Thesis Title

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Kapittel 1

Continuum Mechanics

When studying the dynamics of a mediums with fluid or structure properties under the influence of forces, we need in some sense a good description of how these forces act and alter the system itself.
Any medium on a microscopic scale is built up of a structure of atoms, meaning we can observe empty spaces between each atom or discontinuities in the medium. Discribing any phsycial phenomen on larger scales in such a way are tedious and most often out of bounds due to the high number of particles. Instead we consider the medium to be continously distributed throughout the entire reagion it occupies. Hence we want to study some phsyical properties of the complete volume and not down on atomic scale.
We consider the medium with continuum properties. By a continuum we mean a volume $V(t) \subset \mathbb{R}^3$ consiting of particles, which we observe for some properties. One property of interest could be the veloctity $\mathbf{v}(x,t)$ for some point $x \in V(t)$ in time $t \in (0,T]$, which would mean the average velocity of the particles occupying this point x at time t The intension of this chapter is not to give a thorough introduction of continuum mechanics, but rather present key concepts needed for the evaluation of fluid-strucutre interaction.

1.1 Coordinate system

We assume that our medium is continiously distributed throughout its own volume, and we start our observation of this medium at som time t_0. As this choice is arbitary, we often choose to observe a medium in a stress free initial state. We call this state $V(t_0)$ of the medium as the *reference configuration*. We let $V(t)$ for $t \geq t_0$ denote the *current configuration*.

Central for the coordinate systems introduced in this chapter is the concept of *material* and *spatial* points. *Material* points are simply the points defining the material, moving with it as it undergoes movement. *Spatial* points on the other hand is the relative measure of movement of the *material* points. (Godt nok ??). This concept will be further explained throughout the chapter.

1.1.1 Lagrangian

As some medium is act upon by forces, one of the main properties of interest is the deformation of the medium. Hence we want to know the relative position of some particle from its initial configuration.
Let $\hat{\mathrm{x}}$ be a particle in the reference $\hat{\mathrm{x}} \in \hat{\mathrm{V}}$. Further let $\mathrm{x}(\hat{\mathrm{x}}, \mathrm{t})$ be the new location of a particle $\hat{\mathrm{x}}$ for some time t such that $x \in V(t)$. We assume that no two particles $\hat{\mathrm{x}}_a, \hat{\mathrm{x}}_b \in \hat{\mathrm{V}}$ occupy the same location for some time $V(t)$. Then the transformation $\hat{\mathrm{T}}(\hat{\mathrm{x}}, t) = x(\hat{\mathrm{x}}, t)$ maps a particle $\hat{\mathrm{x}}$ from the *reference configuration* $\hat{\mathrm{V}}$ to the *current configuration* $V(t)$ Assuming that the path for some $\hat{\mathrm{x}}$ is continuous in time, we can define the inverse mapping $\hat{\mathrm{T}}^{-1}(x,t) = \hat{\mathrm{x}}(x,t)$, which maps $x(\hat{\mathrm{x}}, t)$ back to its initial location at time $t = t_0$.
These mappings lets us track each particle from some *reference configuration* to some deformed

state at time t. Such a description of tracking each particle $\hat{\mathrm{x}} \in \hat{\mathrm{V}}$ is often denoted the *Lagrangian Framework* and is a natural choice of describing structure mechanics.
We define the *deformation*

$$\hat{\mathrm{T}}(\hat{\mathrm{x}}, t) = \hat{\mathbf{u}}(\hat{\mathrm{x}}, t) = x(\hat{\mathrm{x}}, t) - \hat{\mathrm{x}} \tag{1.1}$$

and the *deformation velocity*

$$\frac{\partial \hat{\mathrm{T}}(\hat{\mathrm{x}}, t)}{\partial t} = \hat{\mathbf{v}}(\hat{\mathrm{x}}, t) = d_t x(\hat{\mathrm{x}}, t) = d_t \hat{\mathbf{u}}(\hat{\mathrm{x}}, t) \tag{1.2}$$

When tracking each particle as it moves, the *material* and *spatial* points coincide

