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Abstract

The round trip time (RTT) is an important performance measure frequently used in elevator planning, because it is related to the handling capacity of a single elevator or an elevator group. The usual RTT formulas can only be applied to up traffic where passengers enter to the lobby and are destined to upper floors. The presented calculation method can be applied to a general traffic situation, where passengers travel from any floor to any other floor. Such traffic situations are represented by an intensity matrix, which defines the passenger flow between each pair of floors. The algorithm begins from an initial guess of RTT and the calculation is iterated until the value of RTT converges. Iteration consists of the following steps: 1) The lengths of passenger queues between two floors are calculated by using the intensity matrix and an estimate of RTT. 2) The flight probabilities of elevator between each pair of floors are calculated using the queues and by assuming that the arrival is a Poisson process. 3) The RTT is calculated using the flight probabilities. 4) The new RTT is then used as an estimate for RTT in the next iteration.

Keywords: Elevator, Lift, Round trip time, Handling capacity, Collective control, Elevator Planning, Poisson

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1. Introduction

The *round trip time* is the time for a single elevator to complete a cyclic path in a building. The planning of elevator installations is traditionally based on several performance criteria. Two central performance criteria, the *handling capacity* and *the interval* are calculated using the round trip time [2]. Usually the performance criteria are assessed in the up traffic situation, since the morning up-peak is a difficult traffic situation in office buildings. There are several methods for calculating the RTT in the up-peak situation. The simplest method is based on calculating the expected number of stops and the expected highest reversal floor [2]. The RTT is the time needed for the stops, passenger transfers and travels to the highest reversal floor and back to the lobby. There are two variations with respect to loads: one assumes a constant 80% start load at the main floor while the other assumes Poisson distributed loads. More accurate methods do not just calculate the number of stops, but the probability of flights between each pair of floors for up traffic. Roschier and Kaakinen [5] assume constant start load from entrance floor and Siikonen [6] assumes Poisson distributed loads. The RTT is then calculated using the probable flights and the flight times of elevators.

The general traffic situations are complex to calculate and also the group control algorithm affects the results unlike in the up-peak situation. Therefore the performance of an elevator group in general traffic situations is normally evaluated through simulations. Through simulation it is possible to model all the details of call allocation, elevator mechanics and passenger behaviour [3]. However, simulation is a time consuming task, since the simulation series must be long to provide accurate and reliable results. Sometimes the results can be artefacts by the passenger model or the group control algorithm. This means that simulation results can be erratic and give no insight into the performance of similar elevator groups or group control algorithms. For these reasons it is worthwhile to develop statistical models for the behaviour of elevator groups.

There exist formulas also for general traffic situations [1, 4] using the number of stops and the highest reversal floor. This article presents a RTT calculation method for general traffic situations that calculates the probability of flights between each pair of floors and assumes Poisson arrival process. It is a generalization of the up-peak methods presented in [5, 6]. The method of this article is derived for a single-elevator system that uses collective control, but it can be applied to an elevator group by simply dividing the traffic by the number of elevators. In the collective control calls are picked in sequential order in the elevator running direction. The capacity of the elevator is not considered, i.e., it is assumed that all waiting passengers always fit into the elevator.

The RTT calculation method uses the expected lengths of passenger queues to solve the flight probabilities between pairs of floors. The probabilities, flight times, the expected number of passengers and the passenger transfer times result into the RTT. On the other hand, the expected lengths of queues and the expected number of passengers are proportional to the RTT. Therefore iteration is needed to solve the RTT.

2. Definitions

Consider building that has F floors numbered from 0 to F-1. The passengers have 2F-2 entry points: from floor F-1 down, F-2 down, …, 1 down, 0 up, 1 up, …, F-2 up. The passengers have also 2F-2 destination points: down to floor F-2, …, down to 0, up to 1, …, up to floor F-1. The analysis can be simplified by aggregating the floor number and direction into position number. The position numbering is following:

$$i = F + fd\ ,\ 1 \le i \le N \tag{1}$$

where i is the position number, f is floor number [0, F-1] and $d \in \{-1,1\}$ is direction. Up is direction 1 and down is direction -1. The number of positions is N = 2F-1, since positions floor 0 up and floor 0 down use the same position number. The numbering is shown in Table 1.

Table 1: Position numbering

Floor	Direction	Position
F-1	↓	1
F-2	↓	2
…	↓	…
1	↓	F-1
0	↓	F

Floor	Direction	Position
F-1	↑	2F-1
F-2	↑	2F-2
…	↑	…
1	↑	F+1
0	↑	F

Let $\overline{q}_{ij}$ be the expected number of passenger from entry position i to destination position j. The position numbering is such that the entry position is smaller than the destination position.

$$\overline{q}_{ij} = 0, \text{ if } i=1,...,N \text{ and } j \le i \tag{2}$$
$$\overline{q}_{ij} = 0, \text{ if } i < F \text{ and } j > F$$

There is one elevator, which serves the positions, so that if there is a passenger from i to j, then the elevator stops at i, picks the passenger and stops at j to leave the passenger out. It is assumed that all passengers fit into the elevator. The elevator uses the collective control principle:

- The elevator stops at position i if there is a call, i.e. the entry or destination position of some passenger is i.
- The elevator collects the calls in the numerical order: if there are calls at positions i and j and $i < j$ then the elevator stops at i before j.
- After the last stop of the round trip the elevator changes a round trip. This means that if there are no calls after position j and i is the first call of next round trip then the elevator makes a round trip change moving from j to i, $i < j$. Note that it is possible that j is in up-direction and i is in down-direction and they are on the same floor, so that the change does not take any time.

