Proceedings of the 2006 IEEE International Conference on Robotics and Automation
Orlando, Florida - May 2006

Estimation of Optimal Elevator Scheduling Performance*

Jin Sun and Qianchuan Zhao
Center for Intelligent and Networked Systems (CFINS)
Department of Automation, Tsinghua University
Beijing 100084, China
sunjin00@mails.tsinghua.edu.cn, zhaoqc@tsinghua.edu.cn

Peter B. Luh
Department of Electrical and Computer Engineering
University of Connecticut
Storrs, Connecticut 06269-2157, USA
Peter.Luh@uconn.edu

Mauro J. Atalla
United Technologies Research Center
411 Silver Lane, MS 129-48
East Hartford, Connecticut 06108, USA
AtallaMJ@utrc.utc.com

***Abstract* – Group elevator scheduling is important for transportation efficiency in mid-rise and high-rise buildings, and incessant efforts have been made to improve the service efficiency of elevators. Although these efforts have achieved performance improvements, the performance limit remains an open issue. This paper tries to address that goal by estimating the optimal performance of group elevator scheduling with complete knowledge of future traffic information. A two-level minimization formulation is presented, with passenger-to-car at the high level, and single car dispatching at the low level. The low level is formulated as a passenger-to-trip assignment problem by using a concept *trip* to facilitate the description of single car dispatching strategies. In view of the difficulty to obtain the absolute optimal performance, our goal turns into its upper and lower bounds. The upper bound is obtained by finding a good feasible solution to this problem. The lower bound is obtained by finding the lower bound for a newly constructed problem whose optimal performance is less than or equal to that of the original problem. Numerical results demonstrate the effectiveness and the scalability of our method.**

Index Terms - Group Elevator Scheduling, Complete Information, Single Car Dispatching, Lagrangian Relaxation, and the Penalty Surrogate Subgradient Method.

I. Introduction

Group elevator scheduling is important for efficiently transporting occupants to their intended destinations in mid-rise and high-rise buildings. Several efforts have been made to improve the service efficiency of elevators, some focusing on improving real-time scheduling algorithms [1], [2]-[6], and others on collecting traffic information [7]. Although these and several other efforts have achieved performance improvements, the performance limit still remains an open issue. This paper tries to address that goal by estimating the optimal performance of group elevator scheduling under the assumption of complete and deterministic information (i.e., complete knowledge of future passenger arrival times, their origin and destination floors). The problem posed in this fashion becomes an off-line deterministic minimization problem. Note that the major difference between our paper and most of the previous studies on elevator scheduling is that our paper aims at estimating the performance limit of elevator systems rather than providing a particular scheduling algorithm. This problem, however, is extremely difficult because of various traffic patterns, complex car dynamics, and the NP-hardness of single car dispatching and group elevator scheduling even without considering detailed car dynamics [8], [9]. Therefore, it is difficult to obtain the exact value of the optimal performance.

Key characteristics of group elevator scheduling are abstracted in [1]. Individual cars are coupled through serving a common pool of passengers. Provided that these passengers are assigned to individual cars in advance, then the dispatching of one car is independent of others and hence group elevator scheduling turns into several independent single car dispatching problems. Exploiting this "separable" structure, a two-level formulation was developed in [1], with passenger-to-car assignment at the high level and single car dispatching at the low level. Our paper provides an equivalent formulation with the same two-level structure and a novel detailed formulation for the low level. In our formulation, a concept 'trip' that denotes the car movement in a single direction from start until reversal of travel direction is introduced to encode the single car dispatching strategy into the passenger-to-trip assignment. Taking advantage of the encoding scheme, constraints for single car dispatching could be easily set up, and then the low level could be formulated as a passenger-to-trip assignment problem. The detailed car dynamics are embedded in individual car simulation models only for performance evaluation, and hence the complexity of the problem is reduced. Our formulation provides a convenient way to describe dispatching strategies and formulate single car dispatching. The scale of our formulation (i.e., the number of constraints) is linear with the number of passengers and cars, and is much smaller than that of the formulation in [12]. Moreover, the main advantages of the formulation in [1] are inherited, including the following: 1) the objective function of our formulation is flexible within a large range of passenger-wise and car-wise measures (e.g., average passenger service time or wait time); 2) our formulation applies to different building configurations and traffic patterns. The formulation is presented in Section III.

