MEK 4250 Elementmethod
Mandatory Assignment

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1 Exercise 1

In these set of exercises we will study the Stokes problem defined as

$$
\begin{aligned}
-\Delta u + \nabla p &= f \ \text{ in } \ \Omega \\
\nabla \cdot v &= 0 \ \text{ in } \ \partial\Omega \\
u &= g \ \text{ in } \ \partial\Omega_N \\
\frac{\partial u}{\partial x} - pn &= h \ \text{ in } \ \partial\Omega_N
\end{aligned}
$$

Exercise 7.1

First off we will define the weak formulation for the stokes problem. Let u $\in H^1_{D,g}$ and $p \in L^2$. Then the stokes problem can be defined as

$$
\begin{aligned}
a(u,v) + b(p,v) &= f(v) \ \ v \in H^1_{D,0} \\
b(q,u) &= 0 \ \ q \in L^2
\end{aligned}
$$

Where a and b defines the bilinear form, and f defines the linear form as

$$
\begin{aligned}
a(u,v) &= \int \nabla u \nabla v \ dx \\
b(p,v) &= \int p \nabla \cdot v \ dx \\
f(v) = \int fv \ dx &+ \int_{\Omega_N} hv \ ds
\end{aligned}
$$

Further we will define to properties which will be usefull for solving the exercises
Cauchy-Schwarts inequality
Let V be a inner product space, then

$$| < v, w > | \ \leq \ ||u|| \cdot ||w|| \ \ \forall \ v, q \in V$$

Poincare's Inequality Let $v \in H^1_0(\Omega)$

$$||v||_{L^2(\Omega)} \leq C|v|_{H^1}(\Omega) \ \text{ Where } \ \partial\Omega_N$$