1.1.2 Eulerian

Considering a flow of fluid particles in a river, a *Lagrangian* description of the particles would be tidious as the number of particles entring and leaving the domain quickly rise to a immense number. Instead consider defining a view-point V fixed in time, and monitor every fluid particle passing the coordinate $x \in V(t)$ as time elapses. Such a description is defined as the *Eulerian framework*. Therefore the Eulerian formulation is natural for describing fluid dynamics.
We can describe the particles occupying the *current configuration* $V(t)$ for some time $t \geq t_0$

$$x = \hat{\mathrm{x}} + \hat{\mathbf{u}}(\hat{\mathrm{x}}, t)$$

Since our domain is fixed we can define the deformation for a particle occupying position $x = x(\hat{\mathrm{x}}, t)$ as

$$\mathbf{u}(x, t) = \hat{\mathbf{u}}(\hat{\mathrm{x}}, t) = x - \hat{\mathrm{x}}$$

and its velocity

$$\mathbf{v}(x, t) = \partial_t u(x, t) = \partial_t \hat{\mathbf{u}}(\hat{\mathrm{x}}, t) = \hat{\mathbf{v}}(\hat{\mathrm{x}}, t)$$

It is important to mention that the we are not interested in which particle is occupying a certain point in our domain, but only its properties. As such the *material* and *spatial* points doesn't coincide in the *Eulerian formulation*

1.2 Deformation gradients

When studying continuum mechanics we observe continious mediums as they are deformed over time. These deformations results in relative changes of positions due to external and internal forces acting.. These relative changes of postition is called *strain*, and is the primary property that causes *stress* within a medium of interest [6]. We define stress as the internal forces that particles within a continuous material exert on each other.

The equations of mechanics can be derived with respect to either a deformed or undeformend configuration of our medium of interest. The choice of refering our equations to the current or reference configuration is indifferent from a theoretical point of view. In practice however this choice can have a severe impact on our strategy of solution methods and physical of modelling. [8]. We will therefore define the strain measures for both configurations of our medium.

Definition 1.1. Deformation gradient.

$$\hat{\mathbf{F}} = I + \hat{\nabla}\hat{\mathbf{u}} \tag{1.3}$$

Mind that deformation gradient of $\hat{\mathbf{u}}$ is which respect to the reference configuration. From the assumption that no two particles $\hat{\mathrm{x}}_a, \hat{\mathrm{x}}_b \in \hat{\mathrm{V}}$ occupy the same location for some time $V(t)$, the presented transformation must be linear. As a consequence from the invertible matrix theorem found in linear algebra, the linear operator $\mathbf{F}$ cannot be a singuar. We define the *determinant of the deformation gradient* as J, which denotes the local change of volume of our domain.

Definition 1.2. Determinant of the deformation gradient

$$J = det(\hat{\mathbf{F}}) = det(I + \hat{\nabla}\hat{\mathbf{u}}) \neq 0 \tag{1.4}$$

By the assumption that the medium can't be selfpenetrated, we must limit J to be greater than 0 [8]

1.3 Measures of Strain and Stress

The equations describing forces on our domain can be derived in accordinance with the current or reference configuration. With this in mind, different measures of strain can be derived with respect to which configuration we are interested in. We will here by [6] show the most common measures of strain. We will first introduce the right *Cauchy-Green* tensor **C**, which is one of the most used strain measures [8].
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Let $\hat{x}, \hat{y} \in \hat{V}$ be two points in our referemce configuration and let $\hat{a} = \hat{y} - \hat{x}$ denote the length of the line bewtween these two points. As our domain undergoes deformation let $x = \hat{x} + \hat{\mathbf{u}}(\hat{x})$ and $x = \hat{y} + \hat{\mathbf{u}}(\hat{y})$ be the position of our points in the current configuration, and let $a = y - x$ be our new line segment. By [6] we have by first order Taylor expansion

$$\begin{aligned} y - x = \hat{y} + \hat{\mathbf{u}}(\hat{y}) - \hat{x} - \hat{\mathbf{u}}(\hat{x}) = \ \hat{y} - \hat{x} + \hat{\nabla}\hat{u}(\hat{x})(\hat{y} - \hat{x}) + \mathcal{O}(|\hat{y} - \hat{x}|^2) \\ \frac{y - x}{|\hat{y} - \hat{x}|} = [I + \hat{\nabla}\hat{\mathbf{u}}(\hat{x}]\frac{\hat{y} - \hat{x}}{|\hat{y} - \hat{x}|} + \mathcal{O}(|\hat{y} - \hat{x}|) \end{aligned}$$

