Master Thesis

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Continuum Mechanics

When studying the dynamics of a mediums with fluid or structure properties under the influence of forces, we need in some sense a good description of how these forces act and alter the system itself.
Any medium on a microscopic scale is built up of a structure of atoms, meaning we can observe empty spaces between each atom or discontinuities in the medium. Discribing any phsycial phenomen on larger scales in such a way are tedious and most often out of bounds due to the high number of particles. Instead we consider the medium to be continously distributed throughout the entire reagion it occupies. Hence we want to study some phsyical properties of the complete volume and not down on atomic scale.
We consider the medium with continuum properties. By a continuum we mean a volume $V(t) \subset \mathbb{R}^3$ consiting of particles, which we observe for some properties. One property of interest could be the veloctity $\mathbf{v}(x,t)$ for some point $x \in V(t)$ in time $t \in (0,T]$, which would mean the average velocity of the particles occupying this point x at time t

Coordinate system, a matter of perspective

We assume that our medium is continiously distributed throughout its own volume, and we start our observation of this medium at som time t_0. As this choice is arbitary, we often choose to observe a medium in a stress free initial state. We call this state $V(t_0)$ of the medium as the *reference configuration*. We let $V(t)$ for $t \geq t_0$ denote the *current configuration*.

Lagrangian

As the medium is act upon by forces, one of the main properties of interest is the deformation. Let $\hat{\mathrm{x}}$ be a particle in the reference cofiguration $\hat{\mathrm{x}} \in \hat{\mathrm{V}}$. Further let x($\hat{\mathrm{x}}$, t) be the new location of a particle $\hat{\mathrm{x}}$ for time t such that $x \in V(t)$. We assume that no two particles $\hat{\mathrm{x}}_a, \hat{\mathrm{x}}_b \in \hat{\mathrm{V}}$ occupy the same location for some time $V(t)$. Hence the map $\hat{\mathrm{T}}(\hat{\mathrm{x}},t) = x(\hat{\mathrm{x}},t)$ maps a particle $\hat{\mathrm{x}}$ from the *reference configuration* $\hat{\mathbf{V}}$ to the *current configuration* $V(t)$ Assuming that the path for some $\hat{\mathrm{x}}$ is continious in time, we can define the inverse mapping $\hat{\mathrm{T}}^{-1}(x,t) = \hat{\mathrm{x}}(x,t)$, which maps $x(\hat{\mathrm{x}},t)$ back to its initial location at time $t = t_0$.

We now have enough background to define the *deformation*

$$\hat{\mathbf{u}}(\hat{\mathrm{x}},t) = x(\hat{\mathrm{x}},t) - \hat{\mathrm{x}} \tag{1}$$

and the *deformation velocity*

$$\hat{\mathbf{v}}(\hat{\mathrm{x}},t) = d_t x(\hat{\mathrm{x}},t) = d_t \hat{\mathbf{u}}(\hat{\mathrm{x}},t) \tag{2}$$

Such a description of tracking each particle $\hat{\mathrm{x}} \in \hat{\mathrm{V}}$ is often denoted the *Lagrangian Framework* and is a natural choice of describing structure mechanics such as describing the deformation of a steel beam under pressure.

Eulerian

Considering a flow of fluid particles in a river, a *Lagrangian* description of the particles would be tidious as the number of particles entring and leaving the domain quickly rise to a immense number. Instead consider defining a view-point V fixed in time, and monitor every fluid particle passing the coordinate $x \in V(t)$ as time elapses. Such a description is defined as the *Eulerian framework.* It is important to mention that the we are not interested in which particle is occupying a certain point in our domain, but only its properties. Such a description falls natural for describing fluid dynamics.
We can describe the particles occupying the *current configuration* $V(t)$ for some time $t \geq t_0$

$$x = \hat{\mathrm{x}} + \hat{\mathbf{u}}(\hat{\mathrm{x}}, t)$$

Since our domain is fixed can define the deformation for a particle occupying position $x = x(\hat{\mathrm{x}}, t)$ as

$$\mathbf{u}(x, t) = \hat{\mathbf{u}}(\hat{\mathrm{x}}, t) = x - \hat{\mathrm{x}}$$

and its velocity

$$\mathbf{v}(x, t) = \partial_t u(x, t) = \partial_t \hat{\mathbf{u}}(\hat{\mathrm{x}}, t) = \hat{\mathbf{v}}(\hat{\mathrm{x}}, t)$$

0.1 Deformation and deformation gradients next

Fluid Structure Interaction

From the consepts of continuum mechanics we often expand our thoery by observing to mediums interacting with eachother as they are act upon by forces. In this thesis we will look at how to mediums of fluid and structural properties interact. We will let our computational domain Ω in the *reference configuration* be partitioned in a fluid domain $\hat{\mathbf{\Omega}}_{\mathbf{f}}$ and a structure domain $\hat{\mathbf{\Omega}}_{\mathbf{s}}$ such that $\Omega = \hat{\mathbf{\Omega}}_{\mathbf{f}} \cup \hat{\mathbf{\Omega}}_{\mathbf{s}}$. Furhter we define the interface $\hat{\mathbf{\Gamma}}$ as the intersection between these domains such that $\Gamma_i = \partial\hat{\mathbf{\Omega}}_{\mathbf{f}} \cap \partial\hat{\mathbf{\Omega}}_{\mathbf{s}}$

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Lagrangian descritpion of St. Venant Kirchhoff material

Fully Eulerian concept

In this section we will focus on the fully Eulerian formulation approach of FSI. The equations are based on the conservation of mass and momentum within the fluid and structure. We will let $\mathbf{v}_s$ denote the solid velocity and $\mathbf{u}_s$ the solid displacement in the Eulerian formulation of the structure. We define the mapping $\hat{\mathbf{x}} = T_s(x,t) = x - \mathbf{u}(x,t)$ of an Eulerian coordinate of particle $x \in \Omega(t)_s$ back to its coordinate in the *reference configuration* $x \in \Omega(t_0)_s$.

INSERT FIGURE

Fluid

We assume an incrompressible Newtonian fluid, described by the usual Navier-Stokes equations. We define the fluid density as ρ_f and fluid viscosity ν_f to be constant in time. Our phsyical unknowns fluid velocity v_f and pressure p_f both live in the time-dependent fluid domain $\hat{\mathbf{\Omega}}_f(t)$. Let any Dirichlet boundariy conditions be defined as v_f^D, p_f^D on the boundaries of $\hat{\mathbf{\Omega}}_f(t)$, and let g_1 denote the neumann conditions of $\sigma_f \cdot n$ defined on the boundaries of $\hat{\mathbf{\Omega}}_f(t)$.

Structure

For the structure we use the Vernant-Kirchhoff(STVK) model of deformation of solids. We usually describe the material elasticity by two parameters, Lames coefficients λ_s and μ_s or the Poisson ratio ν_s and the Young modulus E_s [1]. INSERT RELATIONS

As mentioned in the continuum chapter, describing deformation falls naturally in the category of the Lagrangian formulation. So we have in

Eulerian descritpion of St. Venant Kirchhoff material

see [2]

Referanser

[1] Thomas Dunne and Rolf Rannacher. Adaptive finite element approximation of fluid-structure interaction based on an eulerian variational formulation. In *Fluid-structure interaction*, pages 110–145. Springer, 2006.

[2] Thomas Richter. A fully eulerian formulation for fluid–structure-interaction problems. *Journal of Computational Physics*, 233:227–240, 2013.