# Exericse 2

```python
import numpy as np

#Dimention of matrix
N = 50

#Construct random Matrix of dimention N
A = np.random.randint(5, size=(N, N))

################### 1.94 ###################

#Construct identity Matrix of dimention N
I = np.eye(N)

#Calculation of I : A
I_leftside = 0
for i in range(N):
    for j in range(N):
        I_leftside += I[i, j]*A[i, j]

#Calculation of trace of matrix A
A_trace = float(np.trace(A))

#Calculation of A : I
I_rightside = 0
for i in range(N):
    for j in range(N):
        I_rightside += A[i, j]*I[i, j]

test_1 = I_leftside == A_trace
test_2 = A_trace == I_rightside
print("Property 1.94")
print("I : A = tr(A) is %r" % test_1)
print("tr(A) = A : I is %r \n" % test_2)

################### 1.96 ###################

u = np.random.randint(100, size=N)
v = np.random.randint(100, size=N)

u_cross_v = np.outer(u, v)
A_leftside = 0
A_rightside = 0
for i in range(N):
    for j in range(N):
        A_leftside  += A[i, j]*u_cross_v[i, j]
        A_rightside += u_cross_v[i, j]*A[i, j]

u_Av = np.dot(u, np.dot(A,v))

test_1 = A_leftside == u_Av
test_2 = u_Av == A_rightside

print("Property 1.96")
print("A : (u (x) v) = u dot Av is %r" % test_1)
```

```
print("u dot Av = (u (x) v) : A is %r" % test_2)
```

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#### TERMINAL OUTPUT ####

Property 1.94
I : A = tr(A) is True
tr(A) = A : I is True

Property 1.96
A : (u (x) v) = u dot Av is True
u dot Av = (u (x) v) : A is True
```

# Exercise 3

Verify $\nabla(uv) = \nabla(u)v + u\nabla v \qquad u, v \in \mathbb{R}$

$$\nabla(uv) = \left(\cancel{\frac{\partial u}{\partial x}v + u\frac{\partial v}{\partial x}}, \cancel{\frac{\partial u}{\partial y}v + u\frac{\partial v}{\partial y}}, \cancel{\frac{\partial u}{\partial z}v + u\frac{\partial v}{\partial z}}\right)$$

$$\nabla(u)v + u\nabla v = \left(\cancel{\frac{\partial u}{\partial x}v + u\frac{\partial v}{\partial x}}, \cancel{\frac{\partial u}{\partial y}v + u\frac{\partial v}{\partial y}}, \cancel{\frac{\partial u}{\partial z}v + u\frac{\partial v}{\partial z}}\right)$$

**Each component cancles out!**

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Verify $\nabla\cdot(\vec{u}\times\vec{v}) = \vec{v}\cdot\nabla\times\vec{u} - \vec{u}\cdot\nabla\times\vec{v} \qquad \vec{u}, \vec{v} \in \mathbb{R}^3$

$$\nabla\cdot\vec{u}\times\vec{v} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)\cdot\left(u_y v_z - u_z v_y, u_x v_z - u_z v_x, u_x v_y - u_y v_x\right)$$

$$\cancel{\frac{\partial u_y}{\partial x}v_z} + \cancel{\frac{\partial v_z}{\partial x}u_y} - \cancel{\frac{\partial u_z}{\partial x}v_y} - \cancel{u_z\frac{\partial v_y}{\partial x}} +$$

$$\cancel{\frac{\partial u_x}{\partial y}v_z} + \cancel{u_x\frac{\partial v_z}{\partial y}} - \cancel{\frac{\partial u_z}{\partial y}v_x} - \cancel{u_z\frac{\partial v_x}{\partial y}} +$$

$$\cancel{\frac{\partial u_x}{\partial z}v_y} + \cancel{u_x\frac{\partial v_y}{\partial z}} - \cancel{\frac{\partial u_y}{\partial z}v_x} - \cancel{u_y\frac{\partial v_x}{\partial z}}$$

$$\vec{v}\cdot\nabla\times\vec{u} - \vec{u}\cdot\nabla\cdot\vec{v} = v_x\cancel{\frac{\partial u_z}{\partial y}} - v_x\cancel{\frac{\partial u_y}{\partial z}} + v_y\cancel{\frac{\partial u_x}{\partial z}} - v_y\cancel{\frac{\partial u_z}{\partial x}} + v_z\cancel{\frac{\partial u_y}{\partial x}} - v_z\cancel{\frac{\partial u_x}{\partial y}}$$

$$-u_x\cancel{\frac{\partial v_z}{\partial y}} + u_x\cancel{\frac{\partial v_y}{\partial z}} - u_y\cancel{\frac{\partial v_x}{\partial z}} + u_y\cancel{\frac{\partial v_z}{\partial x}} - u_z\cancel{\frac{\partial v_y}{\partial x}} + u_z\cancel{\frac{\partial v_x}{\partial y}}$$

**All terms cancel out**

# Exercise 4

$$\vec{v} = \vec{v}(\vec{x},t) = \left(-\alpha(x_1^3 + x_1x_2^2)e^{-\beta t},\ \alpha(x_1^2x_2 + x_2^3)e^{-\beta t}\right)$$

Spatial acceleration : $\vec{a} = \dfrac{\partial \vec{v}}{\partial t} + (\nabla \vec{v})\vec{v}$

$$\frac{\partial \vec{v}}{\partial t} = \left(\alpha\beta e^{-\beta t}(x_1^3 + x_1x_2^2),\ -\alpha\beta e^{-\beta t}(x_1^2x_2 + x_2^3)\right)$$

$$\begin{array}{c|c} & \begin{matrix} x_1^3 + x_1x_2^2 \\ x_1^2x_2 + x_2^3 \end{matrix} \\ \hline (\alpha e^{-\beta t})^2 \begin{pmatrix} -3x_1^2 + x_2^2 & 2x_1x_2 \\ -2x_1x_2 & x_1^2 + 3x_2 \end{pmatrix} & \cancel{(\alpha e^{-\beta t})^2 (-3x_1^2 + x_2^2)(x_1^3 + x_1x_2^2)} \end{array}$$

$(\nabla\vec{v})\vec{v} =$

$$\rightarrow (\alpha e^{-\beta t})^2 \left( \left((-3x_1^2 + x_2^2)(x_1^3 + x_1x_2^2) + 2x_1x_2(x_1^2x_2 + x_2^3)\right)\vec{i} + \left(-2x_1x_2(x_1^3 + x_1x_2^2) + (x_1^2 + 3x_2)(x_1^2x_2 + x_2^3)\right)\vec{j} \right)$$

$$\vec{a}((1,0,0),0) = (\alpha\beta, 0) + (-3\alpha^2, 0) = \underline{\underline{(-3\alpha^3\beta,\ 0)}}$$