In view of the difficulty to obtain the optimal performance, our goal is to establish provable upper and lower bounds. The upper bound is obtained by finding a good feasible solution, i.e., a good passenger-to-car assignment with a good single car dispatching strategy, which is obtained from proprietary dispatching simulators that implement modern

*The work was supported by a grant from United Technologies Research Center. Jin and Zhao received additional support from NSFC (Nos. 60274011, 60574067), and the NCET (No. NCET-04-0094) program of China.

dispatching algorithms. The lower bound is obtained by finding the lower bound for the optimal performance of a newly constructed problem whose optimal performance is less than or equal to that of the original problem. First the original formulation is rewritten in an equivalent form. Then a new problem is constructed by replacing the objective function of the new formulation (i.e., the optimal performance of single car dispatching) with its lower estimate, which is obtained by using Segmentation Approach. In virtue of the separable structure of the new problem, Lagrangian relaxation finally can be applied to obtain the lower bound for its optimal performance. This approach avoids the need to compute the optimal solution to the single car dispatching problem, thus avoiding the associated time and space requirements. The solution methodology is presented in Section IV.

Numerical testing results presented in Section V demonstrate that our method can provide tight bounds for light traffic under various traffic patterns, and for heavy traffic under up-peak traffic, and provide acceptable bounds for heavy traffic under down-peak and inter-floor traffic, and our method is scalable to larger problems.

II. Literature Review

Research on group elevator scheduling has concentrated on the problems toward for the real-time scheduling and/or without exact knowledge of future traffic information [2]-[6], [10], [11]. The deterministic problem with complete information has ever been studied in [12]. In that work, the operation of carrying a passenger from the origin to the destination is divided into two jobs: on-job and off-job. Then a two-level formulation is presented, with the assignment of all jobs to individual cars at the high level and the determination of the processing orders of the assigned jobs for each car at the low level. The formulation, however, is too complicated and does not scale to larger problems due to the very large number of decision variables and constraints involved. In addition, although the Branch and Bound method used in that work can generate the optimal performance, it is very time-consuming for larger problems.

Another related work on on-line scheduling with advance information was presented in [1]. The authors of [1] have extended their method to deal with the off-line performance limit problem with additional elevator behavior assumptions in an updated version of [1] for journal publication. A two-level formulation is established with passenger-to-car assignment at the high level and single car dispatching at the low level. The low level problem including the complex car dynamics is not expressed explicitly but embedded in individual car simulation models. A dynamic programming based approach is developed to solve the low level (e.g., to find an optimal single car dispatching strategy). Note that both [1] and its updated version assume that once the set of "stop floors" in a one-way trip is given, the passengers to be served in this trip can be uniquely determined by introducing additional behavior assumptions. That assumption greatly reduces the search space of dynamic programming. Yet, that assumption loses its validity in our case, for our case does not have those behavior assumptions and the optimal solution of our case might violate those behavior assumptions. Since one cannot take advantage of that assumption in our case, it will require a large amount of computational resources for dynamic programming to obtain an optimal single car dispatching strategy due to the huge search space. For the high level, Lagrangian relaxation is applied to optimize the passenger-to-car assignment. This method, however, requires the resolution of single car dispatching to optimality at least once per iteration. Therefore, the applicability of the method in the updated version of [1] to our case is limited because of the large memory requirements for accomplishing that.

III. Problem Formulation

A. Overview

An equivalent two-level formulation of the one in [1] is presented in this section. The high level is formulated as a passenger-to-car assignment problem. The low level is formulated as a passenger-to-trip assignment problem by using a concept 'trip' to encode single car dispatching strategies.

In a building with F floors, M cars are working together to serve a set P of N passengers. Passenger n arrives at floor O_n on time A_n and will go to floor D_n, where O_n and D_n belong to the set {0, 1, …, F-1}. All the O_n's, A_n's and D_n's are assumed given.

