MEK 4420
Mandatory Assignment

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Mathematical approach

$$\iint_C \Big(\phi \frac{\partial G}{\partial n} - G\frac{\partial \phi}{\partial n}\Big) dS = \begin{Bmatrix} 0 \\ \pi\phi(x,y,z) \\ 2\pi\phi(x,y,z) \end{Bmatrix}$$

$$\pi\phi(x_0) = \int_S \Big(\phi \frac{\partial \psi}{\partial n} - \psi\frac{\partial \phi}{\partial n}\Big) dS$$

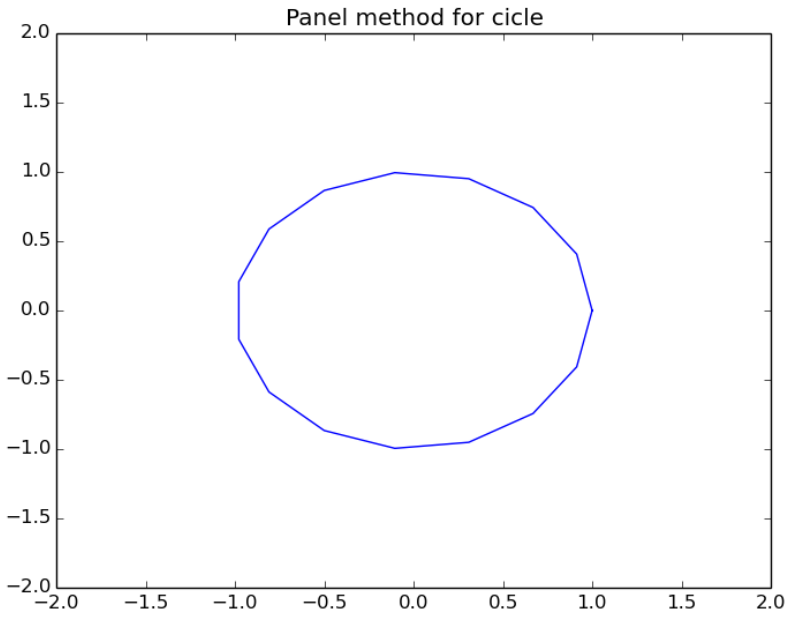
Here $\psi = \ln r$, which is the source potential in 2D.

Numerical approach

$$\pi\phi(X_0) + \sum_{n=1}^{N} \phi(X_n) \int_{Cs} \frac{\partial}{\partial n} lnr \; dS = \sum_{n=1}^{N} \frac{\partial \phi}{\partial n_X} \int_{Cs} lnr \; dS$$

$$\int_{Cs} \frac{\partial}{\partial n_x} lnr \; dS = -\Big(\theta_B - \theta_A\Big)$$

$$\begin{Bmatrix} \pi & (\theta_1 - \theta_2) & (\theta_2 - \theta_3) \cdots \\ (\theta_{N-1} - \theta_N) & \pi & (\theta_1 - \theta_2) \cdots \\ (\theta_{N-2} - \theta_{N-1}) & (\theta_{N-1} - \theta_N) & \pi \cdots \\ & \vdots & \end{Bmatrix} \begin{Bmatrix} \phi(x_0) \\ \phi(x_1) \\ \phi(x_2) \\ \vdots \\ \phi(x_N) \end{Bmatrix} = \begin{Bmatrix} \frac{\partial \phi}{\partial n} \int\limits_{C1} lnr_1 \; dS \\ \frac{\partial \phi}{\partial n} \int\limits_{C2} lnr_2 \; dS \\ \vdots \\ \frac{\partial \phi}{\partial n} \int\limits_{CN} lnr_N \; dS \end{Bmatrix}$$



