Quantum Teleportation Algorithm

Terminology

B	Bell state
H	Hadamard Gate
X	Controlled X
Z	Controlled Z

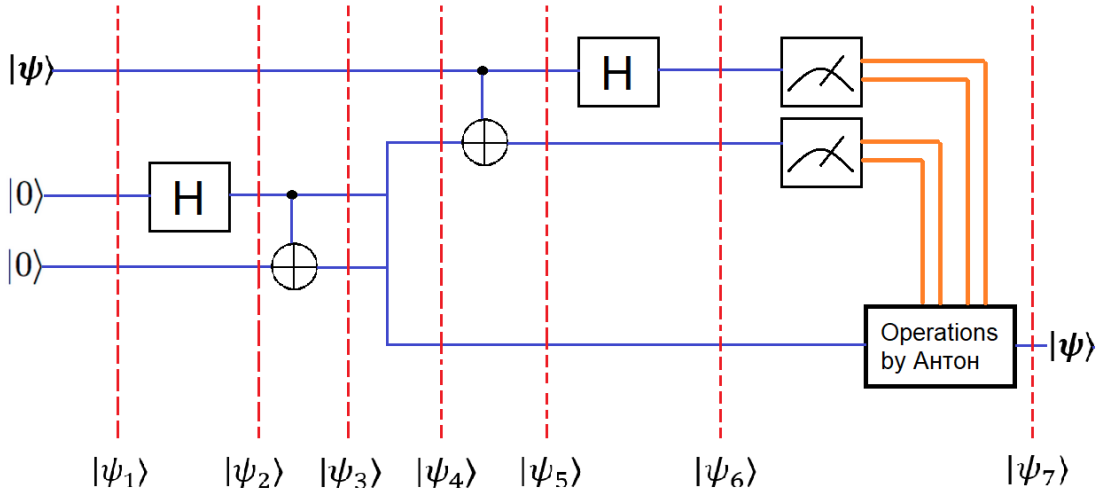
Example

1. Description of the example

Мария and Антон actually are living in Saint Petersburg, Russia. They are working on the Scientific Laboratory «Fiber Optics» in the Peter the Great St. Petersburg Polytechnic University. One day they entangled their qubits for make experiments in the laboratory. Unfortunately, Антон needed to move to Ekaterinburg, Russia. One year after Антон started his research in quantum computing working in the Scientific Laboratory «Fiber Optics» of the city. For research reasons, Мария needs to send the quantum information of her qubit to Антон. Мария and Антон decided to proof the teleportation quantum with objective to proof the teleportation of information. Indicate the quantum circuit with which it will be possible to carry out the transmission of quantum information and indicate which operations Антон will need to do in his laboratory to receive the quantum information.

2. Solution of the problem

2.1. Design of the quantum circuit



2.2. Quantum states of the quantum circuit

2.2.1. First quantum state

$$|\varphi_1\rangle = Second\ qubit_{\text{Мария}}\ First\ qubit_{\text{Антон}}$$

$$|\varphi_1\rangle = |0\rangle|0\rangle$$

2.2.2. Second quantum state

$$|\varphi_2\rangle = \boldsymbol{H}|0\rangle_{\text{Мария}}|0\rangle_{\text{Антон}}$$

$$|\varphi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$$

2.2.3. Third quantum state

$$|\varphi_3\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = |\boldsymbol{B}_{00}\rangle$$

2.2.4. Fourth quantum state

$$|\varphi_4\rangle = First\ qubit_{\text{Мария}}\ \left|\boldsymbol{B}_{\text{Мария}}\right\rangle$$

$$|\varphi_4\rangle = (\alpha|0\rangle + \beta|1\rangle)\left[\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)\right]$$

$$|\varphi_4\rangle = \frac{\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle}{\sqrt{2}}$$

2.2.5. Fifth quantum state

$$|\varphi_5\rangle = \frac{\alpha|000\rangle + \alpha|011\rangle + \beta|110\rangle + \beta|101\rangle}{\sqrt{2}}$$

2.2.6. Sixth quantum state

$$|\varphi_6\rangle = \frac{1}{\sqrt{2}}\boldsymbol{H}(\alpha|000\rangle + \alpha|011\rangle + \beta|110\rangle + \beta|101\rangle)$$

$$|\varphi_6\rangle = \frac{1}{\sqrt{2}}\left\{\frac{1}{\sqrt{2}}[\alpha(|0\rangle + |1\rangle)|00\rangle] + \frac{1}{\sqrt{2}}[\alpha(|0\rangle + |1\rangle)|11\rangle] + \frac{1}{\sqrt{2}}[\beta(|0\rangle - |1\rangle)|10\rangle] + \frac{1}{\sqrt{2}}[\beta(|0\rangle - |1\rangle)|01\rangle]\right\}$$

$$|\varphi_6\rangle = \frac{1}{2}(\alpha|000\rangle + \alpha|100\rangle + \alpha|011\rangle + \alpha|111\rangle + \beta|010\rangle - \beta|110\rangle + \beta|001\rangle - \beta|101\rangle)$$

$$|\varphi_6\rangle = \frac{1}{2}\big(|00\rangle(\alpha|0\rangle + \beta|1\rangle) + |01\rangle(\alpha|1\rangle + \beta|0\rangle) + |10\rangle(\alpha|0\rangle - \beta|1\rangle) + |11\rangle(\alpha|1\rangle - \beta|0\rangle)\big)$$

2.2.7. Seventh quantum state

Measure	Probability	Result State	Operations for Антон
00	$^1/_4$	$\alpha\|0\rangle + \beta\|1\rangle$	$No\ operation$
01	$^1/_4$	$\alpha\|1\rangle + \beta\|0\rangle$	$\boldsymbol{X}\|\boldsymbol{B}_{\text{Антон}}\rangle = \alpha\|0\rangle + \beta\|1\rangle$
10	$^1/_4$	$\alpha\|0\rangle - \beta\|1\rangle$	$\boldsymbol{Z}\|\boldsymbol{B}_{\text{Антон}}\rangle = \alpha\|0\rangle + \beta\|1\rangle$
11	$^1/_4$	$\alpha\|1\rangle - \beta\|0\rangle$	$\boldsymbol{X}\|\boldsymbol{B}_{\text{Антон}}\rangle = \alpha\|0\rangle - \beta\|1\rangle$ $\boldsymbol{Z}(\boldsymbol{X}\|\boldsymbol{B}_{\text{Антон}}\rangle) = \alpha\|0\rangle + \beta\|1\rangle$

2.3. Conclusion

The transmission of the quantum information is possible with the described quantum circuit. Мария is going to lose the quantum state of her first qubit. Антон will need to do certain operations to acquire the quantum information in his qubit.