Reinforcement Learning for Multi-Agent Systems

Robert Babuška and Lucian Buşoniu

Delft Center for Systems and Control
Faculty of Mechanical Engineering
Delft University of Technology
The Netherlands

e-mail: r.babuska@dcsc.tudelft.nl
http://dcsc.tudelft.nl/˜rbabuska
tel: 015-27 85117

Outline

1. Introduction
   - multi-agent systems
   - learning paradigms
2. Single-agent reinforcement learning
   - Markov decision process, learning goal, policy
   - Bellman equation, optimality, solutions
   - Q-learning, actor-critic scheme
3. Multi-agent reinforcement learning
   - stochastic game, learning goal
   - tasks in multi-agent reinforcement learning
   - methods and algorithms
4. Concluding remarks

Multi-Agent Systems

- Agent: an entity that can perceive its environment through sensors and act upon it through actuators (Russell & Norvig, 2003)
- Rational agent: an agent that always tries to optimize an appropriate performance measure
- Multi-agent system: a group of agents that coexist and potentially interact with one another
- Human , software , robotic agents, . . .

Learning: Rather than a priori design agent behaviors that perform well under all conditions $\longrightarrow$ learn / adapt behaviors online.

Learning Systems

Can find solutions

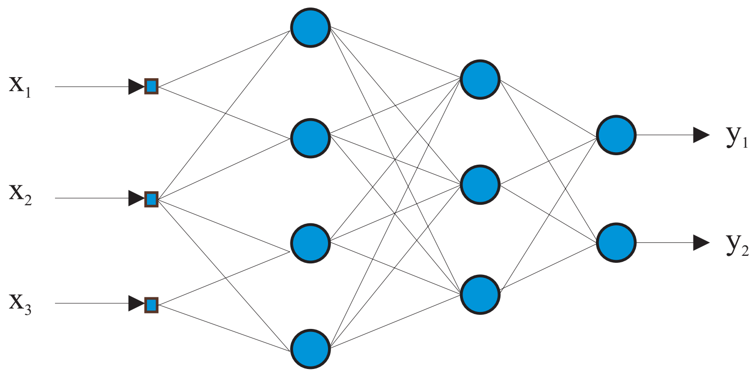
- that are hard to determine a priori
- that are hard to program
- improve over time
- possibly in a (slowly) time-varying process

Learning – an essential feature of "intelligent" systems.

Learning Paradigms (1)

- Supervised learning
  - Learning with teacher (from input–output sample pairs)
  - Mainly used in modeling and identification
  - Example: least-squares fitting, neural nets with back-propagation, operator cloning

Supervised Learning Example: Neural Network

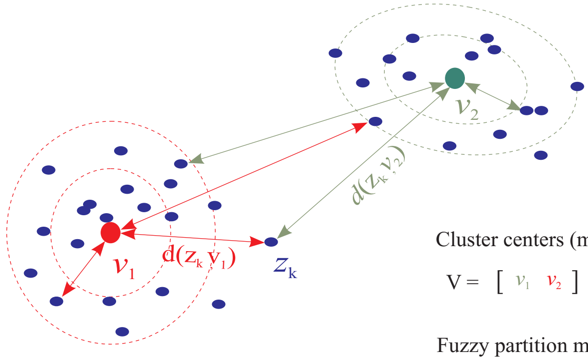


$$y = g(x; w)$$

Learning Paradigms (2)

- Unsupervised learning
  - Without teacher – desired output not used (not available)
  - Discover structures in the data
  - Example: clustering, self-organizing maps, classification

Unsupervised Learning Example: Clustering



Cluster centers (means):

$$V = [\ v_1 \quad v_2\ ]$$

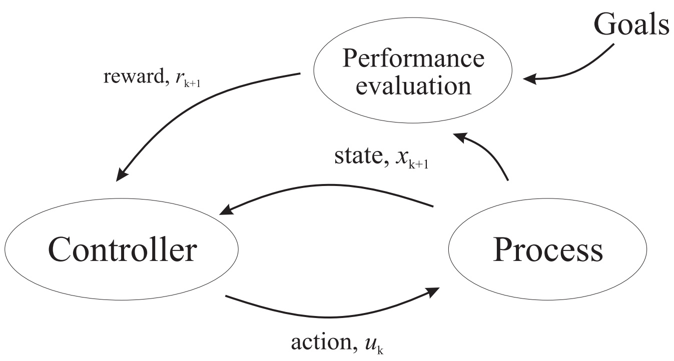
Fuzzy partition matrix:

$$U = \begin{bmatrix} \mu_{11} & \mu_{12} & \cdots & \mu_{1N} \\ \mu_{21} & \mu_{22} & \cdots & \mu_{2N} \end{bmatrix}$$