B. Formulations of the High Level

The high level problem focuses on selecting for each car the appropriate set of passengers to serve. The passenger-to-car assignment is defined as an N×M matrix X of binary variables, where the $(n, m)_{th}$ element X_{nm} equals to 1 if passenger n is assigned to car m and 0 otherwise. These decision variables must subject to the following constraints.

Passenger-to-Car Assignment Constraints. Each passenger must be assigned to one and only one car, i.e.,

$$\sum_{m=1}^{M} X_{nm} = 1 \text{, for } n = 1, \ldots, N. \tag{1}$$

Note that individual cars are coupled by (1) since they have to serve a common pool of passengers.

C. Formulations of the Low Level

Given a passenger-to-car assignment X, individual cars are no longer coupled. Then group elevator scheduling turns into several independent single car dispatching problems. The low level focuses on how to dispatch each car to serve the set of passengers assigned to it.

Since future passenger information is completely known, each single car dispatching strategy corresponds to a specific service process. Then the representation of single car dispatching strategy is equivalent to the representation of its corresponding service process. To facilitate the representation of a service process, a concept 'trip' is introduced to describe the car movement in a single direction, from start until reversal of travel direction. Then passengers are served by the car in several trips. For a given service process, the corresponding passenger-to-trip assignment can be easily obtained by simply recording which passengers are assigned to which trip. Conversely, for a given passenger-to-trip assignment, the corresponding service process can be easily deduced. Therefore, finding the best single car dispatching strategies can be considered equivalent to finding the best passenger-to-trip assignment.

The low level is then to select, for each trip, the best set of passengers in P_m to serve, where P_m denotes the set of passengers assigned to car m and could be identified by the m_{th} column X_m of the matrix X. Sort the passengers in P_m in the ascending order of their arrival times. Let n_i denote the passenger number of the i_{th} passenger in P_m. Let N_m denote the number of passengers in P_m. At most N_m trips are required to serve P_m, as in each trip at least one passenger can be served. Then the passenger-to-trip assignment Y^{X_m} for P_m is defined as an $N_m \times N_m$ matrix of binary variables, where the $(i, t)_{th}$ element $Y_{it}^{X_m}$ equals to 1 if passenger n_i is assigned to trip t and 0 otherwise. Note that the representation of the symbol Y^{Xm} means that the assignment Y depends on X_m. These decision variables $Y_{it}^{X_m}$ must subject to the following constraints.

Passenger-to-Trip Assignment Constraints. Each passenger must be assigned to one and only one trip, i.e.,

$$\sum_{t=1}^{N_m} Y_{it}^{X_m} = 1 \text{, for } i = 1, 2, \ldots, N_m. \tag{2}$$

*Capacity Constraints***.** At any time, the number of passengers in the car must be less than or equal to the capacity of the car, i.e.,

$$\sum_{\{i|O_{n_i} \le f, D_{n_i} \ge f+1\}} Y_{it}^{X_m} \le C_m \text{, for } f = 0, 1, \ldots, F\text{-}2 \text{ and } t = 1, 2, \ldots, N_m, \tag{3}$$

where the left-hand side denotes the number of passengers who stay in car m when the car travels between floor f and floor f+1 in trip t, and C_m denotes the capacity of car m.

Same Direction Constraints. By definition of 'trip', the passengers assigned to the same trip must travel in the same direction, i.e.,

$$\sum_{\{i|O_{n_i} > D_{n_i}\}} Y_{it}^{X_m} \times \sum_{\{i|O_{n_i} < D_{n_i}\}} Y_{it}^{X_m} = 0 \text{, for } t = 1, 2, \ldots, N_m, \tag{4}$$

where the two terms in the left-hand side denote the number of passengers who travel down and the number of passengers who travel up, respectively. Equation (4) means that there must be one zero between the two numbers, implying that all the passengers are in the same direction.