This detour from [6] we have that

$$\begin{aligned} a = y - x = \hat{\mathbf{F}}(\hat{x})\hat{a} + \mathcal{O}(|\hat{a}|^2) \\ |a| = \sqrt{(\hat{\mathbf{F}}\hat{a}, \hat{\mathbf{F}}\hat{a}) + \mathcal{O}(|\hat{a}^3|)} = \sqrt{(\hat{a}^T, \hat{\mathbf{F}}^T\hat{\mathbf{F}}\hat{a})} + \mathcal{O}(|\hat{a}^2|) \end{aligned}$$

We let $\hat{C} = \hat{F}^T\hat{F}$ denote the right *Cauchy-Green tensor*. By observation the Cauchy-Green tensor is not zero at the reference configuration

$$\hat{C} = \hat{F}^T\hat{F} = (I + \hat{\nabla}\hat{u})^T(I + \hat{\nabla}\hat{u}) = 1$$

Hence it is convenient to introduce a tensor which is zero at the reference configuration. We define the *Grenn-Lagrange strain tensor*, which arises from the squard rate of change of the linesegment $\hat{a}$ and a. By using the definition of the Cauchy-Green tensor we have the relation

$$\begin{aligned} \frac{1}{2}(|a|^2 + |\hat{a}|^2) = \frac{1}{2}(\hat{a}^T\hat{\mathbf{C}}\hat{a} - \hat{a}^T\hat{a}) + \mathcal{O}(|\hat{a}^3| = \hat{a}^T\big(\frac{1}{2}(\hat{\mathbf{F}}^T\hat{\mathbf{F}} - I)\big)\hat{a} + \mathcal{O}(\hat{a}^3) \\ \hat{\mathbf{E}} = \frac{1}{2}(\hat{\mathbf{C}} - I) \end{aligned}$$

Both the *right Cauchy-Green tensor* $\hat{\mathbf{C}}$ and the *Green-Lagrange* $\hat{\mathbf{E}}$ are refered to the Lagrangian coordinate system, hence the *reference configuration.*
Using similar arguments (see [6], compsda) Eulerian counterparts of the Lagrangian stress tensors can be derived.
The *left Cauchy-Green* strain tensor

$$\mathbf{b} = \hat{F}\hat{F}^T =$$

and the *Euler-Almansi* strain tensor

$$\mathbf{e} = \frac{1}{2}(I - \hat{\mathbf{F}}^{-1}\hat{\mathbf{F}}^{-T}) = \hat{\mathbf{F}}^{-1}\hat{\mathbf{E}}\hat{\mathbf{F}}^T$$

Strain itself is nothing else than the measurement of line segments under deformation. Therefore strain alone is purely an observation, and it is not dependent on the material of interest. but one expects that a material undergoing strain, will give for forces within the material.

1.4 The fluid

We will throughout this thesis consider incompressible fluids described by Navier-Stokes equations. We define the fluid density as ρ_f and fluid viscosity ν_f to be constant in time. Our phsyical unknowns fluid velocity v_f and pressure p_f both live in the time-dependent fluid domain $\hat{\mathbf{\Omega}}_f(t)$. Let any Dirichlet boundariy conditions be defined as v_f^D, p_f^D on the boundaries of $\hat{\mathbf{\Omega}}_f(t)$, and let g_1 denote the neumann conditions of $\sigma_f \cdot n$ defined on the boundaries of $\hat{\mathbf{\Omega}}_f(t)$.

