PID Control for Mass Spring Damper

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1 Problem Statement

1.1 Consider a basic mass-spring-damper system

$m = 1kg$
$b = 10kg/s$
$k = 5kg/s2$
- Actuator of $A = 10s + 10$
– Assume input-output:
*Input is force
*Output is position of cart

1.2 Design a control with the only objective being specific overshoot in response to a unit-step command to position

- Design K1 so the response shows 10% overshoot
- Design K2 so the response shows 50% overshoot
- Design K3 so the response shows 100% overshoot

1.3 Analyze the system (overlap data for K1, K2, K3)

–Plot data:

- Position in response to a unit-step command
- Position in response to a unit-step command and disturbance of d= 0.5 sin(t)
- Position in response to a unit-step command and noise being random but bounded by ± 0.1m
- Bode plot of loop gain

- Bode plot of sensitivity
- Bode plot of complementary sensitivity

–Discuss data

- Relate data to tracking, disturbance rejection, and noise attenuation
- Correlate data to gain/phase margins
- Comment if crossover (frequency,slope) has any effect

2 Solution

2.1 Plots

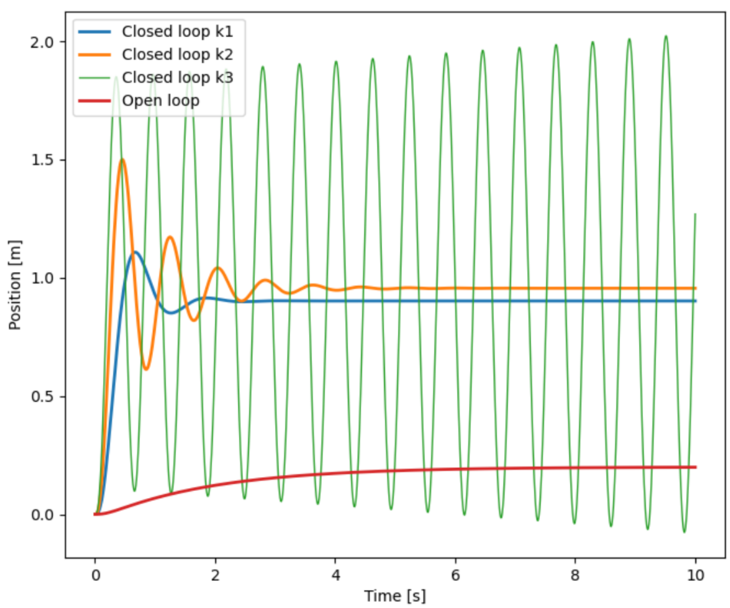

Figure 1: Step Function

Since I implemented a simple P controller, the larger the proportional gain the larger the loop gain PK. k1 has a proportional gain of 46, the k2 of 108 and k3 of 207. As expected, larger proportional gain results in higher overshoot, and the k3 controller exhibits inability to track the reference command because the large proportional gain made the system unstable.

The disturbance plot (Figure 2) shows that the k1 controller has the worst disturbance rejection, the k2 controller showed little signs of errors after the oscillations dissipated, and the k3 controller is barely changed by the disturbance.