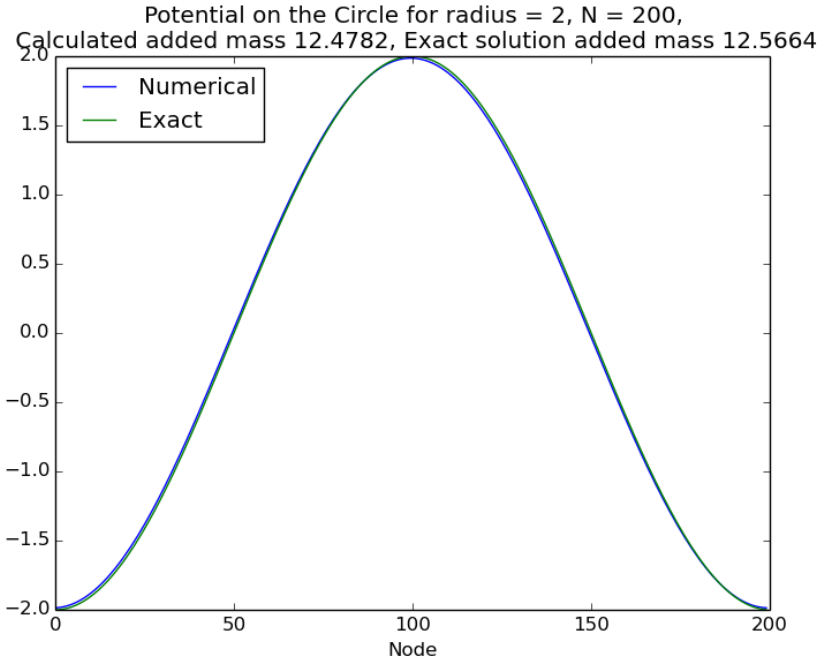
Results

Reference solution circle

- m11: $\rho\pi a^2$
- m22: $\rho\pi a^2$
- m66: 0

Reference solution ellipse

- m11: $\rho\pi b^2$
- m22: $\rho\pi a^2$
- m66: $\frac{1}{8}\pi\rho(a^2 - b^2)^2$



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----------------------------BEGIN SIMULATION-----------------------------

--------------------------- -----CIRCLE-----------------------------------
Radius chosen as 2, with 200 nodes
-------------------------------------------------------------------------
 For direction 11 Numerical solution 12.478, exact solution 12.566,
 error 0.993 %
-------------------------------------------------------------------------
 For direction 22 Numerical solution 12.478, exact solution 12.566,
 error 0.993 %
-------------------------------------------------------------------------
 For direction 66 Numerical solution -0.000, exact solution 0.000,
 error 0.000 %

---------------------------------Ellipse---------------------------------
Radius r_a = 1, r_b = 3 , with 200 nodes
-------------------------------------------------------------------------
 For direction 11 Numerical solution 27.882, exact solution 28.274
Error 0.014 %
-------------------------------------------------------------------------
 For direction 22 Numerical solution 3.127, exact solution 3.142
Error 0.005 %
-------------------------------------------------------------------------
 For direction 66 Numerical solution 24.655, exact solution 25.133
Error 0.019 %

----------------------------BEGIN SIMULATION-----------------------------

--------------------------- -----CIRCLE-----------------------------------
Radius chosen as 2, with 400 nodes
-------------------------------------------------------------------------
 For direction 11 Numerical solution 12.523, exact solution 12.566,
 error 0.997 %
-------------------------------------------------------------------------
 For direction 22 Numerical solution 12.523, exact solution 12.566,
 error 0.997 %
-------------------------------------------------------------------------
 For direction 66 Numerical solution -0.000, exact solution 0.000,
 error 0.000 %

---------------------------------Ellipse---------------------------------
Radius r_a = 1, r_b = 3 , with 400 nodes
-------------------------------------------------------------------------
 For direction 11 Numerical solution 28.078, exact solution 28.274
Error 0.007 %
-------------------------------------------------------------------------
 For direction 22 Numerical solution 3.134, exact solution 3.142
Error 0.002 %
-------------------------------------------------------------------------
 For direction 66 Numerical solution 24.897, exact solution 25.133
Error 0.009 %
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