Equation 1.4.1. *Navier-Stokes equation*

$$\rho \frac{\partial \mathbf{v}_f}{\partial t} + \rho \mathbf{v}_f \cdot \nabla \mathbf{v}_f = \nabla \cdot \sigma + \rho \mathbf{f}_f \tag{1.5}$$

$$\nabla \cdot \mathbf{v}_f = 0 \tag{1.6}$$

Assuming a newtonian fluid the *Cauchy stress sensor* σ takes the form
$\sigma = -p_f I + \mu_f(\nabla \mathbf{v}_f + (\nabla \mathbf{v}_f)^T$

1.5 The solid

For the structure we use the Vernant-Kirchhoff(STVK) model of deformation of solids. We usually describe the material elasticity by two parameters, Lames coefficients λ_s and μ_s or the Poisson ratio ν_s and the Young modulus E_s [3]. INSERT RELATIONS

1.5.1 Conservation of continuum

1.5.2 Conservation of momentum

1.5.3 Material models, St. Venant Kirchhoff materia, incomp neo-Hookean materiall

Kapittel 2

Fluid Structure Interaction

The consepts of Fluid-structure interaction are often introduced in several engineering feelds, for example biomechanics and hydrodynamics. As we will see throughout this chapter, one of the main challenges of this field that our governing equation describing fluid and solids are defined on different coordinate systems. Recall from chapter ?, that the solid equations are often described in the *Lagrangian coordiante system*, while the fluid equations are on the contrary described in the *Eularian coordinate system*.
We define Ω in the *reference configuration* be partitioned in a fluid domain $\hat{\mathbf{\Omega}}_\mathbf{f}$ and a structure domain $\hat{\mathbf{\Omega}}_\mathbf{s}$ such that $\Omega = \hat{\mathbf{\Omega}}_\mathbf{f} \cup \hat{\mathbf{\Omega}}_\mathbf{s}$. Furhter we define the interface $\hat{\mathbf{\Gamma}}$ as the intersection between these domains such that $\Gamma_i = \partial\hat{\mathbf{\Omega}}_\mathbf{f} \cap \partial\hat{\mathbf{\Omega}}_\mathbf{s}$. A natural dilemma arises as the domain Ω undergoes deformation over time. If the natural coordinate system are used for $\hat{\mathbf{\Omega}}_\mathbf{f}$ and $\hat{\mathbf{\Omega}}_\mathbf{s}$, the domains doesn't match and the interface $\hat{\mathbf{\Gamma}}$ doesn't have a general description for both domains. As such only one of the domains can be described in its natural coordinate system, while the other domain needs to be defined in some transformed coordinate system.

The scope of FSI methods can formally be divided into *interface-tracking* and *interface-capturing* methods.[4]. In the *interface-tracking* litterature, the *arbitary Lagrangian-Eulerian* formilation is dominant approach [5], [4]. In this approach the structure is given in its natural *Lagrangian coordinate system*, while the fluid is transformed into an artificial *Lagrangian* coordinate system. From this approach tracking of the interface $\hat{\mathbf{\Gamma}}$ is more trivial, as it is fixed on a *reference system* and can be tracked by mappings defined in Chapter 2.
While *interface-capturing* are also defined on a fixed mesh, the interface $\hat{\mathbf{\Gamma}}$ is now moving over the mesh element. Hence by its name, the inteface must be tracked as it is moving in time.
Initially one of the *interface-capturing* methods, the *fully Eulerian apporach* was considered, but implementation of tracking the interface in time was proven unsuccessful. Therefore ALE approach was finally chosen for this thesis. As such both the *fully Eulerian* and *ALE* consepts, strengths and weaknesses will be introduced following sub-chapters.

2.0.1 Fully Eulerian

This method keeps the fluid in its *Eulerian coordinates*, and such can be seen as the natural counterpart of the ALE method [7]. First proposed by , [2].

