Master Thesis

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Continuum Mechanics

When studying the dynamics of a mediums with fluid or structure properties under the influence of forces, we need in some sense a good description of how these forces act and alter the system itself.
Any medium on a microscopic scale is built up of a structure of atoms, meaning we can observe empty spaces between each atom or discontinuities in the medium. Discribing any phsycial phenomen on larger scales in such a way are tedious and most often out of bounds due to the high number of particles. Instead we consider the medium to be continously distributed throughout the entire reagion it occupies. Hence we want to study some phsyical properties of the complete volume and not down on atomic scale.
We consider the medium with continuum properties. By a continuum we mean a volume $V(t) \subset \mathbb{R}^3$ consiting of particles, which we observe for some properties. One property of interest could be the veloctity $\mathbf{v}(x, t)$ for some point $x \in V(t)$ in time $t \in (0, T]$, which would mean the average velocity of the particles occupying this point x at time t

Coordinate system, a matter of perspective

We assume that our medium is continiously distributed throughout its own volume, and we start our observation of this medium at som time t_0. As this choice is arbitary, we often choose to observe a medium in a stress free initial state. We call this state $V(t_0)$ of the medium as the *reference configuration*. We let $V(t)$ for $t \geq t_0$ denote the *current configuration*.

Lagrangian

As the medium is act upon by forces, one of the main properties of interest is the deformation. Let $\hat{\mathrm{x}}$ be a particle in the reference cofiguration $\hat{\mathrm{x}} \in \hat{\mathrm{V}}$. Further let x($\hat{\mathrm{x}}$, t) be the new location of a particle $\hat{\mathrm{x}}$ for time t such that $x \in V(t)$. We assume that no two particles $\hat{\mathrm{x}}_a, \hat{\mathrm{x}}_b \in \hat{\mathrm{V}}$ occupy the same location for some time $V(t)$. Hence the map $\hat{\mathrm{T}}(\hat{\mathrm{x}}, t) = x(\hat{\mathrm{x}}, t)$ maps a particle $\hat{\mathrm{x}}$ from the *reference configuration* $\hat{\mathbf{V}}$ to the *current configuration* $V(t)$ Assuming that the path for some $\hat{\mathrm{x}}$ is continious in time, we can define the inverse mapping $\hat{\mathrm{T}}^{-1}(x, t) = \hat{\mathrm{x}}(x, t)$, which maps $x(\hat{\mathrm{x}}, t)$ back to its initial location at time $t = t_0$.

We now have enough background to define the *deformation*

$$\hat{\mathbf{u}}(\hat{\mathrm{x}}, t) = x(\hat{\mathrm{x}}, t) - \hat{\mathrm{x}} \tag{1}$$

and the *deformation velocity*

$$\hat{\mathbf{v}}(\hat{\mathrm{x}}, t) = d_t x(\hat{\mathrm{x}}, t) = d_t \hat{\mathbf{u}}(\hat{\mathrm{x}}, t) \tag{2}$$

Such a description of tracking each particle $\hat{\mathrm{x}} \in \hat{\mathrm{V}}$ is often denoted the *Lagrangian Framework*. Such a framework is a natural choice of describing structure mechanics such as describing the deformation of a steel beam under pressure.

Eulerian

Considering a flow of fluid particles in a river, a *Lagrangian* description of the particles would be tidious as the number of particles entring and leaving the domain quickly rise to a immense number. Instead consider defining a view-point V fixed in time, and monitor every fluid particle passing coordinate x as time elapses. We can describe the particles occupying the *current configuration* $V(t)$ for some time $t \geq t_0$

$$x = \hat{\mathrm{x}} + \hat{\mathbf{u}}(\hat{\mathrm{x}}, t)$$

Fluid Structure Interaction

From the consepts of continuum mechanics we often expand our thoery by observing to mediums interacting with eachother as they are act upon by forces. In this thesis we will look at how to mediums of fluid and structural properties interact. We will let our computational domain Ω in the *reference configuration* be partitioned in a fluid domain $\hat{\mathbf{\Omega}}_\mathbf{f}$ and a structure domain $\hat{\mathbf{\Omega}}_\mathbf{s}$ such that $\Omega = \hat{\mathbf{\Omega}}_\mathbf{f} \cup \hat{\mathbf{\Omega}}_\mathbf{s}$. Furhter we define the interface $\hat{\mathbf{\Gamma}}$ as the intersection between these domains such that $\Gamma_i = \partial\hat{\mathbf{\Omega}}_\mathbf{f} \cap \partial\hat{\mathbf{\Omega}}_\mathbf{s}$

0.1 Fully Eulerian concept

In contrast to the Lagrangian description of the structure, we no longer follow an individual partical $x(x_0, t)$ from its initial state. We keep our view-point of the structure domain Ω_s fixed, and observe as the continuum $\Omega_s(t)$ moves in time. This means that for some point $x \in \Omega_s(t)$ will be occupied by different particles $\hat{\mathbf{x}}$ in time.
We will let $\mathbf{v}_s$ denote the solid velocity and $\mathbf{u}_s$ the solid displacement in the Eulerian formulation of the structure. We define the mapping $\hat{\mathbf{x}} = T_s(x, t) = x - \mathbf{u}(x, t)$ of an Eulerian coordinate of particle $x \in \Omega(t)_s$ back to its coordinate in the *reference configuration* $x \in \Omega(t_0)_s$.

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Let $\mathbf{F} = \nabla T = I - \nabla\mathbf{u}$ be defined as the displacement gradient and further let $J = \det\mathbf{F}$, be its determinant.
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[1] Michel Goossens, Frank Mittlebach, and Alexander Samarin. *The Latex Companion*. Addison-Wesley, Reading, Massachusetts, 1993.