OF-1 Report :
Computational Simulations of a Lid-driven Cavity

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Contents

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1 Introduction

The incompressible & steady form of the Navier-Stokes equations is considered for the case of a 2-D lid-driven cavity with height $= H$, length $= L$, and lid velocity $= U$. The non-dimensional forms of the continuity and momentum equations are recorded. After visualizing the solution for Re = 10, we experiment with the size of the grid to compare wallclock times with the refined solutions. We then estimate the relationship between wallclock time and the total number of grid points. We compute and plot the non-dimensional stress along the lid at Reynolds numbers between 10 and 500. Finally, we compute the resulting force on the lid aspect ratios 1/2, 1, and 2 and Reynolds numbers between 10 and 500. "TBR"

2 Nondimensional Navier-Stokes equations

The incompressible, constant density, ρ, and viscocity, μ, steady form of the Navier-Stokes equations govern the prescribed two-dimensional fluid flow problem. The continuity and momentum equations are non-dimensionalized according to the following scales:

- Length scale L $= \dfrac{x}{\tilde{x}} = \dfrac{y}{\tilde{y}}$
- Velocity scale U $= \dfrac{u}{\tilde{u}} = \dfrac{v}{\tilde{v}}$
- Pressure Scale $\tilde{P} = \dfrac{P}{\rho U^2}$
- Reynolds number Re $= \dfrac{\rho U L}{\mu} = \dfrac{UL}{\nu}$ where $\nu = \mu/\rho$ is the kinematic viscocity

Continuity

$$\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} = 0$$

X-Momentum

$$\tilde{u}\frac{\partial \tilde{u}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{u}}{\partial \tilde{y}} = -\frac{\partial \tilde{p}}{\partial \tilde{x}} + \frac{1}{Re}\left(\frac{\partial^2 \tilde{u}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2}\right)$$

Y-Momentum

$$\tilde{u}\frac{\partial \tilde{v}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{v}}{\partial \tilde{y}} = -\frac{\partial \tilde{p}}{\partial \tilde{y}} + \frac{1}{Re}\left(\frac{\partial^2 \tilde{v}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{v}}{\partial \tilde{y}^2}\right)$$

In the momentum equations, the only non-dimensional number that appears is the Reynolds number. When the Reynolds number becomes very large ($Re \to \infty$), the viscous terms on the RHS of the equations become negligible, implying inviscid flow, reducing the momentum equations to the Euler equations. When the Reynolds number becomes very small ($Re \to 0$), the viscous terms on the RHS of the equations become very large, implying a highly viscous flow, reducing the momentum equations to the Stokes equations. The Stokes and Euler equations are listed below.

Euler Equations:

Continuity as $Re \to \infty$:

$$\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} = 0$$

X-Momentum as $Re \to \infty$:

$$\tilde{u}\frac{\partial \tilde{u}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{u}}{\partial \tilde{y}} = -\frac{\partial \tilde{p}}{\partial \tilde{x}}$$

Y-Momentum as $Re \to \infty$:

$$\tilde{u}\frac{\partial \tilde{v}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{v}}{\partial \tilde{y}} = -\frac{\partial \tilde{p}}{\partial \tilde{y}}$$

Stokes Equations:

Continuity as $Re \to 0$:

$$\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} = 0$$

X-Momentum as $Re \to 0$:

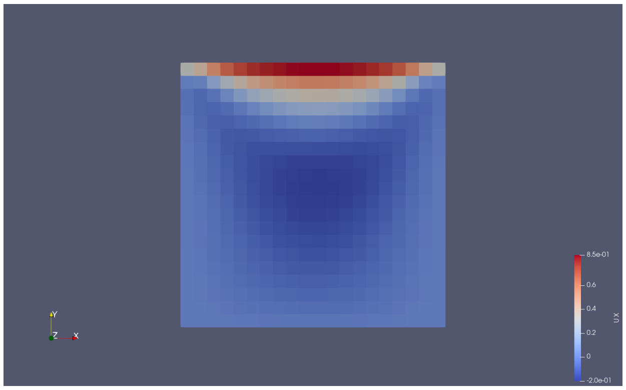
$$0 = -\frac{\partial \tilde{p}}{\partial \tilde{x}} + \frac{1}{Re}\left(\frac{\partial^2 \tilde{u}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2}\right)$$