- If there are no calls in the system, the elevator stays vacant at the position *i*, where the previous call was located. This means that the elevator does not have a parking function like real systems often have. A vacant elevator begins a new round trip every time, when a new call is registered in position *j*. Note that it is possible that $i < j$ in which case the round trip does not consist of two direction changes as it normally does.

Passengers arrive according to the Poisson process. If the expected number of arrivals is $\overline{q}$, then the probability that there are no arrivals is $e^{-\overline{q}}$. The probability of event A_{ij} that there exists a passenger having entry position *i* and destination position *j* is

$$P(A_{ij}) = 1 - e^{-\overline{q}_{ij}}, i = 1,..., N, j = 1,..., N \tag{3}$$

Let $C_{a,b}$, $a < b$ be the event that there are no calls in the open range $]a,b[$ $a=0,...,N+1$, $b=0,...,N+1$. $C_{a,b}$ takes place when no passenger enters at position *i*, $a < i < b$ and no passenger is destined to position *j*, where $a < j < b$. The probability of this event is

$$P(C_{a,b}) = e^{-\sum\limits_{\{i,j | 1 \le i \le a \wedge a < j < b\}} \overline{q}_{ij} - \sum\limits_{\{i,j | a < i < b \wedge 1 \le j \le N\}} \overline{q}_{ij}}, 0 \le a < b \le N+1 \tag{4}$$

Three important special cases of (4) are

- $P(C_{a,a+1}) = 1$, since an empty range does not contain any calls.
- $P(C_{0,N+1}) > 0$, since it is possible that the system does not contain any calls.
- $$P(C_{a-1,a+1}) = e^{-\sum\limits_{1 \le j \le N} \overline{q}_{ja} - \sum\limits_{1 \le j \le N} \overline{q}_{aj}}, 1 \le a \le N \tag{5}$$

  is the probability of not stopping to position *a*.

Let *Q* be an event that the system contains a call. The probability of Q is $P(Q) = 1 - P(C_{0,N+1})$.

It is not necessary to sum over two variables for each $P(C_{a,b})$. A double sum q^s_{ij}:

$$q^s_{ij} = \sum_{1 \le k \le i} \sum_{1 \le l \le j} \overline{q}_{kl}, i = 0, ..., N, j=0, ..., N \tag{6}$$

can be computed incrementally, such that the computation time of $(N+1)\times(N+1)$ matrix will be $O(N^2)$. The probability of $C_{a,b}$ can be expressed in terms of q^s_{ij}:

$$P(C_{a,b}) = e^{-(q^s_{b-1,N} - q^s_{a,N} + q^s_{a,b-1} - q^s_{a,a})}, 0 \le a < b \le N+1 \tag{7}$$

3. Flight Probabilities

The probability that the elevator has a flight from position *a* to *b* (runs from *a* to *b* without stopping between) under the condition that the round trip takes place is

$$P(R_{a,b}|Q) = \frac{P(C_{a,b}) - P(C_{a,b+1}) - P(C_{a-1,b}) + P(C_{a-1,b+1})}{1 - P(C_{0,N+1})}, \ 1 \le a < b \le N \qquad (8)$$

Proof. The event that there are no calls between]a,b+1[is a subset of events that there are no calls between]a,b[. Therefore the probability that there are no calls between]a,b[, but that there are calls between]a,b+1[will be obtained by subtracting the probabilities:

$$P(C_{a,b} \wedge \neg C_{a,b+1}) = P(C_{a,b}) - P(C_{a,b+1})$$

Likewise, the probability that there are no calls between]a-1,b[, but that there are calls between]a-1,b+1[is

$$P(C_{a-1,b} \wedge \neg C_{a-1,b+1}) = P(C_{a-1,b}) - P(C_{a-1,b+1})$$

$R_{a,b}$ means that the elevator runs from position a to b without stopping between. This takes place when there exists no calls between]a,b[, but a call exists between]a,b+1[and exists between]a-1,b[.

$$\begin{aligned}
&P(R_{a,b}) = P(C_{a,b} \wedge \neg C_{a,b+1} \wedge \neg C_{a-1,b}) = \\
&P(C_{a,b} \wedge \neg C_{a,b+1} \wedge (\neg C_{a-1,b} \vee C_{a-1,b+1})) = \\
&P(C_{a,b} \wedge \neg C_{a,b+1} \wedge \neg (C_{a-1,b} \wedge \neg C_{a-1,b+1})) = \\
&P(C_{a,b} \wedge \neg C_{a,b+1}) - P(C_{a-1,b} \wedge \neg C_{a-1,b+1}) = \\
&P(C_{a,b}) - P(C_{a,b+1}) - P(C_{a-1,b}) + P(C_{a-1,b+1}), \ 1 \le a < b \le N \qquad (9)
\end{aligned}$$

Probability $P(R_{a,b})$ is the probability over all situations, also such situations where the system does not have any calls. A conditional probability is used, since the round trip takes place only if there exists at least one call.

$$P(R_{a,b}|Q) = \frac{P(R_{a,b})}{P(Q)}, \ 1 \le a < b \le N.$$

This is equal to (8) □

$P(R_{a,b})$ can be expressed also in an alternative form. By rewriting (4)

$$P(C_{a-1,b}) = e^{-\sum_{1 \le i \le N} \bar{q}_{ia} - \sum_{b \le j \le N} \bar{q}_{aj}} P(C_{a,b}), \ a<b$$

$$P(C_{a,b+1}) = e^{-\sum_{1 \le i \le a} \bar{q}_{ib} - \sum_{1 \le j \le N} \bar{q}_{bj}} P(C_{a,b}), \ a<b.$$

By applying the above, formula (9) is reduced to

$$P(R_{a,b}) = P(C_{a,b})(1 - e^{-\bar{q}_{ab}} + e^{-\bar{q}_{ab}}((1 - e^{-\sum_{1 \le i \le N} \bar{q}_{ia} - \sum_{b < j \le N} \bar{q}_{aj}})(1 - e^{-\sum_{1 \le i < a} \bar{q}_{ib} - \sum_{1 \le j \le N} \bar{q}_{bj}}))),$$
$$1 \le a < b \le N \qquad (10)$$

The parts of (10) have the following interpretations:

$1 - e^{-\bar{q}_{ab}}$ is the probability that there exists a call from position a to b.

