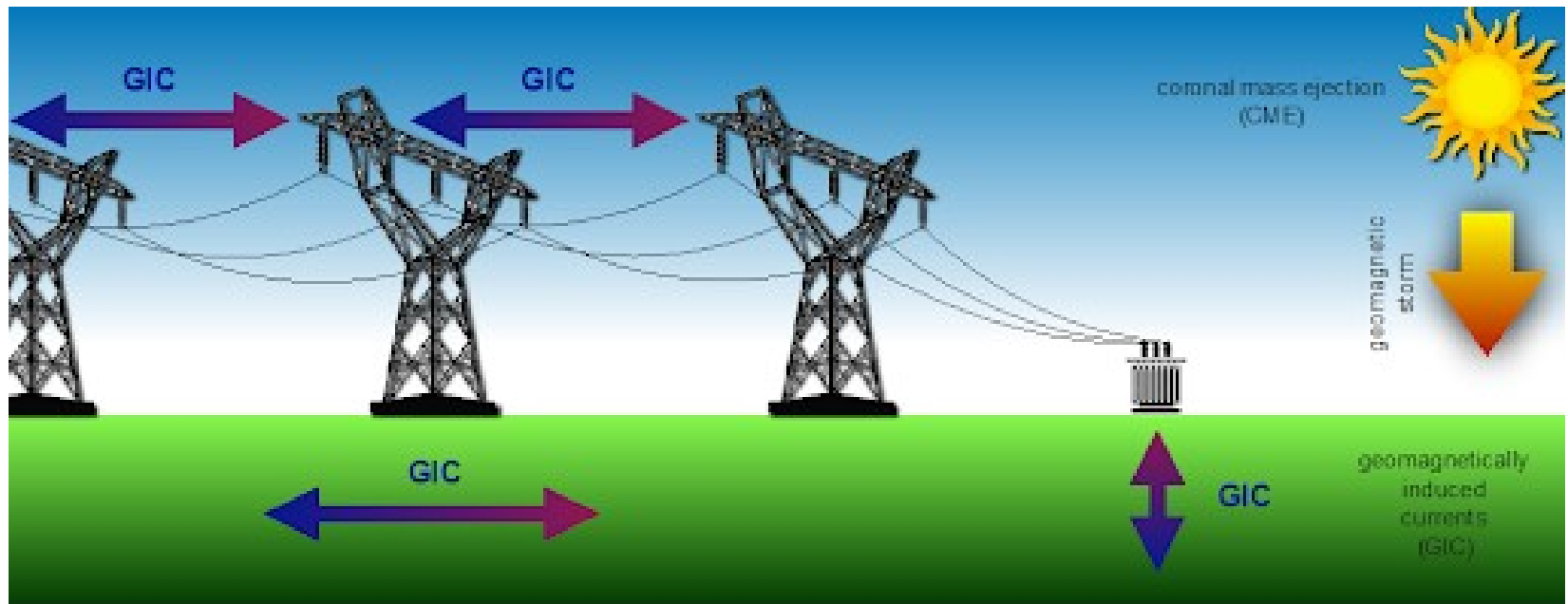
# An Autocorrelation example



# An example: The Wölfer Sunspots (I)



## An example: The Wölfer Sunspots (I)



# An example: The Wölfer Sunspots (II)

Tag

Sonnenflecken-Beobachtungen.

	I.	II.	III.	IV.	V.	VI.	
1							1
2							2
3							3
4							4
5							5
6							6
7							7
8							8
9							9
10							10
11							11
12							12
13							13
14							14
15							15
16							16
17							17
18							18
19							19
20							20
21							21
22							22
23							23
24							24
25							25
26							26
27							27
28							28
29							29
30							30
31							31
32							32

Bemerkungen:

1. Tag der Beobachtung  
 2. Tag der Beobachtung  
 3. Tag der Beobachtung  
 4. Tag der Beobachtung  
 5. Tag der Beobachtung  
 6. Tag der Beobachtung  
 7. Tag der Beobachtung  
 8. Tag der Beobachtung  
 9. Tag der Beobachtung  
 10. Tag der Beobachtung  
 11. Tag der Beobachtung  
 12. Tag der Beobachtung  
 13. Tag der Beobachtung  
 14. Tag der Beobachtung  
 15. Tag der Beobachtung  
 16. Tag der Beobachtung  
 17. Tag der Beobachtung  
 18. Tag der Beobachtung  
 19. Tag der Beobachtung  
 20. Tag der Beobachtung  
 21. Tag der Beobachtung  
 22. Tag der Beobachtung  
 23. Tag der Beobachtung  
 24. Tag der Beobachtung  
 25. Tag der Beobachtung  
 26. Tag der Beobachtung  
 27. Tag der Beobachtung  
 28. Tag der Beobachtung  
 29. Tag der Beobachtung  
 30. Tag der Beobachtung  
 31. Tag der Beobachtung

Sonnenflecken-Beobachtungen.

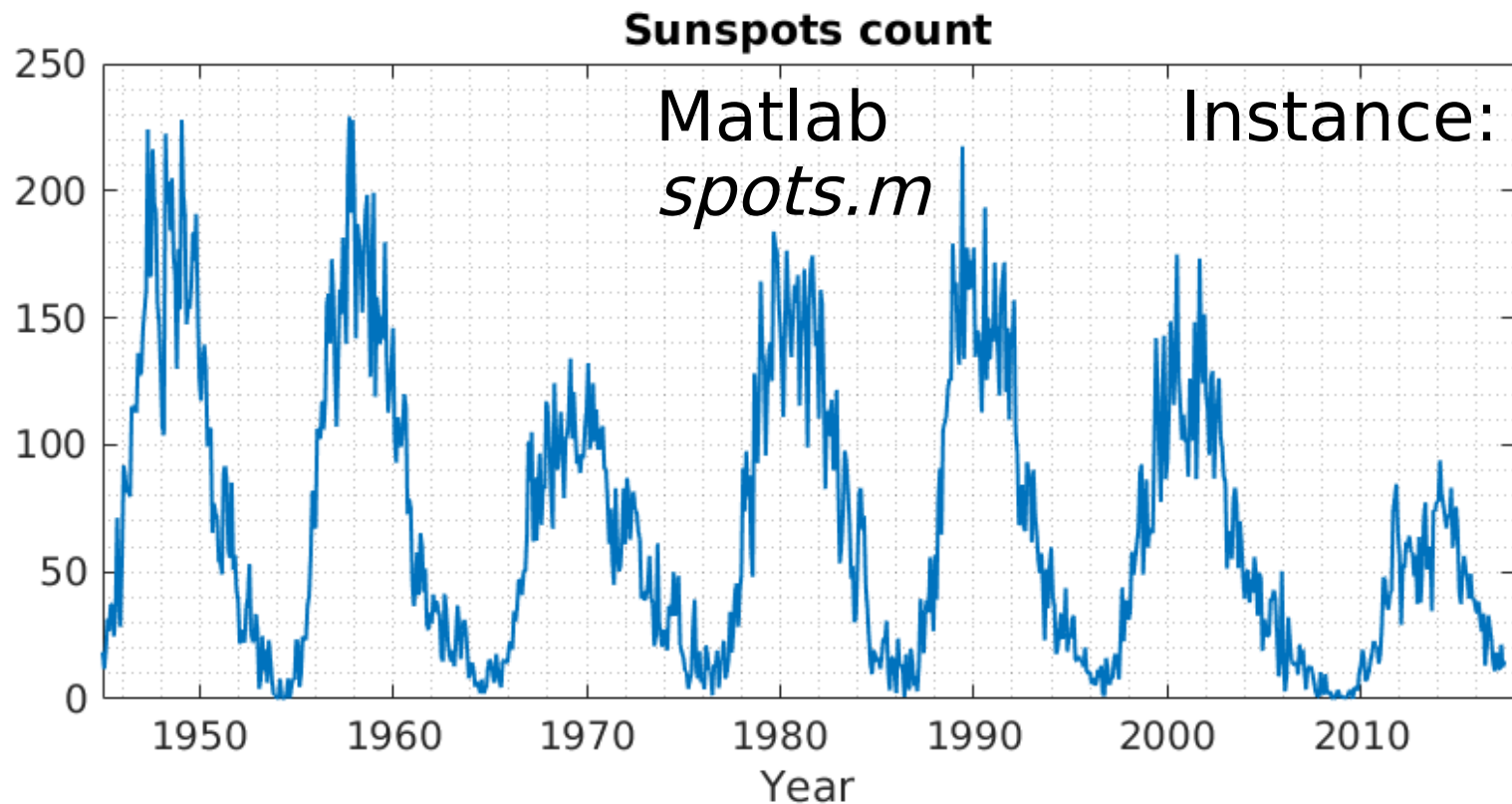
Tag

	VII.	VIII.	IX.	X.	XI.	XII.	
1							1
2							2
3							3
4							4
5							5
6							6
7							7
8							8
9							9
10							10
11							11
12							12
13							13
14							14
15							15
16							16
17							17
18							18
19							19
20							20
21							21
22							22
23							23
24							24
25							25
26							26
27							27
28							28
29							29
30							30
31							31
32							32

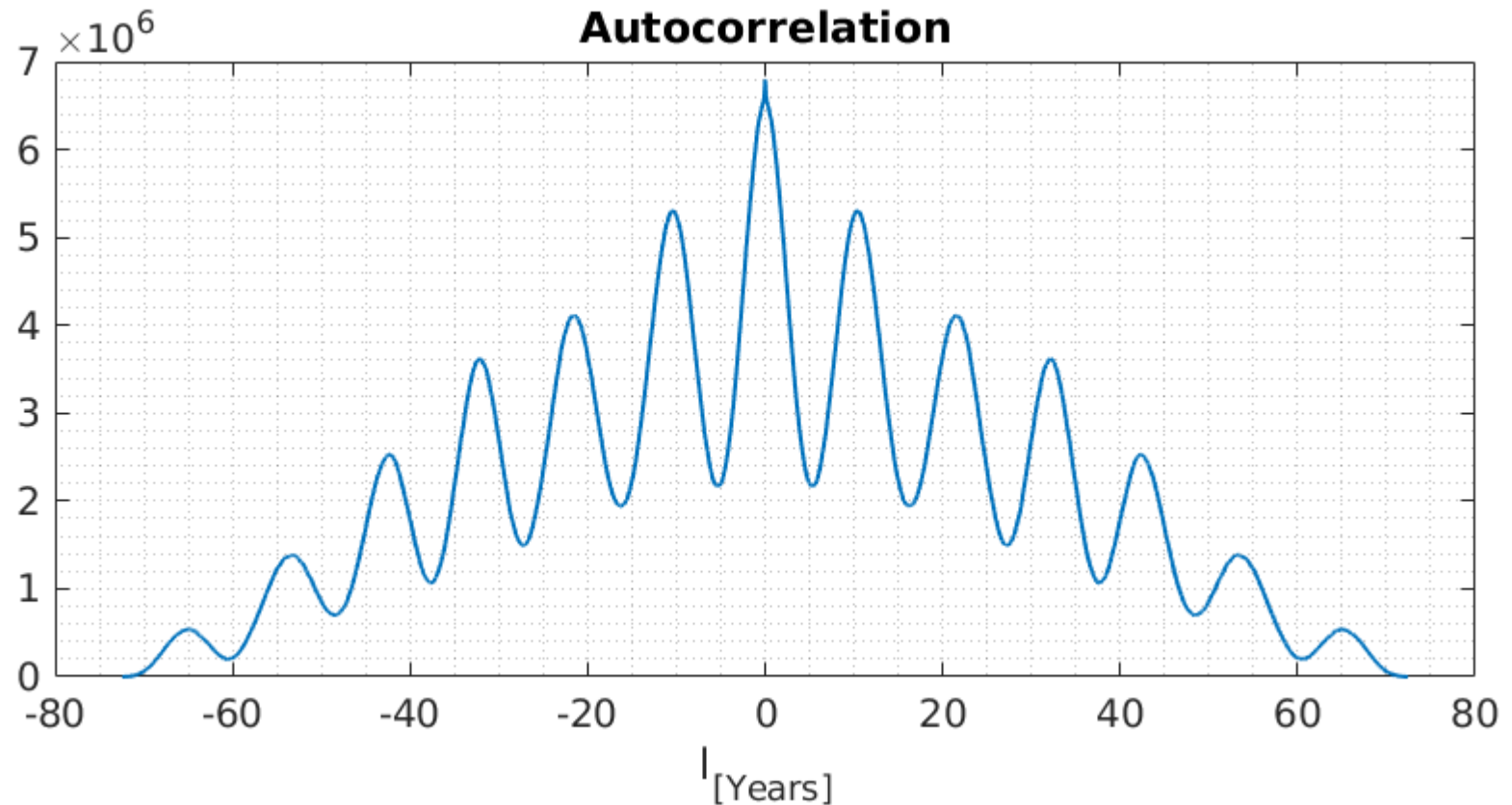
Bemerkungen:



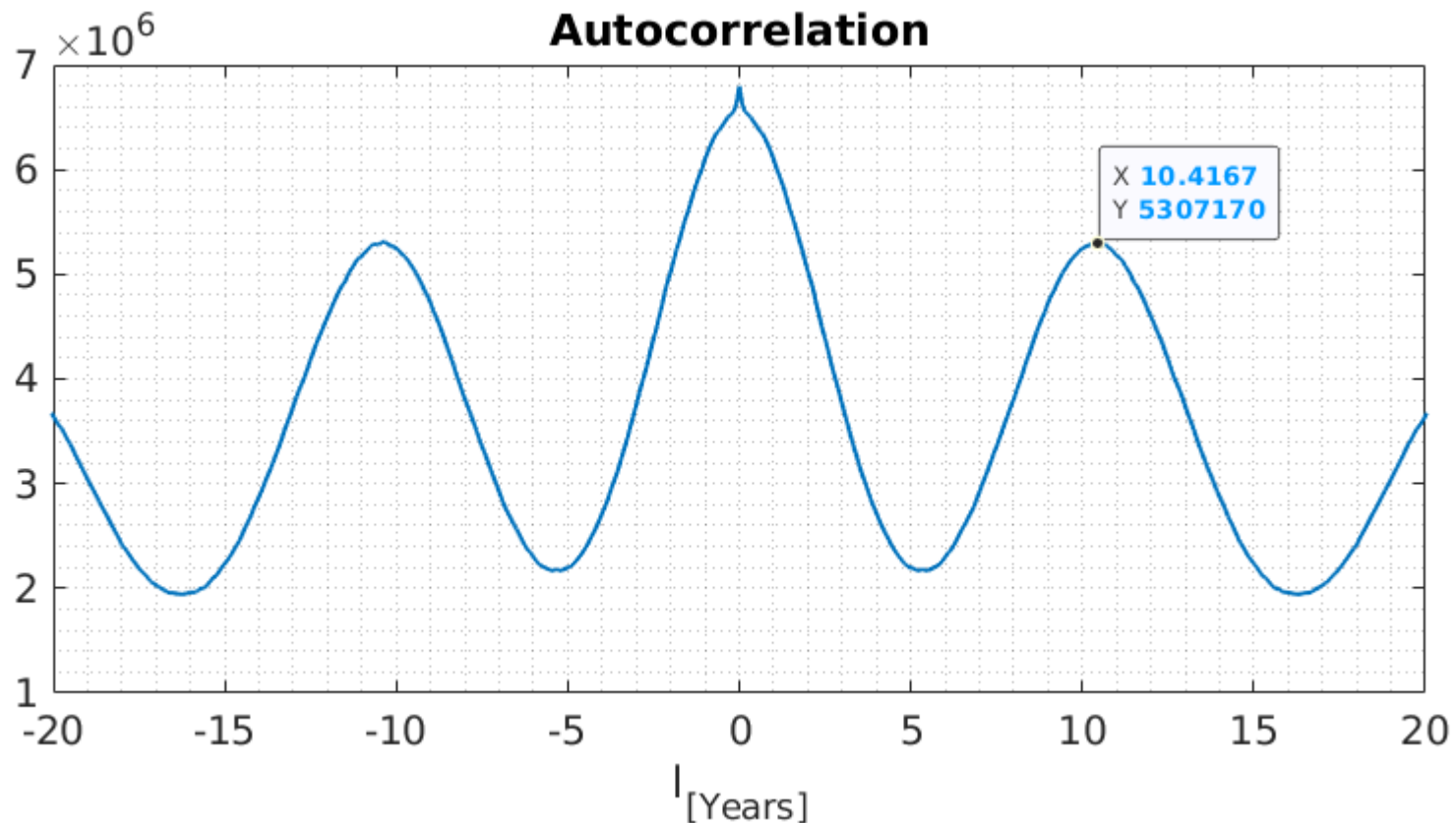
## An example: The Wölfer Sunspots (III)