2.0.2 Arbitary Lagrangian Eulerian

MEANTION ALE CAN BEHAVE EITHER EULERIAN AND LAGRANGIAN The ALE method was initially developed to combine the strengths of the *Lagranngian* and *Eulerian* coordinate systems. As pointed out in chapter 2, the *Lagrangian* description is useful for tracking particles as they are act upon by forces. Hence its main contribution is the ability to track interfaces and materials with history dependent properties. In the ALE method one choose to keep the

structure in its *Lagrangian coordinate system*, while transforming the fluid domain into an artificial coordinate system similar to the *Lagrangian coordinate system*. It is however important to note that there is no natural displacement in the fluid domain, hence this domain has no directly physical meaning [5], [1].
With this in mind, we will derive these transformations with the help of a new arbitary fixed reference system $\hat{\mathrm{W}}$, following the ideas and approaches found in [6]. Further we denote its deformation gradient as $\hat{\mathbf{F}}_w$ and its determinant $\hat{\mathrm{J}}_w$. Following the ideas from chapter 2, we introduce the invertibale mapping $\hat{\mathrm{T}}_w : \hat{\mathrm{W}} \to V(t)$, with the scalar $\hat{\mathrm{f}}(\hat{\mathrm{x}}_W, t) = f(x, t)$ and vector $\hat{\mathbf{w}}(\hat{\mathrm{x}}_W, t) = \mathbf{w}(x, t)$ counterparts.
For $\hat{\mathrm{V}} = \hat{\mathrm{W}}$, $\hat{\mathrm{W}}$ simply denotes the familiar Lagrangian description. In the case $\hat{\mathrm{V}} \neq \hat{\mathrm{W}}$, $\hat{\mathrm{W}}$ as pointed out earlier have no direct physical meaning. Hence it is important to notice that the physical velocity $\hat{\mathbf{v}}$ and the velocity of arbitary domain $\frac{\partial \hat{\mathrm{W}}_w}{\partial t}$ doesn't neseccary coincide. This observation is essential, as we will soon see.

We will first define the transformation of spatial and temporal derivatives from $V(t)$ to $\hat{\mathrm{W}}$ found in [6]

Lemma 2.1. Transformation of scalar spatial derivatives
Let f be a scalar function such that $f : V(t) \to \mathbb{R}$, then

$$\nabla f = \hat{\mathbf{F}}_W^{-T} \hat{\nabla} \hat{\mathrm{f}} \tag{2.1}$$

Lemma 2.2. Transformation of vector spatial derivatives
Let $\boldsymbol{w}$ be a vector field such that $\mathbf{w} : V(t) \to \mathbb{R}^d$, then

$$\nabla \mathbf{w} = \hat{\nabla} \hat{\mathbf{w}} \hat{\mathbf{F}}_W^{-1} \tag{2.2}$$

Lemma 2.3. Transformation of scalar temporal derivatives
Let f be a scalar function such that $f : V(t) \to \mathbb{R}$, then

$$\frac{\partial f}{\partial t} = \frac{\partial \hat{\mathrm{f}}}{\partial t} - (\hat{\mathbf{F}}_W^{-1} \frac{\partial \hat{\mathrm{T}}_W}{\partial t} \cdot \hat{\nabla}) \hat{\mathrm{f}} \tag{2.3}$$

In addition we need a consistent way to transform the induced stresses in the *Eulerian* coordinate system to $\hat{\mathrm{W}}$. Hence we introduce the *Piloa transformation*, found in most introductional courses in structure mechanics (ORANGE BOOK).

Lemma 2.4. T
Let $\boldsymbol{w}$ be a vector field such that $\mathbf{w} : V(t) \to \mathbb{R}^d$, then the Piola transformation of w is defined by

$$\mathbf{w} = \hat{\mathrm{J}}_W \hat{\mathbf{F}}_W^{-1} \hat{\mathbf{w}} \tag{2.4}$$

The Piola transformation can be further extended to transform tensors, see [6], Orange book. This results is essential as it allows us to transform surface forces induced by the *Cauchy stress tensor* on our arbitrary coordinate system $\hat{\mathrm{W}}$. Lemma 1.4 brings us to *the first Piola Kirchhoff stress tensor* $\hat{\mathbf{P}} = \hat{\mathrm{J}}_W \hat{\sigma} \hat{\mathbf{F}}_W^{-T}$, mentioned in chapter 2.
We now have the necessary tools to transform the conservation principles introduced in the fluid problem in chapter 2. Recall the Navier-Stokes equation defined in the *Eulerian coordinate system* V(t).