$e^{-\bar{q}_{ab}}$ is the probability of not having a call from position a to b.

$1-e^{-\sum_{1\le i\le N}\bar{q}_{ia}-\sum_{b<j\le N}\bar{q}_{aj}}$ is the probability that there exists a call destined to position a or starting from position a and destined beyond b.

$1-e^{-\sum_{1\le i<a}\bar{q}_{ib}-\sum_{1\le j\le N}\bar{q}_{bj}}$ is the probability that there exists a call starting before position a and destined to position b or a call starting from position b.

The probability in formula (10) is composed of five distinct cases show in Figure 1.

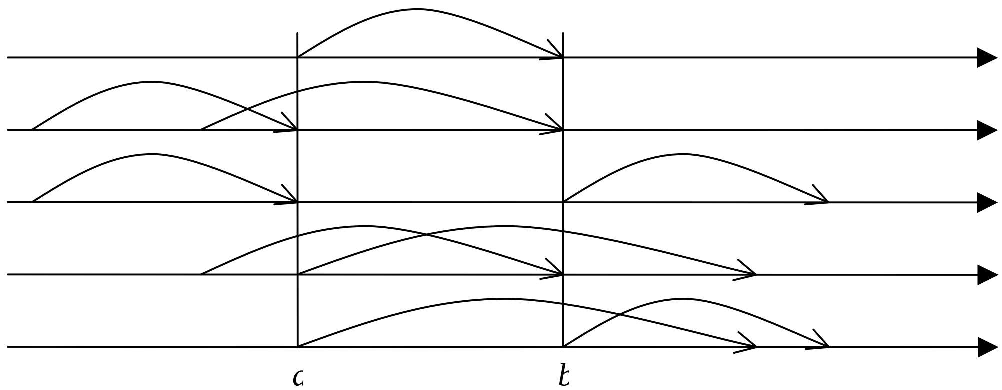

Figure 1. Interpretation of the formula (10).

The probability that an elevator makes a flight from position a to some another position equals the probability of having a call at position a and stopping at some later position. Similar formula holds for the end of flight position b:

$$\sum_{\{j|a<j\le N\}} P(R_{a,j}) = 1 - P(C_{a-1,a+1}) - P(C_{a,N+1}) + P(C_{a-1,N+1}),\ 1\le a\le N \qquad (11)$$

$$\sum_{\{i|1\le i<b\}} P(R_{i,b}) = 1 - P(C_{b-1,b+1}) - P(C_{0,b}) + P(C_{0,b+1}),\ 1\le b\le N$$

Proof.

$$\sum_{\{j|a<j\le N\}} P(R_{a,j}) = \sum_{\{j|a<j\le N\}} (P(C_{a,j}) - P(C_{a,j+1}) - P(C_{a-1,j}) + P(C_{a-1,j+1}))$$

$$= P(C_{a,a+1}) - P(C_{a,N+1}) - P(C_{a-1,a+1}) + P(C_{a-1,N+1}) =$$

$$1 - P(C_{a-1,a+1}) - P(C_{a,N+1}) + P(C_{a-1,N+1})$$

$$\sum_{\{i|1\le i<b\}} P(R_{i,b}) = \sum_{\{i|1\le i<b\}} (P(C_{i,b}) - P(C_{i,b+1}) - P(C_{i-1,b}) + P(C_{i-1,b+1})) =$$

$$P(C_{b-1,b}) - P(C_{b-1,b+1}) - P(C_{0,b}) + P(C_{0,b+1}) = 1 - P(C_{b-1,b+1}) - P(C_{0,b}) + P(C_{0,b+1})$$

4. Ending of a Round Trip

The equation for flight probabilities (8) considers only flights in the direction of the position indexing, where $a < b$. In addition it is necessary to consider the ending of the round trip. Let R_b^{begin} be an event that the first stop of the round trip is at position b and R_a^{end} be an event that the last stop of round trip is at position a.

$$\begin{aligned} R_b^{begin} &= P(C_{0,b}) - P(C_{0,b+1}) \;\; 1 \le b \le N \\ R_a^{end} &= P(C_{a,N+1}) - P(C_{a-1,N+1}) \;\; 1 \le a \le N \end{aligned} \tag{12}$$

The probabilities R_b^{begin} and R_a^{end} have been calculated also over such cases which there are no calls in the system. Therefore the probability that the first stop is somewhere and also the probability that the last stop is somewhere equal the probability of the round trip.

$$\sum_{i=1}^{N} P(R_i^{begin}) = \sum_{i=1}^{N} (P(C_{0,i}) - P(C_{0,i+1})) = P(C_{0,1}) - P(C_{0,N+1}) = 1 - P(C_{0,N+1}) = P(Q)$$
$$\sum_{i=1}^{N} P(R_i^{end}) = \sum_{i=1}^{N} (P(C_{i,N+1}) - P(C_{i-1,N+1})) = P(C_{N,N+1}) - P(C_{0,N+1}) = 1 - P(C_{0,N+1}) = P(Q)$$

Let Q^2 be the condition that the previous and next round trips take place. The probability that the elevator moves from position a to position b during the change of round trip under the condition Q^2 is

$$P(R_{a,b}^{r} | Q^2) = P(R_a^{end} | Q) P(R_b^{begin} | Q) \text{, } 1 \le a \le N,\ 1 \le b \le N$$
$$P(R_{a,b}^{r} | Q^2) = \frac{(P(C_{a,N+1}) - P(C_{a-1,N+1}))(P(C_{0,b}) - P(C_{0,b+1}))}{(1 - P(C_{0,N+1}))^2} \text{, } 1 \le a \le N,\ 1 \le b \le N \tag{13}$$

Usually $a > b$, but it is also possible that the change is in the direction of position indexing. This happens when the elevator stands idle at position a and begins a new round trip when a call arrives to position b, $a \le b$.

