Exercise 3

Verify $\nabla(uv) = \nabla(u)v + u\nabla v \qquad u, v \in \mathbb{R}$

$$\nabla(uv) = \left(\cancel{\frac{\partial u}{\partial x}v + u\frac{\partial v}{\partial x}}, \cancel{\frac{\partial u}{\partial y}v + u\frac{\partial v}{\partial y}}, \cancel{\frac{\partial u}{\partial z}v + u\frac{\partial v}{\partial z}}\right)$$

$$\nabla(u)v + u\nabla v = \left(\cancel{\frac{\partial u}{\partial x}v + u\frac{\partial v}{\partial x}}, \cancel{\frac{\partial u}{\partial y}v + u\frac{\partial v}{\partial y}}, \cancel{\frac{\partial u}{\partial z}v + u\frac{\partial v}{\partial z}}\right)$$

<u>Each component cancles out!</u>

Verify $\nabla\cdot(\vec{u}\times\vec{v}) = \vec{v}\cdot\nabla\times\vec{u} - \vec{u}\cdot\nabla\times\vec{v} \qquad \vec{u}, \vec{v} \in \mathbb{R}^3$

$$\nabla\cdot\vec{u}\times\vec{v} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)\cdot\left(u_y v_z - u_z v_y, u_x v_z - u_z v_x, u_x v_y - u_y v_x\right)$$

$$\cancel{\frac{\partial u_y}{\partial x}}v_z + \cancel{\frac{\partial v_z}{\partial x}}u_y - \cancel{\frac{\partial u_z}{\partial x}}v_y - u_z\cancel{\frac{\partial v_y}{\partial x}} +$$

$$\cancel{\frac{\partial u_x}{\partial y}}v_z + u_x\cancel{\frac{\partial v_z}{\partial y}} - \cancel{\frac{\partial u_z}{\partial y}}v_x - u_z\cancel{\frac{\partial v_x}{\partial y}} +$$

$$\cancel{\frac{\partial u_x}{\partial z}}v_y + u_x\cancel{\frac{\partial v_y}{\partial z}} - \cancel{\frac{\partial u_y}{\partial z}}v_x - u_y\cancel{\frac{\partial v_x}{\partial z}}$$

$$\vec{v}\cdot\nabla\times\vec{u} - \vec{u}\cdot\nabla\cdot\vec{v} = v_x\cancel{\frac{\partial u_z}{\partial y}} - v_x\cancel{\frac{\partial u_y}{\partial z}} + v_y\cancel{\frac{\partial u_x}{\partial z}} - v_y\cancel{\frac{\partial u_z}{\partial x}} + v_z\cancel{\frac{\partial u_y}{\partial x}} - v_z\cancel{\frac{\partial u_x}{\partial y}}$$

$$-u_x\cancel{\frac{\partial v_z}{\partial y}} + u_x\cancel{\frac{\partial v_y}{\partial z}} - u_y\cancel{\frac{\partial v_x}{\partial z}} + u_y\cancel{\frac{\partial v_z}{\partial x}} - u_z\cancel{\frac{\partial v_y}{\partial x}} + u_z\cancel{\frac{\partial v_x}{\partial y}}$$

<u>All terms cancel out</u>

Exercise 4

$$\vec{v} = \vec{v}(\vec{x}, t) = \left(-\alpha(x_1^3 + x_1 x_2^2)e^{-\beta t},\ \alpha(x_1^2 x_2 + x_2^3)e^{-\beta t}\right)$$

Spatial acceleration: $\vec{a} = \dfrac{\partial \vec{v}}{\partial t} + (\nabla \vec{v})\,\vec{v}$

$$\frac{\partial \vec{v}}{\partial t} = \left(\alpha\beta e^{-\beta t}(x_1^3 + x_1 x_2^2),\ -\alpha\beta e^{-\beta t}(x_1^2 x_2 + x_2^3)\right)$$

$$\begin{array}{c|c} & \begin{matrix} x_1^3 + x_1 x_2^2 \\ x_1^2 x_2 + x_2^3 \end{matrix} \\ \hline (\nabla \vec{v})\,\vec{v} = (\alpha e^{-\beta t})^2 \begin{pmatrix} -3x_1^2 + x_2^2 & 2x_1 x_2 \\ -2x_1 x_2 & x_1^2 + 3x_2 \end{pmatrix} & \end{array}$$

$$\to (\alpha e^{-\beta t})^2 \Big(\big((-3x_1^2 + x_2^2)(x_1^3 + x_1 x_2^2) + 2x_1 x_2 (x_1^2 x_2 + x_2^3)\big)\, i + \big(-2x_1 x_2 (x_1^3 + x_1 x_2^2) + (x_1^2 + 3x_2)(x_1^2 x_2 + x_2^3)\big)\, j \Big)$$

$$\vec{a}\big((1,0,0), 0\big) = (\alpha\beta, 0) + (-3\alpha^2, 0) = \underline{\underline{(-3\alpha^3\beta, 0)}}$$