$$\begin{aligned} \rho \frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{v} \cdot \nabla \mathbf{v} &= \nabla \cdot \sigma + \rho \mathbf{f} \\ \nabla \cdot \mathbf{v} &= 0 \end{aligned}$$

Using our newly introduced transformations of derivatives we map the equation to the arbitrary reference system $\hat{\mathrm{W}}$. We will first consider the transformation of the*material derivative*. By

$$\begin{aligned} \frac{d\mathbf{v}}{dt}(x, t) &= \frac{\partial \mathbf{v}}{\partial t}(x, t) + \nabla \mathbf{v}(x, t) \cdot \frac{\partial x}{\partial t} \\ \frac{d\mathbf{v}}{dt}(x, t) &= \frac{\partial \mathbf{v}}{\partial t}(x, t) + \nabla \mathbf{v}(x, t) \cdot \mathbf{v} \end{aligned}$$

Note $\frac{\partial x}{\partial t}$ is the velocity of particles and not the transformation velocity $\frac{\partial \hat{\mathrm{T}}_W}{\partial t}$. By lemma(1.1, 1.2, 1.3) we have

$$\frac{d\mathbf{v}}{dt}(x,t) = \frac{\partial \hat{\mathbf{v}}}{\partial t}(x,t) - (\hat{\mathbf{F}}_W^{-1}\frac{\partial \hat{\mathrm{T}}_W}{\partial t}\cdot\hat{\nabla})\hat{\mathbf{v}} + \hat{\mathbf{F}}_W^{-T}\hat{\nabla}\hat{\mathbf{v}}\cdot\hat{\mathbf{v}}$$
$$\mathbf{v}\cdot\nabla\mathbf{v} = \nabla\mathbf{v}\mathbf{v} = \hat{\nabla}\hat{\mathbf{v}}\hat{\mathbf{F}}_W^{-1}\hat{\mathbf{v}} = (\hat{\mathbf{F}}_W^{-1}\hat{\mathbf{v}}\cdot\hat{\nabla})\hat{\mathbf{v}} \quad \textit{FINN KILDE}$$

These results can be used to show that

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v} = \frac{\partial \hat{\mathbf{v}}}{\partial t} + (\hat{\mathbf{F}}_W^{-1}(\hat{\mathbf{v}} - \frac{\partial \hat{\mathrm{T}}_W}{\partial t})\cdot\hat{\nabla})\hat{\mathbf{v}}$$

By applying *the first Piola Kirchhoff stress tensor* directly we transform the surface stress by

$$\nabla\cdot\sigma = \nabla\cdot(\hat{\mathrm{J}}_W\hat{\sigma}\hat{\mathbf{F}}_W^{-T})$$

In general, σ is presumed on the form of a Newtonian fluid. However special care must be taken, as $\sigma \neq \hat{\sigma}$ due to spatial derivatives within the tensor. Hence

$$\sigma = -pI + \mu_f(\nabla\mathbf{v} + (\nabla\mathbf{v})^T$$
$$\hat{\sigma} = -\hat{\mathrm{p}}I + \mu_f(\hat{\nabla}\hat{\mathbf{v}}\hat{\mathbf{F}}_W^{-1} + \hat{\mathbf{F}}_W^{-T}\hat{\nabla}\hat{\mathbf{v}}^T)$$

For the conservation of continuum we apply the *Piola Transformation* such that

$$\nabla\cdot\mathbf{v} = \nabla\cdot(\hat{\mathrm{J}}\hat{\mathbf{F}}_W^{-1}\hat{\mathbf{v}})$$

Strong and weak coupling

One of the major aspects of Fluid-structure interaction is the coupling of the fluid and solid equations. As the total system is exerted by external forces, the interface $\hat{\mathbf{\Gamma}}$ must fulfill the physical equilibrium of forces given by the two domains. Therefore, is is critical that the transmition of forces from the two domains are fulfilled in a consistent way.

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