The change of round trip begins and ends somewhere at probability 1:

$$\sum_{i=1}^{N}\sum_{j=1}^{N} P(R_{i,j}^{r} | Q^2) = \sum_{i=1}^{N}\sum_{j=1}^{N} P(R_i^{end} | Q) P(R_j^{begin} | Q) = \sum_{i=1}^{N} P(R_i^{end} | Q) \sum_{j=1}^{N} P(R_j^{begin} | Q) = \frac{P(Q)^2}{P(Q)^2} = 1$$

The probability that the elevator either continues the round trip from position a or changes the round trip at position a equals the probability of stopping at position a. Likewise, the probability that the elevator either arrives to position b during the round trip or makes a round trip change to position b equals the probability of stopping at position b.

$$\sum_{\{j|a<j\le N\}} P(R_{a,j}|Q) + \sum_{\{j|1\le j\le N\}} P(R^r_{a,j}|Q^2) = \frac{1-P(C_{a-1,a+1})}{1-P(C_{0,N+1})} \tag{14}$$

$$\sum_{\{i|1\le i<b\}} P(R_{i,b}|Q) + \sum_{\{i|1\le i\le N\}} P(R^r_{i,b}|Q^2) = \frac{1-P(C_{b-1,b+1})}{1-P(C_{0,N+1})}$$

Proof.

$$\sum_{\{j|1\le j\le N\}} P(R^r_{a,j}|Q^2) =$$

$$\sum_{\{j|1\le j\le N\}} \frac{(P(C_{a,N+1})-P(C_{a-1,N+1}))(P(C_{0,j})-P(C_{0,j+1}))}{(1-P(C_{0,N+1}))^2} =$$

$$\frac{(P(C_{a,N+1})-P(C_{a-1,N+1}))(P(C_{0,1})-P(C_{0,N+1}))}{(1-P(C_{0,N+1}))^2} =$$

$$\frac{(P(C_{a,N+1})-P(C_{a-1,N+1}))(1-P(C_{0,N+1}))}{(1-P(C_{0,N+1}))^2} = \frac{P(C_{a,N+1})-P(C_{a-1,N+1})}{1-P(C_{0,N+1})}$$

Using formula (11):

$$\sum_{\{j|a<j\le N\}} P(R_{a,j}|Q) + \sum_{\{j|1\le j\le N\}} P(R^r_{a,j}|Q^2) =$$

$$\frac{1-P(C_{a-1,a+1})-P(C_{a,N+1})+P(C_{a-1,N+1})}{1-P(C_{0,N+1})} + \frac{(P(C_{a,N+1})-P(C_{a-1,N+1}))}{1-P(C_{0,N+1})} = \frac{1-P(C_{a-1,a+1})}{1-P(C_{0,N+1})}$$

$$\sum_{\{i|1\le i\le N\}} P(R^r_{i,b}|Q^2) =$$

$$\sum_{\{i|1\le i\le N\}} \frac{(P(C_{i,N+1})-P(C_{i-1,N+1}))(P(C_{0,b})-P(C_{0,b+1}))}{(1-P(C_{0,N+1}))^2} =$$

$$\frac{(P(C_{N,N+1})-P(C_{0,N+1}))(P(C_{0,b})-P(C_{0,b+1}))}{(1-P(C_{0,N+1}))^2} = \frac{(1-P(C_{0,N+1}))(P(C_{0,b})-P(C_{0,b+1}))}{(1-P(C_{0,N+1}))^2}$$

$$= \frac{P(C_{0,b})-P(C_{0,b+1})}{1-P(C_{0,N+1})}$$

Using formula (11):

$$\sum_{\{i|1\le i<b\}} P(R_{i,b}|Q) + \sum_{\{i|1\le i\le N\}} P(R^r_{i,b}|Q^2) =$$

$$\frac{1-P(C_{b-1,b+1})-P(C_{0,b})+P(C_{0,b+1})}{1-P(C_{0,N+1})} + \frac{P(C_{0,b})-P(C_{0,b+1})}{1-P(C_{0,N+1})} = \frac{1-P(C_{b-1,b+1})}{1-P(C_{0,N+1})}$$

5. Calculation of Round Trip Time

RTT consists of the flights during the round trip, the last flight associated with the end of the round trip, and the passenger transfer times. The number of passengers during the round trip is calculated by summing the expected lengths of queues and dividing by the probability of round trip.

$$E(N^{pass}) = \frac{\sum_{\{i,j | 1 \le i \le N \wedge 1 \le j \le N\}} \bar{q}_{ij}}{1 - \exp(-\sum_{\{i,j | 1 \le i \le N \wedge 1 \le j \le N\}} \bar{q}_{ij})} \quad (15)$$

The expected round trip time is

$$E(t^{RT}) = \sum_{\{i,j | 1 \le i < j \le N\}} P(R_{i,j} | Q) t_{ij} + \sum_{\{i,j | 1 \le i \le N \wedge 1 \le j \le N\}} P(R_{i,j}^{r} | Q^2) t_{ij} + (t^{in} + t^{out}) E(N^{pass}), \quad (16)$$

where t_{ij} is the flight time from position *i* to position *j*, t^{in} and t^{out} are the passenger transfer times into and out of elevator.