Learning Paradigms (3)

- Reinforcement learning
  - Between supervised and unsupervised learning.
  - No input–output pairs: only inputs and reward.
  - Inspired by principles of human and animal learning.
  - Only mild assumptions on the process / environment.
  - A strategy can be learnt from scratch.

Reinforcement Learning



Basic Elements of Reinforcement Learning

- Model of the process – typically unknown.
- Reward function.
- Learning objective (cost function).
- Controller (policy, agent).
- Exploration.

Markov Decision Process

A Markov decision process is a tuple $\langle X, U, f, \rho \rangle$ where:

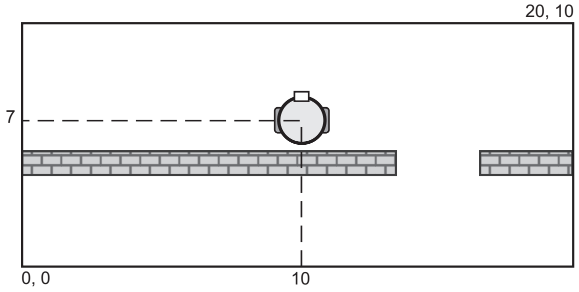
- X is the finite state space
- U is the finite action space
- $f : X \times U \to X$ is the state transition function
- $\rho : X \times U \to \mathbb{R}$ is the reward function

$x_{k+1} = f(x_k, u_k)$ (k - discrete time step)

Note: stochastic formulation possible

Markov Decision Process - Example

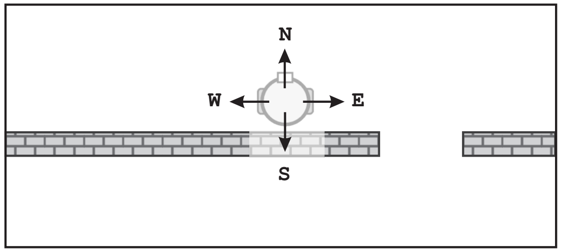
MDP $\ldots \langle {\color{red}X}, U, f, \rho \rangle$



$X = \{0, 1, \ldots, 20\} \times \{0, 1, \ldots, 10\}, \qquad x_k = [10, 7]^{\mathrm{T}}$

Markov Decision Process - Example

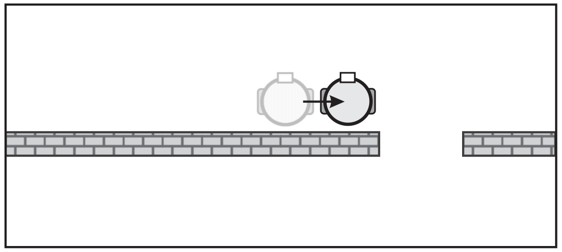
MDP $\ldots \langle X, {\color{red}U}, f, \rho \rangle$



$U = \{\mathtt{N}, \mathtt{E}, \mathtt{S}, \mathtt{W}\}$

Markov Decision Process - Example

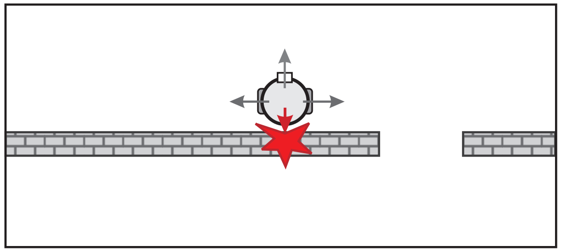
MDP $\ldots \langle X, U, {\color{red}f}, \rho \rangle \qquad x_{k+1} = f(x_k, u_k)$



$x_{k+1} = f([10, 7]^{\mathrm{T}}, \mathtt{E}) = [11, 7]^{\mathrm{T}}$

Markov Decision Process - Example

MDP $\ldots \langle X, U, f, {\color{red}\rho} \rangle \qquad r_{k+1} = \rho(x_k, u_k)$



