Thesis Title

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Kapittel 1

Fluid Structure Interaction

The consepts of Fluid-structure interaction are often introduced in several engineering feelds, for example biomechanics and hydrodynamics. As we will see throughout this chapter, one of the main challenges of this field that our governing equation describing fluid and solids are defined on different coordinate systems.

We define Ω in the *reference configuration* be partitioned in a fluid domain $\hat{\mathbf{\Omega}}_{\mathbf{f}}$ and a structure domain $\hat{\mathbf{\Omega}}_{\mathbf{s}}$ such that $\Omega = \hat{\mathbf{\Omega}}_{\mathbf{f}} \cup \hat{\mathbf{\Omega}}_{\mathbf{s}}$. Furhter we define the interface $\hat{\mathbf{\Gamma}}$ as the intersection between these domains such that $\Gamma_i = \partial\hat{\mathbf{\Omega}}_{\mathbf{f}} \cap \partial\hat{\mathbf{\Omega}}_{\mathbf{s}}$. The fluid-structure interaction problem is then defined by the fluid and solid equations, and the transmittion of the *kinematic* and *dynamic* conditions on the interface $\hat{\mathbf{\Gamma}}$.

$$\mathbf{v}_f = \mathbf{v}_s \tag{1.1}$$

$$\sigma_f \cdot \mathbf{n} = \sigma_s \cdot \mathbf{n} \tag{1.2}$$

A natural dilemma arises as the domain Ω undergoes deformation over time. Recall from chapter ?, that the solid equations are often described in the *Lagrangian coordinate system*, while the fluid equations are on the contrary described in the *Eularian coordinate system*. If the natural coordinate system are used for $\hat{\mathbf{\Omega}}_{\mathbf{f}}$ and $\hat{\mathbf{\Omega}}_{\mathbf{s}}$, the domains doesn't match and the interface $\hat{\mathbf{\Gamma}}$ doesn't have a general description for both domains. As such only one of the domains can be described in its natural coordinate system, while the other domain needs to be defined in some transformed coordinate system.

Fluid-structure interaction problems are formally divided into the *monolithic* and *partitioned* frameworks. In the monolithic framework all equations and interfaceconditions are solved simultaneously. If the *kinematic* (1.1) and *dynamic*(1.2) conditions are satisfied exactly at each timestep, the method denoted as a *strongly coupled* which is typically for a monolithic approach. However this strong coupling yields contributes to a stronger nonlinear behaviour of the whole system [14]. There are several nonlinear solution strategies for the monolithic approach.. Further the monolithic framework provides less modular software since the implementation often *ad hoc* for a certain monolithic approach.

In the *partitioned* framework one solves the equations of fluid and structure subsequently. The strength of such an approach is the wast range of optimized solvers developed for both the fluid and solid. However one major problem is that the interface conditions are not naturally met, and as such certain sub-iterations is required to achieve these.

Independent of framework, one of the major aspects of Fluid-structure interaction is the coupling of the fluid and solid equations through (1.1) and (1.2) . As the total system is exerted by external forces, the interface $\hat{\mathbf{\Gamma}}$ must fulfill the physical equilibrium of forces given by the two domains. Therefore, it is critical that the transmission of forces from the two domains are fulfilled in a consistent way. However as the interface conditions are not naturally met such as in a *monolithic* approach, we one must distinguish between *strongly* and *weakly* coupled schemes. *Partitioned*

solvers often enforce a weakly coupled schemes, meaning (1.1) and (1.2) are not strictly enforced in the calculation. Such an approach is often sufficient for some fields such as computations within aeroelasticity [?]. When these conditions are met exactly, we define the scheme as *strongly* coupled. The scope of FSI methods can formally be divided into *interface-tracking* and *interface-capturing* methods.[?]. In the *Interface-tracking* method, the mesh moves to accommodate for the movement of the structure as it deformes the spatial domain occupied by the fluid. As such the mesh itself tracksthe fluid-structure interface as the domain undergoes deformation. Such an approach is feasable as it allows for better control of mesh resolution near the interface, which in turn yeilds better control of this critical area. Among the *Interface-tracking* methods, the *arbitary Lagrangian-Eulerian* formulation is the most well-known approach [?], [?]. In this approach the structure is given in its natural *Lagrangian coordinate system*, while the fluid is transformed into an artificial *Lagrangian* coordinate system. From this approach tracking of the interface $\hat{\boldsymbol{\Gamma}}$ is more trivial, as it is fixed on a *reference system* and can be tracked by mappings defined in Chapter 2.