## An example: The Wölfer Sunspots (IV)



## An example: The Wölfer Sunspots (IV)



# Properties of Correlation (I)

$$r_{xx}(l) = r_{xx}(-l)$$

$$E_x = r_{xx}(l) \Big|_{l=0}$$

$$\left| r_{xx}(l) \right| \leq r_{xx}(0) = E_x; \quad \forall -\infty < l < \infty$$

$$\left| r_{xy}(l) \right| \leq \sqrt{r_{xx}(0) \cdot r_{yy}(0)} = \sqrt{E_x \cdot E_y}; \quad \forall -\infty < l < \infty$$



# Properties of Correlation (II)

Autocorrelation of a periodic signal:

$$r_{xx}(l) = \frac{1}{N} \cdot \sum_{n=0}^{N-1} x(n) \cdot x(n-l)$$

How to compute correlation by means of a convolution:

$$r_{xy}(l) = x(l) * y(-l)$$

# Normalized Correlation

- Cross Correlation computations might vary very much as a function of signals energy.
- In order to provide a more standard measure of *likeness*, a normalized computation is proposed with magnitudes among 0 and 1.

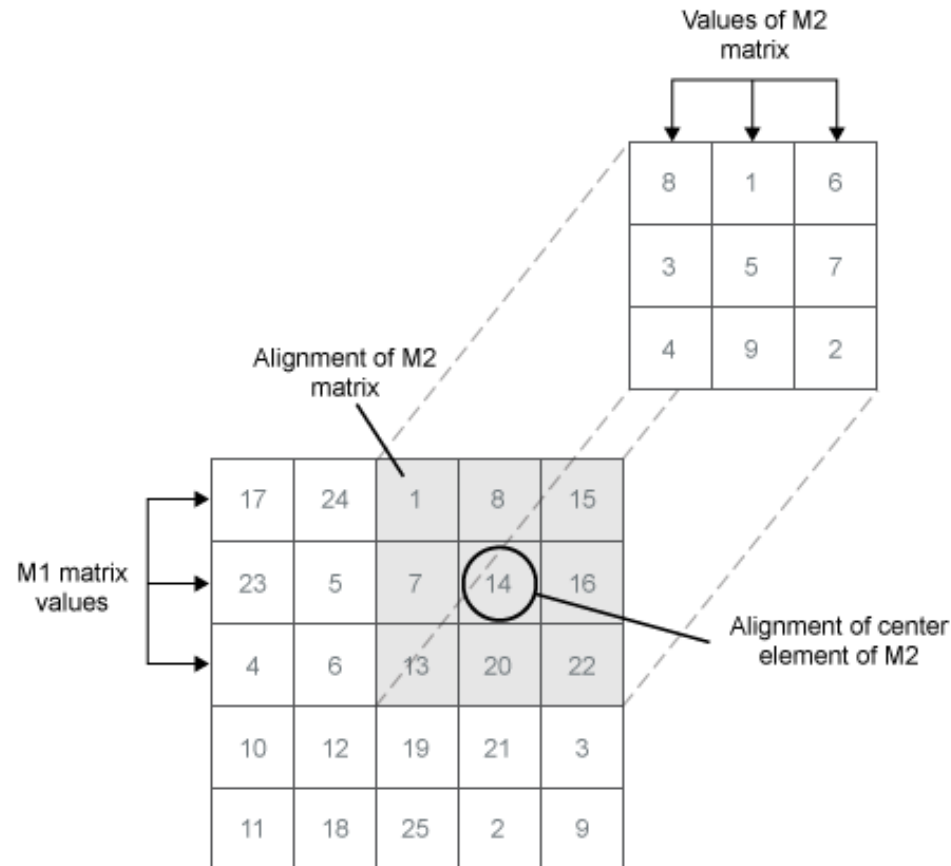
$$\rho_{xy}(l) = \frac{r_{xy}(l)}{\sqrt{E_x \cdot E_y}} = \frac{r_{xy}(l)}{\sqrt{r_{xx}(0) \cdot r_{yy}(0)}}$$

