Quantum Teleportation Algorithm

Terminology

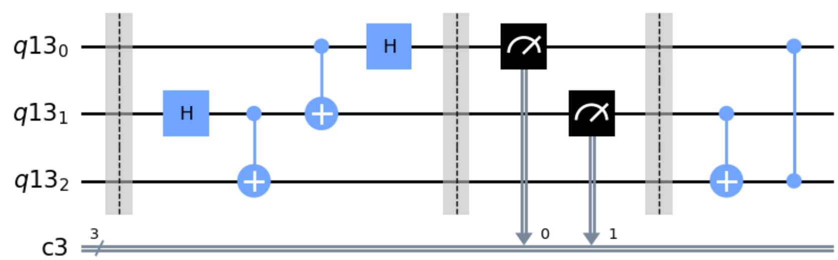
B	Bell state
H	Hadamard Gate
X	Controlled X
Z	Controlled Z

Example

- **Enunciado del ejercicio**

Maria and Alexander actually are living in Saint Petersburg, Russia. They are working on the laboratory of Quantum Computing of Peter the Great St.Petersburg Polytechnic University. One day they entangled their qubits for make experiments on laboratory. Unfortunately, Bob needed to move to Moscow city. After 1 year Bob is working on Moscow University and actually has a laboratory. Alice and Bob decided to proof the teleportation quantum with objective to proof the teleportation of information. In this case the information is the state of entangled qubit that Alice has on laboratory. Here we go!

- **Circuit quantum**



Se debe corregir los estados.

- **Solution of the problem**

- **First status**

$|\varphi_1\rangle = Second\ qubit_{Alice}\ First\ qubit_{Bob}$

$|\varphi_1\rangle = |0\rangle|0\rangle$

- **Second status**

$|\varphi_2\rangle = \boldsymbol{H}|0\rangle_{Alice}|0\rangle_{Bob}$

$$|\varphi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$$

- **Third status**

$$|\varphi_3\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = |\boldsymbol{B}_{00}\rangle$$

- **Fourth status**

$$|\varphi_4\rangle = First\ qubit_{Alice}\ |\boldsymbol{B}_{Alice}\rangle$$

$$|\varphi_4\rangle = (\alpha|0\rangle + \beta|1\rangle)\left[\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)\right]$$

$$|\varphi_4\rangle = \frac{\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle}{\sqrt{2}}$$

- **Fifth status**

$$|\varphi_5\rangle = \frac{\alpha|000\rangle + \alpha|011\rangle + \beta|110\rangle + \beta|101\rangle}{\sqrt{2}}$$

- **Sixth status**

$$|\varphi_6\rangle = \frac{1}{\sqrt{2}}\boldsymbol{H}(\alpha|000\rangle + \alpha|011\rangle + \beta|110\rangle + \beta|101\rangle)$$

$$|\varphi_6\rangle = \frac{1}{\sqrt{2}}\left\{\frac{1}{\sqrt{2}}[\alpha(|0\rangle + |1\rangle)|00\rangle] + \frac{1}{\sqrt{2}}[\alpha(|0\rangle + |1\rangle)|11\rangle] + \frac{1}{\sqrt{2}}[\beta(|0\rangle - |1\rangle)|10\rangle] + \frac{1}{\sqrt{2}}[\beta(|0\rangle - |1\rangle)|01\rangle]\right\}$$

$$|\varphi_6\rangle = \frac{1}{2}(\alpha|000\rangle + \alpha|100\rangle + \alpha|011\rangle + \alpha|111\rangle + \beta|010\rangle - \beta|110\rangle + \beta|001\rangle - \beta|101\rangle)$$

$$|\varphi_6\rangle = \frac{1}{2}\big(|00\rangle(\alpha|0\rangle + \beta|1\rangle) + |01\rangle(\alpha|1\rangle + \beta|0\rangle) + |10\rangle(\alpha|0\rangle - \beta|1\rangle) + |11\rangle(\alpha|1\rangle - \beta|0\rangle)\big)$$

- **Seventh state**

Measure	**Probability**	**Result State**	**Operations for Bob**
00	$^1/_4$	$\alpha|0\rangle + \beta|1\rangle$	$No\ operation$
01	$^1/_4$	$\alpha|1\rangle + \beta|0\rangle$	$\boldsymbol{X}|\boldsymbol{B}_{Bob}\rangle = \alpha|0\rangle + \beta|1\rangle$
10	$^1/_4$	$\alpha|0\rangle - \beta|1\rangle$	$\boldsymbol{Z}|\boldsymbol{B}_{Bob}\rangle = \alpha|0\rangle + \beta|1\rangle$
11	$^1/_4$	$\alpha|1\rangle - \beta|0\rangle$	$\boldsymbol{X}|\boldsymbol{B}_{Bob}\rangle = \alpha|0\rangle - \beta|1\rangle$ $\boldsymbol{Z}(\boldsymbol{X}|\boldsymbol{B}_{Bob}\rangle) = \alpha|0\rangle + \beta|1\rangle$