$\rho(x_k, \mathtt{W}) = -1, \quad \rho(x_k, \mathtt{N}) = -1, \quad \rho(x_k, \mathtt{E}) = -1, \quad \rho(x_k, \mathtt{S}) = -10$

Learning Goal

Performance measure – discounted return:

$$R_k = \sum_{i=0}^{\infty} \gamma^i r_{k+i+1}$$

with:

$r_{k+1} = \rho(x_k, u_k)$ . . . immediate reward

$\gamma \in (0, 1]$ . . . discount factor (lookahead horizon)

finite trial vs. continuous learning

Learning goal: Find a control policy that maximizes the discounted return R_k at every time step k.

Learning the Control Policy

Policy defines what action should be taken in each state:

$$h : X \rightarrow U, \qquad u = h(x)$$

There are various ways to learn the optimal policy.

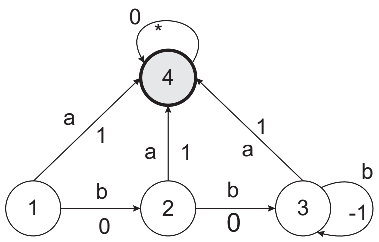
Most straightforward: use the Q-values:

$$Q^h(x, u) = \sum_{i=0}^{\infty} \gamma^i r_{i+1}$$

where the agent starts in x, takes action u and then follows policy h $\quad (r_1 = \rho(x, u),\ x' = f(x, u),\ r_2 = \rho(x', h(x')), \dots)$

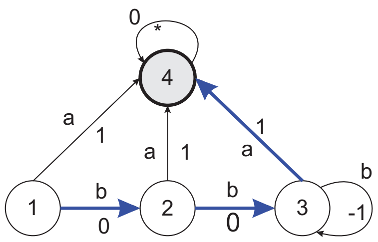
Greedy action: $u = \arg\max_{u' \in U} Q^h(x, u')$

Example: A Simple MDP



- $X = \{1, 2, 3, 4\}$; initial state: 1, goal state: 4
- $U = \{a, b\}$
- a moves to goal, reward 1; b in 1 and 2 moves right, no reward; b in 3 stays, punishment -1
- state 4 is absorbing, episode ends

Example: Policy and Q-Value



Policy: $h(1) = b, \qquad h(2) = b, \qquad h(3) = a, \qquad\qquad \gamma = 0.9$

$$Q^h(1, b) = \gamma^0 \rho(1, b) + \gamma^1 \cdot \rho(2, b) + \gamma^2 \cdot \rho(3, a) + \gamma^3 \cdot \rho(4, *)$$

$$= 1 \cdot 0 + 0.9 \cdot 0 + 0.81 \cdot 1 + 0.729 \cdot 0 = 0.81$$

Recursive Computation of Q-values

$$\begin{aligned} Q^h(x,u) &= \sum_{i=0}^{\infty} \gamma^i r_{i+1} \\ &= r_1 + \gamma \sum_{i=0}^{\infty} \gamma^i r_{i+2} \\ &= \rho(x,u) + \gamma Q^h(x',u') \end{aligned}$$

$$\boxed{Q^h(x,u) = \rho(x,u) + \gamma Q^h(x',u'), \quad x' = f(x,u),\; u' = h(x')}$$

This equation is called the Bellman equation.

Optimality

- Optimal value function:

$$Q^*(x,u) = \max_h Q^h(x,u)$$

- Optimal policy – greedy policy in Q^*:

$$h^*(x) = \arg\max_{u \in U} Q^*(x,u)$$

- Q^* satisfies the Bellman optimality equation:

$$Q^*(x,u) = \rho(x,u) + \gamma \max_{u' \in U} Q^*(x',u'), \quad x' = f(x,u)$$

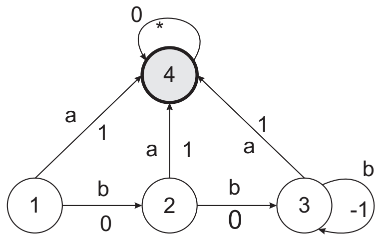
Solution Techniques

Find Q^*, then $h^*(x) = \arg\max_{u \in U} Q^*(x,u)$

1. Model-based:
   - known reward model ρ and state transition model f
   - compute Q^* off-line (DP, value iteration)
2. Model-free:
   - unknown ρ and f
   - learn Q^* online, by interaction with the process
3. Mixed:
   - learn ρ and f models by interaction
   - interleave (1) and (2)