D. Objective Function

The usual objective to measure passenger satisfaction is average passenger service time. For passenger n, the service time T_n^s is defined as the time interval between the arrival time A_n and the departure time T_n^d. Given the passengers assigned to car m, the passenger-to-trip assignment for these passengers, and the elevator physical settings (e.g., travel time between floors, door open time, passenger loading time, etc.), the service process of car m could be deduced, and then the departure times of these passengers could also be obtained, i.e.,

$$\{T_n^d\} = G(X_m, Y^{X_m}) \text{, for } n \in P_m, \tag{5}$$

where G depends on the procedure to recover the service process from a given passenger-to-car assignment. Consequently, the sum of service times of passengers in car m can be easily obtained, and is denoted by $J_m(X_m, Y^{X_m})$. Note that since car dynamics (5) are too complicated to be expressed explicitly, they are embedded in individual car simulation models. Then the average service time is given as follows:

$$J = \frac{1}{N}\sum_{n=1}^{N} T_n^s = \frac{1}{N}\sum_{m=1}^{M} J_m(X_m, Y^{X_m}), \tag{6}$$

where $J_m(X_m, Y^{X_m}) = \sum_{n \in P_m} T_n^s$.

The objective function is car-wise additive and thus separable.

Since the service process could be recovered through simulation models, a large number of passenger-wise and car-wise performance measures (e.g., average passenger waiting time, and average passenger service time, etc.) can be easily derived. Then the objective function is rather flexible with respect to such choices.

E. Overall Formulation

The overall problem is to minimize J (6) subject to passenger-to-car assignment constraints (1), passenger-to-trip assignment constraints (2), capacity constraints (3), and same direction constraints (4). The decision variables are the passenger-to-car assignment and the passenger-to-trip assignment for each car.

Our formulation provides a convenient way to represent single car dispatching strategies and formulate single car dispatching by using the concept 'trip'. The scale of our formulation (i.e., the number of constraints) is linear with respect to the number of passengers and cars, and is much smaller than that of the formulation in [12]. Moreover, the formulation above applies to different building configurations and traffic patterns like the formulation in [1], since no specific assumptions are made about them.

IV. Solution Methodology

A. Overview

In view that it is difficult to obtain the optimal performance for this problem, our aim turns into finding its upper and lower bounds. The upper bound is obtained by finding a good feasible solution to this problem. The lower bound is obtained by finding the lower bound for the optimal performance of a newly constructed problem whose optimal performance is less than or equal to that of the original problem. Our approach to obtain upper and lower bounds does not require the computation of the optimal single car dispatching sequence, and hence avoids the time and space complexities for doing that.

B. Obtaining the Upper Bound

The upper bound is obtained by finding a sufficiently good feasible solution to this problem, i.e., a good passenger-to-car assignment with a good single car dispatching strategy. The solution is generated from by a trip-based dispatching simulation tool that implements the latest real-time dispatchers. The key idea of the simulation tool is to decide which passengers should be assigned to which car and then for each car which passengers should be assigned to which trip by taking advantage of future traffic information. Since the solution is guaranteed to be feasible in the sense that all the constraints are satisfied during generating this solution by using the simulation tool, its corresponding performance is always an upper bound of the optimal performance.

C. Obtaining the Lower Bound

a) Overview

The original formulation is first rewritten in an equivalent form. Based upon this reformulation, a new problem is constructed by replacing its objective function with its lower estimate, which is obtained by using Segmentation Approach. At last Lagrangian relaxation is applied to obtain the lower bound of the newly constructed problem. Since the optimal performance of the new problem is less than or equal to that of the original problem, then the lower bound for the new problem also is the lower bound for the original problem.

b) An Equivalent Formulation

The original formulation could be rewritten in the following form, i.e.,

$$\min J = \min_{X, Y^X} \frac{1}{N} \sum_{m=1}^{M} J_m(X_m, Y^{X_m}) = \min_{X} \frac{1}{N} \sum_{m=1}^{M} \min_{Y^{X_m}} (J_m(X_m, Y^{X_m})), \tag{7}$$

subject to (1), (2), (3), and (4).

Let $J_m^*(X_m)$ or $J_m^*(P_m)$ denote the minimal total service time of passengers in P_m, i.e., the optimal performance of the following single car dispatching problem:

$$\min_{Y^{X_m}} J_m(X_m, Y^{X_m}), \tag{8}$$

subject to (2), (3), and (4).

