

The RTE has the form

$$\frac{d\mathbf{I}}{ds} = \boldsymbol{\epsilon} - \mathbf{K}\mathbf{I}. \quad (1)$$

Let's rotate the field of view around the ray direction by angle α . (see Fig. 1.10 at page 25 in LL04). While I and V are not affected (they are invariant under rotation), the Q and U are affected. Let \mathbf{A}_α be a 4×4 matrix corresponding to the rotation by alpha. The Stokes parameters in the new frame are then $\mathbf{I}' = \mathbf{A}_\alpha \mathbf{I}$, where (see Eq. 1.45)

$$\mathbf{A}_\alpha = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & Q \cos 2\alpha & U \sin 2\alpha & 0 \\ 0 & -Q \sin 2\alpha & U \cos 2\alpha & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (2)$$

We want to express our Eq. 1 in the new reference frame in order to solve it there. We can multiply (from the left) Eq. 1 to get

$$\mathbf{A} \frac{d\mathbf{I}}{ds} = \frac{d\mathbf{I}'}{ds} = \mathbf{A}\boldsymbol{\epsilon} - \mathbf{A}\mathbf{K}\mathbf{I}. \quad (3)$$

The left hand side is now OK and it is clear that $\mathbf{A}\boldsymbol{\epsilon} = \boldsymbol{\epsilon}'$ because the emission vector transform in the same way as the Stokes parameters. The problematic term $\mathbf{A}\mathbf{K}\mathbf{I}$ can be rewritten as

$$\mathbf{A}\mathbf{K}\mathbf{I} = \mathbf{A}\mathbf{K}\mathbf{A}^{-1}\mathbf{A}\mathbf{I} \quad (4)$$

and we are done because $\mathbf{A}\mathbf{I} = \mathbf{I}'$ and we see that

$$\mathbf{K}' = \mathbf{A}\mathbf{K}\mathbf{A}^{-1}. \quad (5)$$

The RTE in the rotated reference frame has now the correct form

$$\frac{d\mathbf{I}'}{ds} = \boldsymbol{\epsilon}' - \mathbf{K}'\mathbf{I}'. \quad (6)$$