Bellman Equation: Example

$$\boxed{Q^*(x,u) = \rho(x,u) + \gamma \max_{u' \in U} Q^*(x',u'), \quad x' = f(x,u)}$$

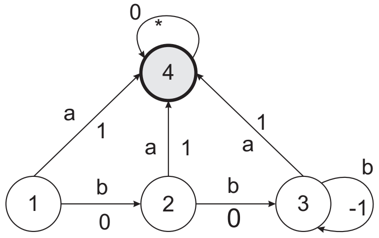


$Q^*(3,a) = \rho(3,a) = 1$

$Q^*(3,b) = \rho(3,b) + \gamma \max\{Q^*(3,a), Q^*(3,b)\} = -1 + 0.9 \cdot 1 = -0.1$

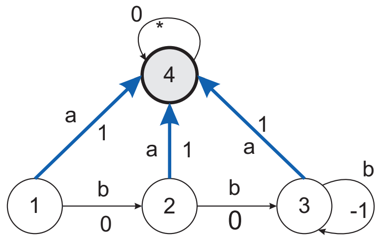
Bellman Equation: Example

$$\boxed{Q^*(x,u) = \rho(x,u) + \gamma \max_{u' \in U} Q^*(x',u'), \quad x' = f(x,u)}$$



$$Q^*(2,a) = \rho(2,a) = 1$$
$$Q^*(2,b) = \rho(2,b) + \gamma \max\{Q^*(3,a), Q^*(3,b)\} = 0 + 0.9 \cdot 1 = 0.9$$

Optimal Solution



Q^*, h^* :

$x \setminus u$	a	b
1	1	0.9
2	1	0.9
3	1	-0.1

u
a
a
a

Value Iteration

Turn the Bellman equation into assignment

$Q(x,u) \leftarrow 0, \quad \forall x \in X, u \in U$
repeat
  for all $x \in X, u \in U$ **do**
    $x' \leftarrow f(x,u)$
    $Q(x,u) \leftarrow \rho(x,u) + \gamma \max_{u' \in U} Q(x',u')$
  end for
until convergence
$Q^* \leftarrow Q$
$h^*(x) \leftarrow \arg\max_{u \in U} Q^*(x,u), \quad \forall x \in X$

- Convergence to Q^* guaranteed

Model-Free: Monte-Carlo

- Estimate returns without using ρ, f (not known):
  - run trials in real world, memorize returns for each pair (x,u)
  - compute the discounted return:

$$Q^h(x,u) = \sum_{i=0}^{N} \gamma^i r_{i+1}$$

- Disadvantages:
  - many tasks are not episodic
  - considerable experience in the world required – costly

Model-Free: Temporal Difference (TD)

- Use experience with the real process (like Monte Carlo)
- Improve value function at each step k:

$$Q(x_k, u_k) \leftarrow Q(x_k, u_k) + \alpha \left[r_{k+1} + \gamma \max_{u' \in U} Q(x_{k+1}, u') - Q(x_k, u_k) \right]$$

- x_k, r_{k+1}, x_{k+1} observed, u_k actually taken action
- Temporal difference: measure of the estimation error – deviation from the Bellman equality $Q(x_k, u_k) = r_{k+1} + \gamma \max_{u' \in U} Q(x_{k+1}, u')$
- α is the learning rate

Exploration

- How to choose u?
- Assume Q is optimal: $u = \arg\max_{u' \in U} Q^*(x, u')$ (greedy action)
- But Q is not (yet) optimal! We need to estimate Q^*!
- $\Rightarrow$ sometimes explore:
  - with probability $(1-\varepsilon)$, choose greedy action $u = \arg\max_{u' \in U} Q(x, u')$
  - with probability ε, choose a random exploratory action u

Q-Learning

$Q(x, u) \leftarrow 0, \quad \forall x \in X, u \in U$
$k \leftarrow 0$
loop
  observe state x_k
  $u_k \leftarrow \arg\max_{u' \in U} Q(x_k, u')$
  with probability ε, $u_k \leftarrow$ random action
  apply u_k, observe r_{k+1} and x_{k+1}
  $Q(x_k, u_k) \leftarrow Q(x_k, u_k) + \alpha \left[r_{k+1} + \gamma \max_{u' \in U} Q(x_{k+1}, u') - Q(x_k, u_k) \right]$
end loop

- Q-learning converges to Q^*

User-Defined Parameters and Choices

- Discount factor γ (typically 0.9–0.99).
- Learning rate α (typically < 0.5).
- Exploration probability ϵ (typically < 0.5).
- State and action space discretization (ad hoc).
- Trial definition, immediate rewards (ad hoc).