Note that it has been proved in [9] that it is NP-hard to compute $J_m^*(X_m)$.

Then an equivalent formulation [1] can be derived from above, i.e.,

$$\min J = \min_{X} \frac{1}{N} \sum_{m=1}^{M} J_m^*(X_m), \tag{9}$$

subject to (1).

c) Constructing a New Problem via Segmentation Approach

A new problem is constructed by replacing the objective function of (9) with its lower estimate. The lower estimate is provided by Segmentation Approach. The key idea is to segment the whole group of passengers which are served by the same car into several small subgroups, each of which is to be served with a different car.

The algorithm works as follows:

Step 1: Segment the passenger group P_m into several small subgroups, each of which has n passengers. Then there are $\lceil N_m/n \rceil$ subgroups together.

Step 2: Each subgroup is to be served by a different car. All the cars have the same physical settings as car m. Let P_m^i denote the i_{th} subgroup. Then $J_m^*(P_m^i)$ denotes the minimum total service time of passengers in P_m^i. Because there are only a few passengers in P_m^i, then $J_m^*(P_m^i)$ can be calculated with trip-based Branch and Bound [15] within a very short time.

Step 3: Denote the sum of the $\lceil N_m/n \rceil$ optimal values by $J_m^S(X_m)$, i.e.,

$$J_m^S(X_m) = \sum_{i=1}^{\lceil N_m/n \rceil} J_m^*(P_m^i). \tag{10}$$

As mentioned above, $J_m^*(X_m)$ is the minimum total service time of passengers in P_m when they are all served by car m. Thereinto the sum of the service times of passengers in P_m^i, denoted by $\hat{J}_m(P_m^i)$, is always greater than or equal to $J_m^*(P_m^i)$. Therefore,

$$J_m^S(X_m) = \sum_{i=1}^{\lceil N_m/n \rceil} J_m^*(P_m^i) \le \sum_{i=1}^{\lceil N_m/n \rceil} \hat{J}_m(P_m^i) = J_m^*(X_m). \tag{11}$$

Based upon Segmentation Approach, a new problem (P) is constructed:

$$\text{(P): } \min J^S = \min_{X} \frac{1}{N} \sum_{m=1}^{M} J_m^S(X_m), \tag{12}$$

subject to (1).

Let J^* and J^{S*} denote the optimal performance of the original problem and the new problem, respectively. Let X^* and X^{S*} denote the optimal solutions of these two problems, respectively. Then,

$$J^{S*} = \frac{1}{N} \sum_{m=1}^{M} J_m^S(X_m^{S*}) \le \frac{1}{N} \sum_{m=1}^{M} J_m^S(X_m^*) \le \frac{1}{N} \sum_{m=1}^{M} J_m(X_m^*) = J^*. \tag{13}$$

Equation (13) shows that the optimal performance of the new problem is always less than or equal to that of the original one. When the segment size n equals to N_m, the two problems are identical.

d) Solving the Newly Constructed Problem

i. The Lagrangian Relaxation Framework

For the new problem (P), different cars are coupled by constraint (1), for they serve a common pool of passengers. Then problem (P) can be decomposed into individual car subproblems through relaxing constraint (1) by using Lagrangian multipliers $\{\lambda\}$:

$$L(\lambda) = \min_{X} \left[\frac{1}{N} \sum_{m=1}^{M} J_m^S(X_m) + \sum_{n=1}^{N} \lambda_n (1 - \sum_{m=1}^{M} X_{nm}) \right]. \tag{14}$$

By pulling out the terms related to car m from the relaxed function L, the subproblem for car m is formulated as follows:

$$L_m(\lambda) = \min_{X_m} \left[\frac{J_m^S(X_m)}{N} - \sum_{n=1}^{N} \lambda_n X_{nm} \right]. \tag{15}$$

The identicalness of those subproblems often causes solution oscillations when the standard Lagarangian relaxation is applied. Nevertheless, surrogate subgradient method (SSG) [16] or its extension penalty surrogate subgradient method (PSS) [17] can alleviate this kind of difficulties. According to PSS, the new Lagrangian dual is given by:

[1] This formulation is actually a bi-level programming formulation [13], [14]. The current research in bi-level programming mainly concentrates on solving linear problems. Most of the existing methods make use of the linearity of the problems, and hence cannot be extended to our case.

$$L_w(\lambda, X) = \frac{1}{N}\sum_{m=1}^{M} J_m^S(X_m) + \sum_{n=1}^{N}\lambda_n(1 - \sum_{m=1}^{M} X_{nm}) + w\sum_{n=1}^{N}(1 - \sum_{m=1}^{M} X_{nm})^2 \quad (16)$$

where w is the penalty weighting factor and a nonnegative constant. And the dual problem is as follows:

$$\max_{\lambda} q(\lambda), \text{ where } q(\lambda) = \min_{X} L_w(\lambda, X). \quad (17)$$

Since the new Lagrangian dual (16) is no longer decomposable, only one subproblem is solved to obtain a subgradient direction per iteration; its cost function is still decomposable, for the solutions to the other subproblems are fixed [17]. Then the subproblem for car m is given as follows:

$$\min_{X_m} L_{wm} = \min_{X_m}(J_m^S(X_m) - \sum_{n=1}^{N}\lambda_n X_{nm} + w\sum_{n=1}^{N}(1 - \sum_{m=1}^{M} X_{nm})^2), \quad (18)$$

ii. Solving individual car subproblems

It is difficult to solve subproblem (18) to optimality because of the huge search space of X_m. Nevertheless, according to SSG or PSS, approximate optimization of (18) suffices to obtain a surrogate subgradient direction to update the multipliers [16, 17]. That is, a better solution to (18) with lower dual cost than the solution in the preceding iteration is good enough to generate a multiplier updating direction. Genetic Algorithm is adopted here to complete the job. The assignment of passengers is encoded as a symbolic string on the set {0, 1} whose length is *N*. Element n equals to 1 if passenger n is assigned to car m and 0 otherwise.

The main steps of Genetic Algorithm are as follows:

Step 1: Initialize population with randomly generated passenger assignments to the car (i.e., 0 or 1). Note that the solution of the preceding iteration should be added into the initial population to ensure that the result of GA is at least no worse than the preceding one. Evaluate individual assignment's fitness.

Step 2: Expand population through mutation and crossover.

Step 3: Select the next population based on fitness.

Step 4: Repeat Steps 1 to 3 for a certain number of times.

The best solution in the final population is used to generate multiplier updating directions.

iii. Solving the Dual Problem

A surrogate subgradient to update multipliers is generated based upon the better solution found in last step. The surroge subgradient $\tilde{g}^k$ at iteration k is then given by:

$$\tilde{g}^k = 1 - \sum_{m=1}^{M} X_m^k. \quad (19)$$

The multipliers are updated in the surrogate subgradient direction:

$$\lambda^{k+1} = \lambda^k + s^k \tilde{g}^k, \quad (20)$$

where s^k is the step size at iteration k. The step sizing formula is as follows:

$$s^k = \alpha^k (q^* - L_w^k) / \left\| \tilde{g}^k \right\|^2, \quad (21)$$

where q^* is the optimal dual value and L_w^k is the surrogate dual value obtained at iteration k. The optimal dual value q^* is unknown, and usually an estimate of q^* (e.g., the performance value of a primal feasible solution) is used. The factor α^0 is set to 1 and α^k is reduced to one half whenever the best Lagrangian dual value found so far has failed to increase in a specified number of iterations.

According to PSS, when the dual solution converges, the final dual cost is a lower bound for the optimal performance of problem (P). Since the optimal performance of (P) is less than or equal to that of the original problem, the final dual cost also is a lower bound for the optimal performance of the original problem.

