Grover's Algorithm

Terminology

$\overline{a}$ Mean
U_s Reflection operator
U_w Unitary operator

Examples

1. Example 1

1.1. Description of the example

How much increases the probability of a marked element of an unstructured database of 8 elements when the Grover operator is applied?

1.2. Solution of the problem

Number of elements of the unstructured database: $N = 8$. The weight of all elements has equal weight: $\langle w|s \rangle = \frac{1}{\sqrt{N}} = \frac{1}{\sqrt{8}}$. U_s changes the sign of amplitude of marked element: $w = -\frac{1}{8}$.

U_w changes the amplitude of all elements $(2\overline{a} - a_x)$, where $\overline{a} = \frac{3}{8\sqrt{2}}$. Every unmarked element will have an amplitude $\frac{1}{4\sqrt{2}}$ and a marked element will have $\frac{5}{4\sqrt{2}}$. The Figure 1 shows the amplification of amplitude.
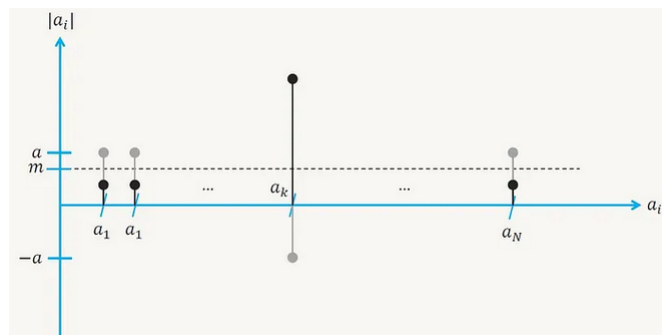

Figure 1: Selective Amplification of Amplitude [1]

1.3. Conclusion

The probability of a marked element increases by 25 times.

2. Example 2

2.1. Description of the example

Solve the following 2x2 Takuzu (binary sudoku) using Grover's algorithm. Finding the solution manually and using the quantum circuit implemented in the platform «$Q|\text{Политех}\rangle$».

2.2. Solution of the problem

To solve the binary sudoku you need the following rules:

- Each cell will either be black (0) or white (1).
- Each column/ow must have an equal number of black and white cells.
- No more than two cells of the same color may be orthogonally adjacent to one another.
- Each row and columns must be unique.

Manually it is easy to solve the problem:

0	1
1	0

1	0
0	1

Using the function for Grover's algorithm implemented in the platform, you can get the same result. Figure 2 shows the quantum circuit that is required to solve the problem. Figure 3 shows the histogram. In this histogram you can see the probabilities of the states (there are 2 states that represent the 2 answers respectively).
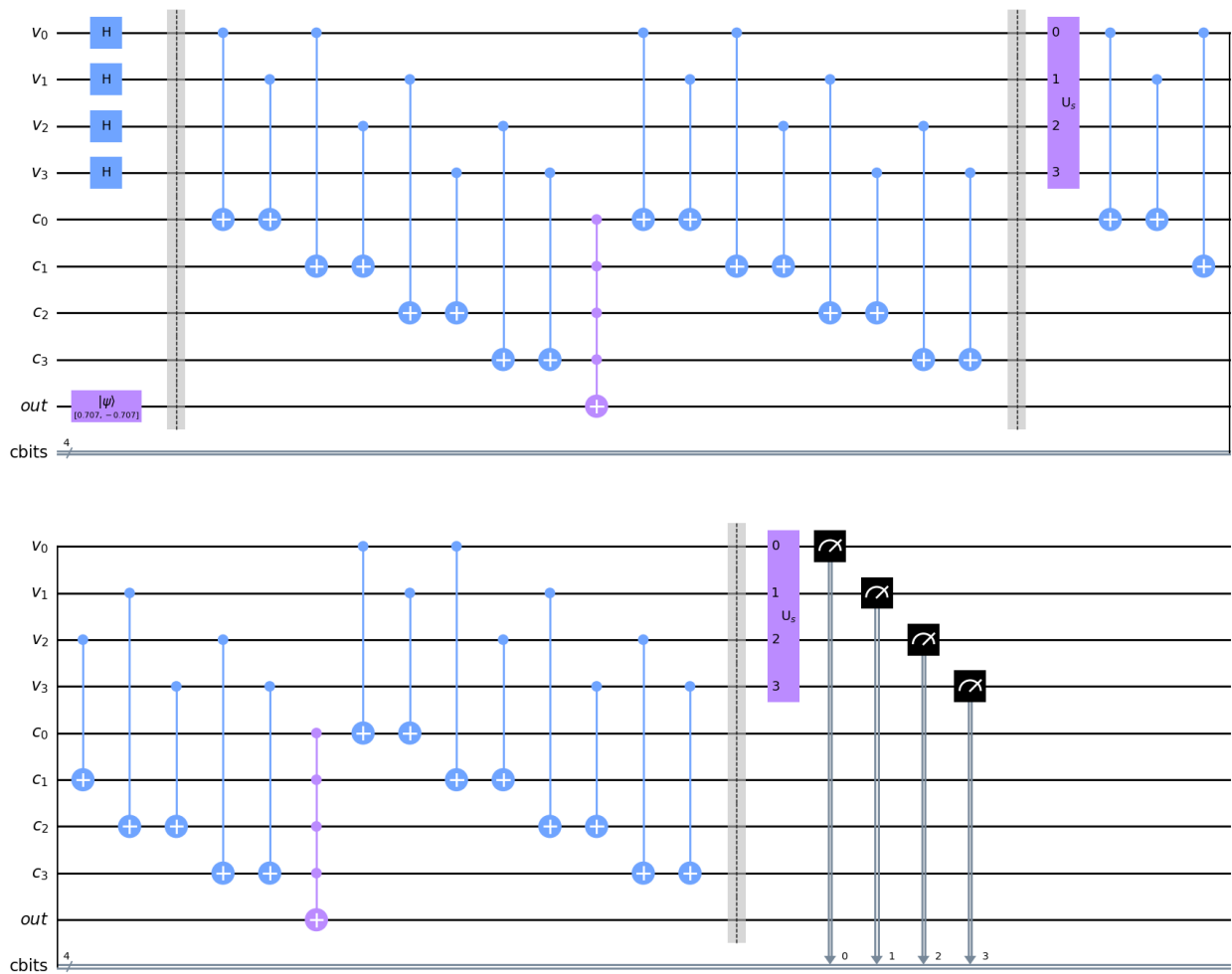
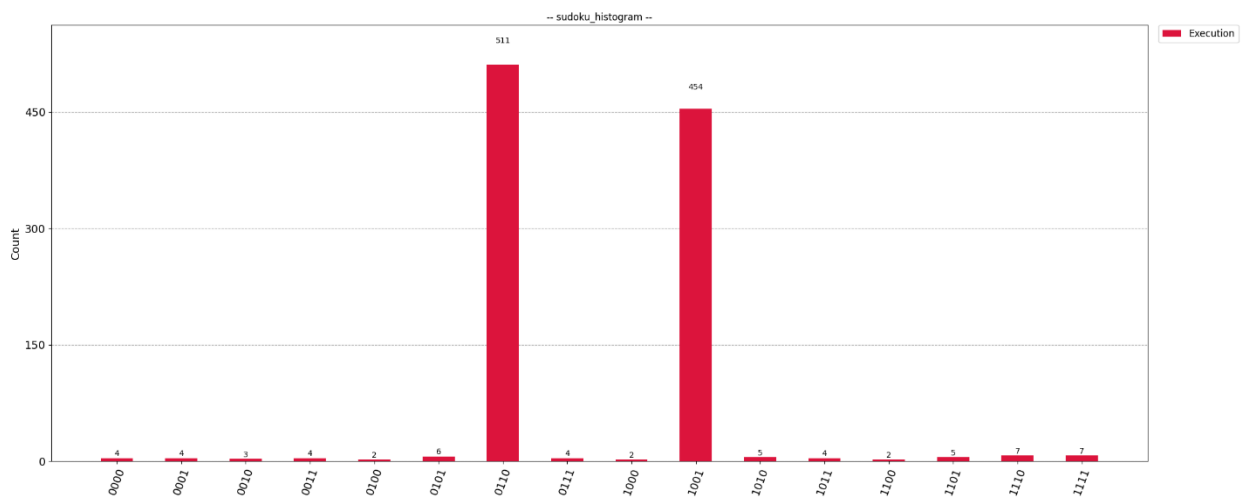Figure 2: Quantum circuit to solve the binary Sudoku.

Figure 3: Histogram of quantum circuit states.

2.3. Solution of the problem

It was easy to find the answer manually. To implement the quantum circuit, is required to understand Grover's algorithm logic. Sudoku is a good example of the type of One-Way Functions. One challenge is to implement a quantum circuit to solve a 3x3 binary sudoku.

References

[1] Salihamajeed (2022). Quantum Grover's Search Algorithm. (Date of consultation: 18.04.2023) Link: https://interviewnoodle.com/quantum-grovers-search-algorithm-f2430451139c