OpenFOAM assignment 1

see Canvas for due date

COE 347

There is absolutely no tolerance for academic misconduct. Late assignments will not be accepted, unless under exceptional circumstances at the instructor's full discretion.

Submit your assignment electronically as a PDF via Canvas by 11:59pm on the due date.

If you are submitting scanned copies of your handwritten notes, rather than a typeset document, please take the time to reduce the file to a manageable size by adjusting the resolution.

Instructions

In this assignment, you will be practicing the usage of OpenFOAM as it applies to the solution of the incompressible Navier-Stokes equations for the steady "lid-driven cavity" flow in two-dimensions.

This is a canonical CFD exercise and test used for verification and validation. If you are interested, you can read the two graduate CFD projects posted on Canvas for more information.

Online tutorial

First, make sure to go over all or most of the OpenFOAM tutorial on the lid-driven cavity online (see Canvas page for link).

Although it is not required for you to include results from the online tutorial, make sure to either go over the whole tutorial before starting the assignment or go back to the tutorial for help with specific topics. It may be helpful to browse through the topics once.

Nondimensional form of the equations

Consider the incompressible ($\rho = const$), constant properties ($\mu = const$) steady form of the Navier-Stokes equations (continuity and momentum).

Let L indicate the length of the cavity wall, U the speed of the lid, μ the dynamic viscosity of the fluid, and ρ the density of the fluid.

Using L, U, μ, ρ, and $P = \rho U^2$ as reference length, velocity, viscosity, density, and pressure, nondimensionalize the continuity and momentum equations.

Using the reference quantities above, one can define nondimensional variables. For example, $\tilde{x} = x/L$ and $\tilde{y} = y/L$ are the nondimensional x- and y- coordinates and $\tilde{u} = u/U$ and $\tilde{v} = v/U$ are the nondimensional velocity components.

A straightforward manner to derive nondimensional equations is to write every variable appearing in the equations as the product of the reference variable and the nondimensional variable, e.g. $x = \tilde{x}L$, $y = \tilde{y}L$, $u = \tilde{u}U$ and $v = \tilde{v}U$, substitute in the dimensional equations, and group the reference quantities together in nondimensional groups.

- Write down the nondimensional continuity and momentum equations. For the momentum equation, write down two equations for the x and y components of velocity. Thus, you should have 3 equations in 3 unknowns, i.e. two nondimensional velocity components $\tilde{u}$ and $\tilde{v}$ and nondimensional pressure $\tilde{p}$.
- What is the (only) nondimensional number that appears in the nondimensional momentum equation? Write it down and comment on what happens to the momentum equation when this nondimensional number becomes large ($\to \infty$) and when it becomes small ($\to 0$).

Description of the flow for $\mathrm{Re} = 10$

Consider the solution to the lid-driven cavity with $U = 1$ m/s, $L = 0.1$ m, and $\nu = \mu/\rho = 0.01$ m^2/s, giving a Reynolds number $\mathrm{Re} = UL/\nu = 10$.

Once you have obtained a first solution to the flow as described in the online tutorial, use the data to produce

- A two-dimensional contour plot of $\tilde{u}$ and $\tilde{v}$, the nondimensional components of velocity.
- A plot of $\tilde{u}$ and $\tilde{v}$ along the direction y through the center of the cavity, $\tilde{u}(\tilde{x} = 0.5, \tilde{y})$ and $\tilde{v}(\tilde{x} = 0.5, \tilde{y})$.

  A table reporting u and v for $U = 1$ m/s, $L = 0.1$ m, and $\mathrm{Re} = 0.01$ is available on Canvas for you to check your results.

Refining the solution

Next, you will repeat the simulations for the case $\mathrm{Re} = 10$ on finer and finer grids. As the grid is refined, i.e. more points are added, the discrete solution to the equations becomes more accurate.

Increase the number of grid points by a factor of two in each direction and run the simulation again. Then again by another factor of two, and again by a factor of two. Each time you increase the number of grid points by two, you should decrease the time step size by a factor of two also.