V. Numerical Results

The method was implemented by Matlab on a Pentium IV 1500M PC with 256M RAM. A building with ten floors and four cars is considered in the following examples. The passenger arrival times are assumed to contain Poisson distribution. Three traffic patterns are considered, including up-peak, down-peak and inter-floor. Up-peak traffic arises when all passengers are moving up from the lobby. Down-peak traffic happens when all the passengers are moving down to the lobby. Inter-floor traffic characterizes that passengers are moving equally likely between floors. For up-peak traffic, the arrival floors are fixed at the lobby and the destination floors contain uniform distribution. For down-peak traffic, the arrival floors are uniformly distributed and the destination floors are set at the lobby. For inter-floor traffic, the arrival floors and the destination floors are uniformly distributed. The passenger loading time and unloading time are all set to one second. The door opening time and closing time are assumed to be zero. The travel time between any two adjacent floors is set to one second. The car capacity is set to 10 in number of passengers.

Example 1

Two load levels (i.e., light, and heavy traffic) are examined for each of the three traffic patterns, respectively. The passenger arrival rates for the two load levels are 25 passengers per 100 seconds, and 50 passengers per 100 seconds, respectively. The number of passengers to be served is 100. The segment size n is 6. The results are summarized in Table I. The optimal performance for each load level and arrival rate pair is estimated by its upper and lower bounds. Note that gap is defined as the relative difference between the upper bound and the lower bound. It can be shown that our method can provide tight bounds (with gap less than 10%) for light traffic under various traffic patterns, and for heavy traffic under up-peak traffic. The bounds for heavy traffic under down-peak and inter-floor patterns are not so tight, but still acceptable. Currently, it is hard to find out the reason why the bounds are not tight, for the methods for obtaining upper and lower bounds are independent. Future research will concentrate on identifying the reason and improving the method for heavy traffic. For the same traffic arrival rate, the inter-floor traffic has the smallest average service time among the three traffic patterns. This is because for inter-floor traffic, elevators are freer to serve passengers in the sense that they can move between any pair of two floors. For the same traffic pattern, the heavy traffic case has larger average service time than light traffic. This is because for heavy traffic, elevators are busier and passengers have to wait for more time. Also for the same traffic pattern, heavy traffic takes more CPU time than light traffic, for it takes the Branch and Bound method in the method for lower bound more CPU time to compute the optimal solution in the heavy traffic case.

TABLE I
RESULTS FOR TWO LOAD LEVELS UNDER VARIOUS PATTERNS

Traffic Pattern	up-peak		down-peak		inter-floor	
Traffic Load	light	heavy	light	heavy	light	heavy
Upper Bounds (s)	6.96	8.87	6.58	8.86	4.86	7.12
Lower Bounds (s)	6.66	8.13	6.45	7.63	4.72	6.02
Gap (%)	4.52	9.09	2.09	16.15	3.08	18.15
CPU Time (h)	3.32	11.72	5.92	14.22	4.53	12.80

Example 2

Three problems of different sizes are examined for up-peak traffic. The numbers of passengers to be served are 50, 100, and 100, respectively. The passenger arrival rate is set to 25 passengers per 100 seconds. The segment size n is set to 7. The results are summarized in Table II. Numerical results show that tight bounds can be obtained for the three problems, and the required CPU time is linear with the number of passengers, thus demonstrating the scalability of our method.

TABLE II
RESULTS FOR THREE PROBLEMS OF DIFFERENT SIZES UNDER UP-PEAK

Number of Passengers	50	100	150
Upper Bounds (s)	7.44	6.91	7.05
Lower Bounds (s)	6.89	6.55	6.86
Gap (%)	7.84	5.46	2.78
CPU Time (h)	3.44	6.43	8.48

VI. CONCLUSIONS

An equivalent formulation of the one in [1] is presented. Our formulation keeps the same two-level structure, while provides a novel detailed formulation for the low level by using a concept ‘trip’ to facilitate the description of single car dispatching strategies. In view of the difficulty to obtain its exact optimal performance, our aim turns into upper and lower bounds. The upper bound is obtained by finding a good feasible solution, i.e., a good passenger-to-car assignment with a good single car dispatching strategy, which is obtained from proprietary dispatching simulators that implement modern dispatching algorithms. The lower bound is obtained by finding the lower bound for a newly constructed problem whose optimal performance is less than or equal to that of the original problem. Our approach to obtain upper and lower bounds avoids the need to compute the optimal solution to the single car dispatching problem, thus avoiding the associated time and space requirements. Numerical results demonstrate the effectiveness and scalability of our method. Future research will concentrate on improving the method for heavy traffic.