Y-Momentum as $Re \to 0$:

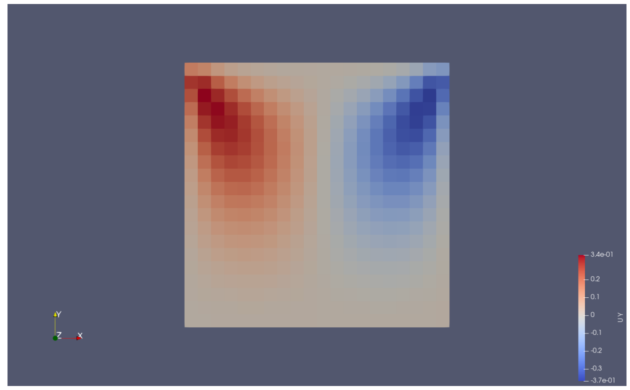
$$0 = -\frac{\partial \tilde{p}}{\partial \tilde{y}} + \frac{1}{Re}\left(\frac{\partial^2 \tilde{v}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{v}}{\partial \tilde{y}^2}\right)$$

3 Flow at Re = 10

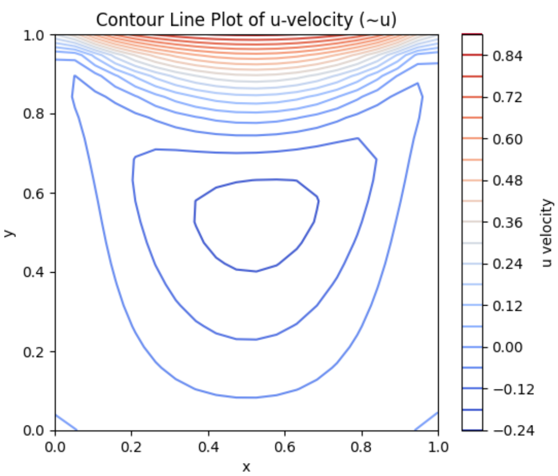