Modifications, Extensions

- On-policy learning.
- Eligibility traces.
- Interpolation methods for continuous spaces.
- Actor-critic methods.
- Mixed model-based – model-free learning.
- Stochastic setting (probabilistic rewards and transitions).
- Multi-agent systems.

Basic Elements of Reinforcement Learning

- Model of the process (includes all agents).
- Reward functions (each agent may have its own).
- Learning objective (may be hard to define).
- Controllers (policies, agents).
- Exploration.

Multi-Agent Case: Stochastic Game

A stochastic game is a tuple $\langle A, X, \{U_i\}_{i \in A}, f, \{\rho_i\}_{i \in A} \rangle$, where:

- $A = 1, \ldots, n$ is the set of agents
- X is the finite state space
- U_i is the finite set of agent's actions, $\boldsymbol{U} = \times_{i \in A} U_i$
- $f : X \times \boldsymbol{U} \times X \rightarrow [0, 1]$ is the state transition probability distr.
- $\rho_i : X \times \boldsymbol{U} \times X \rightarrow \mathbb{R}$ is the agent's reward function

State transitions and rewards depend on the joint action!

Consequences for Learning

- Optimal policy at any moment depends on policies of other agents.
- Other agents are also learning $\longrightarrow$ moving target.
- Need to take special care of convergence.
- Policies found may not be optimal against specific opponent's policies.
- Depending on the goal, different settings (tasks) possible.

Classification According to Tasks

- Cooperative tasks: $\rho_1 = \rho_2 = \cdots = \rho_n$
- Competitive tasks: typically $\rho_1 = -\rho_2$ (cf. zero-sum games)
- Mixed tasks: general case (cf. general-sum games)

RL in MAS: Approaches

- Ignore the presence of other learning agents:
  $\longrightarrow$ apply single-agent reinforcement learning
- Be aware of other learning agents:
  $\longrightarrow$ guarantee convergence independently of other agents
- Adapt to (track) other agents:
  $\longrightarrow$ strive for optimality / rationality

Agents may be heterogeneous: goals, assumptions, algorithms.

Single-Agent Learning

Assumption: Agents interfere weakly

Methods:

- Ignore other agents
- Consider other agents indirectly (through the reward signal)

Some applications reported: (Matarić, 1996; Crites and Barto, 1996)

Convergence not guaranteed.

Cooperative Multi-Agent Learning

Typical approach: use $Q(x, \boldsymbol{u})$, where

x – global world state, including all agents' states

$\boldsymbol{u}$ – joint action of all agents (must be observed)

- Use Q-learning to find Q^*

$$Q_{k+1}(x_k, \boldsymbol{u}_k) = Q_k(x_k, \boldsymbol{u}_k) + \alpha\big[r_{k+1} + \gamma \max_{\boldsymbol{u}'} Q(x_{k+1}, \boldsymbol{u}') - Q_k(x_k, \boldsymbol{u}_k)\big]$$

- Greedy policy $\boldsymbol{u} = \arg\max_{\boldsymbol{u}' \in U} Q^h(x, \boldsymbol{u}')$

However: Agents may break ties differently!