The RTT formula requires knowledge of queue lengths, which depends on intensities and the RTT.

$$\bar{q}_{ij} = \lambda_{ij} t^{RT}, 1 \le i \le N, 1 \le j \le N, \quad (17)$$

where λ_{ij} are arrival intensities of passengers.

Equation $t = E(t^{RT}(t))$ can be solved by iterating

$$t_{i+1} = E(t^{RT}(t_i)), \text{ i} = 1, 2, \ldots \quad (18)$$

until the iteration converges to the RTT. The iteration diverges, if the traffic intensity is too big to handle.

6. Examples

6.1. Calculation of RTT

Compute the RTT of six-floor building. The passenger transfer times are $t^{in} = t^{out} = 1$ s. Table 2 shows the flight time between floors.

Table 2. Flight times [s] between floors.

Floors From \ To	0	1	2	3	4	5
0	0	10	13	16	19	22
1	10	0	10	13	16	19
2	13	10	0	10	13	16
3	16	13	10	0	10	13
4	19	16	13	10	0	10
5	22	19	16	13	10	0

There are 11 positions. The flight times between positions are given in Table 3.

Table 3. Flight times [s] between positions. Matrix t_{ij}.

Floors	To	5	4	3	2	1	0	1	2	3	4	5
From	t_{ij}	1	2	3	4	5	6	7	8	9	10	11
5	1	0	10	13	16	19	22	19	16	13	10	0
4	2	10	0	10	13	16	19	16	13	10	0	10
3	3	13	10	0	10	13	16	13	10	0	10	13
2	4	16	13	10	0	10	13	10	0	10	13	16
1	5	19	16	13	10	0	10	0	10	13	16	19
0	6	22	19	16	13	10	0	10	13	16	19	22
1	7	19	16	13	10	0	10	0	10	13	16	19
2	8	16	13	10	0	10	13	10	0	10	13	16
3	9	13	10	0	10	13	16	13	10	0	10	13
4	10	10	0	10	13	16	19	16	13	10	0	10
5	11	0	10	13	16	19	22	19	16	13	10	0

The lowest floor is the entrance floor and the population is distributed equally between the floors. 50% of the passengers are incoming, 30% are outgoing and 20% of passengers travel between two populated floors. The traffic intensities between floors are given in Table 4.

Table 4. Traffic intensities [persons / 300 s] between floors.

Floors From	To	0	1	2	3	4	5
0		0	1.188	1.188	1.188	1.188	1.188
1		0.748	0	0.088	0.088	0.088	0.088
2		0.748	0.088	0	0.088	0.088	0.088
3		0.748	0.088	0.088	0	0.088	0.088
4		0.748	0.088	0.088	0.088	0	0.088
5		0.748	0.088	0.088	0.088	0.088	0

The traffic intensities between positions are given in Table 5.

Table 5. Traffic intensities [persons / 300 s] between positions. Matrix λ_{ij}.

Floors	To	5	4	3	2	1	0	1	2	3	4	5
From	λ_{ij}	1	2	3	4	5	6	7	8	9	10	11
5	1	0	0.088	0.088	0.088	0.088	0.748	0	0	0	0	0
4	2	0	0	0.088	0.088	0.088	0.748	0	0	0	0	0
3	3	0	0	0	0.088	0.088	0.748	0	0	0	0	0
2	4	0	0	0	0	0.088	0.748	0	0	0	0	0
1	5	0	0	0	0	0	0.748	0	0	0	0	0
0	6	0	0	0	0	0	0	1.188	1.188	1.188	1.188	1.188
1	7	0	0	0	0	0	0	0	0.088	0.088	0.088	0.088
2	8	0	0	0	0	0	0	0	0	0.088	0.088	0.088
3	9	0	0	0	0	0	0	0	0	0	0.088	0.088
4	10	0	0	0	0	0	0	0	0	0	0	0.088
5	11	0	0	0	0	0	0	0	0	0	0	0

The expected lengths of queues are computed from (17). The RTT must be guessed at this point. The guess is 40 s. The resulting lengths are given in Table 6.

Table 6. Queues between positions. Matrix $\bar{q}_{ij}$.

$\bar{q}_{ij}$	1	2	3	4	5	6	7	8	9	10	11
1	0	0.012	0.012	0.012	0.012	0.1	0	0	0	0	0
2	0	0	0.012	0.012	0.012	0.1	0	0	0	0	0
3	0	0	0	0.012	0.012	0.1	0	0	0	0	0
4	0	0	0	0	0.012	0.1	0	0	0	0	0
5	0	0	0	0	0	0.1	0	0	0	0	0
6	0	0	0	0	0	0	0.158	0.158	0.158	0.158	0.158
7	0	0	0	0	0	0	0	0.012	0.012	0.012	0.012
8	0	0	0	0	0	0	0	0	0.012	0.012	0.012
9	0	0	0	0	0	0	0	0	0	0.012	0.012
10	0	0	0	0	0	0	0	0	0	0	0.012
11	0	0	0	0	0	0	0	0	0	0	0

The queue length sums, q_{ij}^{s} computed from (6) are given in Table 7.