In *interface-capturing* methods one distinguish the fluid and solid domains by some phase variable over a fixed mesh. As such one captures the interface $\hat{\boldsymbol{\Gamma}}$ as it´s moving over the mesh element. This method is often emplyed in simulations of multiphase-flow. This ideas was extended in [4] were the authors proposed to transform the lagrangian formulated structure equations in an eulerian formulation, solving the system of equations in a fully eulerian formulation. This approach was considered in this thesis, but implementation of tracking the interface in time was proven unsuccessful. Therefore ALE approach was finally chosen for this thesis. As such both the *fully Eulerian* and *ALE* consepts, strengths and weaknesses will be introduced following sub-chapters.

1.0.1 Fully Eulerian

This method keeps the fluid in its *Eulerian coordinates*, and such can be seen as the natural counterpart of the ALE method [15]. First proposed by , [3]. One motivation of such and approach is the handling of large-deformation, as the transformation to eulerian coordinates are purley natural.

1.0.2 Arbitary Lagrangian Eulerian

MEANTION ALE CAN BEHAVE EITHER EULERIAN AND LAGRANGIAN The ALE method was initially developed to combine the strengths of the *Lagranngian* and *Eulerian* coordinate systems. As pointed out in chapter 2, the *Lagrangian* description is useful for tracking particles as they are act upon by forces. Hence its main contribution is the ability to track interfaces and materials with history dependent properties. In the ALE method one choose to keep the structure in its *Lagrangian coordinate system*, while transforming the fluid domain into an artificial coordinate system similar to the *Lagrangian coordinate system*. It is however important to note that there is no natural displacement in the fluid domain, hence this domain has no directly physical meaning [?], [2].

With this in mind, we will derive these transformations with the help of a new arbitary fixed reference system $\hat{W}$, following the ideas and approaches found in [9]. Further we denote its deformation gradient as $\hat{\mathbf{F}}_w$ and its determinant $\hat{J}_w$. Following the ideas from chapter 2, we introduce the invertibale mapping $\hat{T}_w : \hat{W} \to V(t)$, with the scalar $\hat{f}(\hat{x}_W, t) = f(x, t)$ and vector $\hat{\mathbf{w}}(\hat{x}_W, t) = \mathbf{w}(x, t)$ counterparts.

For $\hat{V} = \hat{W}$, $\hat{W}$ simply denotes the familiar Lagrangian description. In the case $\hat{V} \neq \hat{W}$, $\hat{W}$ as pointed out earlier have no direct physical meaning. Hence it is important to notice that the physical velocity $\hat{\mathbf{v}}$ and the velocity of arbitary domain $\frac{\partial \hat{W}_w}{\partial t}$ doesn't neseccary coincide. This observation is essential, as we will soon see.

We will first define the transformation of spatial and temporal derivatives from $V(t)$ to $\hat{W}$ found in [9]

Lemma 1.1. Transformation of scalar spatial derivatives
Let f be a scalar function such that $f : V(t) \to \mathbb{R}$, then

$$\nabla f = \hat{\mathbf{F}}_W^{-T} \hat{\nabla} \hat{\mathbf{f}} \tag{1.3}$$

Lemma 1.2. Transformation of vector spatial derivatives
Let $\boldsymbol{w}$ be a vector field such that $\mathbf{w} : V(t) \to \mathbb{R}^d$, then

$$\nabla \mathbf{w} = \hat{\nabla} \hat{\mathbf{w}} \hat{\mathbf{F}}_W^{-1} \tag{1.4}$$