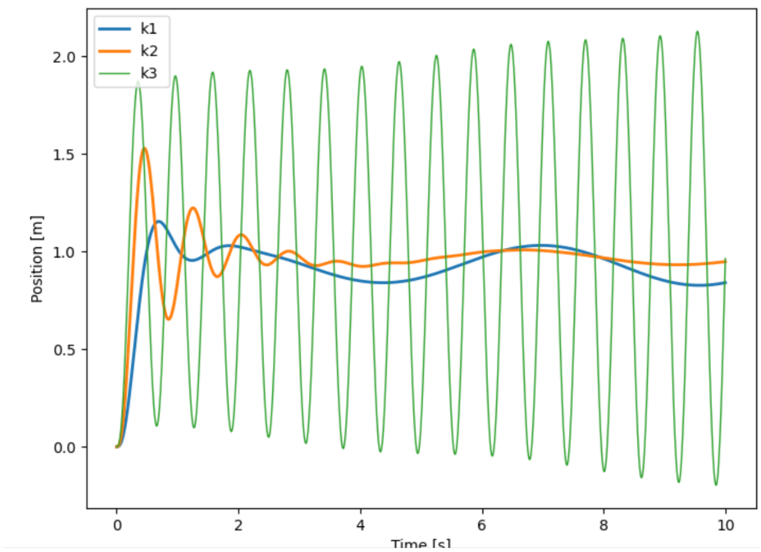

Figure 2: Step Function w/ Disturbance

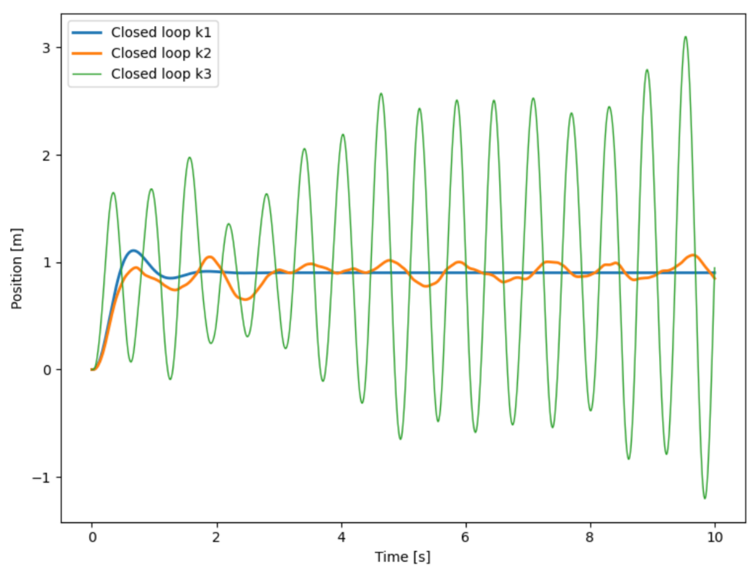

Figure 3: Step Function w/ Noise

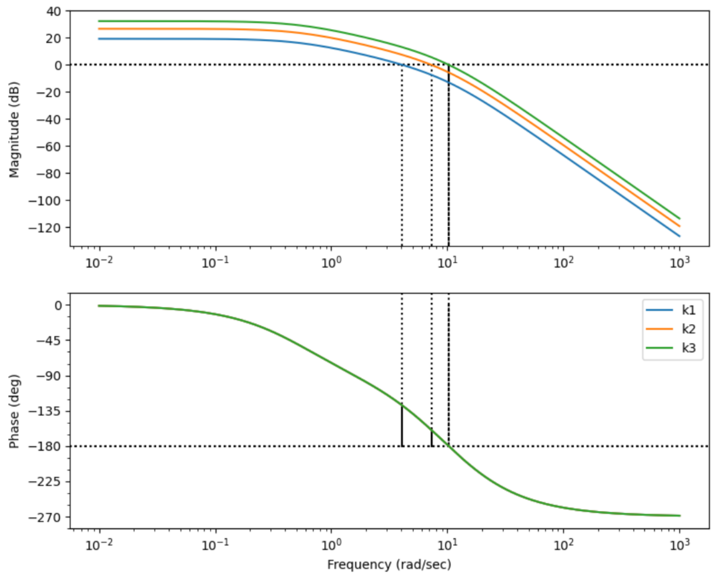

Figure 4: Bode plot of the Loop Gain

The noise plot (Figure 3) shows that k1 is barely affected by noise in the sensor, k2 does a worst job than before by resulting in errors not seen before in 1, and k3 becomes even more unstable.

As the loop gain increases, gain and phase margin are decreased. This makes intuitive sense since gain and phase margins are how far is the system to becoming unstable, which can also be appreciated by noting that crossover frequency increases as the loop gain increases. Therefore the k3 controller, since it is already unstable, has GM and PM or zero seen in Figure 4 as the darker colored or bold dotted lines, and $GM_{k1} > GM_{k2} > GM_{k3}$.

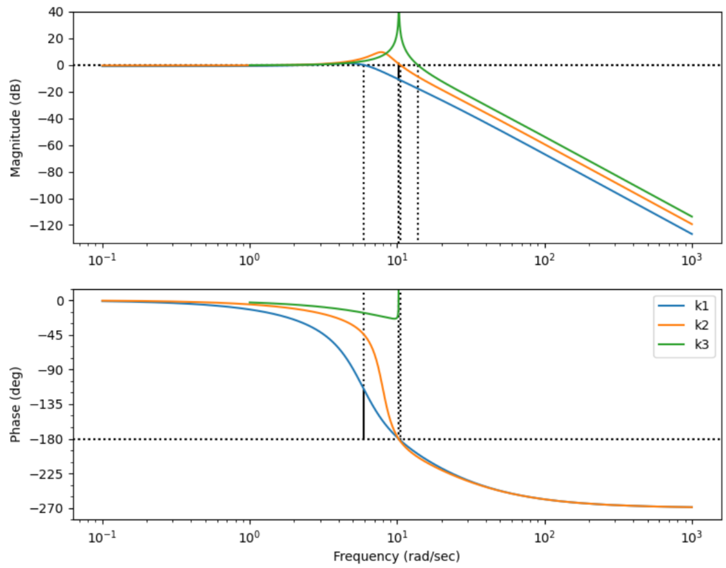

Figure 6: Bode plot of complementary sensitivity

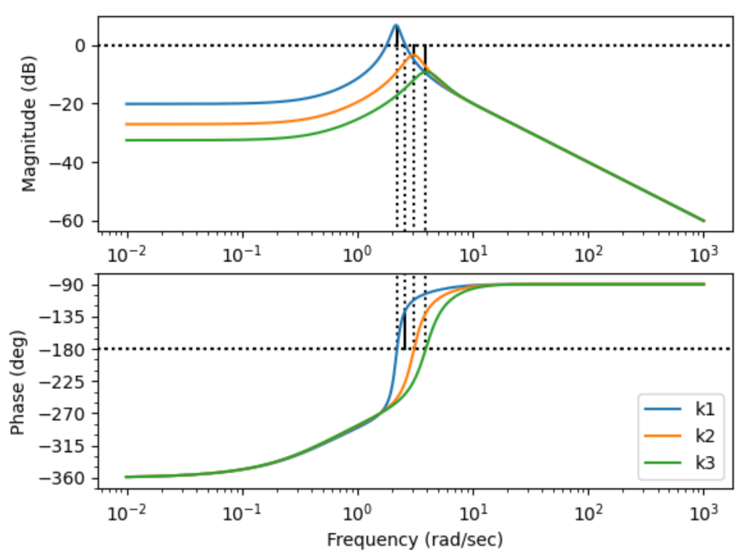

Figure 5: Bode plot of sensitivity

Figure 5 shows that the magnitude of sensitivity is low at low frequency which results in the property of good tracking and disturbance rejection (seen in Figure 2).

The complementary sensitivity bode plot shows that the magnitude of T is high for low frequencies, which results in the good attenuation seen in Figure 3.