## Bonus: Cross Correlation of images

```
M1 = [17 24 1 8 15;  
      23 5 7 14 16;  
      4 6 13 20 22;  
      10 12 19 21 3;  
      11 18 25 2 9];
```

```
M2 = [8 1 6;  
      3 5 7;  
      4 9 2];
```

## Bonus: Cross Correlation of images





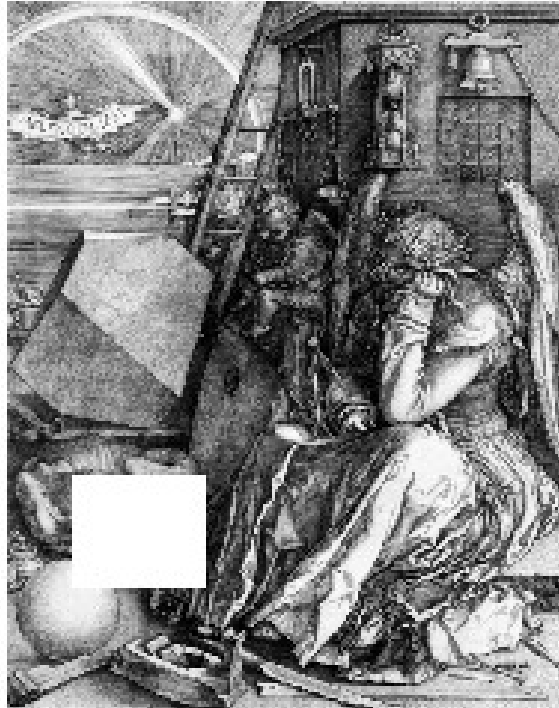
## Bonus: Cross Correlation of images

- Cross Correlation computations will work just fine with bipolar signals, but don't work well with only positive values.
- In images, it is better to subtract a mean to the image matrix data, in order to convert it in a bipolar array.

```
nimg = img-mean(mean(img));  
nSec = nimg(szx,szy);  
  
crr = xcorr2(nimg,nSec);
```