Lemma 1.3. Transformation of scalar temporal derivatives
Let f be a scalar function such that $f : V(t) \to \mathbb{R}$, then

$$\frac{\partial f}{\partial t} = \frac{\partial \hat{\mathbf{f}}}{\partial t} - (\hat{\mathbf{F}}_W^{-1} \frac{\partial \hat{\mathbf{T}}_W}{\partial t} \cdot \hat{\nabla}) \hat{\mathbf{f}} \tag{1.5}$$

In addition we need a consistent way to transform the induced stresses in the *Eulerian* coordinate system to $\hat{\mathbb{W}}$. Hence we introduce the *Piloa transformation*, found in most introductional courses in structure mechanics (ORANGE BOOK).

Lemma 1.4. T
Let $\boldsymbol{w}$ be a vector field such that $\mathbf{w} : V(t) \to \mathbb{R}^d$, then the Piola transformation of w is defined by

$$\mathbf{w} = \hat{\mathbf{J}}_W \hat{\mathbf{F}}_W^{-1} \hat{\mathbf{w}} \tag{1.6}$$

The Piola transformation can be further extended to transform tensors, see [9], Orange book. This results is essential as it allows us to transform surface forces induced by the *Cauchy stress tensor* on our arbitrary coordinate system $\hat{\mathbb{W}}$. Lemma 1.4 brings us to *the first Piola Kirchhoff stress tensor* $\hat{\mathbf{P}} = \hat{\mathbf{J}}_W \hat{\sigma} \hat{\mathbf{F}}_W^{-T}$, mentioned in chapter 2.

We now have the necessary tools to transform the conservation principles introduced in the fluid problem in chapter 2. Recall the Navier-Stokes equation defined in the *Eulerian coordinate system* V(t).

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{v} \cdot \nabla \mathbf{v} = \nabla \cdot \sigma + \rho \mathbf{f}$$
$$\nabla \cdot \mathbf{v} = 0$$

Using our newly introduced transformations of derivatives we map the equation to the arbitrary reference system $\hat{\mathbb{W}}$. We will first consider the transformation of the *material derivative*. By

$$\frac{d\mathbf{v}}{dt}(x,t) = \frac{\partial \mathbf{v}}{\partial t}(x,t) + \nabla \mathbf{v}(x,t) \cdot \frac{\partial x}{\partial t}$$
$$\frac{d\mathbf{v}}{dt}(x,t) = \frac{\partial \mathbf{v}}{\partial t}(x,t) + \nabla \mathbf{v}(x,t) \cdot \mathbf{v}$$

Note $\frac{\partial x}{\partial t}$ is the velocity of particles and not the transformation velocity $\frac{\partial \hat{\mathbf{T}}_W}{\partial t}$. By lemma(1.1, 1.2, 1.3) we have

$$\frac{d\mathbf{v}}{dt}(x,t) = \frac{\partial \hat{\mathbf{v}}}{\partial t}(x,t) - (\hat{\mathbf{F}}_W^{-1} \frac{\partial \hat{\mathbf{T}}_W}{\partial t} \cdot \hat{\nabla}) \hat{\mathbf{v}} + \hat{\mathbf{F}}_W^{-T} \hat{\nabla} \hat{\mathbf{v}} \cdot \hat{\mathbf{v}}$$
$$\mathbf{v} \cdot \nabla \mathbf{v} = \nabla \mathbf{v} \mathbf{v} = \hat{\nabla} \hat{\mathbf{v}} \hat{\mathbf{F}}_W^{-1} \hat{\mathbf{v}} = (\hat{\mathbf{F}}_W^{-1} \hat{\mathbf{v}} \cdot \hat{\nabla}) \hat{\mathbf{v}} \quad \textit{FINN KILDE}$$