Once you obtain these three refined solutions in addition to the original one:

- Plot and compare $\tilde{u}(\tilde{x} = 0.5, \tilde{y})$ and $\tilde{v}(\tilde{x} = 0.5, \tilde{y})$ for $0 \leq \tilde{y} \leq 1$ from the four solutions: original, first refinement, second refinement, and third refinement.
- Keep track of how long each simulation takes and how many time steps are performed. The time it takes for the simulation to execute is called the *wallclock time*. Then compute the wallclock time per step (call it C) on each of the four grids.

  What can you conclude about the increase in wallclock time as you refine the grid? If you define as $N = N_x N_y$, i.e. the overall number of grid points used, you should find that C increases with the power of N, i.e. βN^α. Plot C vs. N on a log-log plot and provide an estimate for the exponent α.

Force on the lid

The fluid exerts a force on the cavity's lid. In this section of the assignment, you will investigate the dependence of the force on the Reynolds number of the flow.

Recall that the fluid exerts a shear stress (force per unit area) $\tau \mathbf{i}_x$ on the lid, where

$$\tau(x) = \mu \frac{\partial u}{\partial y}\bigg|_{x,y=L}. \tag{1}$$

The force is oriented along the unit vector $\mathbf{i}_x$. The integral of $\tau(x)$ provides the net force (per unit depth in the z direction)

$$F = \int_0^L \tau(x) dx. \tag{2}$$

In keeping with the nondimensionalization, define $\widetilde{F}$ as the the nondimensional force

$$\widetilde{F} = \frac{F}{\mu U}, \tag{3}$$

so that

$$\widetilde{F} = \int_0^1 \widetilde{\tau}(\tilde{x}) d\tilde{x}, \tag{4}$$

where the nondimensional stress $\widetilde{\tau}$ on the lid is

$$\widetilde{\tau} = \tau \frac{L}{\mu U} = \frac{\partial \tilde{u}}{\partial \tilde{y}}\bigg|_{\tilde{x},\tilde{y}=1}. \tag{5}$$

Using the solution from the simulation with $\mathrm{Re} = 10$, compute and plot the nondimensional stress $\widetilde{\tau}$ vs. $\tilde{x}$ along the lid.

This will require you to use a one sided finite difference approximation to the derivative (along the $\tilde{y}$ direction) of the $\tilde{u}$ component of velocity at each $0 \leq \tilde{x} \leq 1$ location along the lid.

Alternatively, you can fit a polynomial to the pairs $(\tilde{y}_i, \tilde{u}(\tilde{x}, \tilde{y}_i))$, making sure to enforce the condition $\tilde{u}(\tilde{x}, \tilde{y} = 1) = 1$. Once you have a polynomial, take the (analytical) derivative of the polynomial and evaluate the derivative at $\tilde{y} = 1$. See lecture notes for more information.

To gather the data, you will have to extract the velocity component $\tilde{u}$ along $\tilde{x}$ at various distances $\tilde{y}$ from the lid. To process the data and compute stresses, you may want to use MATLAB or other similar software.

Execute simulations at a few Reynolds numbers between 10 and 500. For each simulation, use whichever grid you think provides an accurate velocity field based on your experience in the section "Refining the solution". Pay attention to the fact that, as the Reynolds number increases, the resolution requirements may become more stringent.

A simple manner to adjust the Reynolds number is to leave L and U unchanged, and adjust ν (kinematic viscosity) instead. This has the advantage that the geometry does not change (L is constant) and the velocity boundary conditions (U is constant) don't change either.

Produce a graph that shows the nondimensional force $\widetilde{F}$ versus the Reynolds number $\mathrm{Re} = UL/\nu$.

(Extra credit - 20 points) Exploring the effect of the aspect ratio

Consider now a cavity that is not square and let H be the length of the vertical side (aligned with y) and L be the length of the horizontal side (aligned with x).

Perform new simulations for $H/L = 0.5$ (a shallow cavity) and $H/L = 2$ (a tall cavity) and the Reynolds numbers that you considered in the section "Force on the lid" above.

Note that you will have to generate new grids for these two geometries. Adjust the number of points in each direction so that the grid spacing in each direction is the same, e.g. if $H = 2L$, you should have twice as many points in y than in x.

For each new simulation, compute the nondimensional force $\widetilde{F}$ and add the new data series on the graph $\widetilde{F}$ vs. Re in order to compare against the base case $H/L = 1$ (a square cavity). Comment on your findings.