# Bonus: Cross Correlation of images

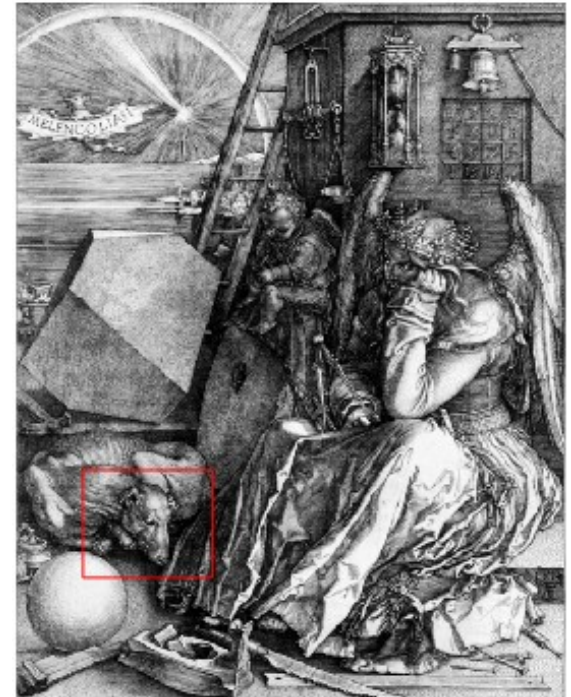
Image



Section



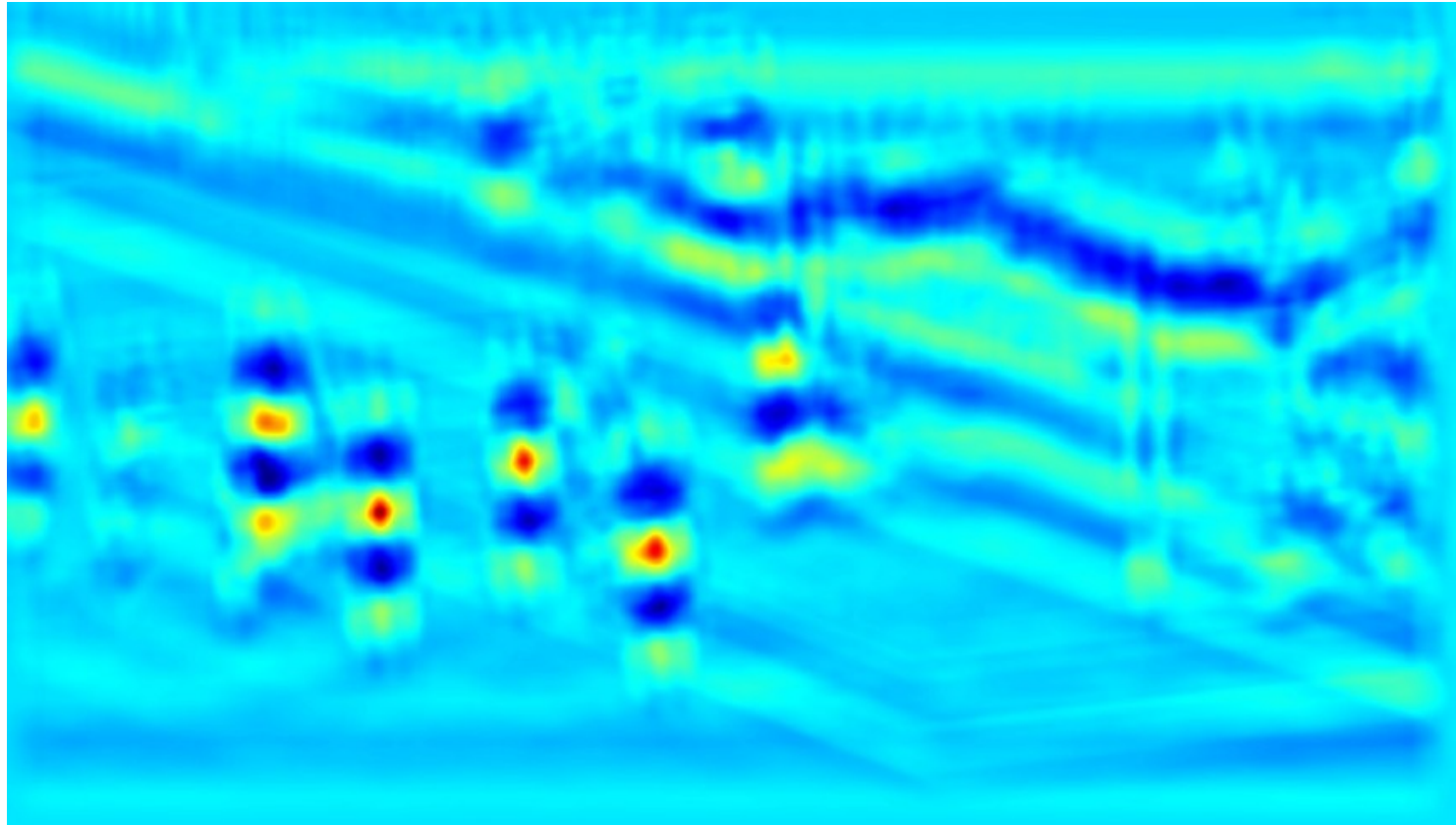
Reconstructed



## Further Example: Cross Correlation (I)



## Further Example: Cross Correlation (II)





## Further Example: Cross Correlation (III)

im1.jpg



im2.jpg



im3.jpg



Imagen compuesta



**Facultad de Minas**  
Sede Medellín



UNIVERSIDAD  
**NACIONAL**  
DE COLOMBIA

***Facultad de Minas***  
***Departamento de Energía Eléctrica y Automática***

*Carrera 80 num. 65-223*  
*Medellín, Colombia*  
*(+57 4) 430 90 00, Ext. 4 5276*  
*fbolanos@unal.edu.co*

***[minas.medellin.unal.edu.co](http://minas.medellin.unal.edu.co)***