Ensuring Convergence

- Assume unique maxima of Q (team Q-learning, Littman, 2001)
- Coordination-free methods – e.g., distributed Q-learning (Lauer and Riedmiller, 2000) – only for deterministic setting
- Coordination-based methods:
  - Direct coordination: social conventions, roles, negotiation (Boutilier, 1996)
  - Indirect coordination: learn models of other agents (Joint Action Learners, Claus and Boutilier, 1998)

Fully Competitive Tasks

Opponent-independent: solve the dynamic task stage-wise

$$\begin{aligned} h_{1,k}(x_k,\cdot) &= \arg \mathbf{m}_1(Q_k, x_k) \\ Q_{k+1}(x_k,u_{1,k},u_{2,k}) &= Q_k(x_k,u_{1,k},u_{2,k}) + \\ &\alpha\big[r_{k+1} + \gamma \mathbf{m}_1(Q_k, x_{k+1}) - Q_k(x_k,u_{1,k},u_{2,k})\big] \end{aligned}$$

where $\mathbf{m}_1$ is the minimax (worst-case) return of agent 1:

$$\mathbf{m}_1(Q,x) = \max_{h_1'(x,\cdot)} \min_{u_2} \sum_{u_1} h_1'(x,u_1) Q(x,u_1,u_2)$$

agent might do better if it has a model of the opponent's policy

Mixed Tasks

Opponent-independent: solve the dynamic task stage-wise

$$\begin{aligned} h_{i,k}(x,\cdot) &= \mathbf{solve}_i\{Q_{\cdot,k}(x_k,\cdot)\} \\ Q_{i,k+1}(x_k,\boldsymbol{u}_k) &= Q_{i,k}(x_k,\boldsymbol{u}_k) + \alpha\big[r_{i,k+1} + \\ &\gamma \cdot \mathbf{eval}_i\{Q_{\cdot,k}(x_{k+1},\cdot)\} - Q_{i,k}(x_k,\boldsymbol{u}_k)\big] \end{aligned}$$

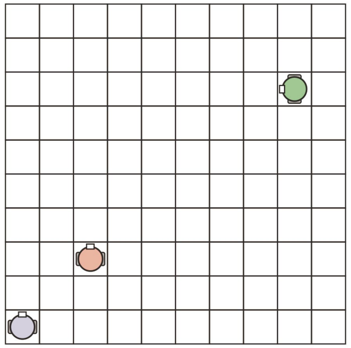
- $\mathbf{solve}$ – game-theoretic equilibrium in stage game
- $\mathbf{eval}$ – expected return for this equilibrium

Target convergence, but cannot exploit a specific opponent's policy

Limitations and Challenges

- Convergence vs. optimality / rationality
- Continuous domains:
  - discretization may not always work
  - function approximation - convergence not guaranteed
- Curse of dimensionality (exponential explosion of Q-tables):
  - computational problems (memory and speed)
  - slow convergence of learning

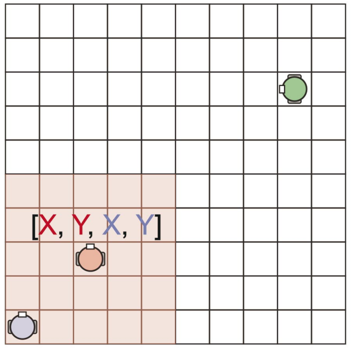
State Space Explosion



Q-table size: $100^3 \cdot 4^3 = 64$ million entries!
Large search space $\Rightarrow$ slow learning speed

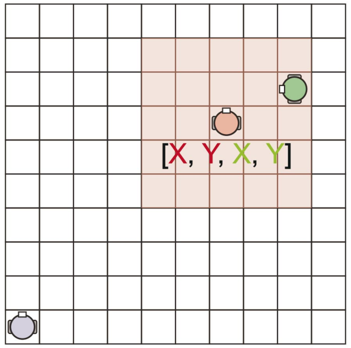
State Space Explosion

Other agents' positions most likely irrelevant to the red agent unless they are nearby:

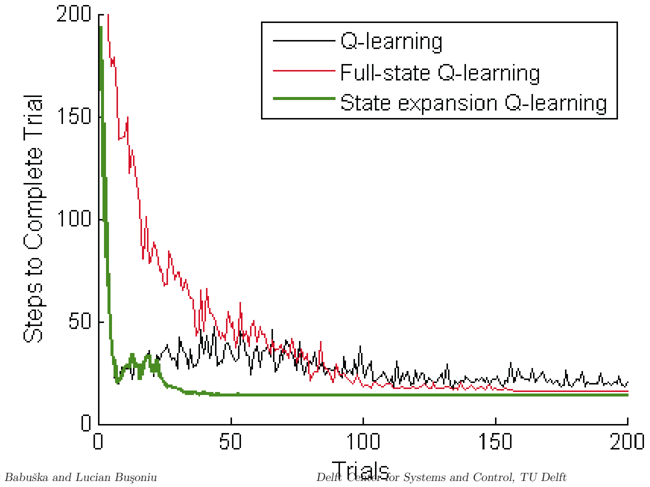


State Space Explosion

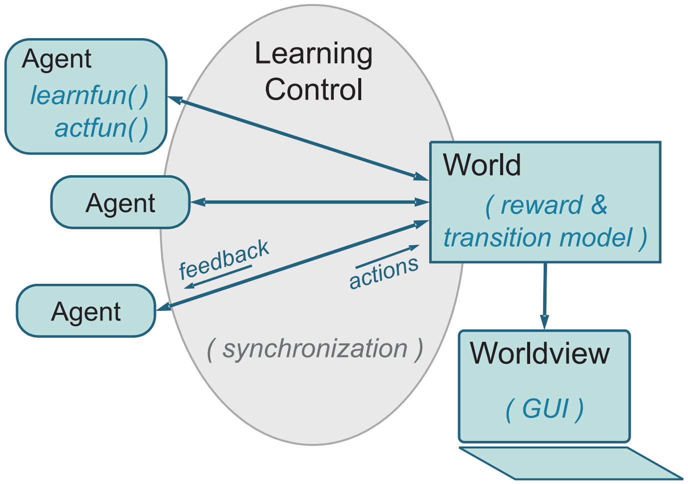
Other agents' positions most likely irrelevant to the red agent unless they are nearby:



MAS Learning: Rate of Convergence



Matlab Toolbox for RL MAS



Concluding Remarks

- Single-agent reinforcement learning
  - well developed for discrete state and action spaces
  - unresolved issues in continuous spaces
  - applicable to moderate-size problems
- Multi-agent reinforcement learning
  - much more intricate than single-agent
  - applicable to small-size problems
- Learning in general
  - not a panacea, the modeling / design problem only shifted
  - performance improvement not monotonous

References

[1] Boutilier, C. (1996). Planning, learning and coordination in multiagent decision processes. In *Proceedings Sixth Conference on Theoretical Aspects of Rationality and Knowledge (TARK-96)*, pages 195–210, De Zeeuwse Stromen, The Netherlands.

[2] Bowling, M. (2004). Convergence and no-regret in multiagent learning. In *Advances in Neural Information Processing Systems 17 (NIPS-04)*, pages 209–216, Vancouver, Canada.

[3] Bowling, M. and Veloso, M. (2002). Multiagent learning using a variable learning rate. *Artificial Intelligence*, 136(2):215–250.

[4] Crites, R. H. and Barto, A. G. (1998). Elevator group control using multiple reinforcement learning agents. *Machine Learning*, 33(2–3):235–262.

[5] Guestrin, C., Lagoudakis, M. G., and Parr, R. (2002). Coordinated reinforcement learning. In *Proceedings Nineteenth International Conference on Machine Learning (ICML-02)*, pages 227–234, Sydney, Australia.

[6] Lauer, M. and Riedmiller, M. (2000). An algorithm for distributed reinforcement learning in cooperative multi-agent systems. In *Proceedings Seventeenth International Conference on Machine Learning (ICML-00)*, pages 535–542, Stanford University, US.

[7] Littman, M. L. (2001). Value-function reinforcement learning in Markov games. *Journal of Cognitive Systems Research*, 2:55–66.

[8] Matarić, M. J. (1996). Learning in multi-robot systems. In Weiß, G. and Sen, S., editors, *Adaptation and Learning in Multi–Agent Systems*, pages 152–163. Springer Verlag.

[9] Panait, L. and Luke, S. (2005). Cooperative multi-agent learning: The state of the art. *Autonomous Agents and Multi-Agent Systems*, 11(3):387–434.

[10] Powers, R. and Shoham, Y. (2004). New criteria and a new algorithm for learning in multi-agent systems. In *Advances in Neural Information Processing Systems 17 (NIPS-04)*, pages 1089–1096, Vancouver, Canada.

[11] Wang, X. and Sandholm, T. (2002). Reinforcement learning to play an optimal Nash equilibrium in team Markov games. In *Advances in Neural Information Processing Systems 15 (NIPS-02)*, pages 1571–1578, Vancouver, Canada.