Table 7. The queue length sums. Matrix q^{s}_{ij}.

q^{s}_{ij}	0	1	2	3	4	5	6	7	8	9	10	11
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0.012	0.023	0.035	0.047	0.147	0.147	0.147	0.147	0.147	0.147
2	0	0	0.012	0.035	0.059	0.082	0.282	0.282	0.282	0.282	0.282	0.282
3	0	0	0.012	0.035	0.07	0.106	0.405	0.405	0.405	0.405	0.405	0.405
4	0	0	0.012	0.035	0.07	0.117	0.516	0.516	0.516	0.516	0.516	0.516
5	0	0	0.012	0.035	0.07	0.117	0.616	0.616	0.616	0.616	0.616	0.616
6	0	0	0.012	0.035	0.07	0.117	0.616	0.774	0.933	1.091	1.25	1.408
7	0	0	0.012	0.035	0.07	0.117	0.616	0.774	0.945	1.115	1.285	1.455
8	0	0	0.012	0.035	0.07	0.117	0.616	0.774	0.945	1.126	1.308	1.49
9	0	0	0.012	0.035	0.07	0.117	0.616	0.774	0.945	1.126	1.32	1.514
10	0	0	0.012	0.035	0.07	0.117	0.616	0.774	0.945	1.126	1.32	1.525
11	0	0	0.012	0.035	0.07	0.117	0.616	0.774	0.945	1.126	1.32	1.525

The non-stopping probabilities computed from (7) are in Table 8.

Table 8. Probability of not stopping between]*i*,*j*[. Matrix *P*(C_{ij}).

$P(C_{ij})$	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	1	0.864	0.755	0.667	0.597	0.54	0.245	0.233	0.225	0.22	0.218	0.218
1	0	0	1	0.864	0.755	0.667	0.597	0.245	0.233	0.225	0.22	0.218	0.218
2	0	0	0	1	0.864	0.755	0.667	0.248	0.236	0.228	0.223	0.22	0.22
3	0	0	0	0	1	0.864	0.755	0.253	0.242	0.233	0.228	0.225	0.225
4	0	0	0	0	0	1	0.864	0.262	0.25	0.242	0.236	0.233	0.233
5	0	0	0	0	0	0	1	0.275	0.262	0.253	0.248	0.245	0.245
6	0	0	0	0	0	0	0	1	0.814	0.671	0.559	0.472	0.403
7	0	0	0	0	0	0	0	0	1	0.814	0.671	0.559	0.472
8	0	0	0	0	0	0	0	0	0	1	0.814	0.671	0.559
9	0	0	0	0	0	0	0	0	0	0	1	0.814	0.671
10	0	0	0	0	0	0	0	0	0	0	0	1	0.814
11	0	0	0	0	0	0	0	0	0	0	0	0	1
12	0	0	0	0	0	0	0	0	0	0	0	0	0

The flight probabilities from (8) are in Table 9:

Table 9. The flight probabilities. Matrix *P*(R_{ij}|*Q*).

$P(R_{ij}\|Q)$	1	2	3	4	5	6	7	8	9	10	11
1	0	0.035	0.028	0.022	0.018	0.072	0	0	0	0	0
2	0	0	0.035	0.028	0.022	0.086	2E-04	1E-04	8E-05	4E-05	0
3	0	0	0	0.035	0.028	0.104	3E-04	2E-04	2E-04	8E-05	0
4	0	0	0	0	0.035	0.128	5E-04	4E-04	2E-04	1E-04	0
5	0	0	0	0	0	0.158	7E-04	5E-04	3E-04	2E-04	0
6	0	0	0	0	0	0	0.221	0.172	0.135	0.108	0.088
7	0	0	0	0	0	0	0	0.054	0.041	0.031	0.024
8	0	0	0	0	0	0	0	0	0.054	0.041	0.031
9	0	0	0	0	0	0	0	0	0	0.054	0.041
10	0	0	0	0	0	0	0	0	0	0	0.054
11	0	0	0	0	0	0	0	0	0	0	0

The round trip change probabilities from (13) are in Table 10:

Table 10. The round trip change probabilities. Matrix *P*(R^r_{ij}|Q^2).

$P(R^r_{ij}\|Q^2)$	1	2	3	4	5	6	7	8	9	10	11
1	0	0	0	0	0	0	0	0	0	0	0
2	6E-04	5E-04	4E-04	3E-04	2E-04	0.001	5E-05	3E-05	2E-05	1E-05	0
3	0.001	9E-04	7E-04	6E-04	5E-04	0.003	1E-04	7E-05	4E-05	2E-05	0
4	0.002	0.001	0.001	9E-04	7E-04	0.004	1E-04	1E-04	7E-05	3E-05	0
5	0.002	0.002	0.002	0.001	0.001	0.005	2E-04	1E-04	1E-04	5E-05	0
6	0.035	0.028	0.023	0.018	0.015	0.076	0.003	0.002	0.001	7E-04	0
7	0.015	0.012	0.01	0.008	0.006	0.033	0.001	9E-04	6E-04	3E-04	0
8	0.02	0.016	0.013	0.01	0.008	0.042	0.002	0.001	7E-04	4E-04	0
9	0.025	0.02	0.016	0.013	0.01	0.054	0.002	0.001	1E-03	5E-04	0
10	0.032	0.026	0.02	0.016	0.013	0.069	0.003	0.002	0.001	6E-04	0
11	0.041	0.033	0.027	0.021	0.017	0.09	0.003	0.002	0.002	8E-04	0

From formula (16): $E(t^{RT}) = 43.7$ s, where the flights of round trip take 27.5 s, the round trip change takes 12.4 s and the passenger transfers take 3.9 s.

The RTT of the example was not equal to the previously guessed 40 s. Formula (18) produces a sequence, which converges to 45.9 s according to Table 11.

Table 11. The iteration of RTT.