These results can be used to show that

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = \frac{\partial \hat{\mathbf{v}}}{\partial t} + (\hat{\mathbf{F}}_W^{-1} (\hat{\mathbf{v}} - \frac{\partial \hat{\mathbf{T}}_W}{\partial t}) \cdot \hat{\nabla}) \hat{\mathbf{v}}$$

By applying *the first Piola Kirchhoff stress tensor* directly we transform the surface stress by

$$\nabla \cdot \sigma = \nabla \cdot (\hat{\mathbf{J}}_W \hat{\sigma} \hat{\mathbf{F}}_W^{-T})$$

In general, σ is presumed on the form of a Newtonian fluid. However special care must be taken, as $\sigma \neq \hat{\sigma}$ due to spatial derivatives within the tensor. Hence

$$\sigma = -pI + \mu_f(\nabla \mathbf{v} + (\nabla \mathbf{v})^T)$$
$$\hat{\sigma} = -\hat{p}I + \mu_f(\hat{\nabla}\hat{\mathbf{v}}\hat{\mathbf{F}}_W^{-1} + \hat{\mathbf{F}}_W^{-T}\hat{\nabla}\hat{\mathbf{v}}^T)$$

For the conservation of continuum we apply the *Piola Transformation* such that

$$\nabla \cdot \mathbf{v} = \nabla \cdot (\hat{\mathbf{J}}\hat{\mathbf{F}}_W^{-1}\hat{\mathbf{v}})$$

Fluid mesh movement

In the ALE framwork one of the most limiting factors is the degeneration of the mesh due to large deformations. Even the most advanced ALE formulated schemes reaches a limit when only re-meshing is nesecarry [12]. Consequently the choice of an appropiate mesh moving tequnice is essential to preserve a feasable mesh quality for the simulation of fluid flow. Let the total domain deformation $\hat{\mathrm{T}}(\hat{\mathrm{x}}, t)$ be divided into the solid and fluid deformation T_s, T_f, were the fluid deformation is mapped to the arbiatry fixed reference system $\hat{\mathrm{W}}$ presented in the last subsection. Then the ALE map T_f on the form

$$\hat{\mathrm{T}}_f(\hat{\mathbf{x}}, t) = \hat{\mathbf{x}} + \hat{\mathbf{u}}_f(\hat{\mathbf{x}}, t)$$

is constructed such that $\hat{\mathbf{u}}_f$ is an extension of the solid deformation $\hat{\mathbf{u}}_s$ from the interface to the fluid domain. The construction of such extensions often involves solving some auxiliary problem on a partial differential equation form. Several extentions have been proposed throuhout the litteratur, and for an overview the reader is refered to [8], and the reference therein. In this thesis the *Laplacian*, *pseudo-elasticity* and *biharmonic* extensions will be considered.

Laplacian model

The main motivation for a *Laplacian* smoothing is due to its simplicity and due to its property of bounding the interior displacements to the boundary values.

$$-\hat{\nabla} \cdot (\alpha^q \hat{\nabla}\hat{\mathbf{u}}) = 0$$
$$\hat{\mathbf{u}}_f = \hat{\mathbf{u}}_s \ \text{ on } \ \Gamma$$
$$\hat{\mathbf{u}}_f = 0 \ \text{ on } \ \partial\hat{\mathbf{\Omega}}_f/\Gamma$$

Most favourable, the largest mesh deformation occuring should be confined to the interal part of the mesh as it causes the least distortion [7]. Therefore the introduced diffusion parameter α, often raised to some power q, is introduced to manipulate this behaviour. The form of this paramter is often problem specific, as selective treatment of the elements may vary from different mesh deformation problems. A jacobian based method was introduced in [10]. In [7], the authors reviewed several distance based options, where α was some function of the distance to the closest moving boundary. This method was adopted in this thesis on the form

$$\alpha(x) = \frac{1}{x^q} \quad q = -1$$

However as pointed out by [6], one of the main disadvantages of using the linear Laplace equation is that the equation solves the mesh deformation components independently of one another. Say one have deformation only in the x-coordinate direction, the interior mesh points will only be moved along this deformation. Such a behaviour restricts the use to the Laplace equation of mesh extrapolation purposes.