3.1 Plots of Velocity



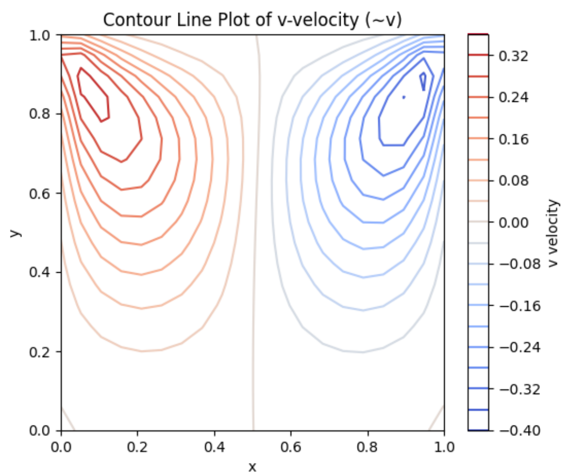
(a) Plot from Paraview of u



(b) Plot from Paraview of v



(a) Contour Plot of u/U



(b) Contour Plot of v/U

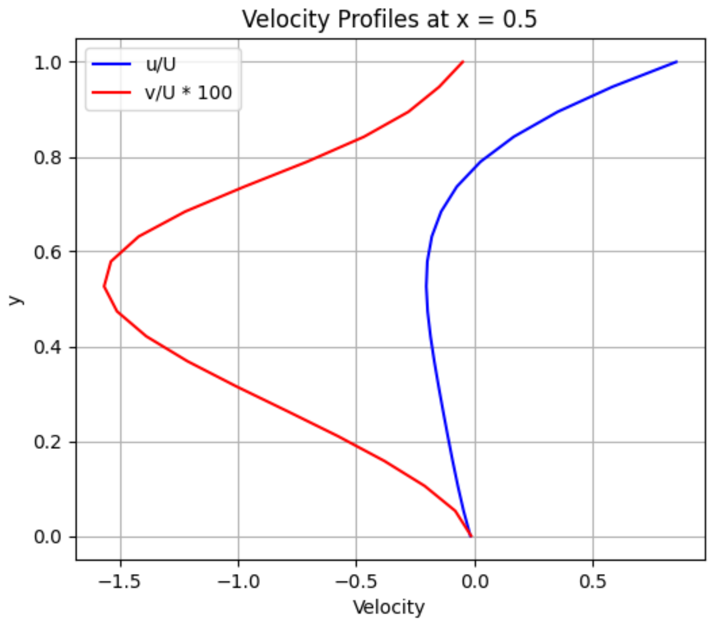
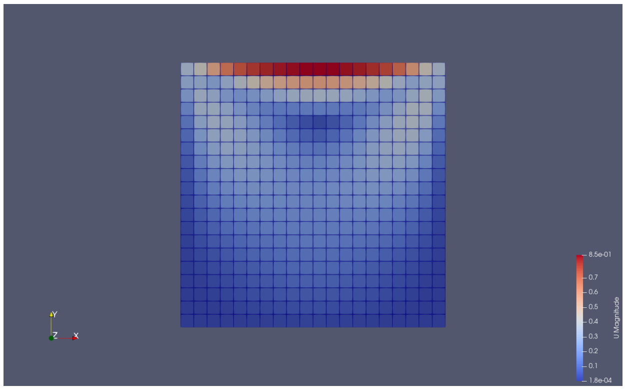
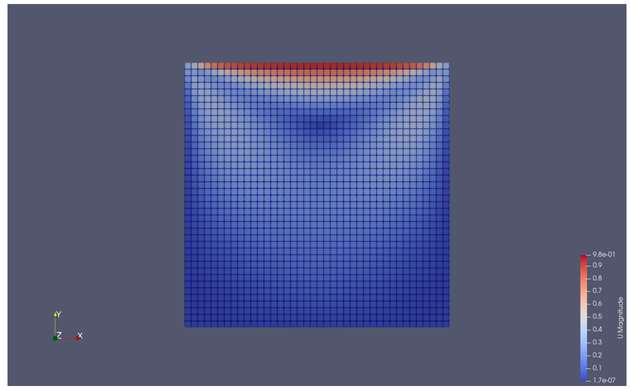