ACKNOWLEDGEMENT

The authors would like to acknowledge the guidance and insights provided by Theresa Christy, Associate Fellow - Dispatching, of Otis Elevator Company. The authors would like to thank Zhen Shen in CFINS for implementing Branch and Bound, Prof. Xiaohong Guan in CFINS for helpful discussions, and two anonymous reviewers for helpful comments on a previous version of this manuscript.

REFERENCES

[1] B. Xiong, P. B. Luh, and S. C. Chang, "Group Elevator Scheduling with Advanced Traffic Information for Normal Operations and Coordinated Emergency Evacuation," *Proceedings of the 2005 IEEE International Conference on Robotics and Automation*, Barcelona, Spain, April 2005.

[2] G. R. Strakosch, *Vertical Transportation: Elevators and Escalators*, Wiley and Sons, New York, 1983.

[3] G. Bao, C. G. Cassandras, T. E. Djaferis, A. D. Gandhi, and D. P. Looze, "Elevators Dispatchers for Down-peak Traffic", ECE Department Technical Report, University of Massachusetts, 1994.

[4] J. Lewis, “A Dynamic Load Balancing Approach to the Control of Multiserver Polling Systems with Applications to Elevator System Dispatching,” Ph.D. dissertation, ECE Department, University of Massachusetts, Amherst, 1991.

[5] S. Tsuji, M. Amano, and S. Hikita, "Application of the Expert System to Elevator Group Supervisory Control," *Proceedings of the 5th IEEE International Conference on Artificial Intelligence Applications*, 1989, pp. 287-294.

[6] H. Aoki and K. Sasaki, "Group Supervisory Control System Assisted by Artificial Intelligence," *Elevator World*, February 1990, pp. 70–80.

[7] J. Gale, "Destination Control and Tower Top Access in Belgium," *Elevator World*, Vol. 10, 2002, pp. 45-49.

[8] G. C. Barney, *Elevator Traffic Handbook*, Spon Press, London, 2003.

[9] D. J. Guan, "Routing A Vehicle of Capacity Greater Than One," *Discrete Applied Mathematics*, Vol. 81, 1998, pp. 41-57.

[10] R. Crites and A. Barto, “Improving Elevator Performance Using Reinforcement Learning,” *In Advances in Neural Information Processing Systems 8*, Cambridge, MA, MIT Press, 1996, pp. 1017-1023.

[11] D. L. Pepyne and C.G. Cassandras, “Optimal Dispatching Control for Elevator Systems During Up-peak Traffic,” IEEE Transaction on Control Systems Technology, Vol. 5, No. 6, November, 1997.

[12] S. Tsuji, M. Amano, and S. Hikita, "Application of the Expert System to Elevator Group Supervisory Control," *Proceedings of the 5th IEEE International Conference on Artificial Intelligence Applications*, 1989, pp. 287-294.

[13] J. F. Bard, *Practical Bilevel Optimization: Algorithms and Applications*. Dordrecht, The Netherlands: Kluwer, 1998.

[14] S. Dempe, *Foundations of Bilevel Programming*. Dordrecht, The Netherlands: Kluwer, 2002.

[15] Z. Shen, and Q. C. Zhao, “Branch and Bound Method of Elevator Systems with Full Information for Continuous Arrival Time”, unpublished.

[16] X. Zhao, P. B. Luh, and J. Wang, "The Surrogate Gradient Algorithm for Lagrangian Relaxation Method," *Journal of Optimization Theory and Applications*, Vol. 100, No. 3, March 1999, pp. 699-712.

[17] X. H. Guan, Q. Z. Zhai, and F. Lai, "New Lagrangian Relaxation Based Algorithm for Resource Scheduling with Homogeneous Subproblems," *Journal of Optimization Theory and Application,* Vol. 113, No. 1, 2002, pp. 65-82