Linear elastic model

Considering a linear elastic model for mesh moving was first introduced in [11]. Both [5]

$$\nabla \cdot \sigma = 0$$
$$\sigma = \lambda Tr(\epsilon)I + 2\mu\epsilon$$
$$\epsilon = \frac{1}{2}(\nabla u + \nabla u^T$$

Where Lamé constants λ and ν are given as

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} \quad \mu = \frac{E}{2(1+\nu)}$$

One of the main motivations for introducing such a model is the manipulation of Young's modulus E, and the poisson´s ration ν. Recall that Young's modulus is the measurement of the a materials stiffness, while the poission's ratio describe the materials stretching in the transverse direction under extension in the axial direction. Manipulating these parameters one can influence the mesh deformation, however the choice of these parameters have proven not to be consistent, and to be dependent of the given problem.

In [13] the author proposed a negative possion ratio, which makes the model mimic an auxetic material. Such materials becomes thinner in the perpendicular direction when they are submitted to compression, and this proparty is feasable for mesh under deformation.
One of the most common approach is to set ν as a constant in the range $\nu \in [0, 0.5)$ and let E be the inverse of the distance of an interior node to the nearest boundary surface [8]. The authors of [1] used this property and also argued that the Young's modulus also could be chosen as the inversely proportional to the cell volume. They also pointed out that both approaches would give the desired result that the small cells around the solid surface would modelled rigid, moving with the surface of the solid as it undergoes deformation. On the other hand cells further away will deform to counter the effects close to the solid surface.

Biharmonic model

Using a biharmonic mesh deformation model provides further freedom in terms of boundary conditions for the operator [14], [?]. The biharmonic model is defined as

$$\hat{\nabla}^2 \hat{u} = 0 \quad \text{on} \ \hat{\mathbf{\Omega}}_f$$

By introducing a second variable on the form $\hat{w} = -\hat{\nabla}\hat{u}$, we get the following system defined by

$$\hat{\mathbf{w}} = -\hat{\nabla}^2\hat{\mathbf{u}}$$
$$- \hat{\nabla}\hat{\mathbf{w}} = 0$$

For the boundary conditions two types has been proposed in [13]. Let $\hat{\mathbf{u}}_f$ be decomposed by the components $\hat{\mathbf{u}}_f = (\hat{u}_f^{(1)}.\hat{u}_f^{(2)})$. Then we have

$$\textbf{Type 1} \quad \hat{u}_f^{(k)} = \frac{\partial \hat{u}_f^{(k)}}{\partial n} = 0 \quad \partial\hat{\mathbf{\Omega}}_f/\Gamma \ \text{for} \ k = 1, 2$$

$$\textbf{Type 2} \quad \hat{u}_f^{(1)} = \frac{\partial \hat{u}_f^{(1)}}{\partial n} = 0 \ \text{and} \ \hat{w}_f^{(1)} = \frac{\partial \hat{w}_f^{(1)}}{\partial n} = 0 \quad \text{on} \ \hat{\mathbf{\Omega}}_f^{in} \cup \hat{\mathbf{\Omega}}_f^{out}$$

$$\hat{u}_f^{(2)} = \frac{\partial \hat{u}_f^{(2)}}{\partial n} = 0 \ \text{and} \ \hat{w}_f^{(2)} = \frac{\partial \hat{w}_f^{(2)}}{\partial n} = 0 \quad \text{on} \ \hat{\mathbf{\Omega}}_f^{wall}$$

The main motivation for second type of boundary condition is for a rectangular domain where the coordinate axes match the Cartesian coordinate system [13]. In such a configuration, the mesh movement is only constrained in the perpendicular direction of the fluid boundary, leading to mesh movement in the tangential direction. This special case reduces the effect of distortion of the cells.

Comparison

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