Figure 3: u/U and v/U through the center of the cavity

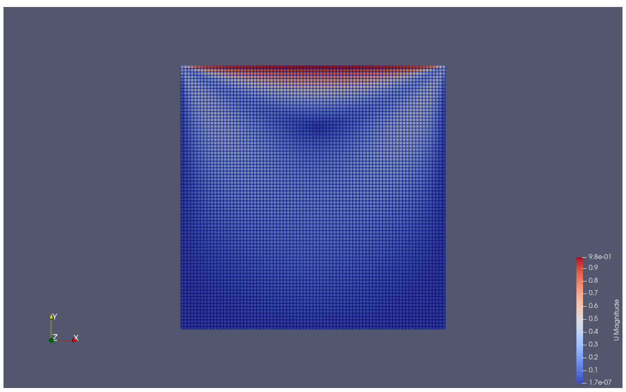
3.2 Solution Refinement



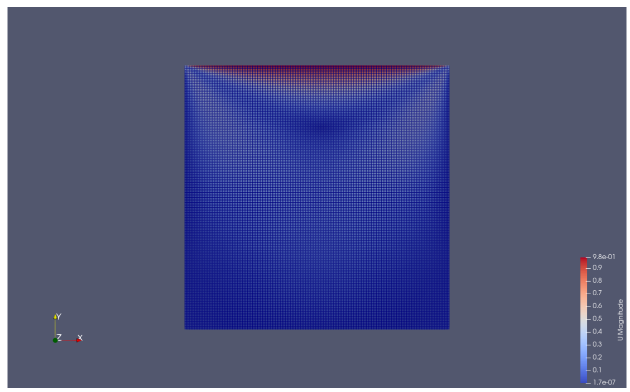
(a) $N = 20 \times 20$



(b) $N = 40 \times 40$

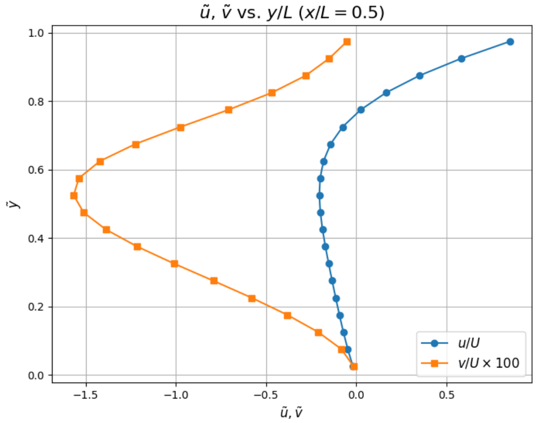


(c) $N = 80 \times 80$

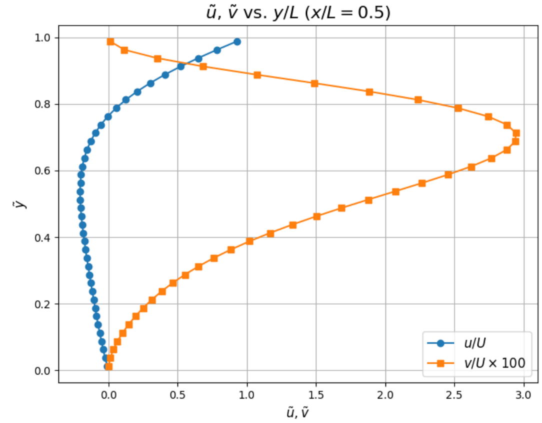


(d) $N = 160 \times 160$

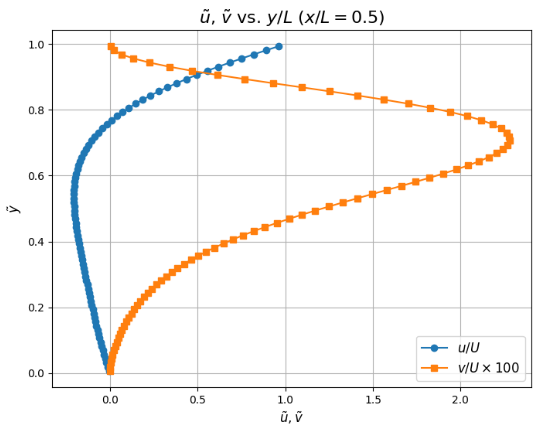
Figure 4: Visualizing effect of gridpoint refinement on U in *Paraview*



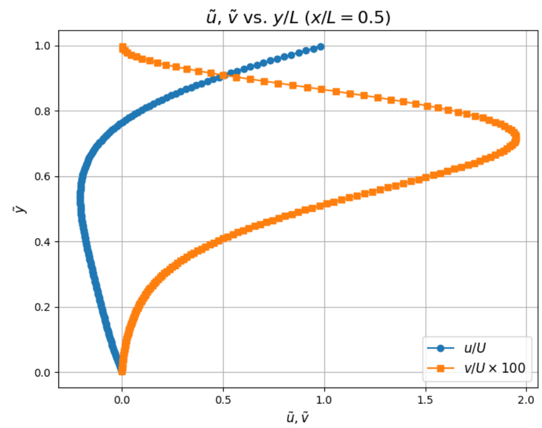
(a) Original



(b) First Refinement



(c) Third Refinement



(d) Fourth Refinement

Figure 5: Effect of increased gridsize and decreased time step size on $\tilde{u}$, $\tilde{v}$ vs. $\tilde{y}$

There is a noticable variation in the scaled up y-component of $\mathbf{U}$ as we refine the mesh further. The x-component of $\mathbf{U}$ seems to curve more and reach closer to $u/U = 1$ at $\tilde{y} = 1$ as the mesh is futher refined too.

