MANDATORY EXERCISE

Everyone is encouraged to define their own project associated with their master- or PhD-projects. The projects must however be approved. The topic must involve testing of a scheme for appropriate numerical behaviour. PhD students (taking Mek9250) will need to present their work by the end of the semester.

Exercise 1
Consider the following problem on the domain $\Omega = (0,1)^2$:

$$
\begin{aligned}
(1) \qquad -\Delta u &= f \text{ in } \Omega, \\
(2) \qquad u &= 0 \text{ for } x = 0 \text{ and } x = 1, \\
(3) \qquad \frac{\partial u}{\partial n} &= 0 \text{ for } y = 0 \text{ and } y = 1.
\end{aligned}
$$

Assume $u = \sin(\pi k x)\cos(\pi l y)$ and compute $f = -\Delta u$.
a) Compute the H^p norm of u.
b) Compute the L_2 and H^1 errors for $k = 1, 10, 100$ and $l = 1, 10, 100$ for $h = 8, 16, 32, 64$ when using first and second order Lagrangian elements.
c) Consider the following error estimates

$$\|u - u_h\|_1 \leq C h^\alpha \|u\|_{p+1}$$

and

$$\|u - u_h\|_0 \leq C h^\alpha \|u\|_{p+1}$$

Estimate C and α and check whether the expected error estimate is valid. (Employ the least square method in the estimation.)

Exercise 2
Consider the following equation on the domain $\Omega = (0,1)^2$:

$$
\begin{aligned}
(4) \qquad -\mu\Delta u + u_x &= 0 \text{ in } \Omega, \\
(5) \qquad u &= 0 \text{ for } x = 0, \\
(6) \qquad u &= 1 \text{ for } x = 1, \\
(7) \qquad \frac{\partial u}{\partial n} &= 0 \text{ for } y = 0 \text{ and } y = 1
\end{aligned}
$$

a) Derive an expression for the analytical solution.
b) Compute the numerical error for $\mu = 1, 0.001, 0.000001$ at $h = 8, 16, 32, 64$.
c) Compare against the expected error estimate, which is on the same form as in Exercise 1 and discuss the result.
d) Implement the Streamwise Upwinding Petrov-Galerkin (SUPG) method and compare against the results in b) and c).

Deadline: March 17. Please include code. Typesetting in LaTeX is prefered.