Iteration	RTT [s]
1	10.000
2	32.621
3	41.015
4	44.124
5	45.265
6	45.683
7	45.836
8	45.892
9	45.912
10	45.920
11	45.922
12	45.923
13	45.924
14	45.924

6.2. Comparison of Up-peak Calculation Methods

The round trip times are calculated in up-peak situation using seven different methods

(A) the method of this article

the frequently used methods [2] with both

(B) constant start load and

(C) Poisson distributed start loads variations,

(D) the method of article [5],

(E) the method of article [6],

(F) simulation of Poisson arrival process and one elevator having infinite capacity.

(G) simulation of Poisson arrival process and one elevator having capacity 17 (rated and bypass load).

RTT is computed with different numbers of floors, elevator speeds and traffic intensities. The height of a floor is 3.5 m and the acceleration of the elevator is 1.0 m/s^2. The door opening time is 1.5 s and closing time is 3.5 s. The simulation period was 10^6 round trips.

Methods B and C:

$$RTT = 2HT_v + (S+1)t_s + 2Pt_p, \qquad (19)$$

where H is the highest reversal floor, t_v is the one-floor transit time at nominal speed, S is the number of stops above the main floor, t_s is the delay associated with a stop and t_p is the passenger transfer time (1.0 s in this example).

t_s = door opening time + door closing time + one-floor flight time - t_v.

Method B:

$$S = N\left(1 - \left(\frac{N-1}{N}\right)^P\right) \tag{20}$$

$$H = N - \sum_{i=1}^{N-1}\left(\frac{i}{N}\right)^P ,$$

where N is the number of floors above the main floor. P is the average start load from the main floor. Usually P is assumed to be 80% of the rated car load, but here only the traffic intensity λ is known and $P = \lambda t$. The round trip time t is solved by iterating.

Method C:

$$S = N\left(1 - e^{-\frac{\lambda}{N}t}\right) \tag{21}$$

$$H = N - \sum_{i=1}^{N} e^{-i\frac{\lambda}{N}t} ,$$

where λ is the traffic intensity, t is the round trip time, which is solved by iterating.

Method C is unrealistic in low traffic intensities, since it does not take into account that the round trip does not occur if there are no passengers. The number of stops and the reversal floor approach zero when the intensity goes towards zero. However this is not a serious problem, if the lowest expected load is at least 2. Method E has the same problem.

Table 12. Up-peak results.

No. of Floors	Speed	Intensity	P (*)	A	B	C	D	E	F	G
	[m/s]	[persons / 300s]		[s]	[s]	[s]	[s]	[s]	[s]	[s]
6	1.0	8.6	1	43.1	35.0	20.6	35.0	0.0	35.0	35.0
6	1.0	12.7	2	48.7	47.4	37.2	47.4	33.6	43.1	43.1
6	1.0	29.8	8	78.4	80.7	78.3	80.7	78.3	77.1	77.2
6	1.0	47.1	16	101.2	102.0	101.2	101.9	101.2	100.8	101.6
12	2.0	11.6	2	54.3	51.7	41.1	51.7	36.7	47.0	47.0
12	2.0	24.1	8	96.4	99.8	96.2	99.8	96.3	94.1	94.2
12	2.0	35.1	16	134.3	136.7	134.2	136.8	134.3	132.9	135.7
18	3.0	11.1	2	58.9	53.9	43.1	56.2	41.1	50.9	50.9
18	3.0	21.9	8	110.0	109.4	105.5	113.8	109.9	107.2	107.3
18	3.0	30.4	16	158.6	157.9	154.7	161.9	158.6	156.4	161.0
24	4.0	10.9	2	63.8	55.0	44.3	61.3	46.9	55.5	55.5
24	4.0	20.9	8	123.9	114.6	110.6	127.8	123.8	120.7	121.0
24	4.0	28.1	16	180.3	170.6	167.0	183.9	180.3	177.5	182.5
Average				96.3	93.4	87.2	96.8	87.8	92.2	93.2

(*) load as persons according to method B.

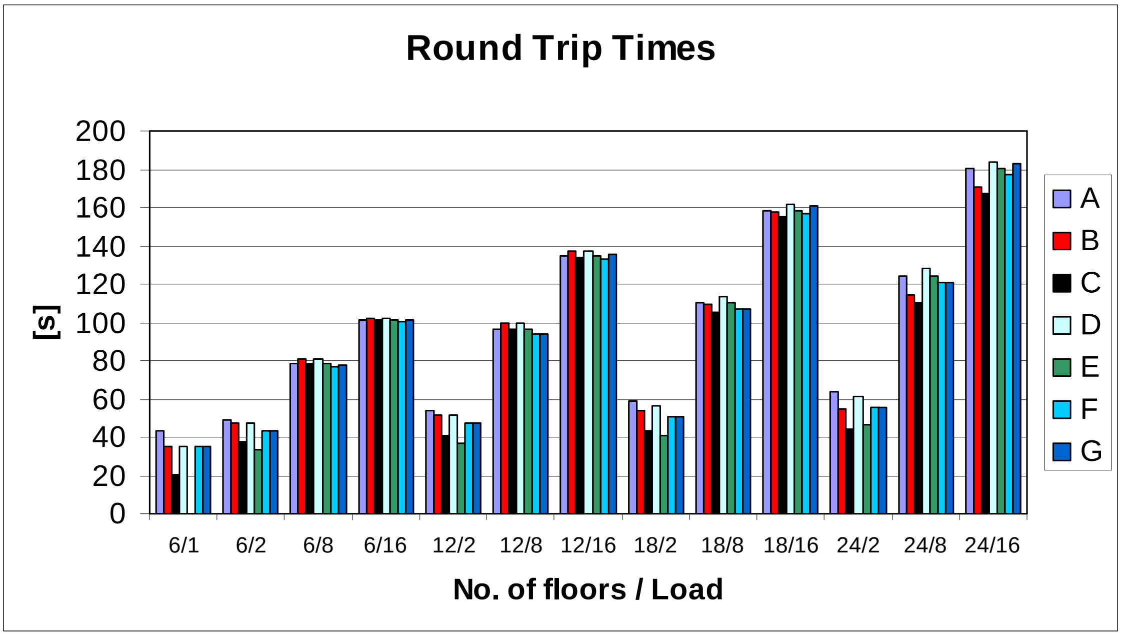

Figure 2. Round trip times obtained with seven different methods in up-peak traffic.