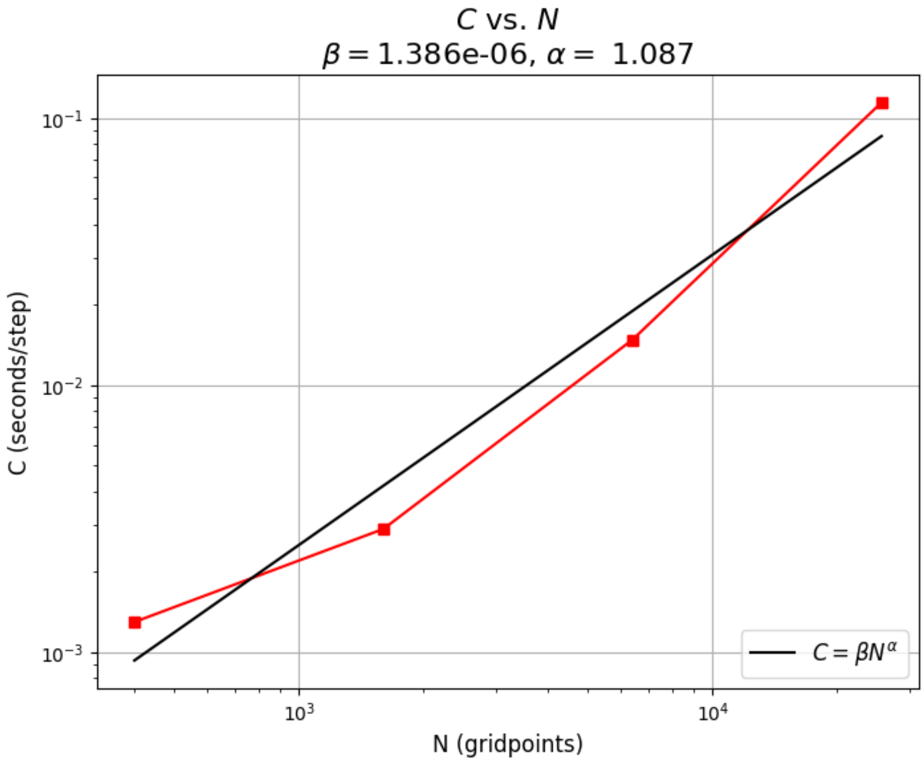


Figure 6: Execution time per step C increases with higher gridpoints N

Q: What can you conclude about the increase in wallclock time as you refine the grid?

A: Higher refinement will take longer per iteration than at lower refinement. It is clear in Fig. 6 with our estimate of $\alpha = 1.087$ in the fit $C = \beta N^{\alpha}$ that with an order of magnitude increase in gridpoints there is an order of magnitude increase in the execution time per step.

4 Force on the Lid

Now we want to investigate the dependance of the force on Reynolds number. We define a few nondimensionalized terms,

- $\tilde{F} = \dfrac{F}{\mu U} = \displaystyle\int_0^1 \tilde{\tau}(\tilde{x})\, d\tilde{x}$

- $\tilde{\tau} = \dfrac{\tau L}{\mu U} = \dfrac{\partial \tilde{u}}{\partial \tilde{y}}$

With these terms and our solution for Re = 10 (using the 80x80 grid), we generate a plot $\tilde{\tau}$ vs. $\tilde{x}$

Appendix

A Code

PDF of code starts on next page.