The results in Table 12 and Figure 2 show that all the methods give about the same results, when the number of floors is small and the intensity is high. The method of this paper (A) seems to overestimate the RTT compared to the simulated results (F). The average difference between A and F is 4.5% and the difference is biggest when the start load is small. The accuracy of method A is comparable to the existing up-peak calculation methods (B, C, D, E).

6.3. Comparison of Lunch-hour Traffic

The calculation method is compared against simulation as in chapter 6.2 using the same parameters and traffic intensities except that the traffic type is not pure up-traffic, but consists of 40 % incoming, 40 % outgoing and 20% inter-floor traffic. This is called lunch-hour traffic, since it happens during lunch-hours in some office buildings.

The results in Table 13 and Figure 3 show that the calculation method (A) slightly overestimates the RTT compared to simulated values (F and G). The average difference between A and F is 4.2%.

Table 13. Lunch-hour results.

No. of Floors	Speed	Intensity	A	F	G
	[m/s]	[persons / 300s]	[s]	[s]	[s]
6	1.0	8.6	41.4	40.8	40.8
6	1.0	12.7	50.8	47.9	47.9

6	1.0	29.8	99.1	95.6	95.7
6	1.0	47.1	135.3	134.0	134.0
12	2.0	11.6	57.6	52.8	52.8
12	2.0	24.1	129.7	123.3	123.3
12	2.0	35.1	199.7	195.1	195.1
18	3.0	11.1	63.5	57.7	57.7
18	3.0	21.9	154.3	146.2	146.3
18	3.0	30.4	250.6	243.2	243.5
24	4.0	10.9	70.1	63.6	63.6
24	4.0	20.9	180.9	171.4	171.6
24	4.0	28.1	298.1	289.1	288.9
Average			133.2	127.7	127.8

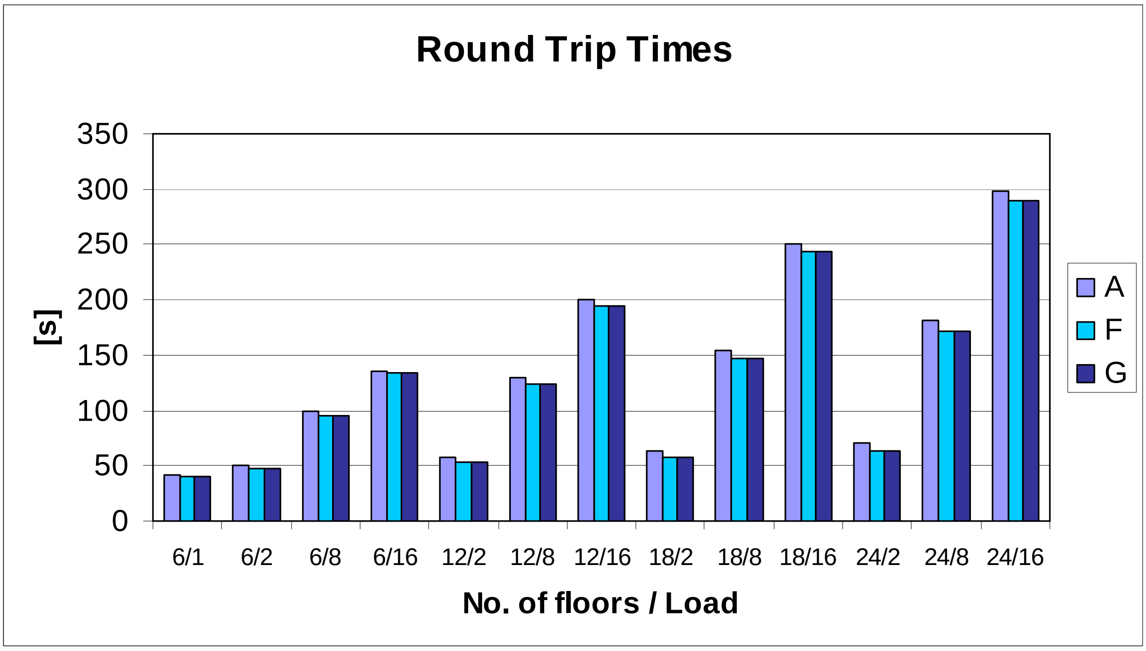

Figure 3. Round trip time comparison for mixed lunch-hour traffic.

7. Conclusions

Round trip time (RTT) is an important performance measure in elevator planning. The method introduced in this paper calculates RTT for a generalization of the up-traffic methods [5] or [6]. The passenger arrivals follow a Poisson process. Infinite car capacity is assumed that is not true in reality, but that does not cause big differences in results. The formulas are derived for a single elevator system that applies the collective control principle. The method can be extended to a group of elevators by dividing the traffic with the number of elevators in group. The calculation is an iterative algorithm, which converges to the RTT. An entry-destination intensity matrix for passengers and some initial guess for the RTT are needed. The algorithm calculates a flight probability matrix, which leads to a new estimate of RTT. Compared to simulation results, this method gives slightly longer round trip times. The average difference in tested cases were 4.4%. The accuracy of the method is comparable to the existing up-peak calculation methods.

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